Module - 2

CAD & Computer Graphic's Software

Computer Aided Design [CAD]

CAD Can be defined as "the creation, modification analysis & Optimisation of a new Component using a computer"

CAD involves three major Elements

1. CAD hardware
2. Software
3. User

In an Engineering sense CAD also incorporates,

i) Finite Element Analysis
ii) Stress Analysis
iii) Heat transfer analysis
iv) Fluid flow analysis

& so on

CAD in actual Sense represents the evolution of graphical representation of information on computer. The origin of CAD technology can be traced back to the development of Interactive Computer graphics (ICG) & Graphical representation of Computers.

The Graphic operation includes

i) Scaling
ii) Translation
iii) Rotation
iv) Animation
v) Simulation

Some of the popular CAD software are

1. CATIA
2. I-DEAS
3. Pro-E
4. Unigraphics
5. Auto Cad

Need for CAD system

The CAD systems are based on the Interactive Computer Graphics (ICG). ICG refers to a "user oriented system in which computers are used to create & modify drawings & relevant data on the display screen"

There are five basic reason for using CAD syst

1. Increasing the productivity of the Designer
2. Improving the Quality of Design
3. Improving the Communication process
4. Creating a large data base for mfg activities
5. Multiple Group performance.

1. Increasing the productivity of the Designer

This is achieved by assisting the designer to visualise the product, its sub assemblies & reducing the time required in synthesising, analysing & storing the design. This helps to reduce the time & cost required to Execute a project

2. Improving the Quality of Design

A CAD system performs a thorough engineering analysis, & the Select the Optimum design among the various design alternatives. It eliminates design errors & brings in accuracy in the designed product thereby resulting in a Quality product

3. Improving the Communication process :- CAD system provides high Quality engineering drawings, results in standardisation, better documentation, fewer errors & greater liability

4. Creating a large database for manufacturing activities

A CAD system is capable of producing the required data base like product dimensions, material specification, bill of material (BOM), Quantity of parts required, while the product is being designed. This data is essential for planning, manufacturing & inspection activities.

5. Multiple/Group performance :- A CAD system also has the capability to allow for multiple users (or) the members of a project team to work on the same project. Like for example a component designed & stored in a library by one member can be accessed by another member for use to produce an assembly. Similarly various users can access other design & make use to execute the project.

Design process

A design performs the complete design process by putting his capabilities into a CAD system. A basic design process is an iteractive procedure which involve various steps. The six steps in a design process, given by shigley are:

1. Recognition of need
2. Definition of problem
3. Synthesis – [Combination of component to form a connected whole]
4. Analysis & Optimisation
5. Evaluation
6. presentation

These steps with iteractive action are illustrated in fig.

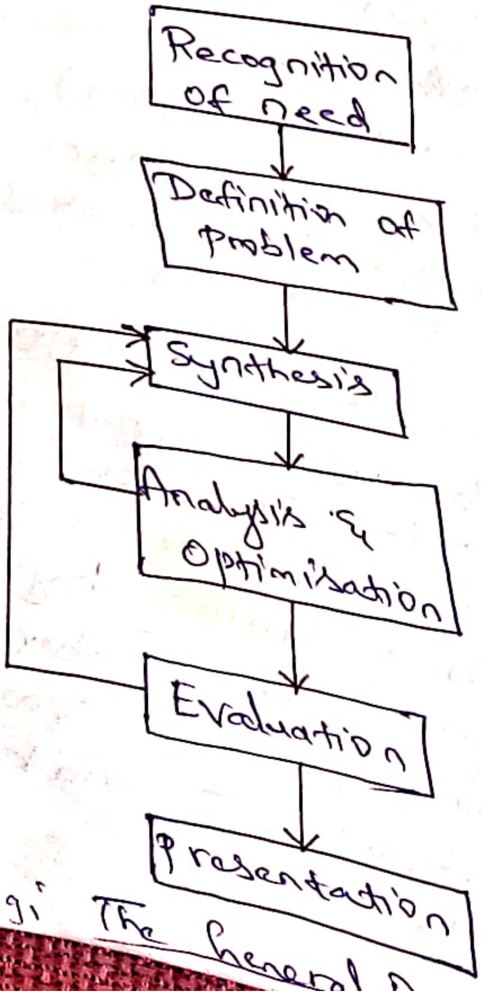


fig: The General ...

1. Recognition of need

- Arises Either from market demand (or) from the problem in an Existing product
- These can be identified by the Sales person (or) Engineer

2. Definition of problem

- Involve setting a definite standard / specification for the part to be designed. This includes physical & functional characteristics, cost, Quality & performance. This again consider both market & Design requirement

3. Synthesis & Analysis are closely related to each other & are highly iterative in design process.

Initially, a product is conceptualised by the designer, analysed, improved & re defined to meet standards specification.

To bring the design to its optimum level within the constraints imposed. Synthesis & Analysis are essential to optimise the product

4. Evaluation : It is the process of Comparing the design with the specification defined in the problem definition stage.

Evaluation generally involves

- Fabrication & testing of a prototype to assess the ~~perf~~performance, Quality & reliability.

5. Presentation :- It is a final phase, This includes documentation of the design using

- Engineering Drawing
- Materials
- Specification
- Bill of materials (BOM) etc.

→ These are design data & useful for further activities

Application of Computer in Design

(or)

Role of Computers in Design process

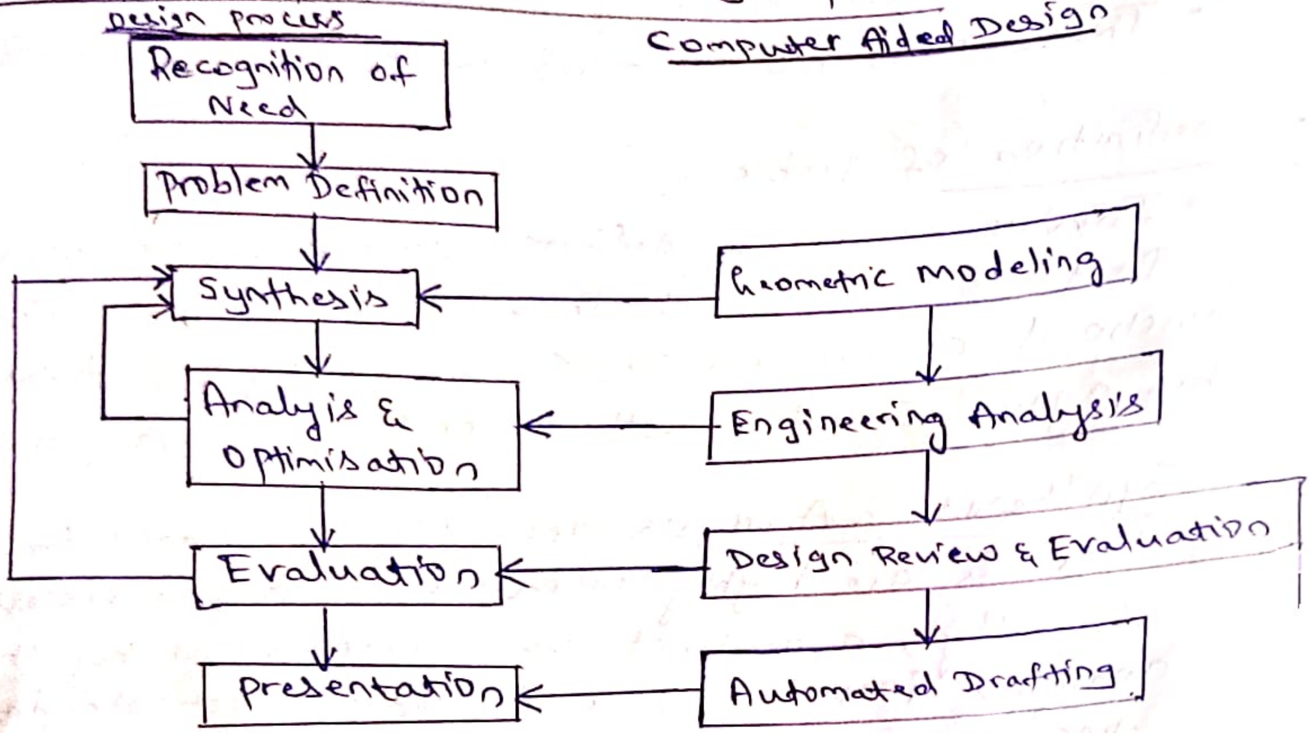


fig:- Role of Computer in Design process

There are four design related activities that are prefered by the computer in CAD system

1. Geometric modeling
2. Engineering Analysis
3. Design Review & Evaluation
4. Automated Drafting

⇒ These four category are related to the last four phases in the general Design process

Geometric modeling :- Related to synthesis process in which physical design is shaped on the ICG system

Engineering Analysis – related to analysis & optimisation phase

Design review & Evaluation – Related to evaluation in 5th phase

Automated Drafting : creating hard copies of Engg drawing using geometric models

1. Geometric Modeling (GM)

In CAD, GM, refers to the creation of geometry of an object, which is nothing but the Computer Compatible mathematic description.

This mathematical description helps in displaying the images of the object created & manipulation on graphic terminal by various software command, executed through CPU & i/p device.

In geometric modeling, the designer creates the graphical image of the required object on the CRT screen of the ICG system by inputing three kinds of command to the computer

i) First type of Command :- Generates basic geometric elements like point, lines, curves etc

ii) Second type of Command :- used to perform transformation like scaling, modifying, moving, rotation etc

iii) Third type of Command :- used to join various element created to obtain desired shape of the product

The created geometric modeling can be
- Stored
- Retrieved
- Analysed
- Modified

There are 3 basic features of ~~[illegible]~~ Geometric modeling

They are :-

a. Wire frame modeling
b. Surface Modeling
c. Solid Modeling

a. <u>Wire Frame Modeling (WFM)</u>

It is the basic form of geometric modeling. In this wire frames are used to represent an object. The object is displayed by inter connecting lines. It consist of point, lines & curves

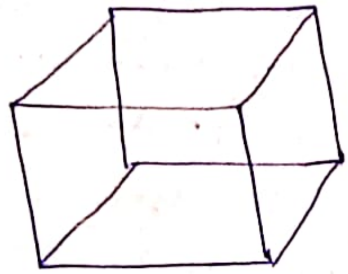


fig:- <u>wire frame modeling</u>

b. <u>Surface Modeling</u>

- In this, object is represented with surfaces, But object will not have any mass (or) volume

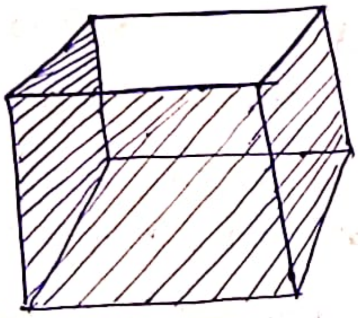


fig: <u>Surface Modeling</u>

c. <u>Solid Modeling</u>

Here the object will occupy volume. This feature uses the solid geometry shape called <u>primitive</u>, to construct object. Here the object will occupy volume

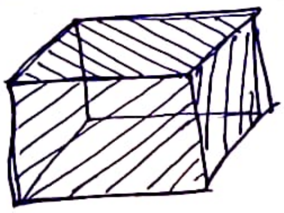


fig: Solid Modeling

2. Engineering Analysis

Analysis may be stress - strain calculation, heat transfer computation, Dynamic Analysis, Thermal analysis, Structural analysis etc

Two different types of Analysis

i) Analysis of mass properties
ii) Finite Element Analysis

i) Analysis of Mass properties

Provides different properties of a solid model like

- Surface Area
- Weight
- Volume
- Centre of Gravity etc

ii) Finite Element Analysis

The model is divided into large number of finite Elements.

Then these Elements are analysed for different properties at each node, & finally for the full part.

Different packages are

Ansys, Nastran, COSMOS, Fluent, Hypermesh etc

3. Design review & Evaluation

Accuracy of the design can be checked conveniently on the graphics terminal with the help of dimensioning, & tolerancing routines built with in the system

Design review & Evaluation technique

i) Layering
ii) Interference checking
iii) Kinematics

i) Layering :- The final image of the product can be compared with starting raw materials. The procedure can be followed in each stages

ii) Interference checking :- To find out, if any parts are intersecting with each other in an assembly model

iii) Kinematics :- Design can be review & evaluate by animating the motion of simple designed mechanism

Software for kinematics is ADAMS { Automatic Dynamic Analysis of mechanical system

4. Automated Drafting

It is a process of creating hard copies of Engineering drawing using geometric model

It helps to generate different Orthographic & Sectional views from a solid models

In addition to 2D Engineering drawing, it also provides

- Bill of material
- Part List
- Specification
- Cost & Other . . .

Computer Graphics

The graphic software is the Collection of Computer programs through which the user can send his Commands to the Computer to make it operates.

The programs include set of instructions, in a manner that the hardware understands; so as to perform:-

- generate drawings on the CRT display screen,
- To manipulate images
- Create Data
- produce output
- Stores on hard disk drives
- Transfer file from one Computer to other
- Storage Device etc

Software Configuration of a Graphic's System

In a graphic system, a number of function are performed, which can be grouped under 3 main categories:-

1) To interact with the graphic display unit & the user to create & Edit images
2) To Construct a physical model, using the designed image. Such model are called application model
3) To store the model in the secondary storage units for further operations

A graphic software for CAD system can be divided into 3 basic modules:-

1) Graphic Module
2) Application Module
3) Application Database

Design workstation

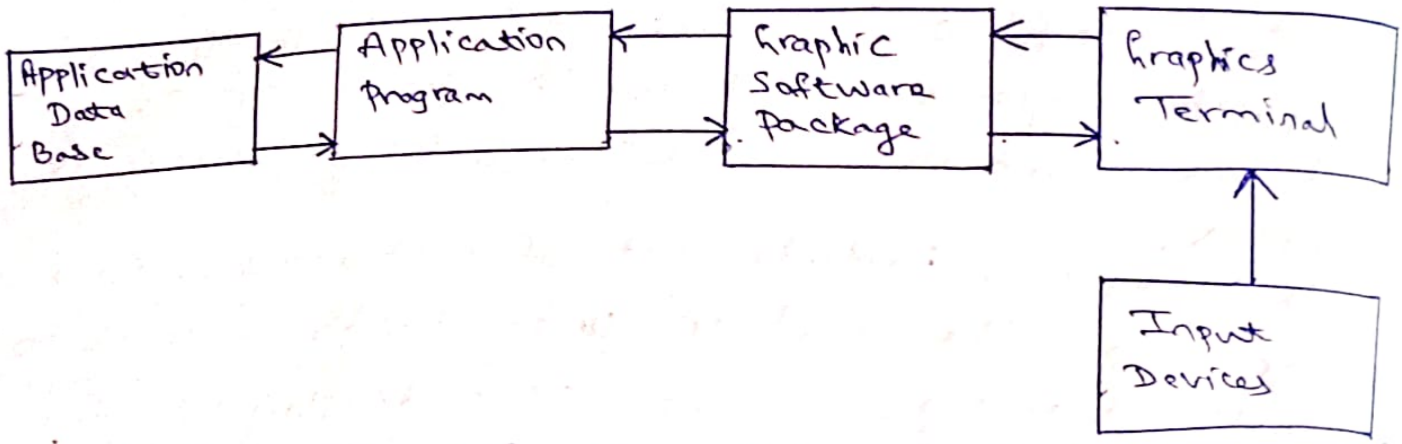


fig: Model of Graphics Software Configuration

1. Graphics Module

The graphics module is the software between the user & the graphics display unit. It performs the graphical interaction b/w the user & the CAD system. It also function as the interface b/w the user & application software.

2. Application Module

The application program is run by the user to create a physical model with the help of graphics terminal & input devices. Each application program is written for a particular problem area, like for design of mechanical components, Electronics, aerospace etc. In each area the application software is developed to deal with images & conventions relevant for that particular field.

3) Application Data Base

It consist of mathematical, numerical & logical definition of the application models, like that of mechanical parts, aircraft, automobile etc. It also include other alphanumeric database associated with the Created models, such as BOM, physical properties etc. The data base is updated as & when the user modifies the created images. The data base contents can be displayed on screen & a hard copy print can be taken for reference

Functions of Graphic Package

A graphic package in a CAD system has to perform the fine basic functions

1. Generation of Graphic Element
2. Transformation
3. Display Control & windowing functions
4. Segmenting Functions
5. User input functions

1. Generation of Graphic Element

- Graphic Element is a basic image Entity like a point, line, segment, circle etc
- Graphic Element also includes alphanumeric character, special symbols etc
- A hardware component called Graphic card helpful for the display of these element on the screen & also speed up the process of generating the Element & hence the image
- primitive is a term used to refer a 3D graphic element like sphere, cube, cylinder, prism etc

Scanned with CamScanner

2. Transformation

- used to modify the image on the graphic terminal & to reposition the item in the data base
- Transformation used to modify the image on the graphic terminal by • increasing/decreasing the size of the existing image (by Scaling)
  - Reposition the image (By translation & Rotation)
  - Assembling [moving one part into another]

3. Display Control & Windowing function

This function in a CAD graphic gives the ability to the user to view the image from the desired angle (rotation) & at the desired magnification (Zooming).

- It makes use of various transformation functions to display the application model as required by the designer for an informative viewing
- windows helps the designer to visualize the model in a better way to get pictorial information of his inputs
- Another important function of display Control is the removal of hidden lines graphic image created on the screen is ... a designed image

is shown clearly with its visible lines and by removing the invisible line. In most of the CAD software the hidden removal lines option are automatically executed, without being specified by the user which lines to be removed.

4. Segmenting Functions

This function provides the designer with the capability to selectively replace, delete & modify portions of the created images.
The segment refers to a particular portions of the image selected for editing (or) modifications
The segment may be a single element or a logical group of element which can be modified together.

5. User Input Functions

The user input function allows the designer to input commands and/or data to the CAD system.
The input is made through a variety of input devices like keyboard, lightpen, digitizer, mouse etc.
The user input functions requires a specific set of instructions for a particular input devices used called device drivers

Construction of Geometry

Geometry construction refers to the process of obtaining the desired image on the graphic terminals.

Geometry construction is possible with the two basic operations. They are

1) Defining the graphic Element
2) Editing the geometry

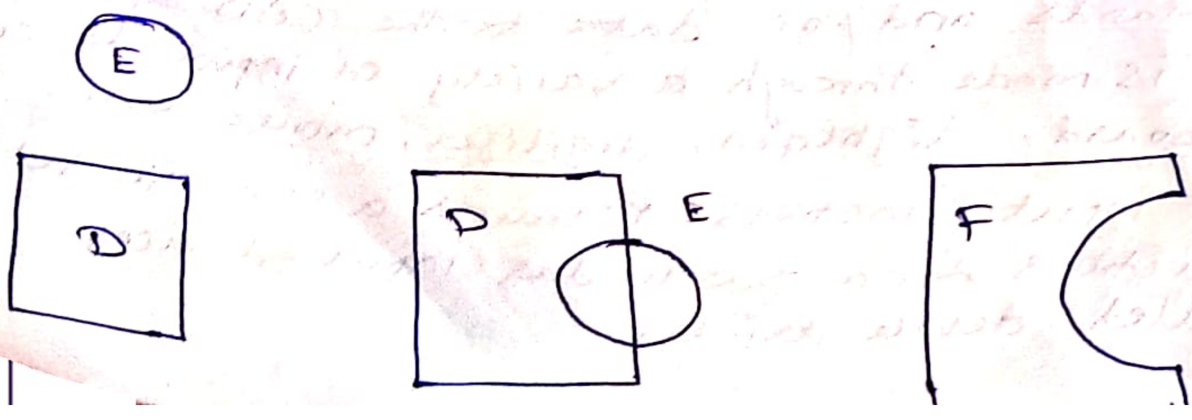
1) Defining the graphic Elements

A model is constructed in a CAD system using a number of graphic element available in the CAD system library. The designer call these Elements & arranges them together in a desired fashion, by adding / subtracting to construct the required images

Ex:

Adding

A+B=C



D-E=F

The graphic Element like points, lines, arcs etc are stored in the database in mathematical form & refered to a 3D coordinate system.

for Example

- A point can be defined by simply by its X, Y, Z Coordinates
- A polygon can be defined as an ordered set of points representing the corners of the polygon.
- Circle by its radius/diameter

mathematically, a circle can be defined in the X-Y plane by the Eqn

$$\boxed{(x-m)^2 + (y-n)^2 = r^2}$$

r = radius

(2) Editing the Geometry

Editing facilities available in a CAD system to modify a model & make corrections to the built geometry.

To modify the Elements, the Editing Commands like

- Delete
- Move
- Copy
- Rotate
- Mirror
- Remove
- Trim
- Duplicate
- Scale

etc

are used for transformation & display control functions.

Transformations

Transformation function is useful to modify the image on the graphic terminal & to reposition the item in the data base. Transformation are applied to graphic Element to help the designer in effectively Constructing the application model.

Transformation operation includes

1) scaling → increasing | decreasing the size
2) Translation → Reposition the image
3) Rotation → Reposition the image
4) Moving → Assembling one part into another

There are Two basic principles of Transformation

1. Denotation
2. Concatenation

Denotation :- A transformation is a single mathematical Entity

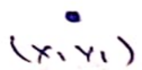
for Ex:- Translation, Rotation, Scaling etc

Concatenation :- Two transformation can be combined (or) concatenated to obtain a single transformation

Two Dimensional (2D) Transformation

Two dimension refers to a two axis Cartesian system, in which a point is located by specifying X & Y coordinates.. These can be represented in matrix form

$$P = \begin{bmatrix} X \\ Y \end{bmatrix}$$



Ill'y matrix representation can be used to specify line connecting b/w two coordinates (x_1, y_1) & (x_2, y_2)

$$L = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$$

The common transformation are

1. Translation
2. Scaling
3. Rotation

1. <u>Translation</u> :- Translation is the operation of simply moving an element from one location to another. The element size, & angle remain unchanged.

In CAD system, the command used for translation are Move, & Copy.

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A simple Translation operation is illustrated

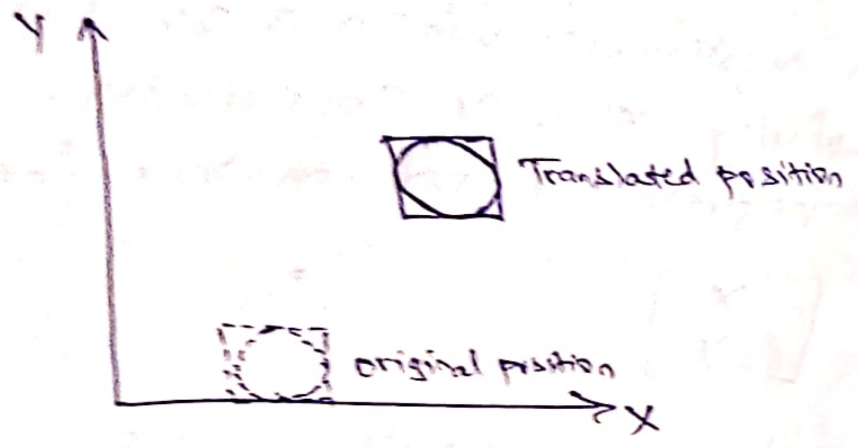


fig: Translation

a) Translation of a point

For a point translation, it is given by

$\boxed{x' = x + m}$ $\boxed{y' = y + n}$

where

x', y' = Coordinates of the translated points
$x\ y$ = Coordinates of the original point
m, n = movement in x & y direction

In a matrix form, translation can be represented as

$$\boxed{(x', y') = (x, y) + T} \rightarrow (1)$$

Where $T = (m, n)$ the translation matrix

Any geometric Element can be translated in space by using Eqn (1)

(b) Translation of a Line

For a line, translation matrix is applied to its End points

for Example. a line given by an end point (x_1, y_1) & (x_2, y_2). Can be represented by the matrix

$$L_1 = \begin{bmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{bmatrix}$$

If the line is incremented by 'm' units in the x direction & 'n' units in Y direction. Then the translated line is given by

$$\boxed{L_2 = L_1 + T}$$

where T = Translation matrix

L_1 = Original line

L_2 = Translated line

i.e $$L_2 = \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} + \begin{vmatrix} m & n \\ m & n \end{vmatrix}$$

(or)

$$\boxed{L_2 = \begin{vmatrix} X_1+m & Y_1+n \\ X_2+m & Y_2+n \end{vmatrix}}$$

1. Translation

Translation is the operation of simply moving an element from one location to another. The element size and angle remain unchanged. Most transformation operations are performed by a *Move* command in CAD system. *Copy* is another translation command where in the original element is retained and a copy of the original is created at the specified location. A simple translation operation is illustrated in Fig. 3-4.

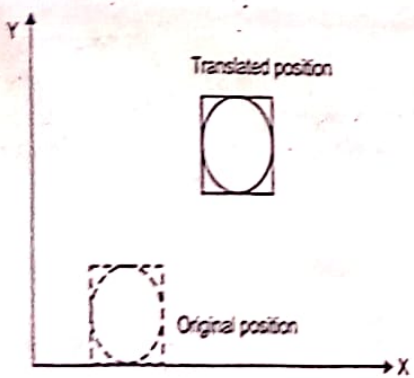


Fig. 3-4. Translation

a) Translation of a point:

For a point, translation is given by-

$$x' = x+m, \qquad y' = y+n \qquad \text{........(3-2)}$$

where,

x', y' = coordinates of the translated point i.e., new location points.

x, y = coordinates of the original point.

m, n = movements (increments/decrements) in the x and y directions, respectively.

In a matrix form, translation can be represented as-

$$(x', y') = (x, y) + T \qquad \text{........(3-3)}$$

where, $T = (m, n)$, the translation matrix ....(3-4)

Any geometric element (that can be represented in a matrix form) can be translated in space by using equation (3-3).

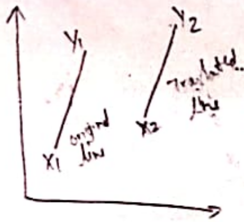
b) Translation of a line

For a line, translation matrix is applied to its end points. For example, a line given by an end points (x_1, y_1) and (x_2, y_2) can be represented by the matrix,

$$L_1 = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \qquad \text{........(3-5)}$$

If the line is incremented by m units in the x direction, then, the translated line is given by-

$$L_2 = L_1 + T \qquad \text{........(3-6)}$$



where, T = translation matrix,

L_1 = Original line

L_2 = Translated line

i.e., $L_2 = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} m & n \\ m & n \end{vmatrix}$

or $L2 = \begin{vmatrix} x_1+m & y_1+n \\ x_2+m & y_2+n \end{vmatrix}$ ....(3-6a)

Example 3-1: *A line is defined by its end points (1, 1) and (3, 4) in a 2-D graphics system. Express the line in matrix notation. Suppose, the line is to be translated in space by 3 units in the x-direction and 2 units in the y-direction, represent the final position of the translated line.*

Solution:

The line can be represented in a matrix form as (before translation)-

$$L_1 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \qquad \text{[Using equation (3-5)]}$$

The line is to be moved by 3 units in x-direction and 2 units in y-direction. Therefore the translation matrix is given by-

$$T = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$$

Now, using equation (3-6), the translated line can be represented as-

$$L_2 = L_1 + T$$

$$= \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$$

Using equation (3-6a),

$$L_2 = \begin{vmatrix} 4 & 3 \\ 6 & 6 \end{vmatrix}$$

Fig. 3-5 illustrates graphically the translation of the original line (1, 1) and (3, 4) to (4, 3) and (6, 6).

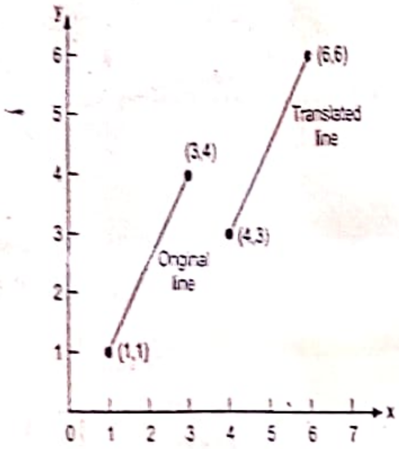


Fig. 3-5.

2. SCALING

Scaling is the operation of increasing or decreasing the size of an element. Scaling need not be equal in x and y direction. For example a circle could be transformed into an ellipse by unequal scaling in x-y direction, and similarly a square can be transformed into a rectangle. Unlike in transformation, here a scaling factor is used. For example a scaling factor of 2 would mean increase in size by 2 times; a scaling factor by 0.8 means reduction in size by 0.2 times.

Zooming is a kind of scaling operation in which only the display on the screen is modified in size, but there is no change in the model data base. Also zooming is performed equally in x and y directions. The scaling operation applied to an element brings in changes to the model data base. The points of an element can be scaled by the scaling matrix as follows:

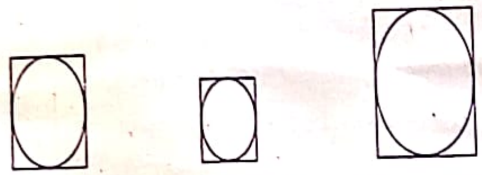
$$(x', y') = (x, y)\, S \qquad \text{......(3-7)}$$

where, S = the scaling matrix

$$= \begin{vmatrix} m & 0 \\ 0 & n \end{vmatrix} \qquad \text{......(3-7a)}$$

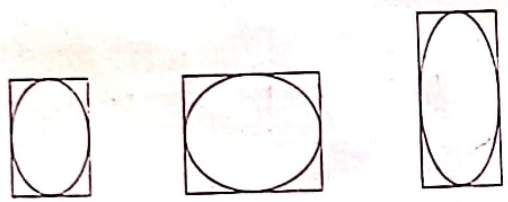
This will result in a change in the size of the element by a factor m in the x-direction and by a factor n in the y-direction. Scaling also brings in the effect of repositioning of the element on the x-y plane with respect to its origin. Obviously, if the scaling factors are less than unity, the element reduces in size, and moves closer to the origin. Similarly, with scaling factors more than unity, the elements get enlarged and move away from the origin.

The scaling factors in x- and y-directions (S_x and S_y) may be equal or different. When the scaling factors are same, then the picture simply reduces in size or increases in size depending upon the scaling factor. The effect of having equal scaling factor is illustrated in **Fig. 3-6**. When the scaling factors are different, they have the effect of distorting the pictures by elongating or shrinking them along the directions parallel to the coordinate axes. The effect of unequal scaling is shown in **Fig. 3-7**.



(a) Original *(b) Scaling factor < 1* *(c) Scaling factor > 1*

Fig. 3-6. Equal scaling factors



(a) Original *(b) $S_{yx} > S_y$* *(c) $S_{yx} < S_y$*

Fig. 3-7. Unequal scaling factors

Another use of scaling is to generate the mirror images of an object using negative values of scaling factors in x- and y-directions (S_x and S_y). **Fig. 3-8** illustrates the different mirror images obtained using three combinations of negative scaling factors of S_x and S_y.

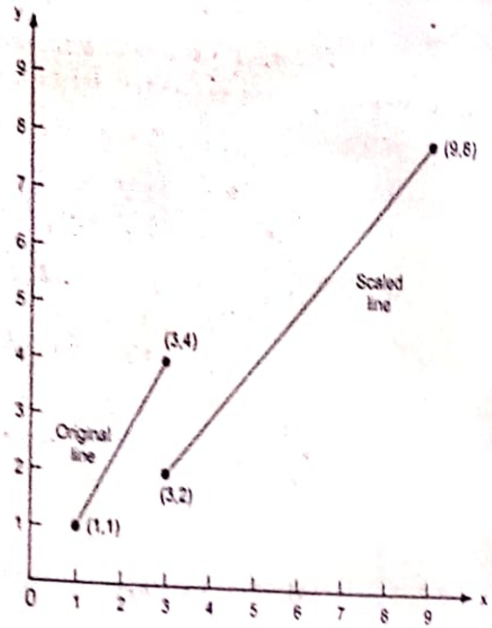


Fig. 3-10

Example 3-4: A line is defined by end points (1, 1) and (3, 4) in a 2-D graphics system. By scaling transformations obtain the different possible mirror images for the line. Use a scaling factor of 1.

Solution:

Mirror images of the original line can be obtained using a combination of negative scaling factors in the x- and y-directions.

Let the original line be,

$$L_1 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$$

Three combinations of scaling factors of S_x and S_y are possible, viz. (i) $-S_x$, $+S_y$ (ii) $-S_x$, $-S_y$ (iii) S_x, $-S_y$

The respective scaling matrices are-

$$S_2 = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$S_3 = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$S_4 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

The scaled and mirrored lines using the scaling transformations are as below:

$$L_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ -3 & 4 \end{vmatrix}$$

$$L_3 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ -3 & -4 \end{vmatrix}$$

$$L_4 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 3 & -4 \end{vmatrix}$$

The original line L_1, and the scaled/mirrored lines L_2, L_3, and L_4 are illustrated in Fig. 3-11.

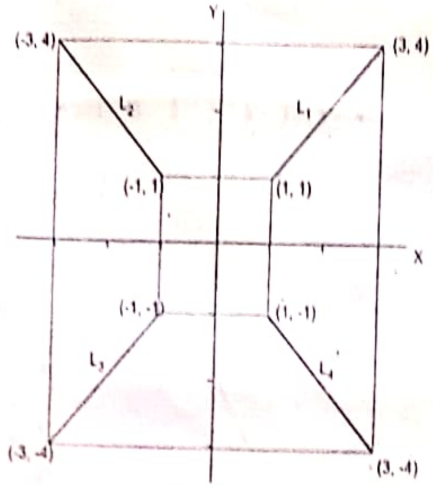


Fig. 3-11.

3 ROTATION

In this transformation operation, the points of an element are rotated about the origin by an angle. Similar to translation in rotation also there is no change in the size of the element except for change in position plus the inclination with respect to the origin. Most CAD systems have the ***rotate*** command where in the element can be rotated in a desired manner.

Rotation can be both for display as well as image modification purpose. In display only the image is rotated for the purpose of proper visualisation. With no change in the model data base, and rotation transformation occurs only on the graphics terminal. On the other hand model rotation to achieve the required shape brings in database change and the transformation occurs in the CAD system itself. In rotation function, for a positive angle, the transformation takes place in the counter clock wise (CCW) direction and for a negative angle, it takes place in the clockwise (CW) direction. Rotation is always accompanied both by object rotation plus object movement with respect to origin. However, it is also possible to rotate an element, say a line, about one of its ends, thus only changing its inclination, but retaining that point at the same distance from the origin as that of the original line.

The common rotation function can be represented in matrix rotation as below:

$$(x', y') = (x, y)\,R \qquad \text{.......(3-8)}$$

where, R = the rotation matrix

$$= \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \qquad \text{.......(3-8a)}$$

→ for CCW

$$= \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \qquad \text{.......(3-8b)}$$

→ for CW

Equation (3-8a) gives the transformation matrix for a counterclockwise (CCW) rotation by angle θ about the origin and equation (3-8b) gives the transformation matrix for a clockwise (CW) rotation by angle θ about the origin.

Example 3-5: A line given by points (1, 1) and (3, 4), is rotated by an angle 30° (CCW). Give the new line position and draw it as a graph.

Solution:

The line matrix is,

$$L_1 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$$

The rotation matrix is,

in Deg

$$R = \begin{vmatrix} \cos 30 & \sin 30 \\ -\sin 30 & \cos 30 \end{vmatrix} = \begin{vmatrix} 0.866 & 0.500 \\ -0.500 & 0.866 \end{vmatrix}$$

Therefore the new line position is-

$$L_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} 0.866 & 0.500 \\ -0.500 & 0.866 \end{vmatrix} = \begin{vmatrix} 0.366 & 1.366 \\ 0.598 & 4.964 \end{vmatrix}$$

The rotated line is given by points (0.366, 1.366) and (0.598, 4.964). This is drawn graphically as in **Fig. 3-12.**

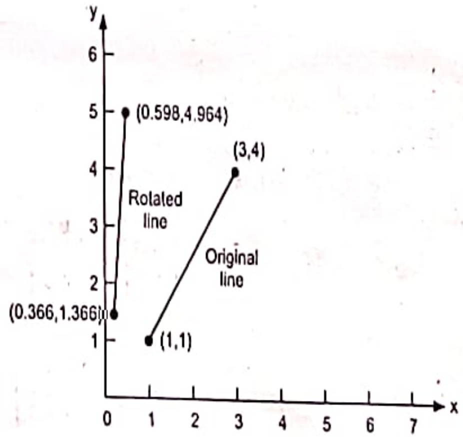


Fig. 3-12.

Example 3-6: ***A line is defined by its end points (3, 5) and (10, 12) in two-dimensional graphic system. Express the line in matrix notation and perform the following transformations:***

a) Translate the line by 6 units in x-direction and 4 units in y-direction.

b) Scale the original line by a factor of 3 in x-direction and 2.0 in the y-direction.

Solution:

The given line can be represented in matrix notation as below-

$$L_1 = \begin{vmatrix} 3 & 5 \\ 10 & 12 \end{vmatrix}$$

a) Translation

The translation matrix, T is given by, $T = \begin{vmatrix} 6 & 4 \\ 6 & 4 \end{vmatrix}$

The translated line is, $L_2 = \begin{vmatrix} 3 & 5 \\ 10 & 12 \end{vmatrix} + \begin{vmatrix} 6 & 4 \\ 6 & 4 \end{vmatrix} = \begin{vmatrix} 9 & 9 \\ 16 & 16 \end{vmatrix}$

b) Scaling

The scaling matrix, S is given by,

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$$S = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix}$$

The scaled line is given by-

$$L_3 = \begin{vmatrix} 3 & 5 \\ 10 & 12 \end{vmatrix} \cdot \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 9 & 10 \\ 30 & 24 \end{vmatrix}$$

Example 3-7: *A line is defined by its end points (0, 0) and (2, 3) in a two-dimensional graphic system. Express the line in matrix notation and perform the following transformation on this line-*

a) Scale the line by a factor of 3.0

b) Rotate the original line by 60° (CCW) about the origin.

Solution:

The given line can be represented in matrix notation as-

$$L_1 = \begin{vmatrix} 3 & 5 \\ 10 & 12 \end{vmatrix}$$

a) Scaling by a factor of 3.0

The scaling matrix is-

$$S = \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix}$$

Therefore, the scaled line is given by-

$$L_2 = \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} \cdot \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 6 & 9 \end{vmatrix}$$

b) Rotation by 60°

The rotation matrix is given by-

$$R = \begin{vmatrix} \cos 60 & \sin 60 \\ -\sin 60 & \cos 60 \end{vmatrix}$$

$$= \begin{vmatrix} 0.500 & 0.866 \\ -0.866 & 0.500 \end{vmatrix}$$

The rotated line is given by-

$$L_3 = \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} \cdot \begin{vmatrix} 0.500 & 0.866 \\ -0.866 & 0.500 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ -1.598 & 3.22 \end{vmatrix}$$

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Example 3-8: *A triangle is defined in a two-dimensional ICG system by its vertices (0,2) and (0,3) and (1,2). Perform the following transformations on this triangle.*

a) Translate triangle in space by 2 units in the x-direction and 5 units in the y-direction.

b) Scale the original triangle by a factor 1.5.

c) Scale the original triangle by a factor of 1.5 in the x-direction and 3.0 in the y-direction.

d) Rotate the original triangle by 45° (CCW) about the origin.

Solution:

a) Translation of the triangle

The triangle with vertices (0,2), (0,3) and (1,2) can be represented by three line matrices.

$$L_1 = \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}, \quad L_2 = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}, \quad \& \ L_3 = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix}$$

The translation matrix is given by,

$$T = \begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix}$$

The translated lines can be represented as,

$$L_{1T} = L_1 + T = \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 7 \\ 2 & 8 \end{vmatrix}$$

$$L_{2T} = L_2 + T = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 7 \\ 3 & 7 \end{vmatrix}$$

$$L_{3T} = L_3 + T = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 7 \\ 2 & 8 \end{vmatrix}$$

The vertices of the translated triangle are now lying at points (2,7), (2,8) and (3,7). The triangle before and after translation is shown in Fig. 3-13a.

b) Scaling the triangle by a factor of 1.5

The scaling matrix is,

$$S = \begin{vmatrix} 1.5 & 0 \\ 0 & 1.5 \end{vmatrix}$$

The scaled lines can be represented by,

$$L_{1S} = L_1 . S = \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 1.5 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 0 & 4.5 \end{vmatrix}$$

$$L_{2S} = L_2 . S = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 1.5 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 1.5 & 3 \end{vmatrix}$$

$$L_{3S} = L_3 . S = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 1.5 \end{vmatrix} = \begin{vmatrix} 1.5 & 3 \\ 0 & 4.5 \end{vmatrix}$$

The vertices of the scaled triangle by a factor of 1.5 are (0,3), (0.4.5) and (1.5, 3). The scaled triangle is shown in **Fig. 3-13b**.

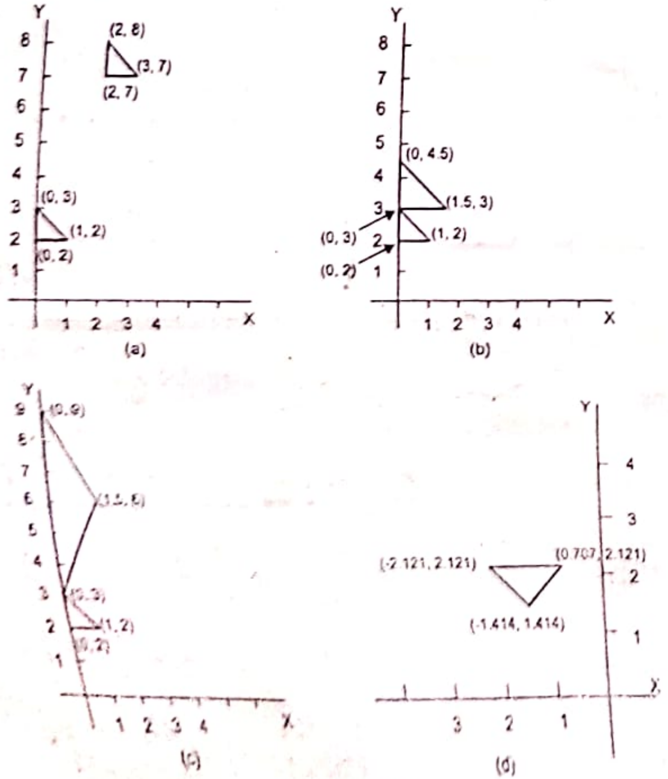

Fig. 3-13.

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c) Scaling by factors $S_x = 1.5$ & $S_y = 3$

The scaling matrix is, $S = \begin{vmatrix} 1.5 & 0 \\ 0 & 3.0 \end{vmatrix}$

The scaled lines can be represented by,

$$L_{1S} = L_1 . S = \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 3.0 \end{vmatrix} = \begin{vmatrix} 0 & 6 \\ 0 & 9 \end{vmatrix}$$

$$L_{2S} = L_2 . S = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 3.0 \end{vmatrix} = \begin{vmatrix} 0 & 6 \\ 1.5 & 6 \end{vmatrix}$$

$$L_{3S} = L_3 . S = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} . \begin{vmatrix} 1.5 & 0 \\ 0 & 3.0 \end{vmatrix} = \begin{vmatrix} 1.5 & 6 \\ 0 & 9 \end{vmatrix}$$

The vertices of the scaled triangle are (0,3), (0,9) and (1.5,6). The scaled triangle is shown in **Fig. 3-13c**.

d) Rotation by 45° about the origin

The rotation matrix is given by,

$$R = \begin{vmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{vmatrix} = \begin{vmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{vmatrix}$$

The rotated lines are represented by,

$$L_{1R} = L_1 . R$$

$$= \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} . \begin{vmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{vmatrix}$$

$$= \begin{vmatrix} 0 - 1.414 & 0 + 1.414 \\ 0 - 2.121 & 0 + 2.121 \end{vmatrix} = \begin{vmatrix} -1.414 & 1.414 \\ -2.121 & 2.121 \end{vmatrix}$$

$$L_{2R} = L_2 . R$$

$$= \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} . \begin{vmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{vmatrix}$$

$$= \begin{vmatrix} 0 - 1.414 & 0 + 1.414 \\ 0.707 - 1.414 & 0.707 + 1.414 \end{vmatrix} = \begin{vmatrix} -1.414 & 1.414 \\ -0.707 & 2.121 \end{vmatrix}$$

$$L_{3R} = L_3 . R$$

$$= \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} \cdot \begin{vmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{vmatrix}$$

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$$= \begin{vmatrix} 0.707 - 1.414 & 0.707 + 1.414 \\ 0 - 2.121 & 0 + 2.121 \end{vmatrix}$$

$$= \begin{vmatrix} -0.707 & 2.121 \\ -2.121 & 2.121 \end{vmatrix}$$

The vertices of the rotated triangle are given by the points (-1.414, 1.414), (-2.121, 2.121) and (-0.707, 2.121). The rotated triangle is shown in Fig. 3-13d.

3.4.2 Three-dimensional (3D) Transformations (or) Homogenous Transformation

Similar to 2D transformations, it is also possible to perform transformations in the three-dimensional space. 3-D transformations are essential in wire-frame, surface and solid modeling of objects. The 3D transformation is similar to 2D operation. With an additional axis, i.e., x, y and z directions. The common transformation functions explained under 2-D transformations are explained briefly here.

Translation

A point in a three-axis system can be located by specifying its x, y and z coordinates. These coordinates can be treated together as a 1x3 matrix, like (x, y, z). For example, the matrix (3, 2, 5) can be interpreted to be a point located at 3 units from the origin in the x-direction, 2 units from the origin in the y-direction and 5 units from the origin in the z-direction. This method of representation can also be extended to identify a line as a 2x3 matrix by giving the x, y and z coordinates of the two end points of the line. Thus, a line in 3-D can be represented by-

$$L = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

The translation matrix for a point in 3-D is given by-

$$T = (m, n, p) \quad \rightarrow (a) \qquad \text{.......(3-9a)}$$

where, m, n and p are the increments in x, y and z directions respectively. In matrix notation, this can be represented for a point as,

$$(x', y', z') = (x_1\, y_1\, z_1) + T \quad \rightarrow (b) \qquad \text{......(3-9b)}$$

Any geometric element in space can be translated by using equation (~~3-9b~~) (b) to each point that defines the element. For a line, the transformation matrix is applied to its two end points.

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Scaling

The scaling transformation in 3-D is given by-

$$S = \begin{vmatrix} m & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & p \end{vmatrix} \qquad \text{.......(3-10)}$$

For equal values of m, n and p, the scaling is linear. It is also possible to give different values for each of m, n and p, so that a different shapes can be obtained from a original image.

Rotation

Rotation in 3-D can be defined for each of the 3 axes x, y and z. Rotation about x axis by an angle is represented by the matrix,

$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix} \qquad \text{.......(3-11a)}$$

Similarly, rotation by angle , about y-axis is-

$$R_y = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix} \qquad \text{.......(3-11b)}$$

and rotation by an angle , about z-axis is-

$$R_z = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \text{.......(3-11c)}$$

3.4.3 CONCATENATION [Combined Transformation]

Individual transformation operations like translation, rotation, scaling, etc., can be combined as a sequence of transformation. This process is called *Concatenation*, and the combined transformations are called *Concatenated Transformations.*

While designing a CAD model, many times, editing becomes very complex, and laborious job, if the transformations are used individually. In such cases, the use of concatenated transformations reduces the designers job and time to a considerable extent. Multiple editing functions can be performed using a single concatenated transformation. However, it is not possible to combine all kinds of transformations. Some combination of concatenated transformations are:

a) *Rotation of an element about an arbitrary point in the element.*

b) *Magnifying the element but maintaining the location of one its points in the same location.*

In case (a) above, the sequence of transformation will be - translation to the origin, rotation about the origin, and then translation back to the original location.

In case (b) above, the element will be scaled (increased) followed by a translation to locate the desired point at its original position.

The main function of concatenation is to obtain a series of editing operations-/image manipulation in one transformation process. This helps to define the concatenated transformation more precisely, assisted with efficient computation.

The simplest way to obtain the sequence of concatenated transformation in one transformation is to express the transformation operations in a matrix form (as explained in 2-D and 3-D transformations). For example, if we have to scale a point by a factor of 3 in a 2-D system, and then rotate it by 60°, then concatenation is simply the product of two transformation matrices. The order of matrix multiplication should be the same as the order in which the transformation functions are desired. Concatenation of a series of transformations involving translation is a complex process to explain. However, the modern CAD system being more powerful, they are capable of performing all such operations.

Example 3-9: *Consider a point in 2-D graphics system given by (3, 2). Transform this line to scale by a factor of 3 and rotate by 45° (CCW) using-*

a) *Individual sequential transformation operation*

b) *Concatenated transformation.*

Solution:

a) Sequential Transformation.

Consider the scaling operation, and using equation [illegible],

$$(x', y') = (x, y)\,S$$

or

$$(x', y') = (3, 2)\begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix}$$

$$= (9+0,\ 0+6) = (9, 6)$$

Now perform the rotation operation, using equation (3-9),

$$(x'', y'') = (x', y')\,R$$

or

$$(x'', y'') = (9, 6)\begin{vmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{vmatrix}$$

$$= (9, 6)\begin{vmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{vmatrix}$$

$$= (2.1213,\ 10.6065) \rightarrow (A) \qquad \text{......(3-12a)}$$

b) Concatenated Transformation

In this both scaling and rotation functions are combined. This is performed by concatenating the two separate transformations into one matrix. The product of the two matrices is-

$$SR = \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} \cdot \begin{vmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{vmatrix} = \begin{vmatrix} 2.1213 & 2.1213 \\ -2.1213 & 2.1213 \end{vmatrix}$$

By using this concatenated transformation matrix, to the original point, we get,

$$(x'', y'') = (3, 2)\begin{vmatrix} 2.1213 & 2.1213 \\ -2.1213 & 2.1213 \end{vmatrix}$$

$x''y'' = (x, y)(S.R)$

$$= (2.1213,\ 10.6065) \rightarrow (B) \qquad \text{......(3-12b)}$$

Both results (3-12a) and (3-12b) are the same, which indicates that concatenation yields the same result as that of individual transformation operations, but the computation is much simpler and faster.

Example 3-10: *A line is defined as 2-D space by its end points (1, 2) and (6, 4). Express this in matrix notation and perform the transformations-*

- rotate the line by 90° (CCW) about the origin
- Scale the line by a factor of 0.5

a) *First by using sequential transformation.*

b) *Then by using concatenated transformation.*

c) *Show the sequence of transformation in sequential transformation.*

Solution:

a) Sequential Transformation

Rotation by 90°

The line in matrix form is-

$$L_1 = \begin{vmatrix} 1 & 2 \\ 6 & 4 \end{vmatrix}$$

The rotation matrix is-

$$R = \begin{vmatrix} \cos 90 & \sin 90 \\ -\sin 90 & \cos 90 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

The rotated line is-

$$L_2 = \begin{vmatrix} 1 & 2 \\ 6 & 4 \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ -4 & 6 \end{vmatrix}$$

Scaling by a factor of 0.5

The scaling matrix is-

$$S = \begin{vmatrix} 0.5 & 0 \\ 0 & 0.5 \end{vmatrix}$$

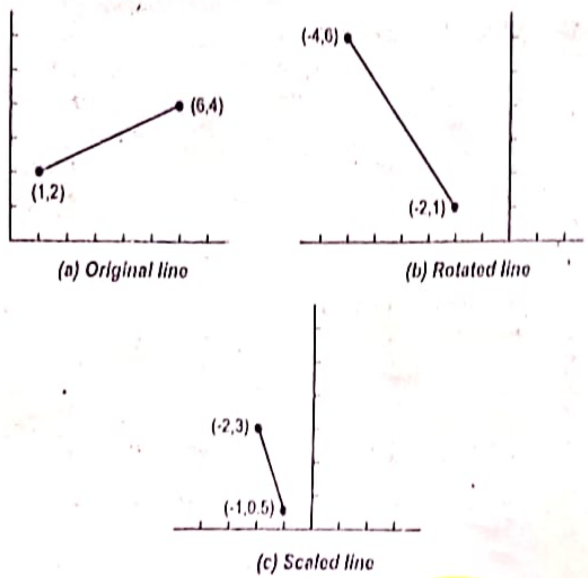

Fig. 3-14. Sequence of transformation

The scaled line is, $L_3 = L_2 . S$

$$\text{or} \quad L_3 = \begin{vmatrix} -2 & 1 \\ -4 & 6 \end{vmatrix} \cdot \begin{vmatrix} 0.5 & 0 \\ 0 & 0.5 \end{vmatrix} = \begin{vmatrix} -1 & 0.5 \\ -2 & 3.0 \end{vmatrix} \quad \text{......(3-12c)}$$

b) Concatenated Transformation

In this both rotation and scaling are combined. The concatenated function matrix is:

$$RS = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0.5 & 0 \\ 0 & 0.5 \end{vmatrix} = \begin{vmatrix} 0 & 0.5 \\ -0.5 & 0 \end{vmatrix}$$

By using this concatenated transformation matrix to the original line, we get:

$$L_2 = \begin{vmatrix} 1 & 2 \\ 6 & 4 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0.5 \\ -0.5 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0.5 \\ -2 & 3.0 \end{vmatrix} \quad \text{......(3-12d)}$$

Both the results (3-12c) and (3-12d) are the same, hence the concatenation result is correct.

c) Sequence of transformation

This is illustrated graphically in **Fig. 3-14.**

Advantages of Concatenated Transformations

1) A number of combined transformations can be computed with minimum number of arithmetic operations.
2) A concatenated transformation is more compact as compared to a sequential transformation.
3) They can be simplified by the use of matrices.

3.4.4 MATRIX REPRESENTATIONS

Two dimensional transformations can be easily represented in a uniform manner using a 3x3 matrix. Hence, the transformation of a point (x, y) to a new point (x', y') for any sequence of translations, rotations, and scalings can be represented as-

$$[x'\ y'\ 1] = [x\ y\ 1] \begin{bmatrix} a & d & 0 \\ b & e & 0 \\ c & f & 1 \end{bmatrix} \quad \text{......(3-13)}$$

The above 3x3 matrix completely specifies the required transformation. The matrix, a single entity represents the transformation.