SATELLITE INJECTION

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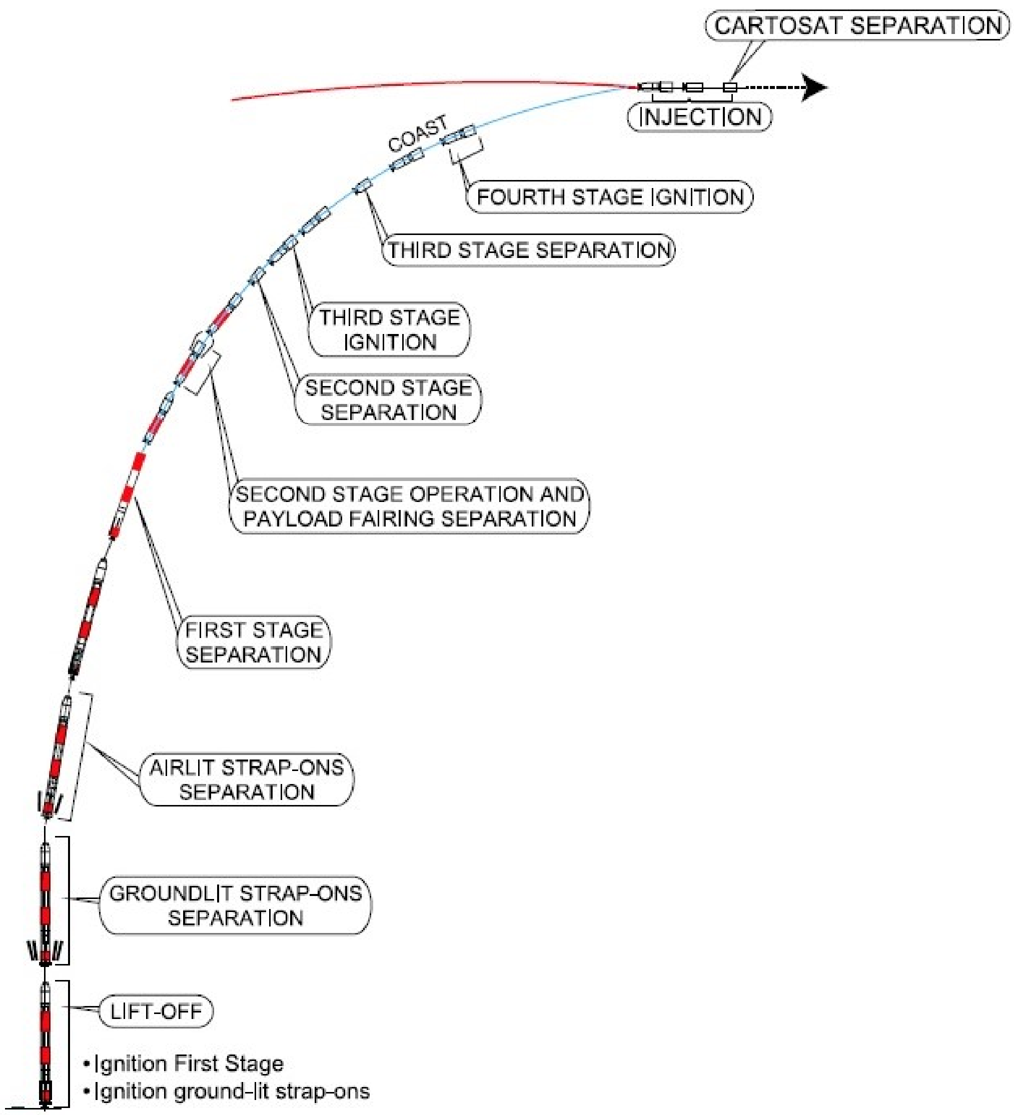
Agenda

Launch Video

Satellite Injection – PSLV XL

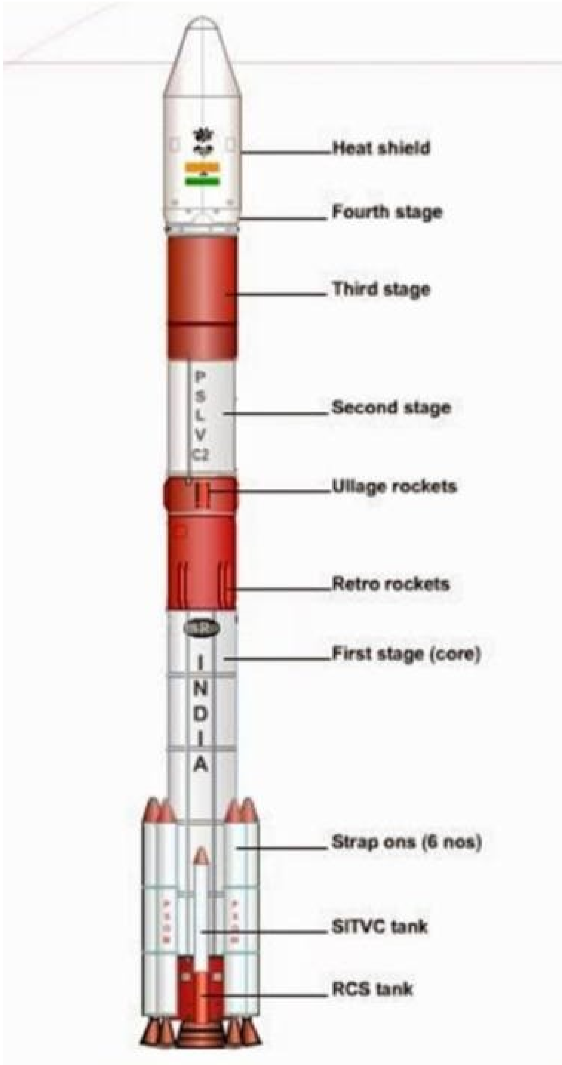


- Launch Vehicle: PSLV XL
- Launch Site: FLP, Satish Dhawan
- Launch Date: June 23, 2017
- Launch Time: 03:59 UTC
- Payload: 31 Satellites (955kg)
- Mission Duration: 23 Minutes & 19 Seconds

- Target Orbit
- Type: Sun Synchronous Orbit
- Altitude: 505 Kilometers
- Inclination: 97.44°

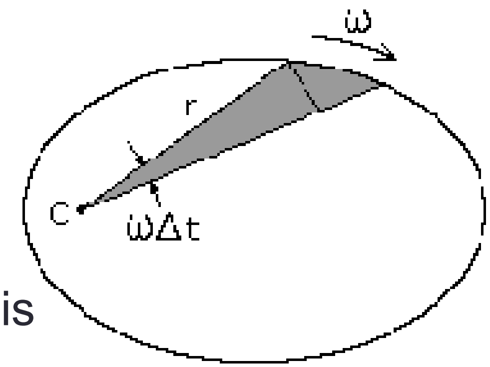
PSLV Event table

Event	Time	Altitude (km)	Velocity (m/s)
PS1 Ignition	0:00.00.0	0.0238	451.89
Booster 1&2 Ignition	0:00.00.42	0.0238	451.89
Booster 3&4 Ignition	0:00.00.62	0.0238	451.89
Booster 5&6 (Air-Lit) Ignition	0:00.25.0	2.767	574.99
Booster 1&2 Separation	0:01:09.9	26.816	1310.61
Booster 3&4 Separation	0:01:10.1	26.973	1315.18
Booster 5&6 Separation	0:01.32.0	47.254	1863.21
PS1 Separation	0:01:50.26	68.241	2158.26
PS2 Ignition	0:01:50.46	68.470	2157.42
Payload Fairing Separation	0:02:38.96	121.683	2456.24
PS2 Separation	0:04:21.72	225.378	4043.33
PS3 Ignition	0:04:22.92	226.596	4040.56
PS3 Separation	0:08:11.22	424.251	5923.04
PS4 Ignition	0:08:21.12	430.405	5914.09
PS4 Cutoff	0:15:58.94	509.854	7604.83
CartoSat-2E Separation	0:16:40.94	510.656	7608.90
NIUSAT Separation	0:16:50.94	510.853	7608.84
CE-Sat 1 Separation	0:17:00.94	511.052	7608.71
CubeSat Separation Sequence	0:17:00.94	511.052	7608.71
Satellite Separation Complete	0:23:18.94	519.266	7604.85



General Aspects of Satellite Injection

- Kepler's second law of planetary motion must, of course, hold true for circular orbits.
- In such orbits both and *r* are constant so that equal areas are swept out in equal times by the line joining a planet and the sun.
- For elliptical orbits, however, both and *r* will vary with time.
- Figure shows a particle revolving around *C* along some arbitrary path. The area swept out by the radius vector in a short time interval Δ*t* is shown shaded..



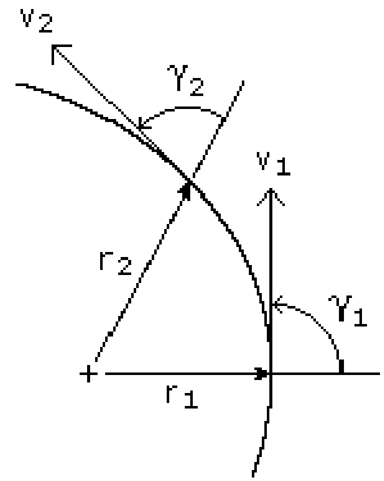
- This area, neglecting the small triangular region at the end, is one-half the base times the height or approximately *r(r ω*Δ *t)/2*. .
- This expression becomes more exact as Δ*t* approaches zero, i.e. the small triangle goes to zero more rapidly than the large one.
- The rate at which area is being swept out instantaneously is therefore

$$\lim_{t \to 0} \left[\frac{r(r\omega\Delta t)}{2} \right] = \frac{\omega r^2}{2}$$

General Aspects of Satellite Injection

- For any given body moving under the influence of a central force, the value ωr^2 is constant.
- Let's now consider two points P_1 and P_2 in an orbit with radii r_1 and r_2, and velocities v_1 and v_2. Since the velocity is always tangent to the path, it can be seen that if γ is the angle between r and v, then

$$v \sin\gamma = \omega r$$



- where $v \sin$ is the transverse component of v. Multiplying through by r, we have

$$rv \sin\gamma = \omega r^2 = \text{Constant}$$

- for two points P_1 and P_2 on the orbital path

$$r_1 v_1 \sin\gamma_1 = r_2 v_2 \sin\gamma_2$$

- Note that at periapsis and apoapsis, γ = 90 degrees. Thus, letting P_1 and P_2 be these two points we get

$$R_p V_p = R_a V_a$$

General Aspects of Satellite Injection

- Let's now look at the energy of the above particle at points P_1 and P_2.
- *Conservation of energy* states that the sum of the kinetic energy and the potential energy of a particle remains constant.
- The kinetic energy T of a particle is given by $mv^2/2$ while the potential energy of gravity V is calculated by the equation $-GMm/r$.
- Applying conservation of energy we have

$$T_1 + V_1 = T_2 + V_2, \quad \text{or}$$

$$\frac{mv_1^2}{2} - \frac{GMm}{r_1} = \frac{mv_2^2}{2} - \frac{GMm}{r_2}, \quad \text{or}$$

$$v_2^2 - v_1^2 = 2GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

- we obtain

$$Vp = \sqrt{\frac{2GMR_a}{R_p(R_a+R_p)}}, \quad \text{and} \quad Va = \sqrt{\frac{2GMR_p}{R_a(R_a+R_p)}}$$

- And

$$R_a = \frac{R_p}{\left(\frac{2GM}{R_pV_p^2} - 1\right)}, \quad \text{and} \quad R_p = \frac{R_a}{\left(\frac{2GM}{R_aV_a^2} - 1\right)} \qquad e = \frac{R_pV_p^2}{GM} - 1$$

Problems

- An artificial Earth satellite is in an elliptical orbit which brings it to an altitude of 250 km at perigee and out to an altitude of 500 km at apogee. Calculate the velocity of the satellite at both perigee and apogee.
- Solution : Given:

$R_p = (6{,}378.14 + 250) \times 1{,}000 = 6{,}628{,}140$ m

$R_a = (6{,}378.14 + 500) \times 1{,}000 = 6{,}878{,}140$ m

$V_p = \text{SQRT}[\, 2 \times GM \times R_a / (R_p \times (R_a + R_p))\,]$

$V_p = \text{SQRT}[\, 2 \times 3.986005\times10^{14} \times 6{,}878{,}140 / (6{,}628{,}140 \times (6{,}878{,}140 + 6{,}628{,}140))\,]$

$V_p = 7{,}826$ m/s

$V_a = \text{SQRT}[\, 2 \times GM \times R_p / (R_a \times (R_a + R_p))\,]$

$V_a = \text{SQRT}[\, 2 \times 3.986005\times10^{14} \times 6{,}628{,}140 / (6{,}878{,}140 \times (6{,}878{,}140 + 6{,}628{,}140))\,]$

$V_a = 7{,}542$ m/s

Problem

- A satellite in Earth orbit passes through its perigee point at an altitude of 200 km above the Earth's surface and at a velocity of 7,850 m/s. Calculate the apogee altitude of the satellite.

- Solution : Given:

$Rp = (6{,}378.14 + 200) \times 1{,}000 = 6{,}578{,}140$ m

$Vp = 7{,}850$ m/s

$Ra = Rp / [2 \times GM / (Rp \times Vp^2) - 1]$

$Ra = 6{,}578{,}140 / [2 \times 3.986005\times10^{14} / (6{,}578{,}140 \times 7{,}850^2) - 1]$

$Ra = 6{,}805{,}140$ m

Altitude @ apogee = 6,805,140 / 1,000 - 6,378.14 = 427.0 km

Problem

- Calculate the eccentricity of the orbit for the satellite in previous problem
- Solution
- Given:

$R_p = 6{,}578{,}140$ m

$V_p = 7{,}850$ m/s

$e = R_p \times V_p^2 / GM - 1$

$e = 6{,}578{,}140 \times 7{,}850^2 / 3.986005 \times 10^{14} - 1$

$e = 0.01696$