ACS COLLEGE OF ENGINEERING

Kambipura, Mysore Road, Bangalore – 560074

DEPARTMENT OF AEROSPACE ENGINEERING

VI SEMESTER B.E.

18ASL67 – AEROSPACE STRUCTURES LABORATORY

LABORATORY MANUAL

ACADEMIC YEAR 2022 – 2023

LIST OF EXPERIMENTS

Experiments for Performance

1. Deflection of a Simply Supported Beam.
2. Deflection of a cantilever Beam
3. Deflection of a beam under combined loading
4. Verification of Maxwell's Reciprocal Theorem.
5. Determination of Young's Modulus using strain gages and Poisson Ratio Determination
6. Determination natural frequency and damping ratio using forcing function.
7. Buckling load of slender Eccentric Columns and Construction of Southwell Plot
8. Shear Failure of Bolted and Riveted Joints using Wagner beam
9. Determination of fundamental frequency of a cantilever beam and harmonics.
10. Frequency spectrum analysis for a cantilever beam.
11. Estimation of the natural frequency for a rigid body-spring system
12. Estimation of the natural frequency for two rotor system
13. Verification of Dunkerley's Equation.
14. Determine the Structural Damping Coefficient of a Composite Material Cantilever Beam and to Draw the Polar Plots of Damping Coefficient.

EXPERIMENT – 1

DEFLECTION OF A SIMPLY SUPPORTED BEAM

Aim: To determine the deflection of a simply supported beam.

Equipment: Beam Test Set-Up with Load cells, steel scale, caliper, flat beam

Theory:

A beam shown in fig-1 shows the section which is simply supported at the ends and is subjected to bending about its major axis with a concentrated load anywhere in the beam. The beam is provided with strain gauge, the deflection of the beam can be determined whenever the load is applied on the beam. Strain gauge values may be noted for several further works.

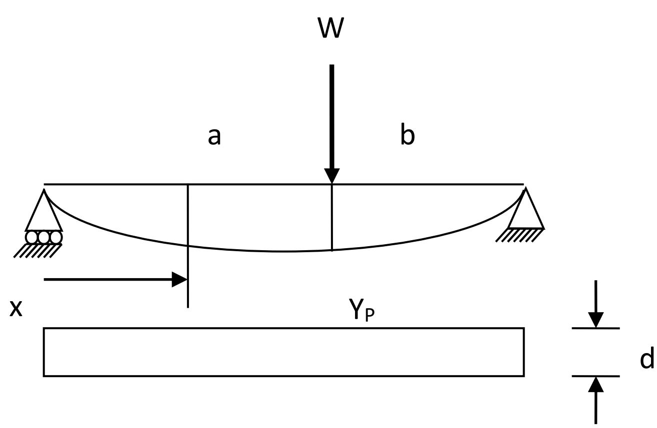


Fig-1: A simply supported beam

Deflections are given by following expressions. These expressions can be derived using Unit Load Method or Castigliano's Theorem.

$$Y_X = W\,b\,\frac{[(L^2 - b^2)\;x\; - x^3]}{(6\,E\,I\,L)}$$

For $0 < x < a$

$$Y_X = W\,b\,\frac{[\,x^3 - \frac{L}{b}(x - a)^3 - (\,L^2 - b^2)]}{(6\,E\,I\,L)}$$

For $a < x < L$

Where,

W is the load placed at a distance `a` from the left support in Newton
L = span of the beam in mm
Y_x = deflection at any point distance x from left end
I = moment of inertia of the beam in mm^4 (I_{xx})
E= Young's modulus in N/mm^2

Procedure:

1) Find the moment of inertia of beam from the following expression:

$$\frac{b_1 d_1^3}{12}$$

Where b_1 is width of beam and d_1 is depth.
2) Place the beam supporting from two wedge supports. The load position can be varied.
3) Set the load cell to read zero in the absence of load.
4) Set the deflection gauge to read zero in the absence of load.
5) Load the beam with 2.5 Kg. Note deflections before and after the load point through deflection gauge.
6) Increase the load to 5.0 Kg and repeat the experiment.
7) Find the deflections from the formula and verify.

Tabular column

Load (N)	I_{xx} (mm^4)	L (mm)	a (mm)	b (mm)	x (mm)	Theoretical value (mm)	Experimental Value (mm)

<u>CAUTION: Never exceed 10 kg load on the beam</u>

Conclusion:

Example:

Determine the deflection of a simply supported beam loaded with W = 50,000 N, Young's Modulus E = 2 x 105 N/mm2; Second moment of Inertia Ixx = 7332.9 x 104 mm4; & the load is placed at a distance a = 4800 mm; and the span of the beam L = 60000 mm. Find the deflection at x = 3000 mm from the left end.

$$Y_X = W\,b\,\frac{[(L^2 - b^2)\;x\;-x^3]}{(6\,E\,I\,L)}$$

$= 50000 \times 1200\times [\,(6000^2 - 1200^2)\,3000 - 3000^3] /(6\times2.0\times10^5\times 7332.9 \times 10^4 \times 6000)$

$$Y_x = 8.714 \text{ mm}$$

EXPERIMENT – 2

DEFLECTION OF A CANTILEVER BEAM

Aim: To determine the deflection of a Cantilever beam.

Equipment: Beam Test Set-Up with Load cells, steel scale, caliper, flat beam

Theory:

A beam shown in fig-1 shows the section which is simply supported at the ends and is subjected to bending about its major axis with a concentrated load anywhere in the beam. The beam is provided with strain gauge, the deflection of the beam can be determined whenever the load is applied on the beam. Strain gauge values may be noted for several further works.

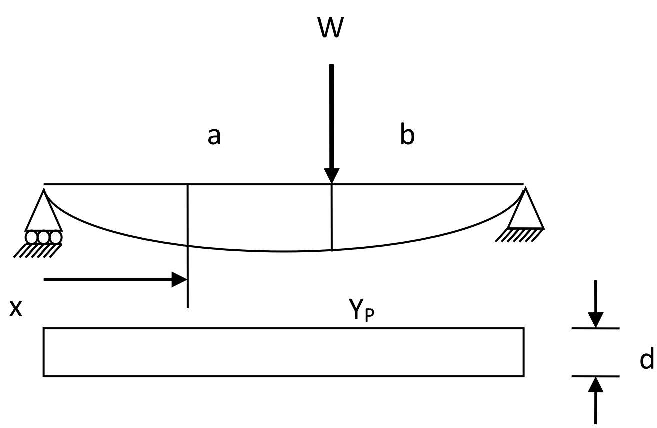


Fig-1: A simply supported beam

Deflections are given by following expressions. These expressions can be derived using Unit Load Method or Castigliano's Theorem.

$$Y_X = W\,b\,\frac{[(L^2 - b^2)\;x\; - x^3]}{(6\,E\,I\,L)}$$

For $0 < x < a$

$$Y_X = W\,b\,\frac{[\,x^3 - \frac{L}{b}(x - a)^3 - (L^2 - b^2)]}{(6\,E\,I\,L)}$$

For $a < x < L$

Where,

W is the load placed at a distance `a` from the left support in Newton
L = span of the beam in mm
Y_x = deflection at any point distance x from left end
I = moment of inertia of the beam in mm^4 (I_{xx})
E= Young's modulus in N/mm^2

Procedure:

1) Find the moment of inertia of beam from the following expression:
$$\frac{b_1 d_1^3}{12}$$
Where b_1 is width of beam and d_1 is depth.
2) Place the beam supporting from two wedge supports. The load position can be varied.
3) Set the load cell to read zero in the absence of load.
4) Set the deflection gauge to read zero in the absence of load.
5) Load the beam with 2.5 Kg. Note deflections before and after the load point through deflection gauge.
6) Increase the load to 5.0 Kg and repeat the experiment.
7) Find the deflections from the formula and verify.

Tabular column

Load (N)	I_{xx} (mm^4)	L (mm)	a (mm)	b (mm)	x (mm)	Theoretical value (mm)	Experimental Value (mm)

<u>CAUTION: Never exceed 10 kg load on the beam</u>

Conclusion:

EXPERIMENT - 3

DEFLECTION OF A BEAM UNDER COMBINED LOADING BY THE THEOREM OF SUPERPOSITION

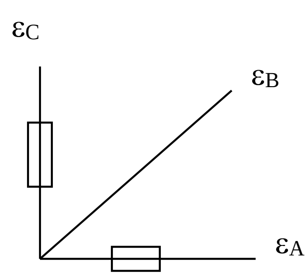
Aim: To verify the theorem of superposition

Equipment: Beam Test Set-Up with multiple loading capability (atleast two load points required), atleast two load cells, cantilever strain gauged beam, strain measuring equipment.

Theory:

Many times, a structural member is subjected to a number of forces acting not only at the ends, but also at the intermediate points along its length. Such a member can be analyzed by the application of the principal of superposition, the resulting strain will be equal to the algebraic sum of the strains caused by individual forces acting along the length of member.

The strain gauge is at a fixed position in the beam and load position can be varied. A strain gauge is mounted on a free surface, which in general, is in a state of plane stress where the state of stress is with regards to a specific xy rectangular rosette. Consider the three element rectangular rosette shown in fig-2, which provides normal strain components in three directions spaced at angles of 45^0. If an xy coordinate system is assumed to coincide with the gauge A and C then $\varepsilon x = \varepsilon_A$ and $\varepsilon y = \varepsilon c$. Gauge B provides information necessary to determine γ_{xy}.



$\varepsilon_x = \varepsilon_A$; $\varepsilon_y = \varepsilon_c$; $\gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_c$

Sl.No	Load (N)	ε_A	ε_B	ε_C	ε_x	ε_y	γ_{xy}

Procedure:

1. Mount the cantilever beam at the left support of beam test set-up. Connect the strain gauges wires with the strain measuring equipment. Use the following color codes:

ε_A	Blue wires
ε_B	Green wires
ε_C	Red wires

2. Set the load cell to read `zero` value in the absence of load. Set the three strains to read `zero` in the absence of load. Now Load the beam with 3.0 Kg at some point from vertical and record the strains in three directions. Record the load value at the load cell. Record the point of loading. **Remove this load.**

3. Set the load cell to read `zero` value in the absence of load. Set the three strains to read `zero` in the absence of load. Now Load the beam with load from horizontal direction. Apply, Load from right side end with load value of 2.0 Kg. , and record the strains in three directions. Record the load value. Remove this load.

4. Set the load cell to read `zero` value in the absence of load. Set the three strains to read `zero` in the absence of load. Now Load the beam with 3.0 Kg at same point from vertical as done earlier.
5. In addition load the beam with load from horizontal direction. Apply, Load from right side end with load value of 2.0 Kg.
6. Record the strains in three directions. Record the load values in the two load cells. Record the vertical load position.

Compute the values of γ_{xy} from the formula:

$$\gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_c$$

CAUTION: NEVER EXCEED 10 Kg LOAD ON THE BEAM.

Tabular Column:

Table I: Strains due to Vertical Load

Sl.No	Vertical Load (N)	Load position mm	ε_A	ε_B	ε_C	ε_x	ε_y	γ_{xy}

Table II: Strains due to Horizontal Load

Sl.No	Horizontal Load (N)	ε_A	ε_B	ε_C	ε_x	ε_y	γ_{xy}

Table III: Strains due to combined loading

Sl.No	Vertical Load (N)	Horizontal load (N)	Vertical Load position mm	ε_A	ε_B	ε_C	ε_x	ε_y	γ_{xy}

Note- The vertical load position should read same in Table I and Table III
Load values should read same in all Tables

Add the values of strains in table I and table II and compare with values in Table III.

Verify the two values should be same, hence the proof of superposition theorem

Conclusion:

EXPERIMENT – 4

VERIFICATION OF MAXWELL'S RECIPROCAL THEOREM

Aim: To verify the Maxwell's Theorem for the structures system

Equipment: Beam Test Set-Up with Load cells, steel scale, caliper, flat beam

Theory:

The displacement at point 'i', in a linear elastic structure, due to concentrated load at point 'j' is equal to the displacement at point 'j' due to a concentrated load of same magnitude at point 'i'. The displacement at each point will be measured in the direction of the concentrated load at that point. The only other restrictions on this statement, in addition to the structure being linear elastic and stable, is that the displacement at either point must be consistent with the type of load at that point. If the load at a point is a concentrated force, then the displacement at that point will be a translation, while if the load is moment, then the displacement will be rotation. The displacement at any point will be in the same direction as the load at that point and its positive direction will be in the same direction as the load.

This theorem often referred to as Maxwell's Reciprocal Displacement theorem. This can be proved through Unit Load Method i.e.; the deflection at A due to unit load at B is equal to deflection at B due to unit load at A.

$$\delta = \int M.m\, dx \;/\; EI$$

where,

M = Bending Moment at any point x due to external load

m= Bending Moment at any point x due to unit load applied at the point where deflection is required

let, $\mathbf{m_{xA}}$ = Bending Moment at any point x due to unit load at A

$\mathbf{m_{xB}}$ = Bending Moment at any point x due to unit load at B

when unit load (external load) is applied at A, $\mathbf{M=m_{xA}}$

To find deflection at B due to unit load at A, apply unit load at B

Then $\mathbf{m=m_{xB}}$

$$\text{Hence, } \delta_{BA} = \int M\,m\,dx \;/\; EI \;\; \delta = \int m_{xA}\, m_{xB}\, dx \;/\; EI \text{ ------(1)}$$

Similarly, when unit load (external load) is applied at B, $M=m_{xB}$

To find deflection at A, then $\mathbf{m=m_{xA}}$

Hence

$$\delta_{AB} = \int M\,m\,dx \;/\; EI = \int m_{xA}\, m_{xB}\, dx \;/\; EI \text{ ------------------(2)}$$

Comparing eqn. (1) and eqn. (2)

$$\delta_{AB} = \delta_{BA} \text{-- (3)}$$

The external load (W) can be taken as a multiple with unit load , therefore, this load W will appear as multiple with m_{xA} in eqn. (1) & as multiple with m_{xB} in eqn. (2). Thereby resulting in

$$W\,\delta_{AB} = W\,\delta_{BA} \text{-- (4)}$$

A beam shown in figure below which is simply supported at the ends and is subjected to bending about its major axis with a concentrated load anywhere in the beam

Deflections δ_x at any distance `x` from left support are given by following expressions; reference may be made to experiment no. 1.

$$\delta_x = W\,b\,[\,(L^2 - b^2)\,x - x^3] / (6\,E\,I\,L) \qquad \text{for} \quad 0<x<a$$

$$\delta_x = W\,b\,[\,X^3 - \; L/b\,(x-a)^3 - (L^2-b^2)\,x\,] / (6\,E\,I\,L) \qquad \text{for} \quad a < x < L$$

Where,

W is the load placed at a distance \`a\` from the left support in Newton
b=distance of load from right side support
L = span of the beam in mm
$\mathbf{Y_x}$ = deflection at any point distance x from left end
I = moment of inertia of the beam in mm^4 (I_{xx})
E= Young\`s modulus in N/mm^2

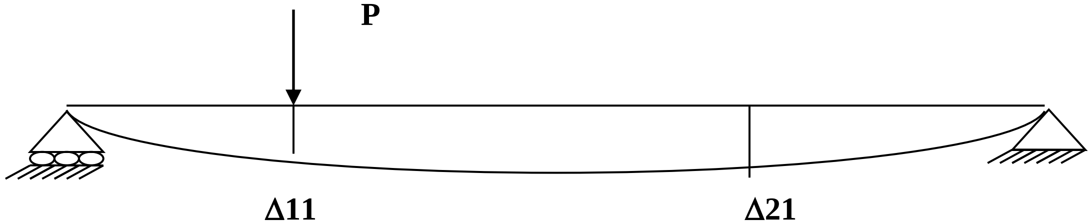


Fig-3: A simply supported Beam loaded at position 1 & 2

The Maxwell's Reciprocal Displacement theorem is very useful in the analysis of statistically indeterminate structures for evaluating the flexibility coefficients.

The displacement relationship can be expressed at point i and j

$$\delta_{i,B} = f_{i,j}\ W \qquad (5)$$

$$\delta_{j,A} = f_{j,i}\ W \qquad (6)$$

Where, $f_{i,j}$ is the displacement at point i due to a unit load at point j and $f_{j,i}$ is the displacement at point j due to a unit load at point 'i'. If we now substitute these expressions in Betti`s law and cancel out the term W on each side, we obtain

$$f_{i,j} = f_{j,i}$$

The theorem can be restated as the displacement at point i, in an elastic structure, due to a unit load at point j is equal to the displacement at point j due to unit load at point i.

Procedure:

1. Find the moment of inertia of beam from the following expression: $1/12\ b_1 d_1{}^3$, where b_1 is width of beam and d_1 is depth.
2. Place the beam supporting from two wedge supports. The load position can be varied.
3. Set the load cell to read zero in the absence of load. Set the deflection gauge to read zero in the absence of load.
4. Load the beam with 2.5 Kg . Note deflections at any point through deflection gauge.
5. Interchange the load location with the point of deflection measurement and repeat the readings.
6. Increase the load to 5.0 Kg and repeat the experiment.
7. Find the deflections from the formula and verify.

CAUTION: NEVER EXCEED 10 Kg LOAD ON THE BEAM .

Tabular column

Load (N)	I_{xx} mm^4	L mm	a mm	b mm	x mm	Theoretical value mm δ_{BA} or δ_{AB}	Experimental value mm δ_{BA} or δ_{AB}

Conclusions

EXPERIMENT – 5

DETERMINATION OF YOUNG'S MODULUS USING STRAIN GAGES, POISSON`S RATIO (ν) DETERMINATION

Aim: To determine the young's modulus of a simply supported beam or a cantilever beam.

Theory:

A beam shown in fig-1 shows the section which is simply supported at the ends and is subjected to bending about its major axis with a concentrated load anywhere in the beam. The beam is provided with strain gauge, the deflection of the beam can be determined wherever the load is applied on to the beam. An available cantilever beam can also be utilized for this experiment.

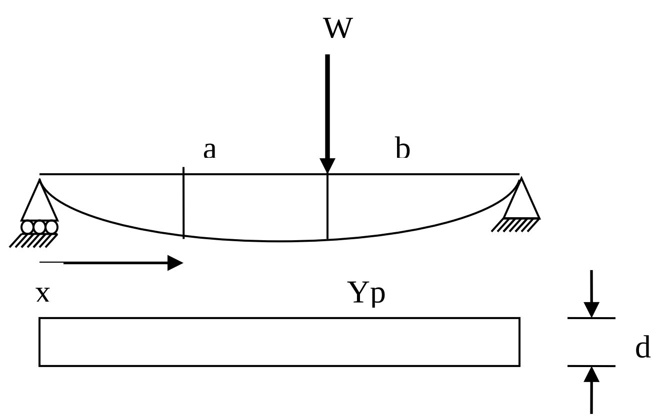


Fig-1: A simply supported beam

The strain gauge is at a fixed position in the beam and load position can be varied. A strain gauge is mounted on a free surface, which in general, is in a state of plane stress where the state of stress is with regards to a specific xy rectangular rosette. Consider the three element rectangular rosette shown in fig-2, which provides normal strain components in three directions spaced at angles of 45^0. If an xy coordinate system is assumed to coincide with the gauge A and C then $\varepsilon x = \varepsilon_A$ and $\varepsilon y = \varepsilon c$. Gauge B provides information necessary to determine γ_{xy}. Once εx, εy and γ_{xy} are known, then Hooke's law can be used to determine σx, $\sigma_{y,}$ and $\tau_{xy.}$ However in this case the requirement is to determine Young`s Modulus (E), which can be determined from equation (1) below.

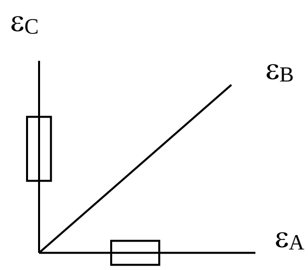


Fig-2

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)\,;\sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)\,;\tau_{xy} = \frac{E}{2(1+\nu\ )}\gamma_{xy}$$

$\varepsilon_x = \varepsilon_A$; $\varepsilon_y = \varepsilon_c$; $\gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_C$;

Sl.No	Load (N)	ε_A	ε_B	ε_C	εx	εy	γ_{xy}	$\nu=\varepsilon y/\varepsilon x$

M/I = σ/y --1

also

Shear Modulus $G = \dfrac{\tau_{xy}}{\gamma_{xy}}$ **---2**

Young's Modulus E = 2G (1+ν) ---3

Procedure: Mount the cantilever beam at the left support of beam test set-up. Connect the strain gauges wires with the strain measuring equipment. Use the following color codes

ε_A Blue wires
ε_B Green wires
ε_C Red wires

Set the load cell to read `zero` value in the absence of load. Set the three strains to read `zero` in the absence of load. Now Load the beam with 2.5 Kg at some point and record the strains in three directions. Record the load value at the load cell.

Repeat the experiment with load value of 5 Kg. Compute the values of Poisson's ratio from:

$$\nu=\varepsilon y/\varepsilon x$$

CAUTION: NEVER EXCEED 10 Kg LOAD ON THE BEAM .

Tabular Column:

Sl.No	Load (N)	ε_A	ε_B	ε_C	εx	εy	γ_{xy}	$\nu=\varepsilon y/\varepsilon x$

Find Young `s modulus through formulas above.

Example:

Sl No.	Load (N)	ε_A	ε_B	ε_C	εx	εy	γ_{xy}	$\nu=\varepsilon y/\varepsilon x$
1	10N	92µ	46µ	-18µ	92µ	-18µ	18µ	0.1957
2	20N	64µ	304µ	-18µ	64µ	-18µ	14µ	0.2188
3	30N	37µ	19µ	-10µ	37µ	-10µ	11µ	0.2973

POISSON`S RATIO (ν) DETERMINATION

Aim: To determine the Poisson's ratio of cantilever beam

Equipment: Beam Test Set-Up with load cells, Cantilever beam with calibrated rosette strain gauge, strain measuring equipment.

Theory: A cantilever beam is subjected to bending about its major axis with a concentrated load anywhere in the beam. The beam is provided with rosette strain gauge.

A calibrated strain gauge rosette is fixed at a location with-in the span of the beam, and load position can be varied. Calibration has been done to read strains in microns (μ). Consider the three element rectangular rosette shown in fig-2, which provides normal strain components in three directions spaced at angles of 45^0. If an xy coordinate system is assumed to coincide with the gauge A and C then $\varepsilon x = \varepsilon_A$ and $\varepsilon y = \varepsilon c$.

Gauge B provides information necessary to determine shear strain (γ_{xy}). Once εx, εy and γ_{xy} are known, then Hooke's law can be used to determine σx, σ_y, and τ_{xy}. Subsequently principal stresses can be determined.

Poisson's ratio (ν) can be determined from:

$$\nu = \varepsilon y / \varepsilon x$$

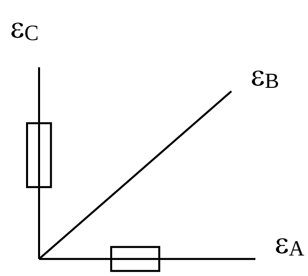


Fig-2

$$\sigma_x = \frac{E}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y)\,;\ \sigma_y = \frac{E}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x)\,;\quad \tau_{xy} = \frac{E}{2(1+\nu\ )}\gamma_{xy}$$

$$\varepsilon_x = \varepsilon_A\ ;\quad \varepsilon_y = \varepsilon_c\ ;\quad \gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_c$$

Principal stress axes:

Principal stress axes is located with the angle θ according to :
$\tan 2\theta = (2\,\varepsilon_B - \varepsilon_A - \varepsilon_c) \,/\, (\varepsilon_A - \varepsilon_c)$

Principal Stresses are given by following expressions:

$$\sigma_1 = A + \sqrt{B^2 + C^2} \quad , \qquad \sigma_2 = A - \sqrt{B^2 + C^2}$$

$$\tau_{max} = (\sigma_1 - \sigma_2)/2 = \sqrt{B^2 + C^2}$$

where, $A = (\sigma_x + \sigma_y)/2$, $B = (\sigma_x - \sigma_y)/2$ and $C = \tau_{xy}$

E= Young's modulus in N/mm^2. Strains are in microns

Procedure: Mount the cantilever beam at the left support of beam test set-up. Connect the strain gauges wires with the strain measuring equipment. Use the following color codes

ε_A	Blue wires
ε_B	Green wires
ε_C	Red wires

Set the load cell to read `zero` value in the absence of load. Set the three strains to read `zero` in the absence of load. Now Load the beam with 2.5 Kg at some point and record the strains in three directions. Record the load value at the load cell.

Repeat the experiment with load value of 5 Kg. Compute the values of Poisson's ratio from: $\boldsymbol{\nu = \varepsilon_y/\varepsilon_x}$

CAUTION: NEVER EXCEED 10 Kg LOAD ON THE BEAM .

Students may determine stresses using formulas above. MATLAB program is provided at the end of this manual to compute stresses from given strain data.

Tabular Column:

Sl.No	Load (N)	ε_A	ε_B	ε_C	εx	εy	γ_{xy}	$\nu=\varepsilon_y/\varepsilon_x$

Example: A typical data is shown for the purpose of computing shear strain and Poisson's ratio is given below.

Sl.No	load	ε_A	ε_B	ε_C	ε_x	ε_y	γ_{xy}	$\nu=\varepsilon_y/\varepsilon_x$
1	10N	92µ	46µ	-18µ	92µ	-18µ	18µ	0.1957
2	20N	64µ	304µ	-18µ	64µ	-18µ	14µ	0.2188
3	30N	37µ	19µ	-10µ	37µ	-10µ	11µ	0.2973

Example: A typical data is shown for the purpose of computing stresses, principal stresses, and planes of principal stresses.

Sl No	ε_A	ε_B	ε_C	σ_x Mpa	σ_y Mpa	τ_{xy} Mpa	Principle stress σ_1 Mpa	Principle Stress σ_2 Mpa
1	200µ	900µ	1000µ	105.58	230.09	46.69	342.04	-6.371

	Normal, shear stress,& principal stress					Angle of principal stress	
Sl No	σ_x Mpa	σ_y Mpa	τ_{xy} Mpa	σ_1 Mpa	σ_2 Mpa	θ_1	θ_2
1	5.6787	0.8159	0.3879	6.8156	-0.242	64.65	-86.00
2	12.064	1.1000	0.9842	13.299	-0.651	51.45	-85.62
3	17.944	1.2762	1.5695	19.347	-1.273	41.80	-85.04

EXPERIMENT - 7

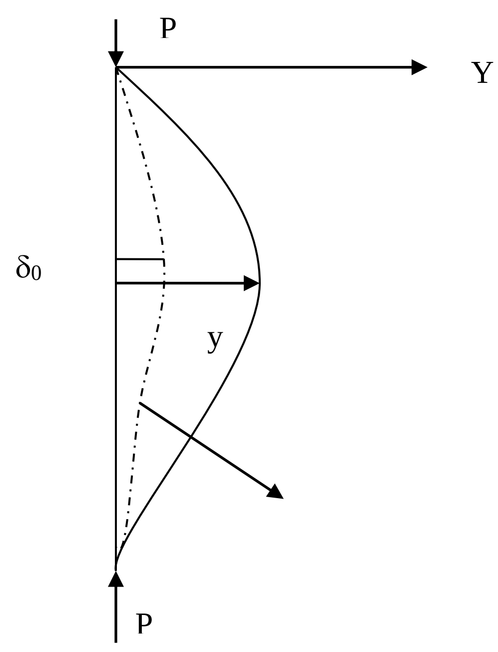
BUCKLING LOAD OF SLENDER ECCENTRIC COLUMNS AND CONSTRUCTION OF SOUTH WELL PLOT

Aim: Practical columns have some imperfections in the form of initial curvature and the buckling of loads of such struts is of real practical value. The experiment aims at measuring the buckling loads of columns and construction of South Well Plot. The imperfection amounts to initial curvature, which shows up in this plot.

Equipment: WAGNER beam set-up, hinged supports, load cells, Long column with initial curvature, mounted dial for deflection measurements.

Theory:

Consider a pin ended strut AB of length L, whose centroidal longitudinal axis is initially curved as shown in fig (1). Under the application of the end load P, the strut will have some additional lateral displacement y at any section.



In this case, Bending Moment at any point is proportional to the change in curvature of the column from its initial bent position y- δ_0. The equation for curvature of column is as follows.

$$M = P(y+y_0) = - EI\, d^2y/dx^2 \quad \ldots (1)$$

$P/EI = k^2$

$$\frac{d^2y}{dx^2} + k^2 y = -k^2 y_0 \quad \ldots (2)$$

Assuming initial curvature (y_o) to be sinusoidal, satisfying the equation

$$y_0 = \delta_0 \sin(\pi x / L) \quad \ldots (3)$$

Where δ_0 <u>equals to the initial displacement</u> at the centre of the strut

The general solution of this differential equation is

$$y = A\cos kx + B\sin kx + \frac{k^2\delta_0}{\pi^2/L^2 - k^2}\sin(\pi in(\pi \quad \ldots (4)$$

If the ends are hinged, for the end conditions, then:
y=0 at x =0 and x =L
This results in values of constants: A=B=0.
The resulting equation is as below:

$$y = \frac{k^2\delta_0}{\frac{\pi^2}{L^2} - k^2}\sin\left(\frac{\pi x}{L}\right) \quad \ldots (5)$$

Substituting Euler's buckling load (P_e)

$$P_e = \Pi^2 EI/L^2 \text{ and } k^2 = P/EI$$

$$y = \frac{\delta_0}{\frac{Pe}{P} - 1}\sin\left(\frac{\pi x}{L}\right) \quad \ldots (6)$$

for x=L/2, at center of column, the deflection at center y_C

$$y_c = \frac{\delta_0}{\frac{Pe}{P} - 1} \qquad \ldots(7)$$

The value P_e represents the buckling load for perfectly straight strut. In the relation for deflection (y) , the additional lateral displacement of the strut, that the effect of end load P is to increase (y) by a factor $1/(p_e/p)-1$; shown by equation (6). When P approaches P_e, the additional displacement at mid length of the strut is expressed by eqn. (7).

The load deflection relationship of eqn. (5) is the basis of *South Well plot* technique for extrapolating for the elastic critical load from experimental measurement.

$$\frac{\delta}{\delta 0} = \frac{\frac{P}{Pe}}{1 - \frac{P}{Pe}} \qquad \ldots (8)$$

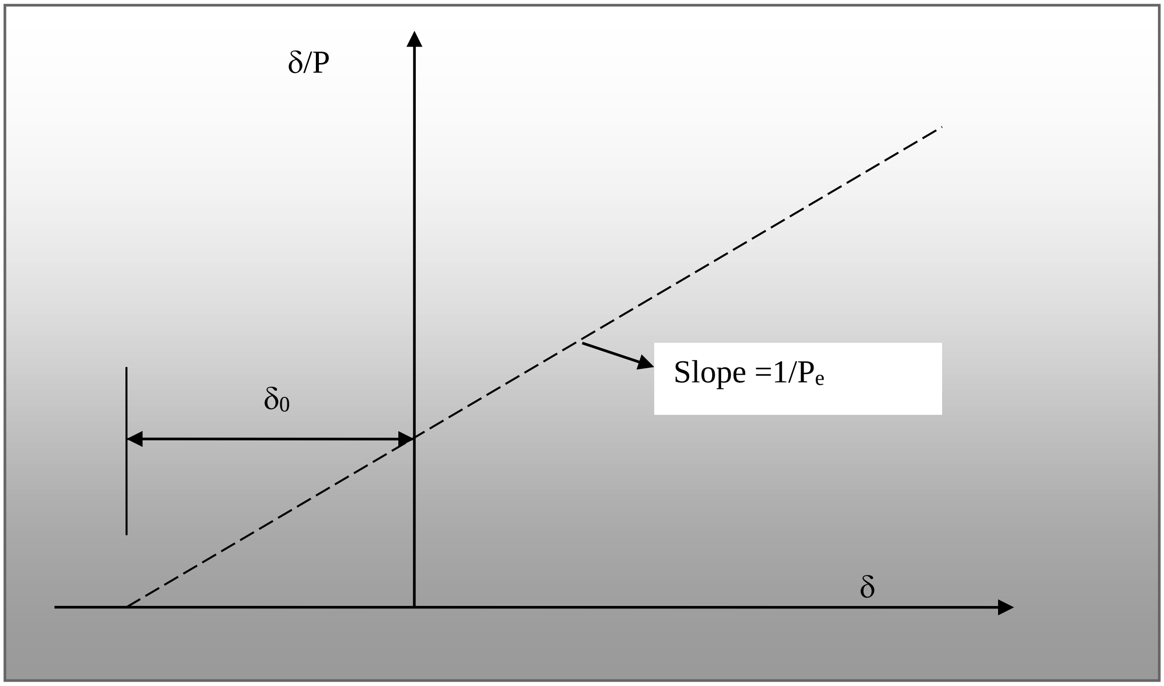
Rearranging the above equation we get

$$\delta = (P_e / P)\ \delta - \delta_0 \qquad \ldots(9)$$

or

$$\delta = (\delta / P)\ P_e - \delta_0$$

The linear relationship between δ/P and δ shown in figure below can be experimentally determined. Thus if a straight line is drawn which best fits the points determined from the experimental measurements of P and δ , the reciprocal of the slope of this line gives an estimate of the magnitude of δ_0 of the initial curvature that can be determined from the intercept on the horizontal axis.



South Well Plot

Procedure:

1. Set up the two hinge supports on the WAGNER beam at the top and bottom supports. Fix the column in the supports.
2. Set the load reading to zero in load cell. Determine the center of column.
3. Set –up the deflection dial gage for reading the column deflections at the center of column. Set the deflection dial gage reading to zero.
4. Apply the vertical load in steps of 5 Kgs. each, in four steps (5Kg, 10Kg, 15Kg, and 20Kg) and record the deflections at each step of load.

Tabular Column:

Load (P) Kgs	Deflection (δ) mm	δ/P

Draw South Well Plot (δ/P vrs δ) .

Determine the slope and estimate P_e. Find out the initial deflection of column.

CAUTION: NEVER EXCEED 20 Kg OF LOAD ON THE COLUMN .

Example:1

A slender strut, 1800mm long, and of rectangular section 30mm x 12mm transmits a longitudinal load P acting at the centre of each end. The strut was slightly bent about its minor principal axis before loading. If the P is increased form 500N to 1500N, the deflection at the middle of the length increases by 4mm. Determine the amount of deflection before loading.

Find also the total deflection and the maximum stress when P is 2000 N.

Take $E = 2.15 \times 10^5$ N/mm^2.

$I = 1/12 \times 30 \times 12^3 = 4320$ mm^4

Let $P_e = \Pi^2 EI / L^2 = \Pi^2\, 2.15 \times 10^5 \times 4320 / (1800 \times 1800) = 2830$ N

Let δ_1 be the central deflection when P =500 N and let δ_2 be the central deflection when P =1500 N

Substituting in $\delta_c = \frac{Pe}{Pe - P} x \delta_0$

$\delta_1 = 2830/(2830-500) \times \delta_0$ …(1)

$\delta_2 = 2830/(2830-1500) \times \delta_0$ …(2)

from eqns.1 & 2

δ_0 = 4.381 mm

P =2000 N $\delta c = 2830/(2830-2000) \times 4.381 = 14.94$ mm

$A = 30 \times 12 = 360$mm^2 $Z = 1/6 \times 30 \times 12^2$

$Mc = P \times \delta c = 2000 \times 14.94 = 298880$ N-mm

$\sigma 0 = P/A = 2000/360$mm^2 $\sigma b = Mc/Z = 29880/ 720 = 41.5$ N/mm^2

σ max $= \sigma 0 + \sigma b = 5.55 + 41.5 = 47.05$ N/mm^2

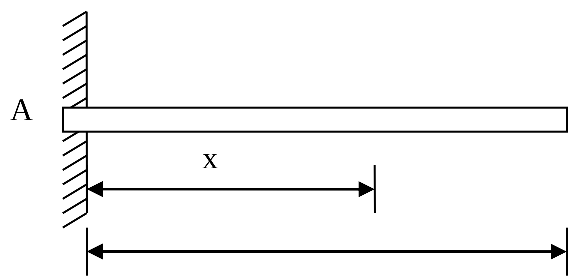
EXPERIMENT-10

VIBRATION OF CANTILEVER BEAM

Objective: To determine the lateral or transverse vibration of a cantilever beam when the beam is fixed at one end and free at the other end.

Introduction:

A beam which is cantilevered of span L with uniform mass w/g per unit run is shown in figure-1and fixed at one end. Assume a deflection function and obtain the first approximation for the fundamental frequency with the origin at the free end.



Cantilever beam

L is the span of the beam in mm
E is the Young's Modulus of the in N/mm^2
$I = bd^3/12$ is the Moment of Inertia of the beam mm^4
w = Weight per unit length
g = Acceleration due to gravity

The fundamental frequency of the cantilever beam
$\omega^2 = 12.39\ E\ I\ /\ wl^4$
The natural frequency for the transverse vibration of a uniform beam fixed at one end and free at the other end.
The four roots of the equation are:
$\beta 1\ L = 1.8751$
$\beta 2\ L = 4.6941$
$\beta 3\ L = 7.8548$
$\beta 4\ L = 10.9955$

$$\omega n = n^2 \pi^2 \sqrt{\frac{EI}{\rho A L^4}} \qquad \text{.... (1)}$$

Example: Determine the three lowest natural frequency for the system shown in fig-1

Given m=10kg,E=200x10^9 N/m2; ρ =7800 kg/m3 ; A =2.6 x 10-3 m2 ; L=1m ; I= 4.7 x10^{-6} m4.

$$\omega n = 1^2 \pi^2 \sqrt{\frac{200x10^9\, x4.7x10^{-6}}{7800x2.6x10^{-3}\, x1^4}} \qquad =215.3\ \phi^2$$

ϕ1=1.423 ; ϕ2=4.113 ; ϕ3=7.192

ω1 = 486.1 rad/ s ; ω2 = 3642 rad/s ; ω3 = 1.114 x 10^4 rad/s.

VIVA QUESTIONS

1. Explain Castigliano`s theorem and its verification through any experimental set-up
2. Explain Maxwell`s reciprocal theorem and its verification though experiment
3. What is Betti`s theorem
4. Explain South well plot
5. What are various modes of failures of riveted joints
6. What is the value of maximum loading allowed in the existing experimental beam test set-up
7. What is the value of maximum loading allowed in the existing Wagner beam set-up
8. What is bending modulus
9. What is a sandwich beam
10. What is strain compatibility
11. Explain unit load method for determining the deflection of beams
12. How Young's modulus and Poisson's ratio can be determined from beam test set-up
13. Explain Superposition theorem
14. What are mode shapes and types of modes in vibrations
15. What is a Rosette strain gauge?
16. Explain unsymmetrical bending of beams
17. What is the purpose of Stiffeners, What is the purpose of Longirons
18. What stresses are taken up by the web and the flange of the spars.
19. Explain how bending and torsion is taken up by the wing structure
20. What is a torsion box
21. Explain how bending and torsion is taken up by the fuselage structure
22. What is shear center and how it can be determined through experimental set-up
23. What is Flexibility matrix
24. What is Power Spectral Density
25. What are Principal stress axes
26. What is simple stress, simple strain
27. What is plain stress, plain strain
28. What is a stress Tensor
29. What is a Wagner Beam test set-up? What are Wagner assumptions?
30. Explain the Energy methods used for structural analysis.

TORSIONAL VIBRATION(UNDAMPED) OF SINGLE ROTOR SHAFT SYSTEM

OBJECTIVE

To study the Torsional vibration (undamped) of single rotor system.

PROCEDURE

1. Fix the bracket at convenient position along the lower beam.
2. Grip one end of the shaft at the bracket by chuck.
3. Fix the rotor on the end of the shaft.
4. Twist the rotor through some angle and release.
5. Note down the time required for n of oscillations.
6. Repeat the procedure for different length of the shaft.
7. Make the following observations.

a) Shaft diameter = mm
b) Diameter of Disc = mm
c) Weight of the Disc = Kgf
d) Modulus of Rigidity = 0.8×10^6 kgf/sq.cm

OBSERVATION TABLE

S.No	Mass of Rotor (Kg)	Length of Shaft (m)	No. of oscillation	Time for n oscillation (sec)	Frequency

SPECIMEN CALCULATIONS

1) Determination of torsional stiffness: Kt?

Kt $= T/\theta = G \times Ip/L$
L = Length of shaft
Ip = polar M.I of shaft $= \pi d^4/32$
d = Shaft Diameter
G = Modulus of Rigidity of shaft 0.8×10^6 Kgf/sq.cm

2) Determine ωn Experimental

ωn = No. of oscillations/Time for n oscillations = c/sec

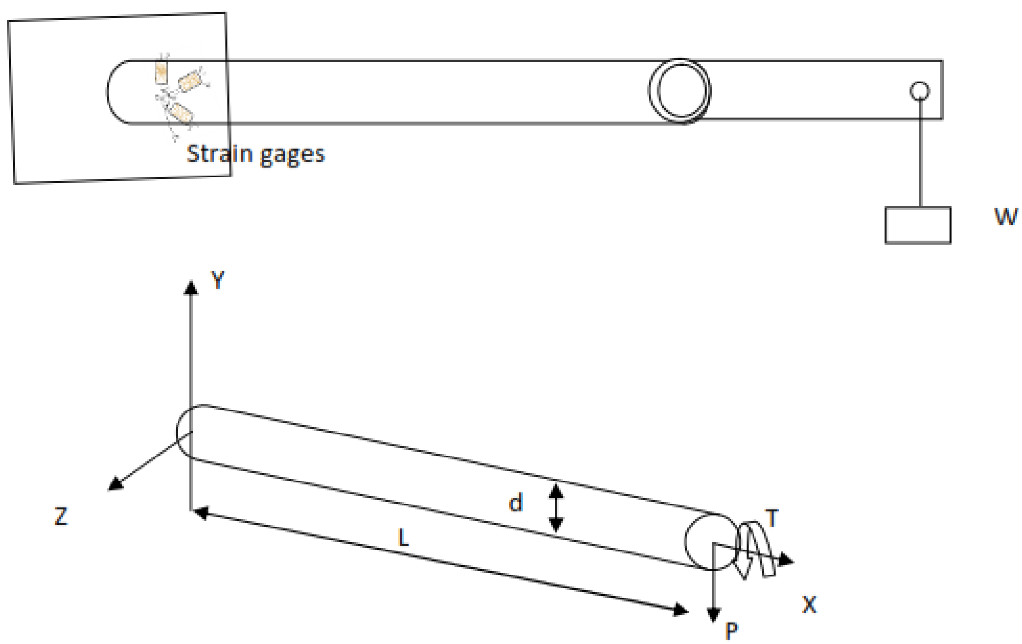
EXPERIMENT - 3

DEFLECTION OF A BEAM UNDER COMBINED LOADING BY THE THEOREM OF SUPERPOSITION

Aim: To determine the principle stresses and principle planed of a hollow circular shaft due to combined loading.

Apparatus Required: Hollow circular shaft fixed as a cantilever, weight hanger with slotted weights, strain gauges, connection wires, strain indicator and micrometer.

Experimental Setup:



Procedure: Two strain gauges are fixed near the root of the tube fixed as a cantilever, one on the top fiber and other at the bottom to measure the bending strain. Another strain gauge is fixed at the same location on the neutral axis at 45° to measure the shear strain. Similarly three more strain gauges are fixed at the middle of the length to verify the result at various location of the tube. The strain gauges on the top and bottom of the tube are connected to half bridge circuit in the strain indicator to increase the circuit sensitivity, since the tension and compression get added up. The strain gauge 45° is connected to the quarter bridge of the strain indicator to measure the shear strain. The outside diameter of the tube is measured using Vernier calipers. Weights are added to the hook attached to the lever in steps of two kg and the strain gauge readings are noted from the strain indicator for each load. From the strains the bending stress, shear stresses are calculated and hence principal stresses and principal angle are calculated. These values are compared with theoretical values. Note: For half bridge the strain readings are multiplied by two and quarter Bridge by four to get the actual strains.

Tabular column:

YOUNG`S Modulus of the tube	=
Outside diameter of the tube	=
Thickness of the tube	=
Length of the tube	=
Strain gauge resistance	=
Gauge factor	=
Distance of the strain gauges near root from tip	=
Distance of the strain gauges at the middle from tip	=
Distance from the center of the tube to the center of the hook	=
Weight of the hook	=

Result:

Principle stress due to axial, bending and torsion are calculated