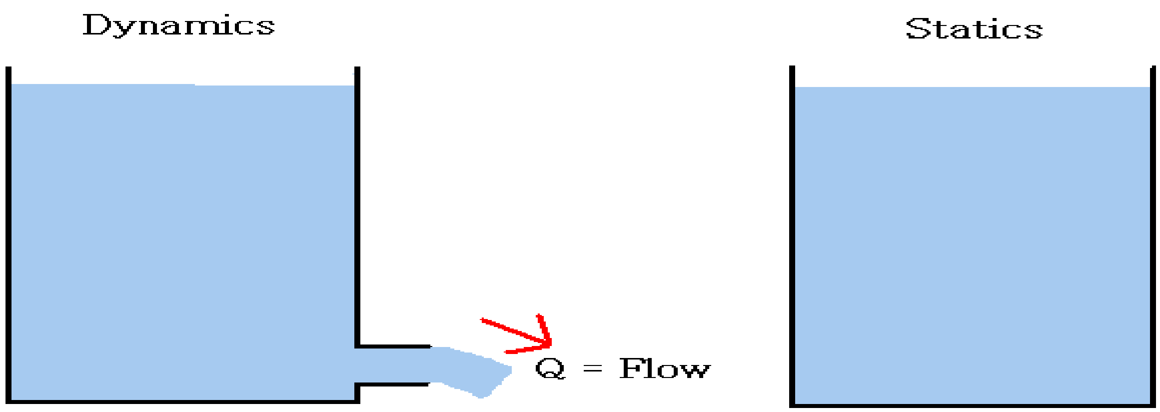
Fluid dynamics:

Introduction:



The laws of Statics that we have learned cannot solve Dynamic Problems. There is no way to solve for the flow rate, or Q. Therefore, we need a new dynamic approach to Fluid Mechanics.

Equations of Motion

The dynamics of fluid flow is the study of fluid motion with forces causing flow. The dynamic behaviors of the fluid flow is analyzed by the **Newton's law of motion (F=ma),** which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

Mathematically, $F_x = m.a_x$

In the fluid flow, following forces are present:

- **pressure force 'F_p'**
- **gravity force 'F_g'**
- **viscous force 'F_v'**
- **turbulent flow 'F_t'**
- **surface tension force 'F_s'**
- **compressibility force 'F_e'**

- The **pressure force 'F_p'** is exerted on the fluid mass, if there exists a pressure gradient between the 2 parts in the direction of flow.

- The **gravity force 'F_g'** is due to the weight of the fluid and it is equal to 'M_g'. The gravity force for unit volume is equal to 'ρg'.

- The **viscous force 'F_v'** is due to the viscosity of the flowing fluid and thus exists in the case of all real fluid.

- The **turbulent flow 'F_t'** is due to the turbulence of the flow. In the turbulent flow, the fluid particles move from one layer to other and therefore, there is a continuous momentum transfer between adjacent layer, which results in developing additional stresses(called Reynolds stresses) for the flowing fluid.

- The **surface tension force 'F_s'** is due to the cohesive property of the fluid mass. It is, however, important only when the depth of flow is extremely small.

- The **compressibility force 'F_e'** is due to elastic property of fluid and it is important only either for compressible fluids or in the cases of flowing fluids in which the elastic properties of fluids are significant.

- If a certain mass of fluid in the motion is influenced by all the above mentioned forces, thus according to Newton's law of motion, the following equation of motion may be written as

$$M\,a = F_g + F_p + F_v + F_t + F_s + F_e = \text{net force } F_x \qquad \text{---- (1)}$$

Further by resolving the various forces and the acceleration along the x, y and z directions, the following equation of motion may be obtained.

$$Ma_x = F_{gx} + F_{px} + F_{vx} + F_{tx} + F_{sx} + F_{ex}$$

$$Ma_y = F_{gy} + F_{py} + F_{vy} + F_{ty} + F_{sy} + F_{ey} \qquad \text{---- (1a)}$$

$$Ma_z = F_{gz} + F_{pz} + F_{vz} + F_{tz} + F_{sz} + F_{ez}$$

The subscripts x, y and z are introduced to represent the component of each of the forces and the acceleration in the respective directions.

In most of the problems of the fluids in motion, the **tension forces** and the **compressibility forces** are not significant. Hence, the forces may be neglected, thus equations (1) and (1a) became.

$$Ma= F_g+F_p+F_v+F_t \qquad \text{--- (2)}$$

And

$$Ma_x= F_{gx}+F_{px}+F_{vx}+F_{tx}$$

$$Ma_y= F_{gy}+F_{py}+F_{vy}+F_{ty} \qquad \text{--- (2a)}$$

$$Ma_z= F_{gz}+F_{pz}+F_{vz}+F_{tz}$$

Equations (2a) are known as **Reynolds's equations of motion which are** useful in the analysis of the turbulent flows. Further, for laminar or viscous flows the turbulent forces are less significant and hence they may be neglected. The eqns.(2) & (2a) may then be modified as,

$$M_a= F_g+F_p+F_v$$

And

$$M_{ax} = F_{gx}+F_{px}+F_{vx}$$

$$M_{ay} = F_{gy}+F_{py}+F_{vy} \qquad \text{---- (3a)}$$

$$M_{az} = F_{gz}+F_{pz}+F_{vz}$$

Equations (3a) are known as **Navier-stokes equations which are** useful in the analysis of viscous flow. Further, if the viscous forces are also of less significance in the problems of fluid flows, then these force may also neglected. The viscous forces will become insignificant if the flowing fluid is an ideal fluid. However, in case of real fluids, the viscous forces may be considered insignificant if the viscosity of flowing fluid is small. In such cases the eqn.(3)&(3a) may be further modified as

$$M_a= F_g+ F_p \qquad \text{------ (4)}$$

And

$$M_{ax} = F_{gx}+ F_{px}$$

$$M_{ay} = F_{gy}+ F_{py} \qquad \text{------- (4a)}$$

$M_{az} = F_{gz} + F_{pz}$

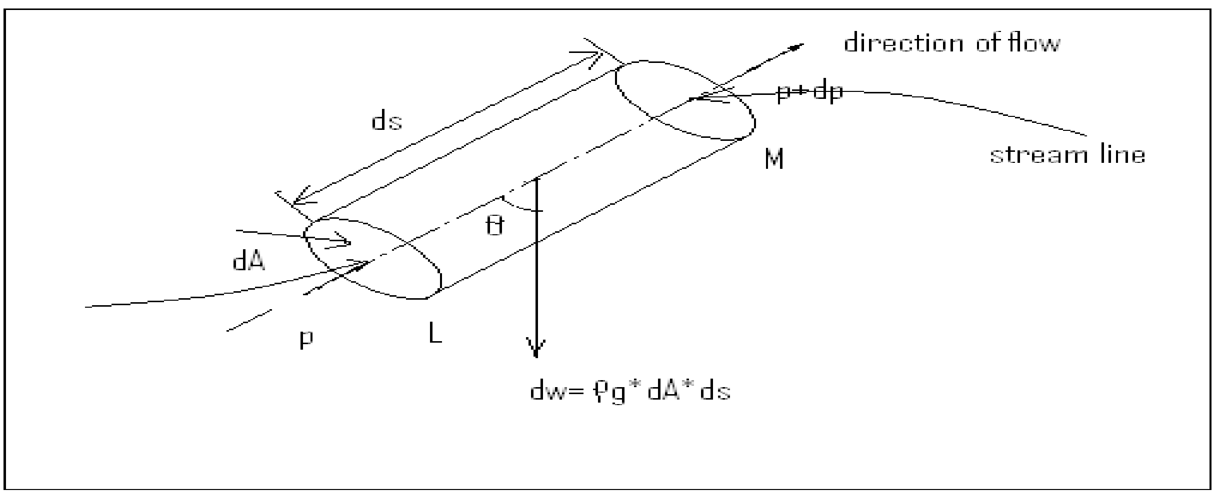
Equation (4a) is known as **Euler's equation of motion**.

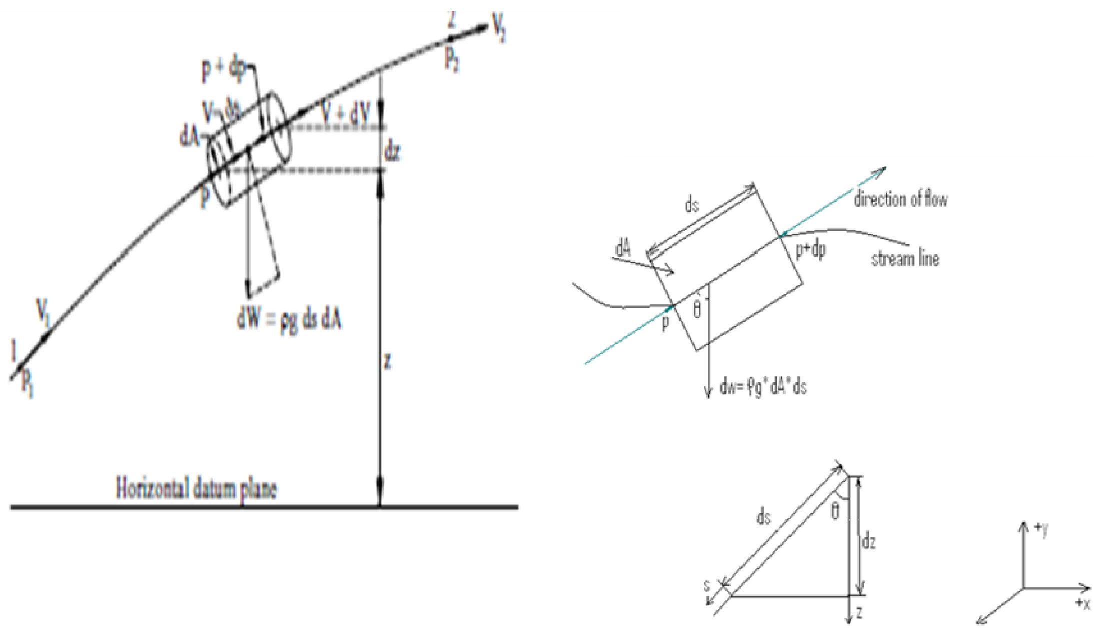
Euler's equation of motion:

Statement: **In an ideal incompressible fluid, when the flow is steady and continuous, sum of the velocity head, pressure head and datum head along a stream line is constant.**

Assumptions:

- The fluid is ideal and incompressible.
- Flow is steady and continuous.
- Flow is along streamline and it is 1-D.
- The velocity is uniform across the section and is equal to the mean velocity.
- Flow is Irrotational.
- The only forces acting on the fluid are gravity and the pressure forces.





Figures 1(b):Forces on a fluid element

Consider a streamline and select a small cylindrical fluid system for analysis as shown in Figs. 1(a) & (b) of length 'ds' and c/s area 'dA' as a free body from the moving fluid,

Let, p = pressure on the element at 'L'

p+dp = pressure on the element at M and

v= velocity of the fluid element.

The forces acting (tending to accelerate) the fluid element in the direction of stream line are as follows,

1) Net pressure force in the direction of flow is

$p.dA-(p+dp)\ dA= -\ dp.dA$ ------- (1)

2) Component of the weight of the fluid element in the direction of flow is

$= -\ \rho g.dA.ds.\cos\theta$

$= -\ \rho g.dA.ds\ (dz/ds)$ (because $\cos\theta=dz/ds$)

$= -\ \rho g.dA.dz$ -- (2)

Mass of the fluid element $= \rho.dA.ds$ -- (3)

The acceleration of the fluid element

$$a= dv/dt = (dv/ds).(ds/dt)= v(dv/ds)$$

Now according to Newton law of motion

Force = mass * acceleration

Therefore $-dp.dA - \rho g.dA.dz= (\rho.dA.ds) (v.dv/ds)$ --- (4)

Dividing both sides by ρdA we get

$$-dp/\rho - gdz = vdv \quad \text{(divide by -1)}$$

$$\boxed{\mathbf{(dp/\rho)+vdv+gdz=0 \quad \text{------ (A)}}}$$

This is the required **Euler's equation for motion**,

<u>Bernoulli's Equation from Euler's equation for motion:</u>

By Integrating **Euler's equation for motion,** we get

$$1/\rho \int dp + \int vdv + \int gdz= \text{constant}$$

$p/\rho + v^2/2 + gz =$constant dividing by 'g' we get

$$p/\rho g + v^2/2g + z= \text{constant}$$

$$p/w+v^2/2g+z =\text{constant}$$

In other words,

$$\mathbf{p_1/w + v_1^2/2g + z_1 = p_2/w_2 + v_2^2/2g + z_2}$$

As points 1 and 2 are any two arbitrary points on the streamline, the quantity

$$\boxed{\mathbf{P/w + v^2/2g + z = H = constant \text{------ B}}}$$

Applies to all points on the streamline and thus provides a useful relationship between *pressure p*, the *magnitude V of the velocity*, and the *height z* above datum. Eqn. B is known as the Bernoulli equation and the Bernoulli constant *H is also termed the total head.*

Bernoulli's equation from energy principle:

Statement: **In an ideal, incompressible fluid, when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy (or datum) energy is constant along a stream line.**

Mathematically, $p/w + v^2/2g + z = \text{constant}$

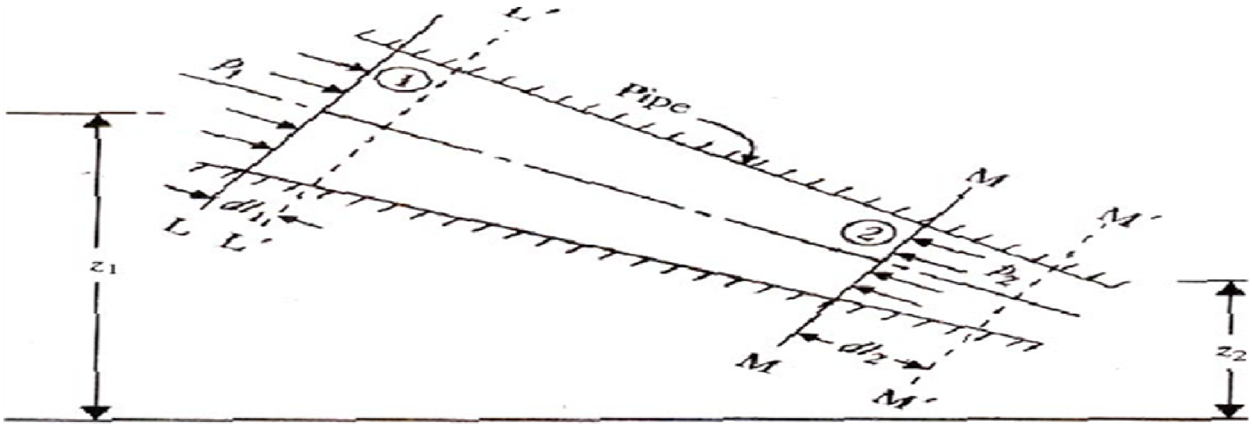


Fig. 2 : Liquid flowing through a non-uniform pipe

Proof: Consider an ideal & incompressible fluid flowing through a non-uniform pipe as shown in fig. 2. Let us consider 2 sections LL&MM and assume that the pipe is running full and there is continuity of flow between the two sections.

Let p_1=pressure at LL

V_1=velocity of liquid at LL

Z_1=height of LL above the datum

A_1=area of pipe at LL and

Similarly, P_2, v_2, z_2, A_2 are the corresponding values at MM

Let the liquid b/w 2sections LL&MM move to L^1L^1&M^1M^1 through very small length dl_1&dl_2 as shown in figure 2. This movement of liquid b/w LL&MM is equivalent to the movement of liquid b/w L^1L^1&M^1M^1 being unaffected

Let W = wt of liquid b/w LL&L^1L^1 as the flow is continuous

$W= wA_1dl_1=wA_2dl_2$ … Volume of fluid

Or $A_1dl_1=W/w$ and $A_2dl_2=W/w$

Therefore $A_1dl_1=A_2dl_2$

Work done by press at LL in moving the liquid to L^1L^1= force * distance=$p_1A_1dl_1$

Similarly, work done by press at MM in moving the liquid to $M_1M_1= P_2A_2dl_2$ (negative sign indicates that direction of p_2 is opposite to that of p_1)

Therefore, work done by the pressure

$=p_1A_1dl_1- p_2A_2dl_2$

$=p_1A_1dl_1-p_2A_2dl_2$ (because $A_1dl_1=A_2dl_2$)

$=A_1dl_1 (p_1-p_2)$

$=W/w (p_1-p_2)$ (because $A_1dl_1=W/w$)

Loss of potential energy (PE) = $W (Z_1-Z_2)$

Gain of kinetic energy (KE) = $W (v_2^2/2g-v_1^2/2g) = W/2g(v_2^2-v_1^2)$

Also, **loss of P.E+ work done by pressure = gain in K.E**

Therefore $W (z_1-z_2) +W/w (p_1-p_2) =W/2g (v_2^2-v_1^2)$

or $(z_1-z_2) +(p_1/w-p_2/w)=(v_2^2/2g-v_1^2/2g)$

$P_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2$

Which prove ***Bernoulli's equation***

P/w = pressure energy per unit weight

= pressure head

$v^2/2g$ = Kinetic energy per unit weight

= kinetic head

Z = datum energy per unit weight

= datum head

Bernoulli's equation for real fluid:

Bernoulli's equation earlier derived was based on the assumption that fluid is non viscous and therefore frictionless. Practically, all fluids are real (and not ideal) and therefore are viscous and as such always some losses in fluid flow. These losses have, therefore, to be taken into consideration in the application of Bernoulli's equation which gets modified (between sections 1 & 2) for real fluids as follows:

$p_1/w + v_1^2/2g + z_1 = p_2/w + v_2^2/2g + z_2 + \mathbf{h_L}$

Where

$\mathbf{h_L}$ = loss of head/energy between sections1 & 2

26/27-3-2010

Problems on Bernoulli's Equation:

1. Water is flowing through a pipe of diameter 5cm under a pressure of 29.43N/cm^2 (gauge) and with mean velocity of 2 m/s. Find the total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution

Diameter of pipe = 5cm = 0.5 m

Pressure p = 29.43N/cm^2 = 29.43 x 10^4 N/m^2

Datum head z =5m

Total head = pressure head + kinetic head+ datum head

Pressure head= $p/\rho g$=29.43 x 10^4 /1000 x9.81 =30m

Kinetic head=$v^2/2g$ = 2x2/2x9.81= 0.204m

Total head= $p/\rho\, g + v^2/2g + z$ = 30+0.204+5 = **35.204 m**

2). A pipe through which the water is flowing, is having diameters 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 40m/s. find the velocity head at sections 1 and 2, and also rate of discharge.

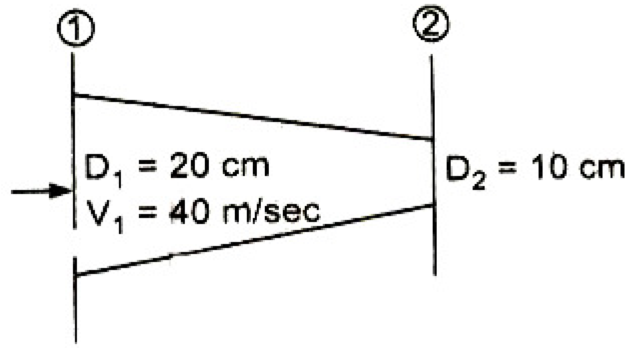


Fig. 2

Solution

D_1=20 cm =0.2m,

A_1=π/4 x D_1^2 = 0.0314 m^2,

v_1=4 m/s

D_2=0.1m,

A_2=π/4 x D_2^2 = 0.00785 m^2

Velocity head at section 1

V_1^2/ 2g = 4 x4 /2x9.81=**0.815 m**

Velocity head at section 2 = V_2^2/2g

To find V_2, apply continuity equation at sections 1&2

$A_1V_1=A_2V_2$

V_2= A_2V_2/A_2= 0.0314 x4/0.00785 =16.0 m/s

Velocity head at sec.2 = V_2^2/2g = 16 x16 /2 x9.81

V_2 = 83.047 m

Rate of discharge = A_1V_1 or A_2V_2= 0.0314 x 4

=0.1256 m3/s

= **125.6 lit/s** [1 m^3 = 1000litres]

3) The water is flowing through a tapering pipe having diameter 300mm and 150mm at section 1 & 2 respectively. The discharge through the pipe is 40lit/sec. the section 1 is 10m above datum and section 2 is 6m above datum. Find the intensity of pressure at section 2, if that at section 1 is $400kN/m^2$

Solution:

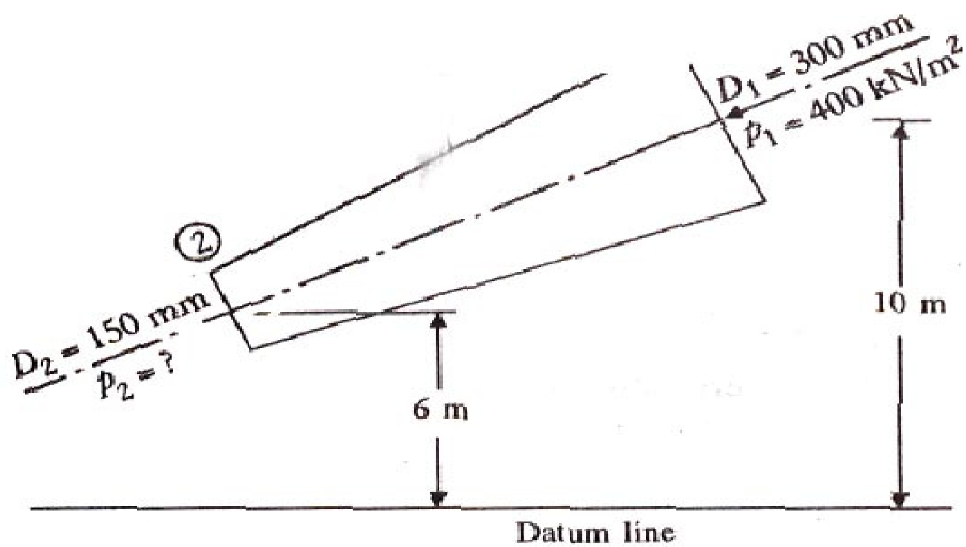


Fig. 3

At section 1

D_1=300mm=0.3m, Area $a_1= \pi/4*0.3^2=0.0707m^2$

Pressure $p_1=400kN/m^2$

Height of upper end above the datum, z_1=10m

At section 2

D_2=150mm=0.15m,

Area $A_2= (\pi/4)*0.15^2=0.01767m^2$

Height of lower end above the datum, z_2=6m

Rate of flow (that is discharge)

Q=40lit/sec=40/1000 (1litre = 1 m^3/sec) =0.04m^3/sec

Intensity of pressure at section 2, p_2

As the flow is continuous, $Q = A_1V_1= A_2V_2$ (Continuity equation)

Therefore, $V_1= Q/A_1 = 0.04/0.0707 \quad = 0.566$m/sec

And $\qquad V_2= Q/A_2 = 0.04/0.01767 = 2.264$m/sec

Apply **Bernoulli's equation** at sections 1 & 2,

We get, $\mathbf{p_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2}$

And $p_2/w = p_1/w + (v_1^2-v_2^2/2g) + z_1-z_2$

$= (400/9.81) + 1/ (2*9.81)*(0.566^2-2.264^2)+(10-6)$

$= 40.77- 0.245+4 \quad$ (as $w =\rho.g = 1000 \times 9.81$ N/m^3)

$= 44.525$m $= 9.81$ kN/m^3

$P_2 = 44.525 * w = 44.525 * 9.81 =$ **436.8 kN/m^2**

4) Water is flowing through a taper pipe of length 100 m, having diameter 600mm and 300mm at the upper end and lower end respectively, at the rate of 50 lit/s. the pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2.

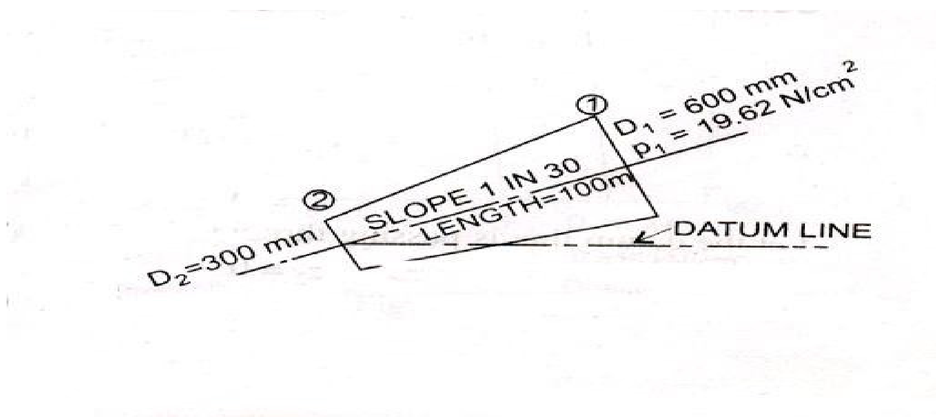


Fig. 4

Solution:

Pipe length L= 100 m

Dia. At the upper end, D_1=600mm =0.6m

$A_1=\pi/4 \times D_1^2 = 0.2827$ m^2

P_1= pr. At the upper end =19.62 N/ cm^2 =19.62 x10^4 N/m^2

Dia. At the lower end, D_2=300mm =0.3m

$A_2=\pi/4 \times D_2^2 = 0.07068\ m^2$

Rate of flow, Q =50 lit/s, Q=50/1000=0.05 m^3/s

Let the datum line is passing through the centre of the lower end, Then $z_2=0$

As slope is 1 in 30 means $z_1=1/30 \times 100 = 10/3$ m

$Q= A_1V_1=A_2V_2$

$V_1=0.05/A_1 =0.1768=0.177$ m/s

$V_2=0.05/A_2=0.7074 =0.707$ m/s

Applying Bernoulli's equation at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + z_2$

$19.62 \times 10^4/1000 \times 9.81 + 0.177^2/2 \times 9.81 + 3.334= P_2/\rho g + 0.707^2/2 \times 9.81+ 0$

$20 + 0.001596 + 3.334= P_2/\rho g + 0.0254$

$23.335 - 0.0254 = P_2/1000 \times 9.81$

$P_2 = 228573\ N/m^2$

= **22.857 N/cm²**

5) A pipe 200m long slopes down at 1 in 100 and tapers from 600mm diameter at the higher end to 300mm diameter at the lower end, and carries 100 lit/sec of oil (specific gravity 0.8). If the pressure gauge at the higher end reads 60 kN/m^2. Determine,

i. Velocities at both ends.

ii. Pressure at the lower end. Neglect the losses

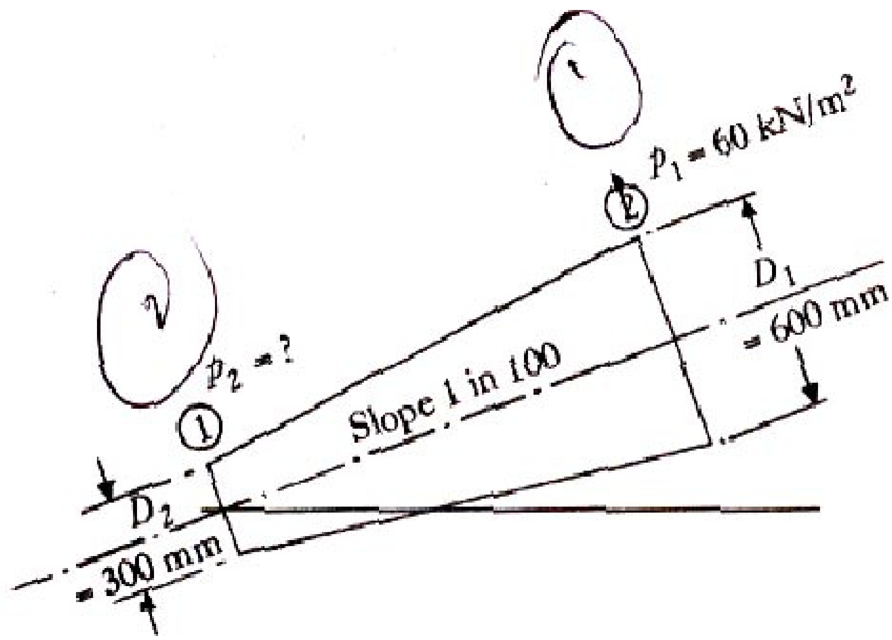


Fig. 5

Solution:

Given: Length of the pipe, l= 200mm

Diameter of the pipe at the higher end,

D_1 =600mm= 0.6m, Area, $A_1 = (\pi/4)*0.6^2 = 0.283\ m^2$

Diameter of the pipe at the lower end,

D_2 =300mm= 0.3m, Area, $A_2 = (\pi/4)*0.3^2 = 0.0707\ m^2$

Height of the lower end, above datum $Z_2 = 0$

Rate of oil flow, Q=100lit/sec= 0.1 m^3/sec

Pressure at the higher end, p_1 =60kN/m^2

(i) Velocities, V_1 & V_2

Now $Q = A_1V_1 = A_2V_2$

Where V_1 & V_2 are the velocities at the higher and lower side respectively.

$V_1 = Q/A_1 = 0.1/0.283 = 0.353$m/sec

$V_2 = Q/A_2 = 0.1/0.0707 = 1.414$m/sec, and

(ii) Pressure at the lower end, p_2

Using Bernoulli's equation for both ends of pipe, we have

$p_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2$

$60/(0.8*9.81)+0.353^2/(2*9.81)+2=p_2/(0.8*9.81)+(1.414^2/2*9.81)+0$

$p_2/(0.8*9.81) = 9.54$,

Pressure at lower end, $p_2 = 74.8$ kN/m^2

6) Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and at upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.

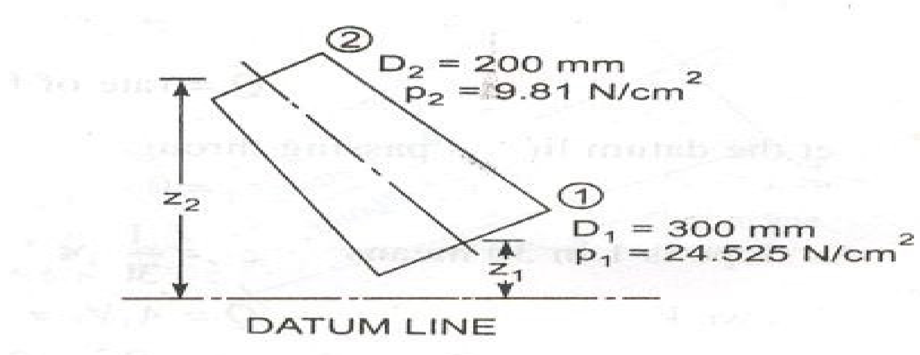


Fig. 6

Solution:

At Section (1), D_1=300mm =0.3m

P_1=24.525 N/cm^2 =24.525 x 10^4 N/m^2

Rate of flow = 40 lit/sec, Q=40/1000=0.04 m^3/s

Now $A_1V_1=A_2V_2=0.04$

$V_1=0.04/A_1=0.5658$ m/s; $V_2=0.04/A_2=1.274$ m/s

Applying Bernoulli's eqn. at (1) and (2) we get

$P_1/ \rho g + V_1^2/ 2g + z_1 = P_2/ \rho g + V_2^2/ 2g + z_2$

$24.525 \times 10^4/1000 \times 9.81 + 0.566 \times 0.566 /2 \times 9.81 + z_1$

$= 9.81 \times 10^4/1000 \times 9.81 + 1.274^2/2 \times 9.81 + z_2$

$25 + 0.32 + z_1 = 10 + 1.623 + z_2$

$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70$ m,

Difference in head, $z_2 - z_1$ = **13.70 m**

7) A non-uniform part of a pipe line 5 m long is laid at a slope of 2 in 5. Two pressure gauges each fitted at upper and lower ends read 20 N/cm^2 and 12.5 N/ cm^2. If the diameters at the upper end and lower end are 15 cm 10 cm respectively. Determine the quantity of water flowing per second.

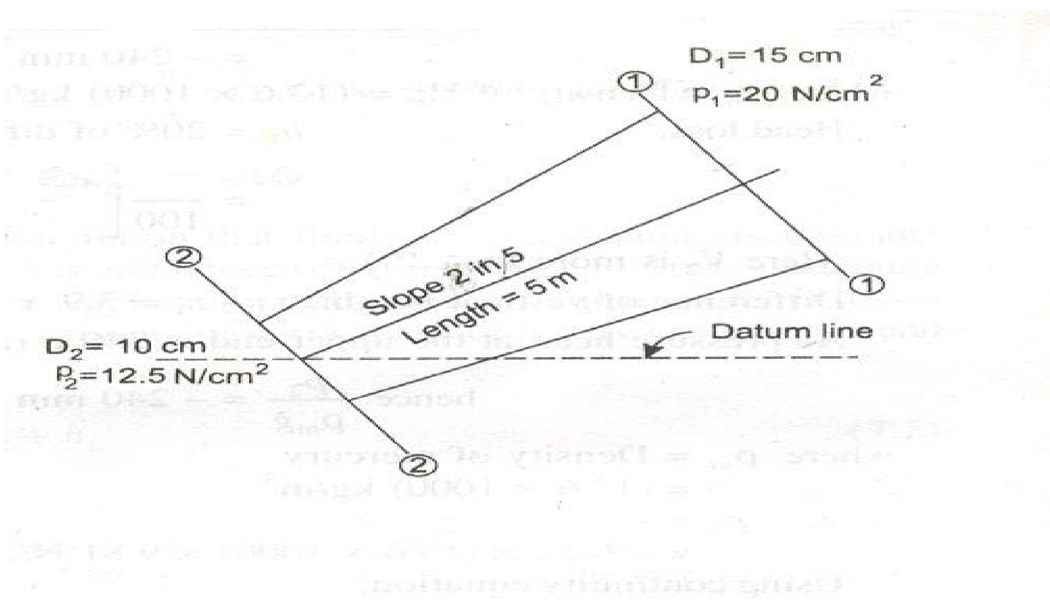


Fig.7

Solution:

L= 5 m, D_1=15cm =0.15m

$A_1 = \pi/4 \times D_1^2 = 0.01767$ m^2

P_1 =20 N/ cm^2 =20x10^4 N/m^2 ,

P_2=12.5N/ cm^2 =12.5x10^4 N/m^2

Dia. At the lower end, D_2=300mm =0.3m

$A_2=\pi/4 \times D_2^2 = 0.00785\ m^2$

Let the datum line is passing through the centre of the lower end

Then $z_2=0$

As slope is 2 in 5 hence, $z_1=2/5 \times 5 = 2$ m

$Q= A_1V_1=A_2V_2$

$V_1 = A_2V_2/A_1 = 0.00785 \times V_2/0.01767$

$V_1 = 0.444\ V_2$

Applying Bernoulli's eqn. at (1) and (2), we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + z_2$

$7.645 + 2 = V_2^2/2g \times 0.8028$

V_2=15.35 m/s

Discharge, $Q= A_2V_2 = 0.00785 \times 15.35 = 0.1205\ m^3/s$

Q = **120.5 lit/s**

<u>Problems on Bernoulli's Eqn. for real fluid:</u>

1) A pipe line carrying oil (specific gravity of 0.8) changes in diameter from 300 mm at position 1 to 600 mm diameter at poison 2, which is 5m at a higher level. If the pressure at position 1 and 2 are 100 kN/m^2 and 60 kN/m^2 respectively and the discharge is 300 lit/sec, determine,

(a) Loss of head, and

(b) Direction of flow.

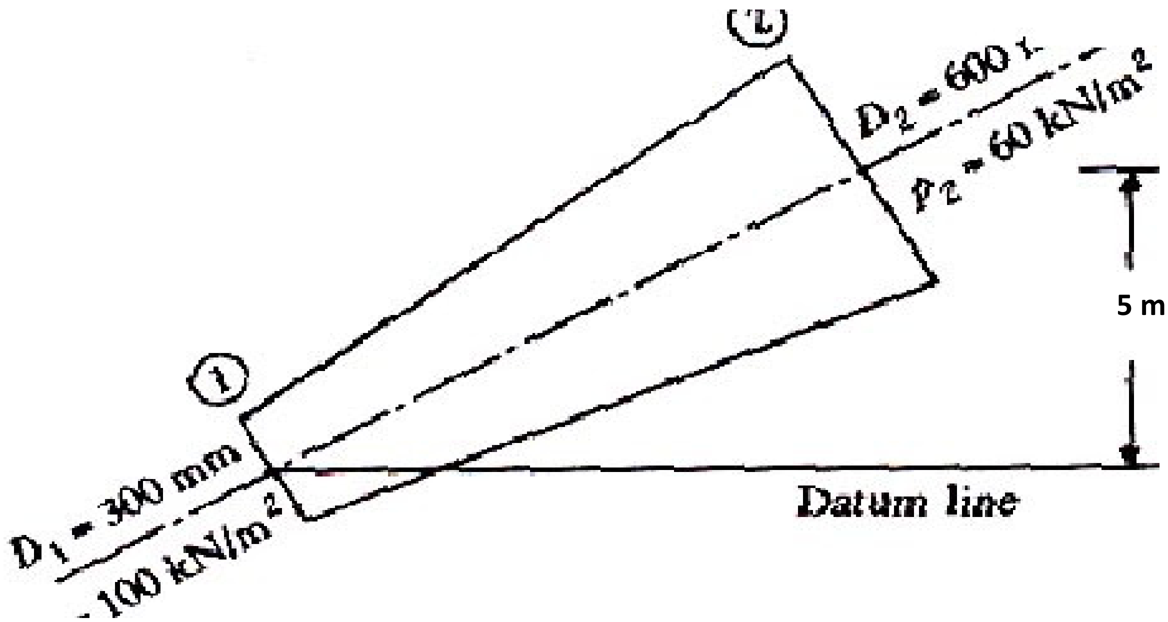


Fig.1

Solution:

Discharge Q= 300 lit/sec = 300/1000= 0.3 m^3/sec

Specific gravity of oil= 0.8

Weight of oil, W_{oil} = 0.8*9.81= 7.85 kN/m^3

At position 1:

Dia of pipe, D_1= 300 mm= 0.3 m

Therefore area of pipe, A_1= $(\pi/4)*(0.3)^2$ = 0.0707 m^2

Pressure at 1, p_1= 100 kN/m^3

If the datum line passes through section 1,

Then Z_1=0

Velocity, V_1= (Q/A_1) = (0.3/0.0707)

V_1= 4.24 m/sec

At position 2

Dia of pipe, D_2= 600 mm= 0.6 m

Therefore area of pipe, A_2= $(\pi/4)*(0.6)^2$ = 0.2828 m^2

Pressure, $p_2 = 60\ kN/m^2$

Datum, Z_2=5m

Velocity, V_2= (Q/A_2) = (0.3/0.2828)= **1.06 m/sec**

(a) <u>Loss of head, h_L</u>

Total energy at position 1,

$E_1 = (p_1/W) + (V_1^2/2g) + Z_1$

$E_1 = (100/7.85) + (4.24^2/2*9.81)+0$

$\mathbf{E_1 = 12.74+0.92 = 13.66m}$

Total energy at position 2,

$E_2 = (p_2/W) + (V_2^2/2g) + Z_2$

$E_2 = (60/7.85) + (1.06^2/2*9.81) + 5 = 7.64+0.06+5$

$\mathbf{E_2 = 12.76m}$

Therefore **loss of head,**

$$h_L = E_1 - E_2 = 13.66-12.76 = 0.9m$$

(b) <u>Direction of flow</u>

Since $E_1 > E_2$, therefore flow taken place from position 1 to position 2

2) A conical tube length 3m is fixed vertically with its small end upwards. The velocity of flow at the smaller end is 10 m/sec. The pressure head at the smaller end is 4m of liquid. The loss of head in the fluid in the tube is $0.4(V_1-V_2)^2/2g$, where V_1 is the velocity at the smaller end and V_2 at the lower/larger end respectively. Determine the pressure head at lower (larger) end.

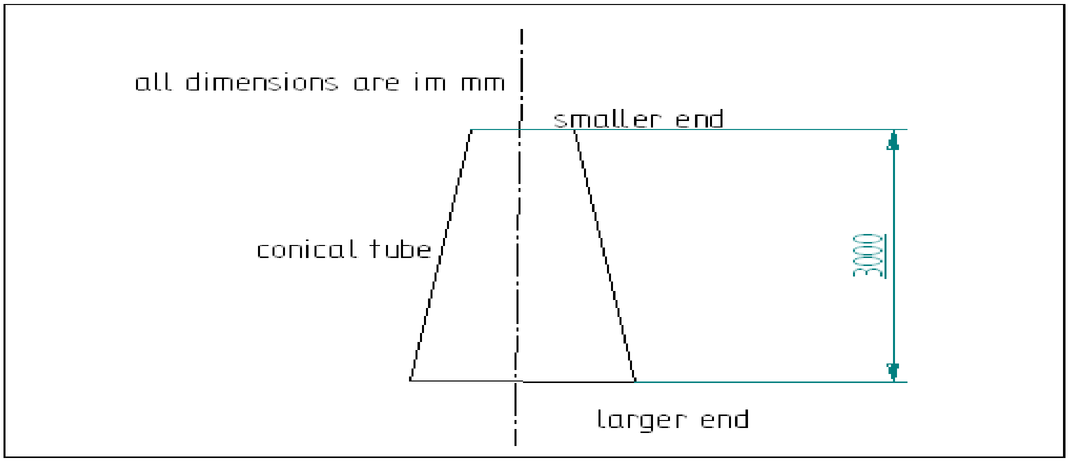


Fig. 2

Given: Length of the tube, $l = 3m$

Velocity, $V_1 = 10$ m/sec

Pressure head, $p_1/w = 4m$ of fluid

Velocity, $V_2 = 4$ m/sec

Solution: Loss of head, $h_L = 0.4(V_1\text{-}V_2)^2/2g$

$= 0.4(10\text{-}4)^2/2*9.81$

$\mathbf{h_L = 0.73\ m}$

<u>Pressure head at the larger end</u>, (p_2/w)

Applying Bernoulli's equation at sections 1 & 2 we get

$(p_1/w) + (V_1^2/2g) + (Z_1) = (p_2/w) + (V_2^2/2g) + (Z_2) + h_2$

Let the datum line through section 2

Then $Z_2=0$, $Z_1=3m$

$4 + (10^2/2g) + 3 = (p_2/w) + 0 + 0.73 + 0.815$

$4 + 5.09 + 3 = (p_2/w) + 0.815 +0.73$

Pressure head $(p_2/w) = 10.55$ m of fluid

3) A conical tube of length 2 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2m/s. the pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is $0.35(v_1 - v_2)^2/2g$, where V_1 is the velocity at smaller end and V_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Solution:

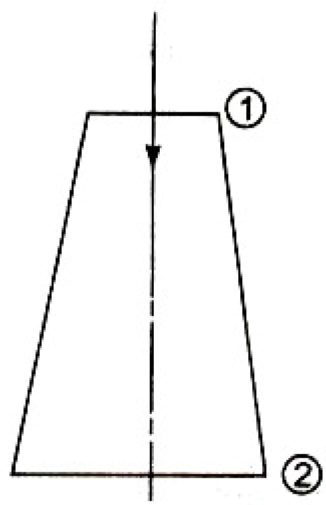


Fig. 3

Length of the tube, L=2m

$V_1 = 5$ m/s

$P_1/\rho g = 2.5$ m of liquid

$V_2 = 2$ m/s

Loss of head = $h_L = 0.35(v_1 - v_2)^2/2g$

$= 0.35(5 - 2)^2/2g = 0.35 \times 9/ 2 \times 9.81$

$= 0.16$ m

Pressure head = $P_2/\rho g$

Applying Bernoulli's equation at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + h_L$

Let the datum line passes through section (2). Then

$z_1 = 2,\ z_2 = 0$

$2.5 + 5^2/2 \times 9.81 + 2 = P_2/\rho g + 2^2/2 \times 9.81 + 0.16 + 0$

$2.5 + 1.27 + 2 = P_2/\rho g + 0.203 + 0.16$

$P_2/\rho g = 5.77 - 0.363$

= 5.047 m of fluid

4) A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 liters/s, determine the loss of the head and the direction of the flow.

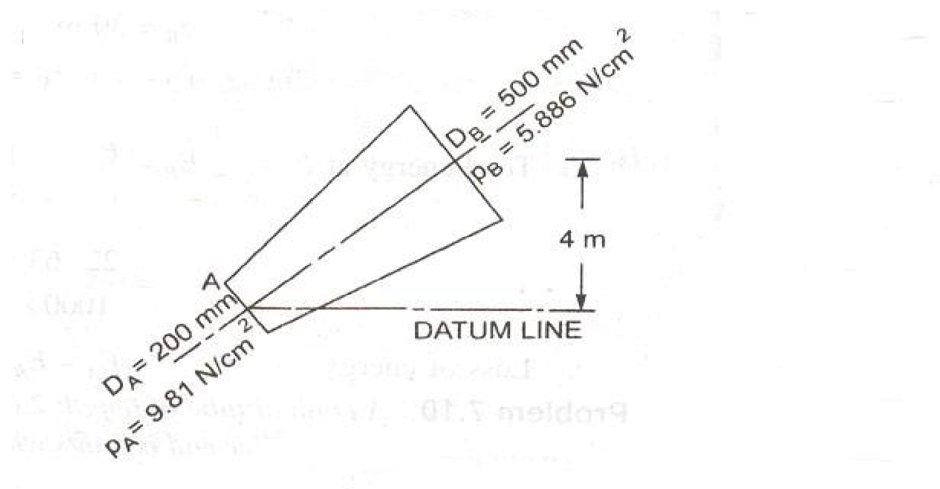


Fig. 4

Solution:

Given, Discharge, $Q = 200$ liters/s $= 0.2\ m^3/s$

Specific gravity of oil = 0.87

ρg for oil $= 0.87 \times 1000 = 870\ kg/m^3$

At Section A, $D_A = 200$ mm $= 0.2$ m

Area, $A_A = \pi/4\ (0.2)^2 = 0.0314\ m^2$

$P_A = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2$ If the Datum line is passing through A, then $Z_A = 0$

$V_A = Q/A_A = 0.2/0.0314 = 6.369$ m/s

At section B, $D_B = 500$ mm = 0.5 m

Area, $A_B = \pi/4\ (0.5)^2 = 0.1963\ m^2$

$P_B = 5.886\ N/cm^2 = 5.886 \times 10^4\ N/m^2$

$Z_B = 4$ m

V_B= Q/ Area = 0.2/0.1963 = 1.018 m/s

Total energy at A, $E_A = P_A/\rho g + V_A^2/2g + z_A$

$= 11.49+2.067+0$

$= 13.557$ m

Total energy at B, $E_B = P_B/\rho g + V_B^2/2g + z_B$

$= 6.896+0.052+4$

$= 10.948$ m

Direction of flow. As E_A is more than E_B and hence flow is taking place from A to B.

Loss of head = $h_L = E_A - E_B =$ **2.609 m**

5) A pump has a tapering pipe running full of water. The pipe is placed vertically with the diameters at the base and the base and top being 1.2 m and 0.6 m respectively. The pressure at the upper end is 240 mm of Hg vacuum, while the pressure at the lower end is $15 kN/m^2$. Assume the head loss to be 20 percent of difference of velocity head. Calculate the discharge, the flow is vertically upwards and difference of elevation is 3.9 m.

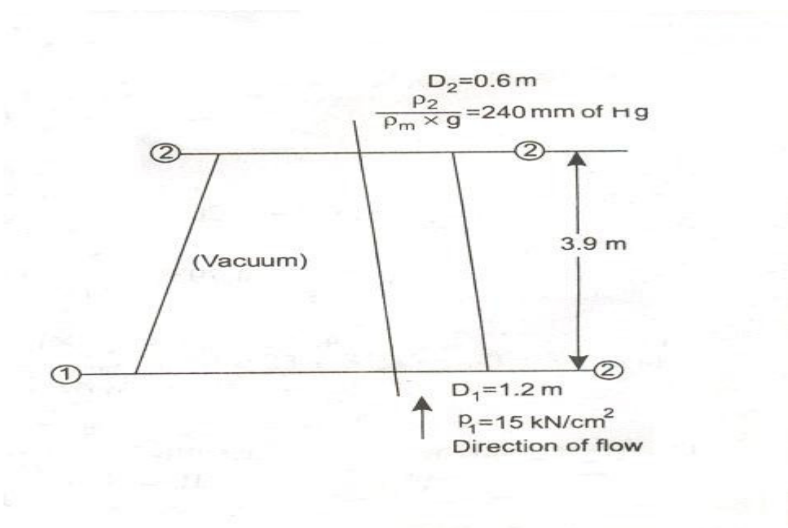


Fig.5

D_1=1.2m, D_2=0.6m

P_1 =15 kN/m^2 =15 x 1000 N/m^2,

P_2/ ρg=240 mm of Hg =0.24 m of Hg

ρ_m=density of Hg =(13.6 x 1000) kg/m^3

Head loss

h_L =20/100 of difference of velocity head,

$= 0.2(V_2^2 – V_1^2) /2g$

Difference of vertical height z_2- z_1=3.9 m

Pressure head at upper end is 240 mm of Hg

Hence P_2/ ρ_mg = -0.24 m of Hg

P_2= -0.24 x 13.6 x 1000 x 9.81

= -32019.8 N/m^2

Using *continuity equation*

$A_1V_1=A_2V_2$

$V_2= A_2V_2/A_{2,} = (D_1 /D_2)^2$ x V_1 = 4 V_1

Applying Bernoulli's eqn. at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + h_L$

$V_2^2/2g - V_1^2/2g + 3.9 + 0.2(V_2^2 - V_1^2/2g$

$1.529+3.264=1.2(V_2^2 - V_1^2)/2g + 3.9$

$4.793 = 1.2((4V_1)^2 - V_1^2)/2g + 3.9$

$0.893 = 9V_1^2/g$

$V_1 = 0.9865$ m/s

Discharge $Q=A_1V_1$ = **1.1157 m^3/s**

Practical applications of Bernoulli's equation:

Although Bernoulli's equation is applicable in all problems of incompressible flow where there is involvement of energy considerations. But we shall consider its application to the following measuring devices.

1) Venturimeter
2) Orifice meter
3) Pitot tube

Differential Pressure Flow Meters

Differential pressure flow meters all infer the flow rate from a pressure drop across a restriction in the pipe. For many years, they were the only reliable methods available, and they remain popular despite the development of higher performance modern devices, mostly on account of exceptionally well researched and documented standards.

The analysis of the flow through a restriction (Fig.1) begins with assuming straight, parallel stream lines at cross sections 1 and 2, and the absence of energy losses along the streamline from point 1 to point 2.

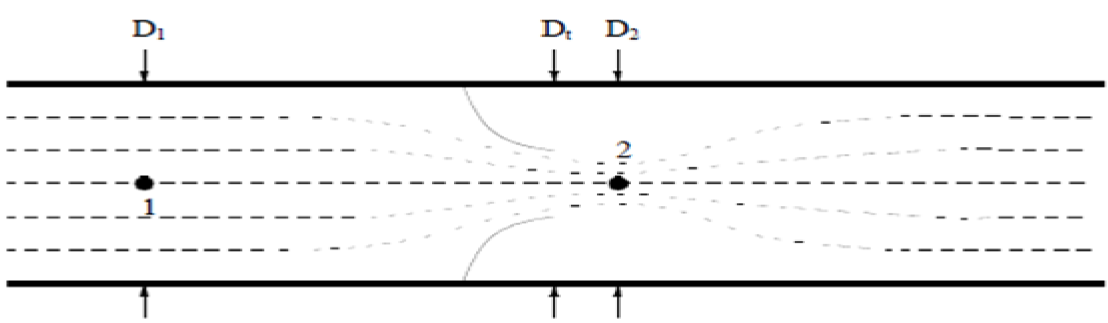


Fig. 1: A generalized restriction/differential pressure flow meter

The objective is to measure the mass flow rate, $\dot{m}$ ***By eqn. continuity***

$\dot{m} = \rho v_1 a_1 = \rho v_2 a_2.$

Bernoulli's equation may now be applied to a streamline down the centre of the pipe from a point 1 well upstream of the restriction to point 2 in the ***vena contracta*** *of the jet immediately* downstream of the restriction where the streamlines are parallel and the pressure across the duct may therefore be taken to be uniform:

Unit 5: Fluid Flow Measurements

Introduction:

Fluid flow measurements means the measuring the rate of flow of a fluid flowing through a pipe or through an open channel. The rate of flow of a fluid through a pipe is measured by four main restriction devices are.

- Venturimeter
- Orifice meter
- Pitot tube
- flow nozzle

Whereas through an open channel the rate of flow is measured by

- Notches
- weirs

The **Venturi effect** is the reduction in fluid pressure that results when a fluid flows through a constricted section of pipe. The Venturi effect is named after Giovanni Battista Venturi (1746–1822), an Italian physicist.

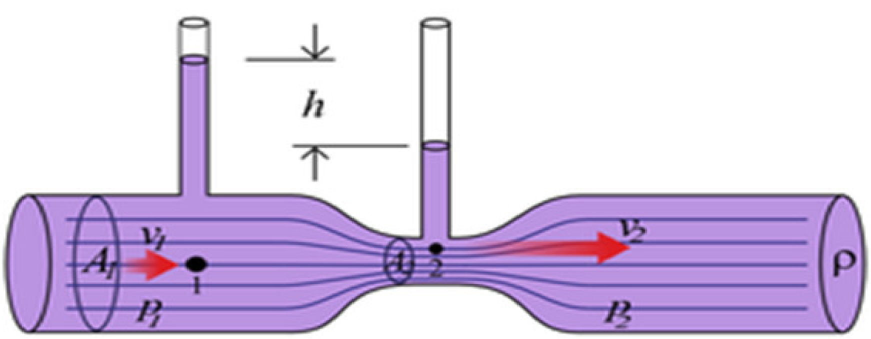


Fig. 1: Venturi effect

The pressure at "1" is higher than at "2" because the fluid speed at "1" is lower than at "2".The **Venturi effect may be observed or used in the following:**

- Inspirators that mix air and flammable gas in grills, gas stoves,

  Bunsen burners and airbrushes

- Atomizers that disperse perfume or spray paint (i.e. from a spray gun).
- Carburetors that use the effect to suck gasoline into an engine's intake air stream
- The capillaries of the human circulatory system, where it indicates aortic regurgitation
- Aortic insufficiency is a chronic heart condition that occurs when the aortic valve's initial large stroke volume is released and the Venturi effect draws the walls together, which obstructs blood flow, which leads to a Pulsus Bisferiens.
- Cargo Eductors on Oil, Product and Chemical ship tankers

- Protein skimmers (filtration devices for saltwater aquaria)
- Compressed air operated industrial vacuum cleaners
- Venturi scrubbers used to clean flue gas emissions
- Injectors (also called ejectors) used to add chlorine gas to water treatment chlorination systems
- Sand blasters used to draw fine sand in and mix it with air
- A scuba diving regulator to assist the flow of air once it starts flowing
- In Venturi masks used in medical oxygen therapy
- In recoilless rifles to decrease the recoil of firing
- Wine aerators, to aerate wine, putatively improving the taste.
- Ventilators
- The diffuser on an automobile

The **main advantages of the Venturimeter over the orifice plate** are:

- Low head loss
- Less affected by upstream flow disturbance
- Good performance at higher β

• Even more robust

• Self-cleaning

• Less affected by erosion

The **disadvantages compared to the orifice** are

• Occupies longer length of pipe

• More expensive (manufacture and installation)

The simplest apparatus, built out of PVC pipe as shown in the photograph is a tubular setup known as a Venturi tube or simply a venturi. Fluid flows through a length of pipe of varying diameter.

Fig. 2: Venturimeter - Experimental apparatus

To avoid undue drag, a Venturi tube typically has an entry cone of 21 to 30 degrees and an exit cone of 5 to 15 degrees. To account for the assumption of an in viscid fluid a coefficient of discharge is often introduced, which generally has a value of 0.98. A venturi can be used to measure the volumetric flow rate Q.

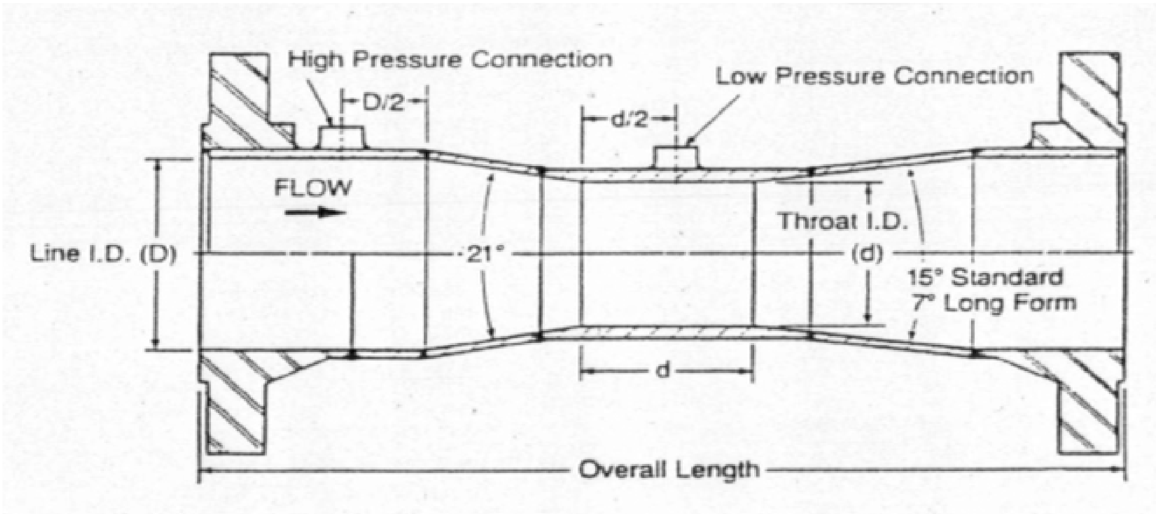


Fig 3 : Venturimeter (Furness 1989)

The fluid velocity must increase through the constriction to satisfy the equation of continuity, while its pressure must decrease due to conservation of energy: the gain in kinetic energy is balanced by a drop in pressure or a pressure gradient force. An equation for the drop in pressure due to the Venturi effect may be derived from a combination of Bernoulli's principle and the equation of continuity.

Expression for rate of flow through venturimeter :

Consider a venturimeter fixed in a horizontal pipe through

which a fluid is flowing (say water) as shown in figure 4.

Let d_1 = diameter at inlet or at section 1

p_1 =Pressure at section 1

v_1 = velocity of fluid at section 1

a_1 = area at section 1= $(\pi/4)*d_1^2$

And d_2, p_2, v_2, a_2 are corresponding values at section 2

Applying Bernoulli's equation at section 1 & 2 we get

$(p_1/\rho g) + (v_1^2/2g) + (z_1) = (p_2/\rho g) + (v_2^2/2g) + (z_2)$

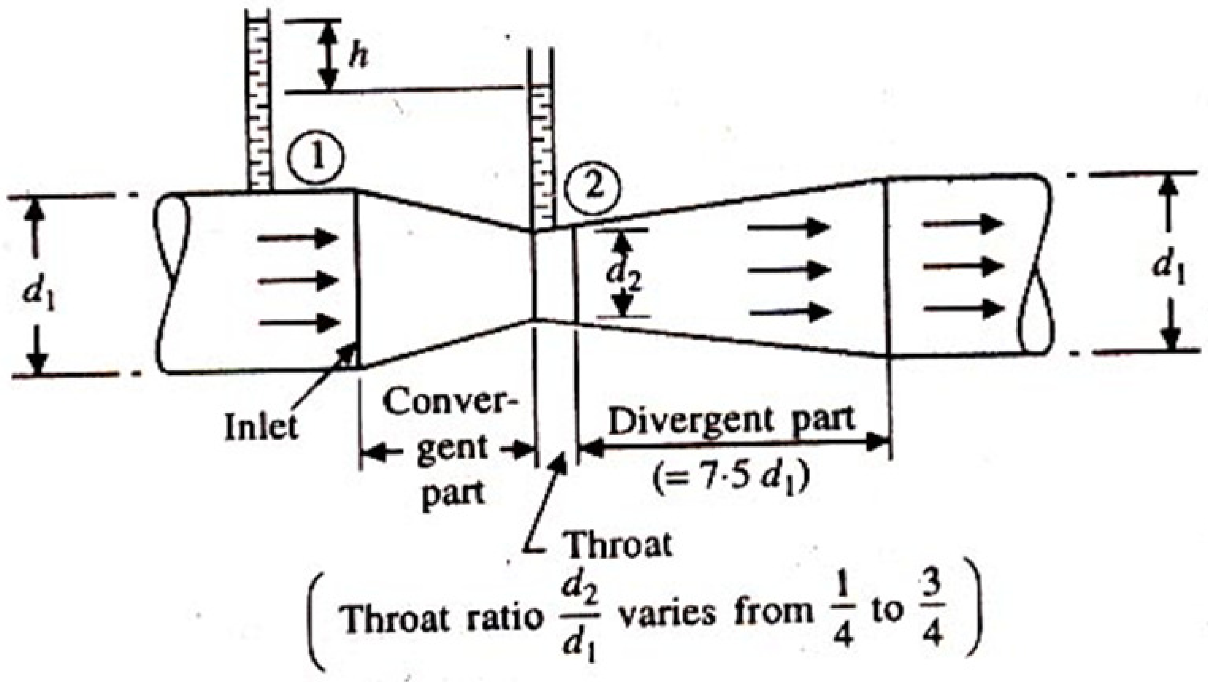


Fig. 4 Typical venturimeter

As pipe is horizontal, hence $z_1=z_2$

$(p_1/\rho g) + (v_1^2/2g) = (p_2/\rho g) + (v_2^2/2g)$ or

$(p_1-p_2)/\rho g = (v_2^2/2g) - (v_1^2/2g)$ ---- (1)

But $(p_1-p_2)/\rho g$, is the difference of pressure head at sections 1 & 2 and it is equal to 'h' or

$(p_1-p_2)/\rho g = h$

Substituting the value of $(p_1-p_2)/\rho g$ in the above eqn. (1) we

Get, $h = (v_2^2/2g) - (v_1^2/2g)$ ---- (2)

now applying **continuous equation at sections 1 & 2**

$a_1v_1= a_2v_2$ or $v_1 = (a_2v_2)/a_1$

substitute the value of v_1 in equation (2)

$h= (v_2^2/2g) - [(a_2v_2/a_1)^2/2g] = (v_2^2/2g)[1-(a_2^2/a_1^2)]$

$= (v_2^2/2g)[(a_1^2- a_2^2)/a_1^2]$

$v_2^2 =2gh\ [a_1^2/(a_1^2- a_2^2)]$

Therefore $v_2 = \sqrt{[2gh \{a_1^2 / (a_1^2 - a_2^2)\}]}$

$$v_2 = [a_1 / \sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}$$

Discharge $Q = a_2 v_2$

$Q_{th} = a_2 * [a_1 / \sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}$ ---- (3)

Equation (3) gives the discharge under ideal conditions and is called theoretical discharge. Actual discharge will be less than theoretical discharge.

$$Q_{act} = C_d [a_1 a_2 / \sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}]$$

Where C_d is coefficient of venturimeter and its value is less than 1.

<u>Value of 'h' by differential 'U' tube manometer</u>

Case 1:

Let the differential manometer contains a liquid which is **heavier than** the liquid flowing through a pipe.

Let

s_h=specific gravity of the heavier liquid.

s_p=specific gravity of the liquid flowing through pipe.

x=difference of the heavier liquid column in U-tube.

$$h = x[(s_h/s_p) - 1]$$

Case 2:

If the differential manometer contains a liquid which is **lighter than** the liquid flowing through the pipe, the value of 'h' is given by

$$h = x[1 - (s_L/s_p)]$$

Where

s_L=sp gravity of lighter liquid in U-tube.

s_p=sp gravity of the liquid flowing through pipe

x= difference of the lighter liquid column in U-tube.

Case 3:

Inclined venturimeter with differential U-tube manometer **(heavier liquid)**

$h = (p_1/\rho g + z_1) - (p_2/\rho g + z_2) = x[(s_h/s_p)-1]$

Case 4:

Inclined venturimeter with differential U-tube manometer**(lighter liquid)**

$h = (p_1/\rho g + z_1) - (p_2/\rho g + z_2) = x[1-(s_L/s_p)]$

Problems on Horizontal Venturimeter:

1) A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the throat and inlet is 20cm of mercury. Determine the rate of flow. Take C_d=0.98.

Solution:

Given:

Dia at inlet, d_1 =30cm, Area at inlet, $a_1 = (\pi d_1^2)/4$

$= (\pi 30^2)/4 = 706.85 cm^2$

Dia at throat, d_2 =15cm, Area at throat, $a_2 = (\pi 15^2)/4 = 176.7 cm^2$

$C_d = 0.98$

Reading of differential manometer x = 20cm of mercury

Therefore difference of pressure head is given by

$h = x [(s_h/s_w)-1]$

Where s_h=specific gravity of mercury =13.6,

s_w=specific gravity of water(assumed) =1

h =20[(13.6/1)-1] =20*12.6cm =252.0 cm of water.

The discharge through venturimeter is given by

$Q = C_d*(a_1 a_2/(\sqrt{a_1^2 - a_2^2}))*(\sqrt{2gh})$

$= 0.98*(706.85*176.7/(\sqrt{706.85^2 176.7^2}))*(\sqrt{2*9.81*25})$

$= 86067593.36/(\sqrt{499636.9-31222.9})$

$= 86067593.36/684.4$

$= 125756 cm^3/s = 125756 lit/s$

Q = 125.756 lit./s

2) An oil of specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat diameter 10cm. The oil(s_o = 0.8)-mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimeter. Take C_d=0.98.

Solution:

Given:

Specific gravity of oil, $s_o = 0.8$

Specific gravity of mercury $s_h = 13.6$

Reading of differential manometer $x = 25cm$

Therefore difference of pressure head, $h = x\,[(s_h/s_o) - 1]$

$= 25[(13.6/0.8) - 1]$ cm of oil $= 25[17-1] = 400$ cm of oil.

Dia at inlet, $d_1 = 20cm$

Area at inlet, $a_1 = (\pi d_1^2)/4 = (\pi 20^2)/4 = 314.16 cm^2$

Similarly at throat, $d_2 = 10cm$

$a_2 = (\pi 10^2)/4 = 78.54 cm^2$

$C_d = 0.98$ (given)

Therefore **discharge Q** is given by

$Q = C_d*(a_1 a_2/(\sqrt{a_1^2 - a_2^2}))*(\sqrt{2gh})$

$= 0.98*(314.16*78.54/(\sqrt{314.16^2 - 78.54^2}))*(\sqrt{2*9.81*400})$

$= 21421375.68/ (\sqrt{98696-6168})$

$= 21421375.68/304\ cm^3/s$

$= 70465 cm^3/s$

Q =**70.465 lit/s**

3) A venturimeter is to be fitted in a pipe of 0.25 m dia. where the pressure head is 7.6 m of flowing liquid and max. flow is 8.1 m^3/min. Find the least dia. of the throat to ensure that the pressure head does not become negative, $c_d = 0.96$.

Solution:

$Q = c_d((a_1a_2)/(\sqrt{a_1^2-a_2^2}))*\sqrt{(2gh)}$

$Q = (8.1/60) = 0.135\ m^3/s$, $c_d = 0.96$

$a_1 = (\pi/4)*(0.25)^2 = 0.049\ m^2$

$h = 7.6$ m

$0.135 = 0.96*((0.049*a_2)/(\sqrt{0.049^2-a_2^2}))*\sqrt{(2*9.81*7.6)}$

$a_2 =$ **$0.0112\ m^2$**

4) A venturimeter is used for measurement of discharge of water in a horizontal pipe line, if the ratio of upstream pipe diameter to that of throat is 2:1, upstream diameter is 300mm, the difference of pressure between the throat is equal to 3m head of water and loss of head through meter is one eighth of the throat velocity head, calculate discharge in pipe

Solution:

Given:

Ratio of inlet dia to throat i.e., $d_1/d_2=2$

d_1=300mm=0.3m

d_2=300/2=150mm=0.15m

$(p_1/\rho g - p_2/\rho g)$=3m of water ,

loss of head, h_f=1/8 of throat velocity head $=1/8*v_2^2/2g$

Using continuity equation

Using **Bernoulli's equation** at inlet and throat, we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2+h_f$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \frac{1}{8} \times \frac{v_2^2}{2g}$$

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + \frac{1}{8} \times \frac{v_2^2}{2g}$$

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 3 \text{ m}$$

$$3 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + \frac{1}{8}\frac{v_2^2}{2g}$$

$$3 = \frac{16v_1^2}{2g} - \frac{v_1^2}{2g} + \frac{2v_1^2}{2g} = \frac{17v_1^2}{2g}$$

$$v_1 = \sqrt{\frac{3 \times 2 \times 9.81}{17}} = 1.86 \text{m/s}$$

$$Q = \left(\frac{\pi}{4} d_1^2\right) \times v_1$$

$$Q = \left(\frac{\pi}{4} \times [.3]^2\right) \times 1.86 = .0.1315 \text{m}^3/\text{sec}$$

$$Q = 131.5 \text{lit/sec}$$

5) A horizontal venturimeter with inlet diameter 200mm and throat diameter 100mm is employed to measure the flow of water. The reading of the differential manometer connected to the inlet is 180mm of Hg. If the coefficient of discharge is 0.98, determine the rate of flow.

Solution:

Inlet dia of venturimeter, D_1= 200mm= 0.2m

Therefore, area of inlet, $A_1= (\pi/4)*0.2^2$

Throat dia D_2= 100mm= 0.1m

Area of throat, $A_2= (\pi/4)*0.1^2$

Reading of differential manometer, x = 180mm

= 0.18m of Hg

Coefficient of discharge, C_d =0.98

<u>Rate of flow</u>, Q

To find the difference of pressure head (h), we have

$$h = x[(s_h/s_p)-1]$$

$h = 0.18[(13.6/1) -1]= 2.268$ m

To find 'Q' using this relation

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$Q = 0.98[0.0314*0.00785 /\sqrt{(0.0314^2-0.00785^2)}]* \sqrt{(2*9.81*2.268)}$

$Q= (0.000241*6.67)/0.0304$

Q = **0.0528 m^3/sec**

6) A venturimeter having a diameter of 75mm at throat and 150mm dia at the enlarged end is installed in a horizontal pipeline 150mm in dia carrying an oil of specific gravity 0.9. The difference of pressure head between the enlarged end and the throat recorded by a U-tube is 175mm of mercury. Determine the discharge through the pipe. Assume the coefficient of discharge of the water as 0.97.

Solution:

The discharge through the venturimeter is given by

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$C_d =0.97$

$d_1 =150mm= 0.15m$

$a_1 =(\pi/4)*0.15^2 = 0.0177m^2$

$d_2 =75mm= 0.075m$

$a_2 = (\pi/4)*0.075^2 = 0.0044m^2$

$x= 175mm = 0.175m$

$h =x[(s_h/s_p) -1] = 0.175[(13.6/0.9)-1]$

$= 2.469m$

by substitution, we get

$Q=0.97[0.0177*0.0044 /\sqrt{(0.0177^2- 0.0044^2)}]*\sqrt{(2*9.81*2.469)}$

$Q= 0.03067\ m^3/sec$

$=$ **30.67 lit/sec**

7) A horizontal venturimeter with inlet diameter 20cm and throat dia 10cm is used to measure the flow of oil of specific gravity 0.8. The discharge of the oil through venturimeter is 60 lit/sec. Find the reading of the oil-Hg differential manometer, take C_d= 0.98.

Solution:

At entry, d_1=20cm

$a_1= (\pi/4)*20^2 = 314.16\ cm^2$

At throat, d_2 = 10cm

$a_2 = (\pi/4)*10^2 = 78.54\ cm^2$

$C_d = 0.98$

$Q = 60$ lit/sec= $60*1000\ cm^3/sec$

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$60*1000 = 0.98[314.16*78.54/\sqrt{(314.16^2-78.54^2)}]*\sqrt{(2*9.81*h)}$

$\sqrt{h}$= 17.029, **h= 289.98 cm of oil**

To calculate reading of the oil-Hg differential manometer

we have

$h =x[(s_h/s_p)-1]$

Where s_h = 13.6 –> sp. gravity of the mercury

s_p = 0.8 --> sp. gravity of the oil

$x = ?$

x= **18.12 cm**

8) A horizontal venturimeter with inlet*throat diameter 300mm and 100mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130 kN/m^2, while the vacuum pressure head at the throat is 350mm of Hg. Assume that 3% of head is lost in between the inlet and throat, find

1) The value of C_d for the venturimeter

2) Rate of flow.

Solution:

Inlet dia of the venturimeter, D_1 = 300mm = 0.3m

Area of inlet, $A_1 = (\pi/4)*0.3^2 = 0.07m^2$

Throat dia, D_2 = 100mm= 0.1m

Area of throat, $A_2 = (\pi/4)*0.1^2 =0.00785m^2$

Pressure at inlet, p_1 =130KN/m^2

Pressure head, p_1/w=130/9.81= 13.25m

Similarly, pressure head at throat

p_2/w = - 350mm of Hg**(vacuum pr. Head)**

= - 0.35*13.6

= - 4.76 m of water

(a) coefficient of discharge, C_d

Differential head, $h = (p_1/w) - (p_2/w) = 13.25 -(- 4.76)$

$h = 18.01m$

head lost, h_f = 3% of h= (3/100)*18.01 = 0.54m

$C_d =\sqrt{[(h- h_f)/h]}= \sqrt{[(18.01-0.54)/18.01]} = 0.985$

(b)Rate of flow, Q

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$Q = 0.985[0.07*0.00785 /\sqrt{(0.07^2- 000785^2)}]*\sqrt{(2*9.81*18.01)}$

Q= **0.146 m^3/sec**

9) The inlet and throat diameter of a horizontal venturimeter are 30cm and 10cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is $13.734N/cm^3$ while the vacuum pressure head at the throat is 37cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.

Solution:

Given:

Dia at inlet, $d_1=30cm$

Area at inlet, $a_1=(\pi d_1^2)/4=(\pi 30^2)/4=706.85cm^2$

Dia at throat, $d_2=10cm$

Area at throat, $a_2=(\pi 10^2)/4=78.54cm^2$

Pressure at entry, $p_1=13.734N/cm^2=13.734*10^4N/m^2$

Therefore pressure head, $p_1/\rho g=13.734*10^4/9.81*1000$

$=14m$ of water

$p_2/\rho g= -37cm$ of mercury

$= (-37*13.6/100)$ m of water

$=-5.032$ m of water

Differential head, $h = p_1/\rho g- p_2/\rho g=14-(-5.032) =14+5.032$

$= 19.032$ m of water

$=1903.2$ cm

Head lost, $h_f= 4\%$ of $h=0.04*19.032=0.7613$ m

$C_d=\sqrt{((h- h_f)/h)} =\sqrt{(19.032-0.7613)/19.032}$

$=0.98$

Therefore discharge

$Q= C_d*(a_1a_2/(\sqrt{a_1^2-a_2^2}))*(\sqrt{2gh})$

$= 0.98*(706.85*78.54/(\sqrt{706.85^2 - 78.54^2}))*(\sqrt{2*9.81*1903.2})$

$= (105132247.8)/\sqrt{(499636.9-6168)}=149692.8cm^3/s$

Q = $\mathbf{0.14969m^3/s}$

10) A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30cm of mercury. Find the discharge of water through venturimeter. Take C_d=0.98

Solution:

Dia at inlet, d_1=20cm

$$a_1=(\pi d_1^2)/4=(\pi 20^2)/4=314.16cm^2$$

Dia at throat,d_2=10cm

$$a_2=(\pi 10^2)/4=78.54cm^2$$

$p_1=17.658\ N/cm^2=17.658*10^4 N/m^2$

density of water = $1000kg/m^3$ and

Therefore, $p_1/\rho g=17.658*10^4/9.81*1000=18m$ of water

$p_2/\rho g= -30cm$ of mercury **(vacuum pr. Head)**

= - 0.30m of mercury

= - 0.30*13.6

= - 4.08 m of water

Therefore **differential head**

$h= p_1/\rho g - p_2/\rho g =18-(-4.08)$

= 18+4.08=22.08 m of water

= 2208 cm of water

The **discharge Q** is given by equation

$Q= C_d*(a_1a_2/(\sqrt{a_1^2-a_2^2}))*(\sqrt{2gh})$

$=0.98*(314.16*78.54/(\sqrt{314.16^2 78.54^2}))*(\sqrt{2*9.81*2208})$

=50328837.21/304

$=1655555\ cm^3/s$

Q =165.55 lit/s

Problems on venturimeter axis vertical/inclined

1) A venturimeter has its axis vertical , the inlet & throat diameter being 150mm & 75mm respectively. The throat is 225mm above inlet and $C_d = 0.96$. Petrol of specific gravity 0.78 flows up through the meter at a rate of 0.029 m^3 /sec. find the pressure difference between the inlet and throat.

Solution:

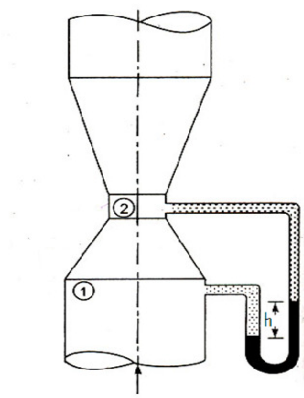


Fig. 1: venturimeter with its axis vertical

The discharge through a venturimeter is given by

$Q = C_d\ [a_1 a_2 /\sqrt{(a_1^2 - a_2^2)}]*\ \sqrt{(2gh)}$

Given:

$C_d = 0.96$

d_1 =150mm= 0.015m

d_2 =75mm= 0.0075m

$a_1 =(\pi/4)*0.015^2 =0.0177m^2$

$a_2 =(\pi/4)*0.0075^2 =0.0044m^2$

$Q = 0.029\ m^3/sec$

By substitution, we have

$0.029 = 0.96[0.0177*0.0044 /\sqrt{(0.0177^2- 0.0044^2)}]*\sqrt{(2*9.81*h)}$

$h= 2.254$m of oil

$h= (p_1/w+z_1) - (p_2/w + z_2)$

$2.254= [(p_1/w)- (p_2/w)]-[z_2-z_1]$

$2.254= [(p_1/w)- (p_2/w)]-[0.225]$

$p_1/\rho g - p_2/\rho g =2.479$

Therefore, $p_1 - p_2= 2.479*0.78*9810$

$= 18969$ N/m^2 =18.969k N/m^2 =18969 Pa

Pr. Difference, $p_1 - p_2$ = 18.96 kPa

2)Determine the rate of flow of water through a pipe of 300mm dia placed in an inclined position where a venturimeter is inserted, having a throat dia of 150mm.The difference of pressure between the main throat is measured by a liquid of specific gravity 0.7 in an inverted u-tube which gives a reading of 260mm.The loss of head the main and throat is 0.3 times the kinetic head of the pipe.

Solution:

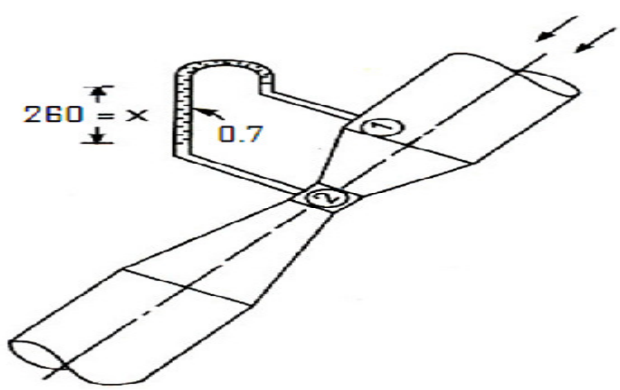


Fig. 2

Given:

Dia of inlet,

D_1=300mm=0.3m

Therefore area of inlet, $A_1= \pi/4*(0.3)^2$=0.07m^2

Throat dia, D_2=150mm=0.15m

Therefore area of throat, $A_2= \pi/4*(0.15)^2=0.01767m^2$

Specific gravity of lighter liquid (u-tube) s_l =0.7

Specific gravity of liquid (water) flowing through pipe,

Reading of differential manometer, x=260mm=0.26m

Difference of pressure head, h is given by

$((p_1/\rho g) +z_1) - (p_2/\rho g) +z_2) =h$

Also, $h= x (1- s_l/s_w) =0.26(1-0.7/1.0)$

$=0.078m$ of H_2O

Loss of head, $h_L=0.3*$kinetic head of pipe $= 0.3 * v_1^2/2g$

Now applying Bernoulli's equation at section '1' and '2',

We get, $(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) +z_2 + (v_2^2/2g) + h_L$

$[(p_1/\rho g) +z_1) - (p_2/\rho g) +z_2]+ [(v_1^2/2g) - (v_2^2/2g)] = h_L$

But $[(p_1/\rho g) +z_1) - (p_2/\rho g) +z_2] =0.078$ m of H_2O

And $h_L =0.3*(v_1^2/2g)$

therefore, $0.078 + [(v_1^2/2g) - (v_2^2/2g)] = 0.3*(v_1^2/2g)$

$0.078 +0.7(v_1^2/2g) - (v_2^2/2g) = 0$ ---- (1)

Applying **continuity equation on section (1) and (2)** ,

we get $A_1v_1=A_2v_2$

$v_1= A_2v_2/A_1 = v_2/4$

Substitute 'v_1' in equation (1), we get

$0.078 + (0.7(v_2^2/4))/2g - (v_2^2/2g) = 0$

$0.078 + (v_2^2/2g) ((0.7/16)-1) = 0$

$(v_2^2/2g) * (-0.956) = - 0.078$

$v_2^2=0.078*2*9.81/0.956=1.6$

v_2 =1.26m/s

Rate of flow Q= A_2v_2=0.01767*1.26

$Q=0.0222m^3/s$

3) 215 litres of gasoline (specific gravity 0.82) flow per second through an inclined venturimeter fitted to a 300 mm dia pipe. The venturimeter is inclined at an angle of 60^0 to the vertical and its 150 mm dia. throat is 1.2 m from the entrance along its length. Pressure at throat =0.077 N/mm^2, calculate C_d.

If instead of pressure gauges the entrance and throat of the venturimeter are connected to the two limbs of a U=tube manometer. Determine its reading in mm of differential mercury column.

Solution:

Discharge, $Q=C_d((a_1a_2)/(\sqrt{}a_1^2-a_2^2))*\sqrt{}(2gh) = 215*10^{-3} = 0.215\ m^3/s$

$a_1 = (\pi/4)*(300/1000)^2 = 0.0707\ m^2$

$a_2 = (\pi/4)*(150/1000)^2 = 0.0177\ m^2$

$h = ((p_1/w) +z_1) - ((p_2/w) +z_2)$

$p_1/w = (0.141 *10^6)/(9810*0.82) = 17.528$ m of gasoline

$p_2/w = (0.077 *10^6)/(9810*0.82) = 9.572$ m of gasoline

$z_1 = 0,\ z_2 = (1.2 \sin 30) = 0.66$m

$h = (17.528 + 0) – (9.572 + 0.66) = 7.356$ m

$0.215 = C_d((0.0707*0.0177)/(\sqrt{}0.0707^2-0.0177^2))*\sqrt{}(2*9.81*7.356)$

$C_d = 0.979$

When a U-tube manometer is connected,

$h = x((s_m/s_o)-1)$

$7.356 = x((13.6/0.82)-1)$

$x = 0.472$ m

x = 472 mm

4) The following data relate to an inclined venturimeter:

Diameter of the pipe line = 400 mm

Inclination of the pipe line with the horizontal =30^0

Throat diameter = 200 mm

The distance between the mouth and throat of the meter = 600 mm

Specific gravity of the oil flowing through the pipe line = 0.7

Specific gravity of the heavy liquid (U-tube) = 13.6

Reading of the differential manometer =50 mm

The co-efficient of the meter = 0.98

Determine the rate of flow in the pipe line.

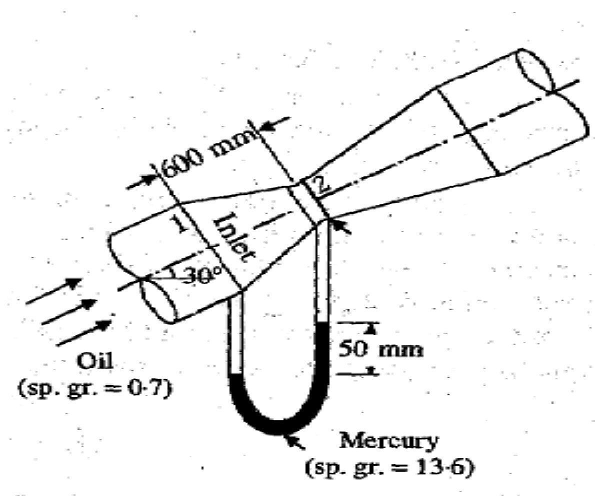


Fig. 4

Difference of pressure head h is given by :

$$h = x[(s_h/s_p)-1]$$

where s_{hl} = specific gravity of heavy liquid (i.e. mercury) in U-tube = 13.6

s_p = specific gravity of liquid (i.e. oil) flowing (sp. gr. = 0.7) trough the pipe = 0.7

Therefore h = 0.05[(13.6/0.7)-1] = 0.92 m of oil

Now applying Bernoulli's equation at section '1' and '2', we get,

$(p_1/w) + (v_1^2/2g) + z_1 = (p_2/w) + (v_2^2/2g) + z_2$ ……. (i)

$((p_1/w) + z_1)) – ((p_2/w) + z_2)) + (v_1^2/2g) - (v_2^2/2g) = 0$

$((p_1/w) + z_1)) – ((p_2/w) + z_2) = h$

$(p_1/w) - (p_2/w) + (z_1 - z_2) = h$

It may be noted that differential gauge reading will include in itself the difference of pressure head and the difference of datum head

Thus equation (i) reduces to :

$h + (v_1^2/2g) - (v_2^2/2g) = 0$ …… (ii)

applying continuity equation at section ‘1’ and ‘2’ we get, $A_1V_1 = A_2V_2$

or $V_1 = (A_2V_2)/ A_1$

$=(0.0314* V_2)/0.1257$

$= V_2/4$

Substituting the value of V_1 and h in eq. (ii) we get,

$0.92 + (v_2^2/16*2g) - (v_2^2/2g) = 0$

$(v_2^2/2g) (1-(1/16)) = 0.92$ or $v_2^2* (15/16)$

$= 0.92$

or $v_2^2 = (0.92*2*9.81*16)/15$

$= 19.52$

or $v_2 = 4.38$ m

rate of flow of oil, $Q = A_2V_2 = 0.0314*4.38$

Q = **0.1375 m³/s**

5) A vertical venturimeter has an area ratio 5. It has a throat diameter of 10 cm. when oil of specific gravity 0.8, flows through it the mercury in the differential gauge indicates a difference in height of 12 cm. Find the discharge through the venturimeter. Take C_d=0.98.

Solution:

Area ratio, $k=a_1/a_2=5$,

throat diameter, $d_2=10cm=0.1m$ and area $a_2=\pi/4*0.12^2$

Specific gravity of oil, $s_0=0.8$,

difference of Hg level, $x=12cm=0.12m$

Now differential head (ii) is given by,

$$h = x\left(\frac{s_h}{s_0} - 1\right)$$

$$h = 0.12\left[\frac{13.6}{0.8}\right] = 1.92\,m$$

The discharge is given by,

$$Q = \frac{C_d . a_1 . a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

The discharge can be expressed in terms of area, Ratio (k) as

$Q= C_d ((a_1a_2)/(\sqrt{}(a_1^2-a_2^2))*\sqrt{}(2gh)$

$= C_d ((a_1/a_2)*a_2)/(\sqrt{}((a_1^2/a_2^2)-a_2^2/a_2^2))*\sqrt{}(2gh)$

$= C_d ((k*a_2)/(\sqrt{}k^2-1))*\sqrt{}(2gh)$

$= 0.98*((5*\pi(0.1)^2/4)/(\sqrt{}(5)^2-1))*\sqrt{}(2*9.81*1.92)$

$Q = 0.0482\ m^3/s$

6) In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 18cm respectively. A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than at A by $0.981N/cm^2$. Neglecting all losses, calculate the flow rate. If the gauges at A and B is replaced by tubes filled with same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

Solution:

Specific gravity of oil s_0=0.8

Density, ρ =0.8*1000=800kg/m^3

Dia at A D_A=16cm =0.16m

area at A, A_1 =0.02 m^2

Dia at B, D_B =18cm =0.18m

area at B,

i)Difference of pressure,

$p_B - p_A$= 0.981 N/m^2= 9810 N/m^2

Difference of pressure head

p_B-p_A=9810/(800*9.81) = 1.25

Applying Bernoulli's theorem at A and B, we get

$$\frac{p_A}{\rho g} + \frac{p_B}{\rho g} + Z_A - Z_B = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$\left(\frac{p_A - p_B}{\rho g}\right) + 2.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$0.75 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

Now applying continuity equation at A and B, we get

V_A*A_1=V_B*A_2

$$V_B = \frac{V_A \times A_1}{A_2} = 4V_A$$

Substituting the value of V_B , we get

$$0.75 = \frac{16V_A^2}{2g} - \frac{V_A^2}{2g} = \frac{15V_A^2}{2g}$$

$$V_A = \sqrt{\frac{0.75 \times 2 \times 9.81}{15}} = 0.99 m/sec$$

$$Q = V_A \times A_1$$

$$Q = \mathbf{0.01989 m^3/sec}$$

Difference level of mercury in the U-tube

Let h=Difference of mercury level

Then

$$h = x\left(\frac{s_h}{s_0} - 1\right)$$

$$0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x \times 16$$

$$x = \frac{0.75}{16} = 0.04687m = 4.687cm$$

7)Estimate the discharge of kerosene (sp gravity=0.8) through the given venturimeter shown in **Fig. 5**. specific gravity of mercury(Hg) is 13.55.

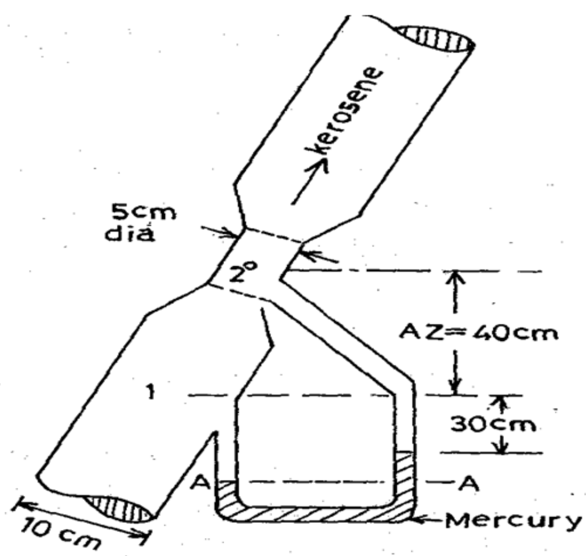


Fig. 5

Solution:

Applying Bernoulli's equation to section 1 and 2

$(p_1/\gamma) + (v_1^2/2g) = (p_2/\gamma) + (v_2^2/2g) + \Delta z$

or $(p_1/\gamma) – ((p_2/\gamma) + \Delta z) = (v_2^2/2g) - (v_1^2/2g)$

Hence $((p_1/\gamma)- (p_2/\gamma) + \Delta z) = 15\ v_1^2/2g$

Equating the pressures at section AA in the two limbs of the manometer

$(p_1/\gamma) + (x + 0.3) = ((p_2/\gamma) + 0.4) + x + (0.3 *(13.85/0.8))$

$(p_1/\gamma) - (p_2/\gamma) + 0.4) = 5.19 – 0.30 = 4.78$ m

$15\ v_1^2/2g = 5.29$ or $v_1 = 2.63$ m/s

Hence Q = 0.785*0.01*2.5 = **0.0196 m^3/s = 19.6 l/s**

<u>Orifice meter or orifice plate</u>

Orifice Flow Measurement – History:

- **The first record** of the use of orifices for the measurement of fluids was by Giovanni B. Venturi, an Italian Physicist, who in 1797 did some work that led to the development of the modern Venturi Meter by Clemons Herschel in 1886.
- It has been reported that an orifice meter, designed by Professor Robinson of Ohio State University was used to measure gas near Columbus, Ohio, about 1890.
- About 1903 Mr. T.B. Weymouth began a series of tests in Pennsylvania leading to the publication of coefficients for orifice meters with flange taps.
- At the same time Mr. E.O. Hickstein made a similar series of tests at Joplin, Missouri, from which he developed data for orifice meters with pipe taps.
- A great deal of research and experimental work was conducted by the American Gas Association and the American Society of Mechanical Engineers between 1924 and 1935 in developing orifice meter coefficients and standards of construction for orifice meters

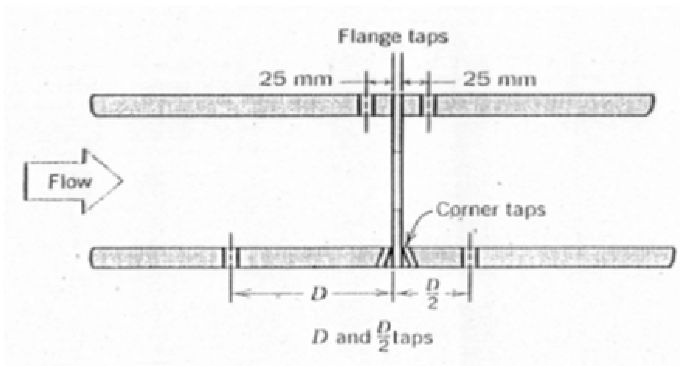


Fig. 1: Tapping arrangements

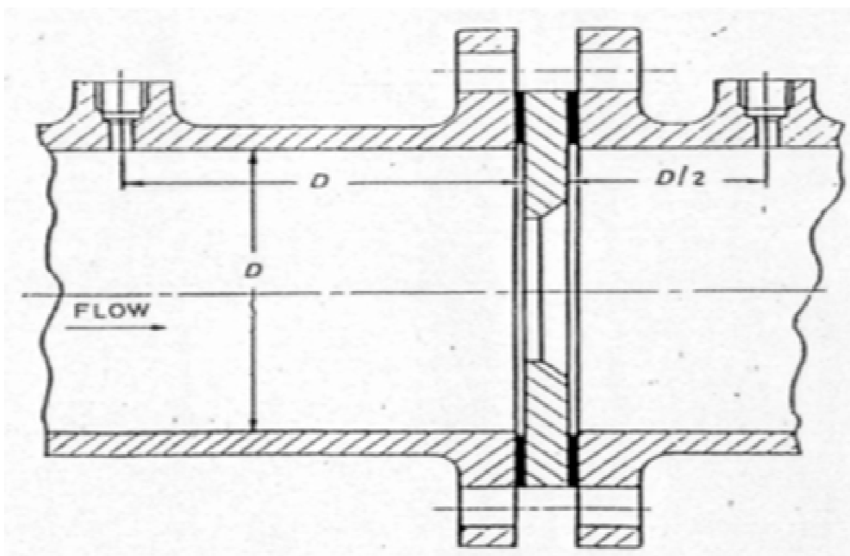


Fig. 2: Orifice profile

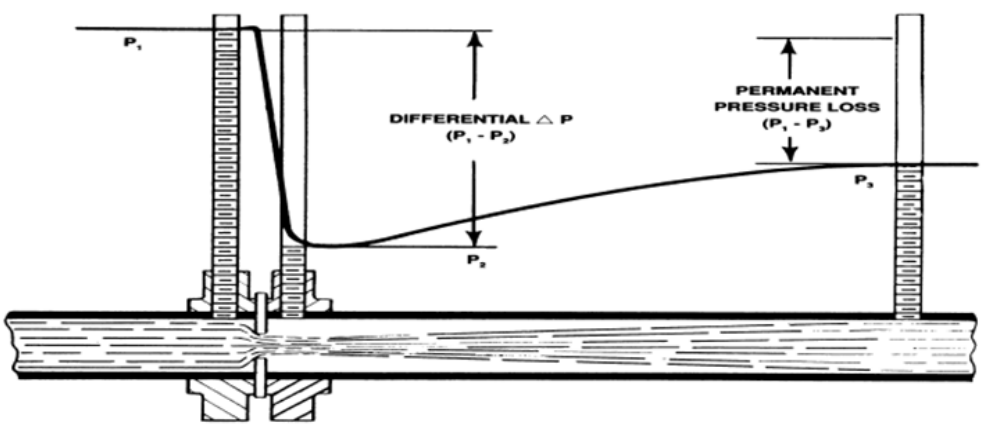


Fig. 3 Typical orifice flow pattern-Flanj taps shown

An orifice in a pipeline is shown in figure 3 with a manometer for measuring the drop in pressure (differential) as the fluid passes thru the orifice. **The minimum cross sectional area of the jet is known as the "vena contracta."**

What is an Orifice Meter?

- An orifice meter is a conduit and a restriction to create a pressure drop. An hour glass is a form of orifice.
- A nozzle, venturi or thin sharp edged orifice can be used as the flow restriction. In order to use any of these devices for measurement it is necessary to empirically calibrate them. That is, pass a known volume through the meter and note the reading in order to provide a standard for measuring other quantities.
- Due to the ease of duplicating and the simple construction, the thin sharp edged orifice has been adopted as a standard and extensive calibration work has been done so that it is widely accepted as a standard means of measuring fluids.
- Provided the standard mechanics of construction are followed no further calibration is required.

Major Advantages of Orifice Meter Measurement

- Flow can be accurately determined without the need for actual fluid flow calibration. Well established procedures convert the differential pressure into flow rate, using empirically derived coefficients.
- These coefficients are based on accurately measurable dimensions of the orifice plate and pipe diameters as defined in standards, combined with easily measurable characteristics of the fluid, rather than on fluid flow calibrations.
- With the exception of the orifice meter, almost all flow meters require a fluid flow calibration at flow and temperature conditions closely approximating service operation in order to establish accuracy.
- In addition to not requiring direct fluid flow calibration, orifice meters are simple, rugged, widely accepted, reliable and relatively inexpensive and no moving parts.

Expression for rate of flow through orifice meter:

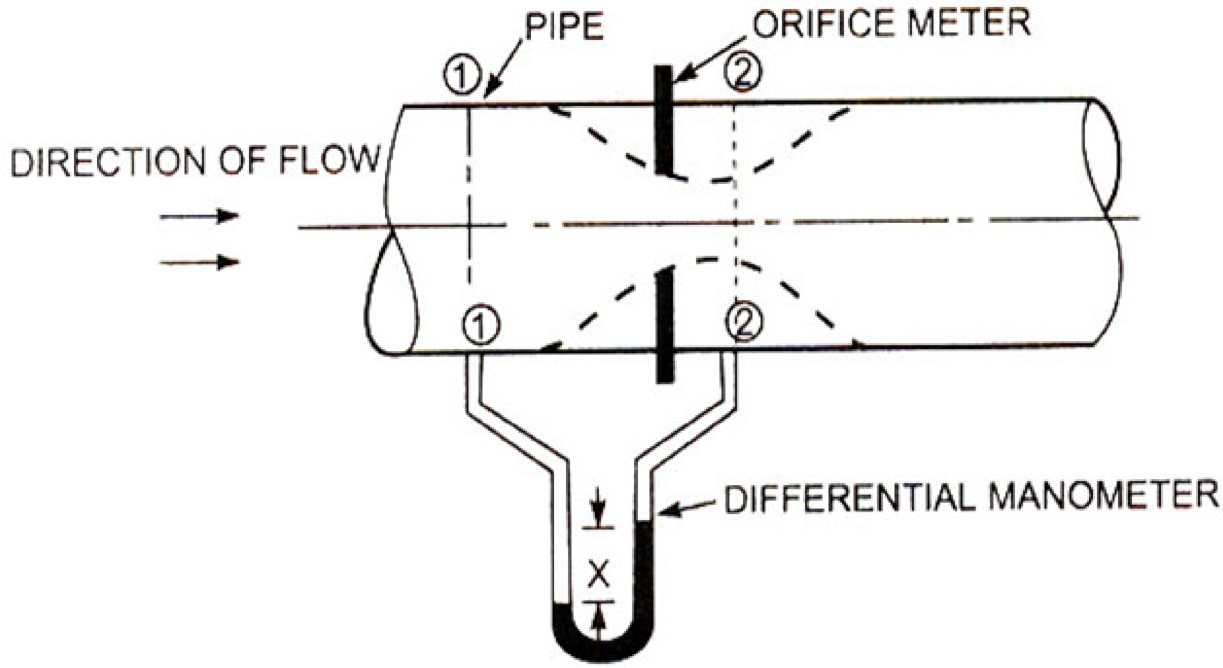


Fig. 4

Orifice meter or orifice plate is a device (cheaper than a venture meter) employed for measuring the discharge of fluid through a pipe. It works on the same principle of a venture meter. It consists of a flat circular plate which has a circular sharp edge hole called orifice, which is concentric with the pipe. The orifice dia is kept generally 0.5 times the dia of the pipe, though it may vary from 0.4 to 0.8 times the pipe dia.

Let p_1, v_1, a_1 at section (1)

p_2, v_2, a_2 at section (2)

Applying Bernoulli's equation at section (1) and (2)

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

$h = ((p_1/\rho g) + z_1) - ((p_2/\rho g) + z_2)$

$h = (v_2^2/2g) - (v_1^2/2g)$

$2gh = v_2^2 - v_1^2$

$v_2 = \sqrt{}\,(2gh) + v_1^2$ ---------(1)

Now section (2) is at the vena contracta and a_2 represents the area at the vena contracta, if a_0 is the area of the orifice,

we have, $C_c = a_2/a_0$

where **C_c=coefficient of contraction**

$$\mathbf{a_2 = C_c\, a_0} \qquad \text{--------(2)}$$

by continuity equation, we have

$v_1 a_1 = v_2 a_2$

$v_1 = a_2/a_1 * v_2 = C_c\, a_0/a_1 * v_2$ [as $a_2 = C_c\, a_0$] -------(3)

Substitute the value of v_1 in eqn.(1)

$v_2 = \sqrt{(2gh) + (C_c\, a_0/a_1 * v_2)^2}$

$v_2^2 = (2gh) + (a_0/a_1)^2\, C_c^2\, v_2^2$

$v_2 = \sqrt{(2gh)}/\sqrt{1-(a_0/a_1)^2\, C_c^2}$

Or $h = (v_2^2/2g) - (v_1^2/2g) \Rightarrow 2gh = (v_2^2 - v_1^2)$

$$2gh = v_2^2 - (C_c\, a_0/a_1 * v_2)^2$$

$$= v_2^2[1 - C_c^2\,(a_0/a_1)^2]$$

$$\mathbf{v_2 = \sqrt{(2gh)}/\sqrt{1-(a_0/a_1)^2\, C_c^2}} \quad \text{......3a}$$

Discharge, Q$= v_2 a_2 = v_2\, \mathbf{C_c\, a_0}$ (since $a_2 = C_c\, a_0$)

Substitute for V_2

$$\mathbf{Q} = \mathbf{C_c\, a_0} \sqrt{(2gh)}/\sqrt{1-(a_0/a_1)^2\, C_c^2} \quad \text{-------(4)}$$

$$\mathbf{Q_{th} = a_0\, a_1 \sqrt{(2gh)}/\sqrt{(a_1^2 - a_0^2)}}$$

$$\mathbf{Q_{act} = C_d\, a_0\, a_1 \sqrt{(2gh)}/\sqrt{(a_1^2 - a_0^2)}}$$

Where, C_d=coefficient of discharge of orifice meter. The coefficient of discharge for orifice meter is much smaller than for a venture meter.

Problems on orifice meter:

1)The following data refers to an orifice meter

Dia of the pipe= 240mm

Dia of the orifice=120mm

Specific gravity of oil=0.88

Reading of differential manometer

$x = 400$ mm of Hg

Coefficient of discharge of the meter,C_d= 0.65

Determine the rate of flow, Q, of oil

Solution:

Dia of the pipe D_1=240mm= 0.24m, A_1=0.0452m^2

Dia of the orifice D_0=120mm= 0.12m,A_2=0.0113 m^2

Coefficient of discharge, C_d= 0.65

Specific gravity of oil, s_0= 0.88

Reading of differential manometer, hg=400mm of Hg = 0.4 m 0f Hg

Therefore differential head, $h=x\,[(s_h/ s_0)-1]$

$=0.4[(13.6/0.88)-1]= 5.78$ m of oil

Discharge, $Q=C_d\, a_0\, a_1\sqrt{(2gh)}/ \sqrt{(a_1^2- a_0^2)}$

$Q = 0.65*0.0113*0.0452*\sqrt{(2*9.81*5.78)}/\sqrt{(0.0452^2- 0.0113^2)}$

=0.08 m^3/s

2) An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give reading of 19.26 N/cm^2 and 9.81N/cm^2 respectively. Co-efficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.

Solution.

Given:

Dia. Of orifice. d_0 =10 cm

Therefore area, $a_0 = (\pi 10^2)/4 = 78.54\ cm^2$

Dia. Of pipe, d_1=20 cm

Therefore area, $a_0 = (\pi 20^2)/4 = 314.16\ cm^2$

$p_1 = 19.62\ N/cm^2 = 19.62*10^4 N/m^2$

$(p_1/\rho g) = (19.62*10^4)/(1000*9.81)$

$= 20$ m of water

Similarly $(p_2/\rho g) = (9.81*10^4)/(1000*9.81)$

$= 10$ m of water

Therefore $h = (p_1/\rho g) - (p_2/\rho g) = 20.0 - 10.0$

$= 10.0$ m of water=1000 cm of water

$C_d = 0.6$

The discharge, Q is given by

$Q = C_d*(a_0 a_1/\sqrt{a_1^2 - a_0^2})*\sqrt{2gh}$

$= 0.6*(78.54*314.16/\sqrt{314.16^2 - 78.54^2})*\sqrt{2*981*1000}$

$= 68213.28\ cm^3/s$

=68.21 lit./s

3) An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter, the pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm mercury. Find the rate of oil of specific gravity 0.9 when the co-efficient of discharge of the meter = 0.64.

Solution:

Given:

Dia. of orifice, $d_0 = 15$ cm

Therefore Area, $a_0=(\pi 15^2)/4 = 176.7 cm^2$

Dia. of pipe, $d_1 = 30$ cm

Therefore Area, $a_1=(\pi 30^2)/4 = 706.85 cm^2$

Specific gravity of oil, $S_0 = 0.9$

Reading of diff. manometer, x= 50 cm of mercury

Differential head, h= $x(s_h/s_0-1)=50(13.6/0.9-1)$

$= 50*14.11=705.5$ cm of oil

Co-efficient of discharge, $C_d = 0.64$

Therefore the rate of the flow, Q is given by

$Q = C_d*(a_0a_1/\sqrt{a_1^2-a_0^2})*\sqrt{2gh}$

$=0.64*(176.7*706.85/\sqrt{706.85^2-176.7^2})*\sqrt{2*981*1000}$

$=137414.25 \ cm^3/s$

= **137.414 lit./s**

Pitot tube

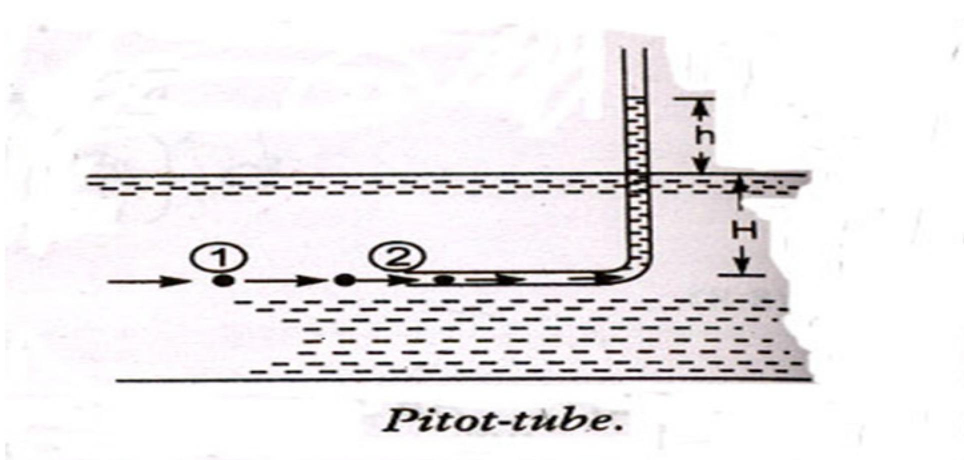


Fig. 1

H= depth of tube in liquid

h=rise of liquid in the tube above the free surface

The Pitot tube (named after the **French scientist Pitot**) is one of the simplest and most useful instruments ever devised. the tube is a small glass tube bent at **right angles** and is placed in flow such that lower end, which is **bent through 90^0 is directed in the upstream direction** as shown in figure. The liquid rises in the tube due to conversion of **kinetic energy** into **potential energy**. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) & (2) at the same level in such a way that the point (2) is just at the inlet of the pitot tube and point (1) is far away from the tube

Let p_1, v_1 & p_2, v_2 are pressure and velocities at point (1) & (2) respectively

H= depth of tube in liquid

h=rise of liquid in the tube above the free surface

Applying Bernoulli's equation at point (1) & (2) we get

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

But $z_1 = z_2$ as point 1 & are on the same line and $v_2=0$

$p_1/\rho g$= pressure head at (1) =H

$p_2/\rho g$ = pressure head at (2) =h+H

Substituting these values, we get

$H + v_1^2/2g = h+H$

$h = v_1^2/2g$

or

$v_1=\sqrt{2gh}$ (theoretical velocity)

Therefore the actual velocity $(v_1)_{act} = C_v\sqrt{2gh})$

C_v =coefficient of pitot tube

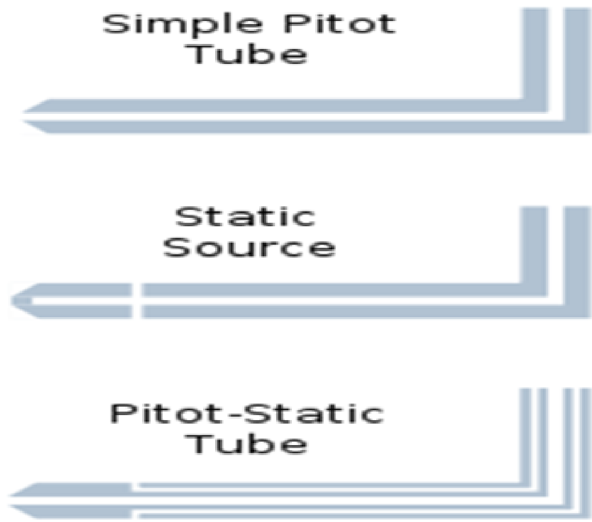


Fig.2: Types of pitot tubes

Stagnation pressure and dynamic pressure

Bernoulli's equation leads to some interesting conclusions regarding the variation of pressure along a streamline. Consider a steady flow impinging on a perpendicular plate (figure 3).

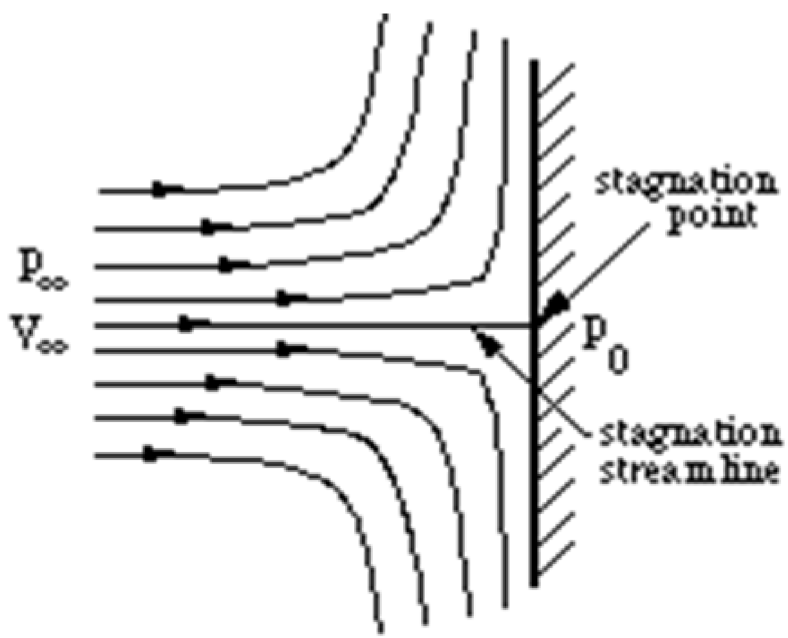


Fig.3: Stagnation point flow.

There is one streamline that divides the flow in half: above this streamline all the flow goes over the plate, and below this streamline all the flow goes under the plate. Along this dividing streamline, the fluid moves towards the plate. Since the flow cannot pass through the plate, the fluid must come to rest at the point where it meets the plate. In other words, it **``stagnates.''** The fluid along the dividing, or **``stagnation streamline''** slows down and eventually comes to rest without deflection at the **stagnation point.**

Bernoulli's equation along the stagnation streamline gives

$$p_e + \tfrac{1}{2}\rho V_e^2 = p_0 + \tfrac{1}{2}\rho V_0^2$$

where the **point e** is for upstream and **point 0** is at the stagnation point. Since the velocity at the stagnation point is zero,

$$\underset{\text{static pressure}}{p_e} + \underset{\text{dynamic pressure}}{\tfrac{1}{2}\rho V_e^2} = \underset{\text{stagnation pressure}}{p_0}$$

Pitot-Static Tubes

The devices for measuring flow velocity directly is the Pitot-static tube. Figure 4 shows the principle of operation

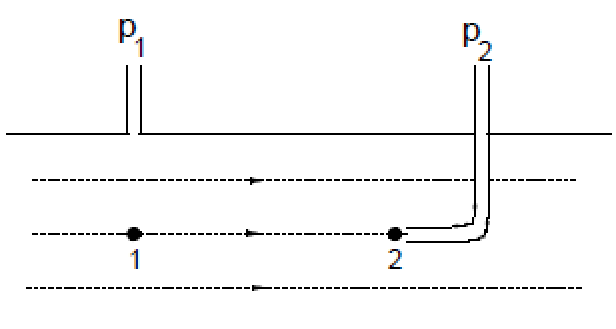


Fig.4: Principle of Pitot-Static tube

By applying Bernoulli's equation to a streamline which meets the tip of the tube. The flow is steady, so there is no flow in the tube. Thus there is a stagnation point, so $u_2 = 0$. *The pressure difference* $p_2 - p_1$ *is the difference between the impact or stagnation pressure at the tip of the tube,* p_2 *, and* the *static pressure in the body of the fluid,* p_1 *. From Bernoulli,*

$$u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

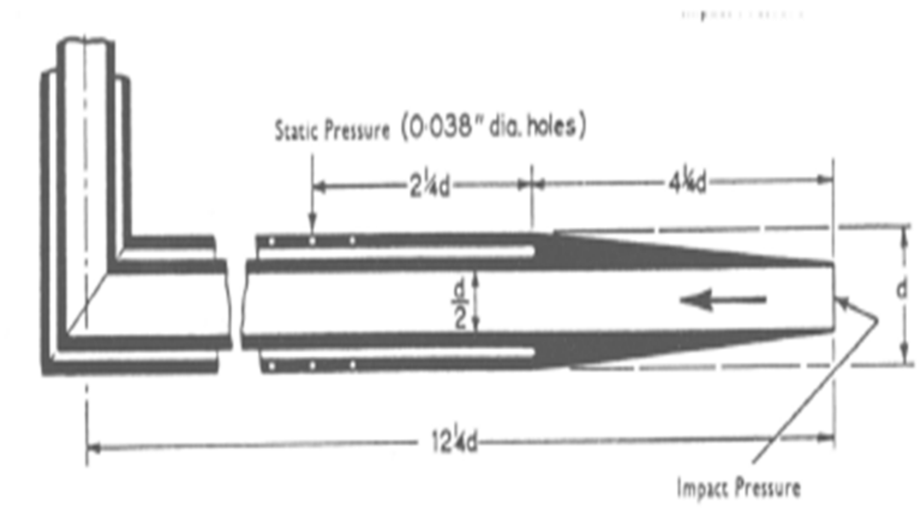


Fig.5- Pitot-Static tube; detail

The most common practical design based upon the above is shown in Figure 5. A pair of concentric tubes is used: the inner tube measured the impact pressure, the outer tube has a number of tiny tappings, flush with the tube, to measure the static pressure. Accuracy is crude, but these devices do provide a very simple and fast estimate of flow velocity.

They are clearly not well suited to dirty flows in which their tappings may become blocked.

Problems on Pitot tube:

1) A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of Pitot tube as $C_v = 0.98$.

Solution.

Given:

Dia. of pipe, $d = 30$ mm $= 0.30$ m

Diff. of pressure head, $h = 60$ mm of water

$= 0.06$ m of water

coefficient of pitot tube, $C_v = 0.98$

Mean velocity, $V = 0.80*$ central velocity

Central velocity,V, is given by

$=C_v\sqrt{2gh}=0.98*\sqrt{2*9.81*0.06}$

$V = 1.063$ m/s

Mean velocity, V = 0.80 * 1.063 = 0.8504 m/s

Discharge, Q =area of pipe * V

$=(\pi d^2)/4*V$

$=(\pi*.30^2)/4*0.8504$

= 0.06 m^3/s

2) A pitot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.

Solution:

Given: Stagnation pressure head, h_g= 6 m

Static pressure head, h_f = 5 m

h = 6-5 =1 m

Velocity of flow, $V=C_v\sqrt{2gh}=0.98\sqrt{2*9.81*1}$

= 4.34 m/s

3) A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum).The stagnation pressure at the centre of the pipe, recorded by the pitot-tube is 0.981 N/cm^2.Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take C_v = 0.98.

Solution: *Given:* dia of pipe, d= 300 mm = 0.3 m

Area, $a= (\pi d^2)/4= \pi(0.3^2)/4=0.07068\ m^2$

Static pressure head = 100 mm of mercury (vacuum)

= -100/1000*13.6 = -1.36 m of water

Stagnation pressure $= 0.981 \text{ N/Cm}^2 = 0.981*10^4 \text{ N/m}^2$

Stagnation pressure head $= (0.984*10^4)/\rho g$

$=(0.984*10^4)/1000*9.81 = 1$ m

h =Stagnation pressure head-Stagnation pressure head

$= 1.0 - (-1.36) = 1+1.36 = 2.36$ m of water

Velocity at centre $= C_v \sqrt{2gh}$

$= 0.98*\sqrt{2*9.81*2.36} = 6.668$ m/s

Mean velocity, $= 0.85*6.668 = 5.6678$ m/s

Rate of flow of water = mean velocity * area of pipe

$= 5.6678*0.07068 \text{ m}^3/\text{s}$

= **0.4006 m³/s**

4) A **submarine** moves horizontally in sea and has its axis 15m below the surface of water. A pitot tube properly placed just in front of the submarine and along its axis is connected to the 2 limbs of U-tube containing mercury. The difference in mercury level is found to be 170mm. Find the speed of the submarine knowing that the specific gravity of mercury is 13.6 and that of sea water is 1.026 with respect of fresh water.

Solution:

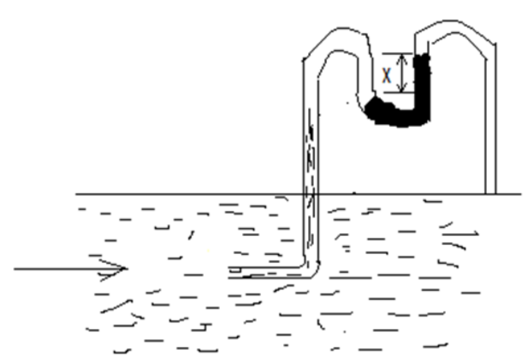


Fig.6: pitot tube

Difference of Hg level, x=170mm=0,17m

Specific gravity of Hg, s_h=13.6

Specific gravity of sea water (in pipe) s_p=1.026

h=x [(s_h/ s_p)-1] =[(13.6/1.026)-1]=2.0834m

v=√2gh=√2*9.81*2.0834=6.393 m/s

speed of submarine,v =6.393*60*60/1000 km/hr

v =**23.01 km/hr**

Notches and weirs:

A **notch** is a device used for measuring the **rate of flow of liquid** through a **small channel or a tank.** The notch is defined as an opening in the side of the tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A **weir** is a **concrete or masonry structure**, placed in an

open channel over which the flow occurs.

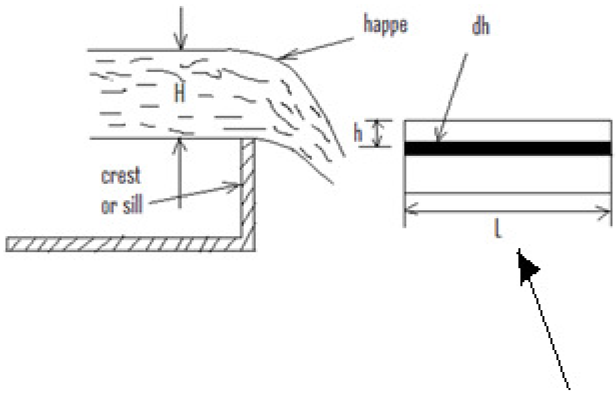
Nappe or Vein: The sheet of water flowing through a notch or over a weir is called Nappe or Vein

Crest or sill: the bottom edge of a notch or a top of a weir over which the water flows, is known as sill or crest

<u>Classification of notches and weirs</u>

- Rectangular notch/weir
- Triangular notch/weir
- Trapezoidal notch
- Stepped notch

Rectangular notch



Section at crest

Fig.1: rectangular notch

Discharge,Q =2/3 $C_d L\sqrt{2g}H^{3/2}$

where H= head of water

L=length of notch

Triangular notch(V-notch)

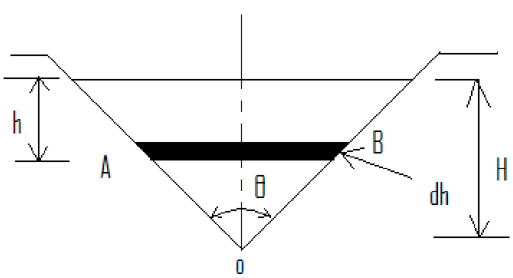


Fig. 2: V-notch

Discharge, $Q=(8/15)* C_d, \tan(\theta/2)* \sqrt{2g}H^{5/2}$

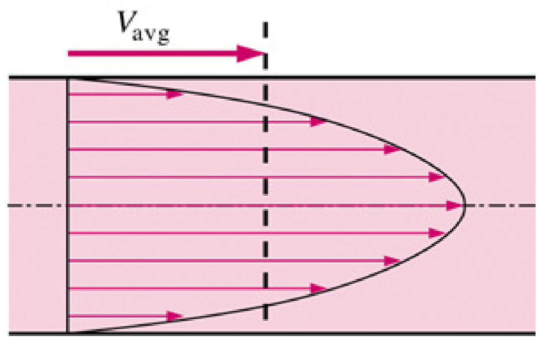
For right angled V-notch, if C_d=0.6, $\theta=90^0$, $\tan(\theta/2)=1$,

$Q=1.417\ H^{5/2}$

Unit 5: Flow through pipes

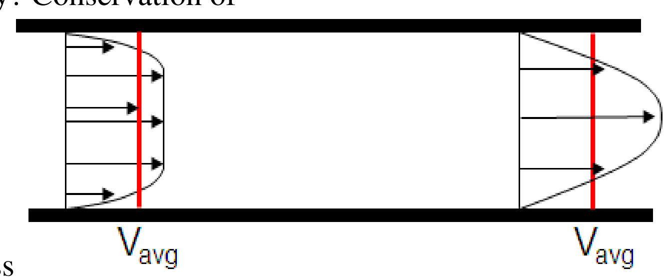
Introduction

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls



Friction force of wall on fluid

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of

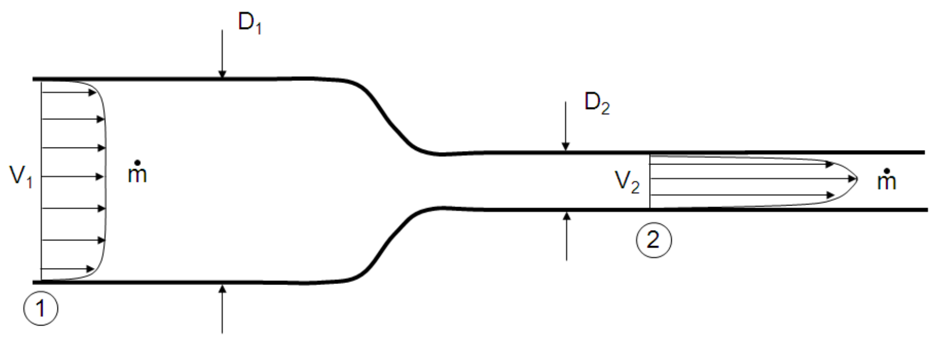


Mass

$$\dot{m} = \rho V_{avg} A = constant$$

same same same

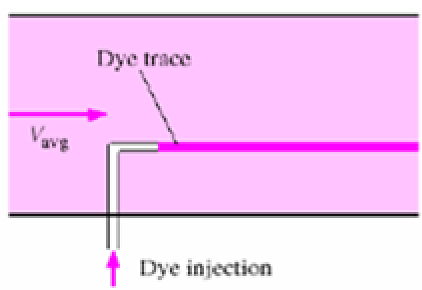
- For pipes with variable diameter, *m* is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

<u>Laminar flow:</u>

- Can be steady or unsteady (steady means the flow field at any instant of time is the same as at any other instant of time)
- Can be one-, two- or three dimensional
- Has regular, predictable behaviour



- Analytical solutions are possible
- Occurs at low Reynold's number

<u>Turbulent flow:</u>

- Is always unsteady.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow

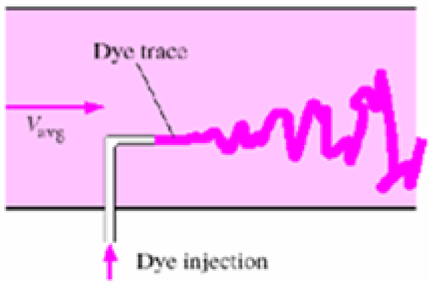
Note: however a turbulent flow can be steady in the mean. We call this a stationary turbulent flow.

- Is always three-dimensional.

Why? Again because of random, swirling eddies, which are in all directions.

Note: however, a turbulent flow can be 1-D or 2-D in the mean.

- Has irregular or chaotic behaviour (cannot predict exactly there is some randomness associated with any turbulent flow.



- No analytical solutions exist! (it is too complicated again because of the 3-D, unsteady, chaotic swirling eddies.)
- Occurs at high Reynold's number.

Definition of Reynolds number

R_e = (inertial force)/(viscous force)

$= (\rho V_{avg}{}^2 L^2)/(\mu V_{avg} L)$

$= (\rho V_{avg} L)/(\mu)$

$= (V_{avg} L)/(\nu)$

- Critical Reynolds number (Re_{cr}) for flow in a round pipe

$Re < 2300 \Rightarrow$ laminar

$2300 \leq Re \leq 4000 \Rightarrow$ transitional

$Re > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.

- For a given application, Re_{cr} depends upon

  - Pipe roughness

  - Vibrations

  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

– Kin. vis. ν(nu) = μ/ρ =viscosity/ density

Loss of energy (or head) in pipe: When a fluid is flowing through a pipe, the fluid experiences some resistance to its motion due to which its **velocity** and ultimately the **head of water available** are reduced. This loss of energy or head is classified as follows

Major energy loss:

This is due to **friction** and it is calculated by the following formula

- Darcy-weisbach equation
- Chezy's equation

Minor energy loss:

This is due to:

- Sudden enlargement of pipe
- Sudden contraction of pipe
- Bend in pipe
- Pipe fittings
- An obstruction in pipe

Darcy-Weisbach equation for loss of head due to friction in pipes

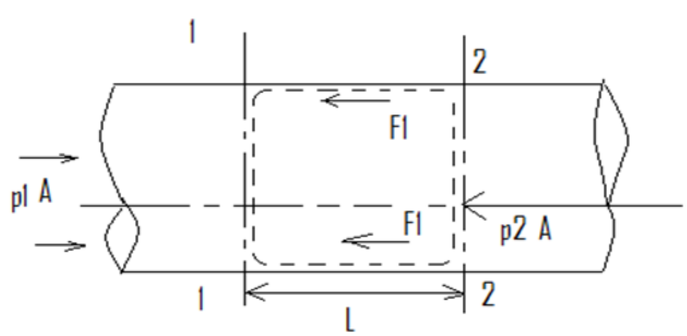


Fig 1: Uniform horozontal pipe

Let, p_1= pressure intensity at section 1-1

v_1= velocity of flow at section 1-1

L= length of the pipe between section 1-1 & 2-2

d= diameter of circular pipe

f^1= frictional resistance per unit wetted area/unit velocity

h_f= loss of head due to friction,

and, p_2 and v_2= are values of **pressure intensity** and **velocity** at section 2-2

Applying **Bernoulli's equation** between sections 1-1 & 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 & 2-2

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

But, $Z_1 = Z_2$ as pipe is horizontal

$v_1 = v_2$ as diameter of pipe is same at 1-1 and 2-2

Therefore $(p_1/\rho g) = \{(p_2/\rho g)+h_f\}$ -----(1)

or $h_f = \{(p_1/w)-(p_2/w)\}$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by **frictional resistance.**

Now frictional resistance = frictional resistance/unit wetted area/ unit

velocity * wetted area * velocity^2

$F_1=f^1*(\pi dL)*V^2$ [because wetted area= $(\pi d*L)$, Velocity=$V=V_1=V_2$]

$= f^1*(P*L)*v^2$ ----(2) [since πd=perimeter=P]

The **forces acting on the fluid between sections 1-1 and 2-2** are:

- Pressure forces at section 1-1 = p_1A [A=area of pressure]
- Pressure forces at section 2-2 = p_2A
- Frictional force F_1 as shown in Fig. 1.

Resolving these forces in horizontal direction, we have, $p_1A - p_2A - F_1 = 0$

$(p_1 - p_2) = F_1 = [f^1*(P*L)*v^2]/A$ [from equation (2)]

$F_1 = f^1 * P* L* V^2]$

But from equation (1)

$p_1 - p_2 = \rho g h_f$

equating the value of $\mathbf{(p_1 - p_2)}$, we get

$\rho g h_f = f^1 * P*L* V^2 / A$ or

$h_f = f^1 /\rho g *P/A * L *V^2$ -------(3)

In equation(3) (P/A)= wetted perimeter(πd)/area (πd_2)/4 = (4/d)

$h_f = f^1 /\rho g * 4/d* * L *V^2$ -------(4)

Putting $f^1/\rho g = f/2$ where **f** is known as **co efficient of friction**

Equation (4) becomes $h_f = 4*f/2g *L V^2/d$ -----(5)

$\boxed{\mathbf{h_f = 4fLV^2/2gd} \quad \textbf{.....} \text{ Darcy-Weisbach equation}}$

Some times (5) is written as $\mathbf{h_f = (f^* L V^2)/2gd}$

Then **f*** is known as **friction factor** **[as f* = 4f]**

co efficient of friction **f** which is function of Reynolds number is given by $f = 16/R_e$ for Re < 2000(viscous flow)

$= 0.079/R_e^{1/4}$ for R_e varying from 4000 to 10^6

Chezy's formula for loss of head due to friction:

An equilibrium between the **propelling force** due to pressure difference and the frictional difference gives

$(P_1-P_2)A = f^1PLV^2$ ÷ through out by **w**

$(P_1-P_2)A/w = f^1PLV^2/w$

Therefore, **Mean velocity**, $V = \sqrt{(w/f^1)} * \sqrt{(A/P * h_f/L)}$

Where the factor $\sqrt{(w/f)}$ is called the Chezy's constant 'c' is the ratio(A/P=area of flow /wetted perimeter.) is called the **hydraulic mean depth or hydraulic radius** and denoted by **m** (or R).

The ratio h_f/L is the loss of head/unit length and is denoted by 'i' or s (slope).

Therefore,

Mean velocity, $\mathbf{v=c\sqrt{(mi)}}$ → **Chezy's formula**

Darcy-Weisbach formula(for loss of head) is generally used for the flow through pipes.

Chezy's formula (for loss of head) is generally used for the flow through open channels.

Problems:

1) In a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2.8 m/s. Find the head lost due to friction using :

(i) Darcy-Weisbach formula;

(ii) Chezy's formula for which C = 55

Assume kinematic viscosity of water as 0.012 stoke .

Solution:

Diameter of the pipe, D = 350mm = 0.35 m

Length of the pipe, L = 75 m

Velocity of flow, V = 2.8 m/s

Chezy's constant C = 55

Kinematic viscosity of water, ν = 0.012 stoke

$= 0.012*10^{-4}\ m^2/s$

Head lost due to friction, h_f :

(i)Darcy-Weisbach formula:

Darcy-Weisbach formula is given by , $h_f=4fLV^2/2gD$

where, f = co-efficeint of friction(a function of Reynolds number R_e)

$R_e = (v*D) / \nu = (2.8*0.35) / 0.012*10^{-4} = 8.167*10^5$

Therefore $f = 0.0719/ (R_e)^{0.25}$ [use when Re > 4000]

$= 0.0719/ (8.167*10^5)^{0.25}$

$= 0.00263$

Therefore head lost due to friction,

$h_f=(4* 0.00263*75*(2.8)^2) /2*9.81*0.35$

$\mathbf{h_f = 0.9\ m}$

(ii) Chezy's formula:

mean velocity $V = C\sqrt{(mi)}$

Where C = 55, $m=A /P = (\pi*D^2/4) /(\pi*D) = D/4 = 0.35/4$

$= 0.0875$ m

Therefore $2.8 = 55\sqrt{(0.0875*i)}$

or $0.0875*i = (2.8/55)^2 = 0.00259$

$i = 0.00296$

But $i = h_f/L = 0.0296$

Therefore $h_f/75 = 0.0296$

$h_f = 0.0296*75$

$h_f =$ **2.22 m**

2) water flows through a pipe of diameter 300 mm with a velocity of 5 m/s. If the co-efficient of friction is given by $f = 0.015 + (0.08/ (R_e)^{0.3})$ where R_e is the Reynolds number, find the head lost due to friction for a length of 10 m. Take kinematic viscosity of water as 0.01 stoke.

Solution:

Diameter of the pipe, D = 300mm = 0.30 m

Length of the pipe, L = 10 m

Velocity of flow, V = 5 m/s

Kinematic viscosity of water, $\nu = 0.01$ stoke

$$= 0.01*10^{-4}\ m^2/s$$

Head lost due to friction, h_f :

Co-efficient of friction, $f = 0.015 + (0.08/ (R_e)^{0.3})$

But Reynolds number, $R_e = \rho VD/\mu = VD/\nu$

$$= 5*0.3/ 0.01*10^{-4} = 1.5*10^6$$

$$f = 0.015 + (0.08/ (1.5*10^6)^{0.3})$$

$$= 0.0161$$

Therefore head lost due to friction,

$$h_f = 4fLV^2/2gD = 4*0.0161*10*5^2/(0.3*2*9.81)$$

$$\mathbf{h_f = 2.735\ m}$$

3) Water is to be supplied to the inhabitants of a college campus through a supply pipe. The following data is given:

Distance of the reservoir from the campus = 3000 m

Number of inhabitants = 4000

Consumption of water per day of each inhabitant = 180 liters

Loss of head due to friction = 18 m

Co-efficient of friction for the pipe, f = 0.007

If the half of the daily supply is pumped in 8 hours, determine the size of the supply main.

Solution:

Distance of the reservoir from the campus = 3000 m

Number of inhabitants = 4000

Consumption of water per day of each inhabitant

$$= 180 \text{ liters} = 0.18\ m^3$$

Therefore total supply per day = 4000*0.18 = 720 m^3

Since half of the daily supply is pumped in 8 hours, therefore maximum flow for which the pipe is to be designed,

$$Q = 720/ (2*8*3600)$$

$$= 0.0125\ m^3/s$$

Loss of head due to friction,

h_f = 18 m

and Co-efficient of friction, f = 0.007

Diameter of the supply line, D:

Using the relation:

$$h_f = 4fLV^2/2gD$$

velocity of flow, $V = Q/A=0.0125/ (\pi*D^2/4) = 0.0159/D^2$

By substitution for loss of head due to friction

$$18=4*0.007*3000*(0.0159/D^2)^2/D*2*9.81$$

or $$D^5 = (4*0.007*3000*0.0159^2)/(18*2*9.81)$$

$$= 6.013*10^{-5}$$

Size of the supply main, D = 0.143 m = **143 mm**

4) In a pipe of 300mm dia and 800m length oil of specific gravity 0.8 is flowing at the rate of 0.45 m^3/s. find Head lost due to friction, and Power required to maintain the flow. Take kinematic viscosity of oil as 0.3 stokes.

Solution:

dia of the pipe, D=0.3m

Length of the pipe, L=800m

Specific gravity of oil =0.8

Kinematic viscosity of oil ν=0.3 stokes=$0.3*10^{-4}$ m^2/s

Discharge $Q=0.45\ m^3/s$

Head lost due to friction, h_f

Velocity, $v= Q/area =0.45/(\pi*0.3^2/4) =6.366m$

Reynolds number, $R_e=v*D/\nu =6.366*0.3/0.3*10^{-4} =6.366*10^4$

Coefficient of friction, $f = 0.0791/(Re)^{0.25} = 0.0791/(6.366*10^4)^{0.25}$

$= 0.00498$

$h_f = 4fLv^2/2gD=4* 0.00498*800*6.366^2/(0.3*2*9.81) = 109.72\ m$

Power required 'P' $=wQh_f$

$w=0.8*9.81=7.848kN/m^3$

$h_f=109.72m$ and $Q=0.43\ m^3/s$

$P=7.848*0.45*109.72$

$P =$ **387.48kW**

5) A pipe conveys 0.25 kg/sec of air at 300K under an absolute pressure of 2.25bar. Calculate minimum diameter of the pipe required if the fluid velocity is limited to 7.5 m/sec.

Solution:

Density of air $\rho= P/RT= (2.25*10^5)/(287*300)= 2.61kg/m^3$

Mass flow of air, $m = \rho AV$

$0.25 =2.61*A*7.5$

Min. area (A) $= (0.25*1) / (2.61*7.5) = 0.01277\ m^2$

Min. dia. $=\sqrt{(0.01277*4)/\pi} = 0.1275m$

= **12.75 mm**

6) A closed tank of a fire engine is partly filled with water, the air space above being under pressure. A 5 cm hose connected to the tank discharges on the roof of building 2 m above the level of water in tank, the friction losses are 50 cm of water. What air pressure must be maintained in the tank to deliver 15 lit/sec on a roof.

Solution:

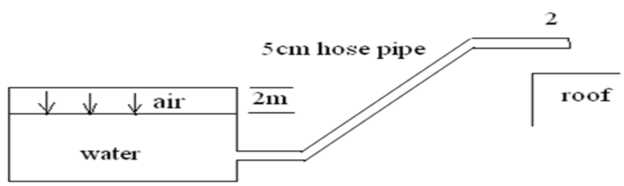


Fig.1

Discharge, $Q= 15$ lit/sec$= 0.015$ m^3/sec

Velocity in 5cm hose pipe$= 0.015/[(\pi/4)*(0.05)^2$

$= 7.64$ m/sec

Applying Bernoulli's theorem to section 1 and 2, taking water surface level in the tank as datum

$$(v_1^2/2g)+(p_1/w)+y_1 = (v_2^2/2g)+(p_2/w)+y_2+h_f$$

The velocity v_1 at the surface is zero.

$$0+(p_1/w)+0 = [(7.64)^2/(2*9.81)]+0+2.05$$

$$(p_1/w)= 5.48 \text{ m of water}$$

$p_1 =$ **0.548 kg f/cm²** (air pressure in tank)

7) Find the head lost due to friction in a pipe of diameter 300mm and length 50m, through which water is flowing at a velocity of 3m/s

(i) Darcy formula

(ii) Chezy's formula for which $C = 60$

Take ν for water $= 0.01$ stoke.

Solution:

Given

Dia. of pipe, $d = 300\text{mm} = 0.30$ m

Length of pipe, L = 50 m

Velocity of pipe, V = 3 m/s

Chezy's constant, C = 60

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01 cm^2/sec$

$$= 0.01 \times 10^{-4} m^2/sec$$

(i) **Darcy Formula** is given by equation as

$$h_f = \frac{4fLV^2}{d \times 2g}$$

Where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by

$$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

Value of

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{0.079}{9 \times 10^5} = 0.00256$$

Head lost, $$h_f = \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = 0.7828 \text{ m}$$

(ii) **Chezy's formula.** Using equation (4)

$$V = C\sqrt{\text{mi}}$$

where c = 60, $m = \frac{d}{4} = \frac{0.3}{4} = 0.075$ m

Therefore $3 = 60\sqrt{0.075 \times i}$ or $i = \left(\frac{3}{60}\right)^2$

But $i = \frac{h_f}{L} = \frac{h_f}{50}$

Equating the two values of i , we have

$$\frac{h_f}{50} = 0.0333$$

$$h_f = 50 \times 0.0333 = 1.665 \text{ m}$$

8)Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 liters/s and the head lost due to friction is 4m.Take the value of c = 50 in chezy's formulae.

Solution:

Length of pipe,

$$L = 2000\ m$$

Discharge,

$$Q = 200 \frac{litres}{s} = 0.2\ m^3/s$$

Head lost due to friction,

$$h_f = 4\ m$$

Value of chezy's constant,

$$c = 50$$

Let the diameter of pipe = d

Velocity of flow,

$$V = \frac{Discharge}{Area} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2\times4}{\pi d^2}$$

Hydraulic mean depth

$$m = \frac{d}{4}$$

Loss of head per unit length.

$$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Chezy's formula is given by equation (4) as

$$V = C\sqrt{mi}$$

Substituting the values of V, m, i and C we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

or $$\sqrt{\frac{d}{4} \times 0.002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{0.00509}{d^2}$$

Squaring both sides,

$$\frac{d}{4} \times 0.002 = \frac{0.00509^2}{d^4} = \frac{0.0000259}{d^4}$$

$$or \quad d^5 = \frac{4 \times 0.0000259}{0.002} = 0.0518$$

= 0.553 m Or

= 553 mm

9) A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300mm litres/s. find the head lost due to friction for a length of 50m of the pipe.

Solution:

given

Kinematic viscosity, v = 0.4 stoke = 0.4 cm^2/s = 0.4 x 10^{-4} m^2/s

Dia. of pipe, d = 300mm = 0.30 m

Discharge, Q = 300 liters/s = $0.3m^3/s$

Length of pipe, L = 50 m

Velocity of flow,

$$V = \frac{Q}{Area} = \frac{0.3}{\frac{\pi}{4}(0.3)^2} = 4.24m/s$$

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{4.24 \times 0.30}{0.4 \times 10^{-4}} = 3.18 \times 10^4$$

As R_e lies between 4000 and 100,000 the value of

f is given by,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(3.18 \times 10^4)^{1/4}} = .00591$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81} = 3.61\ m.$$

10) An oil of sp.gr.0.7 is flowing through a pipe of diameter 300mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000m.Take ν = .29 stokes

Solution:

Given

Sp.gr. of oil, s = 0.7

Dia. of pipe, d = 300mm = 0.30 m

Discharge, Q = 500 liters/s = 0.5m^3/s

Length of pipe, L = 1000m

Velocity of flow,

$$V = \frac{Q}{Area} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 m/s$$

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.30}{0.29 \times 10^{-4}} = 7.316 \times 10^4$$

Co-efficient of friction,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(7.316 \times 10^4)^{1/4}} = .0048$$

Co-efficient of friction,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(7.316 \times 10^4)^{1/4}} = .0048$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} =$$
$$\frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18m$$

$$power\ required = \frac{\rho g.Q.h_f}{1000}\ kW$$

Where,

$$\rho = density\ of\ oil = 0.7 \times 1000$$
$$= 700 kg/m^3$$

Power required P= (700*9.81*0.5*163.18/1000)**=560.28 kW**

6)Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4m of water. Take the value of 'f'=0.009 in the formula,

$$h_f = \frac{4.f.L.V^2}{d \times 2g}.$$

Solution:

Dia. of pipe, d = 200mm = 0.20 m

Length of pipe, L =500 mm

Difference of pressure head,

$h_f = 4\, m\, of\, water$

Co efficient of friction

$f = .009$

Using equation,we have

$$h_f = \frac{4.f.L.V^2}{d \times 2g}$$

$$4.0 = \frac{4 \times 0.009 \times 500 \times V^2}{0.2 \times 2 \times 9.81}\ or$$

$$V^2 = \frac{4 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$$V = \sqrt{.0872} = .9338 \cong .934 m/s$$

Discharge $Q = velocity \times area$

$$= .0934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} 0.2^2 = 0.0293 m^3/s$$

=29.3 litres/s.

7)Water is flowing through a pipe of diameter 200mm with a velocity of 3m/s. Find the head lost due to friction for a length of 5m if the co-efficient of friction is given by

$$f = .002 + \frac{0.09}{{R_e}^{0.3}}$$

where R_e is given Reynolds number. The kinematic viscosity of water = 0.01 stoke

Solution:

Dia. of pipe, d = 200mm = 0.20 m

Length of pipe, L = 5m

Velocity V= 3 m/s

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01\ cm^2/s$

$= 0.01*10^{-4}\ m^2/s$

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{0.01 \times 10^{-4}} = 6 \times 10^5$$

Value of

$$f = .002 + \frac{0.09}{R_e^{\ 0.3}}$$

$$= 0.02 + \frac{0.09}{(6 \times 10^5)^{.3}} = 0.02 + \frac{0.9}{54.13}$$

$$= 0.02 + 0.00166$$
$$= 0.02166$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times 0.02166 \times 5 \times 3^2}{0.20 \times 2 \times 9.81}$$

$$= 0.993 m\ of\ water\ .Answer$$

8)An oil of sp.gr.0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 liters/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.

Solution:

Given: Sp.gr. of oil, $s_{oil} = 0.9$

Viscosity

$$\mu = 0.06\ poise = \frac{0.06}{10}\ Ns/m^2$$

Dia. of pipe, d = 200mm = 0.20 m

Discharge,

$$Q = 60\frac{litres}{s} = 0.06\,m^3/s$$

Length of pipe,

$$L = 500m$$

Density,

$$\rho = 0.9 \times 1000 = 900kg/m^3$$

Reynolds number,

$$R_e = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{\frac{0.006}{10}}$$

Where

$$V = \frac{Q}{Area} = \frac{0.06}{\frac{\pi}{4}d^2} = \frac{0.06 \times 4}{\pi \times 0.2^2}$$
$$= 1.909m/s \cong 1.91m/s$$

$$R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As $\mathbf{R_e}$ lies between 4000 and 10^5

The value of **co-efficient of friction, f** is given by

$$R_e = \frac{0.079}{R_e{}^{0.25}} = \frac{0.079}{(57300)^{0.25}} = 0.0051$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times 0.0051 \times 500 \times 1.91^2}{0.20 \times 2 \times 9.81}$$
$$= 9.48m\ of\ water\ .Answer$$

Power required =

$$\frac{\rho g.Q.h_f}{1000} = \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000} = 5.02 \text{ kW}$$

Minor energy (head) losses:

The minor losses of energy are caused in the velocity of flowing fluid (either in magnitude or direction).In case of long pipes these losses are usually quite small as compared with the loss of energy due to friction and hence there are termed minor losses which may even be neglected without serious error. However in small pipes these losses may sometime overweigh the friction loss. Some of the losses of energy which may be caused due to the change of velocity are indicated below,

Loss of head due to sudden enlargement:

$$h_e = (v_1^2 - v_2^2)/2g$$

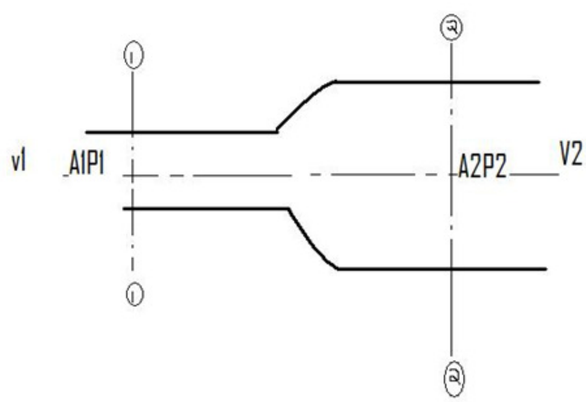


Fig.1: sudden enlargement

Loss of head due to sudden contraction:

when C_c given then use $h_c = v_2^2/2g[v_c/v_2 - 1]^2$

otherwise use $h_c = 0.5 v_2^2/2g$

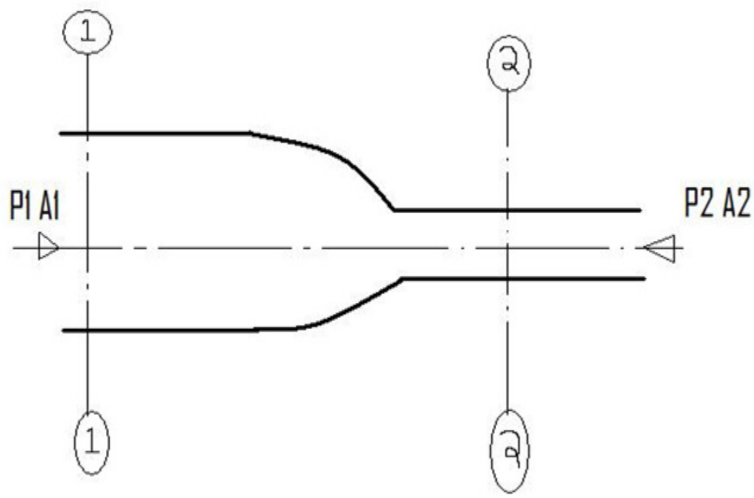


Fig.2: sudden contraction

Loss of energy at the entrance of a pipe:

$$h_i = 0.5v^2/2g$$

Loss of head at the exit of a pipe:

$$h_o = v^2/2g$$

Loss of energy due to gradual contraction or enlargement:

$$h_l = k(v_1 - v_2)^2/2g$$

Loss of energy in bends:

$$h_b = kv^2/2g$$

Loss of head in various pipe fittings:

$=kv^2/2g$ **where V= velocity of flow**

k= co efficient

Problems on head loss due to minor losses:

1)Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 liters/s.

Solution:

Given:

Dia. of smaller pipe,

$$D_1 = 200mm = 0.20m$$

Area

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.20)^2$$
$$= 0.03141m^2$$

Dia. of large pipe,

$$D_2 = 400mm = 0.4m$$

Area

$$A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564m^2$$

Discharge,

$$Q = 250\frac{litres}{s} = 0.25\, m^3/s$$

Velocity,

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.3141} = 7.96m/s$$

Velocity,

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99m/s$$

Loss of head due to enlargement is given by equation

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g}$$
$$= 1.816\, m\, of\, water. answer$$

2) At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Solution:

Given

Dia. of smaller pipe,

$$D_1 = 240mm = 0.24m$$

Area

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.24)^2$$

Dia. of large pipe,

$$D_2 = 480mm = 0.48m$$

Area,

$$A_2 = \frac{\pi}{4} \times (0.48)^2$$

Rise of **hydraulic gradient** ,i.e.

$$\left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = 10mm$$

$$= \frac{10}{1000} = \frac{1}{100}\ m$$

Let the rate of flow = Q

Applying **Bernoulli's equation** to both section,

i.e., smaller pipe section, and large pipe section.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \quad \text{-------(1)}$$

But head loss due to enlargement

$h_c = \frac{(V_1 - V_2)^2}{2g}$ ………..(2)

From continuity equation, we have

$A_1V_1 = A_2V_2$

$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = (\frac{D_2}{D_1})^2 \times V^2 \quad = (\frac{0.48}{0.24})^2 \times V_2 = 2V_2^2 = 4V_2$$

substituting this value in (ii), we get

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g}$$

$$= \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_c and V_1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right)$$

But **hydraulic gradient** rise

$$= \left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = \frac{1}{100}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100} \text{ or } \frac{6V_2^2}{2g} = \frac{1}{100}$$

$V_2 = \sqrt{\frac{2\times9.81}{6\times100}} = 0.1808$ = 0.181 m/s

Discharge, $Q = A_2 \times V_2$

$= \frac{\pi}{4} D_2^2 \times V_1 = \frac{\pi}{4}(0.48)^2 \times .181$

= 0.03275 m^3/s=32.75 liters/sec

3) The rate of flow of water through A horizontal pipe is 0.25 m^3/sec The pipe of diameter 200mm is suddenly enlarged to a diameter of pressure intensity in the smaller pipe is 11.772 N/cm^2 . **Determine:**

- Loss of head due to sudden enlargement
- Pressure intensity in the larger pipe,
- Power lost due to enlargement.

Solution

Given:

Discharge, $Q = 0.25m^3/s$

Dia. of smaller pipe. $D_1 = 200mm = 0.20m$

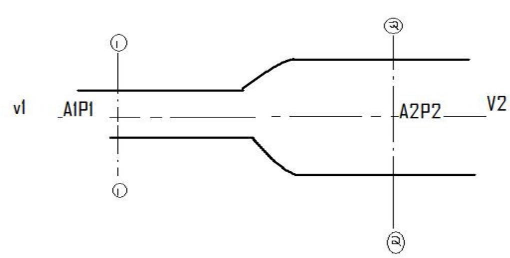
Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141$

Dia. of large pipe. $D_2 = 400mm = 0.4m$

Area $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12566m^2$

Pressure in smaller pipe,

$$p_1 = \frac{11.772\text{N}}{cm^2} = 11.772 \times 10^4 N/m^2$$



Now Velocity. $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.3141} = 7.96 m/s$

Velocity. $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 m/s$

(i)Loss of head due to sudden enlargement

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g}$$
$$= 1.816\ m\ of\ water.answer$$

(ii)Let the pressure intensity in large pipe=p_2

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

[$h_c = h_e$ sudden enlargement]

(Given horizontal pipe)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_c$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178$$

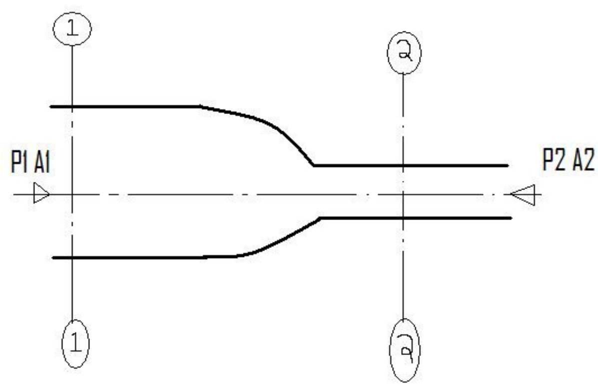
$$= 13.21\ m\ of\ water$$

$$p_2 = 13.21 \times \rho g$$
$$= 13.21 \times 1000 \times 9.81 N/m^2$$
$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} N/cm^2$$
$$= 12.96\ N/cm^2\ .$$

Power lost due to sudden enlargement

$$p = \frac{\rho g.Q.h_c}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$
$$= \mathbf{4.453 \quad KW \qquad Answer}$$

4)A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if C_e=0.62.Aiso determine the rate of flow of water.



Solution:

Given:

Diameter of large pipe, D_1=500 mm =0.5m

$$\text{Area } A_1 = \frac{\pi \times d^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.1963\ m^2$$

Diameter of smaller pipe, D_2=250 mm=0.25 m

Therefore

$$\text{Area } A_2 = \frac{\pi \times d^2}{4} = \frac{\pi \times 0.25^2}{4} = 0.04908\ m^2$$

Pressure in larger pipe,p_1=13.734N/cm^2=13.734 10^{4}N/m^2

Pressure in larger pipe, p_2=11.772N/cm^2=11.772 10^{4}N/m^2

C_c=0.62

Head loss due to contraction,

$$\frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1.0\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.62} - 1.0\right)^2 = 0.375\,\frac{V_2^2}{2g}$$

From **continuity equation**, we have A_1V_1=A_2V_2

$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V^2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Appling Bernoulli's equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_{2+h_c}$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$h_c=0.375\frac{V_2^2}{2g}$ and

$$V_1 = \frac{V_2}{4}$$

Substituting these values in the above equation,
we get

$$\frac{13.734\times10^4}{9.81\times1000} + \frac{(\frac{V_2}{4})^2}{2g} = \frac{11.772\times10^4}{9.81\times1000} + \frac{V_2^2}{2g} + 0.375\ \frac{V_2^2}{2g}$$

$$14 + \frac{V_2^2}{16*2g} = 12.0 + 1.3125\frac{V_2^2}{2g}$$

or $$14 - 12 = 1.375\ \frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.375\ \frac{V_2^2}{2g}$$

or $$2.0 = 1.3125 \times \frac{V_2^2}{2g}\ or\ V_2 = \sqrt{\frac{2.0\times2\times9.81}{1.3125}} = 5.467\frac{m}{s}$$

1. Loss of head due to contraction,

$$h_c = 0.375\ \frac{V_2^2}{2g}$$

$$= \frac{0.375\times(5.467)^2}{2\times9.81} = 0.571\text{ m}$$

2. Rate of flow of water,

Q $= A_2V_2 = 0.04908 \times 5.476 = 268.3$ lit/s.

5)If in the previous problem, the rate of flow of water is 300 liters/s, other data reaming the same, find the value of co-efficient of contraction,c_c.

Solution:

Given,

D_1=0.5 m, D2=0.25 m

p_1=13.734 10^4N/m^2

p_2=11.772 10^4N/m^2

Q=300 lit/s=0.3 m^3/sec

Also, from the previous problem,

$V_1 = \frac{V_2}{4}$, where $V_1 = \frac{Q}{A_1} = \frac{0.30}{\frac{\pi}{4}(0.5)^2}$ =1.528 m/s

$V_2 = 4 \times V_1$ = 4 x 1.528 = 6.112m/s

From **Bernoulli's equation**, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e \qquad [\text{as } Z_1 = Z_2]$$

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(1.528)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{9.81 \times 1000} + \frac{(6.112)^2}{2 \times 9.81} + h_c$$

$$h_c = 14.119 - 13.904 = 0.215$$

But $h_c = \frac{V_2^2}{2g}(\frac{1}{C_c} - 1)^2$

Hence equating the two values of h_c, we get

$\frac{V_2^2}{2g}(\frac{1}{C_c} - 1)^2 = 0.215$

V_2=6.112

Therefore $\frac{6.112^2}{2 \times 9.81}(\frac{1}{C_c} - 1)^2$=0.215

$[1/c_c - 1]^2$ = 0.215 x 2.0 x 9.81/6.112^2 = 0.1129

C_c = 1.0/1.336 = 0.748

6)150mm diameter pipe reduces in diameter abruptly to 100mm diameter. If the pipe carries Water at 30 liters per second, calculate the pressure loss across the contraction. Take the co-efficient of contraction as 0.6.

Solution

Given:

Dia of large pipe. D_1=150 mm = 0.15 m

Area of large pipe. $A_1 = \pi(0.15)^2/4 = 0.01767\ m^2$

Dia.of smaller pipe D_2=100 mm=1.00 m

Area of smaller pipe, $= A_1 \frac{\Pi}{4}(.10)^2$ =0.007854 m^3

Discharge Q=30 liters/s=0.3 m^3/s

Co-efficient of contraction, $C_c = 0.6$

From **continuity equation**, we have

$A_1V_1 = A_2V_2 = Q$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\frac{m}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\frac{m}{s}$$

Applying **Bernoulli's equation** before and after the contraction,

$(p_1/\rho g) + (v_1^2/2g) + z_1 = (p_2/\rho g) + (v_2^2/2g)z_2 + h_c$ ----(1)

But $z_1 = z_2$

And the h_c head loss due to contraction is given by

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{3.82^2}{2\times 9.81}\left(\frac{1}{0.6} - 1\right)^2 = 0.33$$

Substituting these values in eqn.1, we get

$$(p_1/\rho g) + \frac{1.697^2}{2\times 9.81} = (p_2/\rho g) + \frac{3.82^2}{2\times 9.81} + 0.33$$

$(p_1/\rho g) - (p_2/\rho g)$ =0.7438+.33-0.1467=0.9271 m of water

$p_1 - p_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271$ N/m^2

Pressure loss across contraction

$p_1 - p_2$ = 0.909*10^4 N/m2 = 0.909 N/cm^2

7) In Fig. (3) show, when a sudden contraction is introduced in a horizontal pipe line from 50 cm to 25cm, the pressure changes from 10,500kg/m^2 (103005 N/m^2) to 6900 kg/m^2(67689 N/m^2). Calculate the rate of flow. Assume co-efficient of contraction of jet to be 0.65.

Following this If there is a sudden enlargement from 25 cm to 50 cm and if the pressure at the 25 cm section is 6900 kg/m^2(67689 N/m^2) what is the pressure at the 50cm enlarged section?

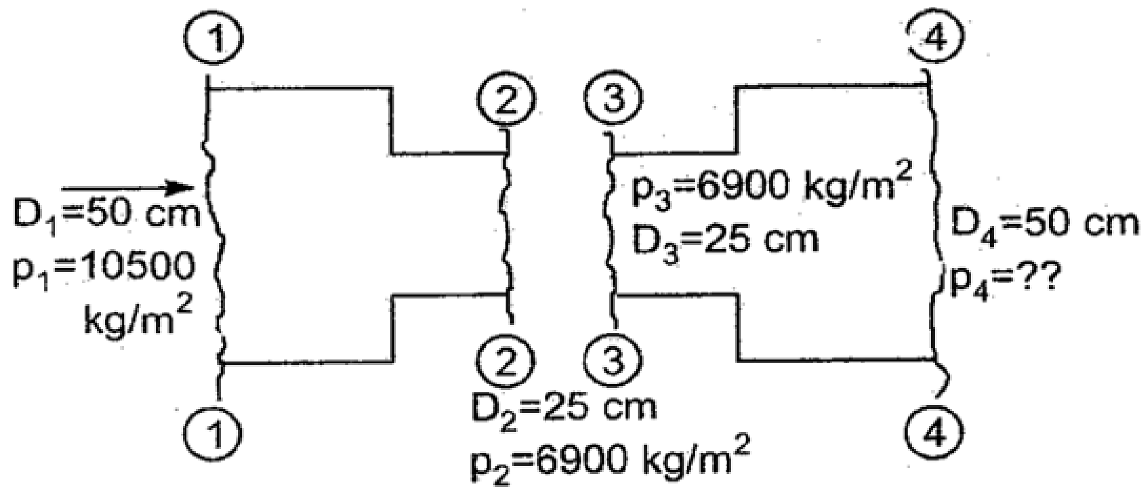


Fig 3

Solution:

Dia of large pipe d_1 = 50 cm = 0.5m

Area A_1 = 0.1963 m^2

Dia of smaller pipe , D2 = 25cm =0.25m

Area A_2 = 0.04908 m^2

Pressure in large pipe,p_1 = 10500 kg/m^2 = 103005 N/m^2

Pressure in smaller pipe, p_2 = 6900 kg/m^2 = 67689 N/m^2

Co-efficient of contraction, C_c = 0.65

Head lost due to contraction is given by equation

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1\right)^2 = 0.2899\,\frac{V_2^2}{2g}$$

From **continuity equation**, we have

$$A_1V_1 = A_2V_2$$

$$\text{or } V_1 = \frac{A_2V_2}{A_1} \quad \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2}$$

$$\left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{.5}{.25}\right)^2 \times V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation to sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

(as pipe is horizontal) $z_1 = z_2$

But $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$

Substituting the values of p_1, p_2, h_c and V_1 we get

$$\frac{103005}{1000\times 9.81} \times \frac{\left(\frac{V_2}{4}\right)^2}{2g} = \frac{67689}{1000\times 9.81} + \frac{V_2^2}{2g} + .2899\,\frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{3.6\times2\times9.81}{1.2274}} = 7.586\, m/s$$

(i)Rate of flow of water ,

Q = $A_2V_2 = 0.04908 \times 7.586\ m^3/s$ or 372.3 lit/s

(ii) Appling Bernoulli's equation to section 3-3 and 4-4

$$\frac{p_3}{\rho g} + \frac{V_3^2}{2g} + z_3 =$$

$$\frac{p_4}{\rho g} + \frac{V_4^2}{2g} + z_4 + head\ loss\ due\ to\ sudden\ enlargement(h_e)$$

But $p_3 = 6900\frac{kg}{m^2}, or\ 67689\frac{N}{m^2}$ $Z_3 = Z_4$

$V_3 = V_2 = 7.586$ m/s $V_4 = V_1 = \frac{V_2}{4} = \frac{7.586}{4} = 1.896$ m/sec

And

head loss due to sudden enlargement is given by

$$h_e = \frac{(V_3 - V_4)^2}{2g} \quad \frac{(7.586-1.8965)^2}{2\times9.81} = 1.65\ \text{m}$$

Substituting these values in Bernoulli's equation, we get

$$\frac{67689}{1000\times9.81} + \frac{7.586^2}{2\times9.81} = \frac{P_4}{1000\times9.81} + \frac{1.8965^2}{2\times9.81} + 1.65$$

$$P_4 = 8\times1000\times9.81 = 78480\ \text{N/m}^2$$

8) Determine the rate of flow of water through a pipe diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the Height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f=0.009 in the formula,

$$\frac{4.f.L.V^2}{d\times2g}.$$

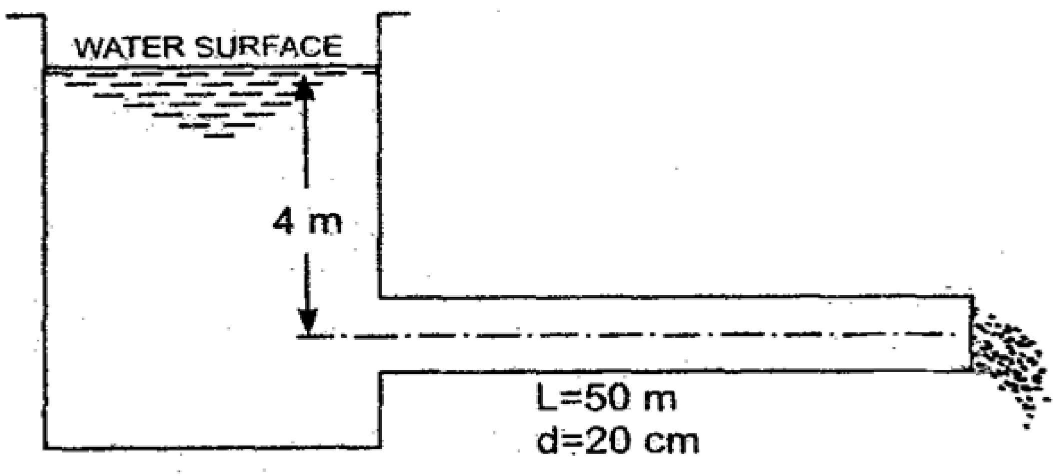


Fig 4

Solution:

Dia of pipe d = 20cm = 0.20m

Length of pipe L= 50 m

Height of water H=4 m

Co-efficient of friction, f=0.009

Let the velocity of water in the pipe =V m/s

Applying Bernoulli's equation at the top of water surface in the tank and at the outlet of pipe, we have [taking point 1 on the top point 2 at the outlet of pipe]

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + all\ losses$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + h_i + h_f$$

or $$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe =V,
therefore V= V_2

Therefore $$4.0 = \frac{V^2}{2g} + h_i + h_f$$

From the equation for loss of head at the entrance of pipe we have,

$h_i = 0.5\frac{V^2}{2g}$ and $h_f = \frac{4.f.L.V^2}{d \times 2g}$

Substituting these values, we have

$$4.0 = \frac{V^2}{g} + 0.5\frac{V^2}{2g} + \frac{4.f.L.V^2}{d \times 2g}$$

$$\frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}\right] = \frac{V^2}{2g}[1.0 + 0.5 + 9.0]$$

$$\therefore \quad 4 = 10.5 \times \frac{V^2}{2g}$$

$$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734\ m/sec$$

Rate of flow, $Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734$

$$= 0.08589\ m^3/s$$

= 85.89 liters/s

9) Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300mm and length 400m. The rate of flow of water through the pipe is 300 liters/s. Consider all losses and take the value of f = 0.008.

Solution:

Dia. of pipe d = 300mm=0.30m

Length L = 400m, Discharge Q = 300 liters/s = 0.3 m^3/s

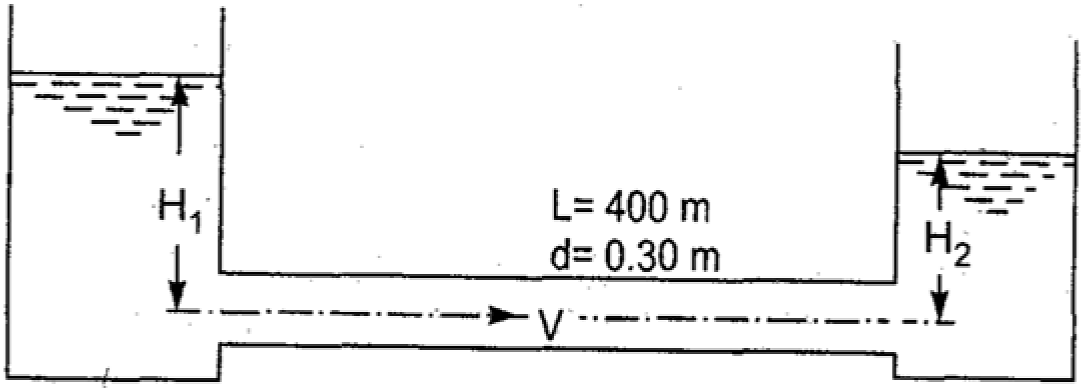


Fig. 5

Co-efficient of friction, f=0.008

Velocity, V = $\frac{Q}{Area} = \frac{0.3}{\frac{\pi}{4}\times(0.3)^2} = 4.224\frac{m}{s}$

Let the two tanks are connected by a pipe as shown in figure 3.

Let H_1 = height of water in 1st tank above t

H_2= height of water in 2nd tank above the centre of pipe

Then the difference in elevations between water surfaces= $H_1 - H_2$

Appling Bernoulli's equation to the free surface of water in the two tanks, we have

$$H_1 = H_2 + \text{losses}$$

$$= H_2 + h_i + H_{f_1} + h_0 \quad \text{........ (i)}$$

Where h_i = loss of head at entrance $= 0.5\frac{V^2}{2g}$

$= \frac{0.5\times4.244^2}{2\times9.81} = .459\ m$

h_{f1} = Loss of head due to friction = $\frac{4\times f\times L\times V^2}{d\times2g}$

$= \frac{4\times.008\times400\times4.244^2}{0.3\times2\times9.81}$ = 39.16 m

10) The friction factor for turbulent flow through rough pipes can be determined by karman-prandtl $\frac{1}{\sqrt{f}} = 2\ log_{10}\left(\frac{R_0}{k}\right) + 1.74$

Where **f = friction factor**, R_0 = pipe radius, k = average roughness Two reservoirs with a surface level difference of 20 m are to be connected by 1 m diameter pipe6 km long. What will be the discharge when a cast iron pipe of roughness k = 0.3 mm is used? What will be the percentage increase in the discharge if the cast iron pipe is replaced by a steel pipe of Roughness k = 0.1 mm? Neglect all local losses.

Solution:

Difference in levels, h=20 mm

Dia of pipe, d=1.0 m

Length of pipe, L =6km=6 x 1000=6000m

Roughness of cast iron pipe, k=0.3 mm

Roughness of steel pipe, k=0.1 mm

1st case. Cast iron pipe.

First find the value of friction factor using

$$\frac{1}{\sqrt{f}} = 2\ log_{10}\left(\frac{R_0}{k}\right) + 1.74 \quad ....(i)$$

$$= 2\ log_{10}\left(\frac{500}{0.3}\right) + 1.74 = 8.1837$$

f = $\left(\frac{1}{8.1837}\right)^2 = 0.0149$

Local losses are to be neglected. This means only head loss due to friction is to be consider, head loss due to friction is

$$20 = \frac{f \times L \times V^2}{d \times 2g}$$

[Here f is the friction factor and not co-efficient of friction because Friction factor = 4 x co-efficient of friction]

$$20 = \frac{0.0149 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 4.556\, V^2$$

$$V = \sqrt{\frac{20}{4.556}} = 2.095\ m/\ \text{sec}$$

$$\therefore \text{Discharge}, Q = V \times \text{Area} = 2.095 \times \frac{\pi}{4} \times d^2$$

$$= 2.095 \times \frac{\pi}{4} \times 1^2 = 1.645 \frac{m^3}{s}$$

2nd case. Steel pipe. K=0.1mm, R_0 = 500 mm

Substituting these values in equation (i), we get

$$= 2\ log_{10}\left(\frac{500}{0.1}\right) + 1.74 \quad = 9.1379$$

f = $\left(\frac{1}{9.1379}\right)^2 = 0.0119$

Head loss due to friction, $20 = \frac{f \times L \times V^2}{d \times 2g}$

$$20 = \frac{0.0119 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 3.639V^2$$

$$\therefore V = \sqrt{\frac{60}{3.639}} = 2.344 \text{ m/s}$$

∴ Discharge,

$$Q = V \times \text{Area} = 2.344 \times \frac{\pi}{4} \times 1^2$$

$$= 1.841 \text{ m}^3/\text{s}$$

Percentage increase in the discharge

$$= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{(1.841 - 1.645)}{1.645} \times 100$$

$$= 11.91\%.$$

11).A horizontal pipeline 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head which occurs determine the rate of flow, take f=0.01 for both section of the pipe.

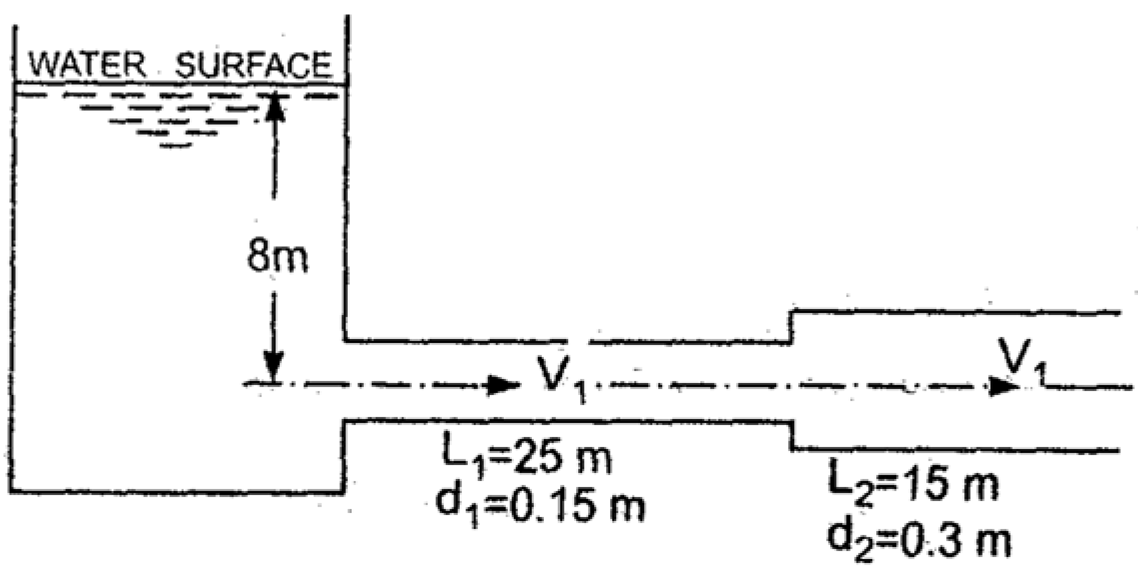


Fig. 6

Solution:

Given:

Total length of pipe, L = 40m

Length of 1^{st} pipe L_1= 25m

Dia. Of the 1^{st} pipe, d1 = 150mm = 0.15m

Length of 2^{nd} pipe, L_2 = 40 – 25 = 15m

Dia. Of 2^{nd} pipe d_2 = 300mm = 0.3 m

Height of water, H = 8m,

Co-efficient of friction, f = 0.01

Applying Bernoulli's equation to the **free surface of water on the tank** and **outlet of pipe** as shown in Fig. 4 and

Taking reference line passing through the centre of pipe.

[Taking point 1 on the top point 2 at the outlet of pipe]

$\mathbf{P_1/w+v_1^2/2g+z_1= p_2/w+v_2^2/2g+0+all\ losses}$

$0 + 0 \quad + 8 = \quad 0+ V_2^2 /2g +0+\mathbf{h_i+h_{f1}+h_e+h_{f2}}$------(1)

Where $\mathbf{h_i}$=loss of head at entrance=$0.5v_1^2/2g$

$\mathbf{h_{f1}}$=head loss due to friction in pipe 1= $\mathbf{4f\ L_1v_1^2/2gd_1}$

$\mathbf{h_e}$=loss of head due to sudden enlargement

= $\mathbf{(v_1\text{-}v_2)^2/2g}$

$\mathbf{h_{f2}}$= head loss due to friction in pipe 2= $\mathbf{4f\ L_2v_2^2/2gd_2}$

From continuity equation, we have

$A_1V_1= A_2V_2$

$\mathbf{V_1}= A_2V_2/V_1= (D_2/D_1)^2*V_2 = (0.3/0.15)^2*V_2= \mathbf{4V_2}$

Substituting the value of V_1 in different head losses, we have

$h_i = 0.5v_1^2/2g=0.5(4v_2^2)/2g$

$= 8v_2^2/2g$

$h_{f1}=4f\ L_1v_1^2/2gd_1=(4*0.01*25*(4v_2)^2)/(0.15*2g)$

$=106.6v_2^2/2g$

$h_e = (v_1 - v_2)^2/2g = (4v_2 - v_2)^2/2g$

$= 9v_2^2/2g$

$h_{f2} = 4fl_2v_2^2/2gd_2 = 4*0.01*15*v_2^2/(0.3*2g)$

$= 2v_2^2/2g$

Substituting the value of the losses in equation (1), we get

$8 = v_2^2/2g + 8v_2^2/2g + 106.6v_2^2/2g + 9v_2^2/2g + 2v_2^2/2g$

$= v_2^2/2g(1+8+106.6+9+2)$

$= 126.6v_2^2/2g$

$V_2 = \sqrt{(8*2g)}/126.6$

$= \sqrt{(8*2*9.81)}/126.6$

$\mathbf{V_2 = 1.11\ m/s}$

Hence rate of flow '$\mathbf{Q' = A_2v_2 = (\pi*(0.3)^2/4)*1.11}$

$\mathbf{Q = 0.078 m^3/s}$

Q= 78.67 liters/s

12) Design the diameter of a steel pipe to carry water having kinematic viscosity $\nu = 10^{-6}\ m^2/sec$ With a mean velocity of 1 m/s. The head loss is to be limited to 5 m per 100 m length of pipe. Consider the equivalent sand roughness height of pipe $k_s = 45 \times 10^{-4}$ cm. Assume that the Darcy weisbach friction factor over the whole range of turbulent flow can be expressed as

$$f = 0.0055\,[1 + (20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e})^{\frac{1}{3}}]$$

Where D =Diameter of pipe and R_e=Reynolds number.

Solution:

Given:

Kinematic viscosity, $\nu = \frac{10^{-6} m^2}{s}$

Mean velocity, V =1 m/s.

Head loss, h_f = 5 m in a length L = 100m

Value of k_s= 45 x 10^{-4} cm=45 x 10^{-6} m

Value of f = 0.0055[1+ $(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e})^{\frac{1}{3}}$]................(i)

Using Darcy weisbach equation, $h_f = \frac{4 \times f \times L \times V^2}{D \times 2g}$

or $f = \frac{h_f \times D \times 2g}{4 \times L \times V^2}$ $\frac{5 \times D \times 2 \times 9.81}{4 \times 100 \times 1^2}$ = 0.2452 D

Now the Reynolds number is given by,

$R_e = \frac{\rho V D}{\mu} = \frac{V \times D}{\gamma}$ As (v= μ/ρ)

$= \frac{1 \times D}{10^{-6}} = 10^6 D$

Substituting the values of f, R_e, and k_s in equation (i), we get

0.2452 D = 0.0055 [1+ $(20 \times 10^3 \times \frac{45 \times 10^{-6}}{D} + \frac{10^6}{10^6 D})^{\frac{1}{3}}$]

$\frac{0.2452}{0.0055} D = [1 + (\frac{0.9}{D} + \frac{1}{D})^{\frac{1}{3}}]$

or 44.58 D = [1+$(\frac{1.9}{D})^{\frac{1}{3}}$] or 44.58 D -1=$(\frac{1.9}{D})^{\frac{1}{3}}$

or $\frac{1.9}{D}$

or D$(44.58 D - 1)^3 = 1.9$ .... (ii)

The equation (ii) is solved by hit and trail method.

$\therefore$ correct value of D = **0.0854 m**

13) A pipe line AB of diameter 300 mm and length 400m carries water at the rate of 50 liters/s. The flow takes place from A to B where point B is 30 m above A.

Find the pressure at A if the pressure at B is 19.62 N/cm^2 Take f = 0.008

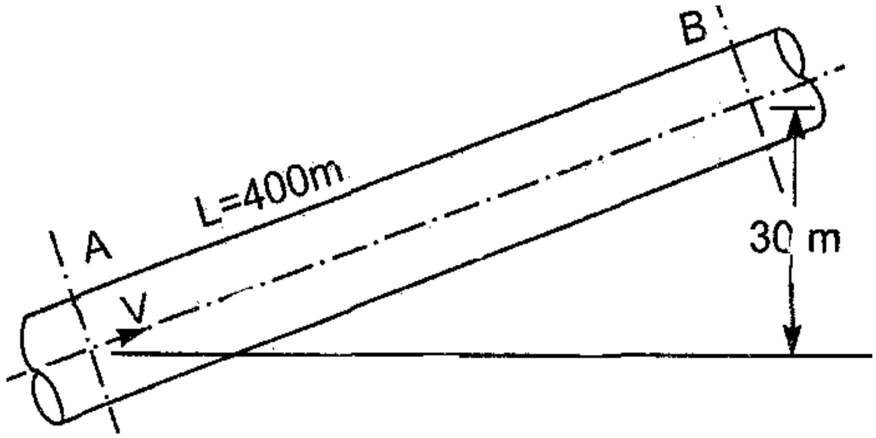


Fig 7

Solution:

Given:

Dia.of pipe, $d = 300$ mm = 0.30 m

Length of pipe, $L = 400$ m

Discharge, $Q = 50$ liters/s = 0.05 m^3/sec

$$\text{Velocity,}\quad V = \frac{Q}{Area} = \frac{0.05}{\frac{\pi}{4}d^2} = \frac{.05}{\frac{\pi}{4}\times(.3)^2} = 0.7074\ m/s$$

Pressure at B, $P_B = 19.62 N/cm^2 = 19.62 \times 104\ N/m^2$

F = 0.008

Appling Bernoulli's equation at points A and B

And taking datum line passing through A, we have

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+z_B+h_f$$

But $V_A = V_B$ $\quad\quad\because Dia. is\ same]$

$z_A = 0, z_B = 30$

and $h_f = \frac{4\times f\times L\times V^2}{d\times 2g}$

$$\therefore \frac{P_A}{\rho g}+0=\frac{19.62\times 10^4}{1000\times 9.81}+30+\frac{4\times .008\times 400\times .7074^2}{0.3\times 2\times 9.81}$$

= 20+30+1.088 = 51.088

$$P_A = 51.088\times 1000\times 9.81\ N/m^2$$

$$P_A = \frac{51.088\times 1000\times 9.81}{10^4} = 50.12\ N/cm^2$$

Hydraulic gradient line (H.G.L):

It is defined as the line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in pipe with respect to some reference or it is line which is obtained by joining the top of all vertical ordinates, showing the pressure head(p/w) of a flowing fluid in a pipe from the centre of the pipe. The line so obtain is called the H.G.L.

Total energy loss (TEL or EGL)

It is known that the total head (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head.

Total head = $P/w+Z+v^2/2g$

When the fluid flows along the pipe there is loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various parts along the axis of the pipe is plotted and joined by a line, the line so determined is called the Energy gradient (E.G.L) is also called total energy line (TEL).

Points are worth noting

- Energy gradient like always drops in direction of flow because of loss of head.
- HGL may rise or fall depending upon the pressure change.

- HGL is always below the Energy Gradient

Problems on H.G.L and T.E.L

1) Determine the rate of flow of water through a pipe diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the Height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f=0.009 in the formula,

$$\frac{4.f.L.V^2}{d\times 2g}.$$

Draw the hydraulic gradient line (H.G.L) and total energy line (T.E.L)

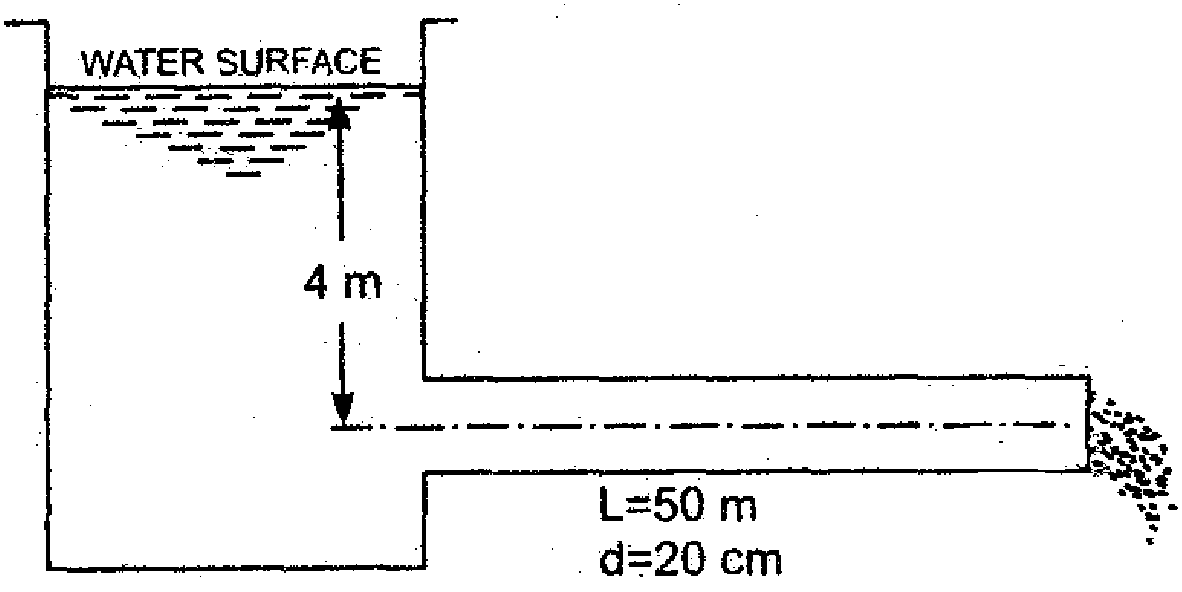


Fig 1

Solution:

Dia of pipe d = 20cm = 0.20m

Length of pipe L= 50 m

Height of water   H=4 m

Co-efficient of friction,   f=0.009

Let the velocity of water in the pipe =V m/s

Applying Bernoulli's equation at the top of water

Surface in the tank and at the outlet of pipe, we have

[Taking point 1 on the top point 2 at the outlet of pipe]

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + all\ losses$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + h_i + h_f$$

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe =V,

Therefore V= V_2

Therefore

$$4.0 = \frac{V^2}{2g} + h_i + h_f$$

From the equation for loss of head at the entrance of pipe

We have,

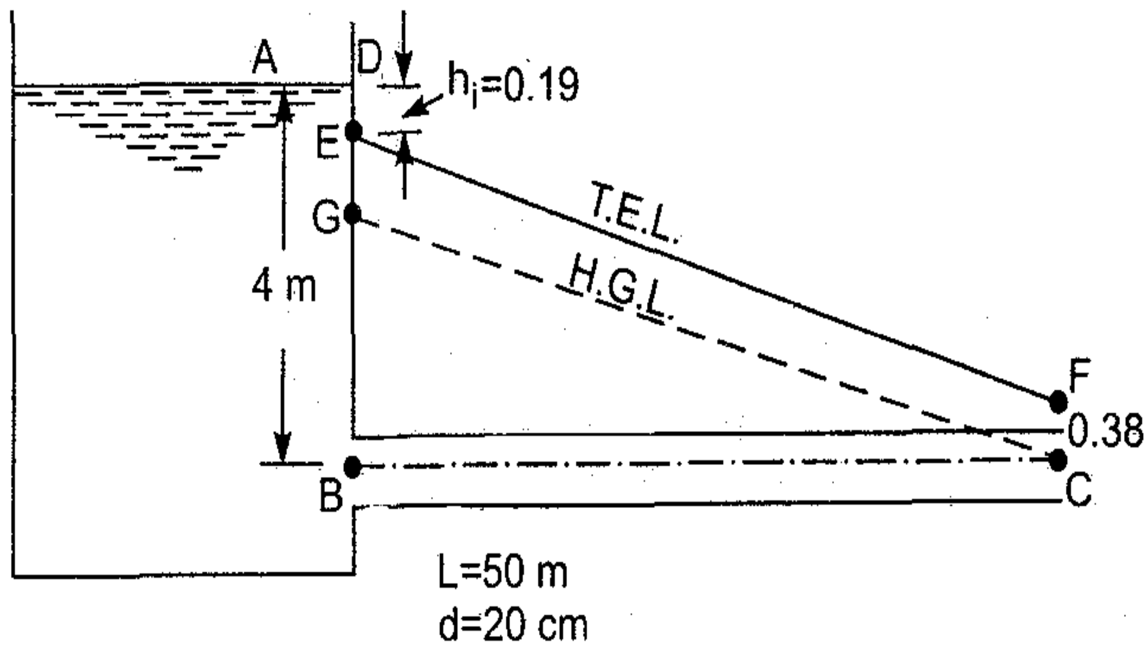
$$h_i = 0.5\frac{V^2}{2g}$$

and

$$h_f = \frac{4.f.L.V^2}{d \times 2g}$$

Substituting these values, we have

$$4.0 = \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}\right] = \frac{V^2}{2g}[1.0 + 0.5 + 9.0]$$



Velocity, V through pipe is calculated and its value is V=2.734 m/s.

$$= 0.5\frac{V^2}{2g} = \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19\ m$$

And

h_f = head loss due to friction

$$\frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428\ \text{m}$$

(a) Total energy line (T. E. L.). consider three points, A, B, and C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in the fig. us find total energy at these points, taking the centre of pipe as the reference line.

1) Total energy at A

A $= \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4\ m$

2) Total energy at B = Total energy at A- h_i

3) Total energy at C,

$$\frac{p_c}{\rho g}+\frac{V_c^2}{2g}+z_c=0+\frac{V^2}{2g}+0=\frac{2.734^2}{2\times 9.81}=0.38\ m.$$

Hence total energy line will be coinciding with free surface of water in the tank. At the inlet of the pipe, it will decrease by

h_i (=.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in the fig.

- Point D represents total energy at A
- Point E, where DE = h_i represents total energy at inlet of the pipe
- Point F, where CF = 0.38 represents total energy at outlet of the pipe

Join D to E and E to F .Then

DFE represents the total energy line.

(b) Hydraulic gradient line (H.G.L.). H.G.L. gives the sum

$(\frac{p}{w}+z)$ with reference to the datum- line.

Hence hydraulic gradient line is obtained by subtracting from total energy line. $V^2/2g$

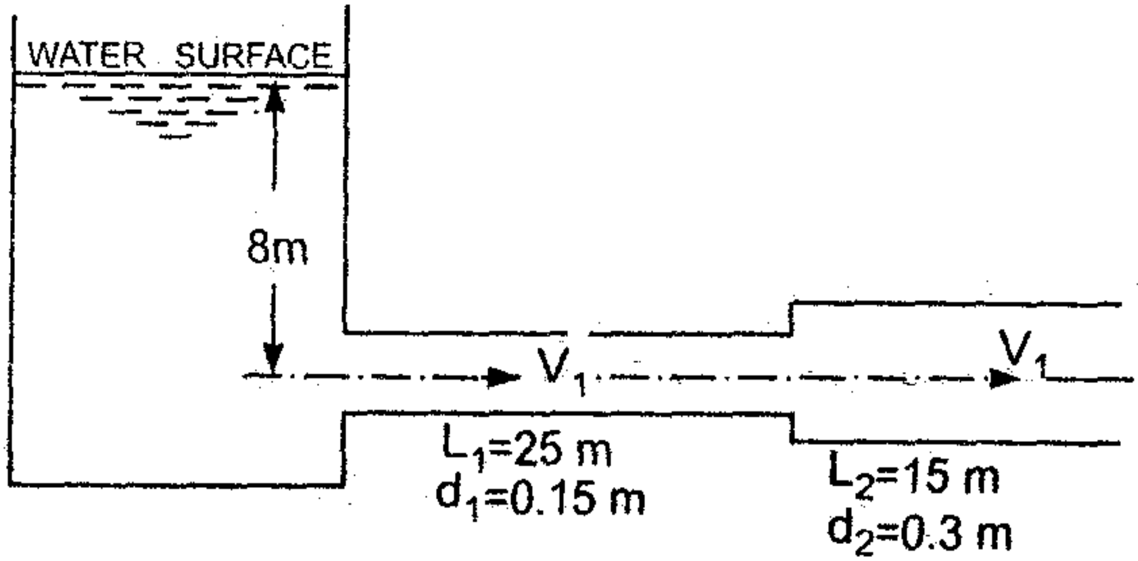
At the outlet of the pipe, total energy is $V^2/2g$ By subtracting $V^2/2g$

From total energy at this point, we shall get point C. which lies on the center line of pipe.

From C, draw a line CG parallel to EF.

Then CG represents the hydraulic gradient line.

2)A horizontal pipeline 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head which occurs determine the rate of flow, take f=0.01 for both section of the pipe. Draw the hydraulic gradient and total energy line.



Solution:

Given:

Total length of pipe, L = 40m

Length of 1st pipe L_1= 25m

Dia. of the 1st pipe, d1 = 150mm = 0.15m

Length of 2nd pipe, L_2 = 40 – 25 = 15m

Dia. of 2nd pipe d_2 = 300mm = 0.3 m

Height of water, H = 8m,

Co-efficient of friction, f = 0.01

Applying Bernoulli's equation to the free surface of water on the tank and outlet of pipe as shown in Fig. 4 and

Taking reference line passing through the centre of pipe.

[Taking point 1 on the top point 2 at the outlet of pipe]

$P_1/w+v_1^2/2g+z_1= p_2/w+v_2^2/2g+0+\text{all losses}$

$0 + 0 \quad + 8 = \quad 0+ V_2^2 /2g +0+h_i+h_{f1}+h_e+h_{f2}$------ (1)

Where h_i=loss of head at entrance=$0.5v_1^2/2g$

h_{f1}=head loss due to friction in pipe 1= $4f\ L_1v_1^2/2gd_1$

h_e=loss of head due to sudden enlargement

$= (v_1-v_2)^2/2g$

h_{f2}= head loss due to friction in pipe 2= $4f\ L_2v_2^2/2gd_2$

From continuity equation, we have

$A_1V_1= A_2V_2$

$V_1= A_2V_2/V_1= (D_2/D_1)^2*V_2 = (0.3/0.15)^2*V_2= 4V_2$

Substituting the value of V_1 in different head losses, we have

$h_i = 0.5v_1^2/2g=0.5(4v_2^2)/2g$

$= 8v_2^2/2g$

$h_{f1}=4f\ L_1v_1^2/2gd_1=(4*0.01*25*(4v_2)^2)/(0.15*2g)$

$=106.6v_2^2/2g$

$h_e = (v_1-v_2)^2/2g = (4v_2-v_2)^2/2g$

$= 9v_2^2/2g$

$h_{f2} = 4fL_2v_2^2/2gd_2 = 4*0.01*15*v_2^2/(0.3*2g)$

$= 2v_2^2/2g$

Substituting the value of the losses in equation (1), we get

$8 = v_2^2/2g + 8v_2^2/2g + 106.6v_2^2/2g + 9v_2^2/2g + 2v_2^2/2g$

$= v_2^2/2g\ (1+8+106.6+9+2)$

$= 126.6v_2^2/2g$

$V_2 = \sqrt{(8*2g)/126.6}$

$= \sqrt{(8*2*9.81)/126.6}$

$V_2 = 1.11$ m/s

Hence rate of flow 'Q' $= A_2v_2 = (\pi*(0.3)^2/4)*1.11$

$Q = 0.078 m^3/s$

Q= 78.67 liters/s

$V_1 = 4V_2 = 4 \times 1.113 = 4.452$ m/s

The various head losses are

$$h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5 \times 4.452^2}{2 \times 9.81} = 0.50\ m$$

$$h_{f_1} = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .01 \times 25 \times 4.452^2}{.15 \times 2 \times 9.81} = 6.73\ m$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(4.452 - 1.11)^2}{2 \times 9.81}$$

$$= 0.568\ m$$

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$$

$$= \frac{4 \times .01 \times 15 \times 1.113^2}{.3 \times 2 \times 9.81} = 0.126\ m$$

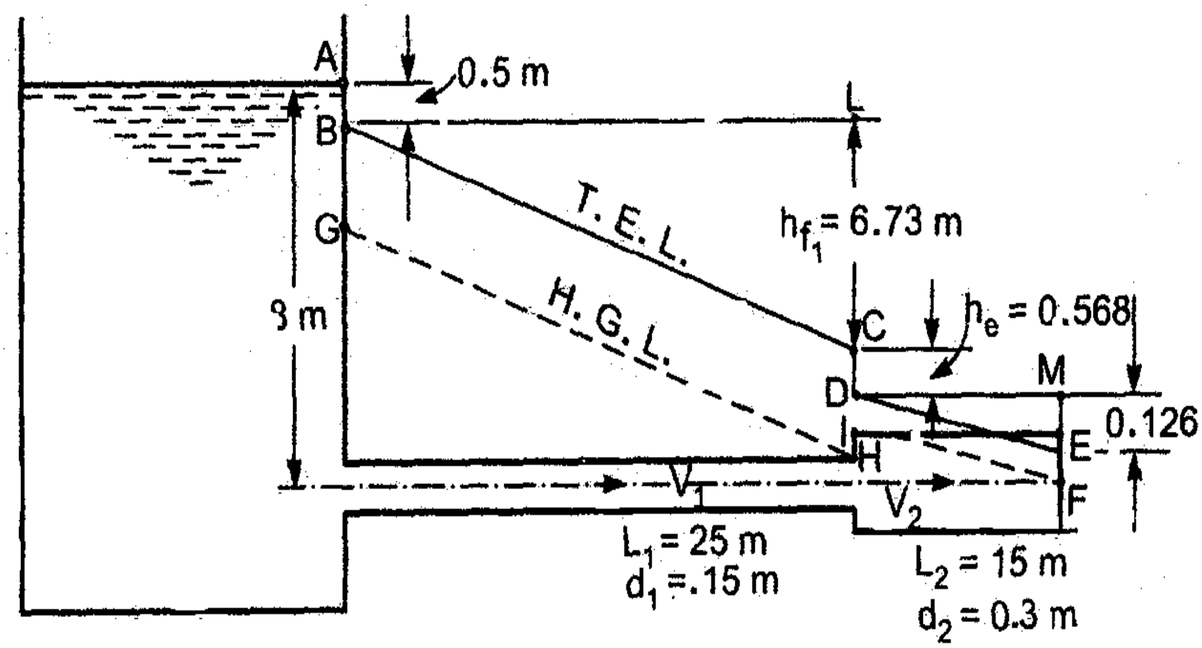
$$ho = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063\ m$$

$$\frac{V_1^2}{2g} = \frac{4.452^2}{2 \times 9.81} = 1.0\ m.$$

Total Energy Line

- Point A lies on free surface of water.
- Take AB = h_i = 0.5 m

- From B, draw a horizontal line. Take BL equal to the
- Length of Pipe i.e., L_1. From L draw a vertical line downward.
- Cut the line LC = h_{f1} = 6.73m
- Join the point B to C. take a line CD vertically downward equal to h_e = 0.568 m
- From D, draw DM horizontal and from point F which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at M. From M, take a distance ME = h_{f2} = 0.126. Join DE. Then line ABCDE represents the total energy line (TEL).
  Hydraulic gradient line (H.G.L.):
- From B, take BG = $V_1^2 / 2g$ = 1.0m
- Draw the line GH parallel to the line BC
- From F, draw a line Fl parallel to the ED.
- Join the point H and I.
- Then the line GHIF represents the hydraulic gradient line (HGL).

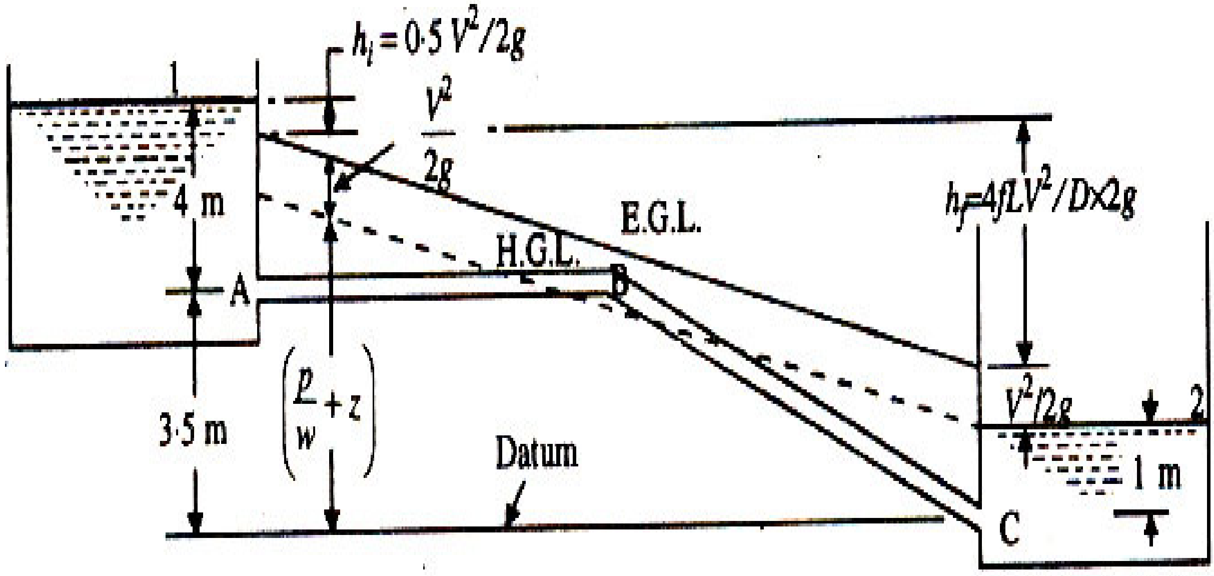


3) A pipe ABC connecting two reservoirs is 80 mm in diameter. From A to B the pipe is horizontal as shown in fig. 12.7 and from B to C it falls by 3.5 meters. The lengths AB and BC are 25m and 15 m respectively. If the water surface in the reservoirs at A is 4 m above the centerline of the pipe and at C 1 m above the centre line of the pipe, *calculate:*

(1)The rate of flow and

(2)The pressure head in the pipe at B

Neglect the loss at the bend but consider all other losses. Also draw the energy and hydraulic gradient lines. Take Darcy friction factor F = 0.024 and $E_{ntrance}$ = 0.5



Solution:

Diameter of the pipe, D = 80 mm = 0.8 m

Area, A = $\pi/4 * 0.08^2 = 0.00026\ m^2$

Friction factor (=4f), = 0.024

$K_{entrance} = 0.5$

(1) The rate of flow Q:

Applying Bernoulli's equation between the water surfaces 1 and 2 in the two reservoirs (considering horizontal plane through C as datum), we get

$p_1/w + v_1^2/(2g) + z_1 = p_2/w + v_2^2/(2g) + z_2$ + loss at entrance + h_f (loss due to friction) + $v^2/(2g)$

$0+0+(4+3.5-1)=0+0+0+0.5v^2/(2g)+\{4fLv^2/(D*2g)\}+v^2/(2g)$

(Where v = velocity of flow in the pipe)

or $6.5 = 0.5v^2/(2g)+ \{0.024*(25+15)*v^2\}/(0.08*2g) + v^2/(2g)$

$= v^2/(2g)(0.5+12+1) + 13.5\ v^2/(2g)$

$v^2 = 6.5*2*9.81/13.5= 9.446$

$v = 3.073$ m/s

Therefore, flow rate= A* V = 0.005026*3.073 = 0.01544 m^3/s

(2)Pressure head in the pipe at B,

Applying Bernoulli's equation at 1 and B, we get

$p_1/w+v_1^2/(2g) +z_1 = p_B/w+v_B^2/(2g)+z_B+0.5v^2/(2g)+h_f$

$0+0+4 = p_B/w+v^2/(2g)+z+0.5v^2/(2g)+4fLv^2/(D*2g)$

$4 = p_B/w+(v^2/2g)+(0.5v^2/2g)+\{0.024*25*v^2/(0.08*2g)\}$

$(v_B = v = 3.073m/s)$

$4 = p_B/w+(v^2/2g)+(0.5v^2/2g)+(7.5v^2/2g)$

$= p_B/w+(9v^2/2g)$

$p_B/w = 4 - (9v^2/2g)$

$= 4 - (9*3.073^2/2*9.81)$

$p_B/w = - 0.33$ m of water (below atmosphere)

Energy gradient and hydraulic gradient lines (E.G.L. and H.G.L):

For plotting E.G.L and H.G.L., we require the velocity head, (same throughout)

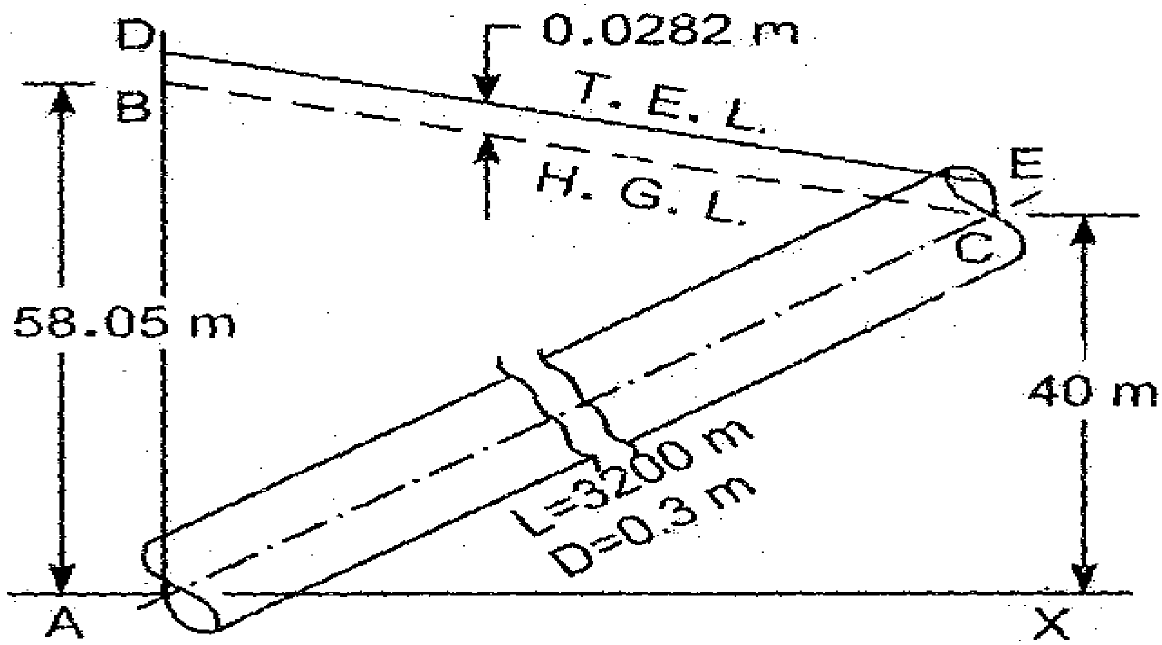
$v^2/2g = 3.073^2/(2*9.81) = 0.481$ m

Total energy at B with respect to horizontal datum through C

$= 3.5+p_B/w+v^2/2g = 3.5- 0.33+ 3.073^2/(2*9.81)$

$= 3.65$ m

4) A pipe line, 300 mm in diameter and 3200 m long is used to pump up 50kg per second of oil whose density is 950 kg/m^3 and whose kinematic viscosity is 2.1 strokes. The centre of the pipe line at the upper end is 40m above than that at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.



Solution

Given:

Dia. of pipe. $D = 300mm = 0.3m$

Length of pipe, $L = 3200m$

Mass, $M = 50kg/s = \rho. Q$

Density, $\rho = 950kg/m^3$

Discharge, $Q = 50/\rho = 50/950 = 0.0523\ m^3/s$

Kinematic viscosity, $\nu = 2.1$ stokes $= 2.1\ cm^2/s$

$= 2.1*10^{-4}\ m^2/s$

Height of upper end = 40m

Pressure at upper end = atmospheric = 0

Reynolds number, $R_e = V*d/\nu$

Where V=Discharge/area= $0.0526/3.14*0.3^2=0.744$ m/s

$R_e = 0.744*0.30/2.1*10^{-4}=1062.8$

Coefficient of friction, f =16/R_e=16/1062.8 = 0.015

Head lost due to friction, $h_f = 4*f*L*V^2/d*2g$

$h_f = 4*0.015*3200*0.744^2/0.3*2*9.81 = 18.05$ m of oil

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$p_2 = 0$, $h_f = 18.05$m

But $Z_1 = 0$, $Z_2 = 40$m, $V_1 = V_2$ as diameter is same

Substituting these values, we have

$\frac{P_1}{\rho g} = 40+18.05 = 58.05$ m of oil

$p_1 = 58.05*\rho\ g = 58.05*950*9.81$ [ρ for oil =950]

= 540997 N/m^2 =540997/10^{-4} N/cm^2

= 54.099 N/cm^2. Ans

H.G.L and T.E.L

$V_2/2g = 0.744^2 / 2 \times 9.81 = 0.0282$ m

H.G.L and T.E.L

$V_2/2g = 0.744^2 / 2 \times 9.81 = 0.0282$ m

$P_1/\rho g = 58.05$ m of oil

$P_2/\rho g = 0$

Draw a horizontal line AX as shown in fig.8 . From A, draw a centre line of the pipe in such a way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05m. Join B with C. Then BC is the hydraulic gradient line .

Draw a line DE parallel to BC at a height of 0.0282 m above the hydraulic gradient line. Then DE is the total energy line.

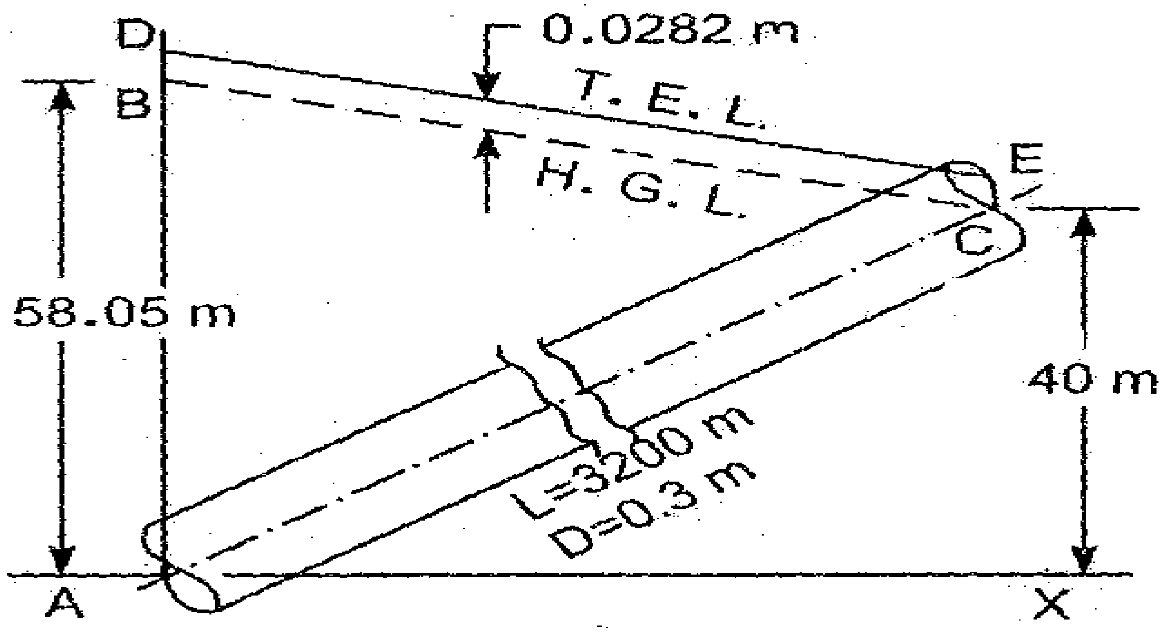


Fig 8

5)Two reservoirs A and C having a difference of level of 15.5 m are connected by a pipe line ABC the elevation of point B being 4.0 m below the level of water in reservoir A. The length AB of the pipe line is 250 m, the pipe being made of mild steel having a friction co-efficient f_1 , while the length BC is 450 m the pipe having made of cast iron having a friction co-efficient f_2. Both the lengths AB and BC have a diameter of 200mm. A partially Closed valve is located BC at a distance of 150m from reservoir C. If the flow through the pipeline is 3m3/min, the pressure head at B is 0.5m and the head loss at the valve is 5.0m.

Find the friction co- efficient f_1 and f_2; Draw the hydraulic grade line of the pipe line and indicate on the diagram head loss values at significant points. Take into account head loss at entrance and exit points of the pipeline.

Solution:

Difference of water level between two reservoirs = 15.5m

Diameter of the pipeline ABC, D=200mm=0.2m

Length AB, L_{AB}=250m

Length BC, L_{BC}=450m

Discharge through the pipe,$Q=3m^3/min=0.05m^3/sec$

Pressure head at B,

$h_B= (p_B/w) = 0.5m$

Head loss the valve =5.0m

Friction co-efficients f_1 and f_2:

Velocity in the pipes ABC,

$V=Q/A=1.59$ m/sec

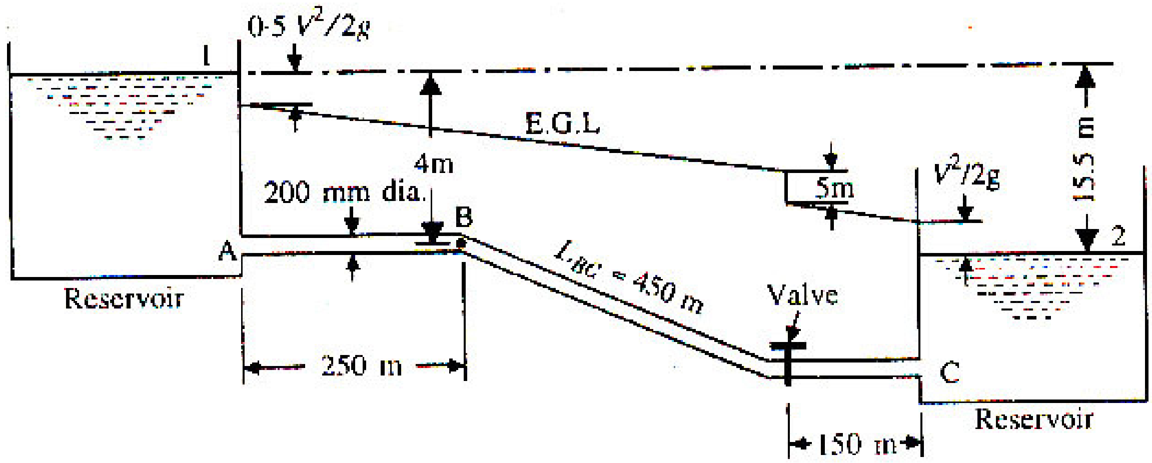
Applying Bernoulli's equation at 1 and at B we get

$P_1/w+V_1^2/2g+z_1 = p_B/w+V_B^2/2g+z_2(0.5\ V_B^2/2g+(h_f)_{AB}$

$f_1=0.0051$

Applying Bernoulli's equation between 1 and 2 and considering all losses in the pipeline ABC in the exit loss, we have

$p_1/w+V_1^2/2g+z_1=p_2/w+V_2^2/2g+z_2+0.5V^2/2g+(4f_1L_{AB}V^2/D*2)+ (4f_2L_{BC}V^2/D*2g)+5.0+V^2/2g.$



$15.5=0.0644+3.28+1159.6f_2+5.0+0.1288$

$f_2=0.0066$

H.G.L (hydraulic gradient line):

Above figure shows the E.G.L(energy gradient line), H.G.L will be $V^2/2g$ below the E.G.L

6) The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if the co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively, considering:

(i) Minor losses

(ii) Neglecting minor losses

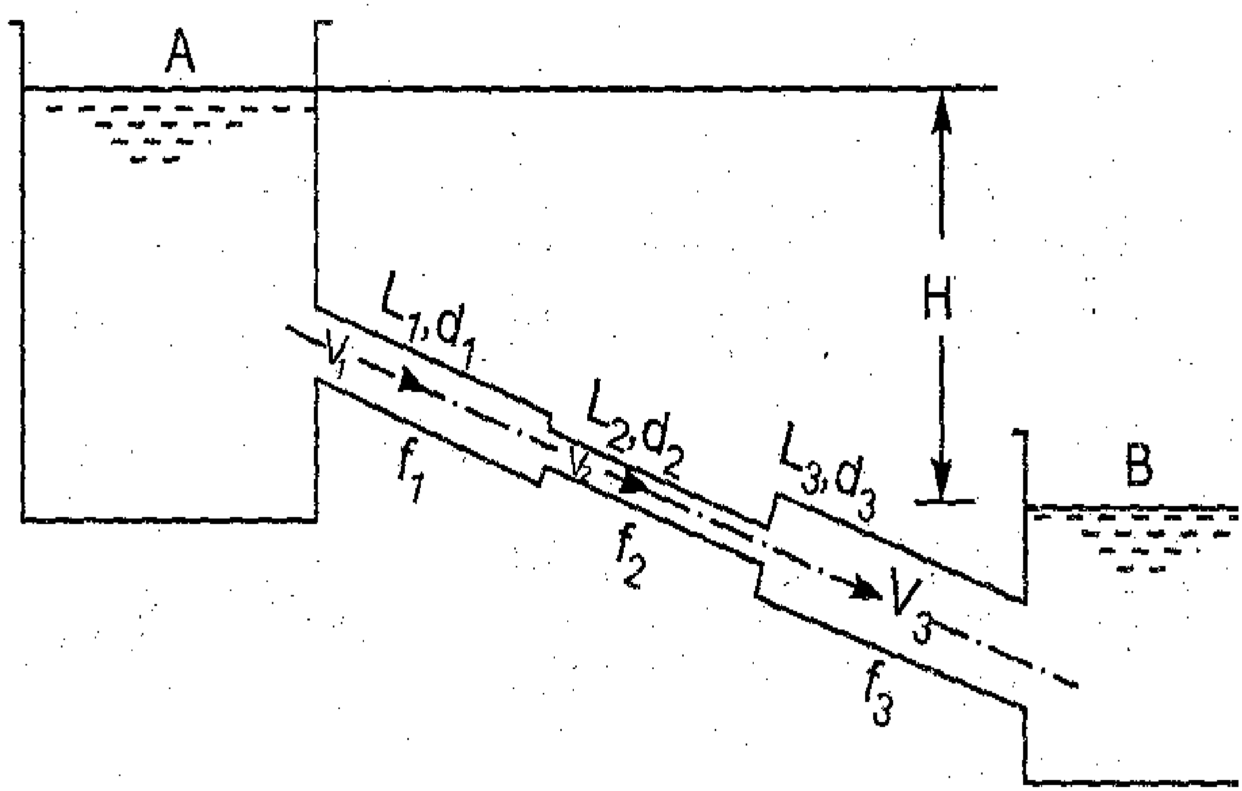


Fig 9

Solution

Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and

dia. of pipe 1, $d_1 = 300$ mm $= 0.3$ m

Length of pipe 2, $L_2 = 170$ m and

dia. of pipe 2 $d_2 = 200$ mm $= 0.2$ m

Length of pipe 3, L_3 = 210 m and

dia. of pipe 3, d_3 = 400 mm = 0.4 m

Also,

$f_1 = 0.005$, $f_2 = 0.0052$ and $f_3 = 0.0048$

(i) Considering Minor Losses. Let V_1, V_2 and V_3 are the

Velocities in the first second and third pipe respectively

From continuity equation, we have

$A_1V_1 = A_2V_2 = A_3V_3$

$V_2 = A_1V_1/A_2 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} V_1 = d_1^2 / d_2^2 (V_1)$

$= (0.3/0.2)^2 * V_1 = 2.25V_1$

$V_3 = A_1V_1/A_3 = d_1^2/d_3^2(v_1)$

$= (90.3/0.4)^2\ V_1 = 0.5625\ V_1$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes

$$H = \left[\frac{0.5V_1^2}{2g} + \frac{4\ f_1\ \ L_1\ V_1^2}{d_1\ 2g} + \frac{0.5\ V_2^2}{2g} + \frac{4\ f_2\ \ L_2\ V_2^2}{d_2 * 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 * f_3 * L_3 * V_3^2}{d_3 * 2g} + \frac{V_3^2}{2g} \right]$$

Substituting V_2 and V_3,

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 * 0.005 * 300 * V_1^2}{0.3 * 2g} + \frac{0.5*(2.25\ V_1^2)^2}{2g} + 4 * 0.0052 * 170$$

or

$$12.0 = \frac{V_1^2}{2g}\,[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$12 = \frac{V_1^2}{2g}[118.887]$$

$$V_1 = \sqrt{\frac{12*2*9.81}{118.887}} = 1.407 \text{ m/s}$$

Therefore,

Rate of flow, $Q = \text{Area} * \text{velocity} = A_1 * V_1$

$$= \pi/4\,(d_1)^2 * V_1 = \pi/4\,(0.3)^2 * 1.407 = 0.09945 \text{ m}^3/\text{s}$$

Q = 99.45 liters/s.

(ii) Neglecting Minor Losses. Using equation we have

$$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81$$

$$+ 4 \times f_3 \times L_2 \times v_2^2 / d_3 \times 2 \times 9.81$$

$$12.0 = \frac{V_1^2}{2g}\left(\frac{4*0.005*300}{0.3} + \frac{4*0.0052*170*2.25^2}{0.2} + \frac{4*0.0048*210*0.5625^2}{0.4}\right)$$

$$= \frac{V_1^2}{2g}[20.0 + 89.505 + 3.189] = \frac{V_1^2}{2g} * 112.694$$

$$V_1 = \sqrt{\frac{2*9.81*12.0}{112.694}} = 1.445 \text{ m/s}$$

Discharge, $Q = V_1 * A_1$

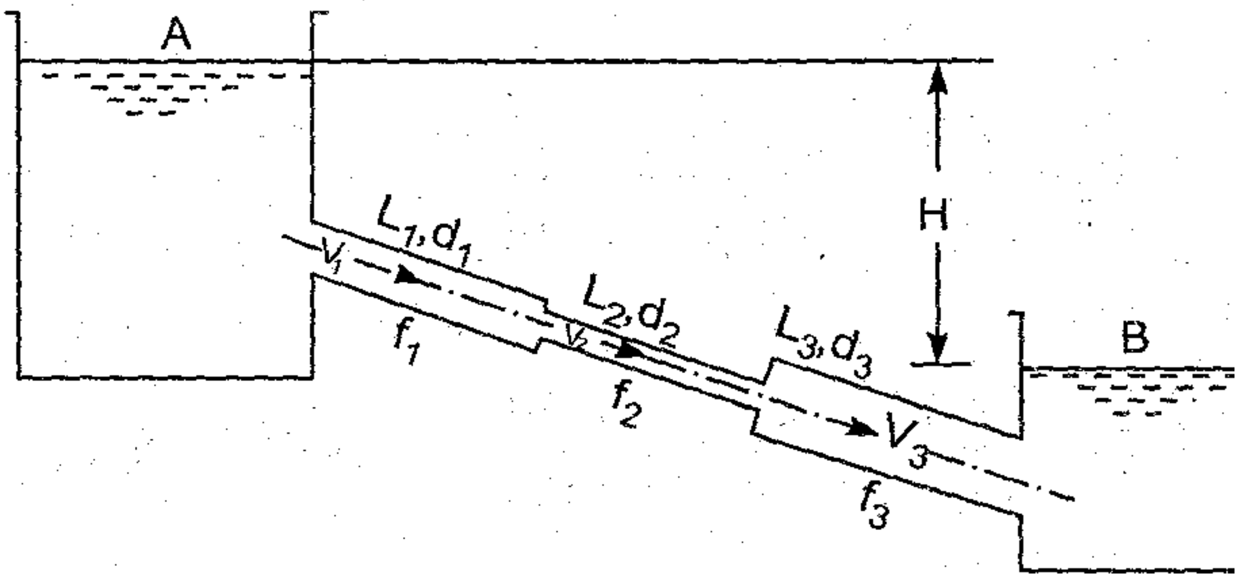
$= 1.445 * \pi / 4\,(0.3^2)$

$= 0.1021 \text{ m}^3/\text{s}$

Q = 102.1 litres /s

7) Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400m, 200m and 300 m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16 m.

If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.



Solution:

Given:

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and

$d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_1 = 200$ m and

$d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_1 = 300$ m and

$d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

Discharge through the compound pipe

(i) first neglecting minor losses

Let V_1, V_2 and V_3 are the velocities in the first, second and third pipe respectively.

From continuity equation, we have

$A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = A_1V_1/A_2 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}(V_1) = d_1^2 \ / d_2^2 (V_1)$$

$$= (0.4/0.2)^2\ V_1 = 4V_1$$

$$V_3 = A_1V_1/A_3 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_3^2}(V_1) = d_1^2 \ / d_3^2 (V_1)$$

$$= (0.4/0.2)^2\ V_1 = 1.77\ V_1$$

Using equation, we have

$h = 4 \times f_1 \times L_1 \times v_1^2 \ / \ d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 \ / \ d_2 \times 2 \times 9.81 + 4 \times f_3 \times L_2 \times v_2^2 \ / \ d_3 \times 2 \times 9.81$

$$16 = \frac{V_1^2}{2*9.81}\left(\frac{4*0.005*400}{0.4} + \frac{4*0.005*200*200*16}{0.2} + \frac{4*0.005*300*3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2*9.81}\ (20 + 320 + 63.14) = \frac{V_1^2}{2*9.81} * (403.14)$$

$$\mathbf{V_1} = \sqrt{\frac{16*2*9.81}{403.14}} = \mathbf{0.882\ m/s}$$

Therefore, Discharge, $Q = A_1 * V_1 = \pi / a * (0.4)^2 * 0.882$

$$= \mathbf{0.1108\ m^3/s}$$

Discharge through the compound pipe

(ii) Considering minor losses

Minor losses are:

(a) At inlet, $\mathbf{h_i}$ $(0.5*V_1^2)/2*g$

(b) Between first pipe and second pipe, due to contraction,

$h_c=(0.5V_2^2)/(2*g)=(0.5*(4V_1^2))/(2*g)$

$$h_c = \frac{0.5V_2^2}{2*g} = \frac{0.5*(4V_1^2)}{2*g}$$

$$h_c = \frac{0.5*16*V_1^2}{2*g} = 8 * \frac{V_1^2}{2*g}$$

(c) Between second pipe and third pipe, due to sudden enlargement,

$$h_e = \frac{(V_2 - V_3)^2}{2*g} = \frac{(4V_1 - 1.77V_1)^2}{2*g}$$

$$\frac{V_1^2}{2*g} = 4.973\frac{V_1^2}{2*g}$$

(d) At the outet of third pipe,

$$h_o = \frac{(1.77 * V_1^2)^2}{2*g}$$

$$= 1.77^2 * \frac{V_1^2}{2*g} = 3.1323\frac{V_1^2}{2*g}$$

The major losses are =

$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81 + 4 \times f_3 \times L_2 \times v_2^2 / d_3 \times 2 \times 9.81$

$$= \frac{V_1^2}{2g}\quad \frac{4*0.005*400}{0.4} + \frac{4*0.005*200*200*16}{0.2} + \frac{4*0.005*300*3.157}{0.3}]$$

$$= 403.14 * \frac{V_1^2}{2*9.81}$$

Therefore, Sum of minor losses and major losses

$$= [\ \frac{0.5V_1^2}{2*g} + 8 * \frac{V_1^2}{2*g} + 4.973\ \frac{V_1^2}{2*g} + 3.1329\ \frac{V_1^2}{2*g} + 403.14\ \frac{V_1^2}{2*g}\ ]$$

$= 419.746 \quad \frac{V_1^2}{2*g}$

But total loss must be equal to H

Therefore $419.746 * \frac{V_1^2}{2*g} = 16 \;\therefore\; V1 = \sqrt{\frac{16*2*9.81}{419.746}}$

$= 0.864$ m/s

Therefore Discharge, $Q = A_1 V_1$

$= \pi / 4 (0.4)^2 * 0.864$

$= 0.1085$ m^3/s

Problem on equivalent pipe

8) Three pipes of lengths 800 m 500m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 170 m. Find the diameter of the single pipe.

Solution:

Given :

Length of pipe 1, $L_1 = 800$ m and

Dia., of pipe 1, $d_1 = 500$ mm $= 0.5$ m

Length of pipe 2, $L_2 = 500$ m and

Dia., of pipe 2, $d_2 = 400$ mm $= 0.4$ m

Length of pipe 3, $L_3 = 400$ m and

Dia., of pipe 3, $d_3 = 300$ mm $= 0.3$ m

Length of the single pipe, $L = 1700$ m

Let the diameter of equivalent single pipe = d

Applying equation , $\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$

$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5} = 25600 + 48828.125$$

$$+164609$$

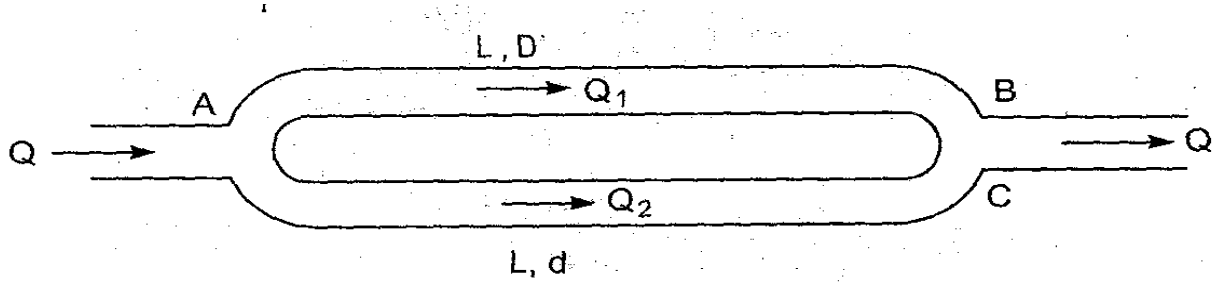
$$= 239037$$

Therefore, $d^5 = \frac{1700}{239037} = 0.007118$

Therefore $d = (0.007188)^{0.2} = 0.3718$ m

d = 371.8 mm

9).A main pipe divided into two parallel pipes which again forms one pipe as shown in fig. 3. The length and diameter for the first parallel pipe are 2000m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m^3/s . The co efficient of friction for each parallel pipe is same and equal to 0.005.



Solution:

Given:

Length of pipe 1,

Dia of pipe 1,

Length of pipe 2,

Dia of pipe 2 $d_2 = 0.8\ m$

Total flow, $Q = 3.0\ m^3/s$

$f_1 = f_2 = f = .005$

Let $Q_1 = discharge\ in\ pipe\ 1$

$Q_2 = discharge\ in\ pipe\ 2$

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes =

We have, $Q = Q_1 + Q_2$ ……(i)

$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81$

or $\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8}$ $or\ V_1^2 = \frac{V_2^2}{0.8}$

Therefore $V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$

Now $Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4}(1)^2 \times \frac{V_2}{.894}$ [As $V_1 = \frac{V_2}{.894}$)

and $Q_1 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4}(0.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$

Substituting the value of Q_1 and Q_2 in equation (i),we get

$$\frac{\pi}{4} \times \frac{V_2}{.894} + \frac{\pi}{4} \times .64V_2 = 3.0\ or\ 0.8785V_2 + .5026V_2 = 3.0$$

or $V_2[.8785 + .5026] = 3.0\ OR\ V = \frac{3.0}{1.3811} = 2.17\frac{m}{s}.$

Substituting this value in equation (ii)

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{.894} = 2.427\ \frac{m}{s}.$$

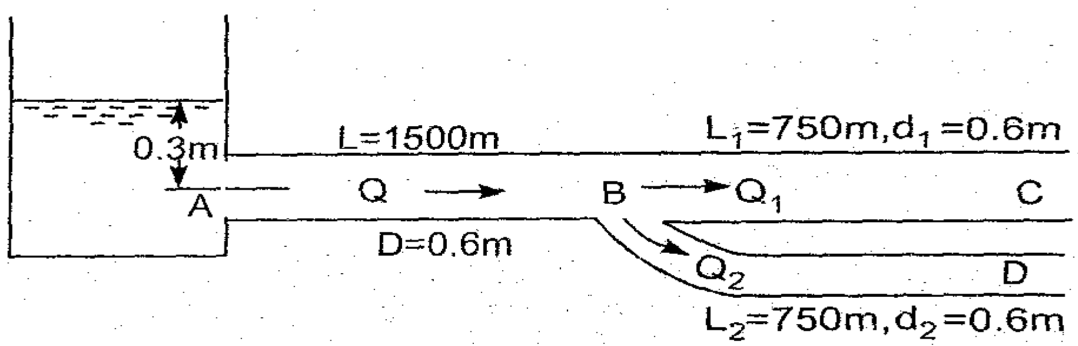
Hence $Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427$

$= \mathbf{1.906}\ \frac{m^3}{s}.\ \boldsymbol{Ans.}$

$Q_2 = Q - Q_1 = 3.0 - 1.906 =$

$\mathbf{1.094}\frac{m^3}{s}.\ \boldsymbol{Ans.}$

10). A pipe line of 0.6 m diameter is 1.5 km long. To increase the discharge, another line of the same diameter is introduced parallel to the first in the second half of the length. Neglecting minor losses. Find the increase in discharge if 4f = 0.04. The head at inlet is 300mm.



1^{st} case: Discharge for a single pipe of length 1500m and dia. = 0.6m

This loss of head due to friction in single pipe is $h_f = 4flv^{*2}/2gd$

Where v* = velocity of flow for single pipe

or $0.3 = 4 \times .01 \times 1500 \times v^{*2} / 0.6 \times 2g$

$$v^* = \sqrt{\frac{0.3*0.6*2*9.81}{4*0.01*1500}} = 0.2426\, m/s$$

Discharge Q* = v* x Area = 0.2426 x π x $0.6^2/4$ = **0.0685 m^3/s**

2^{nd} case: When an addition pipe of length 750m and diameter 0.6 m is connected in parallel with the last half length of the pipe

Let Q_1 = discharge in 1^{st} parallel pipe

Let Q_2 = discharge in 2^{nd} parallel pipe

Therefore $Q = Q_1 + Q_2$

Where Q = discharge in main pipe when pipes are parallel

But as the length and diameters of each parallel is same

$Q_1 = Q_2 = Q/2$

Consider the flow through pipe ABC or ABD

Head loss through ABC = Head lost through AB + head lost through BC ……….(ii)

But head lost due to friction through ABC = 0.3 m given

Head lost due to friction through AB = $4 \times f \times 750 \times v^2 / 0.6 \times 2 \times 9.81$

Where v = velocity of flow through AB

$= Q/\text{Area} = Q/\pi \times 0.6^2/4 = 40/\pi \times 0.36$

Head lost due to friction through AB

$$= 4 \times 0.01 \times 750/0.6 \times 2 \times 9.81 \times (4Q/\pi \times 36)^2$$

$= 31.87\ Q^2$

Head lost due to friction through BC $= 4 \times f \times L_1 \times v_1^2\ / d \times 2 \times g$

$=4 \times 0.01 \times 750/0.6 \times 2 \times 9.81\ [(Q/2 \times \pi/4 \times 0.6^2\ ]$

(as v1 = Discharge / $(\pi/4(0.6)^2 = Q/2 \times \pi/4(0.6)^2$)

$= 7.969\ Q^2$

Substituting these values in equation (ii), we get

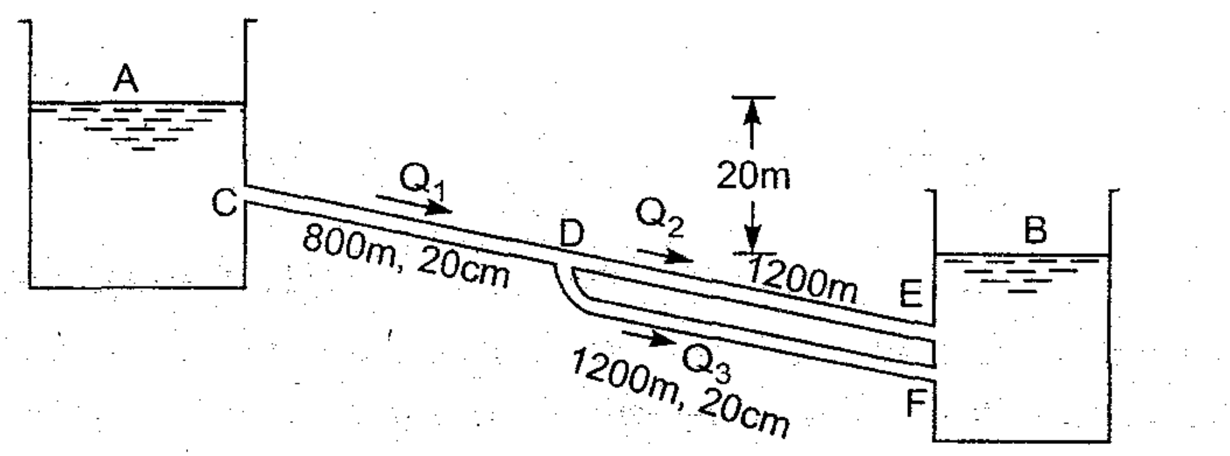
$0.3 = 31.87\ Q^2 + 7.969\ Q^2 = 39.839$

$Q = \sqrt{0.3/39.839}$

$Q = 0.0867\ m^3/s$

Increase in discharge = $Q - Q^* = 0.0867 - 0.0685$
$= 0.0182\ m^3/s$

11). A pipe of diameter 20 cm and length 2000m connects two reservoirs, having difference of water levels as 20. Determine the discharge through the pipe. If an additional pipe of diameter 20 cm and length 2000 m is attached to the last 1200m length of the existing pipe, find the increase in the discharge . Take f = 0.015 and Neglect minor losses



1st case: When a single pipe connects the reservoirs

$H = 4\,f\,L\,v^2/2gd = 4f\,LV^2/2gd\,(Q/\,\pi\,/4\,d^2\,)^2$

$[\text{ as } V = Q/\,\pi \times d^2/4\,]$

$= 32\,f\,L\,Q^2/\,\pi^2\,g\,d^5$

$20 = 32 \times 0.015 \times 2000 \times Q^2\,/\pi^{\,2} \times 9.81 \times (0.2)5$

$Q \; = 0.0254\ m^3/s$

2nd case:

Let Q_1 = discharge through pipe CD

Q_2 = discharge through pipe DE

Q_3 = discharge through pipe DF

Length of pipe CD, L_1 = 800m and its dia, d_1 = 0.20 m

Length of pipe DE, L_2 = 800m and its dia, d_2 = 0.20 m

Length of pipe DF L_3 = 800m and its dia, d_3 = 0.20 m

Since the diameters and lengths of the pipes DE and DF are equal. Hence Q_2 will be equal to Q_3.

Also, for parallel pipes, we have

$Q_1 = Q_2 + Q_3 \; = Q_2 + Q_2 \; = \; 2Q_2$ **[as $Q_2 = Q_3$]**

Therefore ,$Q_2 = Q_1/2$

Applying **Bernoulli's equation to points A and B and taking the flow through CDE,** we have

$20 = 4 \times f \times L_1 \times v_1^2\; / d_1 \times 2 \times 9.81 + 4 \times f \times L_{\,2} \times v_2^2\; / d_2 \times 2 \times 9.81$

Where $v_1 = Q_1/\,\pi \times 0.2^2/4 = 4 \times Q_1 / \pi \times 0.04$

$v_2 = Q_2/\,\pi \times 0.2^2/4 = 4 \times Q_2 / \pi \times 0.04 = 4 \times Q_1/2 / \pi \times 0.04$

$= 2 \times Q_1/2 / \pi \times 0.04$

$$= 4 \times .015 \times 800/0.2 \times 2 \times 9.81 \times (4 \times Q_1 / \pi \times 0.04)^2 + 4 \times .015 \times 1200/0.2 \times 2 \times 9.81 \times (2 \times Q_1 / \pi \times 0.04)^2$$

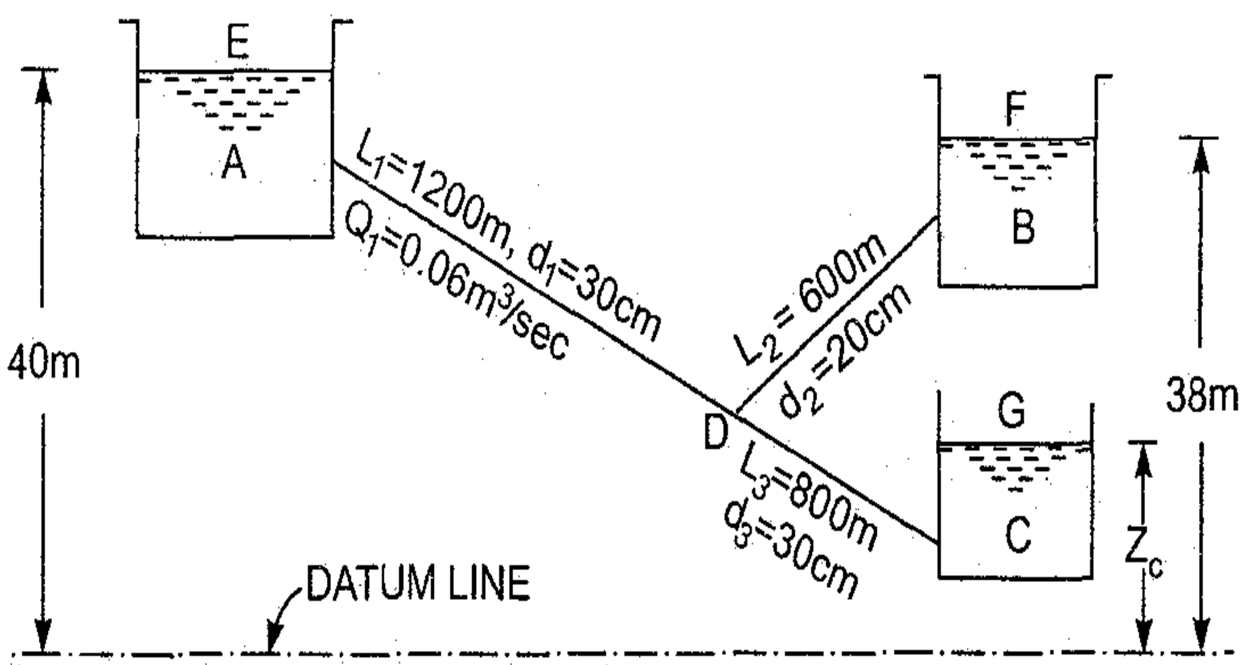
$$= 12394\, Q_1^2 + 4647\, Q_1^2 = 17041\, Q_1^2$$

$$Q_1 = \sqrt{20/17041}$$

Increase in discharge = $Q_1 - Q = 0.0342 - 0.0254$
$= 0.0088\ m^3/s$

Flow Through branched pipes

12) Three reservoirs A, B, and C are connected by a pipe system shown in the fig 17.find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 liters/s. **Find the height of water level in the reservoir C.** Take f=.006 for all pipes.



Solution:

Given:

Length of pipe AD, L_1=1200m

Dia. of pipe AD,

D_1 = 30 cm =0.30 m

Discharge through AD, Q_1 = 60 liters = 0.06 m^3/s

Height of water level in A from reference line, z_A = 40m

For pipe DB, length, L_2 = 600m, d_2 = 20cm=0.20m, Z_B = 38.0

For pipe DC, length, L_3 = 800m, d_3 = 30cm=0.30m

Appling Bernoulli's equations to point E and D,

Where

$$Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1}$$

where $h_{f_1} = \frac{4.f.L_1.V_1^2}{d_1 \times 2g}$

Where $V_1 = \frac{Q_1}{Area} = \frac{0.06}{\frac{\pi}{4}(0.3)^2}$ $0.848\frac{m}{sec}$.

$$h_f = \frac{4 \times 0.006 \times 1200 \times .848^2}{0.3 \times 2 \times 9.81} = 3.518 \text{ m}$$

Therefore

$$Z_A = Z_D + \frac{p_D}{\rho g} + 3.518$$

$$40.0 = Z_D + \frac{p_D}{\rho g} + 3.518$$

$$\therefore \left(Z_D + \frac{p_1}{\rho g}\right) = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482.

but Z_B=38m

Hence water flows from B to D.

Appling **Bernoulli's equations to point B and D**

$$Z_B = \left(Z_D + \frac{p_D}{\rho g}\right) + h_{f_2}$$

$$38 = 36.482 + h_{f_2}$$

$$\therefore h_{f_2} = 38 - 36.482 = 1.518\ m$$

But $h_{f_2} = \frac{4.f.L_2.V_2^2}{d_2 \times 2g} = \frac{4 \times 0.006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$

Therefore $1.518 = \frac{4 \times 0.006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$

$$V_2 = \sqrt{\frac{1.518 \times 0.2 \times 2 \times 9.81}{4 \times .006 \times 600}}$$

$$= 0.643 \frac{m}{s}.$$

Therefore, discharge,

$$Q_2 = V_2 \times \frac{\pi}{4}(d_2)^2 = 0.643 \times \frac{\pi}{2} \times (0.2)^2$$

$Q_2 = 0.0202\ m^3/s$ = **20.2 liters/s**

Appling Bernoulli's equations to point D and C,

$$Z_D + \frac{p_D}{\rho g} = Z_c + h_{f_3}$$

or $36.482 = \frac{4.f.L_3.V_3^2}{d_3 \times 2g}$

where, $V_3 = Q_3 / (\pi/4 \text{ x } d_3^2)$

But from continuity $Q_1 + Q_2 = Q_3$

$$Q_3 = Q_1 + Q_2 = 0.06 + .0202 = .0802\ m^3/s$$

$$V_3 = \frac{Q_3}{\frac{\pi}{4}(0.3)^2} = \frac{0.0802}{\frac{\pi}{4}(.09)}$$

$$= 1.134\ m/s$$

Therefore $36.482 = Z_C + \frac{4 \times .006 \times 800 \times 1.134^2}{0.3 \times 2 \times 9.81} = Z_C + 4.194$

$$Z_C = 36.482 - 4.194 = \mathbf{32.288\ m. Ans.}$$

Unit 7: Intoduction to compressible flow

Introduction :

Consider a *ideal gas equation*

$$\rho = \frac{p}{RT}.$$

It seen that density is depends directly on pressure and inversely on temperature. Thus density changes in the flow can in fact occur . Such flows called compressible flows

Compressible flow is defined as the flow in which the density of the fluid **does not remain constant during flow**. This means that the density changes from point to point in compressible flow. But in case of incompressible flow, the density of the fluid is assumed to be constant. In fluid flow measurements, flow passed immersed bodies, viscous flow etc,

Attributes of Compressible Flow

- **Density can no longer be regarded as constant.**
- **Bernoulli's principle** doesn't hold for compressible flow.
- **Coupling between Internal energy and Kinetic energy** can

no longer be ignored.

- **The change in density of a fluid** is accompanied by the changes in pressure and temperature and hence the thermodynamic behavior of the fluid will have to be taken into consideration

A study of compressible flow is so important because of the wide range examples that exist:

- natural gas piped from producer to consumer,
- high speed flight through air,
- discharging of compressed gas tanks,
- flow of air through compressor,

- flow of gases/steam through turbine, in machines , and many others
- Flow of gases through orifices and nozzles,
- Projectiles and airplanes flying at high altitudes with high

velocities, the density of the fluid changes during the flow.

Basic Thermodynamic Relations

(1)Equation of state- is defined as the equation which gives the relationship between the pressure, temperature ans specific volume of gas. For the perfect gas, the equation of state is

where

Vs = Specific volume or volume per unit mass = 1/ρ

p = Absolute pressure of gas in kgf/m^2 abs

T = Absolute temperature=(t+273)˚C ,absolute =Degrees Kelvin (˚K)

R = Gas constant in kgf-m/kg ˚K or (j/kg K)

- The value of **gas constant R is different for each gas**. For air having specific weight w of 1.293 (12.68) at a pressure of 760 mm of Hg (or 10,332 kgf/ or 101,300) and temperature 0˚C, the gas constant will be

$$R = \frac{pV_s}{T} = \frac{p}{\gamma T} = \frac{10,332}{1.293*273} \text{ or } \frac{101,300}{12.68*273}$$

Therefore **R = 29.27** if the value of R is given as 29.27 kgf-m/kg ˚K for air, value of **p and ρ** should be taken Kgf/m^2 and kg/m^3

- If the weight of gas is known as W, then total volume of the gas is v. Then equation (1) can be written in the form

$$pv = WRT \qquad \text{………. (2)}$$

In addition to above equation of state, certain other basic relationships of thermodynamics are also required. They are briefly described below for the purpose of revision.

2 (a) Isothermal Process - In this process the temperature remains always constant. It means that when a gas is compressed sufficient time is allowed to dissipate the amount of heat generated due to compression, to the atmosphere.

Equation of state for this case is

pv_s= constant or

$p_1v_{s1} = p_2v_{s2}$ = constant….(when t = constant) …….(3)

which is also known as Boyle's Law.

(b) Adiabatic process- In this process no heat is added or taken away from the flow system by its surroundings, that is, in this process, the expansion or compression of gas is done without allowing sufficient time. A good insulated system is an example from which neither heat can go out nor enter in. The relationship between pressure and specific volume is given by the following relationship:

$$(\Upsilon = k) = \text{Bulk modulus} = 1.4 \text{ for air}$$

Pv_s^k = constant

or $p_1 v_{s1}^k = p_2 v_s^k$ = constant ……4

$$\frac{p_1}{p_2} = \left(\frac{v_{s2}}{v_{s1}}\right)^k \quad \text{…. (4.a)}$$

The **equation of state** gives the following relationship for this process

$$\frac{p_1}{p_2} = \left(\frac{V_{s2}}{V_{s1}}\right)^k \quad \textbf{………….. (5)}$$

This is also known as Charles' Law.

Substituting the value of $\frac{p_1}{p_2}$ from Equation (4.a) in (5), the following relationships are obtained : $\frac{T_1}{T_2} = \left(\frac{V_{s2}}{V_{s1}}\right)^{k-1}$ … (6)

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} \quad \text{…..(7)}$$

(c) specific Heats- In above equations, k is given by

$k = c_p/c_v$

Where cp =specific heat at constant pressure and

c_v = specific heat at constant volume

(a)c_p is defined as that amount of heat which is required to change the temperature of a unit weight of fluid throug 1˚C, when **pressure remains constant.**

$$c_p = (\frac{1}{k-1})^R \qquad .....(8)$$

(b) c_v is defined as the amount of heat required to change the temperature of a unit weight of fluid through 1˚C when **volume remains constant**

$$c_v = (\frac{1}{k-1})^R \qquad ......(9)$$

<u>Physical properties of Various Gases</u>

Gas	Chemical formula	c_p	c_v	K	R
Air	---	0.242	0.171	1.4	29.57
Carbon dioxide	CO_2	0.203	0.156	1.3	19.50
Carbon monoxide	CO	0.244	0.174	1.4	30.64
Hydrogen	H_2	3.410	2.420	1.41	425.00
oxygen	O_2	0.217	0.155	1.39	26.80
Nitrogen	N_2	0.244	0.173	1.4	30.60
Water vapour	H_2O	0.451	0.339	1.33	47.55

Entropy is defined as a property of a gas which **measures the availability of heat energy for conversion into work.** It is denoted by **S**. It cannot be measured by instruments but like moment of inertia it is quite real and very useful in thermodynamic calculations. **Like heat it has no**

definite value, but it is measured above an arbitrary chosen datum. Its absolute value is not important. The change of entropy during a given process is a quantity which is of interest.

If $\mathbf{d_\theta}$ is the amount of heat absorbed or transferred by a unit weight of gas in a small time interval and **T** is the absolute temperature of gas at that instant, then the change of entropy during the process is

$$d_s = \frac{d\theta}{T} \quad \text{and} \quad s_2 - s_1 = \int_{T_1}^{T_2} \frac{d\theta}{T} \qquad \text{.........(10)}$$

Basic Equations of Compressible Fluid Flow:

These are the same as continuity equation and momentum equation derived from three basic principles. The only change from incompressible fluid cases is that **thermodynamics of mass, energy and momentum.**

Equation of Continuity:

In deriving the equation $a_1V_1 = a_2V_2 = Q$ = constant, it was assumed that flowing fluid is incompressible i.e. $\rho_1=\rho_2$, hence the volumetric rate of i.e. flow volumetric discharge passing through any section

This is based on law of conservation of mass which states that matter cannot be created nor be destroyed. Or in other words, the matter or mass is constant. For 1-D steady flow, the mass per second = ρAV

Where ρ = mass density, A = area of cross section, V= velocity

As mass or mass per second is constant according to law of conservation of mass, Hence

ρAV = constant ----(a)

Differentiating eqn. (a), $d(\rho AV) = 0$ **or** $\rho \mathbf{d(AV)} + AV\mathbf{d\,\rho} = 0$

or $\rho[AdV + VdA] + AV\,d\,\rho = 0$ **or** $\rho AdV + AVd\,\rho = 0$

Dividing by ρAV, we get $\mathbf{dV/V + dA/A + d\,\rho/\,\rho = 0}$ **-→ continuity equation in differential form**

Bernoulli's equation

Total energy = pressure energy (p/ρ)+ potential or elevation energy (z) +kinetic energy ($V^2/2g$)

In case of compressible flow , with the change of density ρ, the pressure p also changes for compressible fluids.The Bernoulli's equation will be different for **isothermal process** and for **adiabatic process**

(a) Bernoulli's equation for Isothermal process:

For isothermal process, the relation between pressure(p) and density(ρ) is given by equation

p/ρ = constant

Finally, $p/\rho g \log_e p + V^2/2g + Z$ = constant

Bernoulli's equation for compressible flow under going Isothermal process.

For the two points 1 and 2, this equation is written as

$$\mathbf{p_1/\rho_1 g \log_e p_1 + V_1^2/2g + Z_1 = p_2/\rho_2 g \log_e p_2 + V_2^2/2g + Z_2}$$

(b) Bernoulli's equation for Adiabatic (or Isentropic) Process:

For adiabatic process, the relation between pressure(p) and density(ρ) is given by equation

p/ρ^k = constant

Finally,
$$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g}+V_1^2+Z_1=\left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g}+V_2^2+Z_2$$

Bernoulli's equation for compressible flow under going Isothermal process

For the two points 1 and 2, this equation is written as

$$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g}+V_1^2+Z_1=\left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g}+V_2^2+Z_2$$

Momentum Equations:

The momentum per second of a flowing fluid (momentum flux) Is equal to the product of mass per second and the velocity of the flow.

Mathematically, the momentum per second of a flowing fluid (compressible or incompressible) is = ρAV x **V, where** ρAV = mass per second

The term **ρAV** is constant at every section of flow due to continuity equation. This means the momentum per second at any section is equal to the **product of a constant quantity** and the **velocity**. This also implies that momentum per second is independent of compressible effect. Hence the momentum equation for incompressible and compressible fluid is the same. The momentum

Equation for compressible fluid for any direction may be expressed as,

Net force in the direction of S

= rate of change of momentum in the direction of S

= Mass per second [change of velocity]

= $\rho AV [V_2 - V_1]$

where v2 = Final velocity in the direction of S

V1 = Initial velocity in the direction of S

Propagation of Disturbances in Fluid and Sonic Velocity of Flow:

Both solid and fluid as transmitting media consist of molecules with a difference that in case of a solid the molecules are close together and in a fluid the molecules are relatively apart. Whenever a minor disturbance takes place, it is transmitted through a solid body instantaneously, but in case of fluid its molecules change in position before the disturbance is transmitted or propagated.

Thus the propagation of disturbance depends upon the elastic properties of a fluid. **This propagation of disturbance is similar to the propagation of sound through a media.** The speed of propagation of sound in a media is known as acoustic or sonic velocity which is due to pressure difference. The **sonic velocity** is considered as an important factor in compressible flow.

Derivation of sonic velocity:

Consider one-dimensional flow through a long straight cylinder made of rigid material and having uniform thickness (refer Fig 2). Let a frictionless piston work in the cylinder with **velocity v**. It compresses the fluid and produces disturbance propagating along its length in the form of a pressure wave travelling with the **velocity of sound C.**

Let A = cross sectional area of the pipe

V = velocity of the piston

p = pressure of the fluid in pipe before movement of the

piston

ρ = Density of fluid before the movement of the piston

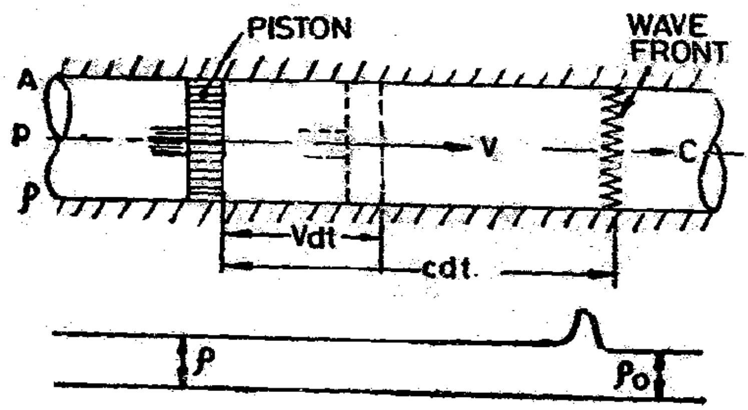


Fig 2. One dimensional pressure wave propagation

Let C = Velocity pressure wave or sound wave travelling in fluid

vdt = distance travelled by the piston in time dt

dp = Increase in pressure

Dρ =Increase in fluid density between piston and wave front

dt = A small interval of time with which piston is moved

Neglect friction and heat transfer, if any.

By applying momentum equation

$$(p + dp)dA - pdA = dA\rho c\{c - (c - dv)\}$$

$$dp = \rho c dv \qquad \text{……(a)}$$

Now, ρVA = constant ….Equation of continuity

(neglecting terms of higher order)

i.e $\rho vA = (\rho + d\rho)(c - dv).A$

or

$c d\rho = \rho dV$ (neglecting terms of higher order)

Therefore $$dv = \frac{c.\,d\rho}{\rho}$$

Substituting the value of **dv** in eqn. (a)

$$dp = \rho c.\,c\frac{d\rho}{\rho} = c^2 d\rho$$

Or $$C = \sqrt{\frac{dp}{d\rho}} \qquad \text{........(b)}$$

Equation (b) gives the velocity of sound wave which is the square root of the ratio of change of pressure to the change of density of a fluid disturbance

Velocity of sound in terms of bulk modulus.

Bulk modulus k is defined as K = (Increase in pressure/(Decrease in volume/Original volume)

$= dp/-(dv_s/v_s)$ …(c)

Where dv_s = Decrease in volume,

v_s= Original volume

Negative sign is taken as with the increase of pressure, volume decreases

Now we know mass of the fluid is constant. Hence ρ * volume = constant (since mass = ρ * volume) $\rho * v_s$= constant

Differentiating the above equation (ρ and v_s are variables)

$\rho dv_s+v_s d\rho = 0$ **or** $\rho dv_s= -v_s d\rho$ **or** $dv_s/v_s= d\rho/\rho$

substituting the value $(-dv_s/v_s)$ in equation (c), we get $K=dp/(d\rho/\rho) = \rho(dp/d\rho)$ **or** $(dp/d\rho) = (K/\rho)$

the velocity of sound wave is given by

$$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \qquad \text{.....(d)}$$

Equation (d) gives the velocity of sound wave in terms of bulk modulus and density. This equation is applicable for liquids and gases.

Velocity of sound for isothermal process

For isothermal process , p/ρ = constant

$p\ \rho^{-1}$ = constant

Differentiating the above equation, we get

$P^{(-1)}\ \rho^{-2}\ d\rho + \rho^{-1}\ dp = 0$

Dividing by ρ^{-1} , we get $-p\ \rho^{-1}\ d\rho + dp$ **or** $-p\ d\rho + dp = 0$

$dp = p/\rho\ dp$ **or** $dp/d\rho = p/\rho = RT$ (as $p/\rho = RT$…Eqn. of state)

Substituting the value of **dp/dρ** in equation $C = \sqrt{\dfrac{dp}{d\rho}}$

We get $C = \sqrt{\dfrac{K}{\rho}} = \sqrt{RT}$

Velocity of sound for adiabatic (isentropic process)

p/ρ^{k} = constant

$p\ \rho^{-k}$ = constant

Differentiating the above equation, we get

$P^{(-k)}\ \rho^{-k-1}\ d\rho + \rho^{-k}\ dp = 0$

Dividing by ρ^{-k} , we get $-p^{k}\ \rho^{-1}\ d\rho + dp$ or $dp = p^{k}/\rho\ d\rho$

$dp/d\rho = p/\rho\ k = RTk$ (as $p/\rho = RT$…Eqn. of state)

Substituting the value of **dp/dρ** in equation $C = \sqrt{\dfrac{dp}{d\rho}}$

We get $c = \sqrt{kRT}$ **…….(e)**

Note 1. For the propagation of the minor disturbances through air, the process is assumed to be adiabatic. The velocity of disturbances (pressure wave) through air is very high and hence there is no time for any appreciable heat transfer.

2. isothermal process is considered for calculation of the velocity of the sound waves (or pressure waves) only when it is given in the numerical problem that process is isothermal. If no process is mentioned, it is assumed to be adiabatic.

Mach number:

Mach number is defined as the square root of the inertia force of a flowing fluid to the elastic force.Then,

Mach number = $M = \sqrt{\frac{Inertiaforce}{Elasticforce}} = \sqrt{\frac{\rho A V^2}{KA}}$

$$= \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \qquad \because \sqrt{\frac{K}{\rho}} = C$$

Thus mach number = M [From Equation eqn. (d)]

Mach number, M = Velocity of fluid or body moving in fluid / Velocity of sound in the fluid

$$\mathbf{M} = \frac{\mathbf{V}}{\mathbf{c}}$$

If the Mach number of fluid flow is less than one($M<1$), it is flowing with a velocity which is less than velocity of sound.Such a flow is called subsonic flow. For $M>1$, the flow is known as Supersonic flow. For $M=1$, the flow is sonic flow.

Problems on Mach number

1)Find the **sonic velocity** of the following fluid :

(i)Crude oil of specific gravity 0.8 and bulk modulus 153036 N/cm^2

(ii)Mercury having a bulk modulus of 2648700 N/cm^2

Solution:

Given:

(i) Crude oil: Specific gravity =0.8

Therefore density of oil, $\rho = 0.8 * 1000 = 800\ kg/m^3$

Bulk modulus, $K = 153036\ N/cm^2 = 153036 * 10^4\ N/m^2$

Using the equation for sonic velocity, as

$C = \sqrt{(k/\rho)} \quad = \sqrt{((153036 * 10^4)/800)}$

= 1383.09 = **1383 m/s**

(ii) Mercury: Bulk modulus, K= 2648700 N/cm^2 = 2648700 * 10^4 N/m^2

Specific gravity = 13.

Density of mercury, ρ =13.6 * 1000 =13600 kg/m^3

The sonic velocity, C is : given by C = $\sqrt{(k/\rho)}$

= $\sqrt{((2648700 * 10^4)/13600)}$

= **1395.55 m/s**

2) Find the **sonic velocity** for the following fluids:

(i) Crude oil of specific gravity 0.8 and bulk modulus 1.5 GN/m^2

(ii) Mercury having a bulk modulus of 27 GN/m^2

Solution:

Crude oil: Specific gravity =0.8

Therefore density of oil, ρ =0.8 * 1000 =800 kg/m^3

Bulk modulus, K = 1.5 GN/m^2

Mercury: Bulk modulus, K= 27 GN/m^2

Density of mercury, ρ =13.6 * 1000 =13600 kg/m^3

Sonic velocity, C_{oil} , C_{Hg} :

Sonic velocity is given by the relation :

C = $\sqrt{(k/\rho)}$

C_{oil} = $\sqrt{((1.5 * 10^9)/800)}$ = 1369.3 m/s

C_{Hg} = $\sqrt{((27 * 10^9)/13600)}$ = 1409 m/s

3) Find the **speed of the sound wave** in air at sea-level where the pressure and temperature are 10.1043 N/cm^2 (absolute) and 15^{o}C respectively. Take R =287 J/kg K and k=1.4.

Solution: Given :

Pressure, $p = 10.1043 \text{ N/cm}^2$

$= 10.1043 * 10^4 \text{ N/m}^2$

Temperature, $t = 15^oC$

Therefore $T = 273+15 = 288$ K

$R = 287$ J/kg K, $k=1.4$.

For **adiabatic process**, the velocity of sound is given by

$$C=\sqrt{(kRT)} = \sqrt{(1.4 * 287 * 288)}$$

$$= \mathbf{340.17\ m/s}$$

4) Calculate the **Mach number** at a point on a jet propelled aircraft, which is flying at 1100 km/hour at sea level where air temperature is 20^oC. Take $k = 1.4$ and $R = 287$ J/kg K.

Solution: Given :

Speed of aircraft, $V = 1100$ km/hour

$= (1100 * 1000)/(60 * 60) = 305.55$ m/s

Temperature, $t = 20^oC$

Therefore $T = 273+20 = 293$ K

$k = 1.4$, $R = 287$ J/kg K.

The **velocity of sound** is given by the equation

$$C = \sqrt{(kRT)}$$

$$= \sqrt{(1.4 * 287 * 293)}$$

$$= \mathbf{343.11\ m/s}$$

Mach number is given as

$$M = (V/C) = (305.55/343.11) = \mathbf{0.89}$$

5) An aero plane is flying at an height of 14 km where the temperature is -50ºC. The speed of the is corresponding to M = 2.0. Assuming k = 1.4 and R = 287 k/kg K, find the speed of the plane

Solution:

Height of plane, Z = 15 km(extra data)

Temperature, t = -50oC

Therefore, T = -50+273 = 223oC

Mach number, M = 2.0, k = 1.4, R=287 j/kg K

Using equation, we get the velocity of sound as C = √(kRT)

$$= \sqrt{1.4 \times 287 \times 223} = 299.33 m/s$$

We have Mach number M = V/C

2.0= V/299.33

V = 2.0 x 299.33 = 598.66 m/s

= 598.66 x 60x60/1000 = 2155.17 km/hr

6)Find the Mach number of rocket travelling in standard air with a Speed of 1600 km/hr.

Solution:

Same in MKS and SI units

$$v = 1600 \frac{km}{hr} = \frac{1600 \times 1000}{60 \times 60} = 444.4 m/sec$$

Standard air has the following values at sea level;

$$p = 1.0332 \frac{kgf}{cm^2 \left(or\ 101.3 \frac{kN}{m^2}\right)}$$

$$t = 15°C;\ \gamma = 1.226 \frac{kgf}{m^3} (or 12\ N/m^3)$$

$$\rho = 0.125 \frac{msl}{m^3} \left(or\ 1.226 \frac{kgm}{m^3}\right)$$

$$R = 29.27 \frac{m}{°K}$$

Nature of propagation of pressure Waves or disturbances) in a compressible Fluid

Whenever any disturbance is produced in a compressible fluid, the disturbance is propagated in all direction with a Velocity of sound. The nature of propagation of the disturbance depends upon the Mach number. Let a projectile travel in a straight line with a steady velocity V. It will produce disturbance propagating in all directions.

At time $t = 0$ sec, the body is at point A

At time $t = 1$ sec, the body is at point 1, then distance $s_1 = V\Delta t$

At time $t = 2$ sec, the body is at point 2, then $s_2 = 2V\Delta t$

At time $t = 3$ sec, the body is at point 3, then $s_3 = 3V\Delta t$ etc.

With point 3 as center, draw a circle with radius CΔ t, and with point 2 as center, draw another circle with radius 2cΔt. similarly with point A as center draw circle with radius to 4c Δt. From this it will be seen that point B remains within the sphere of radius 4c

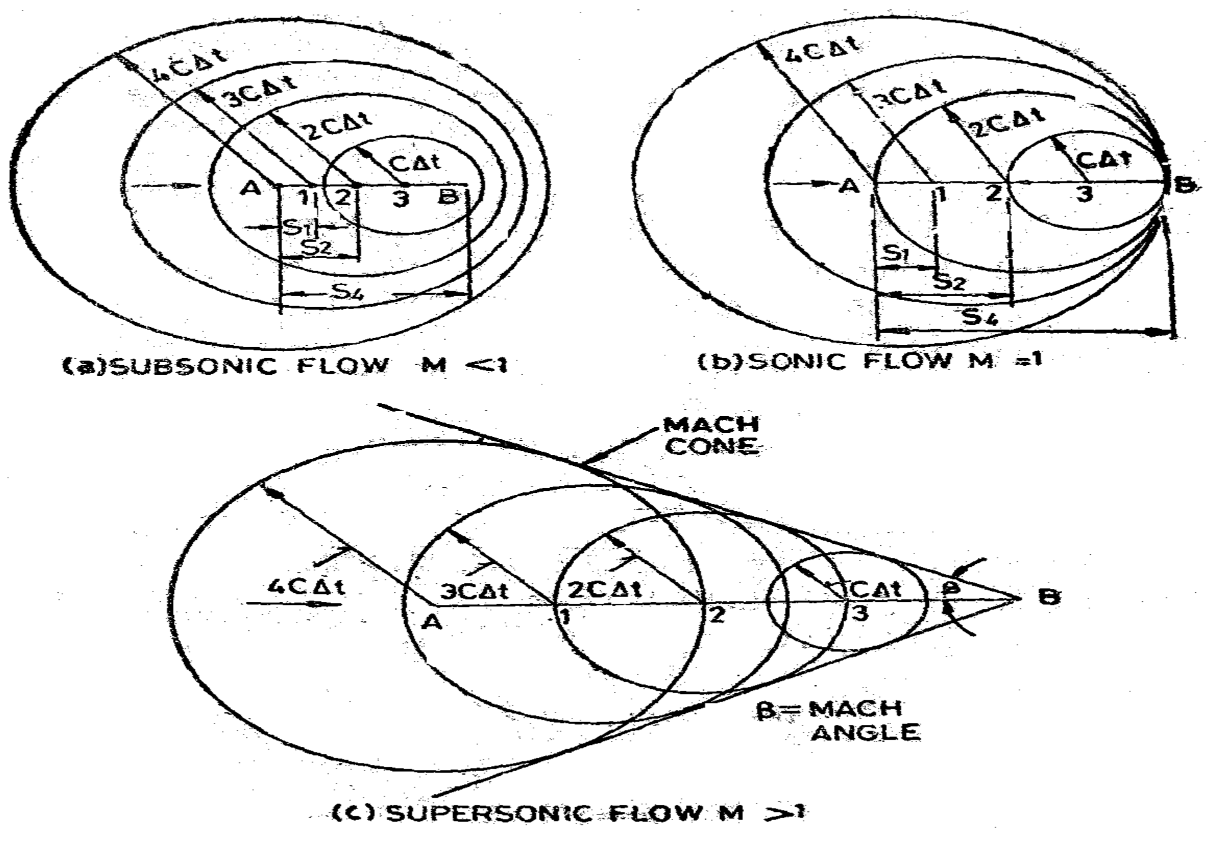


Figure 3. Nature of disturbances in compressible flow

(a) M<1, when V is less than the velocity of sound C,which means the projectile lags behind the pressure wave.

(b) M=1 Since V=C, then circles drawn join to points as shown in Fig 1.2(b). The circle drawn with center A will pass through B.

(c) M>1, i.e V>C, then the sphere of propagation of disturbance is smaller and the velocity of projectile is higher.

Drawing the circles as earlier, if tangents are drawn to the circles, the spherical pressure waves form a cone with its vertex at B. It is known as Mach cone. Half cone angle is known as mach angle and denoted by β and $\beta = \sin^{-1}\frac{c}{v} = \sin^{-1}\frac{1}{Ma}$

In such a case the disturbance takes place inside the coneand outside it there is no disturbance which is then called Silence zone. It is seen that when an aero plane is moving with supersonic speed, the noise of the plane is heard only after the plane has already passed over us.

When M >1, the effect of the disturbance is felt only in the region Inside the mach cone. This region is called the zone of action

Stagnation Properties:

When the fluid flowing past an immersed body, and at a point on the body if the resultant velocity becomes zero, the value of pressure, temperature and density at that point are called **Stagnation point.** The values of pressure, temperature and density are called **stagnation pressure, stagnation temperature and stagnation density** respectively. They are denoted as **p_s, ρs and Ts** respectively.

Expression for stagnation pressure (p_s)

Consider a compressible fluid flowing past an immersed body under frictionless adiabatic conditions.

Consider points 1 and 2 on a stream line.

Let p_1 = pressure of compressible fluid at point 1

V_1 = Velocity of fluid at 1 and

ρ_1= Density of fluid at 1

p_1, v_2 ρ_2 = corresponding values of pressure, velocity and density at point 2

By applying Bernoulli's equation for adiabatic flow by equation at 1 and 2, we get

$$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g}+V_1^2+Z_1=\left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g}+V_2^2+Z_2$$

Finally

$$p_s = p_1\left[1+\frac{k-1}{2}M_1^2\right]^{\left(\frac{k}{k-1}\right)}$$

Where P_s =stagnation pressure

Expression for stagnation Density (ρ_s)

$p_s / \rho_s = RT_s$

Expression for stagnation (T_s)

Equation fo state is given by $p_s/\rho_s = RT$

Finally

$$T_s = T_1\left[1+\frac{k-1}{2}M_1^2\right]$$

1) A projectile is travelling in air having pressure and temperature as 88.3 kN/m^2 and -2^{o}C. If the Mach angle is 40^o, find the velocity of the projectile. Take k =1.4 and R = 287 J/kg K.

Solution:

Pressure, p= 88.3 kN/m^2

Temperature of air, t= -2ºC

T = -2+273 = 271K

Mach angle, α = 40^o

k =1.4, R = 287 J/kg K.

L et **Velocity of the projectile = V**

Sonic velocity, C = √(kRT)

= √(1.4*287 *271)

= 330 m/s

Now, sinα= C/V

or $\sin 40^o = 330/V$

or $V = 330/\sin 40^o$

$\mathbf{V = 513.4\ m/s}$

2) A projectile travels in air of pressure 10.1043 N/cm^2 at 10^{o}C at a speed of 1500 km/hour. Find the **Mach number** and the **Mach angle**. Take k = 1.4 and R = 287 J/kg K.

Solution:

Given :

Pressure, $p = 10.1043$ N/cm^2

$= 10.1043 * 10^4$ N/m^2

Temperature, $t = 10^o$C

$T = 10+273 = 283$ K

Speed of projectile, V=1500 km/hour

$= (1500 * 1000)/(60 * 60)$ m/s

$= 416.67$ m/s

$k = 1.4$, $R = 287$ J/kg K.

For **adiabatic process** velocity of sound C is given by

$C = \sqrt{(kRT)}$

$= \sqrt{(1.4 * 287 * 283)}$

$= 337.20$ m/s

Therefore Mach number, $M = (V/C)$

$= (416.67/337.20)$

$= \mathbf{1.235.}$

Therefore Mach angle is obtained as

$\sin\alpha = (C/V)$

$= (1/M)$

$= (1/1.235)$

$= 0.8097$

Therefore Mach angle, $\alpha = \sin^{-1}(0.8097)$

$= \mathbf{54.06^o}$

3) Find the velocity of bullet fired in standard air if the Mach angle is 30^o. Take R = 287.14 J/kg k and k = 1.4 for air. Assume temperature as 15^oc.

Solution:

Given :

Mach angle $\alpha = 30^o$

$R = 287.14$ J/kg k

$k = 1.4$

Temperature, $t = 15^oC$

Therefore $T = 15 + 273 = 288$ k

Velocity of sound is given as

$C = \sqrt{(kRT)}$

$= \sqrt{(1.4 * 287.14 * 288)}$

$= 340.25$ m/s

Using the relation, $\sin \alpha = C/V$

$\sin 30^o = 340.25/V$

$V = 340.25/\sin 30$

$= \mathbf{680.5\ m/s}$

4)An air plane is flying at an altitude of 15 km where the temperature is -50°C. The speed of the plane corresponds to Mach number of 1.6. Assuming k= 1.4 and R = 287 J/kg K for air. Find the **speed of the plane and Mach angle α.**

Solution:

Given :

Height of plane, H = 15 km = 15 * 1000 = 15000 m

Temperature, t = -50°C

therefore T = -50+273 = 223 K

Mach number, M = 1.6, k=1.4 and R = 287 J/kg K

Find : (i) speed of plane (V)

(ii) Mach angle, α

Velocity of sound wave is given as

$C = \sqrt{(kRT)} = \sqrt{(1.4 * 287 * 223)} = 229.33$ m/s

(ii) Mach angle, α

Using the relation for Mach angle, we get

$\sin \alpha = C/V = 1/(V/C) = 1/M = 1/1.6 = 0.625$

$\alpha = \sin^{-1} 0.625 = \mathbf{38.68^{o}}$

(i) Speed of plane, V

We know, $M=V/C$

$1.6 = V/299.33$

$V = 1.6 * 299.33 = 478.928$ m/s

$= (478.98 * 3600)/1000 =$ **1724.14 m/s**

5)Find the Mach number when an aeroplane is flying at 1100 km/hour through still air having a pressure of 7 N/cm^2 and temperature -5^oC. Wind velocity may be taken as zero. Take R = 287.14 J/kg K. **calculate the pressure, temperature and density of air at stagnation point** on the nose of the plane. Take k=1.4.

Solution:

Given :

Speed of aeroplane, V=1100 km/hour

= (1100 * 1000)/(60 *60)

= 305.55 m/s

Pressure of air, p_1=7 N/cm^2 = 7*10^4 N/m^2

Temperature, t_1 = -5^oC

Therefore T_1 = -5 +273 = 268 K

R = 287.14 J/kg K

K=1.4

Using relation C = √(kRT) for velocity of sound for adiabatic process, we have

C_1 = √(1.4 *287.14 *268) = 328.2 m/s

Therefore Mach number, $M_1 = (V_1/C_1)$

= (305.55/328.20) = 0.9309 = **0.931**

Stagnation pressure, p_s. using equation for stagnation pressure,

$p_s= p_1[1+((k-1)/2)M_1^{\,2}]^{k/(k-1)}$

$= 7.0 *10^4[1+((1.4-1)/2)(0.931)^2]^{1.4/(1.4-1)}$

$= 7.0 *10^4[1+0.1733]^{1.4/0.4}$

$= 7.0 *10^4[1.1733]^{3.5} = 12.24 * 10^4\ N/m^2$

= 12.24 N/cm^2

Stagnation temperature, T_s. using the equation for stagnation temperature,

$T_s = T_1[1+((k-1)/2)\ M_1^2]$

$= 268\ [1+((1.4-1)/2)\ (0.931)^2]$

$= 268\ [1.1733] = 314.44$ k

Therefore

$t_s = T_s - 273 = 314.44 - 273 =$ **41.44 ^oC**

Stagnation density , ρ_s. Using equation of state for stagnation density, $p_s/\rho_s = RT_s$

$$\rho_s = p_s/(RT_s)$$

In the above equation given, if R is taken as 287.14 J/kg K, then pressure should be taken in N/m^2 so that the value of ρ is in kg/m^3. Hence $p_s = 12.24 * 10^4\ N/m^2$ and Ts = 314.44 k.

ρ_s $= 12.24 * 10^4/(287.14 * 314.44)$ **= 1.355 kg/m^3**

6) Calculate the stagnation pressure, temperature, density on the stagnation point on the nose of a plane, which is flying at 800 km/hour through still air having a pressure 8.0 N/cm^2 (abs.) and temperature -10 ^0C.Take R=287 J/Kg K and k = 1.4.

Solution:

Given:

Speed of plane, V = 800 km/hour

$= (800*1000)/(60*60) = 222.22$ m/s

Pressure of air, $p_1 = 8.0\ N/cm^2 = 8.0*10^4\ N/cm^2$

Temperature, $t_1 = -10\ ^0C$

$T_1 = -10 + 273 = 263^0$ C

$R = 287\ J/Kg^0\ K$

$k = 1.4$

For adiabatic flow, the velocity of sound is given by

$$C = \sqrt{(KRT)}$$

$$= \sqrt{(1.4*287*263)}$$

=325.07 m/s.

Mach number, $M = V/C$

$= 222.22/325.07 = 0.683$

This **Mach number is the local Mach number and hence equal to M_1.**

Therefore $M_1 = 0.683$

Using equation for **stagnation pressure**,

$p_s = p_1 [1+ ((k-1)/2) M_1^2] (k/(k-1))$

$= 8.0*104[1+((1.4-1.0)/2.0)*(0.683)^2](1.4/(1.4-1.0))$

$=8.0*104[1.0933]3.5 = 10.93*104$ N/m2

$= 10.93$ N/cm2

Using equation for **stagnation temperature**

$T_s = T_1 [1+ ((k-1)/2) M_1^2] =263 [1+ ((1.4-1.0)/2.0)*(0.683)^2]$

$=263 [1.0933] =287.5$ K

Therefore $t_s = T_S - 273 = 287.5 – 723$

$=14.5\ ^0C$

Using equation of state, $p/\rho = RT$

For stagnation point, $p_s/\rho_s=RT_S$

Therefore $\rho_s= p_s / RT_S$

As R =287 J/Kg K, the value of p_s should be taken in N/m^2 so that the value of ρ_s is obtained in Kg/m^3.

$p_s =10.93*10^4$ N/m^2

Therefore **Stagnation density**,

$\rho_s = (10.93*10^4) / (287*287.5)$

$= 1.324$ Kg/m^3

<u>Area-velocity Relationship for Compressible Flow</u>:

The are velocity relation ship for incompressible fluid is given by the continuity equation as A * V = constant

From the above equation, it is clear that with the increase of area, velocity decreases. But in case of compressible fluid, the continuity equation is given by, ρAV = constant ……(i)

Differentiating equation(i), we get

$\rho d(AV) + Avd\rho = 0$ or $\rho[AdV + VdA] + Avd\rho = 0$

or $\rho AdV + \rho VdA + Avd\rho = 0$

Dividing by ρAV, we get $dV/V + dA/A + d\rho/\rho = 0$ …..(ii)

The **Euler's equation for compressible fluid** is given by equation, as $dp/\rho + VdV + gdZ = 0$

Neglecting the Z term, the above equation is written as

$$dp/\rho + VdV = 0$$

This equation can also be written as

$$dp/\rho \times d\rho/d\rho + VdV = 0$$

(Dividing and multiplying by dρ)

or $dp/d\rho \times d\rho/\rho + VdV = 0$

But $\mathbf{dp/d\rho = C^2}$

Hence above equation becomes as $C^2\, d\rho/\rho + VdV = 0$

or $C^2\, d\rho/\rho = -VdV$

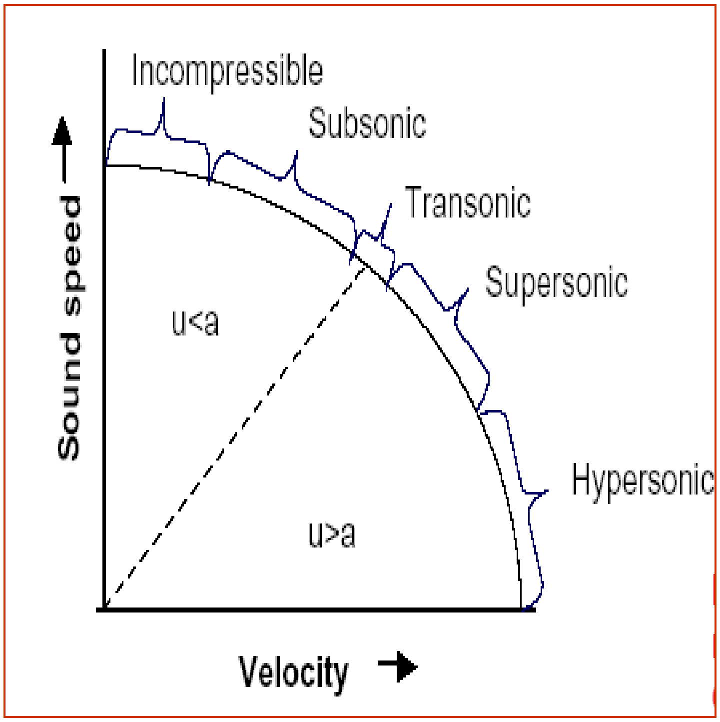
or $d\rho/\rho = VdV/C^2$

Substituting the value of **dρ/ρ** in equation (ii), we get

$dV/V + dA/A - VdV/C^2 = 0$

or $dA/A = VdV/C^2 - dV/V = dV/V[V^2/C^2 - 1]$

$$\mathbf{dA/A = dV/V[M^2-1]}$$



Ma < 1 : Subsonic

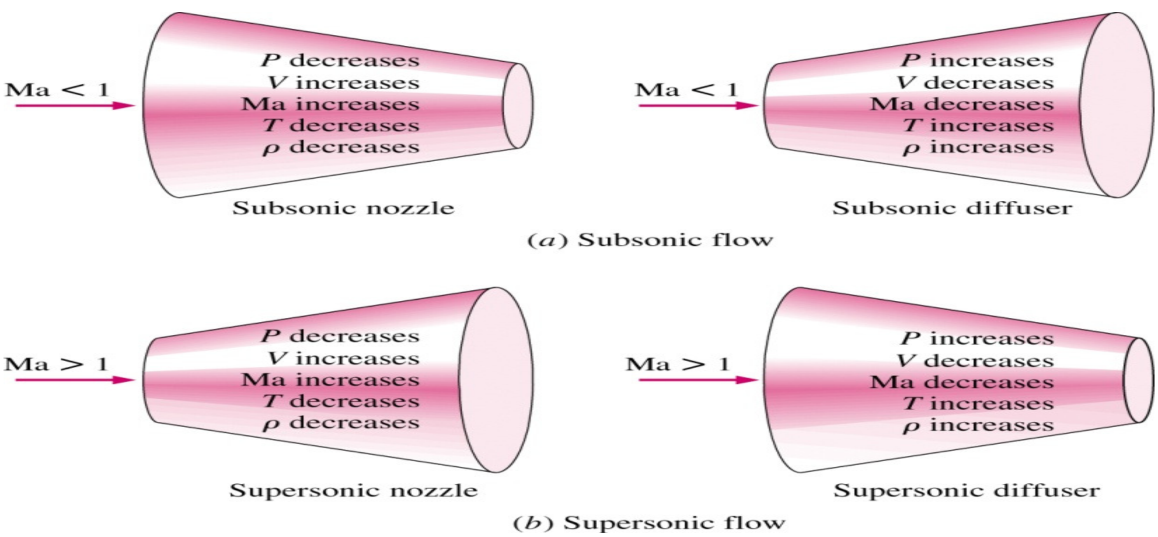
Ma = 1 : Sonic

Ma > 1 : Supersonic

Ma >> 1 : Hypersonic

Ma ≈ 1 : Transonic

<u>Comparison of flow properties in subsonic and supersonic nozzles and diffusers</u>



<u>Pitot-static tube in a compressible flow</u>

The pitot - static tube, when used for determining the velocity at any point in a compressible fluid, gives only the difference between the stagnation head and static head. From this difference, the velocity of the incompressible fluid at that point is obtained from the relation

$V = \sqrt{(2gh)}$, where h = difference in two heads.

But when the pitot - static tube is used for finding velocity at any point in a compressible fluid, the actual pressure difference shown by the gauges of the pitot – static should be multiplied by a factor, for obtaining correct velocity at that point. The value of the factor depends upon the mach number of the flow. Let us find an expression for the correction factor for sub-sonic flow.

At a point in pitot-static tube, the pressure becomes stagnation pressure, denoted by p_s. The expression for stagnation pressure, p_s is given by equation, as

$$\mathbf{p_s = p_1 [1+ ((k-1)/2)\, M_1^2]^{(k/(k-1))}} \quad \ldots\ldots\text{(i)}$$

Where, p_1 = pressure of fluid far away from stagnation point,

M_1 = Mach number at point 1, far away from

stagnation point,

For M<1,the term $(k-1)/2 * M_1^2$ will be the less than 1 and hence the right-hand side of the equation (i) can be expressed by binomial theorem as

$p_s = [1 +((k-1)/2)M_1^2 * k/(k-1)*((k/(k-1)*((k/(k-1)-1))/2)$

$* (((k-1)/2)\, M_1^2)+((k/k-1)(k/(k-1)-1)(k/(k-1)-2)((k-1/2)M_1^2)^3+\ldots.]$

$=p_1 [1+ (k/2)\, M_1^2+ (k/8)\, M_1^4+ (k\,(2-k)/48)\, M_1^6+\ldots]$

$=p_1+p_1 [(k/2)\, M_1^2+ (k/8)\, M_1^4+ (k\,(2-k)/48)\, M_1^6+\ldots]$

$p_s-p_1=p_1*(k/2\,)\, M_1^2[1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots.]$ ..(ii)

But $M_1^2=V_1^2$ where $C_1^2=kp_1/\rho_1$

$=(V_1^2/ (kp_1/\rho_1))= V_1^2\rho_1/kp_1$

Substituting the value of M_1^2 in equation, we get

$p_s-p_1=p_1*(k/2\,) * (V_1^2\rho_1/kp_1\,)[1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots.]$

$= V_1^2\rho_1/2[1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots]$

The term $[1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots]$ is known as compressible correction factor. And $(V_1^2\rho_1/2)$ is the **reading of the pitot-static tube**. Thus the readings of the pitot-tube must be multiplied by a correction factor given below for correct value of velocity measured by pitot-tube.

Compressibility Correction Factor,

$C.C.F=[1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots]$

1) Calculate the **numerical factor** by which the actual pressure difference shown by the gauge of a pitot-static tube must be multiplied to allow for compressibility when the value of the Mach number is 0.9. Take k=1.4

Solution :

Mach number, $M_1 = 0.9$

$K = 1.4$

Using equation for, **Compressibility Correction Factor** is

C.C.F $=[\ 1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots]$ $=1+((0.9^2)/4)+((21.4)/24)(0.9)^4+\ldots.$

$=1.0+0.2025+0.0164+\ldots. =$ **1.2189.**

Numerical factor by which the actual pressure difference is to be multiplied = 1.2189

============@@@@@@@@@@@@@===================