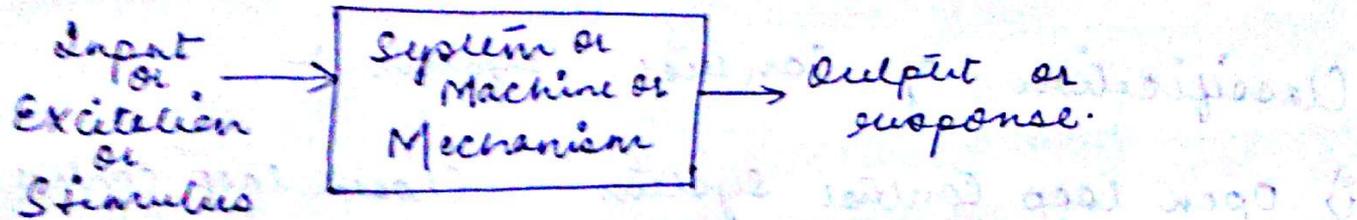


System: It is the arrangement of different component arranged in a particular fashion for accomplishing a desired job.

A system can be represented by the block diagram as shown -



Control System: It is that quantity of interest in a machine or mechanism / system which may have to be maintained constant or altered in accordance with desired manner is defined as control system.

Automatic Control System: (It should not draw human attention to operate).

It is one which measures the existing value compares it with the desired value and generates an error and initiates an action to eliminate the error.

Uses:

Control systems are used in many applications :-

- i) Domestic application: Refrigerator, AC, Washing machine, electric iron etc.
- ii) Industrial application: Numerically Controlled (NC) machines, Computerised numerical control (CNC) machines, Boilers, Robots etc.
- iii) Defense application: Guided Missile, pilotless aircraft, automatic gun.

iv) Space Applications: Satellite control, Radar system, Sonar.

v) Meteorological: Weather forecasting.

vi) Communication system: For transmitting and receiving the signals.

Classification of Control System :-

i) Open loop Control System Closed loop control system

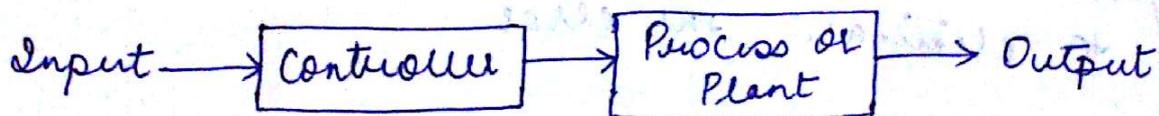
ii) Time invariant Control system Time Variant Control system.

iii) Linear Control system Non-linear control system

iv) Continuous data CS Discrete data CS.

v) Single input single output Control system (SISO) Multiple input and multiple output Control system (MIMO)

Open Loop Control System :- (OLCS)



Open loop control system is a system in which the control action does not depend on the output.

These are the systems without feedback.

Open loop control system does not have any measuring device. Eg: Washing machine, Traffic signaling, Coffee vendor, electric iron, electric lift.

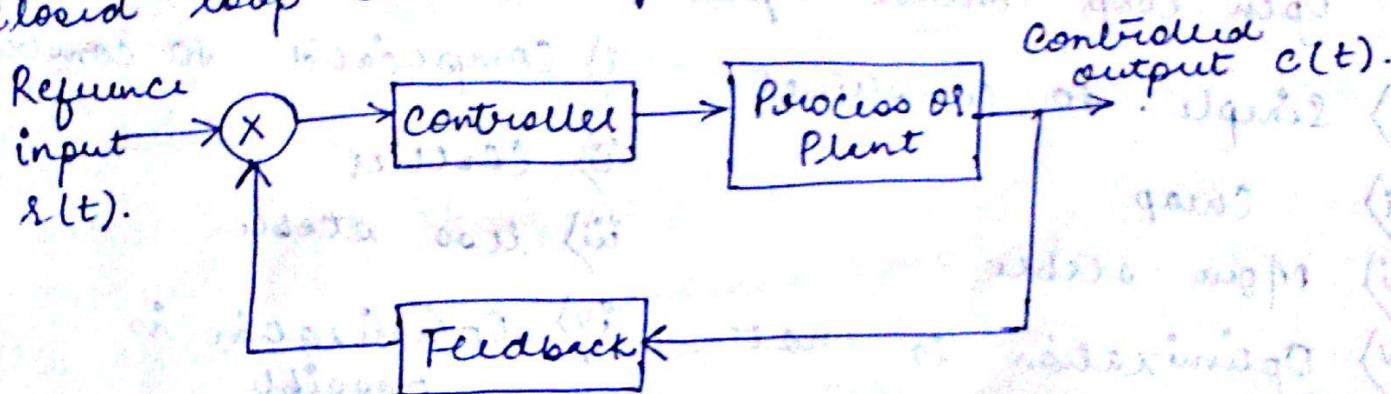
Advantages of Open loop control system :-

- i) These systems are simple in construction and design.
- ii) These systems are economical.
- iii) These systems require less maintenance.
- iv) These systems are more stable.

Disadvantages of OLCS :-

- i) These systems are not accurate and reliable.
- ii) Inaccurate results may be obtained.
- iii) Optimization is not possible.

Closed loop control system (CLCS) :-



It is a system in which the control action is somehow dependent on the output or error changes in the output.

CLCS are systems with feedback.

CLCS will have a measuring device. eg: AC, refrigerator, automatic electric iron, automatic doors, automatic escalators.

Advantages of CLCS :

- i) Accuracy is very high due to correction made if there is any error.

- i) There are facilities of automation
- ii) These are faster system
- iii) Optimization is possible. (Make the system to work to its max efficiency)

Disadvantages of CLCS :-

- i) These systems are complicated to design and construct.
- ii) The system are costly.
- iii) The maintenance of these system are difficult.
- iv) These are less stable.

Open loop control system

- i) Simple to construct.
- ii) Cheap
- iii) More stable.
- iv) Optimization is not possible.
- v) Accuracy depends on calibration.

Closed loop control system

- i) Complicated to construct
- ii) Costlier
- iii) Less stable.
- iv) Optimization is possible.
- v) More accurate due to feedback.

2) Time Variant Control System :

Time variant control system are those in which the parameters of the system varies with time. Eg: space vehicle whose mass decreases with time as it leaves earth surface.

Time Invariant control system :

In these the parameters of the control system does not vary with time. Eg: electrical

networks consisting of the elements such as resistors, inductors and capacitors are the time invariant systems. The value of the element of such system are constant and not function of time.

3) Linear Control System :-

A linear Control system is one which is characterised by a linear differential equation.

$$\frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 4y = 0$$

It is very difficult to obtain linear system.

Non linear Control system :-

A non linear Control system is one which is characterised by a non linear differential equation.

$$\frac{d^2y}{dt^2} + 55 \frac{dy}{dt} + y^2 = 0.$$

All systems are non linear.

4) Analog / Continuous Time Control system :-

In a Continuous time control system all systems are the functions of continuous time variable (t). Eg: motor, generator.

Discrete / Digital time Control system :-

In a discrete time system one or more system variables are known only at certain discrete intervals of time. Eg: microprocessor or micro computer or microcontroller based system.

5.) Single input, single output control system:
A system is said to be single input single output control system if it has only one input and one output. Eg: NOT gate, Diode, switch.

Multiple input, Multiple output control system:
A system is said to be multiple input and multiple output control system if the system has many inputs and many outputs.
Eg: half adder, flip flops.

Analogous systems: Two equations of similar form are said to represent analogous systems.

The advantage of analogous system is that by knowing the solution of one system the solution of other system is also known without actually solving for the system.

Analogous system are not same as equivalent systems.

One of the most important application of analogous system comes in the analysis of electromechanical system.

In the analysis of electromechanical system either the mechanical system is converted into its electrical analogous or vice versa.

Advantages of electrical systems over mechanical system:

a) Standard, simple symbols are available in electrical systems using which any system can be represented in compact form in the form of circuit diagram.

b) Standard, simple laws and theorems are available in electrical system using which it is easier to analyse a system.

c) It is easier to analyse the system for different parameter values as an electrical system because electrical parameter like L , C and R not only can be varied.

Linear Mechanical System :-

- i) Translational mechanical System.
- ii) Rotational mechanical system.

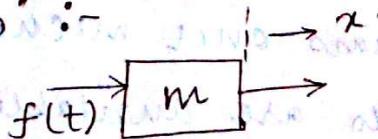
i) Translational mechanical system : In this system the motion of a body is along a straight line or over a fixed path.

ii) Rotational mechanical system : In this system the motion of the body is about its own axis.

i) Translational mechanical system :-

There are three elements which are dominantly involved in the analysis of translational system.

a) Mass "m" :-



where $f(t)$ is force applied
 $x \rightarrow$ displacement

Mass stores kinetic energy.

We know that

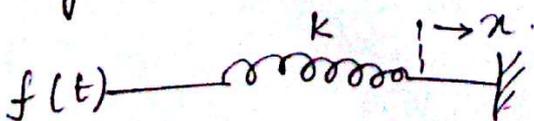
$$f(t) = \text{mass} \times \text{acceleration}$$

$$= m \times \frac{dv}{dt} \quad \text{acc} = \frac{dv}{dt}$$

$$= m \times \frac{d}{dt} \left(\frac{dx}{dt} \right) \quad v = \frac{dx}{dt}$$

$$f(t) = m \frac{d^2x}{dt^2}$$

b) Spring "k" :-



$$f(t) \propto x$$

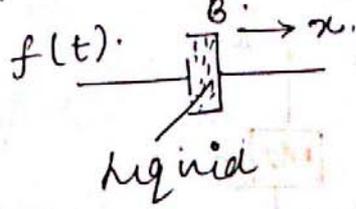
$$f(t) = kx$$

Spring stores potential energy.

where $k =$ spring constant

$k =$ stiffness of spring.

c) Damper / Dash pot "B" :-



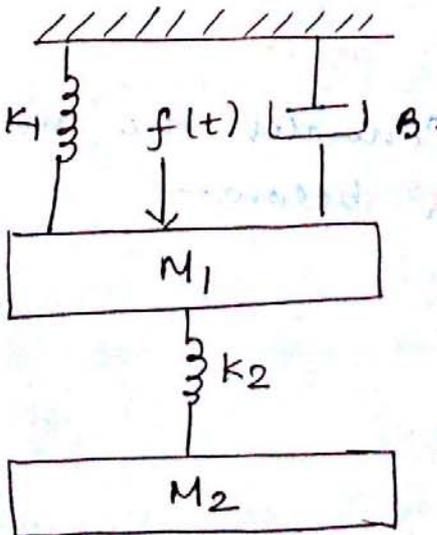
$$f(t) = B \frac{dx}{dt}$$

where $B =$ coefficient of viscous friction.

De - Alembert's law for Translational mechanical system :-

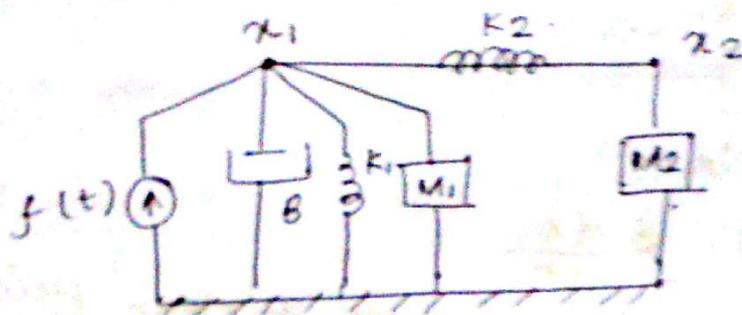
De - Alembert's principle states that for any body the algebraic sum of externally applied forces and the forces resisting the motion of the mechanical system in any direction at any given instant of time is equal to zero.

Write the differential equation of the motion for the mechanical system given below.



Solution: The no. masses in a mechanical system =
NO. of displacements in Mech. S/m.

NO. of displacements in Mech. System = NO. of
nodes in mechanical system.



Applying de Alembert's principle at node x_1 :-

$$f(t) = B \frac{dx_1}{dt} + k_1 x_1 + M_1 \frac{d^2 x_1}{dt^2} + k_2 (x_2 - x_1)$$

at node x_2 -

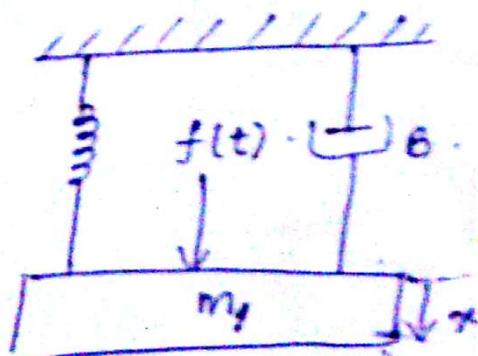
$$0 = M_2 \frac{d^2 x_2}{dt^2} + k_2 (x_2 - x_1)$$

Mechanical - Electrical analogous systems :-

There are two types of analogies -

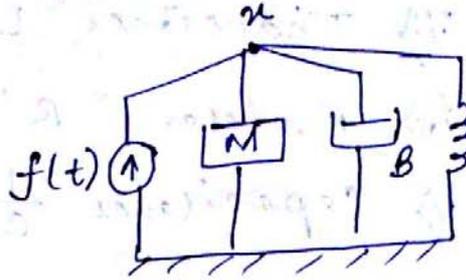
- i) Force voltage analogy
- ii) Force current analogy

i) Force voltage analogy: Consider the mechanical system given below -



No. of masses in mech system = no. of displacements in mech system.

No. of displacements in mech system = No. of nodes in mechanical system.



using de Alberto principle at node x —

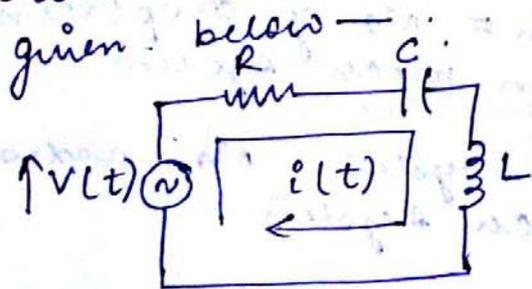
$$f(t) = M \frac{dx^2}{dt^2} + B \cdot \frac{dx}{dt} + kx \quad (\text{Force displacement eqn})$$

Force velocity equation —

$$f(t) = m \frac{dv}{dt} + Bv + k \int v dt \quad (i)$$

$$\left. \begin{aligned} v &= \frac{dx}{dt} \\ dx &= v dt \\ \int dx &= \int v dt \\ x &= \int v dt \end{aligned} \right\}$$

Consider series RLC circuit



Applying KVL for the path traced by $i(t)$.
[Integral differential eqn]

$$v(t) = i(t)R + \frac{1}{C} \int i(t) dt + L \frac{di}{dt}(t)$$

$$v(t) = L \frac{di(t)}{dt} + R i(t) + \frac{1}{C} \int i(t) dt \quad (ii)$$

Since force of a mechanical system is made analogous to voltage of the electrical system. This type of analogous system is known as force voltage analogy.

Force-voltage analogous table —

Mechanical System.

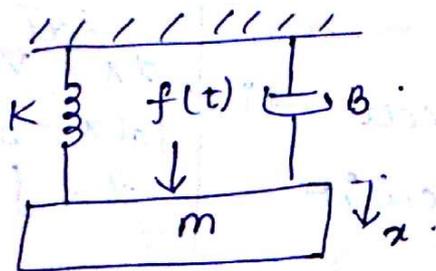
- i) Force " $F(t)$ "
- ii) Velocity " v "
- iii) Mass " m "
- iv) Damper " B "
- v) Spring " k "

Electrical system.

- i) Voltage " $v(t)$ "
- ii) Current " $i(t)$ "
- iii) Inductance " L "
- iv) Resistor " R "
- v) Capacitance " $\frac{1}{C}$ "

Force Current Analogy:

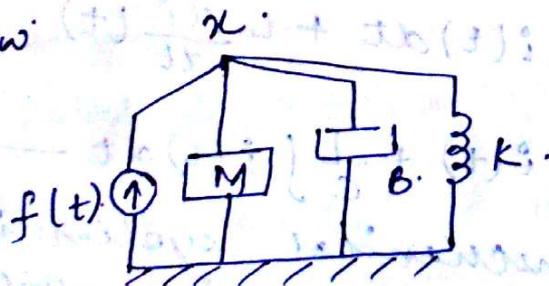
Consider mechanical system shown below:



No. of masses in mech system = No. of displacements in mech system.

No. of displacements in mech system = No. nodes in mech system.

Mech n/w



$$f(t) = m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx \quad \text{--- (i)}$$

$$f(t) = m \frac{dv}{dt} + Bv + k \int v dt \quad \text{--- (ii)}$$

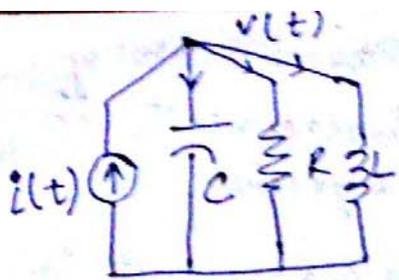
$$v = \frac{dx}{dt}$$

$$dx = v dt$$

$$\int dx = \int v dt$$

$$x = \int v dt$$

Consider the parallel RLC circuit as shown below —



Applying KCL at node $v(t)$,

$$i(t) = C \frac{dv(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L} \int v(t) dt - i(t)$$

Eqⁿ (ii) & (iii) represents analogous system. Since the force of mechanical system is made analogous to current of electrical system, this is called as force current analogy.

Mechanical system

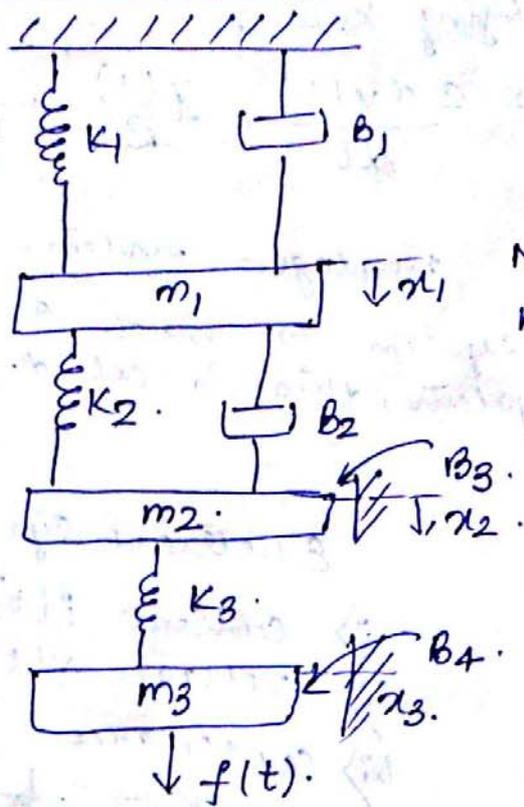
- i) Force "F(t)"
- ii) velocity "v"
- iii) Mass "m"
- iv) Damper "B"
- v) spring "K"

Electrical system

- i) current $i(t)$
- ii) voltage $v(t)$
- iii) Capacitance "C"
- iv) Resistor " $\frac{1}{R}$ "
- v) Inductance " $\frac{1}{L}$ "

$f(t)$	$v(t)$	$i(t)$
v	$i(t)$	$v(t)$
m	L	C
B	R	$\frac{1}{R}$
K	$\frac{1}{C}$	$\frac{1}{L}$

Q Write the differential equation of the motion for the mechanical system shown below. Draw the analogous electrical circuit (Force current analogy and force voltage analogy) for the given mechanical system.



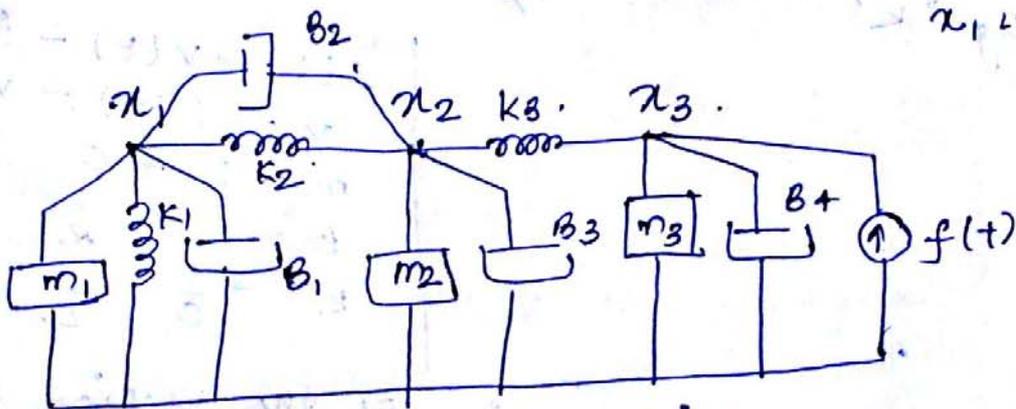
No. of masses = No. of displacements in mech system.

No. of displacements in mech system = no. of nodes in mech network

Mechanical Network :-

$$V_1 = \frac{dx_1}{dt}$$

$$x_1 = \int v_1 dt$$



Using de Alembert's equation

at node x_1 :-

$$0 = m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_2 (x_1 - x_2) + B_2 \frac{d}{dt} (x_1 - x_2)$$

Force-velocity equation -

$$0 = m_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_1 v_1 + K_2 \int (v_1 - v_2) dt + B_2 (v_1 - v_2)$$

Force voltage analogous eqn :-

$$0 = L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_1 i_1(t) + \frac{1}{C_2} \int (i_1(t) - i_2(t)) dt + R_2 (i_1(t) - i_2(t)) \quad \text{--- (i)}$$

Force current analogous eqⁿ :-

$$0 = C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_1} v_1(t) + \frac{1}{L_2} \int (v_1(t) - v_2(t)) dt + \frac{1}{R_2} [v_1(t) - v_2(t)] \quad \text{--- (ii)}$$

at node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + b_3 \frac{dx_2}{dt} + k_2 (x_2 - x_1) + k_2 \frac{d}{dt} (x_2 - x_1) + k_3 (x_2 - x_3)$$

Force-velocity equation -

$$0 = m_2 \frac{dv_2}{dt} + b_3 v_2 + k_2 \int (v_2 - v_1) dt + b_2 (v_2 - v_1) + k_3 \int (v_2 - v_3) dt$$

Force velocity analogous eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + R_3 i_2(t) + \frac{1}{C_2} \int (i_2(t) - i_1(t)) dt + R_2 (i_2(t) - i_1(t)) + \frac{1}{C_3} \int (i_2(t) - i_3(t)) dt \quad \text{--- (iii)}$$

Force-current equation

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R_3} v_2(t) + \frac{1}{L_2} \int (v_2(t) - v_1(t)) dt + \frac{1}{R_2} (v_2(t) - v_1(t)) + \frac{1}{L_3} \int v_2(t) - v_3(t) dt \quad \text{--- (iv)}$$

at node x_3 :-

$$f(t) = m_3 \frac{d^2 x_3}{dt^2} + b_4 \frac{dx_3}{dt} + k_3 (x_3 - x_2)$$

Force velocity equation :-

$$f(t) = m_3 \frac{dv_3}{dt} + b_3 v_3 + k_3 \int (v_3 - v_2) dt$$

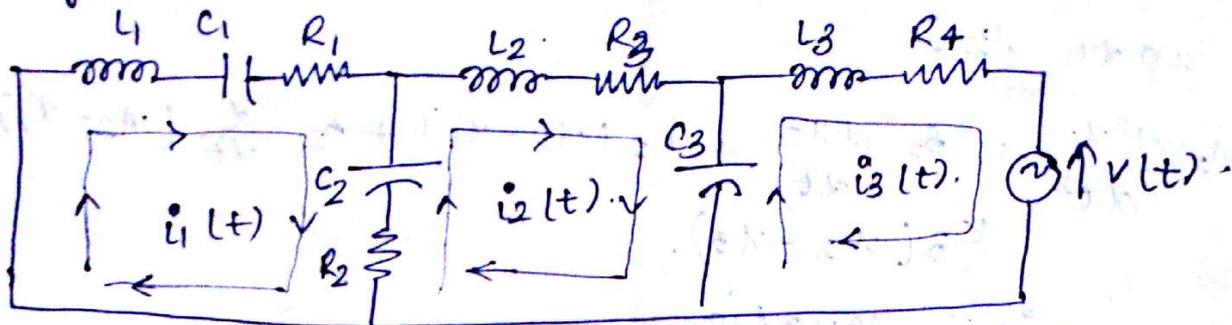
Force velocity analogous equation -

$$v(t) = L_3 \frac{di_3(t)}{dt} + R_4 i_3(t) + \frac{1}{C_3} \int (i_3(t) - i_2(t)) dt \quad \text{--- (v)}$$

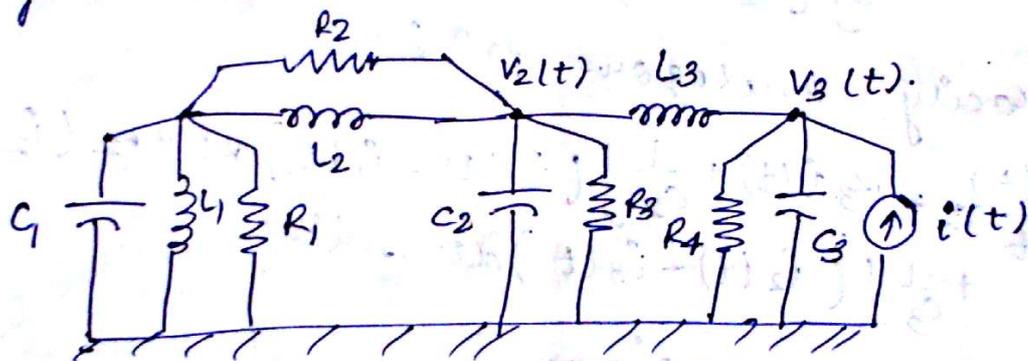
Force current analogous equation-

$$i(t) = C_3 \frac{dv_3(t)}{dt} + \frac{1}{R_4} v_3(t) + \frac{1}{L_3} \int (v_3(t) - v_2(t)) dt + v_2(t)$$

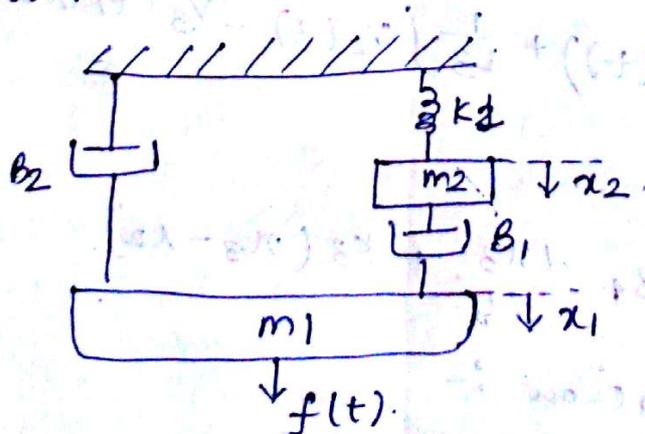
Using eqn (i) (iii) and (v).



Using eqn (ii) (iv) and (vi) the force current analogous network is -

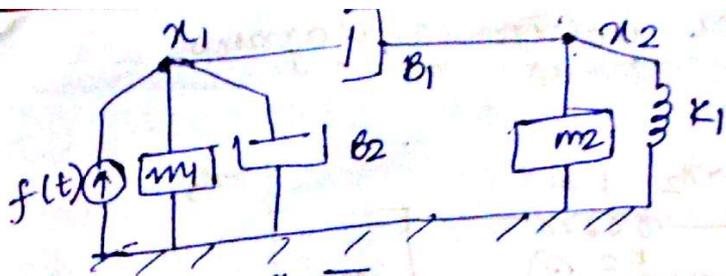


Q Draw the force current analogous network for the given mechanical network.



No. of masses in a mech system = no. of displacement in a mech system.

No. of displacement in mech system = no. of nodes in the mech system.



at node x_1

$$f(t) = m_1 \frac{d^2 x_1}{dt^2} + B_2 \frac{dx_1}{dt} + B_1 \frac{d}{dt} (x_1 - x_2).$$

Force velocity eqⁿ :-

$$f(t) = m_1 \frac{dv_1}{dt} + B_2 v_1 + B_1 (v_1 - v_2).$$

Force current eqⁿ :-

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_2} v_1(t) + \frac{1}{R_2} [v_1(t) - v_2(t)].$$

at node x_2 :-

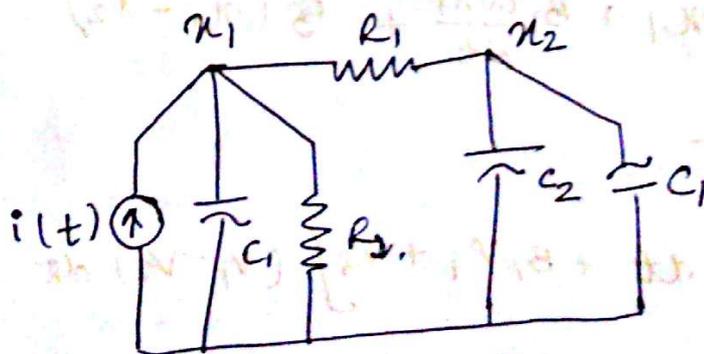
$$0 = m_2 \frac{d^2 x_2}{dt^2} + k_1 x_2 + B_1 \frac{d}{dt} (x_2 - x_1).$$

Force velocity eqⁿ :-

$$0 = m_2 \frac{dv_2}{dt} + k_1 \int v_2 dt + B_1 (v_2 - v_1).$$

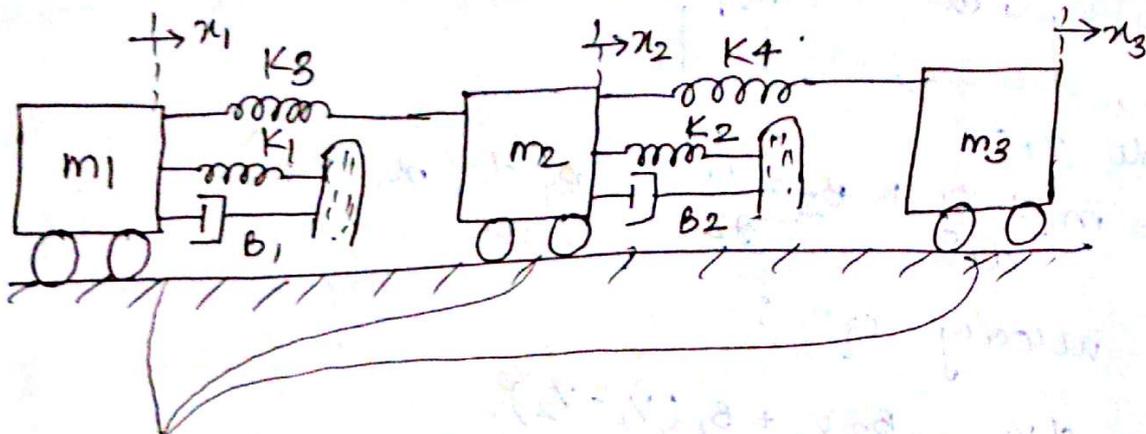
Force current eqⁿ :-

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_1} \int v dt + \frac{1}{R_1} [v_2(t) - v_1(t)].$$



Force voltage eqⁿ :-

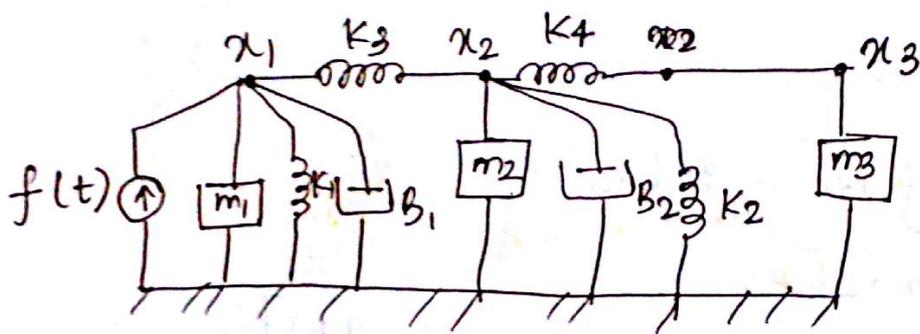
Draw the force current force voltage analogues for the mechanical circuit shown below.



Frictionless.

No. of masses in mechanical system = No. of displacement in mech system = 3
 No. of nodes in mech system = 3

Mech network -



at node x_1 -

$$f(t) = m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_3 (x_1 - x_2)$$

Force velocity eqⁿ :-

$$f(t) = m_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_1 v_1 + K_3 \int (v_1 - v_2) dt$$

Force voltage eqⁿ :-

$$v(t) = L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_1 i_1(t) + \frac{1}{C_3} \int (i_1(t) - i_2(t)) dt \quad \text{--- (i)}$$

force current equation.

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_1} v_1(t) + \frac{1}{L_3} \int v_1(t) - v_2(t) dt \quad \text{--- (ii)}$$

at node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + K_3 (x_2 - x_1) + K_4 (x_2 - x_3)$$

force velocity eqⁿ :-

$$0 = m_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2 dt + K_3 \int (v_2 - v_1) dt + K_4 \int (v_2 - v_3) dt$$

force voltage eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C_2} \int i_2(t) dt + \frac{1}{C_3} \int (i_2(t) - i_1(t)) dt + \frac{1}{C_4} \int (i_2(t) - i_3(t)) dt \quad \text{--- (iii)}$$

force current eqⁿ :-

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R_2} v_2(t) + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{L_3} \int (v_2(t) - v_1(t)) dt + \frac{1}{L_4} \int (v_2(t) - v_3(t)) dt \quad \text{--- (iv)}$$

at node x_3 :-

$$0 = m_3 \frac{d^2 x_3}{dt^2} + K_4 (x_3 - x_2)$$

force velocity eqⁿ :-

$$0 = m_3 \frac{dv_3}{dt} + K_4 \int (v_3 - v_2) dt$$

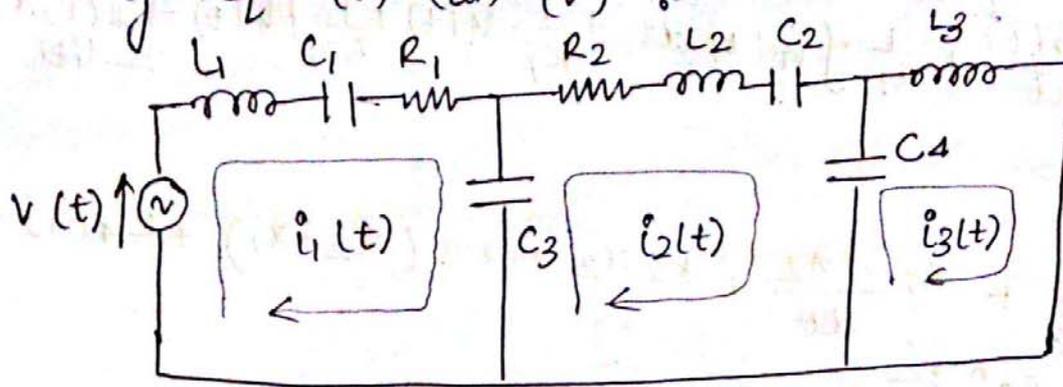
force voltage eqⁿ :-

$$0 = L_3 \frac{di_3(t)}{dt} + \frac{1}{C_4} \int (i_3(t) - i_2(t)) dt \quad \text{--- (v)}$$

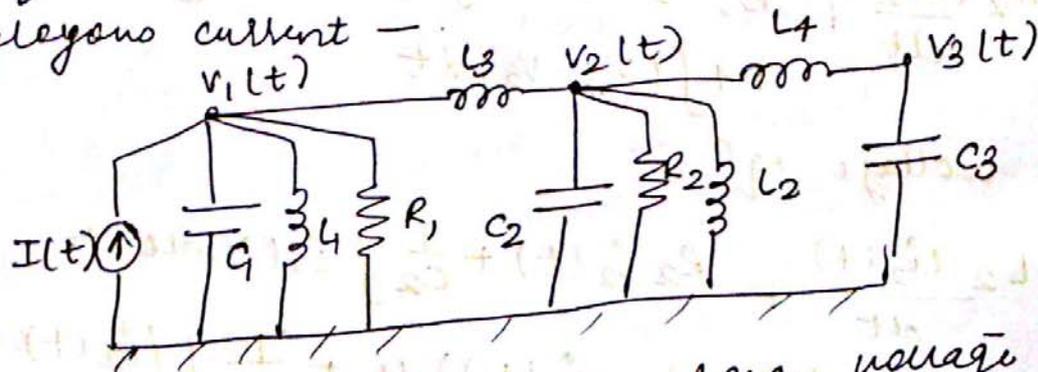
force current eqⁿ :-

$$0 = C_3 \frac{dv_3(t)}{dt} + \frac{1}{L_4} \int (v_3(t) - v_2(t)) dt \quad \text{--- (vi)}$$

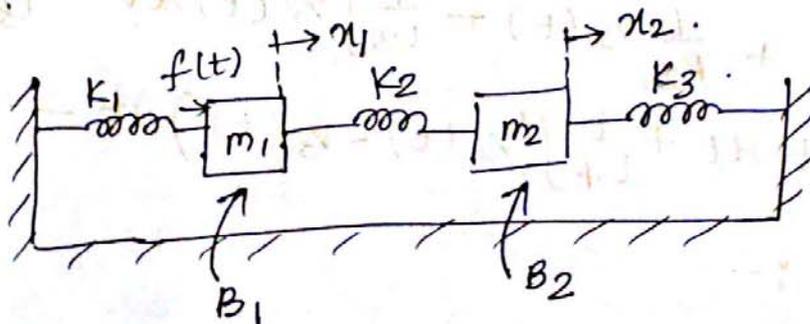
Using eqⁿ (i) (ii) (iii) (v) :-



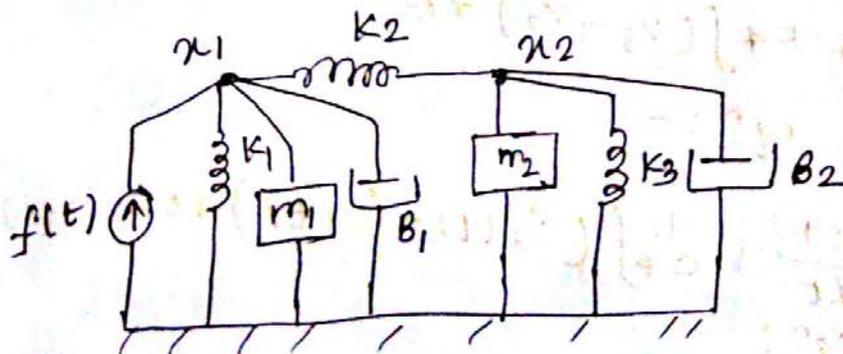
Using equation (ii) (iv) & (vi) force current analogous circuit -



Draw the force current and force voltage analogous electrical network for the mechanics network.



No. of masses = no. of displacement = no. of nodes in such system = 2.



at node x_1 :-

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_2 (x_1 - x_2)$$

force velocity eqⁿ :-

$$f(t) = m_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_1 v_1 + K_2 \int (v_1 - v_2) dt$$

force voltage eqⁿ :-

$$v(t) = L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_1 i_1(t) + \frac{1}{C_2} \int (i_1(t) - i_2(t)) dt$$

force current eqⁿ :-

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_1} v_1(t) + \frac{1}{L_2} \int (v_1(t) - v_2(t)) dt$$

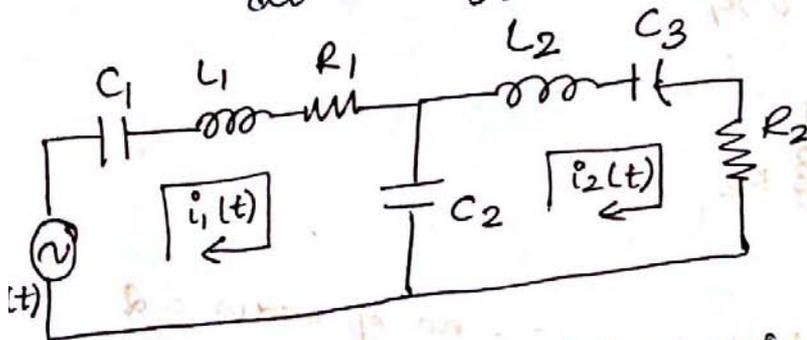
at node x_2 :-

$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_3 x_2 + B_2 \frac{dx_2}{dt} + K_2 (x_2 - x_1)$$

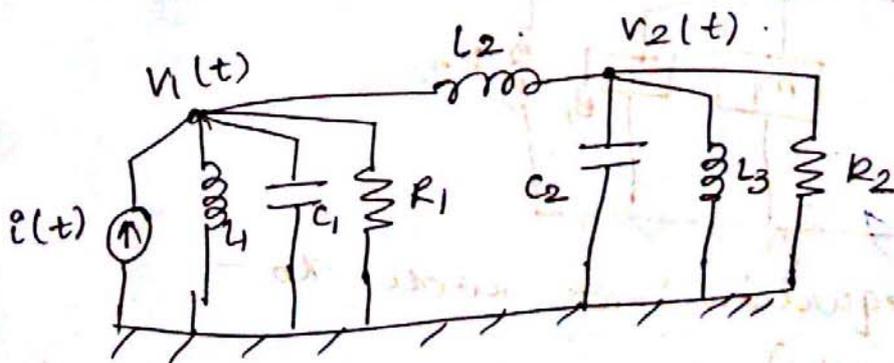
force velocity eqⁿ :-

$$0 = M_2 \frac{dv_2}{dt} + K_3 \int v_2 dt + B_2 v_2 + K_2 \int (v_2 - v_1) dt$$

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_3} \int i_2(t) dt + R_2 i_2(t) + \frac{1}{C_2} \int (i_2(t) - i_1(t)) dt$$

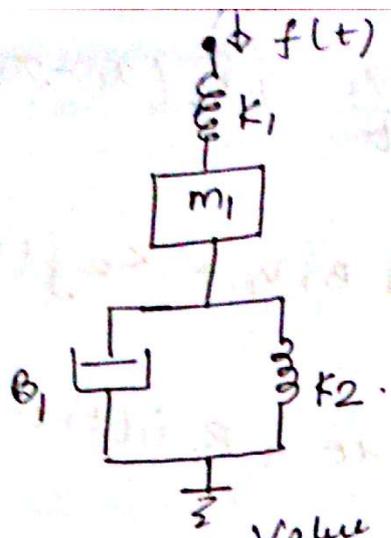


force current analogous circuit -

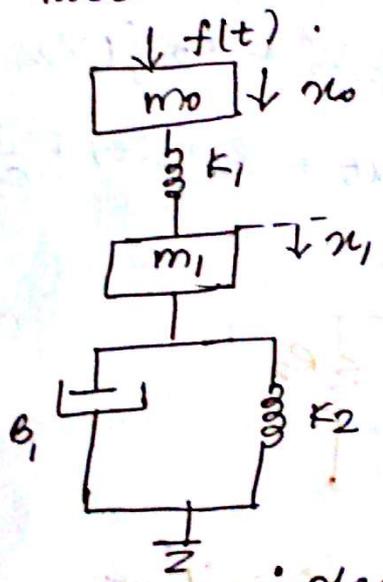


$$0 = C_1 \frac{dv_2(t)}{dt} + \frac{1}{L_3} \int v_2(t) dt + \frac{1}{R_2} v_2(t) + \frac{1}{L_2} \int (v_2(t) - v_1(t)) dt$$

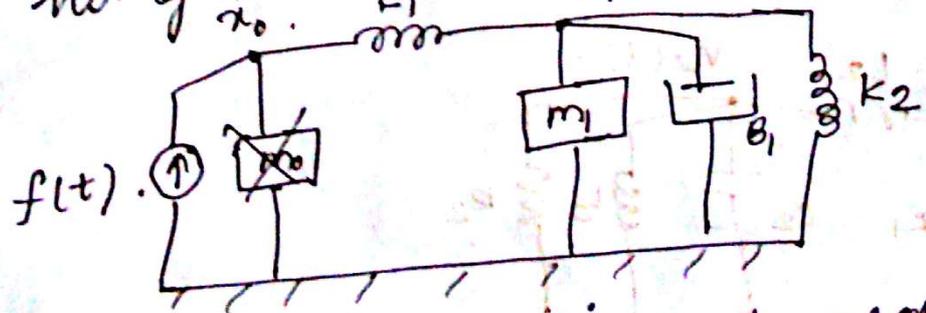
Q



The mass of zero value (dummy mass). (m_0)
 The dummy mass is used to write the mechanical network.
 The dummy mass is not taken into consideration to write the electrical n/k's.
 The modified mechanical network is -



no. of mass = no. of displacements = no. of nodes = 2.



The differential equation at node x_0 -
 $f(t) = k_1(x_0 - x_1)$.

force velocity eqⁿ :-

$$f(t) = K_1 \int (v_0 - v_1) dt$$

force velocity eq voltage eqⁿ :-

$$v(t) = \frac{1}{C_1} \int (i_0(t) - i_1(t)) dt \quad \text{--- (i)}$$

force current eqⁿ :-

$$i(t) = \frac{1}{L_1} \int (v_0(t) - v_1(t)) dt$$

At node x_1 -

$$0 = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_2 x_1 + K_1 (x_1 - x_0)$$

force velocity eqⁿ :-

$$0 = m_1 \frac{dv_1}{dt} + b_1 v_1 + K_2 \int v_1 dt + K_1 \int (v_1 - v_0) dt$$

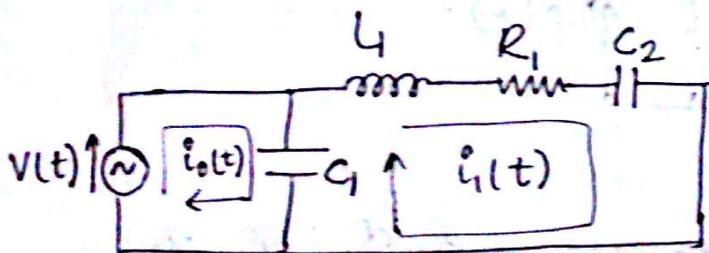
force voltage eqⁿ :-

$$0 = L_1 \frac{di_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_2} \int i_1(t) dt + \frac{1}{C_1} \int (i_1(t) - i_0(t)) dt \quad \text{--- (ii)}$$

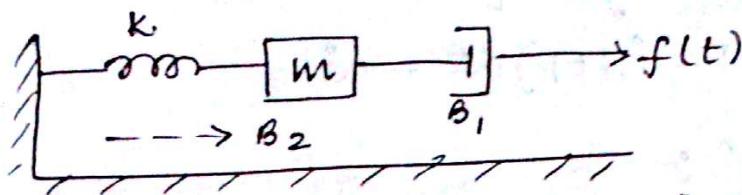
force current eqⁿ :-

$$0 = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} v_1(t) + \frac{1}{L_2} \int v_1(t) dt + \frac{1}{L_1} \int (v_1(t) - v_0(t)) dt$$

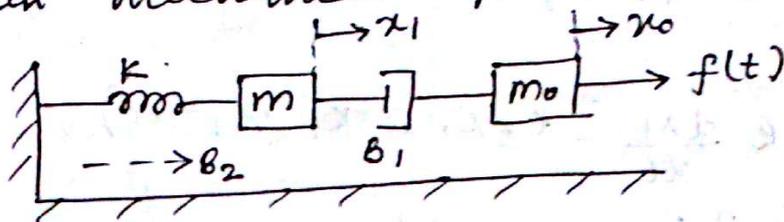
using eqⁿ (i) and (ii).



Q For the mechanical system shown below, write the force current and force voltage analogous networks.



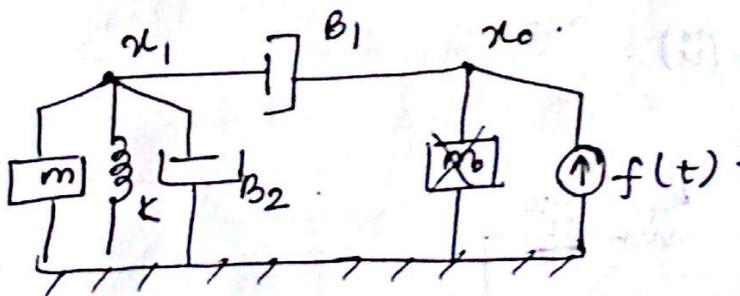
The given mechanical system is re-written as-



$m_0 =$ dummy mass [mass of "0" value].

m_0 is not taken into consideration to write the F-V and F-I analogous electrical n/w.

no. of masses = no. of displacements = no. of nodes in mech network = 2.



at node 1 -

$$0 = m \frac{d^2 x_1}{dt^2} + k x_1 + B_2 \frac{dx_1}{dt} + B_1 \frac{d}{dt} (x_1 - x_0)$$

Force velocity eqⁿ :-

$$0 = m \frac{dv_1}{dt} + k_1 \int v_1 dt + B_2 v_1 + B_1 (v_1 - v_0)$$

Force voltage eqⁿ :-

$$0 = L \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_2 i_1(t) + R_1 (i_1(t) - i_2(t))$$

force current eqⁿ :-

$$0 = C \frac{dv_1}{dt} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_2} v_1(t) + \frac{1}{R_1} (v_1(t) - v_0(t)) \quad \text{--- (i)}$$

at node x₀ :-

$$f(t) = B_1 \frac{d}{dt} (x_0 - x_1)$$

force velocity eqⁿ :-

$$f(t) = B_1 (v_0 - v_1)$$

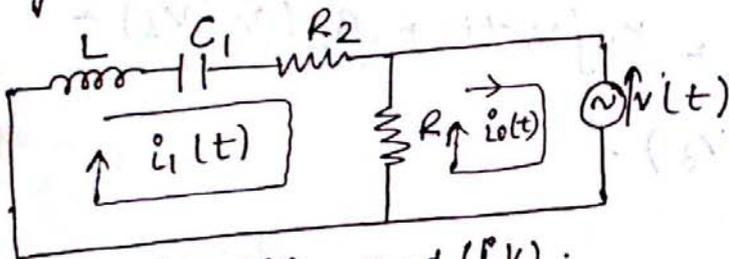
force voltage eqⁿ

$$v(t) = R_1 (i_0(t) - i_1(t)) \quad \text{--- (ii)}$$

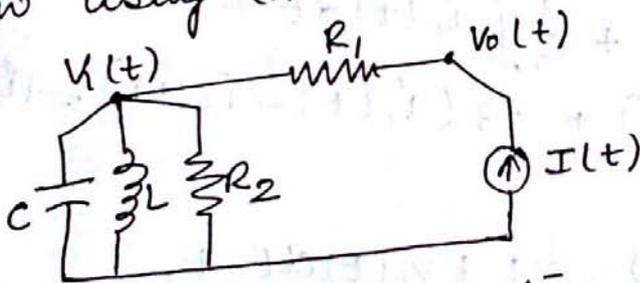
force current eqⁿ -

$$i_1(t) = \frac{1}{R_1} (v_0(t) - v_1(t)) \quad \text{--- (iv)}$$

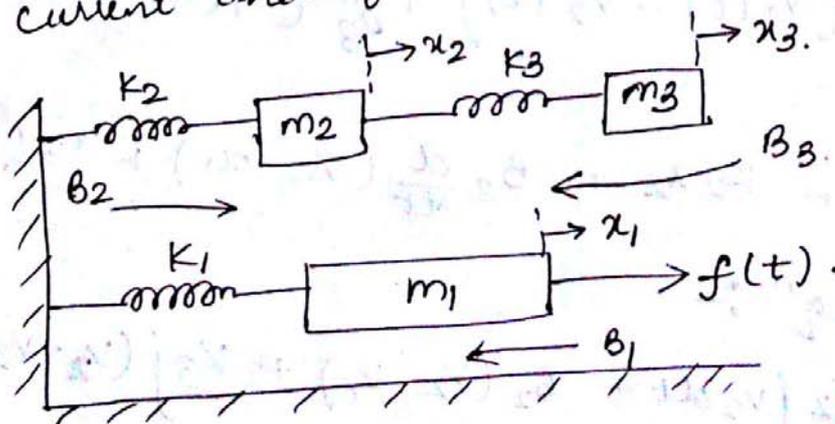
using eqⁿ (i) and (ii).



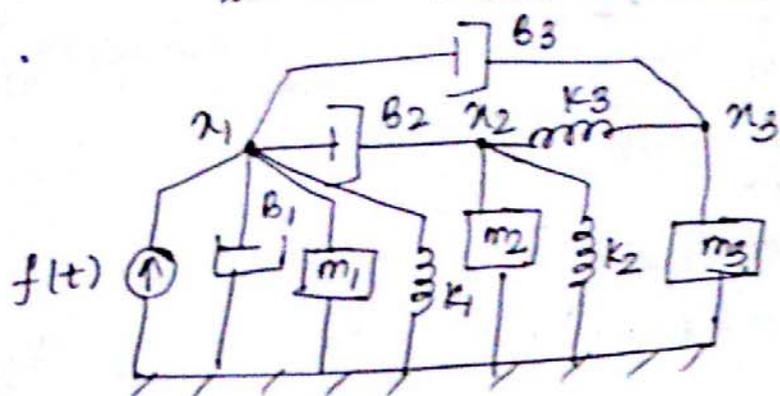
Now using (ii) and (iv).



Q. Show the mech system shown below with the force current and force voltage analogous n/w.



no. of masses = no. of displacement = no. of nodes
in the mesh network = 3.



Force at node x_1 -

$$f(t) = B_1 \frac{dx_1}{dt} + m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_2 \frac{d}{dt} (x_1 - x_2) + B_3 \frac{d}{dt} (x_1 - x_3)$$

Force velocity eqⁿ :-

$$f(t) = B_1 v_1 + m_1 \frac{dv_1}{dt} + K_1 \int v_1 dt + B_2 (v_1 - v_2) + B_3 (v_1 - v_3)$$

Force voltage eqⁿ :-

$$v(t) = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_2 (i_1(t) - i_2(t)) + R_3 (i_1(t) - i_3(t)) \quad \text{--- (i)}$$

Force current eqⁿ :-

$$i(t) = \frac{1}{R_1} v_1(t) + C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_2} (v_1(t) - v_2(t)) + \frac{1}{R_3} (v_1(t) - v_3(t)) \quad \text{--- (ii)}$$

at node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + B_2 \frac{d}{dt} (x_2 - x_1) + K_3 (x_2 - x_3)$$

Force velocity eqⁿ :-

$$0 = m_2 \frac{dv_2}{dt} + K_2 \int v_2 dt + B_2 (v_2 - v_1) + K_3 \int (v_2 - v_3) dt$$

force voltage eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2} \int i_2(t) dt + R_2 (i_2(t) - i_1(t)) + \frac{1}{C_3} \int (i_2(t) - i_3(t)) dt \quad \text{--- (iii)}$$

force current eqⁿ :-

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{R_2} (v_2(t) - v_1(t)) + \frac{1}{L_3} \int (v_2(t) - v_3(t)) dt \quad \text{--- (iv)}$$

at node x₃ :-

$$0 = m_3 \frac{d^2 x_3}{dt^2} + k_3 (x_3 - x_2) + B_3 \frac{dx_3}{dt} (x_3 - x_1)$$

force velocity eqⁿ :-

$$0 = m_3 \frac{dv_3}{dt} + k_3 \int (v_3 - v_2) dt + B_3 (v_3 - v_1)$$

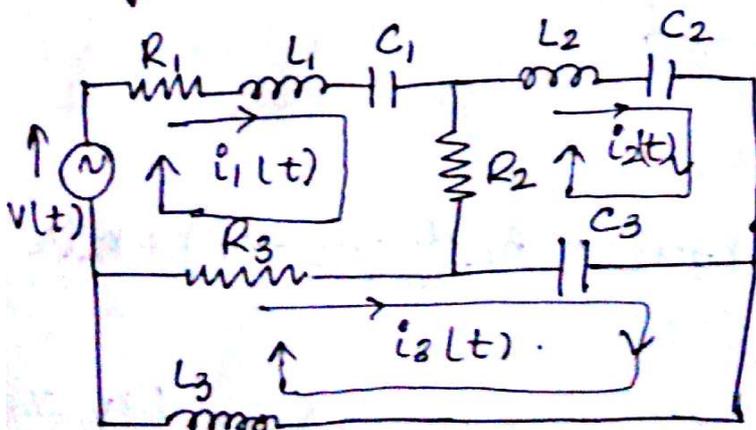
force voltage eqⁿ :-

$$0 = L_3 \frac{di_3(t)}{dt} + \frac{1}{C_3} \int (i_3(t) - i_2(t)) dt + R_3 (i_3(t) - i_1(t)) \quad \text{--- (v)}$$

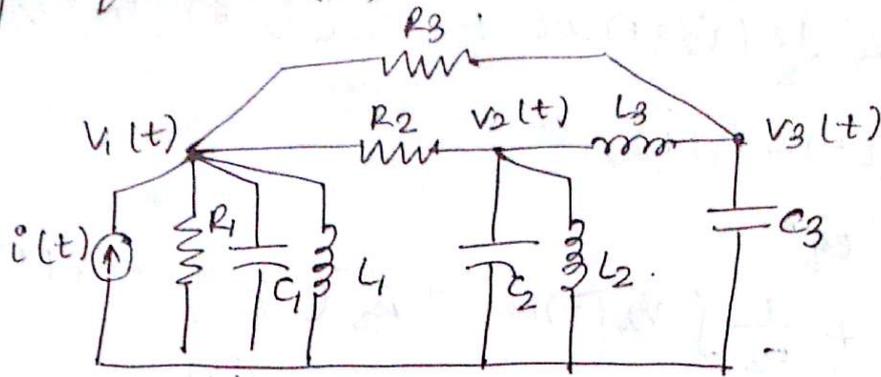
force current eqⁿ -

$$0 = C_3 \frac{dv_3(t)}{dt} + \frac{1}{L_3} \int (v_3(t) - v_2(t)) dt + \frac{1}{R_3} (v_3(t) - v_2(t)) \quad \text{--- (vi)}$$

Using eqⁿ (i) (iii) & (v)

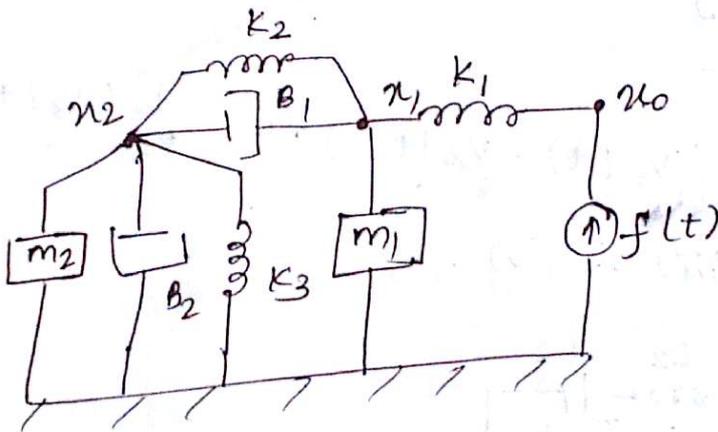
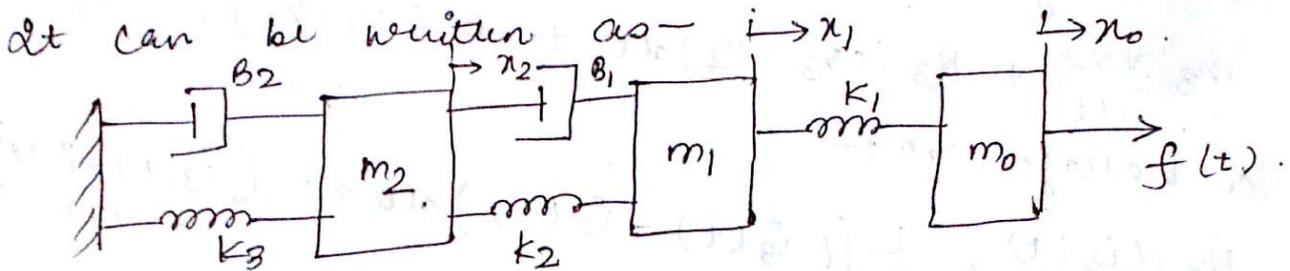
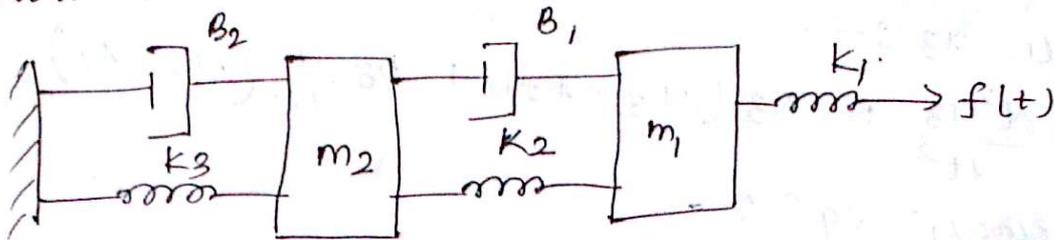


using eqn (ii) (iii) and (vi) -



force $f(t)$ cannot be connected to spring it should be connected to mass

Q Draw the mech system for the mech system shown below:



at node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_3 x_2 + B_1 \frac{d}{dt} (x_2 - x_1) + k_2 (x_2 - x_1)$$

force velocity - eqn :-

$$0 = m_2 \frac{dv_2}{dt} + B_2 v_2 + k_3 \int v_2 dt + B_1 (v_2 - v_1) + k_2 \int (v_2 - v_1) dt$$

Force voltage eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C_3} \int i_2(t) dt + R_1 (i_2(t) - i_1(t)) + \frac{1}{C_2} \int (i_2(t) - i_1(t)) dt \quad \text{--- (i)}$$

Force current eqⁿ :-

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R_2} v_2(t) + \frac{1}{L_3} \int v_2(t) dt + \frac{1}{R_1} (v_2(t) - v_1(t)) + \frac{1}{L_2} \int (v_2(t) - v_1(t)) dt \quad \text{--- (ii)}$$

at node x_1 :-

$$0 = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d}{dt} (x_1 - x_2) + K_2 (x_1 - x_2) + K_1 (x_1 - x_0)$$

Force velocity eqⁿ :-

$$0 = m_1 \frac{dv_1}{dt} + B_1 (v_1 - v_2) + K_2 \int (v_1 - v_2) dt + K_1 \int (v_1 - v_0) dt$$

Force voltage eqⁿ :-

$$0 = L_1 \frac{di_1(t)}{dt} + R_1 (i_1(t) - i_2(t)) + \frac{1}{C_2} \int (i_1(t) - i_2(t)) dt + \frac{1}{C_1} \int (i_1(t) - i_0(t)) dt \quad \text{--- (iii)}$$

Force current eqⁿ :-

$$0 = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} (v_1(t) - v_2(t)) + \frac{1}{L_2} \int (v_1(t) - v_2(t)) dt + \frac{1}{L_1} \int (v_1(t) - v_2(t)) dt \quad \text{--- (iv)}$$

4)

at node x_0 :-

$$dt \quad f(t) = K_1 (x_0 - x_1)$$

Force velocity eqn :-

$$f(t) = k_1 \int (v_0 - v_1) dt$$

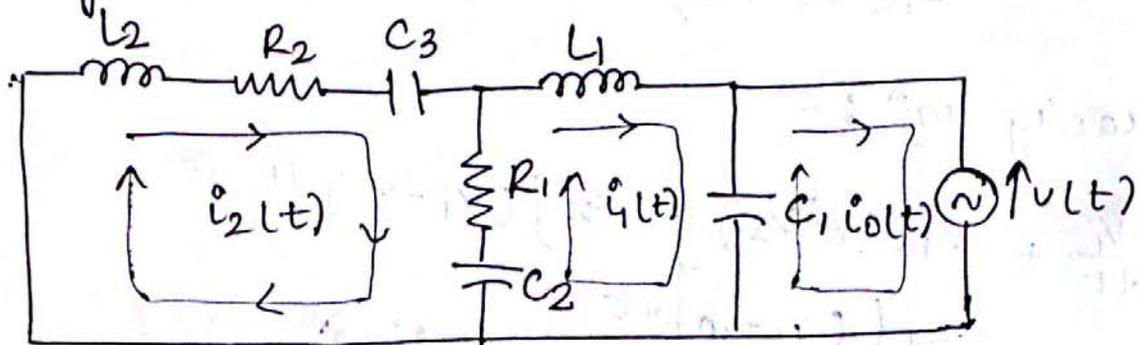
Force voltage eqn :-

$$v(t) = \frac{1}{c_1} \int (i_0(t) - i_1(t)) dt \quad \text{--- (v)}$$

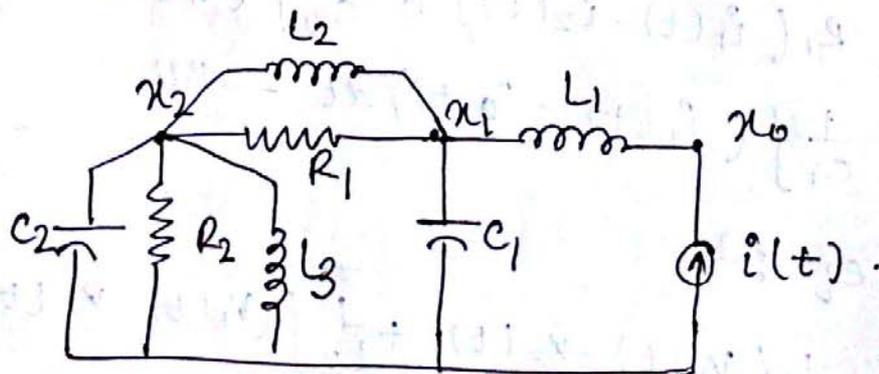
Force current eqn :-

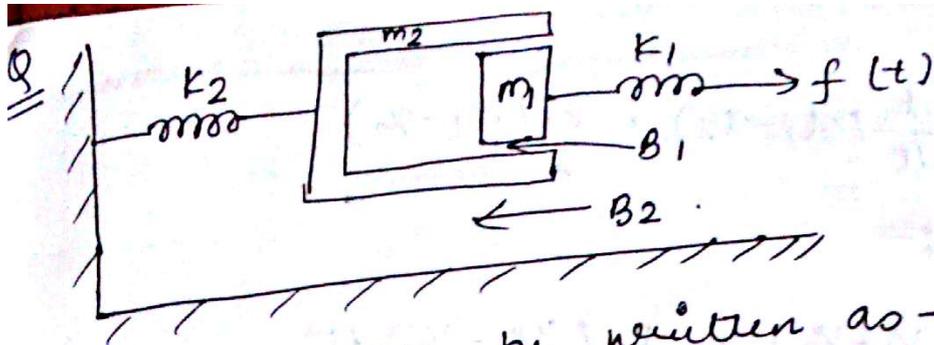
$$i(t) = \frac{1}{L_1} \int (v_0(t) - v_1(t)) dt \quad \text{--- (vi)}$$

using eqn (i) (iii) & (v) :-

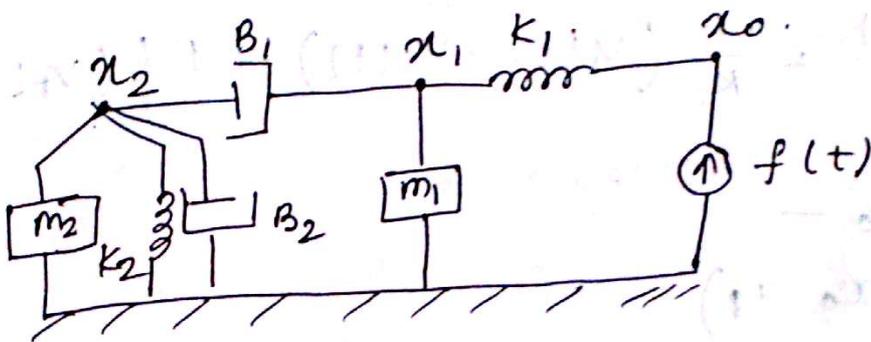
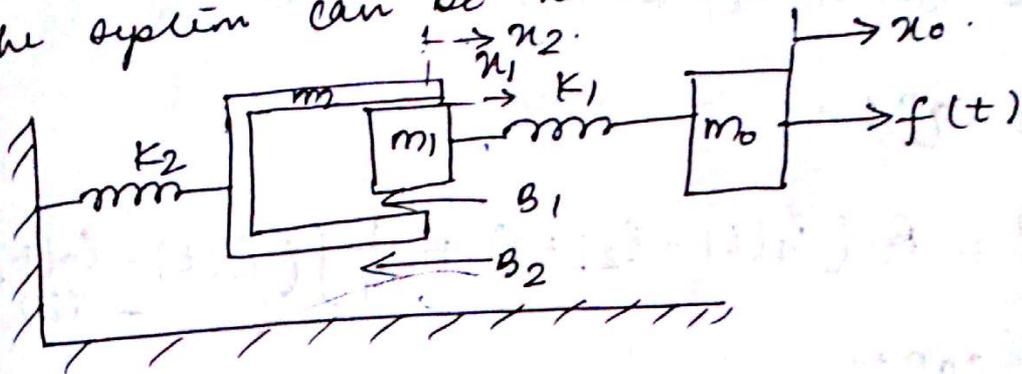


using eqn (ii) (iv) & (vi) -





The system can be written as -



at node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + k_2 x_2 + B_2 \frac{dx_2}{dt} + B_1 \frac{d}{dt} (x_2 - x_1)$$

Force velocity eqⁿ :-

$$0 = m_2 \frac{dv_2}{dt} + k_2 \int v_2 dt + B_2 v_2 + B_1 (v_2 - v_1)$$

Force voltage eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2} \int i_2(t) dt + R_2 i_2(t) + R_1 (i_2(t) - i(t))$$

Force current eqⁿ :-

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{R_2} v_2(t) + \frac{1}{R_1} (v_2(t) - v_1(t))$$

at node x_1 :-

$$0 = m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d}{dt} (x_1 - x_2) + K_1 (x_1 - x_0)$$

Force velocity eqⁿ :-

$$0 = m_1 \frac{dv_1}{dt} + B_1 (v_1 - v_2) + K_1 \int (v_1 - v_0) dt$$

Force voltage eqⁿ :-

$$0 = L_1 \frac{di_1(t)}{dt} + R_1 (i_1(t) - i_2(t)) + \frac{1}{C_1} \int (i_1(t) - i_0(t)) dt \quad \text{--- (ii)}$$

Force current eqⁿ :-

$$0 = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} (v_1(t) - v_2(t)) + \frac{1}{L_1} \int (v_1(t) - v_0(t)) dt \quad \text{--- (v)}$$

at node x_0 -

$$f(t) = K_1 (x_0 - x_1)$$

Force velocity eqⁿ :-

$$f(t) = K_1 \int (v_0 - v_1) dt$$

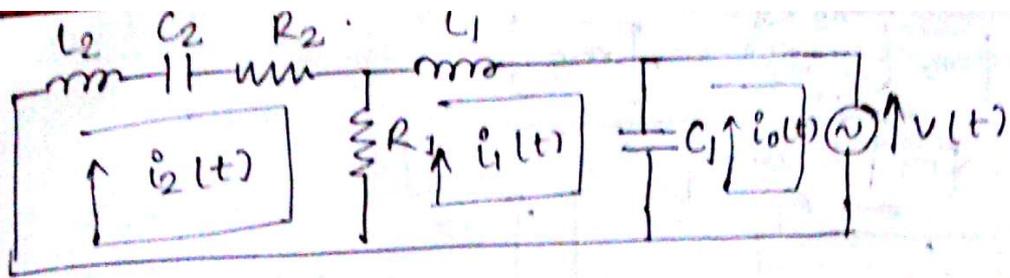
Force voltage eqⁿ :-

$$v(t) = \frac{1}{C_1} \int (i_0(t) - i_1(t)) dt \quad \text{--- (v)}$$

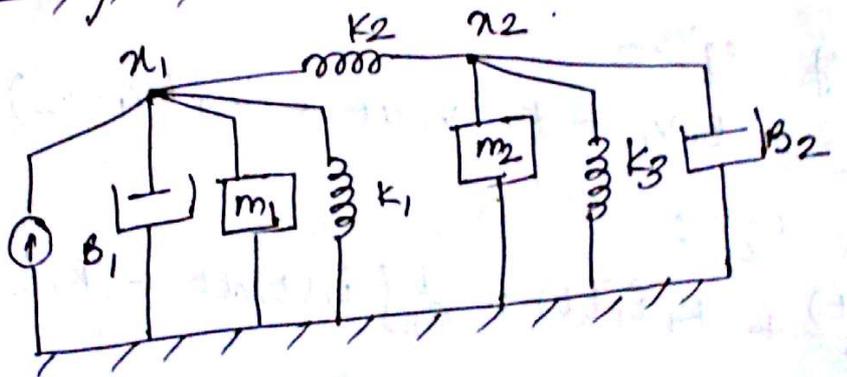
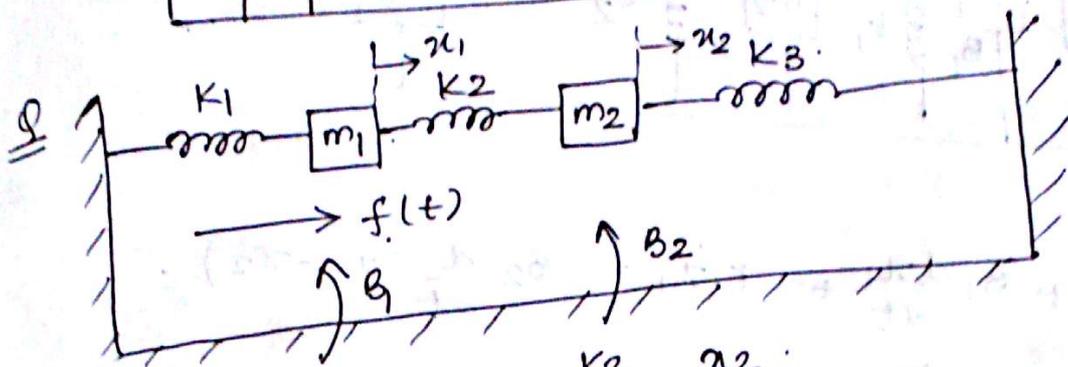
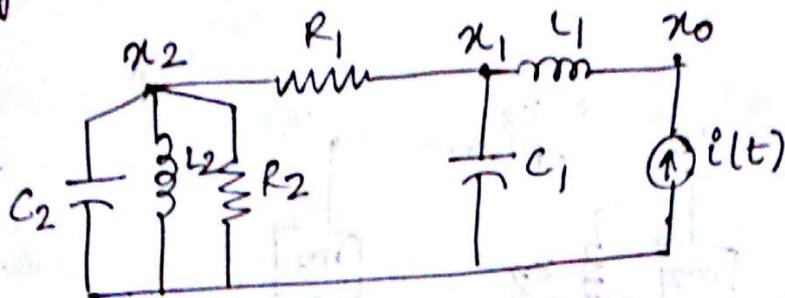
Force current eqⁿ :-

$$i(t) = \frac{1}{L_1} \int (v_0(t) - v_1(t)) dt \quad \text{--- (vi)}$$

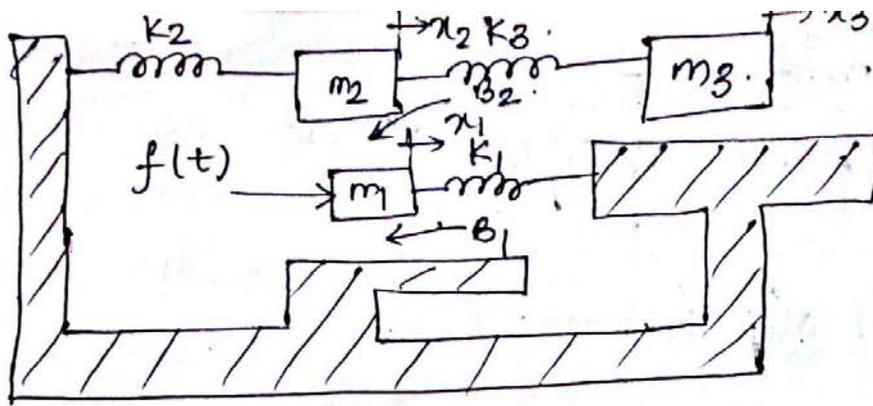
Using eqⁿ (i) (ii) and (v) :-



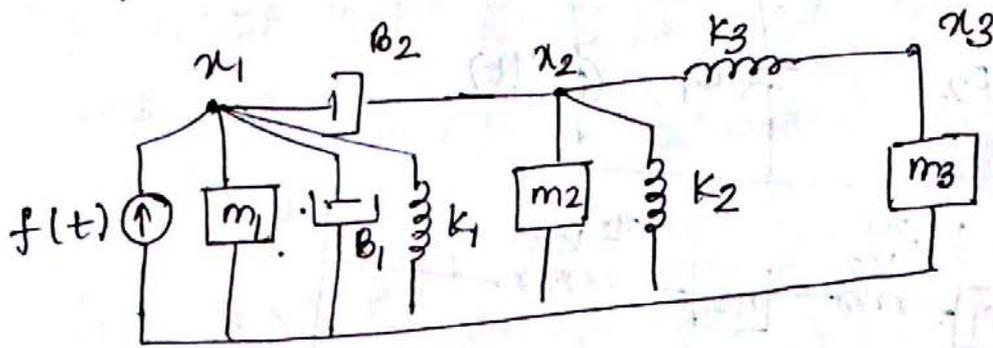
using eqⁿ (ii) (iv) and (vi) —



Q



Mech n/w :-



At node x_1 -

$$f(t) = m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1 + B_2 \frac{d}{dt} (x_1 - x_2)$$

Force velocity eqⁿ -

$$f(t) = m_1 \frac{dv_1}{dt} + B_1 v_1 + k_1 \int v_1 dt + B_1 (v_1 - v_2)$$

Force voltage eqⁿ :-

$$v(t) = L_1 \frac{di_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1} \int i_1(t) dt + R_1 (i_1(t) - i_2(t))$$

Force current eqⁿ :-

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} v_1(t) + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{R_1} (v_1(t) - v_2(t))$$

At node x_2 :-

$$0 = m_2 \frac{d^2 x_2}{dt^2} + k_2 x_2 + B_2 \frac{d}{dt} (x_2 - x_1) + k_3 (x_2 - x_3)$$

$$0 = m_2 \frac{dv_2}{dt} + k_2 \int v_2 dt + B_2 (v_2 - v_1) + k_3 \int (v_2 - v_3) dt$$

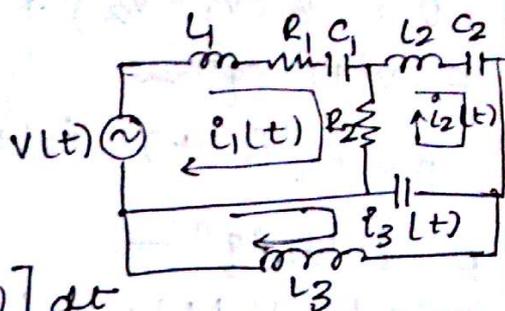
f-v(t) eqⁿ :-

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2} \int i_2(t) dt + R_2 [i_2(t) - i_1(t)] + \frac{1}{C_2} \int [i_2(t) - i_1(t)] dt$$

$$f - i(t) \text{ eqn} - \\ 0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{R_2} [v_2(t) - v_1(t)] + \frac{1}{L_3} \int [v_2(t) - v_3(t)] dt$$

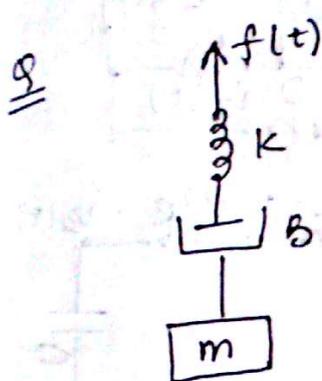
$$\text{At node } x_3 - \\ 0 = m_3 \frac{d^2 x_3}{dt^2} + k_3 (x_3 - x_2)$$

$$0 = m_3 \frac{dv_3}{dt} + k_3 \int (v_3 - v_2) dt$$

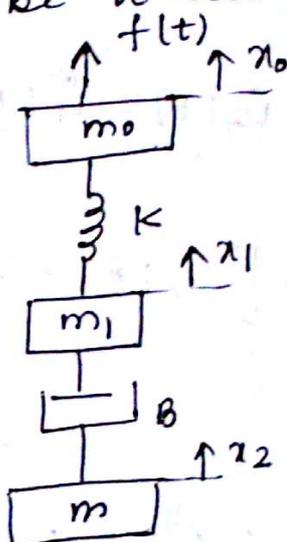


$$f - v(t) \text{ eqn} - \\ 0 = L_3 \frac{di_3(t)}{dt} + \frac{1}{C_3} \int [i_3(t) - i_2(t)] dt$$

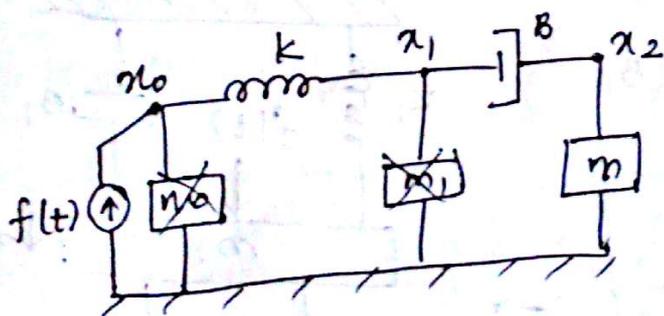
$$f - i(t) \text{ eqn} - \\ 0 = C_3 \frac{dv_3(t)}{dt} + \frac{1}{L_3} \int (v_3(t) - v_2(t)) dt$$



It can be written as -



Here m_0 and m_1 are dummy mass.



At node x_0 :

$$f(t) = k(x_0 - x_1)$$

$$f(t) = k \int (v_0 - v_1) dt$$

f - v(t) eqn -

$$v(t) = \frac{1}{C} \int [i_0(t) - i_1(t)] dt \quad \text{--- (i)}$$

f - i(t) eqn -

$$i(t) = \frac{1}{L} \int [v_0(t) - v_1(t)] dt \quad \text{--- (ii)}$$

At node x_1 :-

$$0 = B \frac{d}{dt} (x_1 - x_2) + K(x_1 - x_0)$$

$$0 = B(v_1 - v_2) + K \int (v_1 - v_0) dt$$

F-v(t) eqn -

$$0 = R[i_1(t) - i_2(t)] + \frac{1}{C} \int [i_1(t) - i_0(t)] dt$$

F-i(t) eqn -

$$0 = \frac{1}{L} [v_1(t) - v_2(t)] + \frac{1}{L} \int [v_1(t) - v_2(t)] dt$$

At node x_2 -

$$0 = m \frac{d^2 x_2}{dt^2} + B \frac{d(x_2 - x_1)}{dt}$$

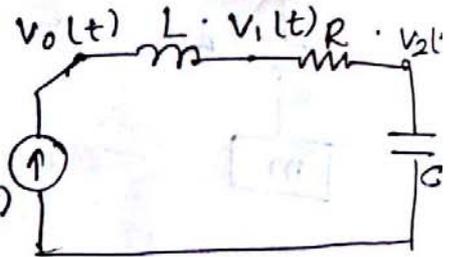
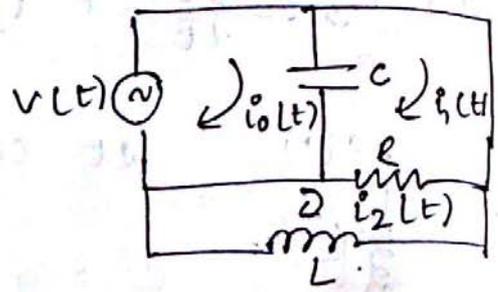
$$0 = m \frac{dv_2}{dt} + B(v_2 - v_1)$$

F-v(t) eqn -

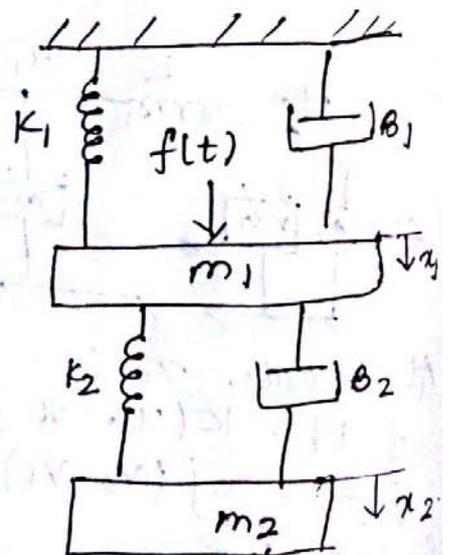
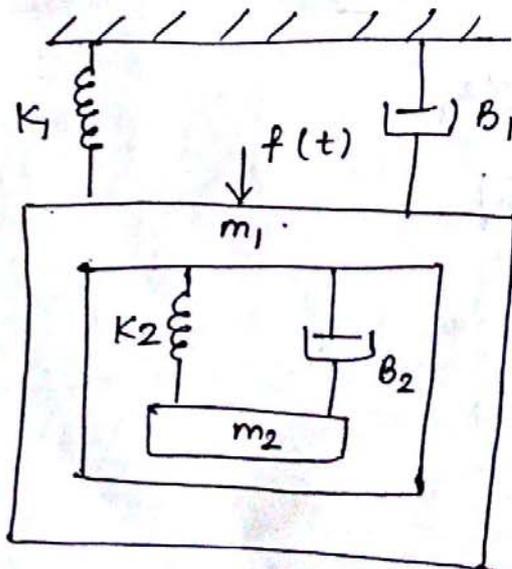
$$0 = L \frac{di_2(t)}{dt} + R[i_2(t) - i_1(t)]$$

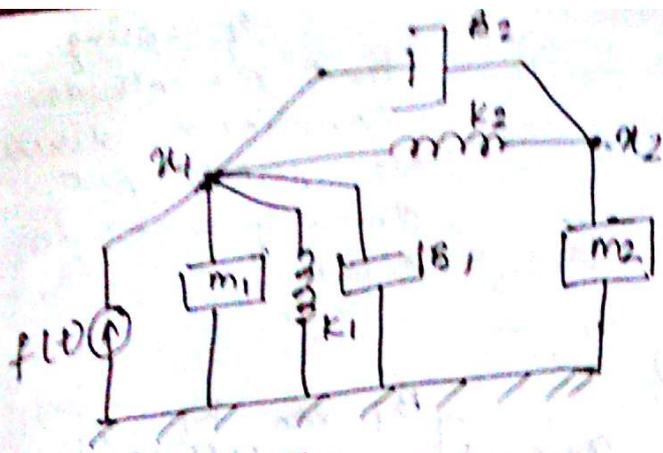
F-i(t) eqn -

$$0 = C \frac{dv_2(t)}{dt} + \frac{1}{R} [v_2(t) - v_1(t)]$$



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At node \$x_1\$ -

$$f(t) - m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1 + k_2 (x_1 - x_2) + B_2 \frac{d(x_1 - x_2)}{dt}$$

$$f(t) = m_1 \frac{dv_1}{dt} + B_1 v_1 + k_1 \int v_1 dt + k_2 \int (v_1 - v_2) dt + B_2 (v_1 - v_2).$$

F-v(t) eqn -

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1} \int i_1(t) dt + \frac{1}{C_2} \int (i_1(t) - i_2(t)) dt + R_2 [i_1(t) - i_2(t)].$$

F-i(t) eqn -

$$i_1(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R} v_1(t) + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{L_2} \int [v_1(t) - v_2(t)] dt + \frac{1}{R_2} [v_1(t) - v_2(t)].$$

At node \$x_2\$ -

$$0 = m_2 \frac{d^2 x_2}{dt^2} + k_2 (x_2 - x_1) + B_2 \frac{d}{dt} (x_2 - x_1).$$

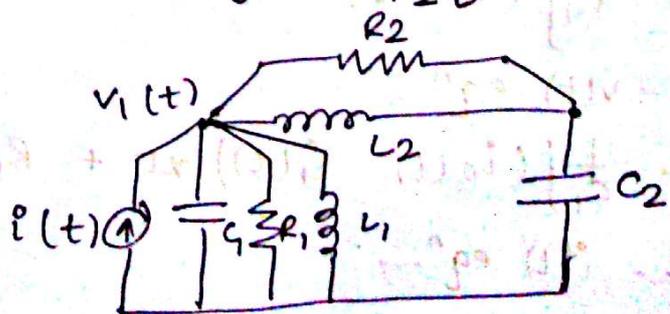
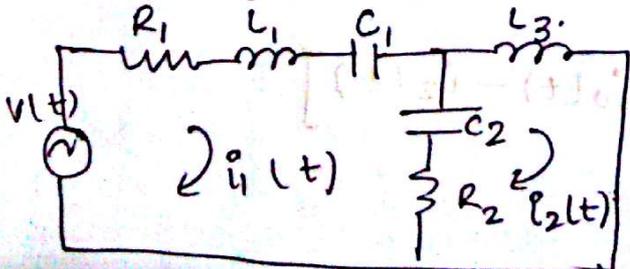
$$0 = m_2 \frac{dv_2}{dt} + k_2 \int (v_2 - v_1) dt + B_2 (v_2 - v_1)$$

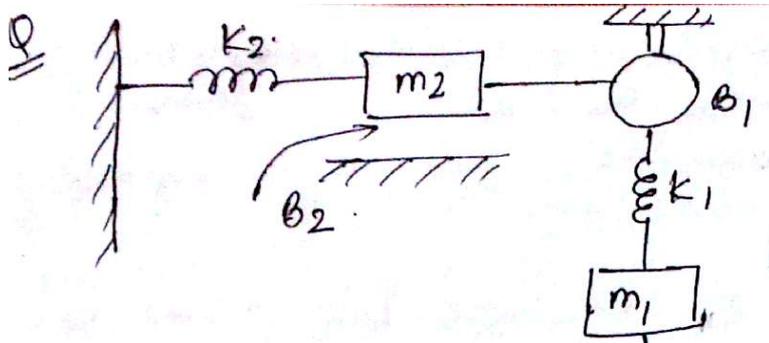
F-v(t) eqn -

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2} \int [i_2(t) - i_1(t)] dt + B_2 [i_2(t) - i_1(t)].$$

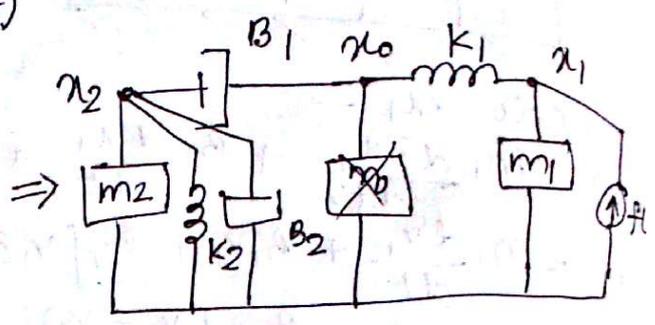
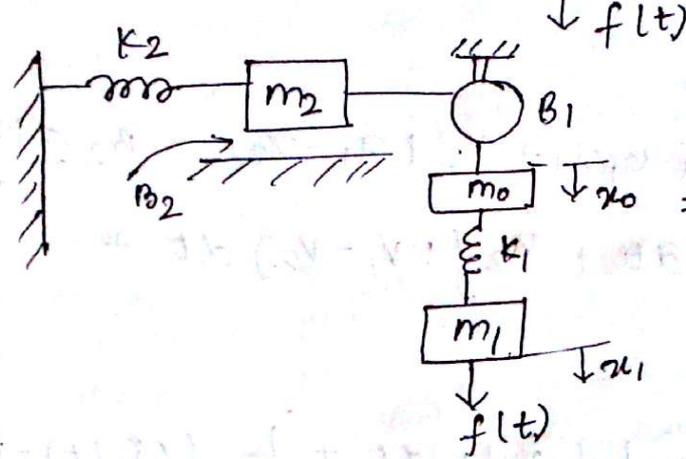
F-i(t) eqn -

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2} \int [v_2(t) - v_1(t)] dt + \frac{1}{R_2} [v_2(t) - v_1(t)].$$





If spring and damper are connected directly we should put dummy mass in between.



At node x_2 -

$$0 = m_2 \frac{d^2 x_2}{dt^2} + k_2 x_2 + B_2 \frac{dx_2}{dt} + B_1 \frac{d}{dt} (x_1 - x_0)$$

$$0 = m_2 \frac{dv_2}{dt} + k_2 \int v_2 dt + B_2 v_2 + B_1 (v_1 - v_0)$$

F - v(t) eqⁿ

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2} \int i_2(t) dt + R_2 i_2(t) + R_1 [i_1(t) - i_2(t)]$$

F - i(t) eqⁿ

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{R_2} v_2(t) + \frac{1}{R_1} [v_1(t) - v_0(t)]$$

At node x_0 :

$$0 = k_1 (x_0 - x_1) + B_1 \frac{d}{dt} (x_0 - x_2)$$

$$0 = k_1 \int (v_0 - v_1) dt + B_1 (v_0 - v_2)$$

f - v(t) eqⁿ

$$0 = \frac{1}{C_1} \int (i_0(t) - i_1(t)) dt + R_1 [i_0(t) - i_2(t)]$$

f - i(t) eqⁿ

$$0 = \frac{1}{L_1} \int [v_0(t) - v_1(t)] dt + \frac{1}{R_1} [v_0(t) - v_2(t)]$$

At node x_1 -

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + k_1 (x_1 - x_0)$$

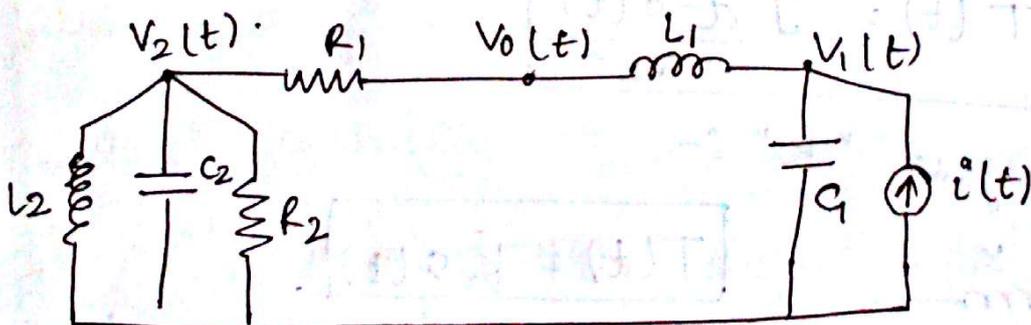
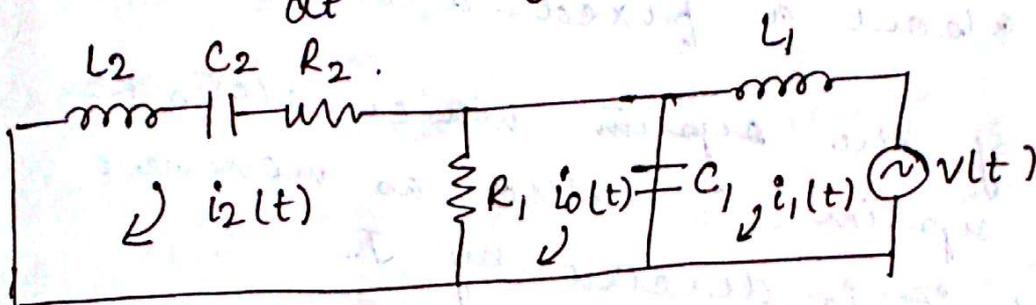
$$f(t) = m_1 \frac{dv_1}{dt} + k_1 \int (v_1 - v_0) dt$$

F - v(t) eqn -

$$v(t) = L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int (i_1(t) - i_0(t)) dt$$

F - i(t) eqn -

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{L_1} \int (v_1(t) - v_0(t)) dt$$

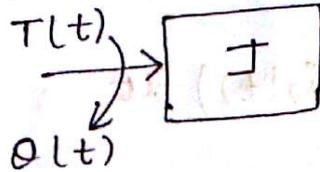


Rotational mechanical system :-

The motion/movement of the body is about its own axis.

The three elements which are dominantly involved in rotational mechanical system -

i) Inertia Torque "J".



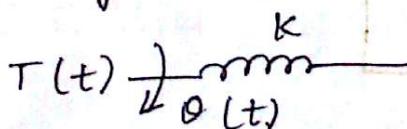
where $T(t)$ = Torque $\theta(t)$ = angular displacement
 J = Inertia torque.

The moment about a fixed axis is called as torque.

The property of the system which stores KE in rotational system is called as moment of inertia and it is denoted by J .

$$T(t) = J \frac{d^2\theta(t)}{dt^2}$$

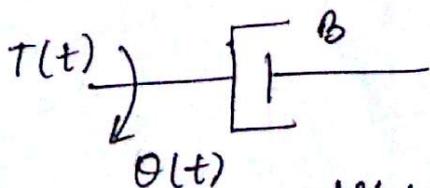
ii) Spring Torque "k" :-



$$T(t) = k\theta(t)$$

where k = stiffness of spring or spring constant
 $\theta(t)$ = angular displacement

iii) Damping Torque "B" :-



$$T(t) = B \frac{d\theta(t)}{dt}$$

where = coefficient of viscous friction

$\theta(t)$: angular displacement
 $T(t)$: Torque.

D'Alembert's principle for Rotational mechanical system :-

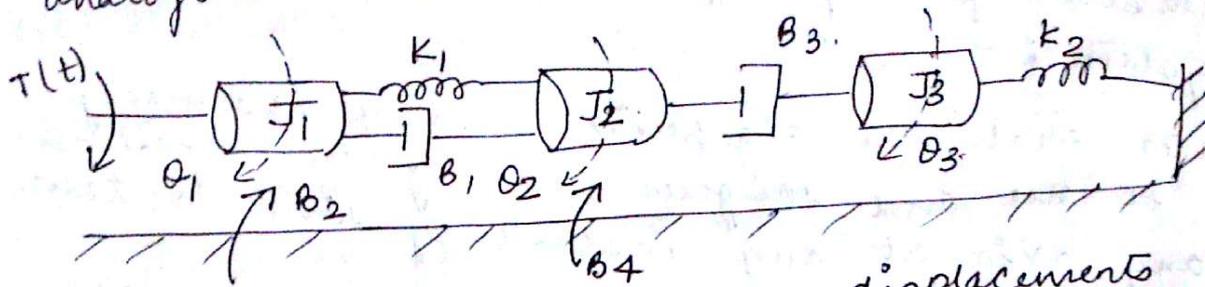
It states that the algebraic sum of externally applied torque and torques resisting the rotation about any axis at any instant of time is zero.

Comparison of translational and Rotational mech system -

- | | |
|---------------------------|--|
| Translational mech system | Rotational mech system |
| i) Force " $f(t)$ " | i) Torque " $T(t)$ " |
| ii) Displacement " x " | ii) Angular displacement " $\theta(t)$ " |
| iii) Velocity " v " | iii) Angular velocity " $\omega(t)$ " |
| iv) Mass " m " | iv) Inertia Torque " J " |
| v) Spring " k " | v) Spring " k " |
| vi) Damper " B " | vi) Damper " B " |

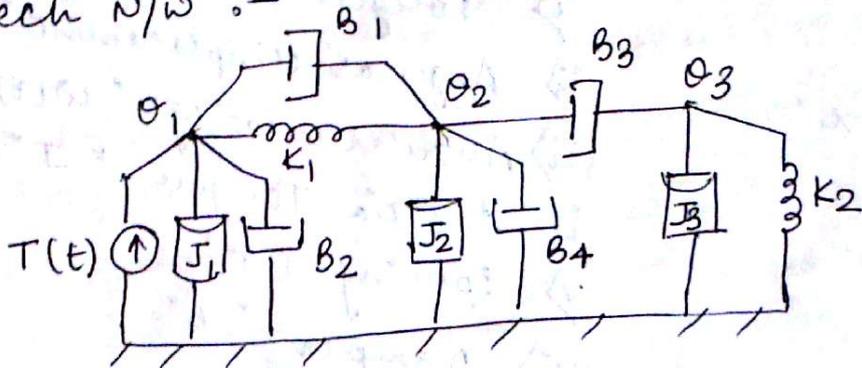
Translational MS	Rotational MS	T-V Analogy	T-I Analogy
Force " $F(t)$ "	Torque " $T(t)$ "	Voltage " $V(t)$ "	Current
Velocity " v "	Angular velocity " $\omega(t)$ "	Current " $i(t)$ "	Voltage
Mass " M "	Moment of Inertia " J "	Inductance " L "	Capacitance " C "
Spring " k "	Spring " k "	Capacitance " $\frac{1}{C}$ "	Inductance " $\frac{1}{L}$ "
Damper " B "	Damper " B "	Resistor " R "	Resistance " $\frac{1}{R}$ "

Q Write the differential eqn of motion for the system shown below. Draw T-I and T-V analogous electric circuit.



No. of MI = No. of angular displacements
 No. of angular displacements = No. of nodes in mech n/w = 3.

Mech n/w :-



at node θ_1 -

$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + B_2 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) + B_1 \frac{d}{dt}(\theta_1 - \theta_2)$$

Torque angular velocity eqn :-

$$T(t) = J_1 \frac{d\omega_1}{dt} + B_2 d\omega_1 + K_1 \int (\omega_1 - \omega_2) dt + B_1 (\omega_1 - \omega_2) + \int \omega_2 dt$$

Torque voltage analogous eqn -

$$V(t) = L_1 \frac{di_1(t)}{dt} + R_2 i_1(t) + \frac{1}{C_1} \int (i_1(t) - i_2(t)) dt + R_1 (i_1(t) - i_2(t))$$

Torque current analogous eqn -

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} i_1(t) + \frac{1}{L_1} \int (v_1(t) - v_2(t)) dt + \frac{1}{R_1} (v_1(t) - v_2(t))$$

at node θ_2 :-

$$0 = J_2 \frac{d^2 \theta_2}{dt^2} + B_4 \frac{d\theta_2}{dt} + K_1 (\theta_2 - \theta_1) + B_1 \frac{d}{dt} (\theta_2 - \theta_1) + B_3 \frac{d}{dt} (\theta_2 - \theta_3)$$

Torque angular velocity eqⁿ :-

$$0 = J_2 \frac{d\omega_2}{dt} + B_4 \omega_2 + K_1 \int (\omega_2 - \omega_1) dt + B_1 (\omega_2 - \omega_1) + B_3 (\omega_2 - \omega_3)$$

Torque voltage analogous eqⁿ -

$$0 = L_2 \frac{di_2(t)}{dt} + R_4 i_2(t) + \frac{1}{C_1} \int (i_2(t) - i_1(t)) dt + R_1 (i_2(t) - i_1(t)) + R_3 (i_2(t) - i_3(t)) \text{---(ii)}$$

Torque current analogous eqⁿ -

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R_4} v_2(t) + \frac{1}{L_1} \int (v_2(t) - v_1(t)) dt + \frac{1}{R_1} (v_2(t) - v_1(t)) + \frac{1}{R_3} (v_2(t) - v_3(t)) \text{---(iv)}$$

at node θ_3 -

$$0 = J_3 \frac{d^2 \theta_3}{dt^2} + K_2 \theta_3 + B_3 \frac{d}{dt} (\theta_3 - \theta_2)$$

Torque angular velocity eqⁿ -

$$0 = J_3 \frac{d\omega_3}{dt} + K_2 \int \omega_3 dt + B_3 (\omega_3 - \omega_2)$$

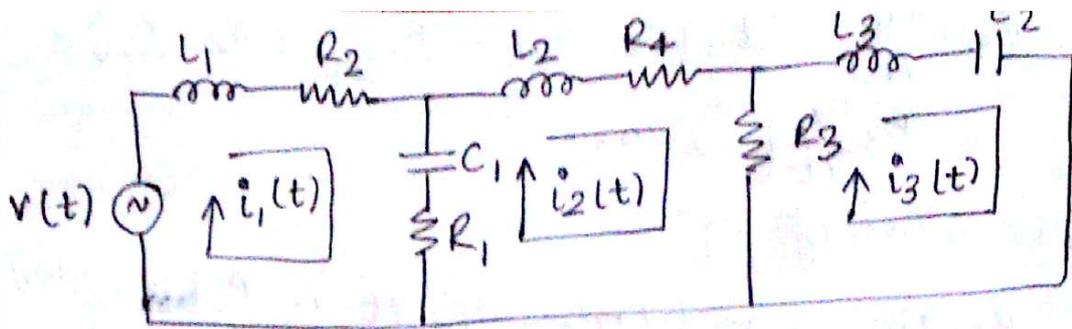
Torque voltage analogous eqⁿ -

$$0 = L_3 \frac{di_3(t)}{dt} + \frac{1}{C_2} \int i_3(t) dt + R_3 (i_3(t) - i_2(t)) \text{---(v)}$$

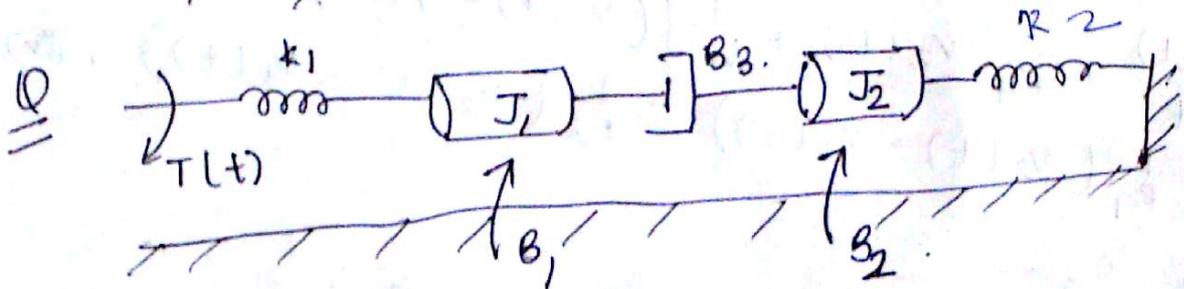
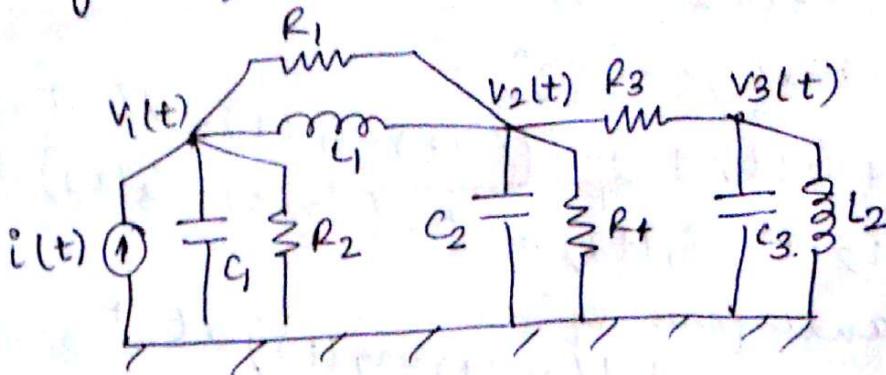
(b). Torque current analogous eqⁿ -

$$0 = C_3 \frac{dv_3(t)}{dt} + \frac{1}{L_2} \int v_3(t) dt + \frac{1}{R_3} (v_3(t) - v_2(t)) \text{---(vi)}$$

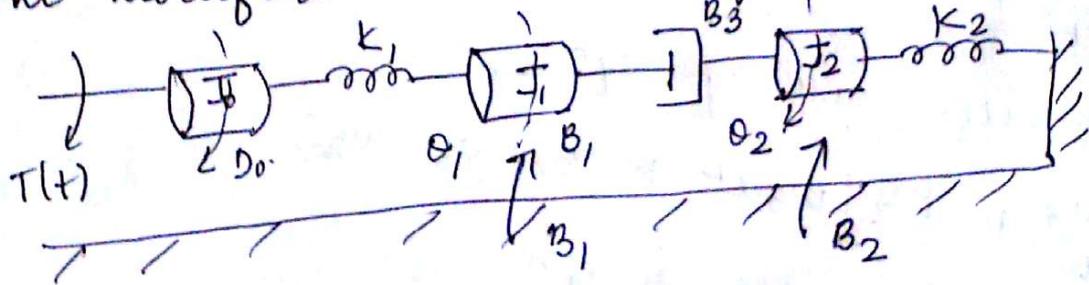
using eqⁿ (i) (ii) & (v) -



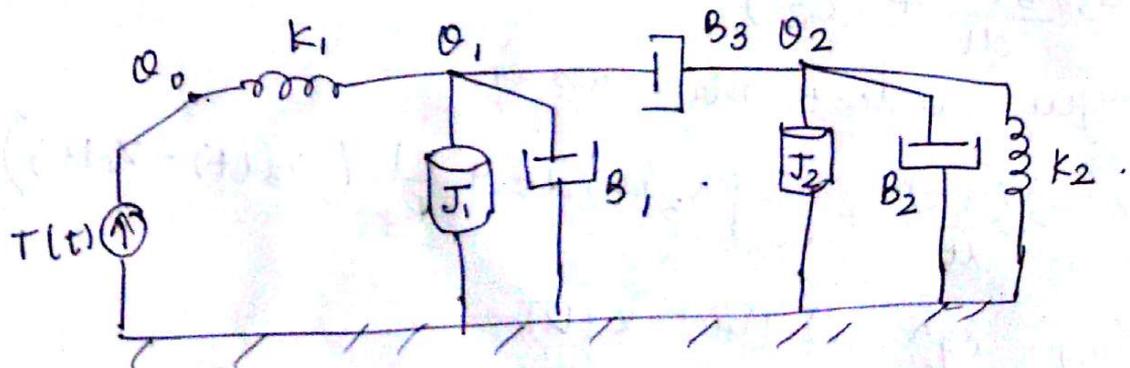
using eqⁿ (ii) (iv) (vi)



The modified mechanical system is -



No. of MI = No. of angular displacements = No. of nodes in mech $n/w = 3$.



at node θ_0 .

$$T(t) = k_1(\theta_0 - \theta_1)$$

Torque angular velocity equation —

$$T(t) = k_1 \int (\omega_0 - \omega_1) dt$$

Torque voltage —

$$V(t) = \frac{1}{C_1} \int (i_0(t) - i_1(t)) dt \quad \text{--- (i)}$$

Torque current —

$$i(t) = \frac{1}{L_1} \int (v_0(t) - v_1(t)) dt \quad \text{--- (ii)}$$

at node θ_1 —

$$0 = J_1 \frac{d^2 \theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + B_3 \frac{d}{dt} (\theta_1 - \theta_2)$$

$$0 = J_1 \frac{d\omega_1}{dt} + B_1 \omega_1 + B_3 (\omega_1 - \omega_2)$$

$$0 = L_1 \frac{di_1(t)}{dt} + R_1 i_1(t) + R_3 (i_1(t) - i_2(t)) \quad \text{--- (iii)}$$

$$0 = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} v_1(t) + \frac{1}{R_3} (v_1(t) - v_2(t)) \quad \text{--- (iv)}$$

at node θ_2 —

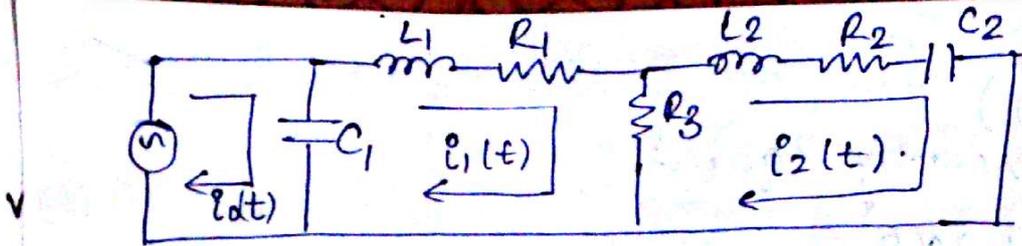
$$0 = J_2 \frac{d^2 \theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + k_2 \theta_2 + B_3 \frac{d}{dt} (\theta_2 - \theta_1)$$

$$0 = J_2 \frac{d\omega_2}{dt} + B_2 \omega_2 + k_2 \int \omega_2 dt + B_3 (\omega_2 - \omega_1)$$

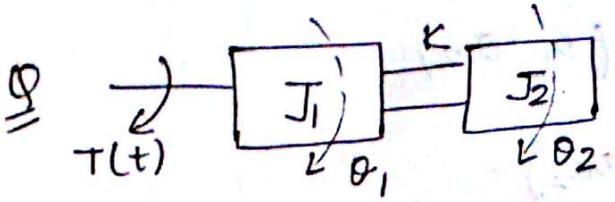
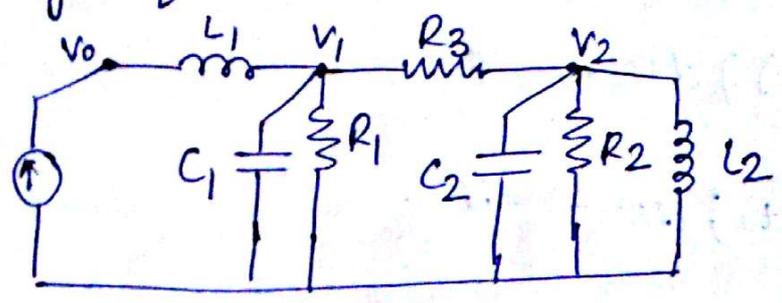
$$0 = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C_2} \int i_2(t) dt + R_3 (i_2(t) - i_1(t)) \quad \text{--- (v)}$$

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R_2} v_2(t) + \frac{1}{L_2} \int v_2(t) dt + \frac{1}{R_3} (v_2(t) - v_1(t)) \quad \text{--- (vi)}$$

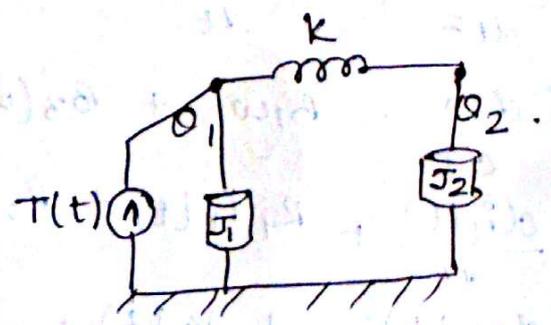
Using eqⁿ (i) (iii) & (v).

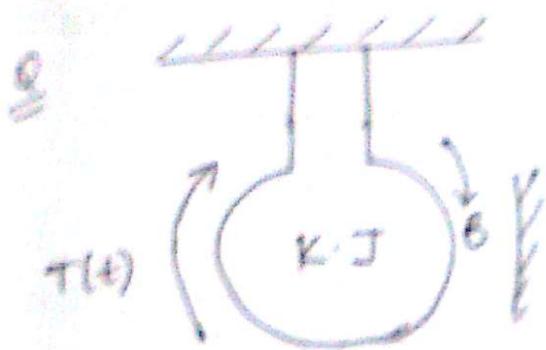


using eqn (ii) (iv) (vi) —

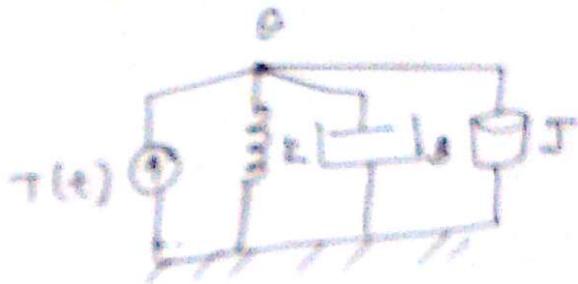


Mech network —

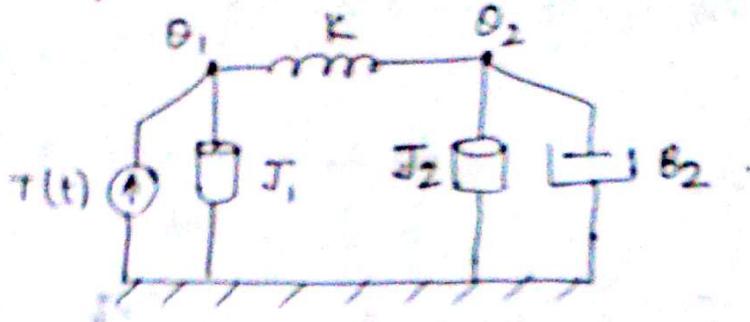
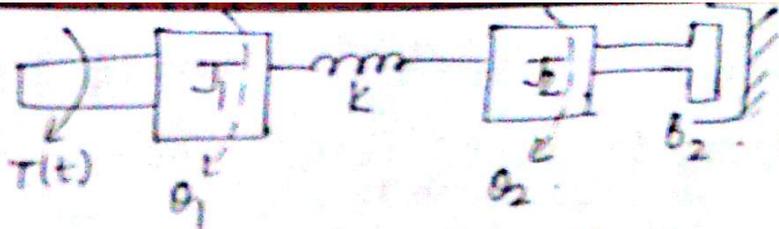




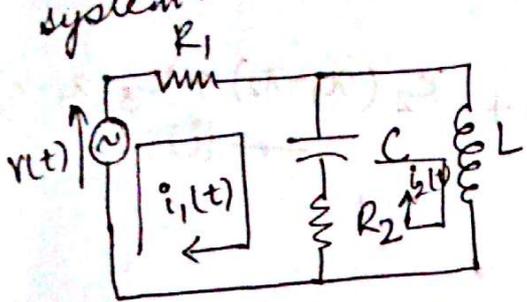
Mechanical n/w -



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Draw the force voltage analogous mechanical system for the electrical circuit shown below by writing the loop eqⁿ for the electrical circuits and then transforming it into mechanical system.



Apply KVL for the path traced by $i_1(t)$.

$$v(t) = R_1 i_1(t) + \frac{1}{C} \int (i_1(t) - i_2(t)) dt + R_2 (i_1(t) - i_2(t)).$$

Force voltage Analogous eqⁿ -

$$F(t) = B_1 \frac{dx_1}{dt} + k(x_1 - x_2) + B_2 \frac{d(x_1 - x_2)}{dt} \quad (i)$$

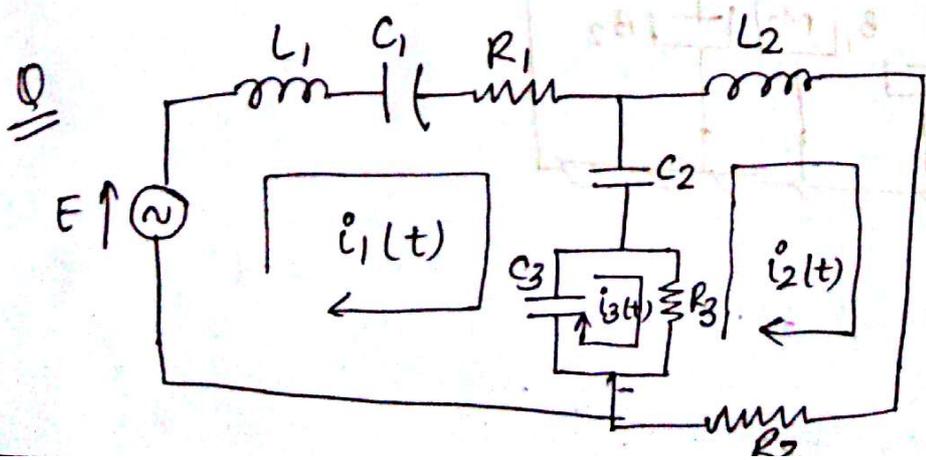
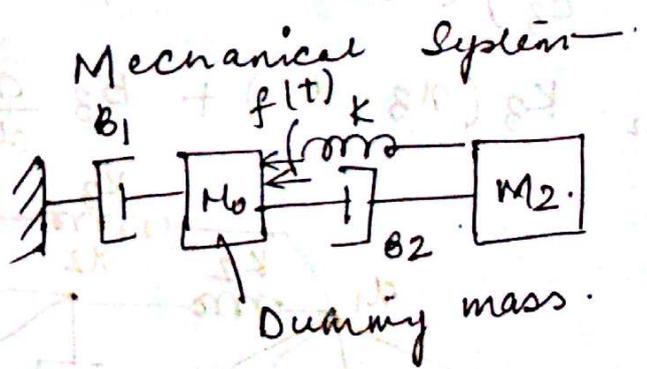
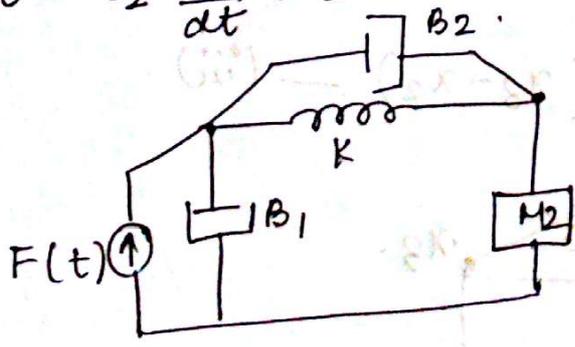
$$\begin{aligned} F(t) &= V \\ V &= i(t) \\ M &= L \\ K &= \frac{1}{C} \\ B &= R \end{aligned} \quad \begin{aligned} i(t) &= v \\ i(t) &= \frac{dx}{dt} \end{aligned}$$

Apply KVL for the path traced by $i_2(t)$ -

$$0 = R_2 (i_2(t) - i_1(t)) + \frac{1}{C} \int (i_2(t) - i_1(t)) dt + L \frac{di_2(t)}{dt}$$

Force voltage analogous eqⁿ -

$$0 = B_2 \frac{d(x_2 - x_1)}{dt} + k(x_2(t) - x_1(t)) + M \frac{d^2 x_2}{dt^2} \quad (ii)$$



Applying KVL for the loop 1 -

$$E_1 = L_1 \frac{di_1(t)}{dt} + \frac{1}{C_1} \int i_1(t) dt + R_1 i_1(t) + \frac{1}{C_2} \int (i_1(t) - i_2(t)) dt + \frac{1}{C_3} \int (i_1(t) - i_3(t)) dt$$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + K_2 (x_1 - x_2) + K_3 (x_1 - x_3) \quad \text{--- (i)}$$

Applying KVL for loop 2 -

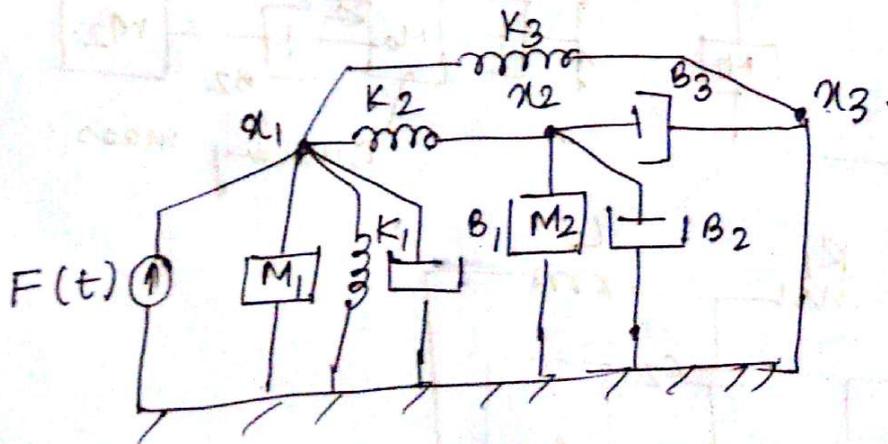
$$0 = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C_2} \int (i_2(t) - i_1(t)) dt + R_3 (i_2(t) - i_3(t))$$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 (x_2 - x_1) + B_3 \frac{d}{dt} (x_2 - x_3) \quad \text{--- (ii)}$$

Applying KVL for loop 3 -

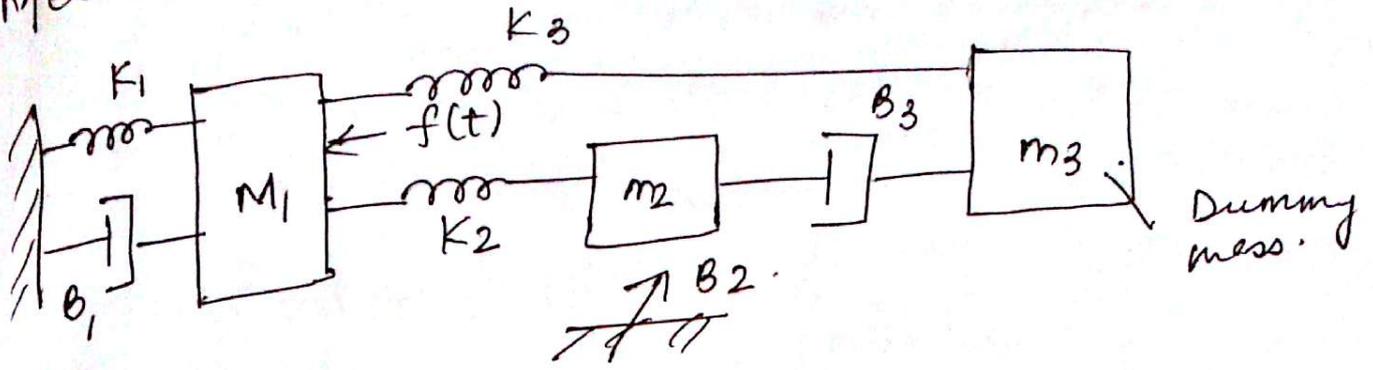
$$0 = \frac{1}{C_3} \int (i_3(t) - i_1(t)) dt + R_3 (i_3(t) - i_2(t))$$

$$0 = K_3 (x_3 - x_1) + B_3 \frac{d}{dt} (x_3 - x_2) \quad \text{--- (iii)}$$

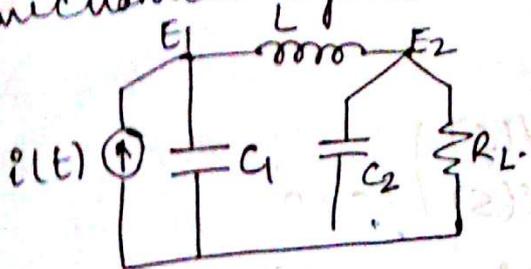


Mechanical system

3 nodes - 3 mass



Q For the electrical network shown below using Torque current analogy draw the mechanical r/w and mechanical system.



at node E_1 -

$$i(t) = C_1 \frac{dE_1(t)}{dt} + \frac{1}{L} \int (E_1 - E_2) dt$$

Torque current analogous eqⁿ -

$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + k(\theta_1 - \theta_2) \quad \text{--- (i)}$$

at node E_2 -

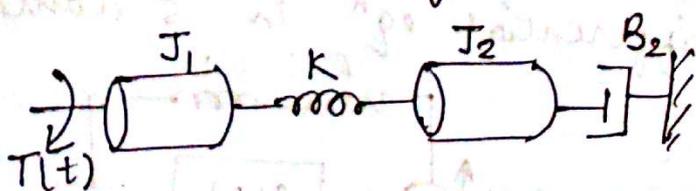
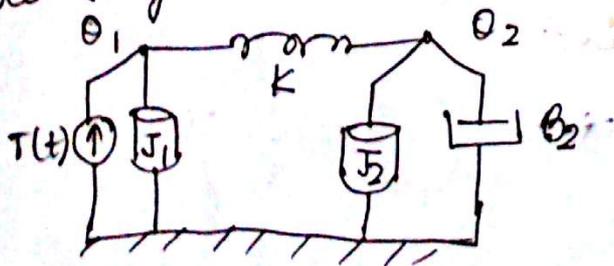
$$0 = \frac{E_2}{R_2} + C_2 \frac{dE_2}{dt} + \frac{1}{L} \int (E_2 - E_1) dt$$

Torque current analogous eqⁿ -

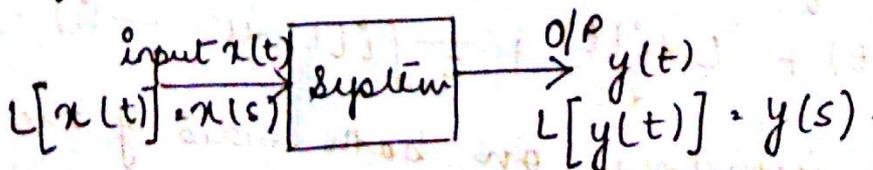
$$0 = B_2 \frac{d\theta_2}{dt} + J_2 \frac{d^2\theta_2}{dt^2} + k(\theta_2 - \theta_1) \quad \text{--- (ii)}$$

using eqⁿ (i) & (ii).

The torque current analogous mechanical network is



Transfer Function -



It is defined as the ratio of Laplace transform of O/P variable to the Laplace transform of I/P variable with all the initial conditions set equal to zero.

$$TF = \frac{L[y(t)]}{L[x(t)]} = \frac{Y(s)}{X(s)} \Big|_{IC=0}$$

Laplace transform of electrical network -

$$L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

Element Time domain expression Laplace domain expression

Voltage $v(t)$ $V(s)$

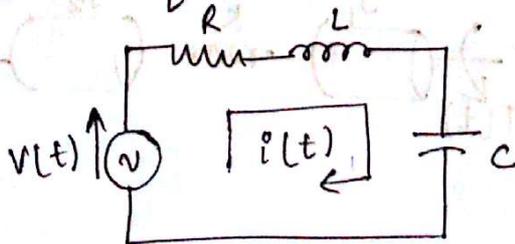
Current $i(t)$ $I(s)$

Resistance $v(t) = i(t)R$ $V(s) = I(s)R$

Inductance $v(t) = L \frac{di(t)}{dt}$ $V(s) = LS I(s)$

Capacitance $v(t) = \frac{1}{C} \int i(t) dt$ $V(s) = \frac{1}{Cs} I(s)$

Q Write the integral differential eqⁿ for the network shown below also write the integral differential eqⁿ in s domain.



Applying KVL for the path traced by $i(t)$ -

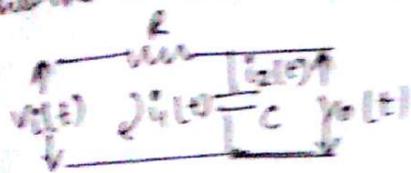
$$v(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt \quad \text{--- (i)}$$

Taking Laplace transform on both sides of the above eqⁿ we get differential eqⁿ in s domain -

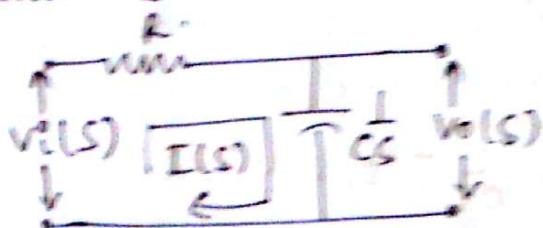
$$V(s) = RI(s) + sL I(s) + \frac{1}{Cs} I(s)$$

$$V(s) = I(s) \left[R + sL + \frac{1}{Cs} \right]$$

Q Write the transfer function for the electrical network shown below -



The network in the transfer domain is -



Apply KVL for the path traced by $I(s)$

$$V_i(s) = RI(s) + \frac{1}{Cs} I(s)$$

$$V_i(s) = I(s) \left[R + \frac{1}{Cs} \right]$$

$$I(s) = \frac{V_i(s)}{R + \frac{1}{Cs}} \quad \text{--- (i)}$$

$$V_o(s) = I(s) \cdot \frac{1}{Cs} \quad \text{--- (ii)}$$

put (i) in (ii) -

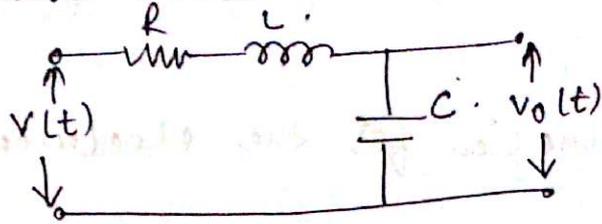
$$V_o(s) = \left(\frac{V_i(s)}{R + \frac{1}{Cs}} \right) \cdot \frac{1}{Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{1}{R + \frac{1}{Cs}} \right) \cdot \frac{1}{Cs}$$

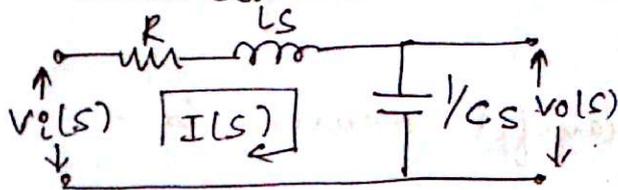
$$= \left(\frac{1}{Rcs + 1} \right) \cdot \frac{1}{Cs}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{Rcs + 1}}$$

Q. Write the transfer function for the network shown below -



in s domain -



KVL for I(s) -

$$V_i(s) = R I(s) + sL I(s) + \frac{1}{Cs} I(s)$$

$$V_i(s) = I(s) \left[R + sL + \frac{1}{Cs} \right]$$

$$I(s) = \frac{V_i(s)}{R + sL + \frac{1}{Cs}} \quad \text{--- (i)}$$

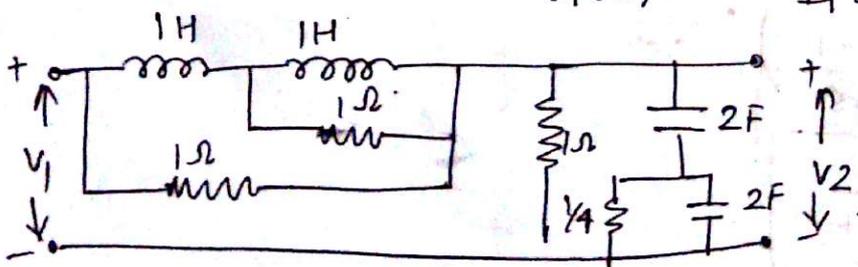
$$V_o(s) = I(s) \cdot \frac{1}{Cs} \quad \text{--- (ii)}$$

Sub (i) in (ii)

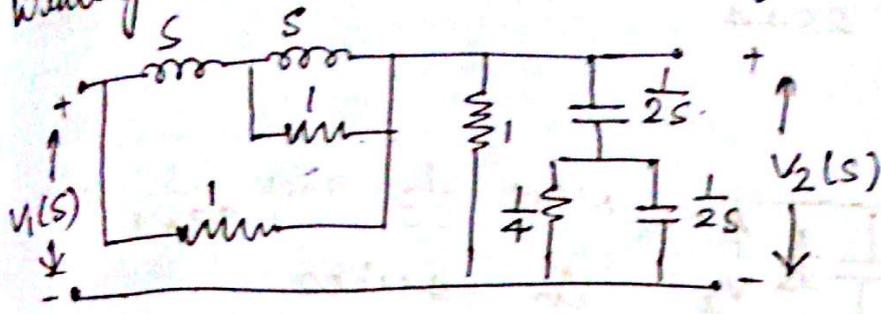
$$V_o(s) = \frac{V_i(s)}{R + sL + \frac{1}{Cs}} \cdot \frac{1}{Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + sL + \frac{1}{Cs}} = \frac{1}{RCS + Ls^2 + 1}$$

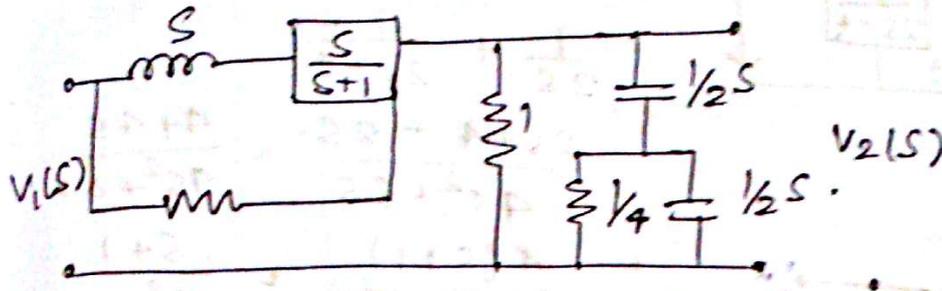
Q. For the 2-port network shown below obtain the transfer function $\frac{V_2(s)}{V_1(s)}$ & $\frac{V_1(s)}{I_1(s)}$.



Writing the network in transform is —

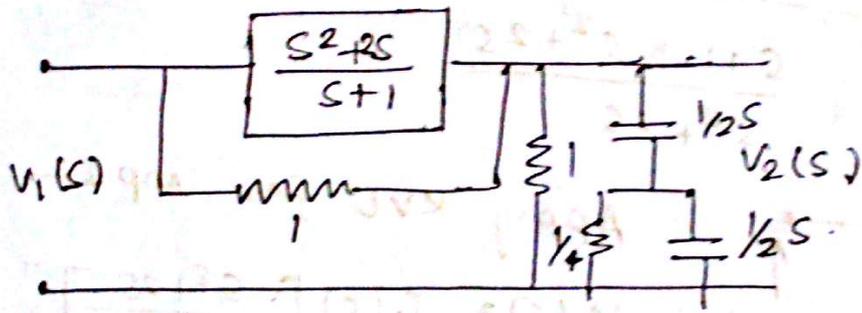


s and 1 are in parallel \therefore equivalent is $\frac{s \times 1}{s+1}$



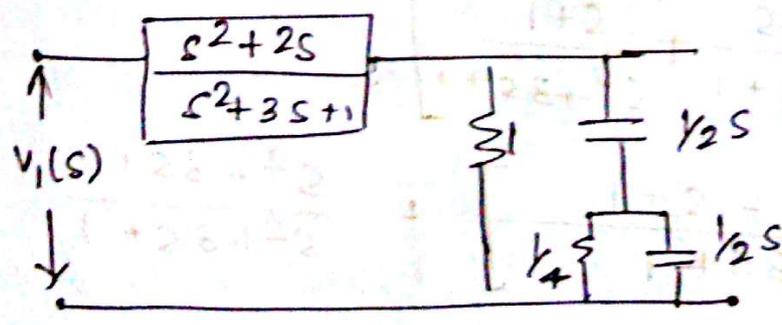
s and $\frac{s}{s+1}$ blocks are in series $\frac{s + \frac{s}{s+1}}{s+1}$
 $= \frac{s(s+1) + s}{s+1} = \frac{s^2 + s + s}{s+1}$
 $= \frac{s^2 + 2s}{s+1}$

Now this network becomes —



Now $\frac{s^2 + 2s}{s+1}$ and 1 are in parallel.

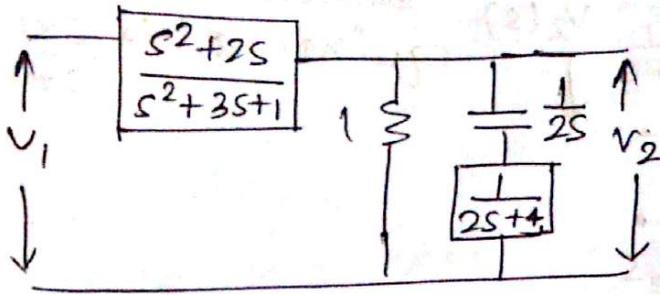
$$\therefore \frac{\frac{s^2 + 2s}{s+1}}{\frac{s^2 + 2s}{s+1} + 1} = \frac{\frac{s^2 + 2s}{s+1}}{\frac{s^2 + 2s + s + 1}{s+1}} = \frac{s^2 + 2s}{s^2 + 3s + 1}$$



Now $1/4$ and $1/2s$ are in parallel.

$$\frac{1}{4} \times \frac{1}{2s} = \frac{1}{8s} = \frac{1}{2s+4}$$

$$\frac{1}{4} + \frac{1}{2s} = \frac{2s+4}{8s}$$

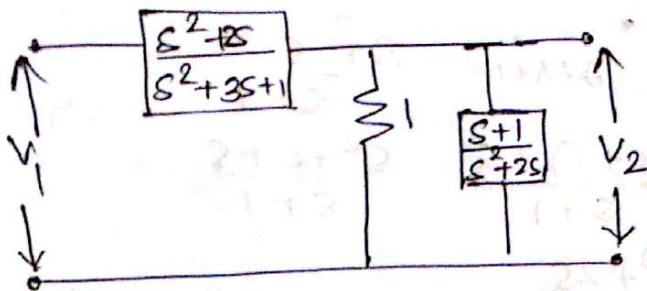


Now $\frac{1}{2s}$ and $\frac{1}{2s+4}$ are in series.

$$\frac{1}{2s} + \frac{1}{2s+4}$$

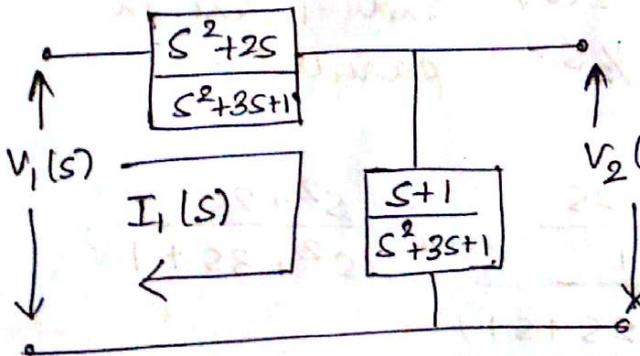
$$\frac{2s+4 + 2s}{4s^2+8s} = \frac{4+4s}{4s^2+8s}$$

$$= \frac{4(s+1)}{4(s^2+2s)} = \frac{s+1}{s^2+2s}$$



Now 1 and $\frac{s+1}{s^2+2s}$ are in parallel.

$$\frac{\frac{s+1}{s^2+2s} \cdot 1}{\frac{s+1}{s^2+2s} + 1} = \frac{\frac{s+1}{s^2+2s}}{\frac{s+1 + s^2+2s}{s^2+2s}} = \frac{s+1}{s^2+3s+1}$$



Apply KVL for loop 1 -

$$V_1(s) = I_1(s) \left[\frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1} \right]$$

$$V_1(s) = I_1(s) \left[\frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1} \right]$$

$$\frac{V_1(s)}{I_1(s)} = \left[\frac{s^2+2s+s+1}{s^2+3s+1} \right] = \frac{s^2+3s+1}{s^2+3s+1}$$

$$\boxed{\frac{V_1(s)}{I_1(s)} = 1} \quad \text{--- (i)}$$

$$V_2(s) = I_1(s) \left[\frac{s+1}{s^2+3s+1} \right]$$

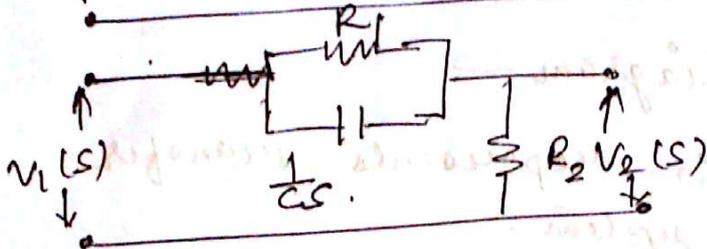
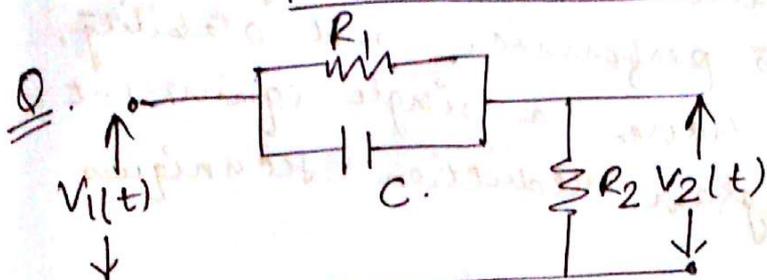
$$\frac{V_2(s)}{I_1(s)} = \frac{s+1}{s^2+3s+1} \quad \text{--- (ii)}$$

From eqⁿ (i) $\frac{V_1(s)}{I_1(s)} = 1$

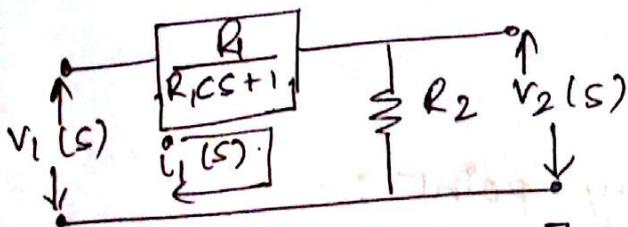
$$V_1(s) = I_1(s) \quad \text{--- (3)}$$

Sub (3) in (ii)

$$\frac{V_2(s)}{V_1(s)} = \frac{s+1}{s^2+3s+1}$$



Now R_1 and $\frac{1}{Cs}$ are in parallel.

$$\frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{\frac{R_1}{Cs}}{\frac{R_1 Cs + 1}{Cs}} = \frac{R_1}{R_1 Cs + 1}$$


$$V_1(s) = I_1(s) \left[\frac{R_1}{R_1 Cs + 1} \right] + I_1(s) R_2$$

$$V_1(s) = I_1(s) \left[\frac{R_1}{R_1 Cs + 1} + R_2 \right] \quad \frac{V_1(s)}{I_1(s)} = \left[\frac{R_1}{R_1 Cs + 1} + R_2 \right]$$

$$V_1(s) = I_1(s) \left[\frac{R_1 + R_2(R_1 Cs + 1)}{R_1 Cs + 1} \right] \quad I_1(s) = \frac{V_1(s)}{\left[\frac{R_1 + R_2(R_1 Cs + 1)}{R_1 Cs + 1} \right]}$$

$$V_2(s) = I_1(s) R_2$$

$$\frac{V_2(s)}{I_1(s)} = R_2$$

Now putting the value of $I_1(s)$:

$$V_2(s) = R_2 \left[\frac{V_1(s)}{R_1 + R_2(R_1Cs + 1)} \right]$$

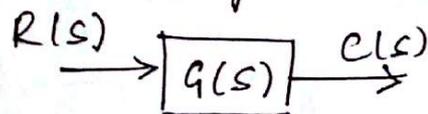
$$\frac{V_2(s)}{V_1(s)} = R_2 \left[\frac{R_1Cs + 1}{R_1 + R_2(R_1Cs + 1)} \right]$$

BLOCK DIAGRAM REDUCTION TECHNIQUE :-

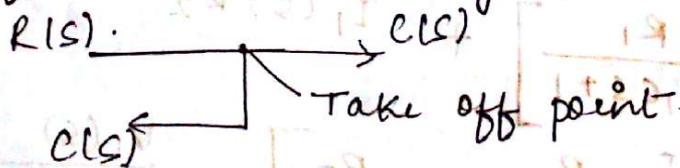
Practical system consists of no. of blocks in series or in parallel. For analysing the system in terms of its performance and stability, it is compulsory to have a single equivalent block hence block diagram reduction techniques are used.

Components of block diagram —

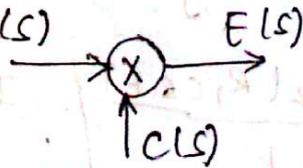
i) Functional block : This represents transfer function of a system.



ii) Take off point or Tapping point :-



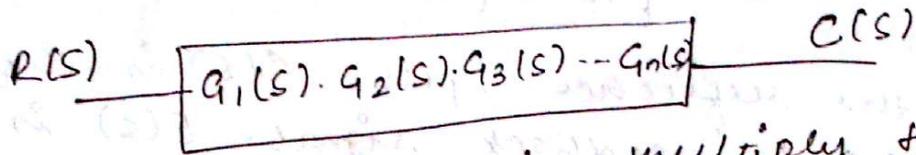
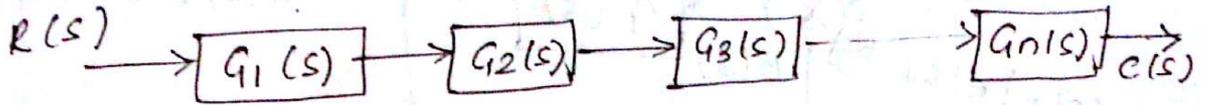
iii) Summer : $E(s) = e(s) \pm e(s)$



iv) Arrow : Represents the flow of signal.

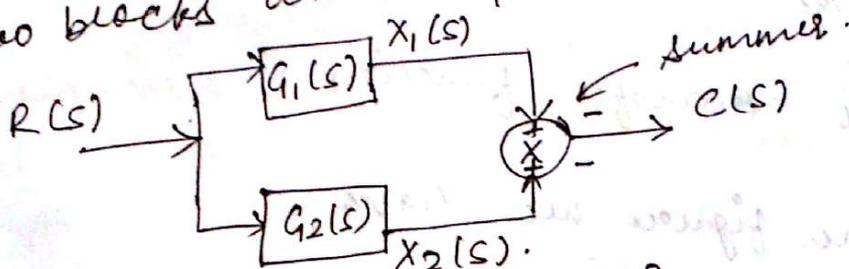
Rules for block diagram construction —

i) Two or more blocks in series/cascade.



When blocks are in series multiply the values of all the blocks and enclose the value in a single block.

ii) Two blocks are in parallel —



$$X_1(s) = R(s) G_1(s)$$

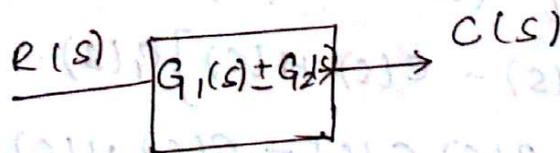
$$X_2(s) = R(s) G_2(s)$$

$$C(s) = X_1(s) \pm X_2(s)$$

$$C(s) = R(s) G_1(s) \pm R(s) G_2(s)$$

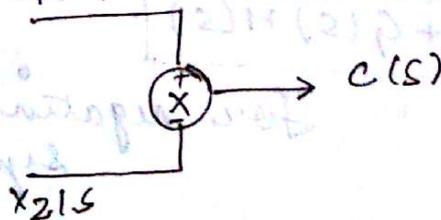
$$C(s) = R(s) [G_1(s) \pm G_2(s)]$$

$$\frac{C(s)}{R(s)} = G_1(s) \pm G_2(s)$$



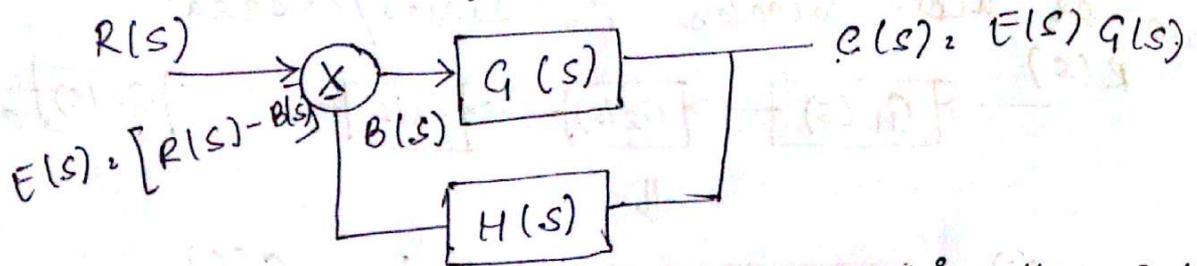
When blocks are in parallel add the values of all blocks and enclose the value in a single block.

NOTE :



$$C(s) = X_1(s) - X_2(s)$$

iii) To eliminate negative feedback loop -



$R(s)$ is the reference input. $C(s)$ is the output signal. $B(s)$ is feedback signal. $E(s)$ is error signal. $G(s) = \frac{C(s)}{E(s)}$ is called forward path transfer function.

$H(s) = \frac{B(s)}{C(s)}$ is called feedback transfer function.

$\frac{C(s)}{R(s)}$ is called transfer function.

From the above figure we have

$$E(s) = R(s) - B(s) \quad \text{--- (i)}$$

$$B(s) = C(s) \cdot H(s) \quad \text{--- (ii)}$$

$$C(s) = E(s) \cdot G(s) \quad \text{--- (iii)}$$

Substitute eqⁿ (i) in (iii) -

$$C(s) = [R(s) - B(s)] G(s) \quad \text{--- (iv)}$$

Substitute eqⁿ (ii) in (iv).

$$C(s) = [R(s) - C(s) H(s)] G(s)$$

$$C(s) = R(s) G(s) - G(s) H(s) C(s)$$

$$C(s) + C(s) H(s) G(s) = R(s) G(s)$$

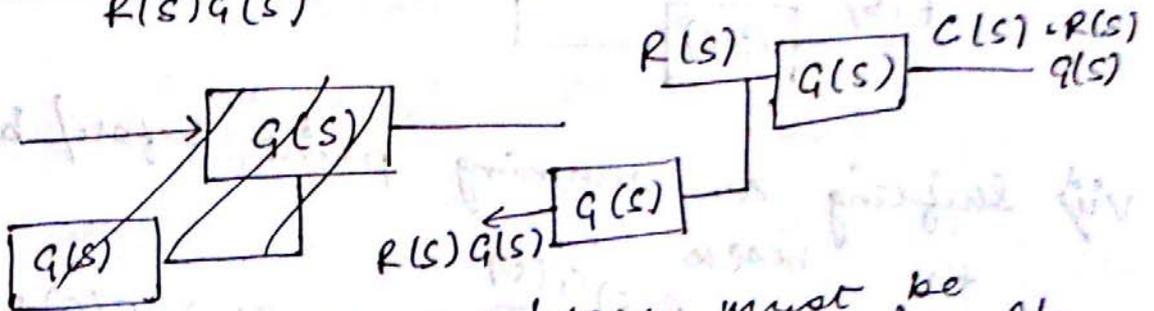
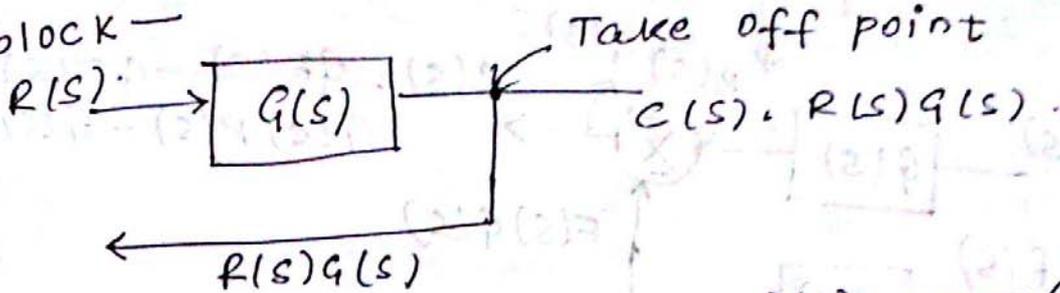
$$C(s) [1 + G(s) H(s)] = R(s) G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}}$$

For negative feedback system.

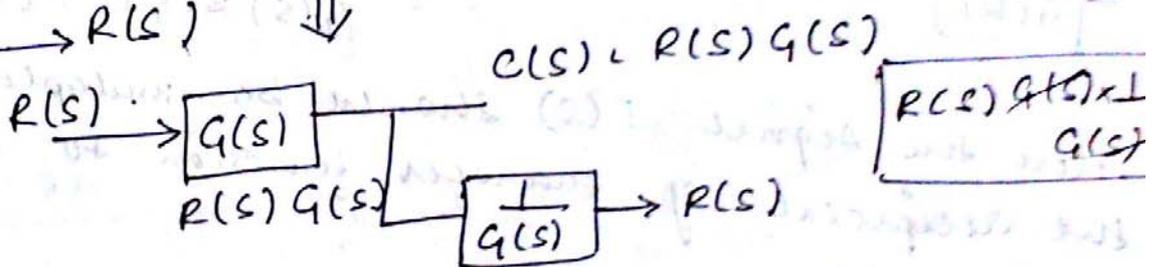
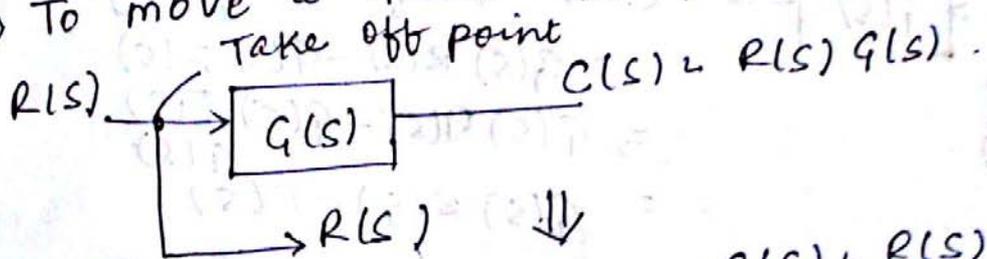
$$\left[\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \right] \text{ for positive feedback system}$$

iv) To move a take-off point behind the block —



Signal taking off after a block must be multiplied with the transfer function of that block while shifting the block.

v) To move a take off point after the block:

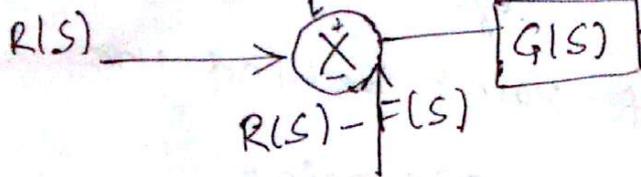


Signal taking off the block must be multiplied with the reciprocal of transfer function of that block while shifting beyond.

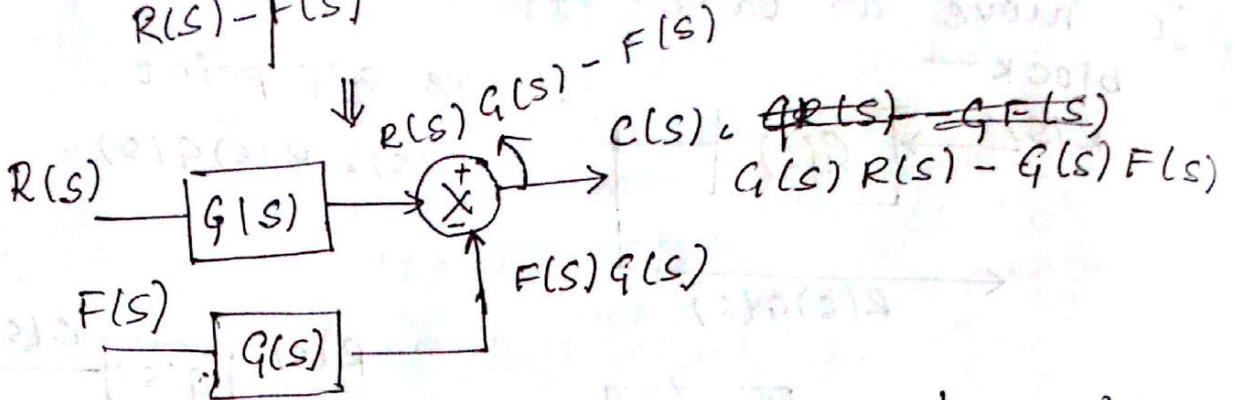
Left to Right — dividing the system

Right to left — multiplying the system

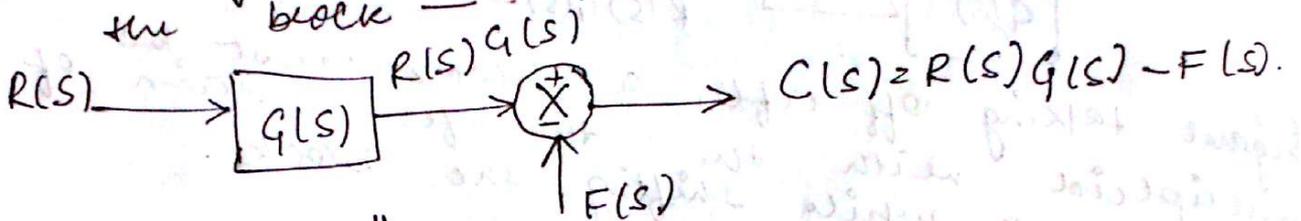
vi) Shifting the summing point after / beyond —



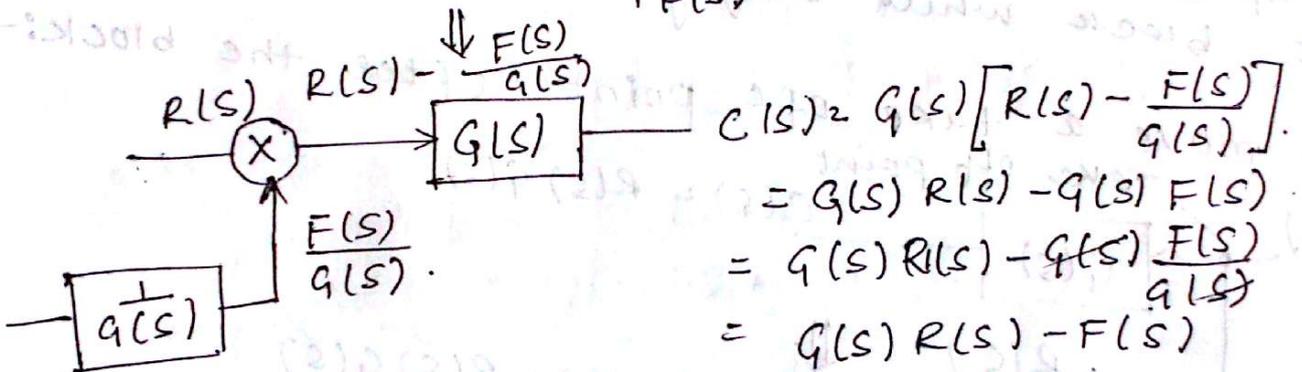
$$C(s) = G(s)[R(s) - F(s)] \\ = G(s)R(s) - G(s)F(s)$$



vii) Shifting the summing point before / behind the block —



$$C(s) = R(s)G(s) - F(s)$$



$$C(s) = G(s) \left[R(s) - \frac{F(s)}{G(s)} \right] \\ = G(s)R(s) - G(s) \frac{F(s)}{G(s)} \\ = G(s)R(s) - F(s)$$

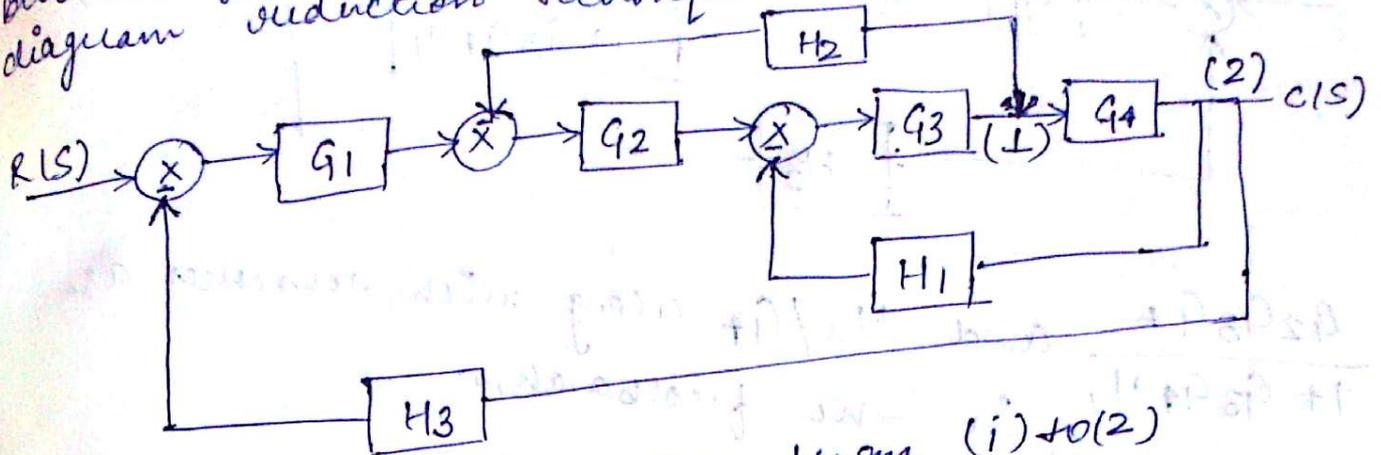
Here the signal $F(s)$ should be multiplied by the reciprocal of transfer function to keep the output same.

For summer —

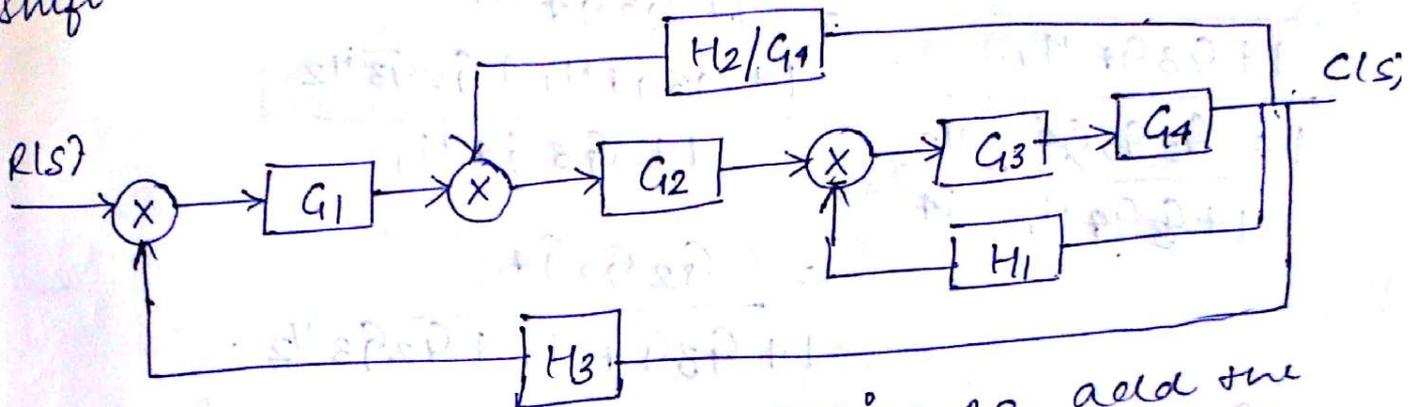
Left to Right — Multiplication

Right to left — Division

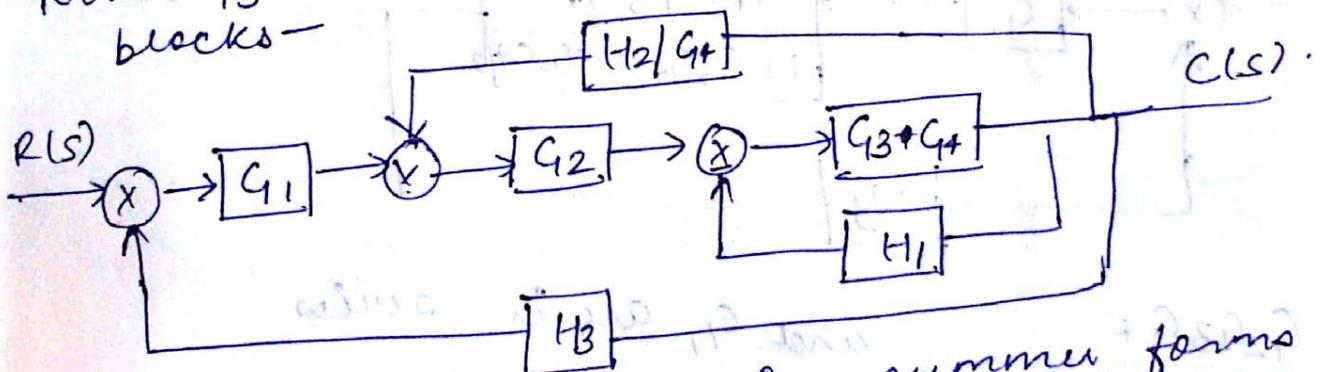
Q Find the closed loop transfer function of the block diagrams given below by using block diagram reduction technique.



Shift the take off point from (1) to (2)

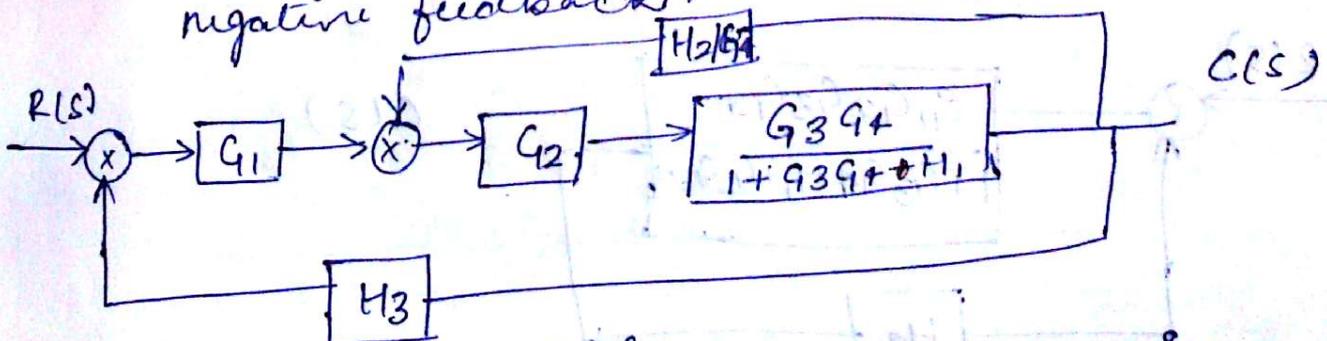


Now G_3 and G_4 are in series so add the blocks -

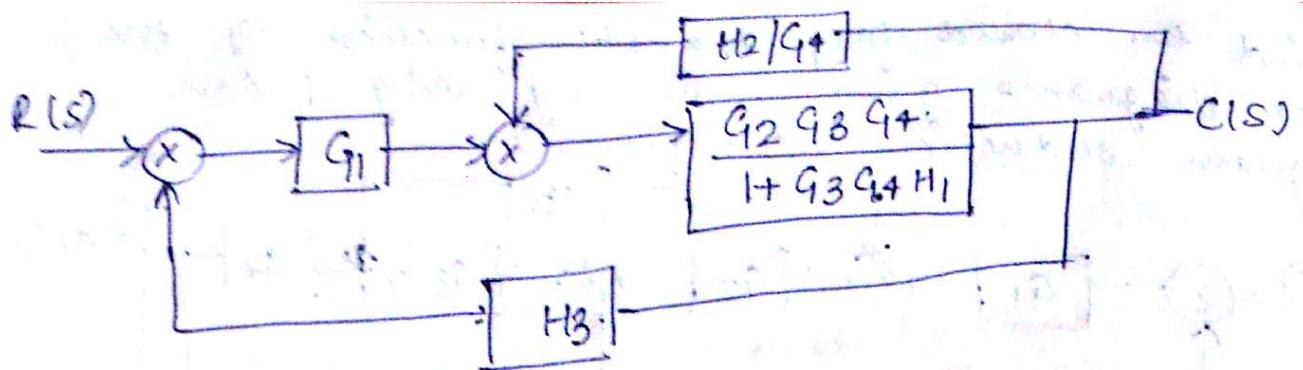


Now $G_3 G_4$ and H_1 along with negative feedback summer forms

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



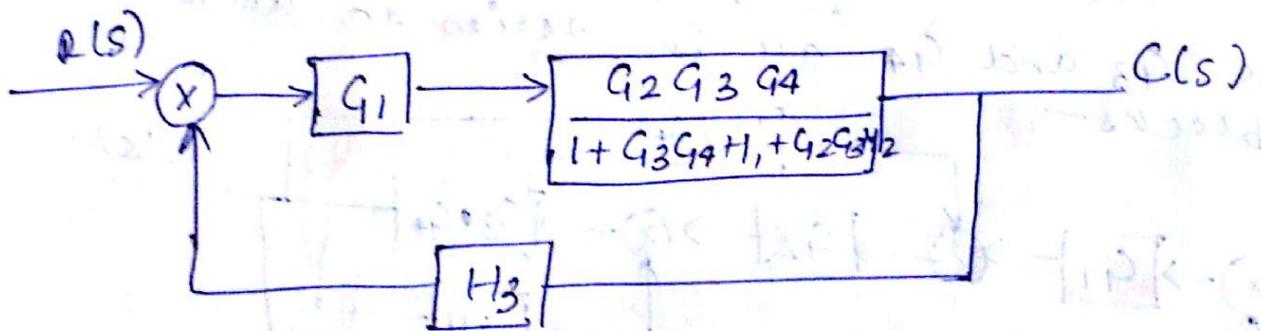
Now G_2 and $\frac{G_3 G_4}{1 + G_3 G_4 + H_1}$ are in series



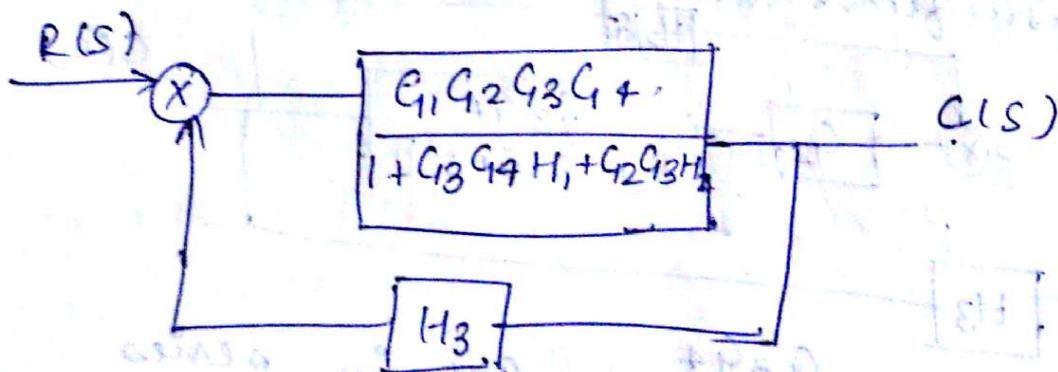
$\frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1}$ and H_2/G_4 along with summer are in -ve feedback:

$$\frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1} = \frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$$

$$\frac{1 + \frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1} \cdot H_2}{1 + G_3 G_4 H_1} = \frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$$



$\frac{G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$ and G_1 are in series.



$\frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$ and H_3 and summer
are in negative feedback.

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

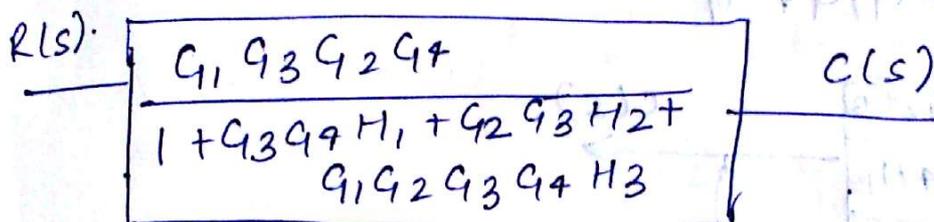
$$= \frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$$

$$1 + \frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2} \times H_3$$

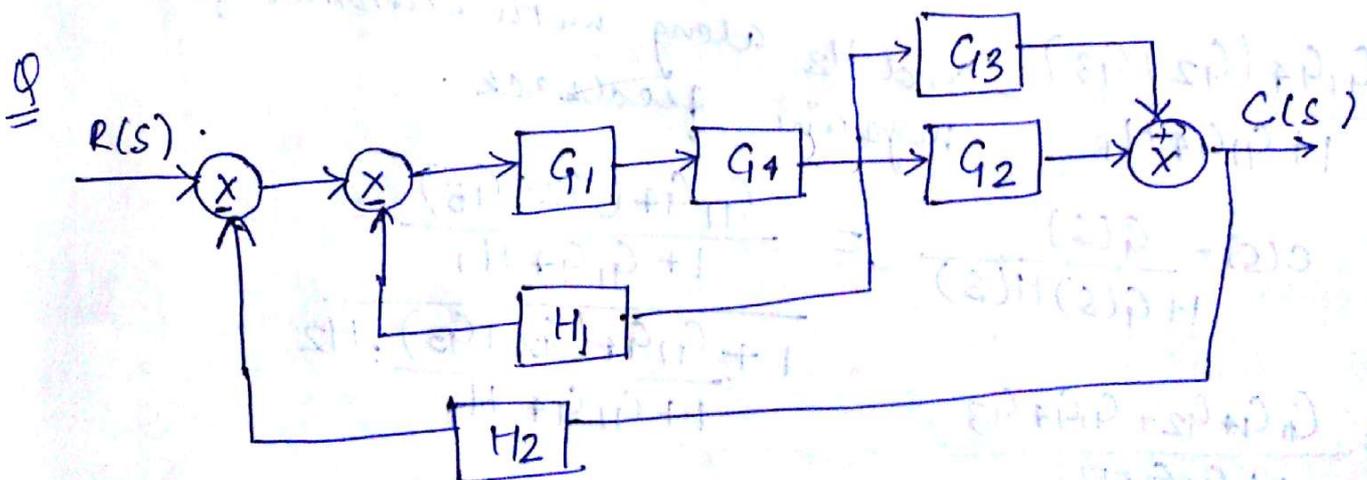
$$= \frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2}$$

$$= \frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$

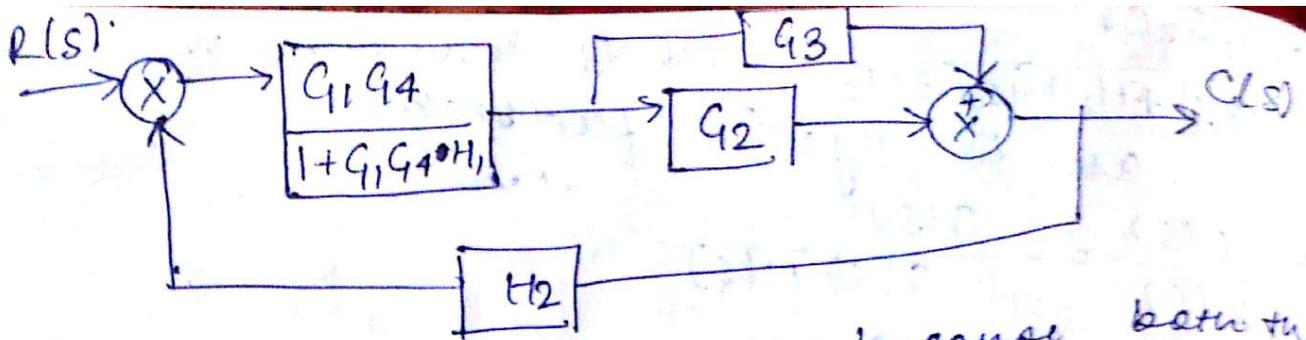
$$\frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$



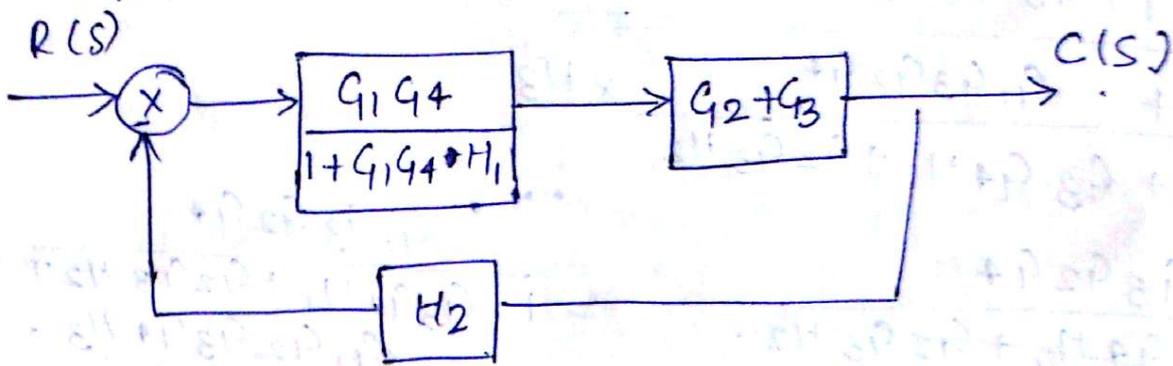
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_3 G_2 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$



G_1 and G_4 are in series. and $G_1 G_4$, H_1 and summer will form negative feedback.

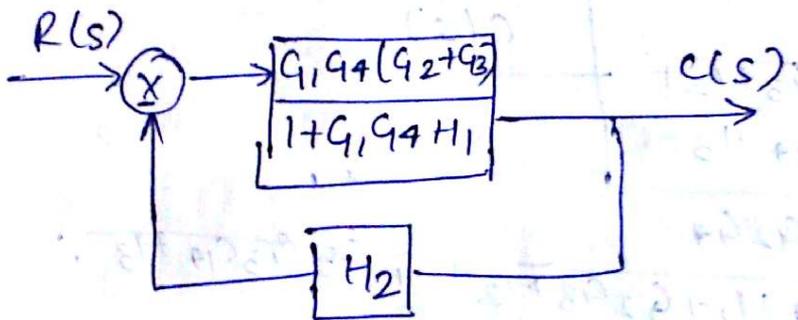


G_2 and G_3 are in parallel because both the arrows are towards the summer.



$\frac{G_1 G_4}{1 + G_1 G_4 H_1}$ and $G_2 + G_3$ are in series

$$\frac{(G_1 G_4)(G_2 + G_3)}{1 + G_1 G_4 H_1}$$



$\frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_4 H_1}$ and H_2 along with summer form negative feedback.

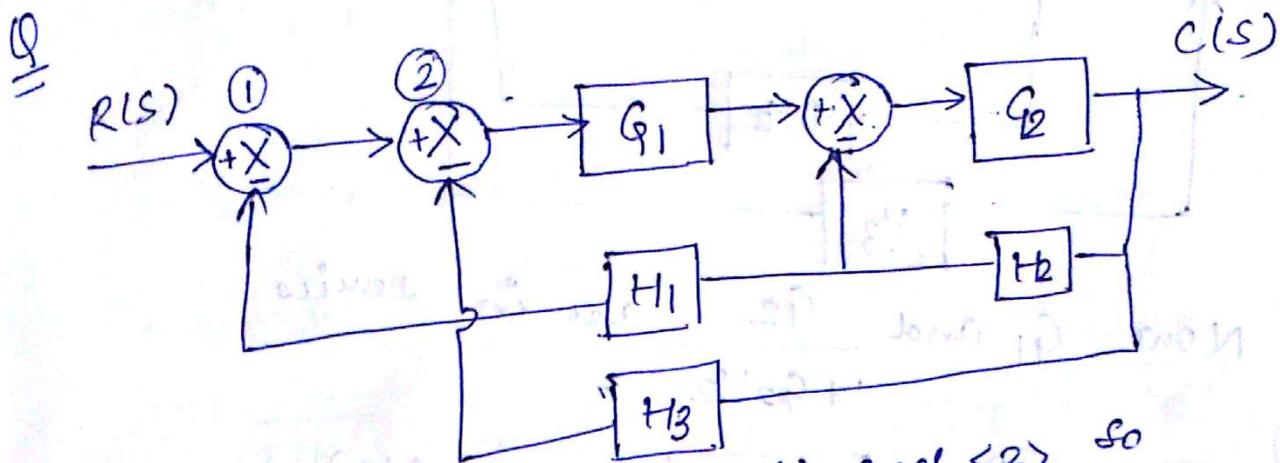
$$C(s) = \frac{R(s)}{1 + G(s)H(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_4 H_1}$$

$$= \frac{G_1 G_4 G_2 + G_1 G_4 G_3}{1 + G_1 G_4 H_1 + (G_1 G_2 G_4 + G_1 G_4 G_3) H_2}$$

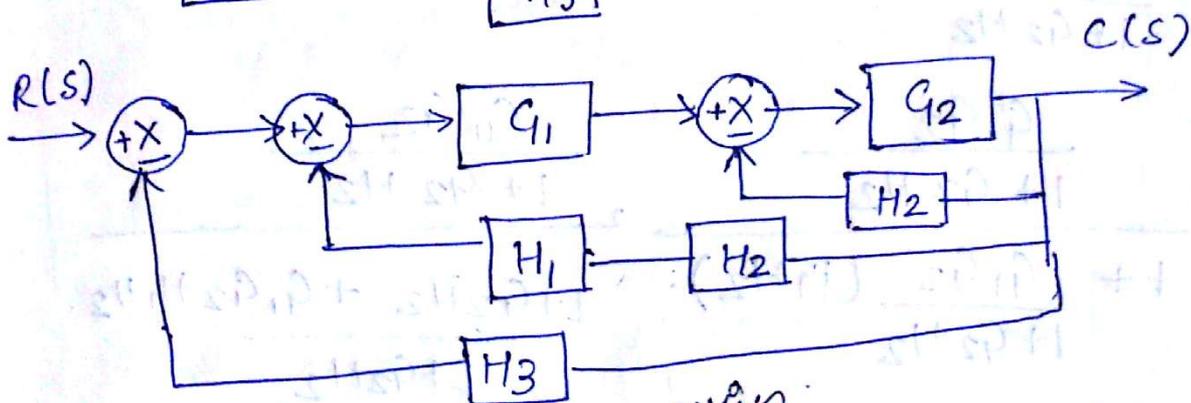
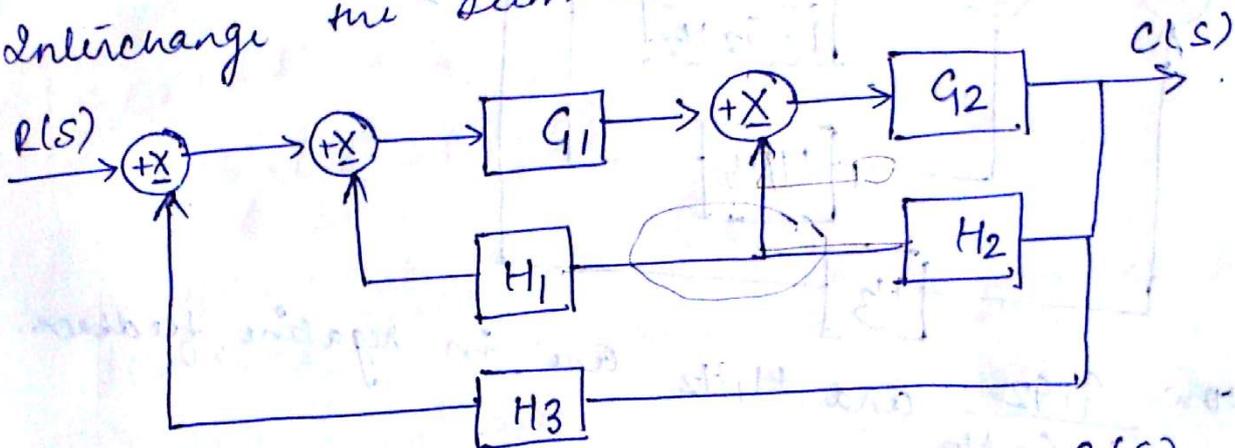
$$\frac{G_1 G_4 G_2 + G_1 G_4 G_3}{1 + G_1 G_4 H_1 + (G_1 G_2 G_4 + G_1 G_4 G_3) H_2}$$

$$= \frac{G_1 G_4 G_2 + G_1 G_4 G_3}{1 + G_1 G_4 H_1 + G_1 G_4 G_2 H_2 + G_1 G_4 G_3 H_2}$$

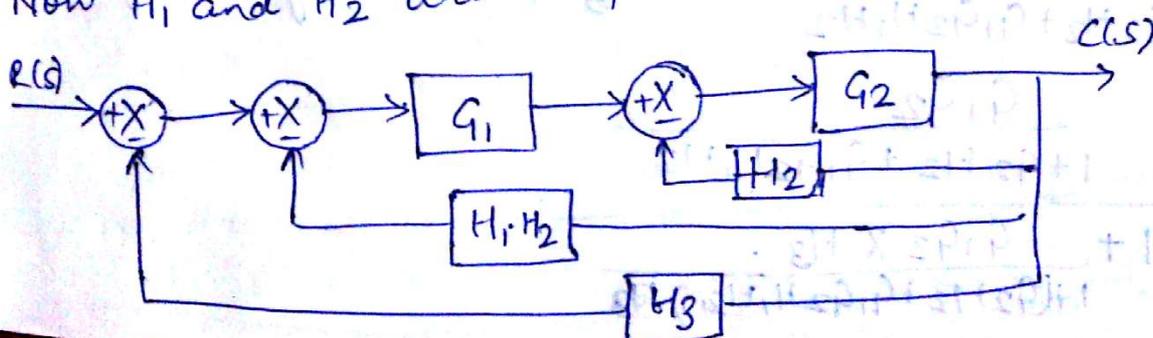
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_4 G_2 + G_1 G_4 G_3}{1 + G_1 G_4 H_1 + G_1 G_4 G_2 H_2 + G_1 G_4 G_3 H_2}$$



Interchange the summer (1) and (2) so

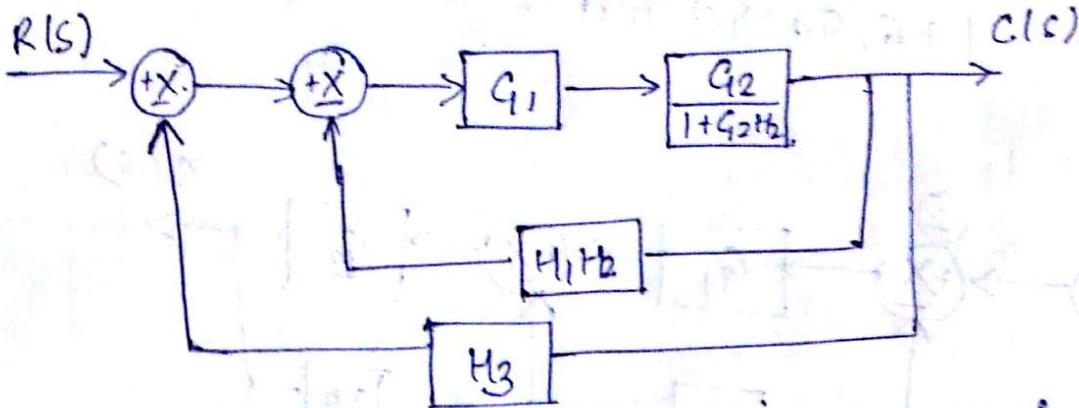


Now H_1 and H_2 are in parallel

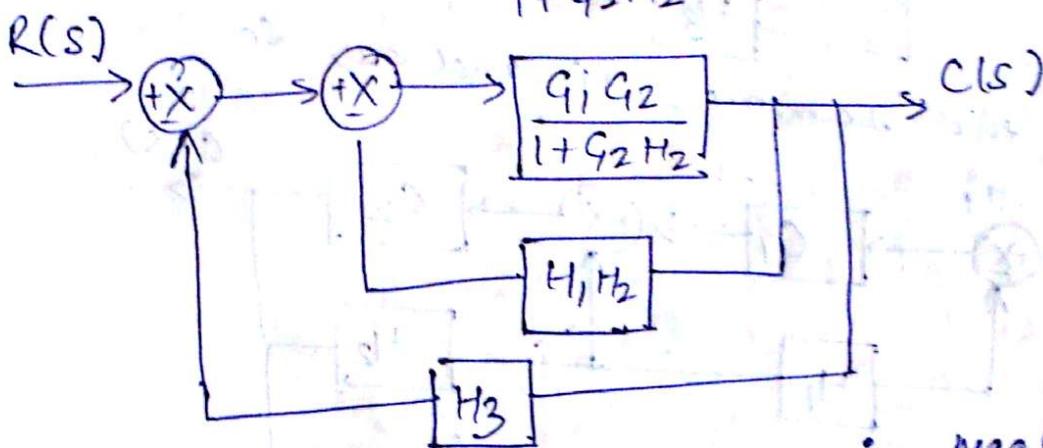


Now H_2 and G_2 and summer are in negative feedback.

$$\frac{G_2}{1+G_2H_2}$$



Now G_1 and $\frac{G_2}{1+G_2H_2}$ are in series.

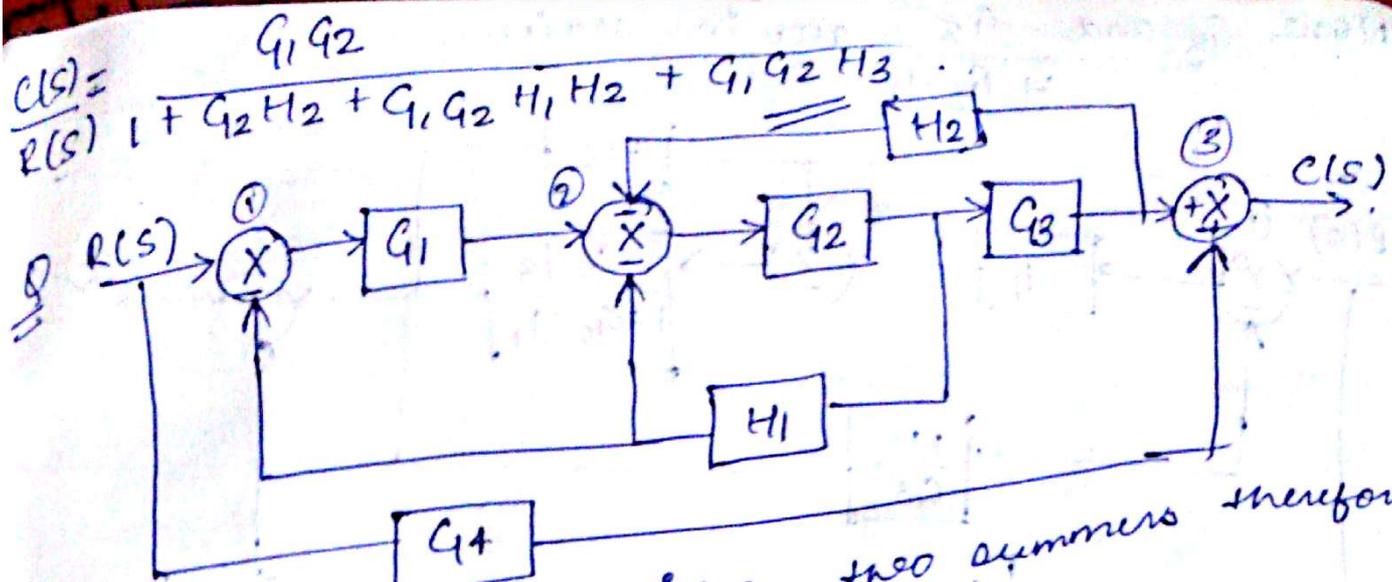


Now $\frac{G_1G_2}{1+G_2H_2}$ and H_1H_2 are in negative feedback.

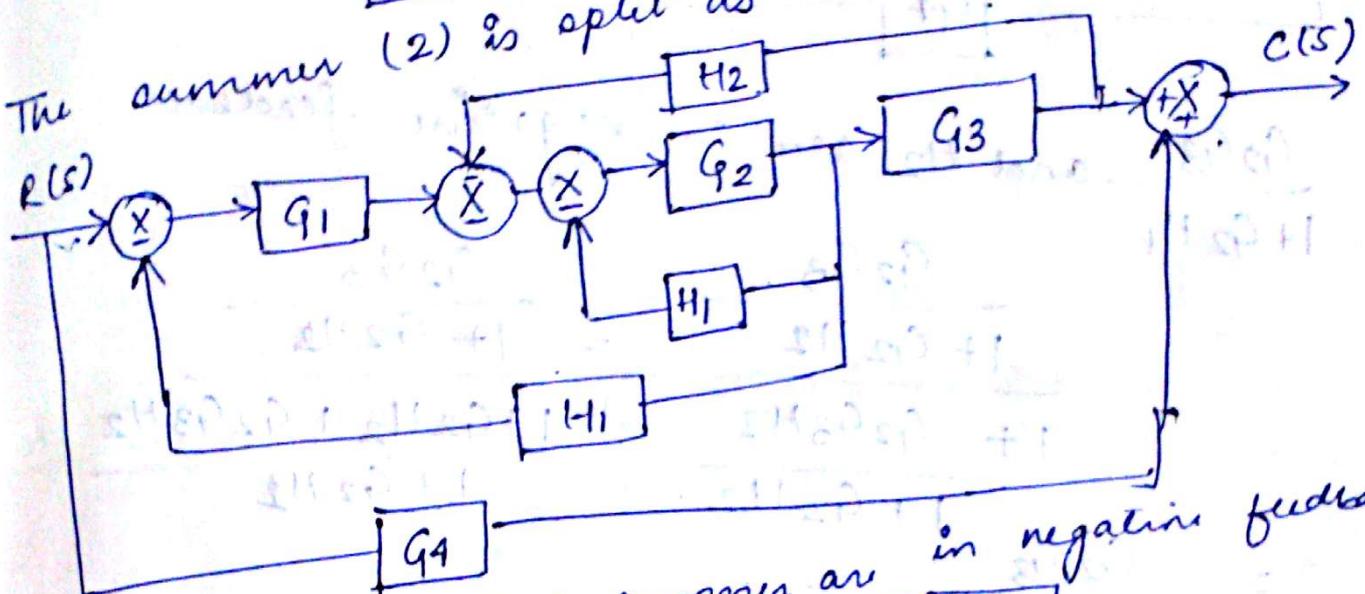
$$\frac{\frac{G_1G_2}{1+G_2H_2}}{1 + \frac{G_1G_2}{1+G_2H_2} (H_1H_2)} = \frac{G_1G_2}{1+G_2H_2 + G_1G_2H_1H_2}$$

Now $\frac{G_1G_2}{1+G_2H_2 + G_1G_2H_1H_2}$ and H_3 are in negative feedback.

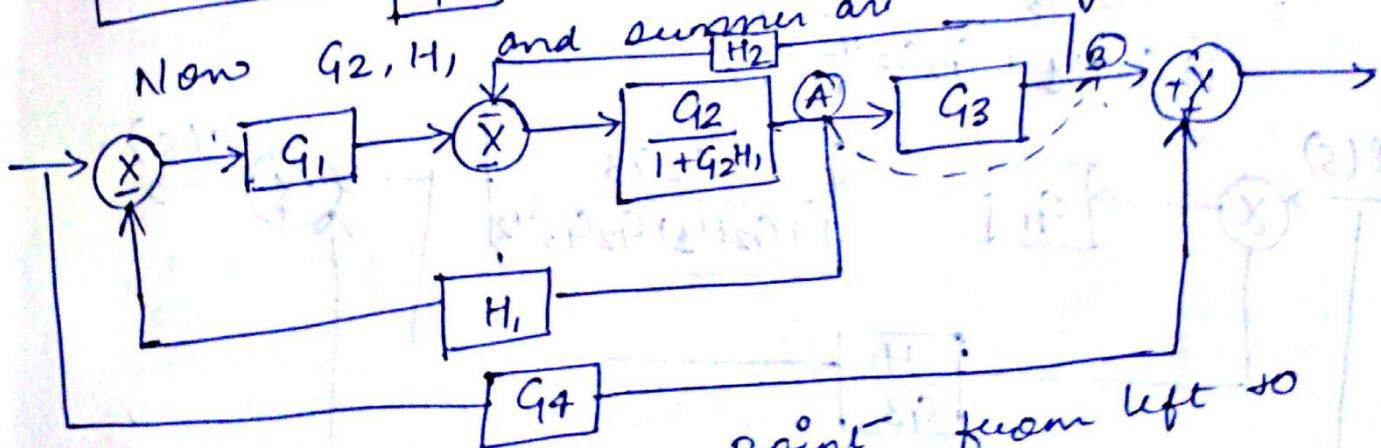
$$\frac{\frac{G_1G_2}{1+G_2H_2 + G_1G_2H_1H_2}}{1 + \frac{G_1G_2 \times H_3}{1+G_2H_2 + G_1G_2H_1H_2}}$$



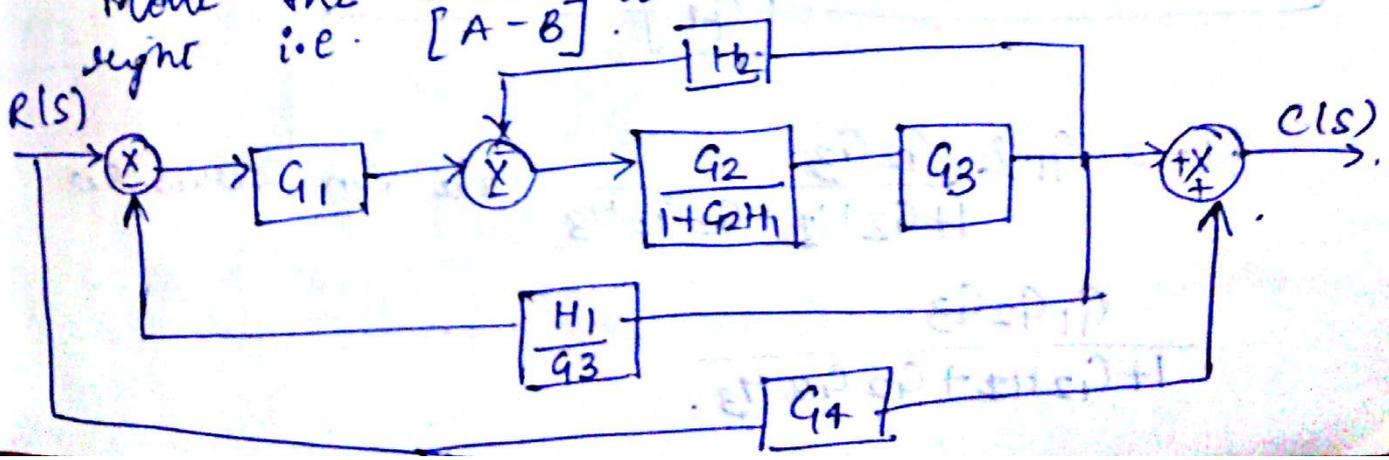
The summer (2) is split as two summers therefore



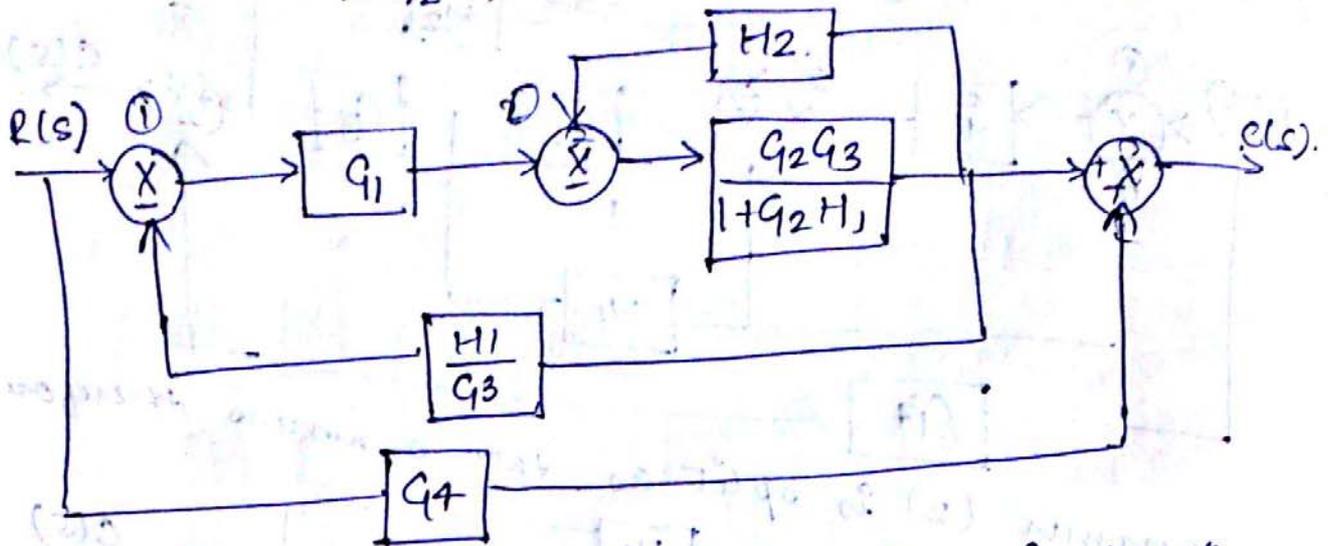
Now G_2, H_1 and summer are in negative feedback



Move the take off point from left to right i.e. $[A-B]$.



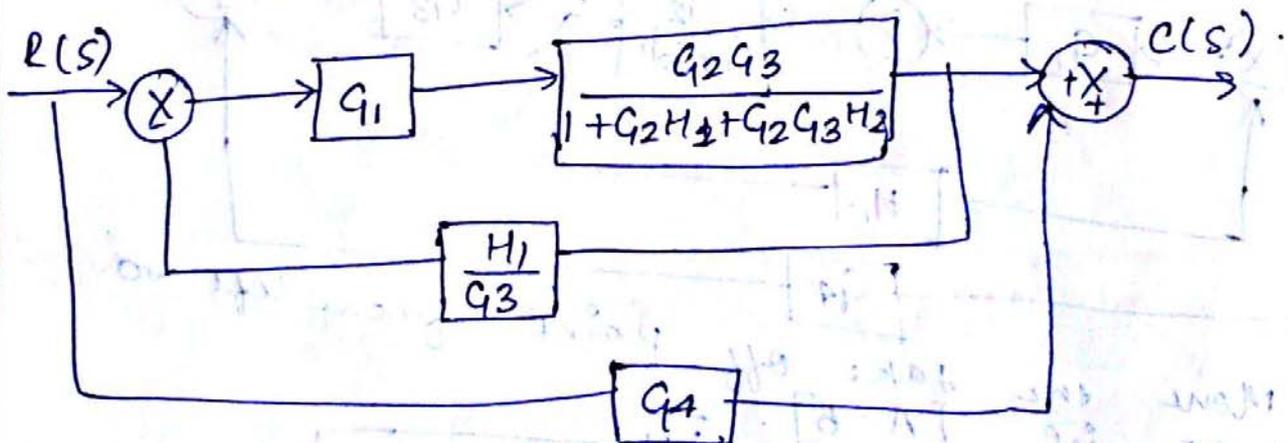
Now G_3 and $\frac{G_2}{1+G_2H_1}$ are in series.



$\frac{G_2G_3}{1+G_2H_1}$ and H_2 are in negative feedback

$$\frac{\frac{G_2G_3}{1+G_2H_1}}{1 + \frac{G_2G_3H_2}{1+G_2H_1}} = \frac{\frac{G_2G_3}{1+G_2H_1}}{\frac{1+G_2H_1+G_2G_3H_2}{1+G_2H_1}}$$

$$= \frac{G_2G_3}{1+G_2H_1+G_2G_3H_2}$$



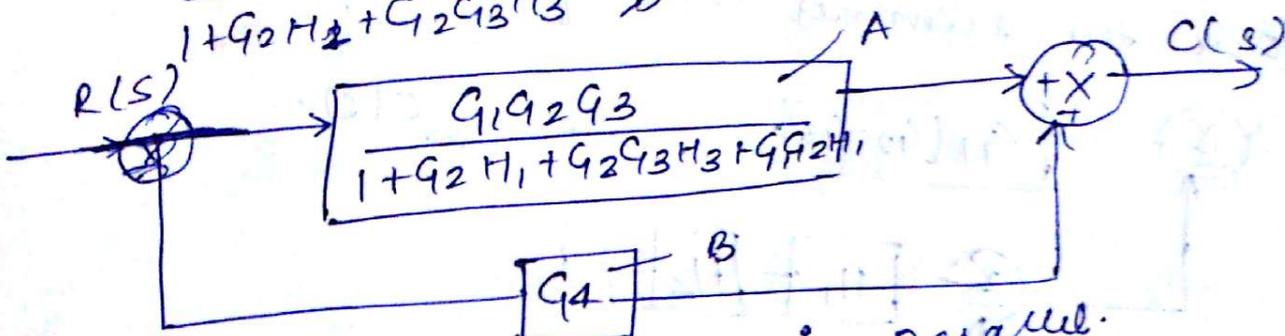
Now G_1 & $\frac{G_2G_3}{1+G_2H_1+G_2G_3H_2}$ are in series so

$$\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2}$$

Now $\frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3}$ and $\frac{H_1}{G_3}$ are negative feedback

$$\frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3} = \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3 + G_1 G_2 H_1}$$

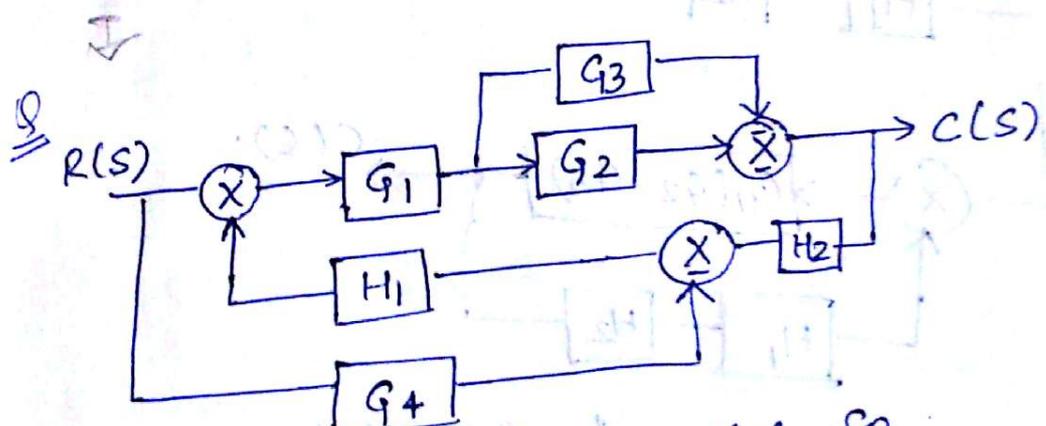
$$1 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3} \cdot \frac{H_1}{G_3}$$



Now A and B are in parallel.

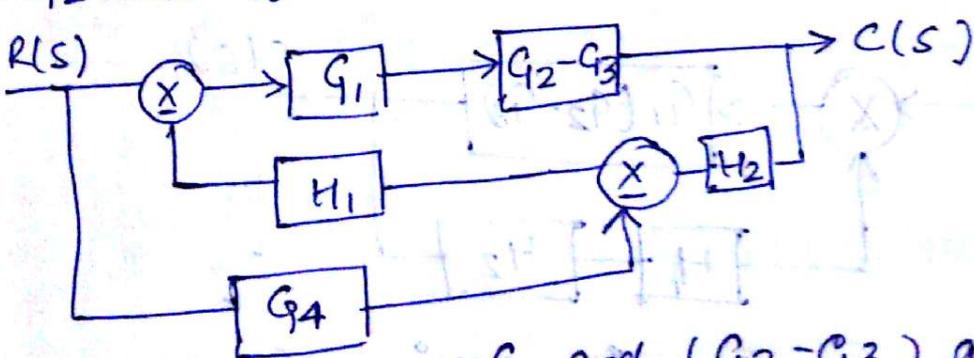
$$G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3 + G_1 G_2 H_1}$$

$$\frac{G_4 (1 + G_2 H_1 + G_2 G_3 H_3 + G_1 G_2 H_1) + G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_3 + G_1 G_2 H_1}$$

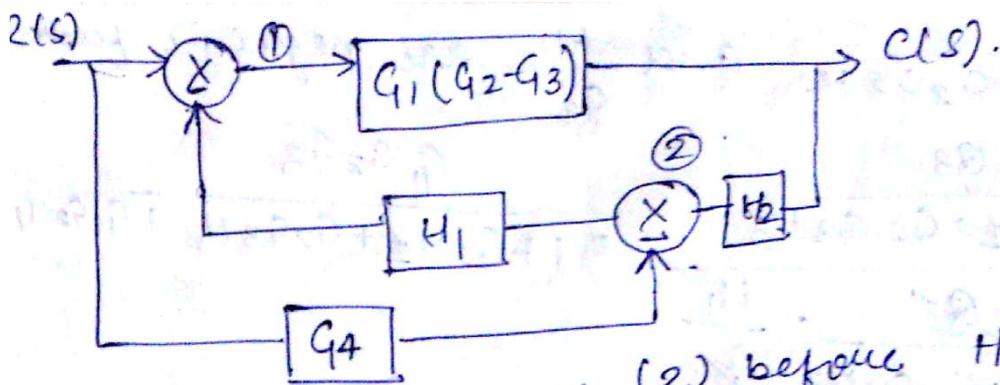


If - sign will be there then for parallel (G2-G3)

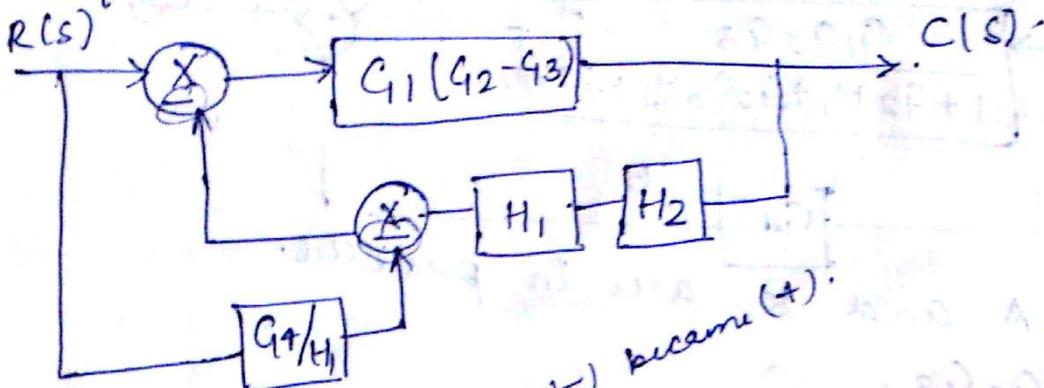
G2 and G3 are in parallel so



G1 and (G2-G3) are in series.

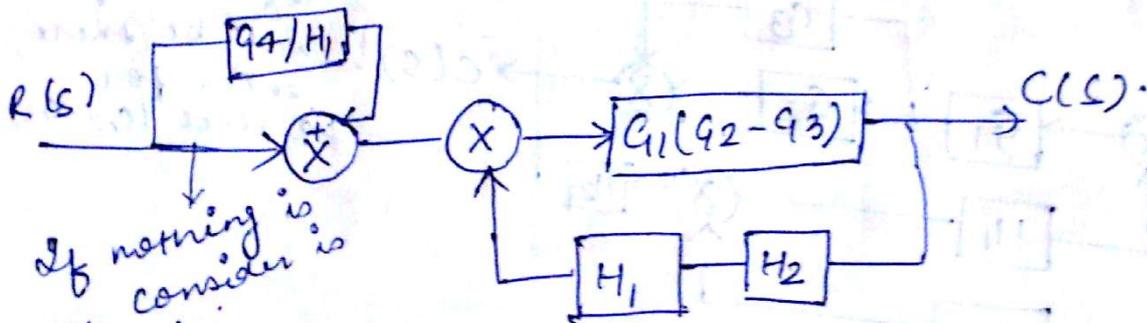
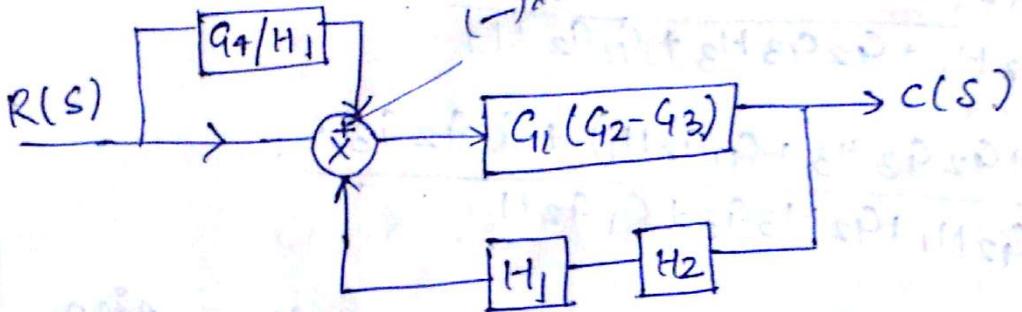


Move the summer (2) before H_1 .



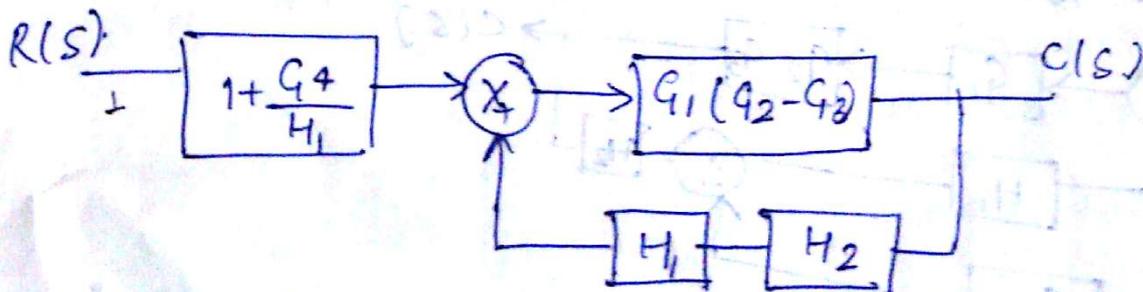
(\rightarrow \times) became (+).

If nothing is there between two summer we can multiply or club.



If nothing is there consider is at 1.

so 1 and G_4/H_1 are in parallel.

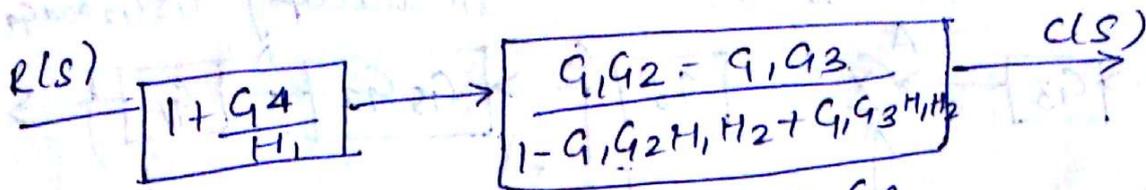


Now H_1 , H_2 are in series so $H_1 H_2$.

Now $G_1(G_2 - G_3)$ and $H_1 H_2$ and summer are in positive feedback.

$$\text{So } \frac{C_1(G_2 - G_3)}{1 - (G_1 G_2 H_1 H_2 - G_1 G_3 H_1 H_2)} \cdot \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

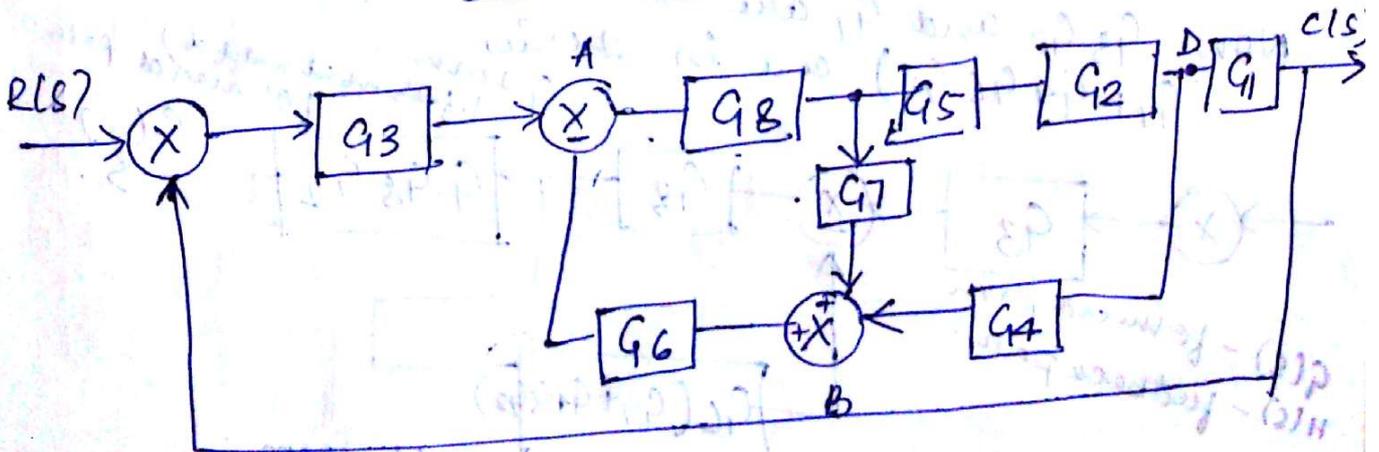
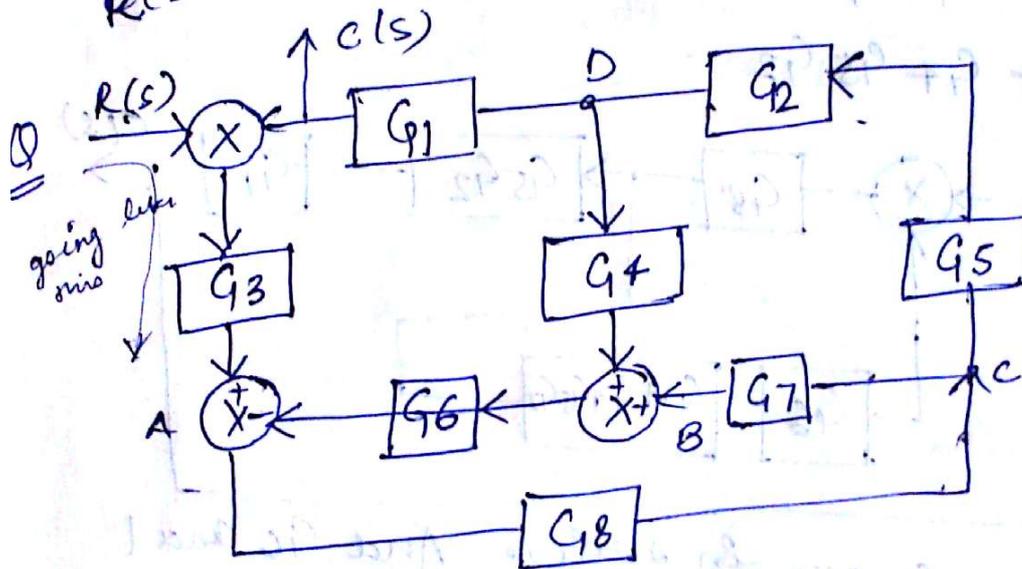
$$= \frac{G_1 G_2 - G_1 G_3}{1 - G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2}$$



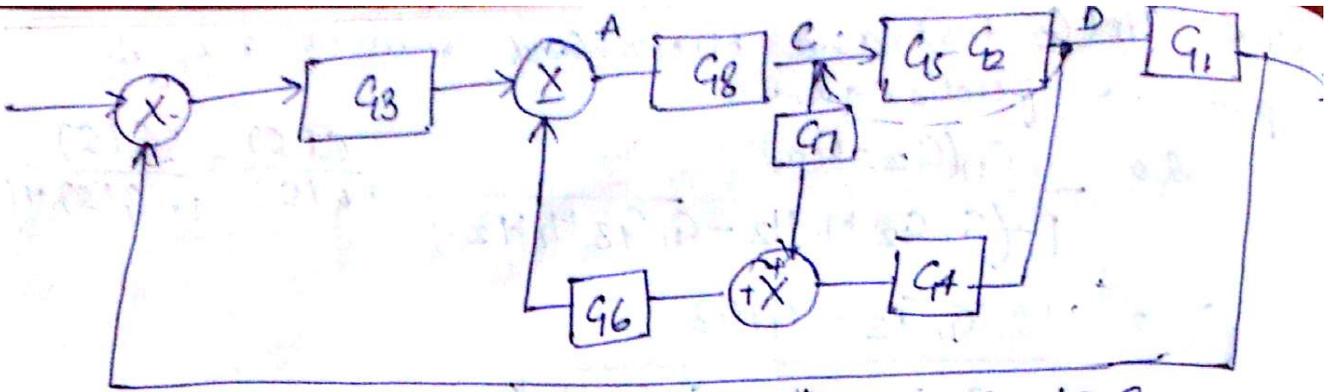
Now $1 + \frac{G_4}{H_1}$ and $\frac{G_1 G_2 - G_1 G_3}{1 - G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2}$ are

in series.

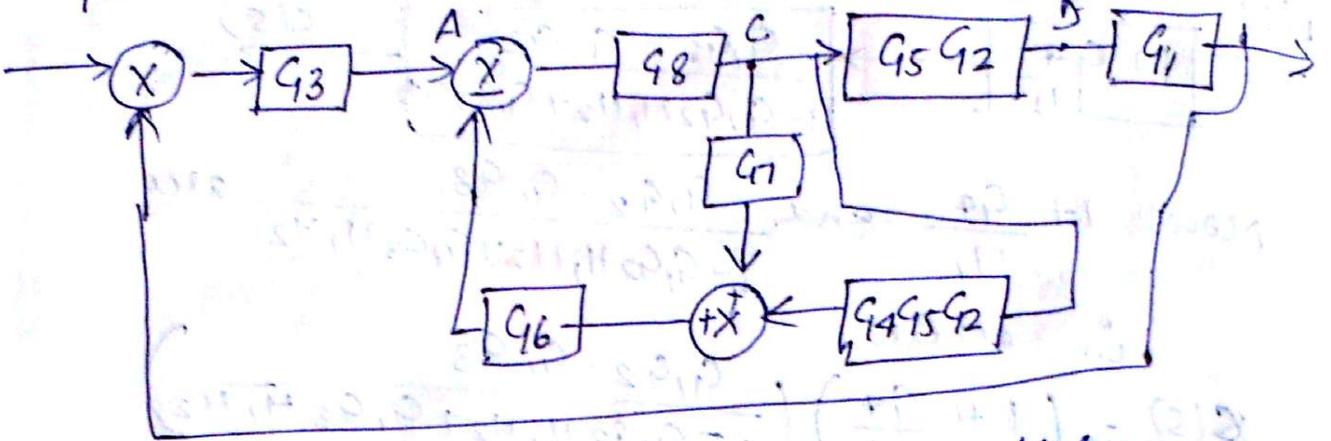
$$\frac{C(s)}{R(s)} = \left(1 + \frac{G_4}{H_1}\right) \left(\frac{G_1 G_2 - G_1 G_3}{1 - G_1 G_2 H_1 H_2 + G_1 G_3 H_1 H_2}\right)$$



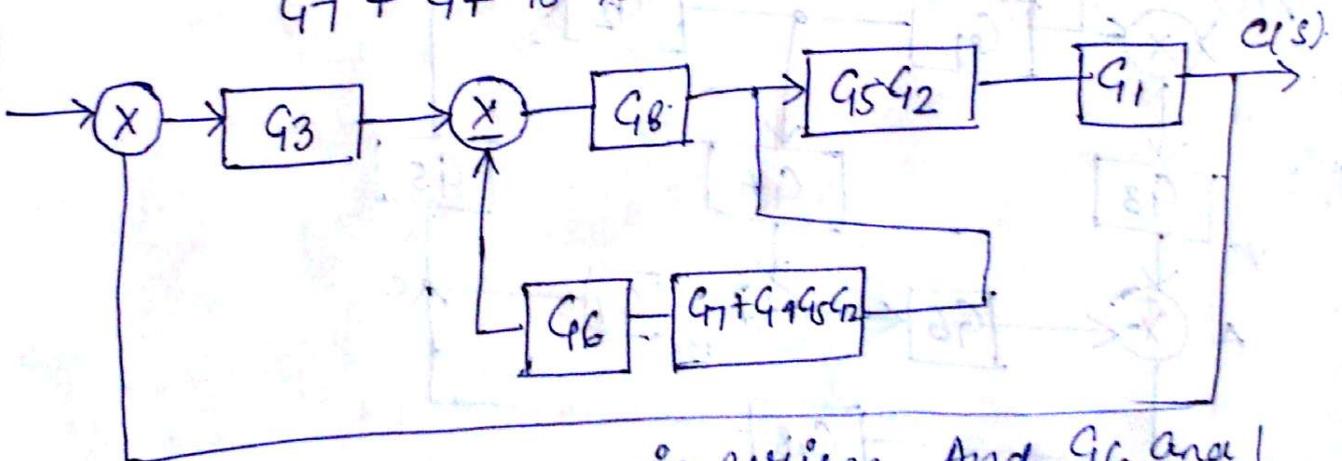
G_5 and G_2 are in series.



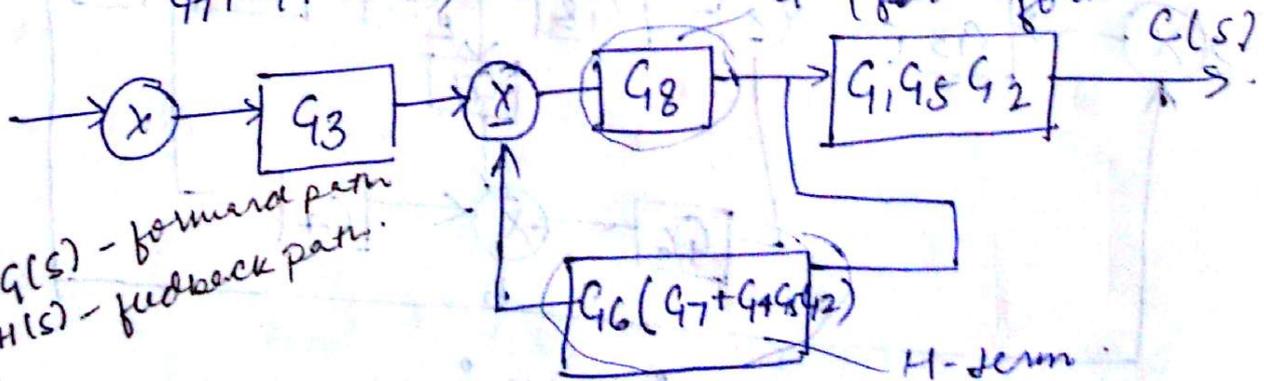
Move the take off point from D. to C. so multiply G_5G_2 and G_4



Now G_7 & $G_4G_5G_2$ are in parallel.
 $G_7 + G_4G_5G_2$



Now G_5G_2 and G_1 are in series. And G_6 and $G_7 + G_4G_5G_2$ are in series.
 G_1 term (forward path)
 G_6 term (feedback path)



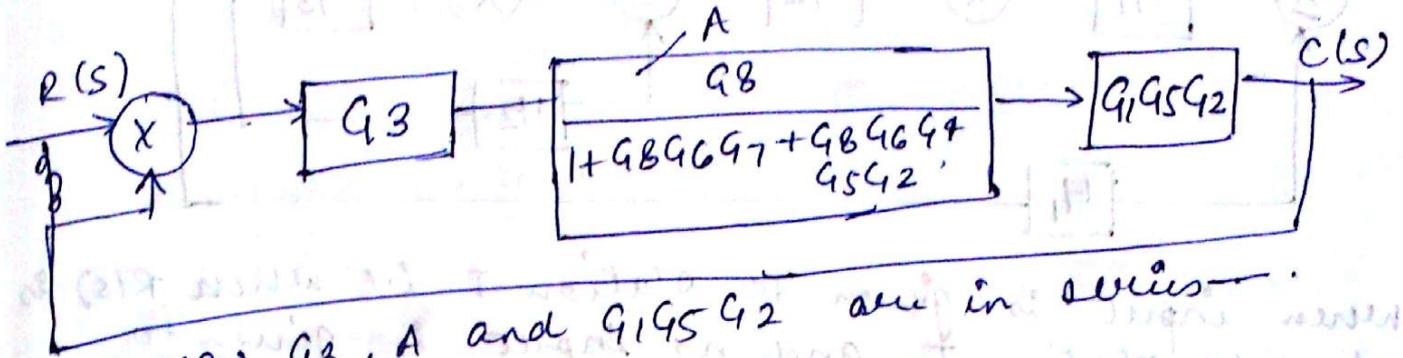
$G(s)$ - forward path
 $H(s)$ - feedback path

H-term

Now G_8 and $G_6(G_7 + G_4G_5G_2)$ are in negative feedback.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{G_8}{1 + G_8(G_6G_7 + G_6G_4G_5G_2)}$$

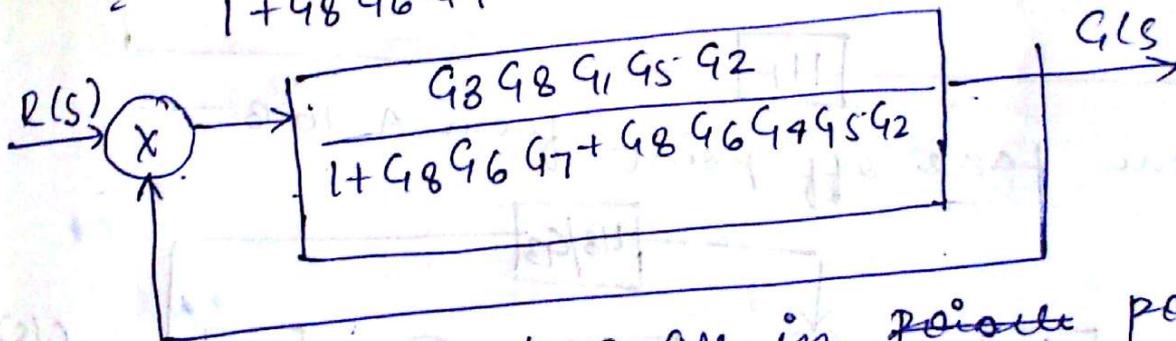
$$= \frac{G_8}{1 + G_8G_6G_7 + G_8G_6G_4G_5G_2}$$



Now G_3 , A and $G_1G_5G_2$ are in series.

$$G_3 \left[\frac{G_8}{1 + G_8G_6G_7 + G_8G_6G_4G_5G_2} \right] \times G_1G_5G_2$$

$$= \frac{G_3G_8G_1G_5G_2}{1 + G_8G_6G_7 + G_8G_6G_4G_5G_2}$$



Now these two are in positive feedback so.

$$G_3G_8G_1G_5G_2$$

$$1 + G_8G_6G_7 + G_8G_6G_4G_5G_2$$

$$1 - \frac{G_3G_8G_1G_5G_2}{1 + G_8G_6G_7 + G_8G_6G_4G_5G_2}$$

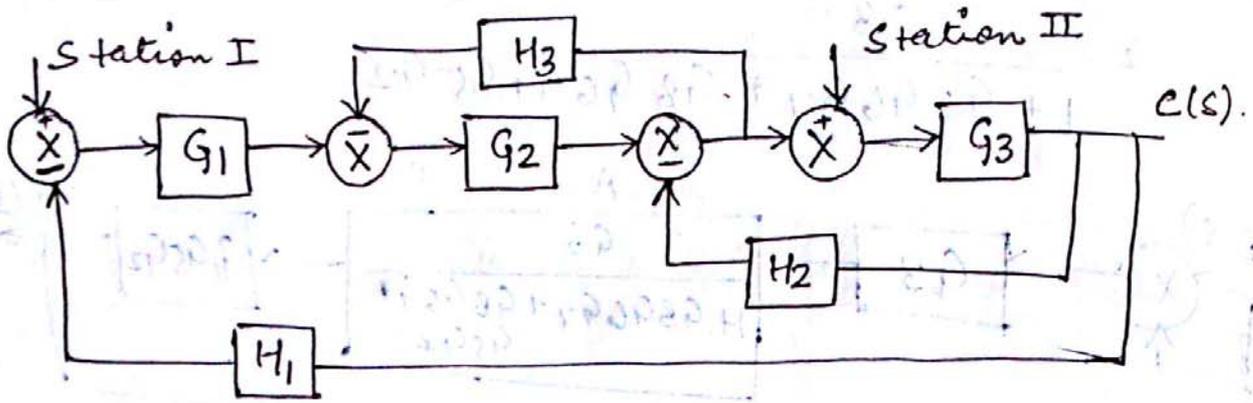
$$= \frac{G_3G_8G_1G_5G_2}{(1 + G_8G_6G_7 + G_8G_6G_4G_5G_2) - G_3G_8G_1G_5G_2}$$

$$\frac{C(s)}{R(s)} =$$

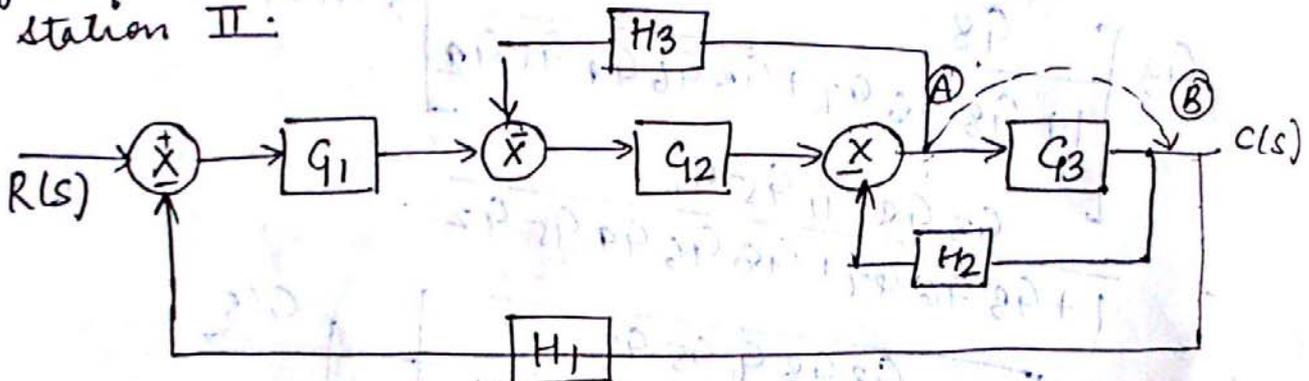
$$\frac{G_3G_8G_1G_5G_2}{(1 + G_8G_6G_7 + G_8G_6G_4G_5G_2) - G_3G_8G_1G_5G_2}$$

Q For the block diagram given below find the closed loop transfer function with

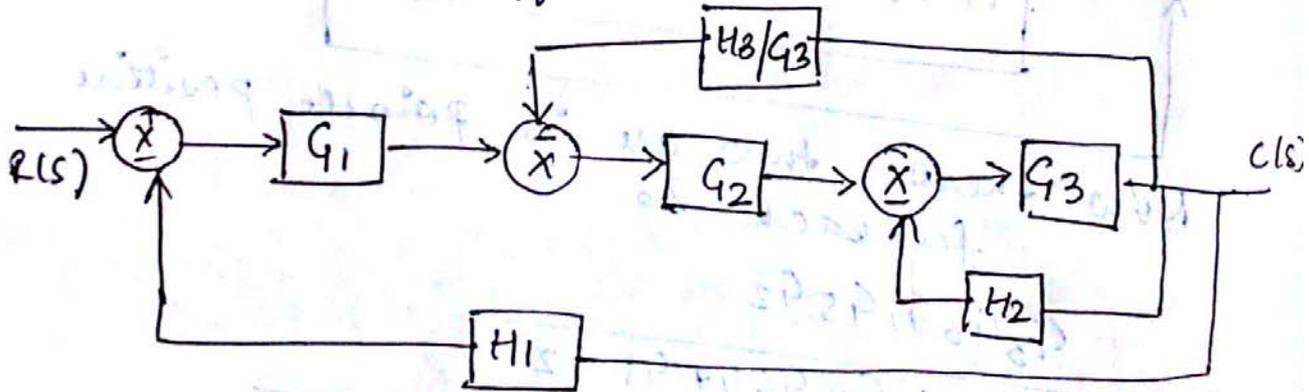
- i) input $R(s)$ is given to station I.
- ii) input $R(s)$ is given to station II.



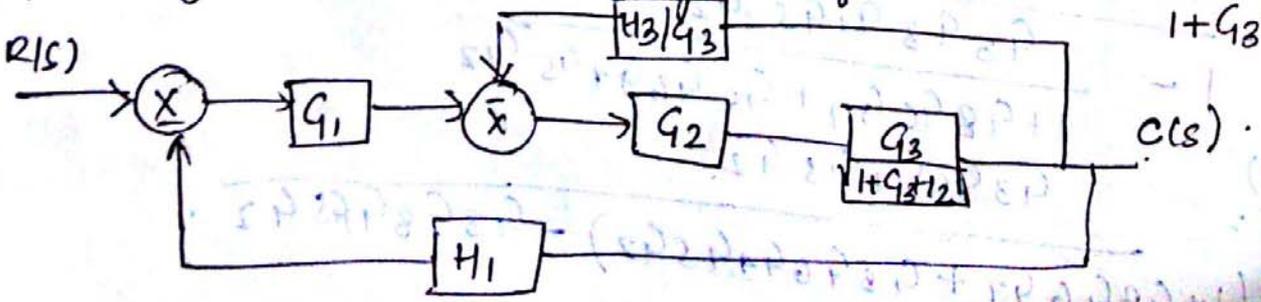
When input is given to station I i.e. when $R(s)$ is given to station I and no input is given to station II:



Shift the take off point from A to B —



Now G_3 and H_2 are in negative feedback so $\frac{G_3}{1+G_3H_2}$

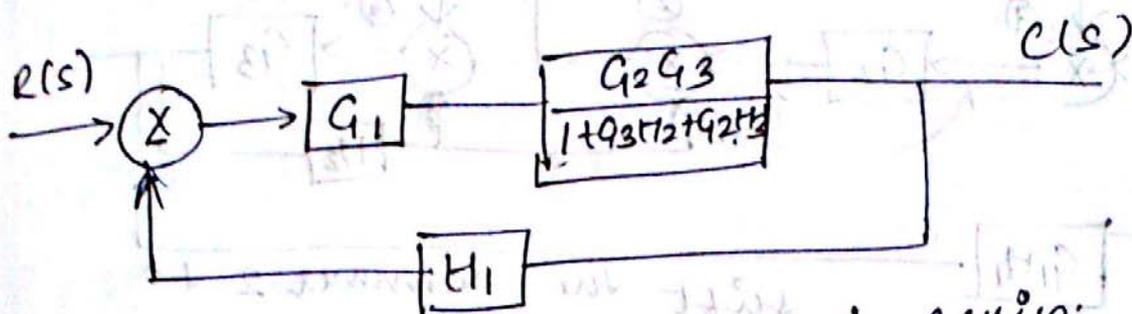


Now G_2 and $\frac{G_3}{1+G_3H_2}$ are in series so $\frac{G_2G_3}{1+G_3H_2}$.

Now H_3/G_3 and $\frac{G_2G_3}{1+G_3H_2}$ are in negative feedback.

$$\text{So } \frac{G_2G_3}{1+G_3H_2} \cdot \frac{1}{1 + \frac{G_2G_3}{1+G_3H_2} \times \frac{H_3}{G_3}} = \frac{G_2G_3}{1+G_3H_2 + G_2H_3}$$

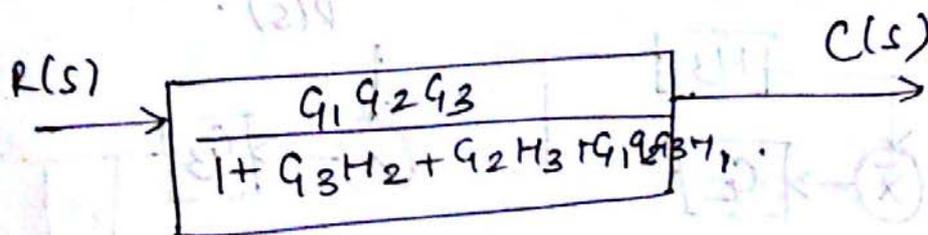
$$\therefore \frac{G_2G_3}{1+G_3H_2 + G_2H_3}$$



Now G_1 and $\frac{G_2G_3}{1+G_3H_2+G_2H_3}$ are in series.

$\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}$ and H_1 are in negative feedback.

$$\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3} \cdot \frac{1}{1 + \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3} \cdot H_1} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

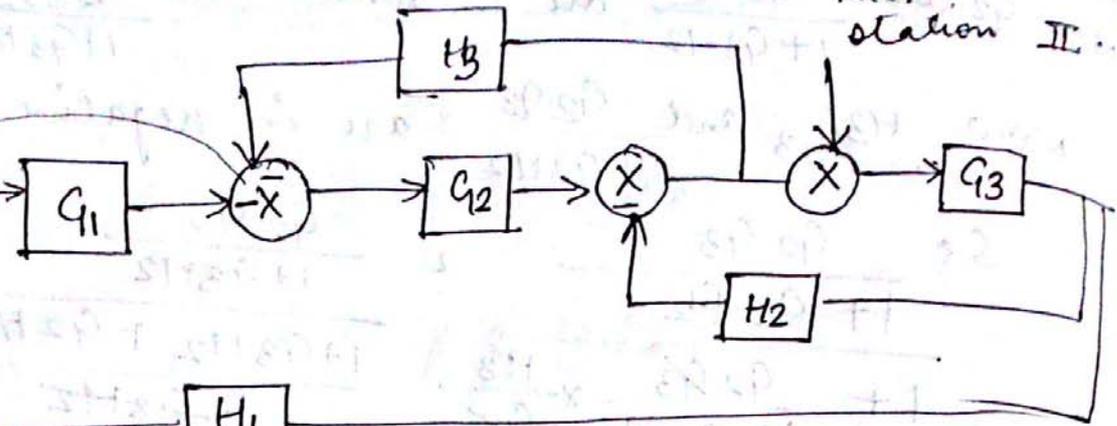


Now when input is given to station II i.e. when $R(s)$ is given to station II and no input is given to station I.

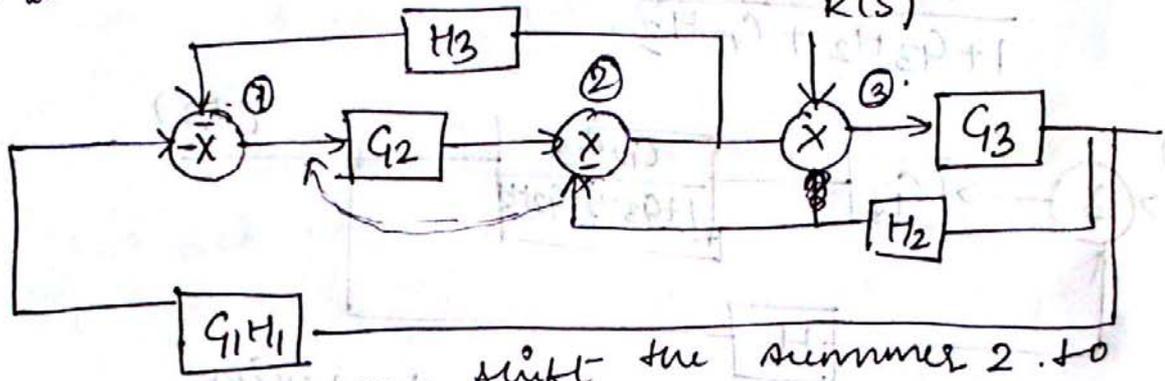
R(s) station II.

station I

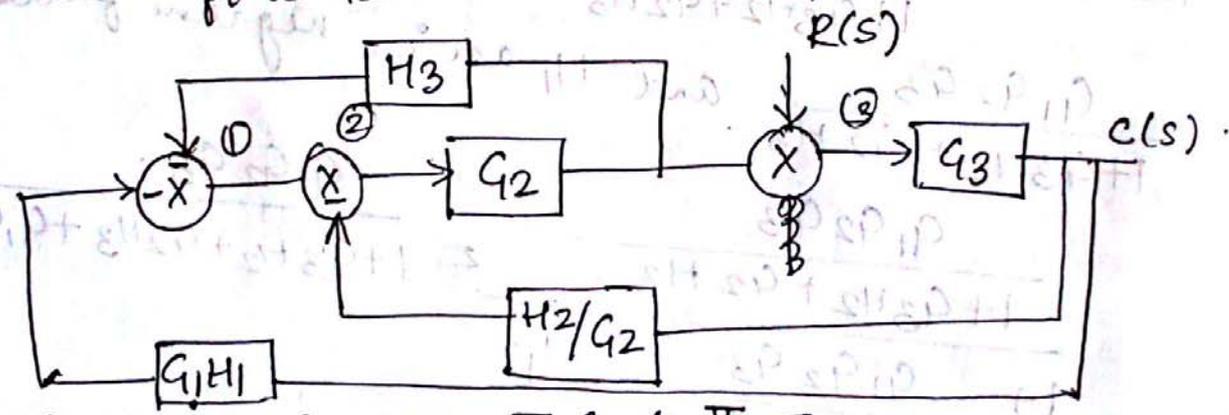
Here, negative sign was there so it will go to H_1 i.e. $-H_1$ or it can be moved to the summer also.



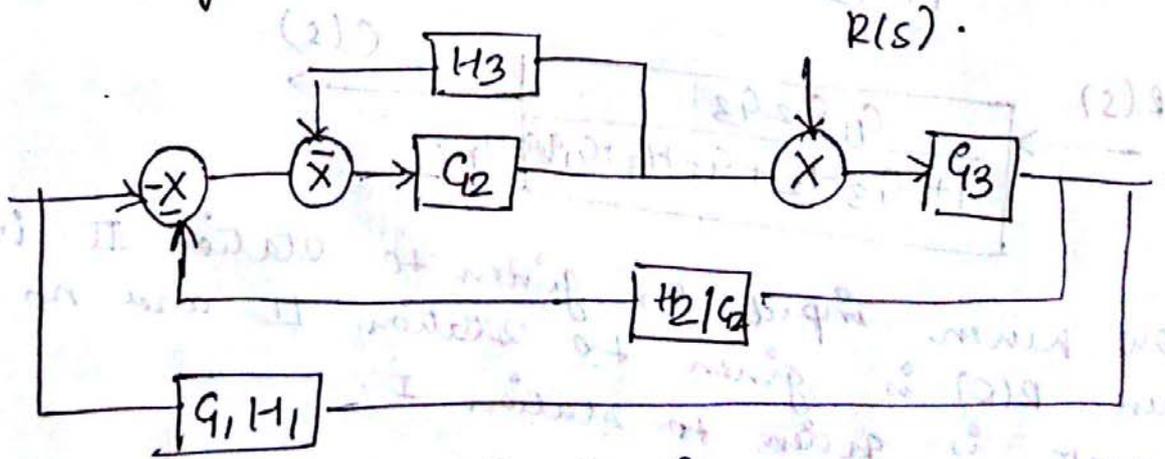
Now G_1 and H_1 are in series.



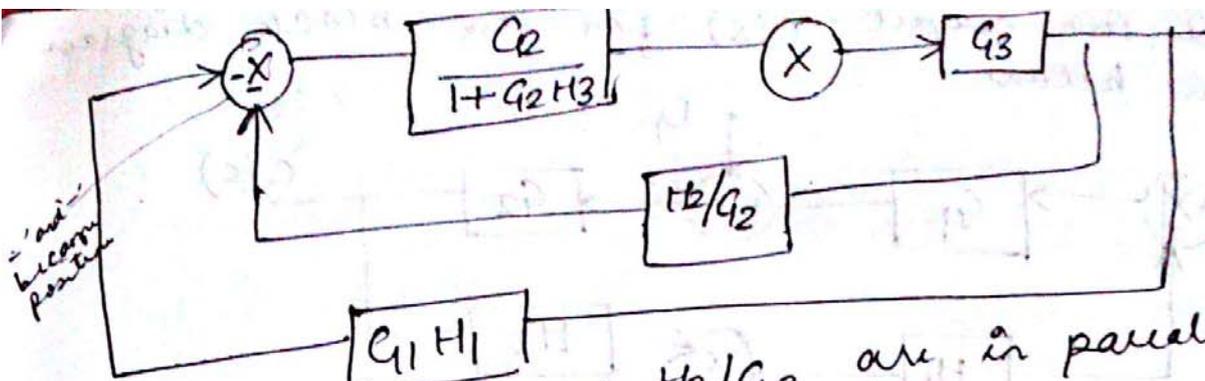
Now shift the summer 2 to before G_2 .



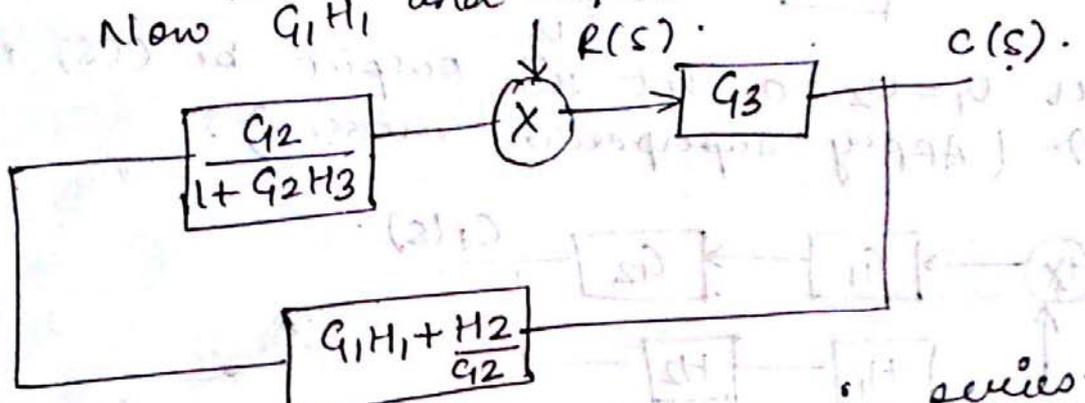
Interchanging summer I and II



Now H_3 and G_2 are in negative feedback.

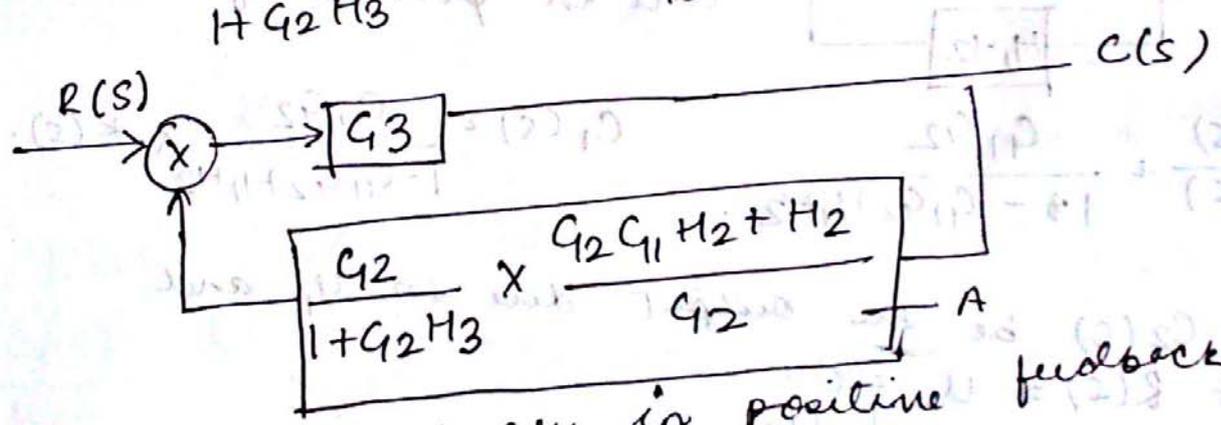


Now G_1H_1 and H_2/G_2 are in parallel.



Now $\frac{G_2}{1+G_2H_3}$ and $\frac{G_1H_1 + H_2}{G_2}$ are in series.

$$\frac{G_2}{1+G_2H_3} \times \frac{G_1G_2H_2 + H_2}{G_2}$$



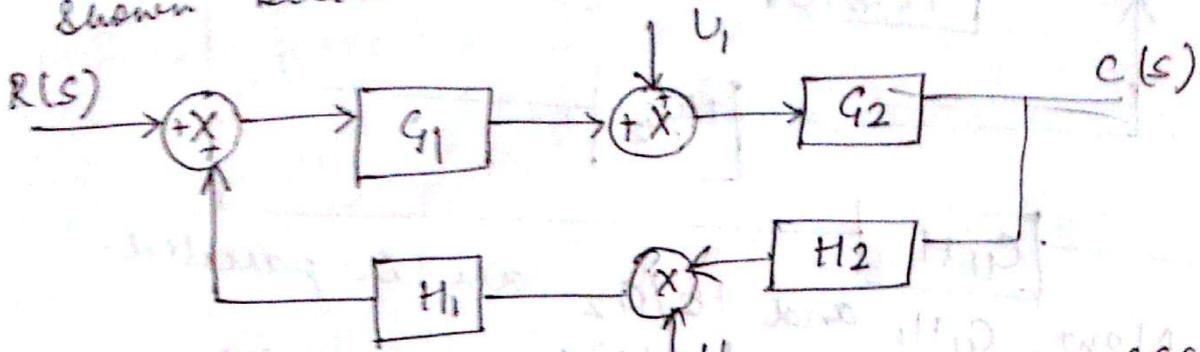
Now G_3 and A are in positive feedback.

$$\text{So } \frac{C(s)}{R(s)} = \frac{G_3}{1 - G_3 \left(\frac{G_2}{1+G_2H_3} \times \frac{G_1G_2H_2 + H_2}{G_2} \right)}$$

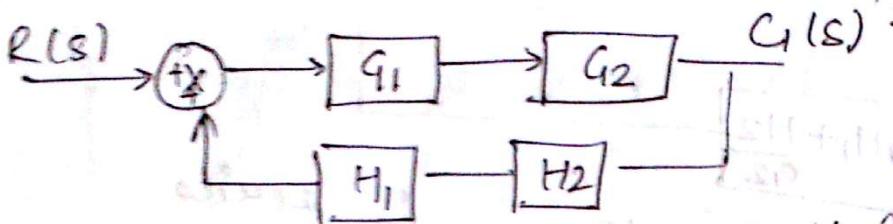
$$\text{So } \frac{C(s)}{R(s)} = \frac{G_3}{1 - \left(\frac{G_3}{1+G_2H_3} \times \frac{G_3G_1G_2H_2 + G_3H_2}{G_2} \right)}$$

$$\frac{C(s)}{R(s)} = \frac{G_3}{1 + G_2H_3 - \frac{G_3G_1G_2H_2 + G_3H_2}{G_2}}$$

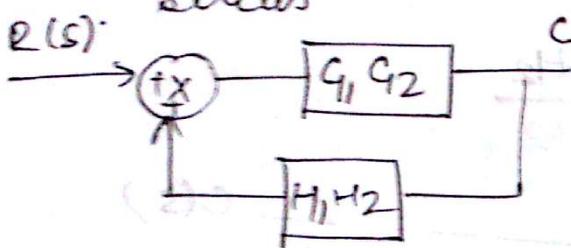
Q Find the output $C(s)$ for the block diagram shown below



Consider $U_1 = U_2 = 0$ let the output be $C(s)$ due to $R(s)$. (Apply superposition theorem).



G_1 and G_2 are in series and H_1 & H_2 are in series.

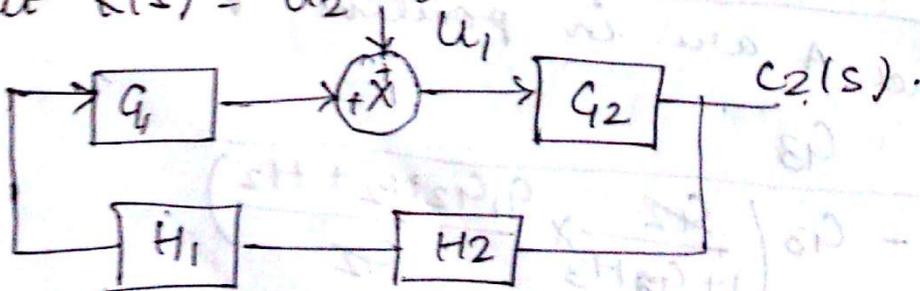


No G_1, G_2 and H_1, H_2 are in positive feedback.

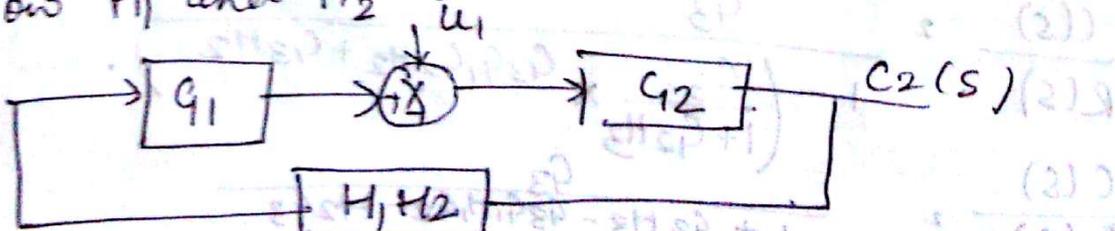
$$\frac{C_1(s)}{R(s)} = \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2}$$

$$C_1(s) = \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2} \times R(s)$$

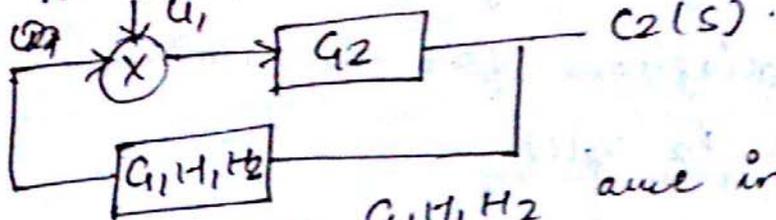
Now $C_2(s)$ be the output due to U_1 and let $R(s) = U_2 = 0$.



Now H_1 and H_2 are in series.



New G_1 and $H_1 H_2$ are in series —

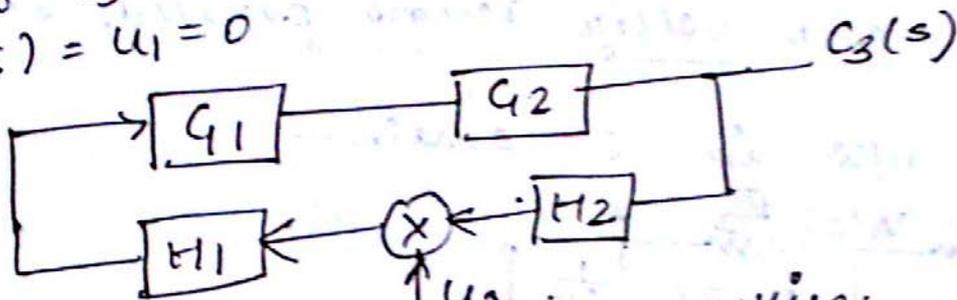


New G_2 and $G_1 H_1 H_2$ are in positive feedback.

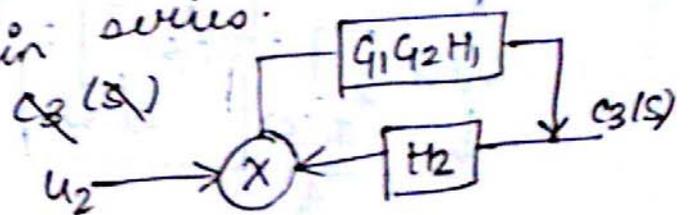
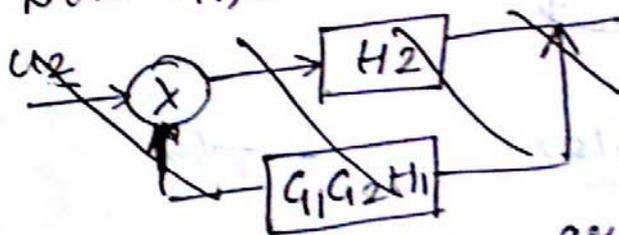
$$\text{So } \frac{C_2(s)}{U_1} = \frac{G_2}{1 - G_2 G_1 H_1 H_2}$$

$$C_2(s) = \frac{G_2}{1 - G_2 G_1 H_1 H_2} \cdot U_1$$

New $C_3(s)$ be the output due to U_2 and $R(s) = U_1 = 0$



New G_1, G_2 and H_1 are in series.



New H_2 and $G_1 G_2 H_1$ are in positive feedback.

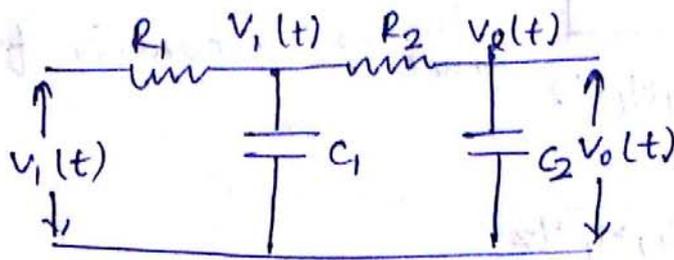
$$\frac{C_3(s)}{U_2} = \text{So } \frac{G_1 G_2 H_1}{1 - G_1 G_2 H_1 H_2}$$

$$C_3(s) = \frac{G_1 G_2 H_1}{1 - G_1 G_2 H_1 H_2} U_2$$

$$\therefore \text{output } C(s) = C_1(s) + C_2(s) + C_3(s)$$

Construction of block diagram :-

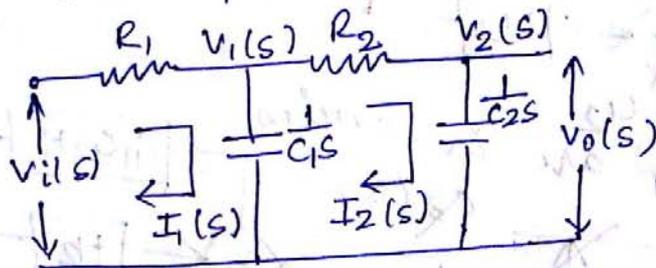
1.) Draw block diagram for the electrical n/w.



Assume the loop current as $i_1(t)$ and $i_2(t)$ and the node voltages as $v_1(t)$ and $v_2(t)$

Consider the current flowing through elements in series and voltage across parallel elements

Write the n/w in s-domain -



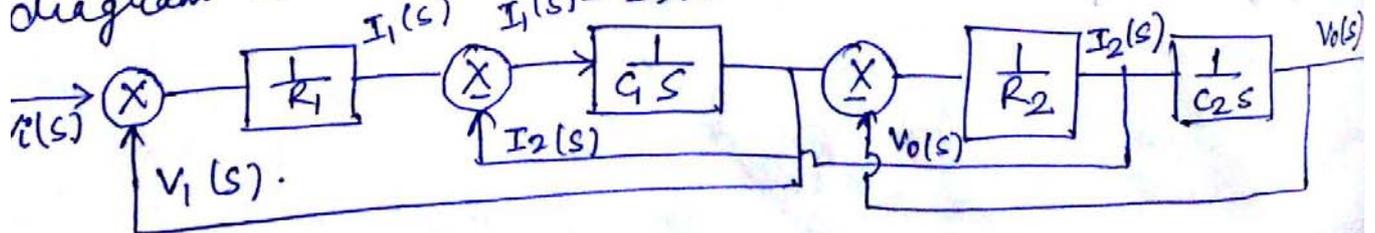
For R_1 , $I_1(s) = \frac{V_i(s) - V_1(s)}{R_1}$ — (i)

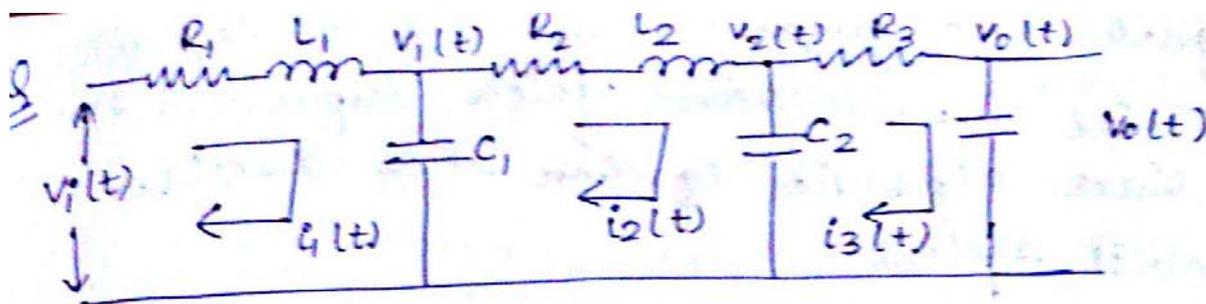
For C_1 , $V_1(s) = \frac{1}{C_1 s} [I_1(s) - I_2(s)]$ — (ii)

For R_2 , $I_2(s) = \frac{V_1(s) - V_0(s)}{R_2}$ — (iii)

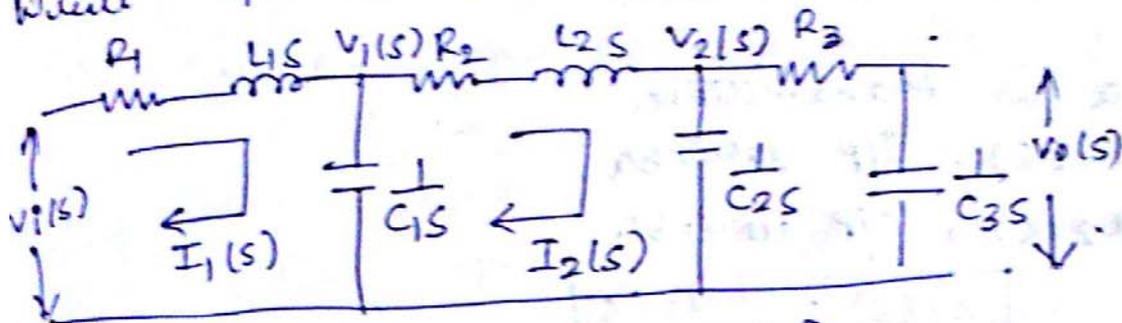
For C_2 , $V_0(s) = \frac{1}{C_2 s} [I_2(s)]$ — (iv)

By making use of (i), (ii), (iii) & (iv) the block diagram is written as -





Write n/w in s-domain



For R_1 & L_1 ; $I_1(s) = \frac{V_1(s) - V_1(s)}{R_1 + sL_1}$

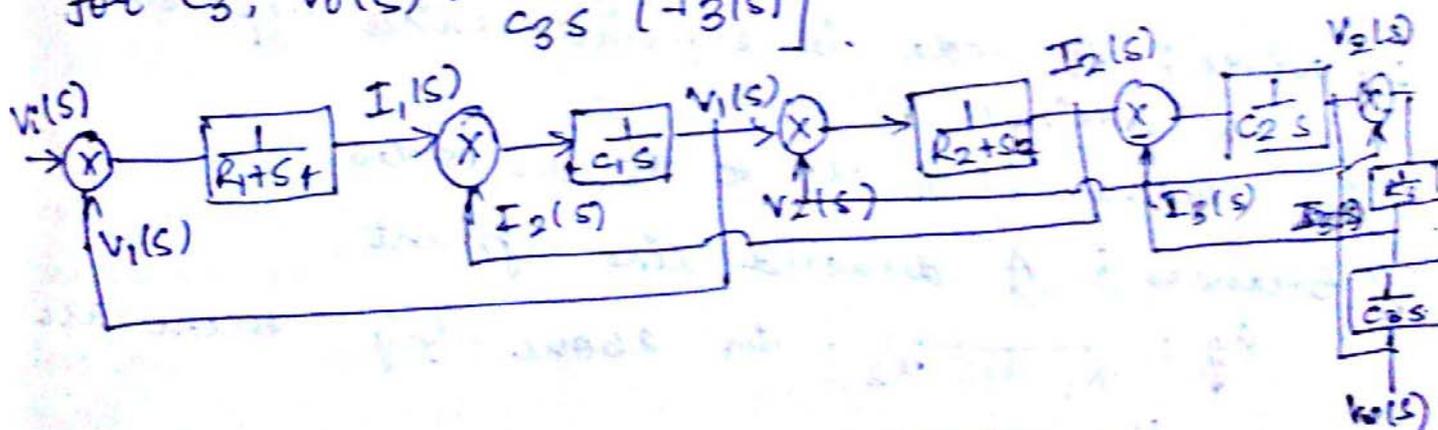
For C_1 , $V_1(s) = \frac{1}{C_1 s} [I_1(s) - I_2(s)]$

For R_2 & L_2 $I_2(s) = \frac{V_1(s) - V_2(s)}{R_2 + sL_2}$

For C_2 , $V_2(s) = \frac{1}{C_2 s} [I_2(s) - I_3(s)]$

For R_3 , $I_3(s) = \frac{V_2(s) - V_0(s)}{R_3}$

For C_3 , $V_0(s) = \frac{1}{C_3 s} [I_3(s)]$



Signal flow graph:

The pictorial representation which represents the set of linear algebraic equation which describes the control system.

Eg: $x_1(s) \xrightarrow{a} x_2(s)$

where a is transmittance

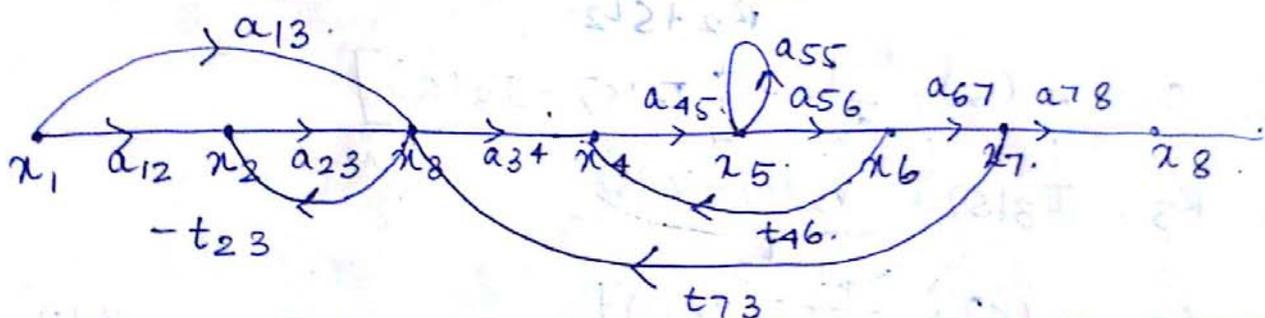
$x_1(s)$: i/p variable

$x_2(s)$: o/p variable

$$x_2(s) = a x_1(s)$$

The transmittance is the gain acquired by signal when it travels from one node to another in a signal flow graph.

Terminologies used in signal flow graph -



1) Node: A node is a point which represents a variable.

In above SFG x_1 to x_8 are nodes.

2) Branch: A directed line segment.

Eg: $x_1 \xrightarrow{a_{12}} x_2$ In above fig there are 12 branches.

3) Input/Source node: The node having only out going branches is called as input source node.

4) Output / Sink node: The node having 1 or more incoming branches is called as output / sink node. AB.

5) Chain node: The node having both incoming and outgoing branches.

Eg: In given fig: $x_2, x_3, x_4, x_5, x_6, x_7$ are chain nodes.

6) Forward path: A path from i/p node to o/p node is called forward path.

In identifying the forward path, no node should be traced twice.

In the fig, 3 f.p. i.e., $x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8$.

$x_1 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8$.

$x_1 - x_2 - x_3 - x_4 - x_6 - x_7 - x_8$.

NOTE: Self loop should not be taken into consideration in determining the forward path.

Loop: Loop is a closed path means a feedback path or feedback loop.

If a loop originates and terminates at the same node is known as feedback loop or feedback path.

In the above fig. there are two feedback paths or two loops.

From fig $x_2 - x_3 - x_2$

$x_3 - x_4 - x_5, x_6 - x_7 - x_3$.

Self node: By taking only one node the loop has to be formed.

In the above eg there is one self loop. a_{55} at node n_5 is a self loop.

Forward path gain:-

The product of gain along the forward path is called the forward path gain.

$$P_1 = a_{12} a_{23} a_{34} a_{45} a_{56} a_{67} a_{78}$$

gain of second forward path

$$P_2 = a_{13} a_{34} a_{45} a_{56} a_{67} a_{78}$$

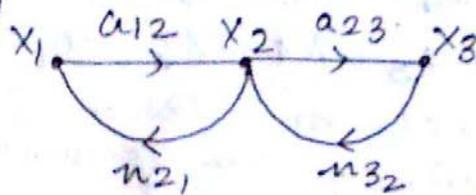
Gain of third forward path -

$$P_3 = a_{12} a_{23} a_{34} a_{46} a_{67} a_{78}$$

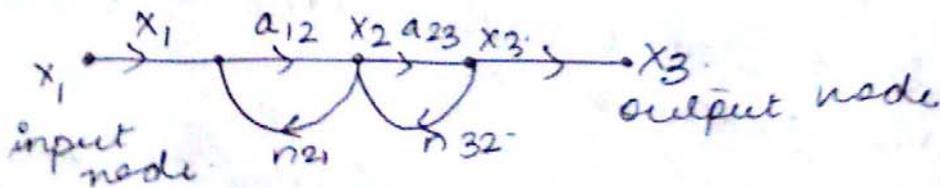
Gain of fourth forward path -

$$P_4 = a_{13} a_{34} a_{46} a_{67} a_{78}$$

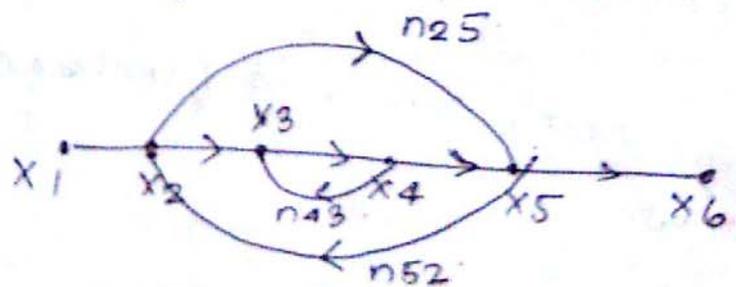
Dummy node:-



Without input and output nodes.



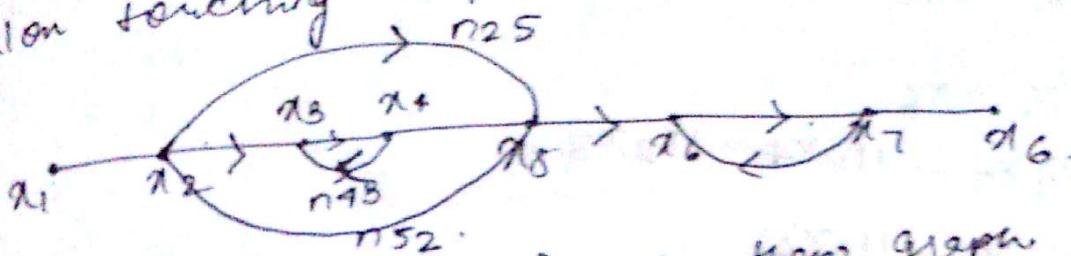
Non touching loops:-



If there is no node common b/w two or more loops these loops are known as non touching loops.

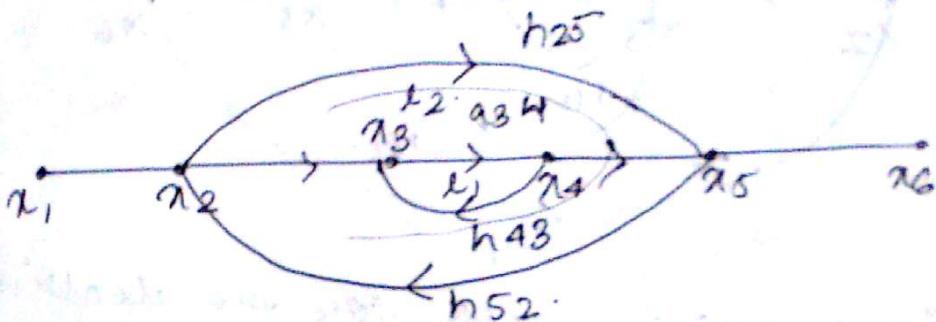
eg: $x_3 - x_4 - x_3$
 $x_2 - x_5 - x_2$

Non touching loops -



In the above signal flow graph there is one combination of 3 non-touching loops.
 $x_3 - x_4 - x_3$, $x_2 - x_5 - x_2$, $x_6 - x_7 - x_6$.

Loop gain: It is a product of all gains of the branches forming a loop.



gain of loop 1, $L_1 = a_{34}h_{43}$
 loop 2, $L_2 = h_{25}h_{52}$

∴ Draw the signal flow graph to satisfy the set of linear equations given below.

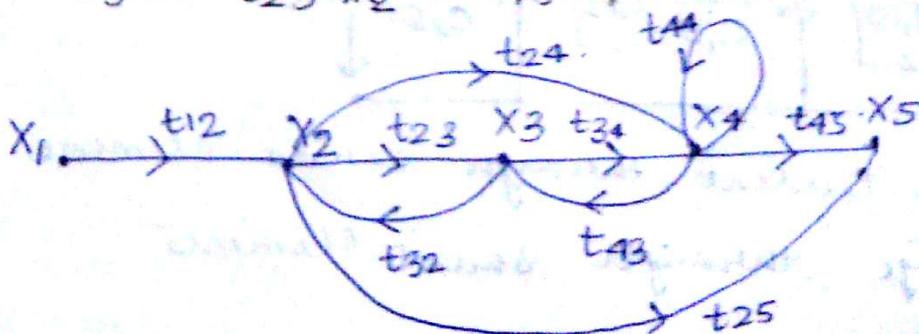
$$x_2 = t_{12}x_1 + t_{32}x_3$$

$$x_3 = t_{23}x_2 + t_{43}x_4$$

$$x_4 = t_{24}x_2 + t_{34}x_3 + t_{44}x_4$$

$$x_5 = t_{25}x_2 + t_{45}x_4$$

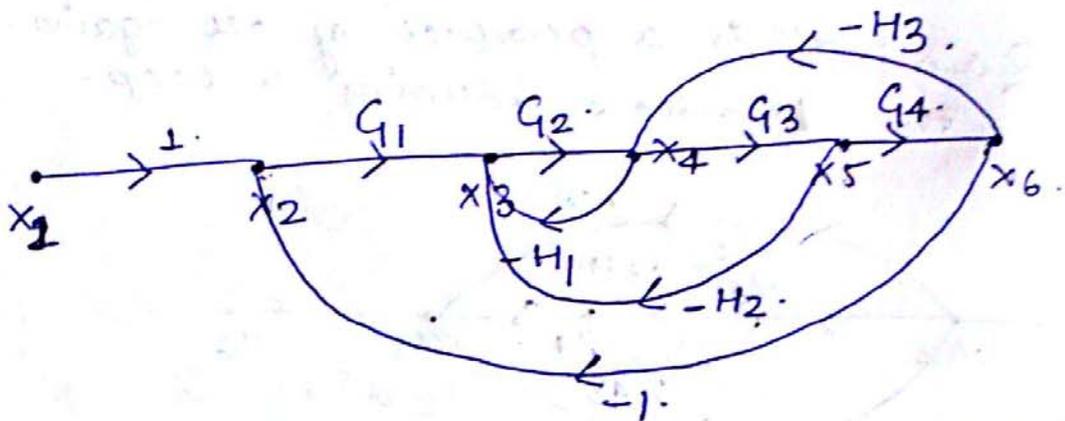
x_1, x_2, x_3, x_4, x_5 represent the nodes.



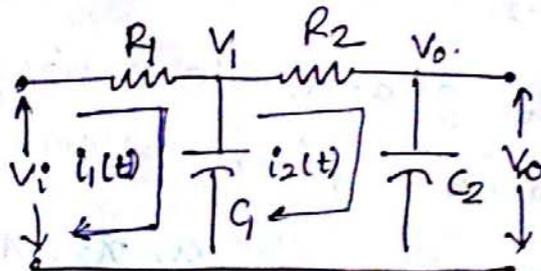
x_1 represents input node voltage
 x_5 represents output node voltage
 t_{12}, t_{32}, t_{43} etc represents transmittance

110

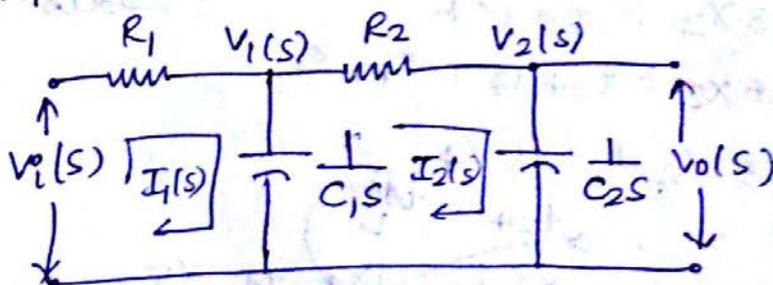
$$\begin{aligned}
 x_2 &= x_1 - x_6 \\
 x_3 &= G_1 x_2 - H_1 x_4 - H_2 x_5 \\
 x_4 &= G_2 x_3 - H_3 x_6 \\
 x_5 &= G_3 x_4 \\
 x_6 &= G_4 x_5
 \end{aligned}$$



Draw the signal flow graph for the electrical network shown below.



The n/w in the s-domain is written as-



Find the current through series elements and voltage through shunt elements.

For R_1 :-

$$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1}$$

$$I_1(s) = \frac{V_i(s)}{R_1} - \frac{V_1(s)}{R_1}$$

For C_1 :-

$$V_1(s) = \frac{1}{C_1 s} [I_1(s) - I_2(s)]$$

$$V_1(s) = \frac{I_1(s)}{C_1(s)} - \frac{I_2(s)}{C_1 s}$$

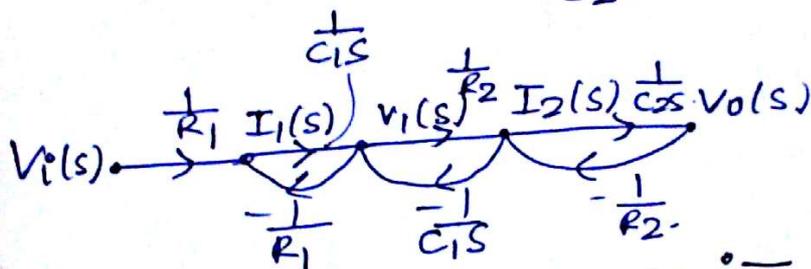
For R_2 :-

$$I_2(s) = \frac{V_1(s) - V_o(s)}{R_2}$$

$$I_2(s) = \frac{V_1(s)}{R_2} - \frac{V_o(s)}{R_2}$$

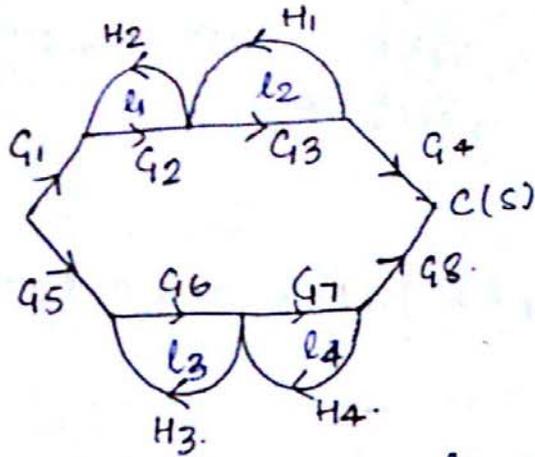
For C_2 :-

$$V_o(s) = \frac{1}{C_2 s} I_2(s)$$



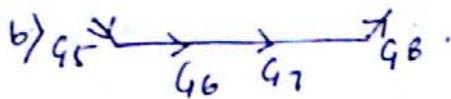
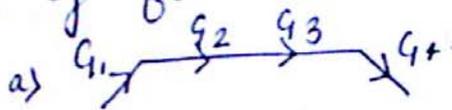
Mason's Gain Formula :-

Q Find $\frac{C(S)}{R(S)}$ for the signal shown below using Mason's gain formula -



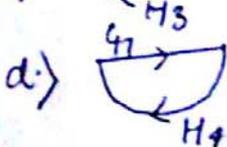
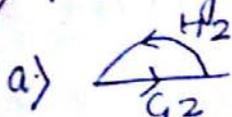
Solution :-
M.G.F, $\frac{C(S)}{R(S)} = \sum_{k=1}^L \frac{P_k D_k}{\Delta}$

i) NO. of forward paths : 02



Gain of I forward path $P_1 = G_1 G_2 G_3 G_4$
Gain of II forward path $P_2 = G_5 G_6 G_7 G_8$

ii) NO. of loops : 04



Gain of loop 1 $l_1 = G_2 H_2$

Gain of loop 2 $l_2 = G_3 H_1$

Gain of loop 3 $l_3 = G_6 H_3$

Gain of loop 4 $l_4 = G_7 H_4$

iii) Non touching loops:

a) 2 non touching loops : 04

i) $l_1 l_3 = G_2 H_2 G_6 H_3$

ii) $l_1 l_4 = G_2 H_2 G_7 H_4$

iii) $l_2 l_3 = G_3 H_1 G_6 H_3$

iv) $l_2 l_4 = G_3 H_1 G_7 H_4$

b) 3 non touching loops : 0

$$\Delta = 1 - \{l_1 + l_2 + l_3 + l_4\} + \{l_1 l_3 + l_1 l_4 + l_2 l_3 + l_2 l_4 + l_3 l_4\} - l_1 l_2 l_3 l_4$$

$$\Delta = 1 - G_2 H_2 - G_3 H_1 - G_6 H_3 - G_7 H_4 + G_2 G_6 H_2 H_3 + G_2 G_7 H_2 H_4 + G_3 G_6 H_1 H_3 + G_3 G_7 H_1 H_4$$

v) $\Delta_k \Rightarrow k \rightarrow 1 \text{ to } 2$

$$\Delta_1 = 1 - \{0 + 0 + l_3 + l_4\} + \{0 l_3 + 0 l_4 + l_3 \cdot 0 + l_4 \cdot 0\}$$

$$= 1 - l_3 - l_4$$

$$\Delta_1 = 1 - G_6 H_3 - G_7 H_4$$

$$\Delta_2 = 1 - \{l_1 + l_2 + 0 + 0\} + \{l_1 \cdot 0 + l_1 \cdot 0 + 0 \cdot l_3 + l_2 \cdot 0\}$$

$$= 1 - l_1 - l_2$$

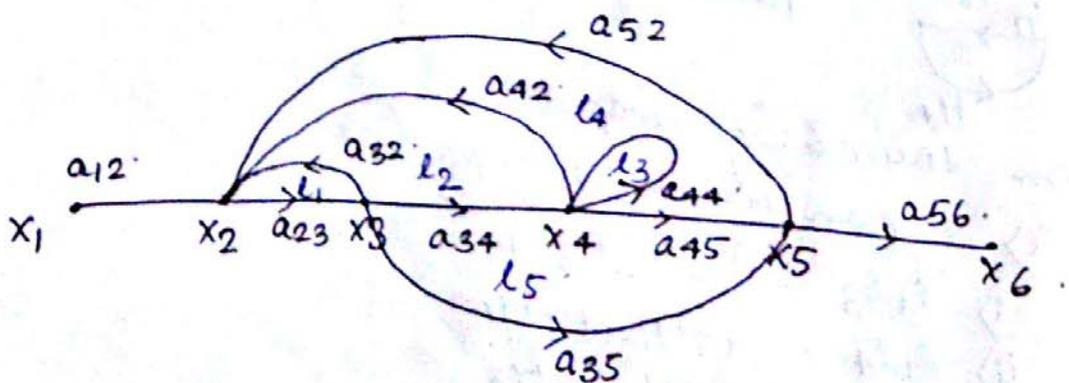
$$= 1 - G_2 H_2 - G_3 H_1$$

$$\frac{C(s)}{R(s)} = \sum_{k=1}^2 \frac{P_k \Delta_k}{\Delta}$$

$$= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

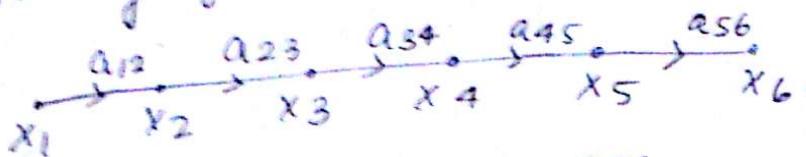
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 (1 - G_6 H_3 - G_7 H_4) + G_5 G_6 G_7 G_8 (1 - G_2 H_2 - G_3 H_1)}{1 - G_2 H_2 - G_3 H_1 - G_6 H_3 - G_7 H_4 + G_2 G_6 H_2 H_3 + G_2 G_7 H_2 H_4 + G_3 G_6 H_1 H_3 + G_3 G_7 H_1 H_4}$$

||



Soln: MGF, $\frac{X_6}{X_1} = \sum_{k=1}^L \frac{P_k \Delta_k}{\Delta}$

i) No. of forward paths: 02



Gain of forward path $P_1 = a_{12} a_{23} a_{34} a_{45} a_{56}$.

Gain of II forward path $P_2 = a_{12} a_{23} a_{35} a_{56}$.

ii) No. of loops:



Gain of loop 1 $l_1 = a_{23} a_{32}$



Gain of loop 2 $l_2 = a_{23} a_{34} a_{42}$



Gain of loop 3 $l_3 = a_{44}$



Gain of loop 4 $l_4 = a_{23} a_{34} a_{45} a_{52}$



Gain of loop 5 $l_5 = a_{35} a_{52} a_{23}$

iii) Non touching loops:

a) 2 non touching loops - 2

a) $l_1 l_3 = a_{23} a_{32} a_{44}$

b) $l_3 l_5 = a_{44} a_{35} a_{52} a_{23}$

3 non touching loops - 0

iv) $\Delta = 1 - \{l_1 + l_2 + l_3 + l_4 + l_5\} + \{l_1 l_3 + l_3 l_5\}$

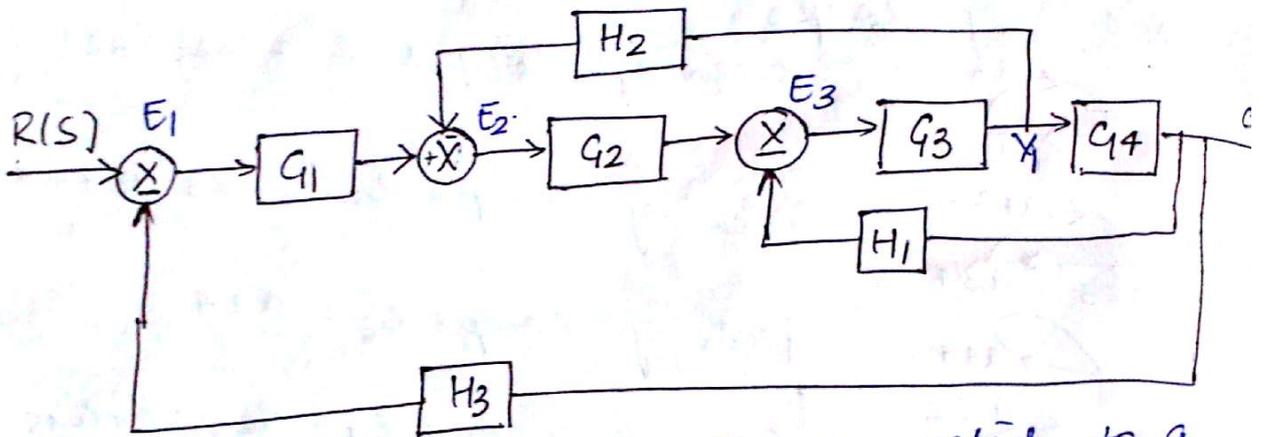
$\Delta = 1 - a_{23} a_{32} - a_{23} a_{34} a_{42} - a_{44} - a_{23} a_{34} a_{45} a_{52} - a_{35} a_{52} a_{23}$

$$\Delta_k = k \rightarrow 1 + 0.2$$

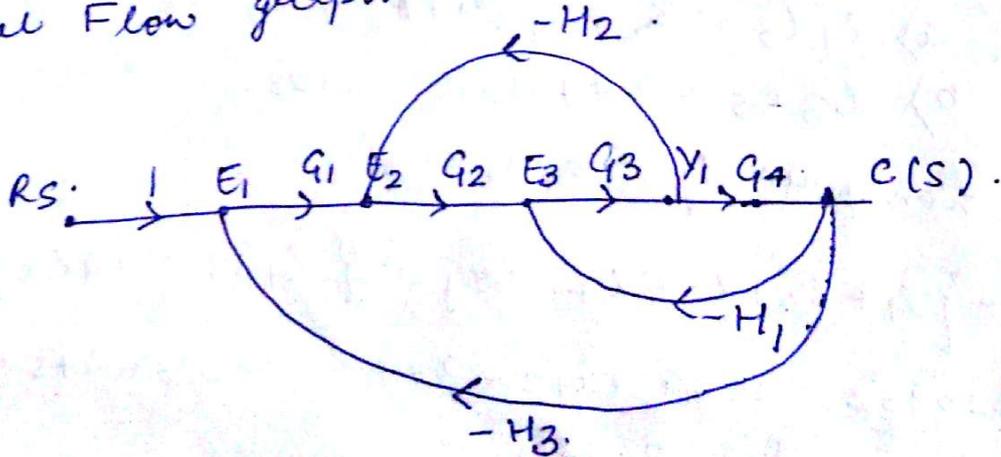
$$\Delta_1 = 1 - \{ l_1 + l_2 + l_3 + l_4 + l_5 \} + \{ l_1 l_3 + l_3 l_5 \}$$

$$= 1 - \{ 0 + \}$$

110



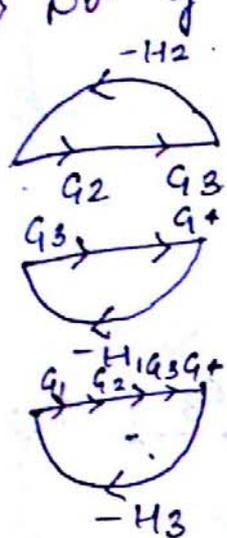
The above block diagram is converted to a signal flow graph -
 Name the summing points and take off points.
 Each summing point and take off point is represented as a node in the signal flow graph.
 Signal Flow graph :-



$$\frac{C(s)}{R(s)} = \sum_{k=1}^L \frac{P_k \Delta_k}{\Delta}$$

1) NO. of forward paths: 01
 a) Gain of forward path: $G_1 G_2 G_3 G_4$.

2) NO. of loops: -



l_1 Gain of loop 1 $l_1 = -G_2 G_3 H_2$.

l_2 Gain of loop 2 $l_2 = -G_3 G_4 H_1$.

l_3 Gain of loop 3 $l_3 = -G_1 G_2 G_3 G_4 H_3$.

3) NO. of non touching loops:
 2 non touching loops: 0.
 3 non touching loops: 0.

4) $\Delta = 1 - \{l_1 + l_2 + l_3\} + \{0\}$
 $= 1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4 H_3$

5) $\Delta_k \rightarrow k=1$

$$\Delta_1 = 1 - \{l_1 + l_2 + l_3\} + \{0\}$$

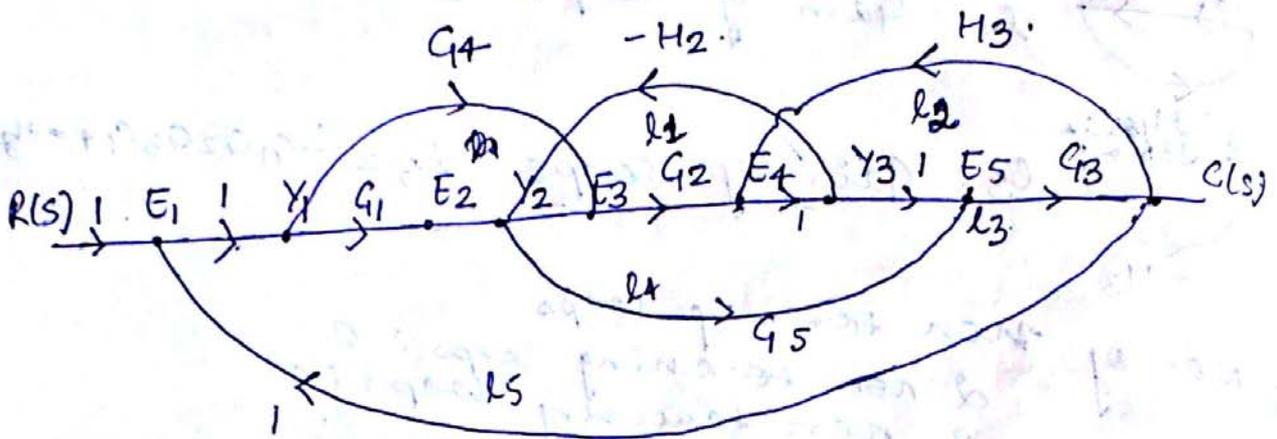
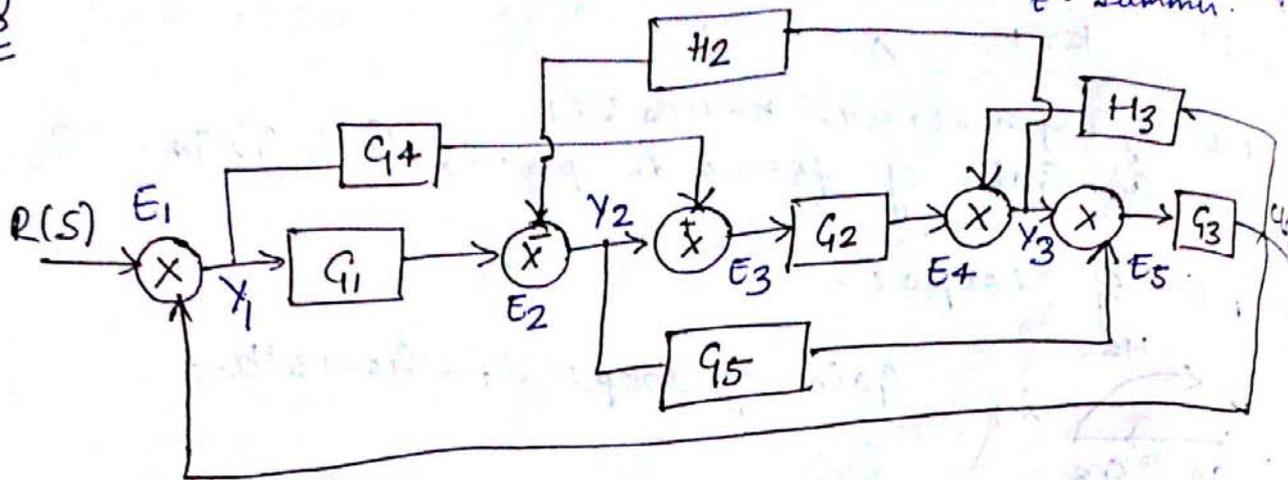
$$= 1 - \{0 + 0 + 0\}$$

$$\Delta_1 = 1$$

$$\frac{C(s)}{R(s)} = \sum_{k=1}^L \frac{P_k \Delta_k}{\Delta}$$

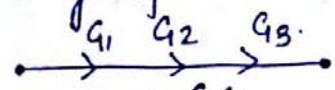
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 \cdot 1}{1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4 H_3}$$

V - take off point
E - summer

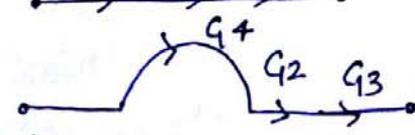


$$\frac{C(s)}{R(s)} = \sum_{k=1}^l \frac{P_k \Delta_k}{\Delta}$$

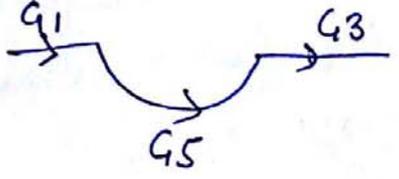
No. of forward paths: 03.



Gain of I forward path $P_1 = G_1 G_2 G_3$

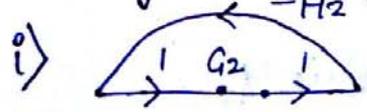


Gain of II forward path $P_2 = G_4 G_2 G_3$

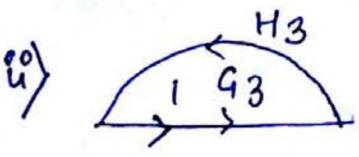


Gain of III forward path $P_3 = G_1 G_5 G_3$

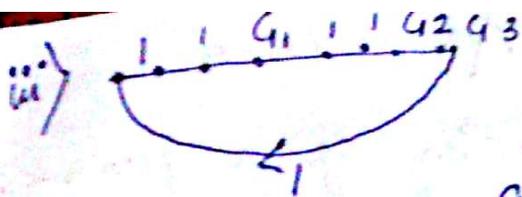
No. of loops: 04.



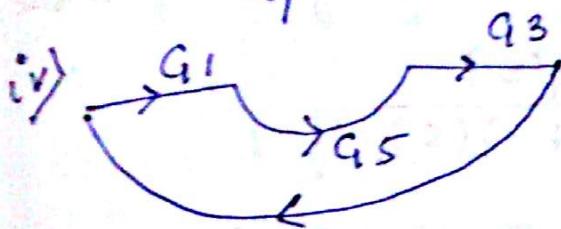
Gain of I loop $l_1 = -G_2 H_2$



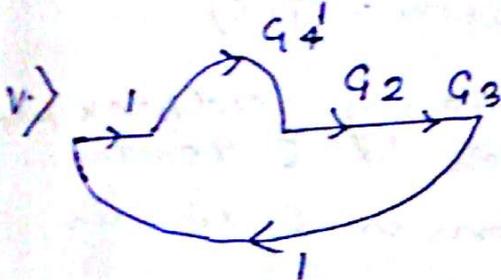
Gain of II loop $l_2 = G_3 H_3$



Gain $l_3 = G_1 G_2 G_3$



Gain $l_4 = G_1 G_3 G_5$



Gain $l_5 = G_4 G_2 G_3$

NO. of non touching loop :-

a) 2 non touching loops : 0.

$$\Delta = 1 - \{ l_1 + l_2 + l_3 + l_4 + l_5 \} + \{ 0 \}$$

$$= 1 + G_2 H_2 - G_3 H_3 - G_1 G_2 G_3 - G_1 G_3 G_5 - G_4 G_2 G_3$$

$\Delta K \rightarrow K = 3$

$$\Delta_1 = 1 - \{ l_1 + l_2 + l_3 + l_4 + l_5 \}$$

$$= 1 - \{ 0 + 0 + 0 + 0 + 0 \}$$

$\Delta_1 = 1$

$$\Delta_2 = 1 - \{ 0 + 0 + 0 + 0 + 0 \} = 1$$

$$\Delta_3 = 1 - \{ 0 + 0 + 0 + 0 + 0 \} = 1$$

$$\frac{C(s)}{R(s)} = \sum_{k=1}^3 \frac{P_k \Delta_k}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4 G_2 G_3 + G_1 G_3 G_5}{1 + G_2 H_2 - G_3 H_3 - G_1 G_2 G_3 - G_1 G_3 G_5 - G_4 G_2 G_3}$$