Module-2 :- Open Channel Flow Hydraulics

1. Uniform Flow:-

An open channel flow is defined as a passage in which liquid flows with its upper surface exposed to atmosphere.

Classification of flow in channels :-

The flow in channels is classified into the following types.

1. Steady and unsteady flow.
2. Uniform flow and Non-uniform flow.
3. Laminar flow & turbulent flow.
4. Sub-critical, critical & super critical flow.

1. Steady flow and unsteady flow:-

The flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to time, the flow is said to be steady flow.

Steady flow is expressed as,

$$\frac{\partial V}{\partial t} = 0, \quad \frac{\partial Q}{\partial t} = 0 \quad (\text{or}) \quad \frac{\partial y}{\partial t} = 0$$

where, V = velocity, Q = rate of flow &

y = depth of flow

Unsteady flow:-

The type of flow characteristics such as depth of flow, velocity of flow & flow rate at any point in channel channges with respect to time, the flow is said to be unsteady flow.

Unsteady flow is expressed as,

$$\frac{\partial V}{\partial t} \neq 0, \quad \frac{\partial Q}{\partial t} \neq 0, \quad \frac{\partial y}{\partial t} \neq 0.$$

③ Uniform flow:-

Flow in a channel is said to be uniform if the velocity of flow, depth of flow, slope of the channel and C/s^n remain constant over a given length of the channel.

ie, $\frac{\partial y}{\partial s} = 0, \quad \frac{\partial v}{\partial s} = 0.$

Non-uniform flow:

Flow in a channel is said to be non-uniform if ~~when channel~~ the depth of flow, velocity of flow, slope of the channel & C/s^n do not remain constant over a given length of the channel

ie, $\frac{\partial y}{\partial s} \neq 0, \quad \frac{\partial v}{\partial s} \neq 0.$

Non-uniform flow may be further classified into.

(i) Rapidly varied flow (R.V.F) &

(ii) Gradually varied flow (G.V.F).

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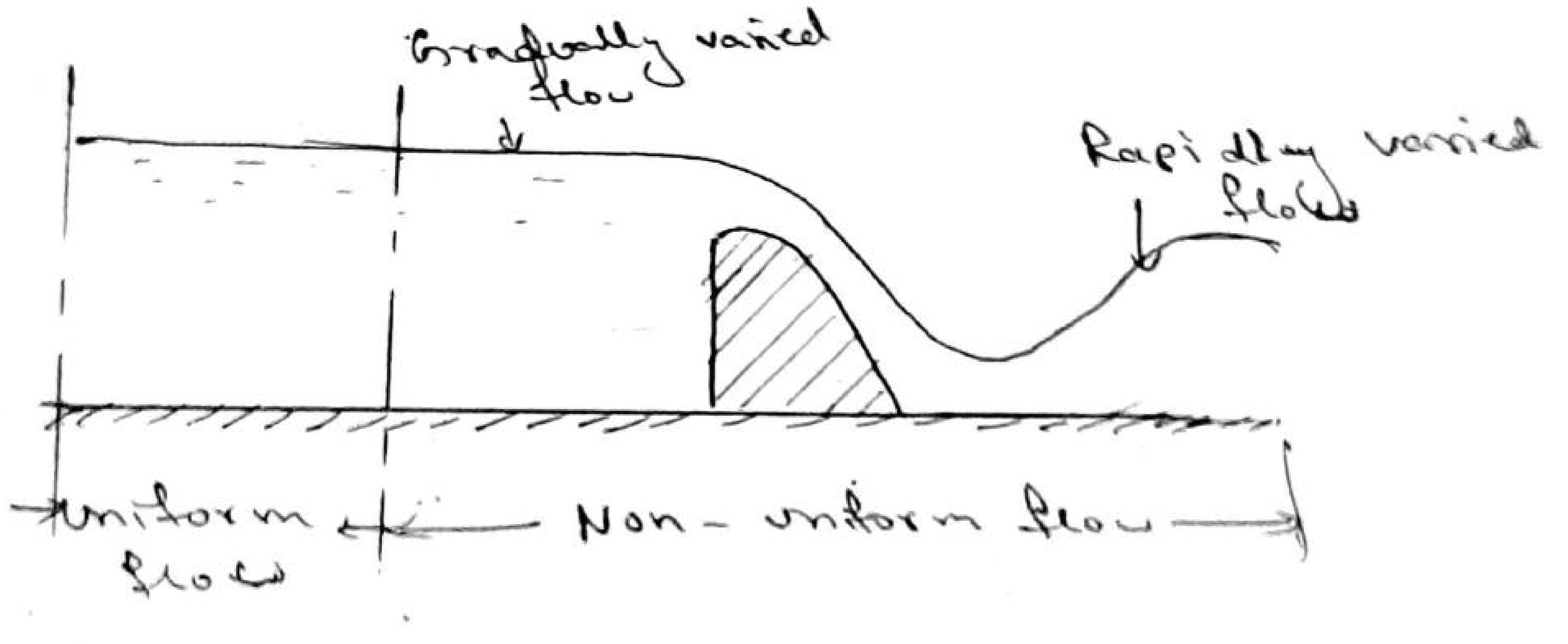
(i) Rapidly varied flow (R.V.F):-

· It is defined as that flow in which depth of flow changes abruptly over a small length of the channel.

Ex: Hydraulic Jump, Hydraulic drop

(ii) Gradually varied flow (G.V.F):-

If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow.



③ Laminar flow & Turbulent flow:-

The flow in open channel is said to be laminar if the Reynold number (R_e) is less than 500 or 600.

Reynold number in case of open channels is defined as: $R_e = \frac{\rho V R}{\mu}$

where, V = Mean velocity of flow of water.

R = Hydraulic radius (or) Hydraulic mean depth

$= \frac{\text{C/s}^n \text{ area of flow normal to the direction of flow}}{\text{Wetted perimeter,}}$

ρ & μ = Density & viscosity of water.

If the Reynold number is more than 2000, the flow is said to be turbulent in open channel flow. R_e lies b/w 500 to 2000, the flow is considered to be in transition state.

④ Sub-critical, critical & super-critical flow:-

The flow in open channel is said to be subcritical if the froude number (F_e) is less than 1.0. The froude number is defined as

$$F_e = \frac{V}{\sqrt{gD}}$$

where, V = Mean velocity of flow.

D = Hydraulic depth of channel

$= \frac{A}{T}$

T = Top width of channel.

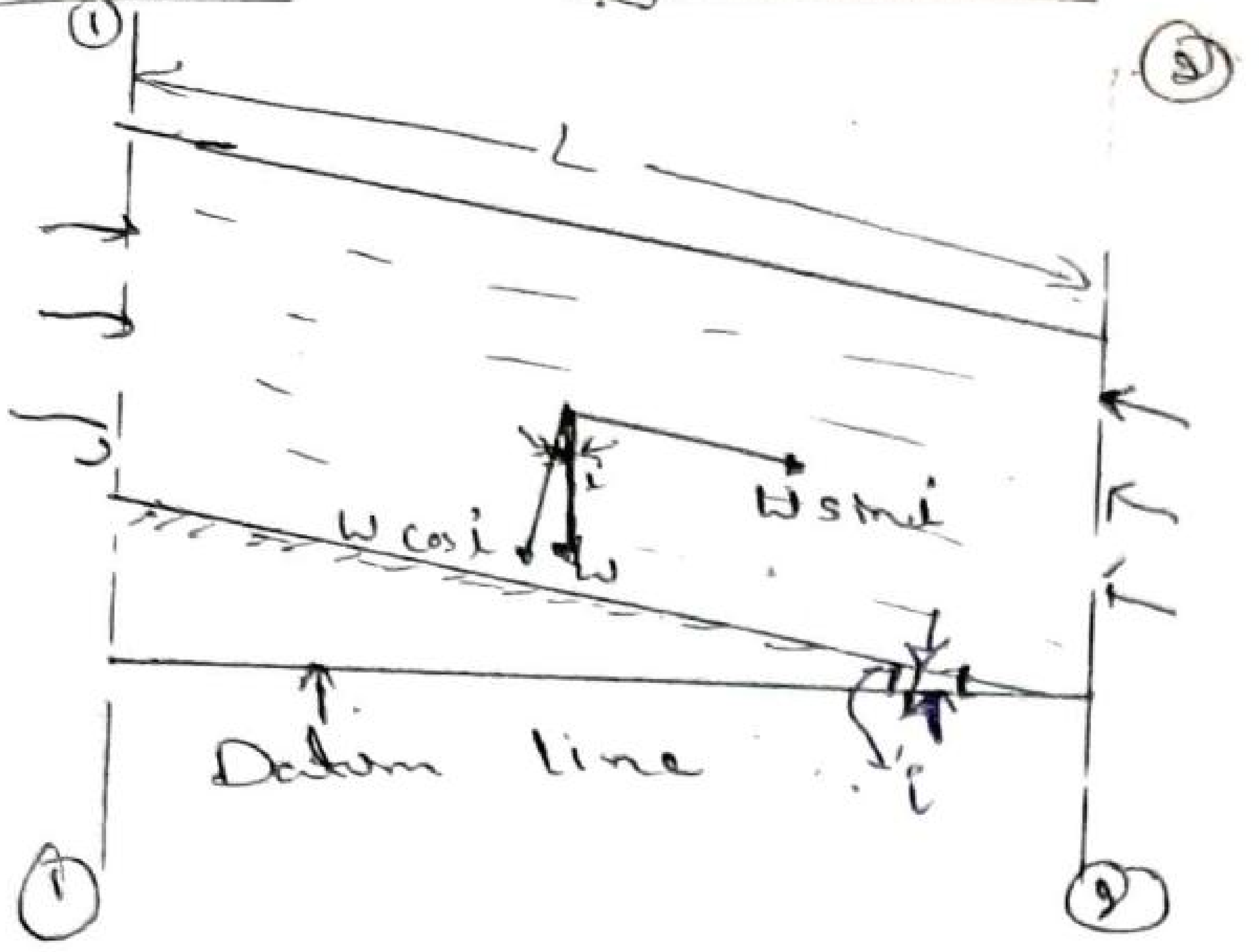
A = wetted area.

g = acceleration due to gravity.

Sub-critical flow is also called tranquil or streaming flow. For sub-critical flow, $F_e < 1.0$.

The flow is called critical if $F_e = 1.0$. & if $F_e > 1.0$, the flow is called super-critical flow or shooting or rapid or torrential.

Discharge through open channel by Chezy's Formula:



Consider a Steady and uniform flow of water in open channel as shown in fig. hence the depth of flow, velocity & area of flow will be const for a given length of channel.

Consider section 1 - 1 & 2 - 2.

Let, L = Length of the channel.

A = c/s^n area of flow of water

i = Slope of bed.

V = mean velocity.

P = wetted perimeter of the c/s^n

f = frictional resistence per unit velocity per unit area.

The weight of the water b/w the section 1 - 1 & 2-2

W = specific weight of water X volume of water

(vol) x density $W = w \times A \times L$

Components of the weight of water along the direction of ~~water~~ flow = W sin i

$= w \times A \times L \times \sin i$

Frictional resistance against motion of water =

f X surface area x (velocity)^2

The value of $n = 2$ which is found experimentally & surface area $= P \times L$.
wetted perimeter

$\therefore$ Frictional resistance against motion of water
$$= f \times P \times L \times (V)^2$$

The force acting on the water between section ①–① & ②–② are :-

1. Component of weight of water along the direction of flow.
2. Friction resistance against flow of water.
3. Pressure force at section ①–①
4. Pressure force at section ②–②.

The depth of water at section ①–① & ②–② are same, the pressure force at these two section are same but acting opposite in direction. Hence they cancel each other. In case of uniform flow the velocity of flow is constant for given length of the channel. Hence there is no acceleration acting on the water. Hence the resultant force acting in the direction of flow must be zero.

$\therefore$ Resolving all forces in the direction of flow, we get

$$w \times A \times L \times \sin i - f \times P \times L \times V^2 = 0$$

$$w A \cancel{L} \sin i = f P \cancel{L} V^2$$

$$\therefore V^2 = \frac{w A \sin i}{f P} = \frac{w}{f} \times \frac{A}{P} \times \sin i$$

$$V = \sqrt{\frac{w}{f} \times \frac{A}{P} \times \sin i} \qquad \left(\because m = A/P \right)$$

$$\& \; C = \sqrt{\frac{w}{f}}$$

where m = Hydraulic mean depth.
C = Chezy's constant

Substituting the values of $\frac{A}{P}$ & $\sqrt{\frac{w}{f}}$ in above

$\therefore V = C\sqrt{m \sin i}$

For small values of i, $(\sin i \approx \tan i \approx i)$

$\therefore \boxed{V = C\sqrt{mi}}$

$\therefore$ Discharge = Area x velocity

$$Q = A \times V$$

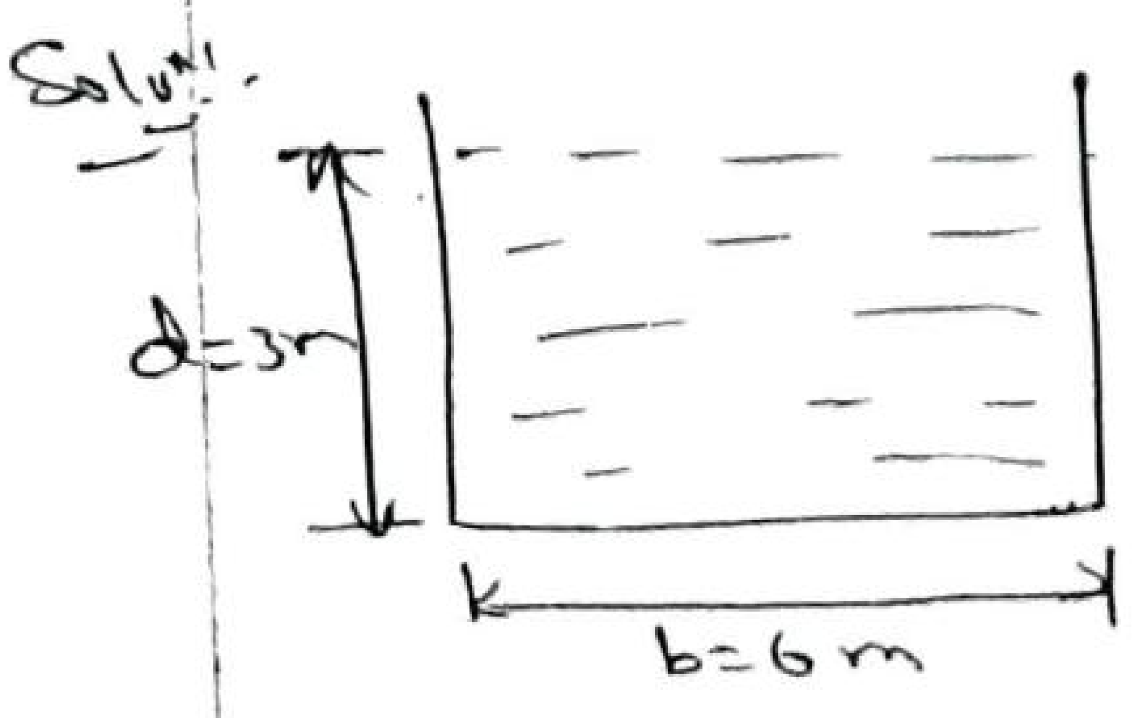
$$\boxed{Q = A\,C\sqrt{mi}}$$

$\therefore Q = K\sqrt{i}$, where, K = conveyance of Channel = $AC\sqrt{m}$

Problems

1) Find the velocity of flow and rate of flow of water through a rectangular channel ~~to~~ of 6 m wide and 3m deep. when it is running full. The channel is having bed slope as 1 in 2000, Take Chezy's constant C=55.

Solu^n^:



$V = ?$

$Q = ?$

$b = 6m$

$d = 3m$

$i = 1/2000$, $C = 55$

$\Rightarrow m = A/P \Rightarrow A = b \times d = 6 \times 3 = 18\ m^2$

$\Rightarrow P = b + 2d = 6 + (2 \times 3) = 12\ m$

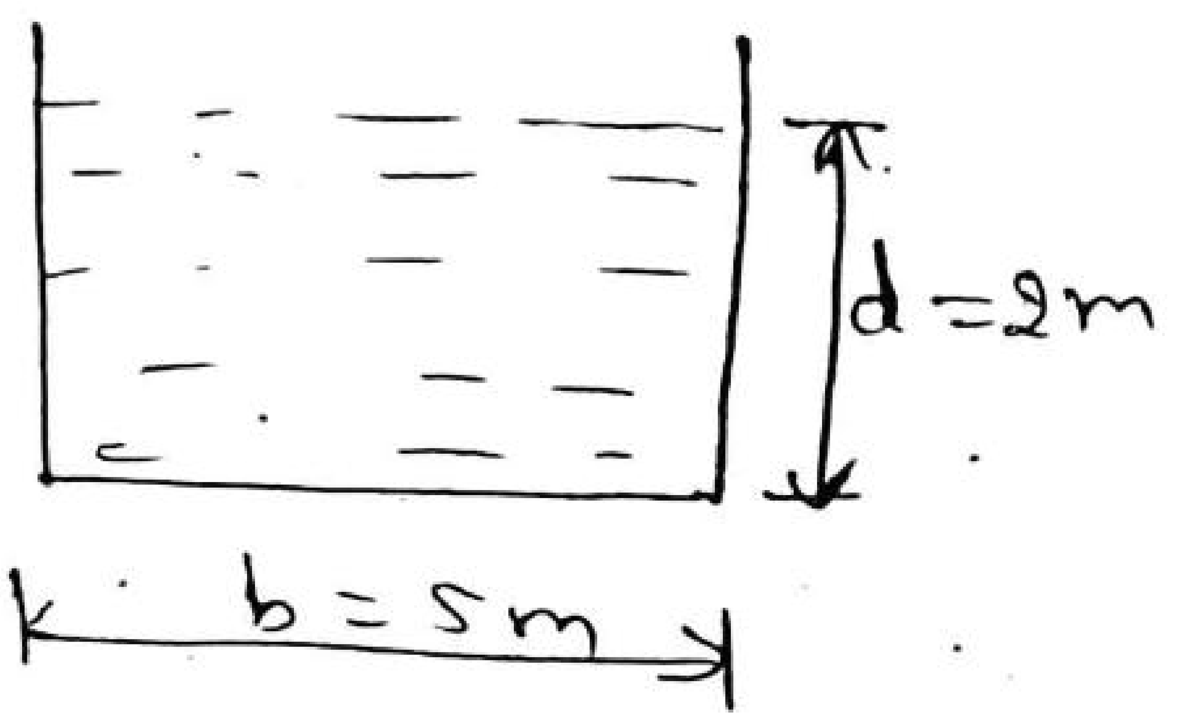
$$\therefore m = \frac{A}{P} = \frac{18}{12} = 1.5\ m$$

$$\Rightarrow V = C\sqrt{mi} = 55\sqrt{1.5 \times 1/2000} = 1.5062\ m/sec$$

$$\Rightarrow Q = A \times V = 18 \times 1.5062 = 27.11\ m^3/sec$$

2) Find the slope of the bed of a rectangular cha (i) of width 5 m when depth of water is and rate of flow is given as $20\ m^3/sec$ take $C = 50$?

Solun:



$Q = 20\ m^3/sec$

$C = 50$

$i = ?$

$b = 5m,\ d = 2m$

$$\Rightarrow m = A/P$$

$$\Rightarrow A = bd = 5 \times 2 = 10\ m^2$$

$$\Rightarrow P = b + 2d = 5 + (2 \times 2) = 9m$$

$$\Rightarrow m = \frac{A}{P} = \frac{10}{9} = 1.11m$$

$$\Rightarrow V = C\sqrt{mi} \quad \& \quad Q = AV.$$

$$\therefore Q = AV$$

$$20 = 10 \times V$$

$$\therefore \boxed{V = 2\ m/sec}$$

$$\therefore V = C\sqrt{mi}$$

$$2 = 50\sqrt{1.11} \times \sqrt{i}$$

$$\sqrt{i} = 0.0379$$

$$\therefore i = 1.441 \times 10^{-3} = 1 \text{ in } 694$$

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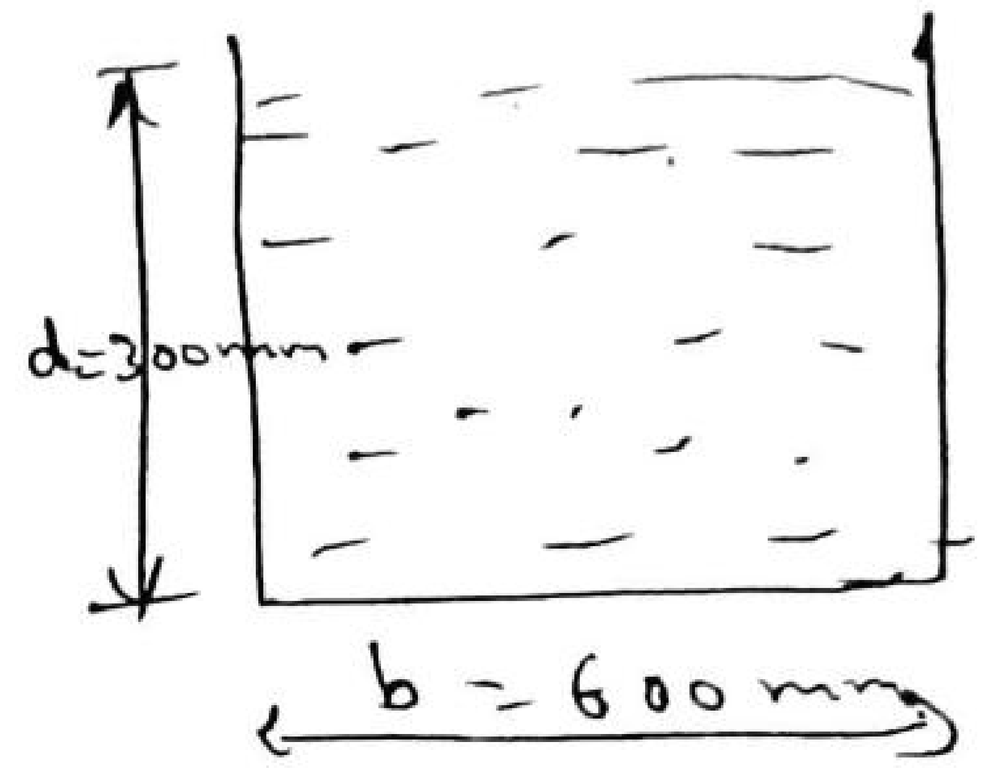
(b) A flow of water 100 litres/sec flows in a rectangular flume of width 600mm & having adjustable bottom flow if Chezy's constant is equal to 56. Find the bottom slope necessary for uniform flow with a depth of flow of 300mm also find the conveyance 'K' of the flume.

Solution: $Q = 100$ litres/sec $= 0.1\ m^3/sec$

$b = \frac{600}{1000} = 0.6\,m$

$C = 56$

$d = \frac{300}{1000} = 0.3\,m$



$\Rightarrow A = bd = 0.6 \times 0.3 = 0.18\,m^2$

$\Rightarrow P = b + 2d = 0.6 + (2 \times 0.3) = 1.2\,m$

$\Rightarrow m = A/P = \frac{0.18}{1.2} = 0.15\,m$

$\Rightarrow Q = AC\sqrt{mi}$

$0.1 = 0.18 \times 56 \sqrt{0.15 \times i}$

$\sqrt{i} = 0.0256$

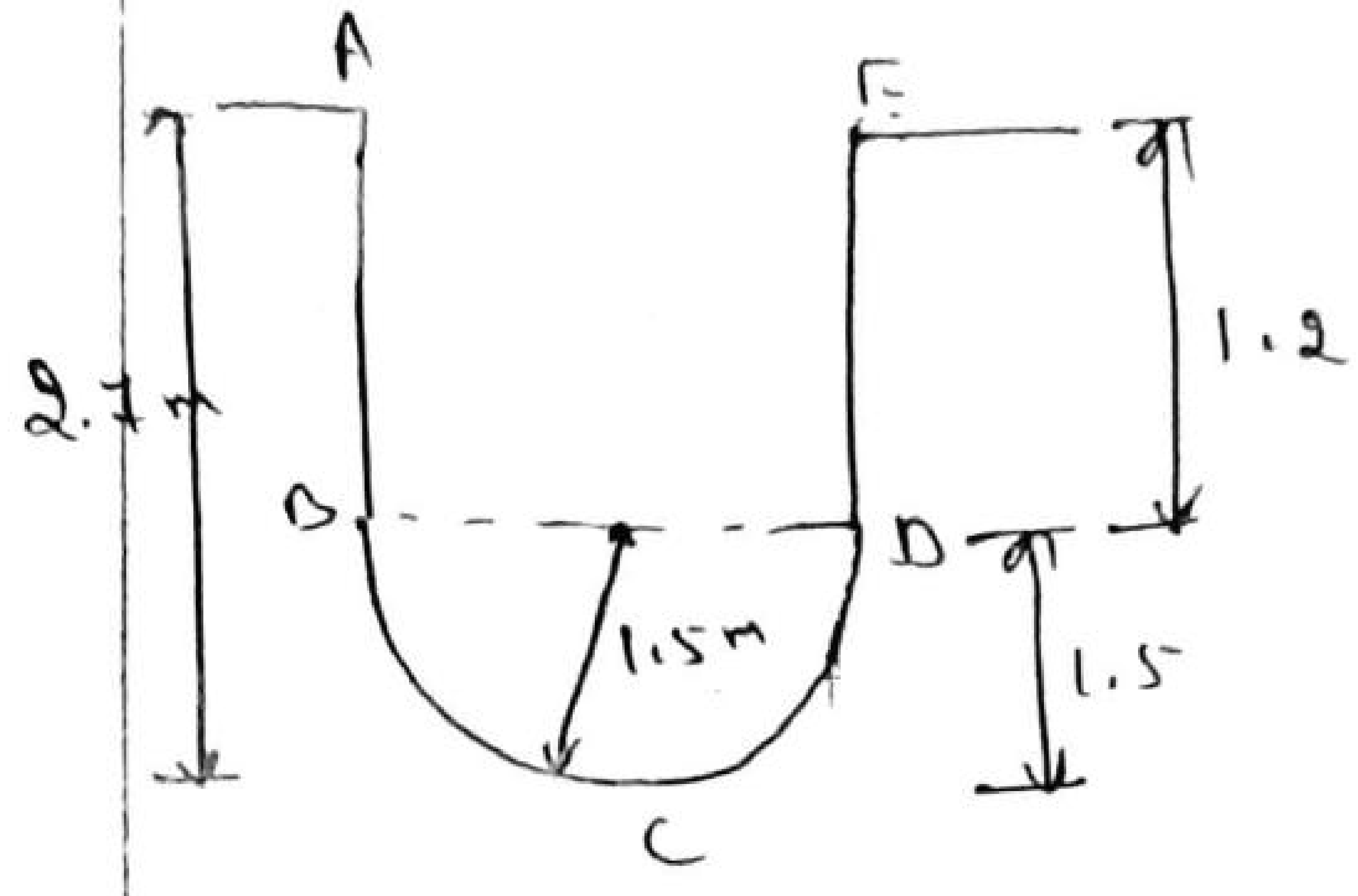
$i = 6.561 \times 10^{-4} = 1$ in 1524

$\Rightarrow K = AC\sqrt{m}$

$K = 0.18 \times 56 \times \sqrt{0.15}$

$\boxed{K = 3.9039\ m^3/sec}$

4) Find the discharge of water through the channel shown in fig. take C = 60 & slope of bed has 1 in 2000.



$C = 60$

$i = 1/2000$

Soln. ⇒ A = Area of ABDE + Area of BCD

$= (1.2 \times 3) + \left(\frac{\pi r^2}{2}\right)$

$= 3.6 + \frac{\pi \times (1.5)^2}{2}$

$\boxed{A = 7.134\ m^2}$

⇒ $P = AB + BCD + DE$

$= 1.2 + (\pi r) + 1.2$

$= 1.2 \times (\pi \times 1.5) + 1.2$

$\boxed{P = 7.1123\ m}$

⇒ $m = A/P = \frac{7.134}{7.1123} = 1.003\ m$

⇒ $Q = AC\sqrt{mi}$

$Q = 7.134 \times 60 \times \sqrt{1.003 \times 1/2000}$

$\boxed{Q = 9.5856\ m^3/sec}$

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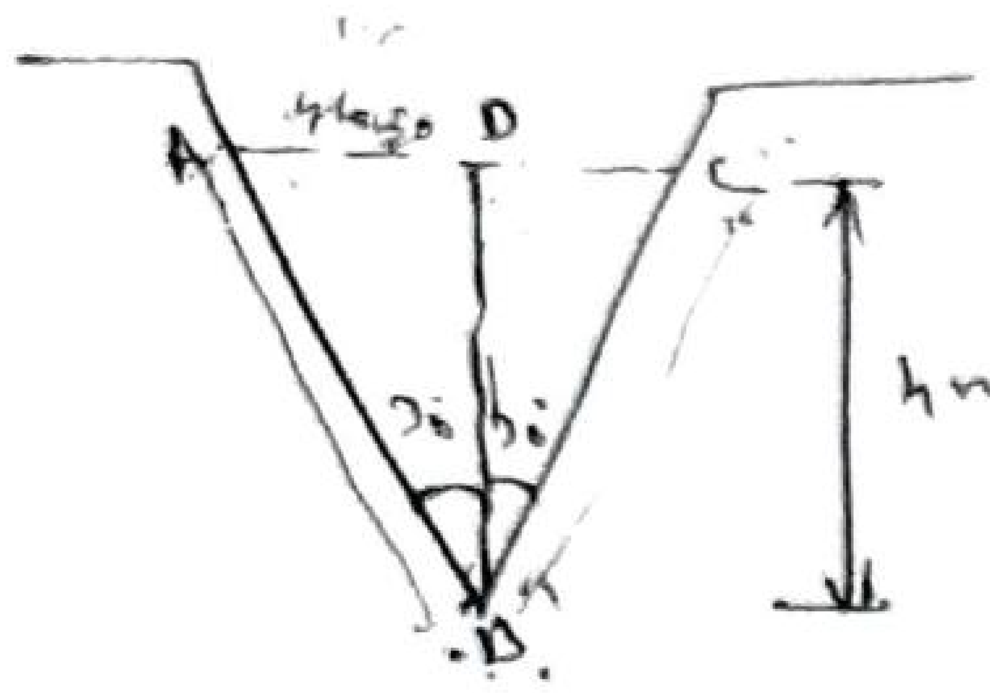
5) Find the rate of flow of water through a V - shaped channel shown in fig. take value of C = 55 and slope of the bed is 1 in 1000

Soln:-



$C = 55$

$i = 1/1000$

$\therefore$ Δ^{le} ABD Considers

$\tan 30° = \frac{AD}{BD}$ $\frac{opp}{adj}$

$\therefore AD = BD \tan 30° = \boxed{4 \tan 30°} = CD$

$\therefore AC = 2AD = 2DC$

$AC = 2 \times 4 \tan 30° = \boxed{8 \tan 30°} =$

$\therefore A = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 2(4 \tan 30°) \times 4$

$$\boxed{A = 9.2376 \ m^2}$$

$\therefore AB = \sqrt{BD^2 + AD^2}$

$AB = \sqrt{(4)^2 + (4 \tan 30)^2}$

$AB = 4.618 \ m = BC$

$\therefore P = AB + BC$

$P = 4.618 + 4.618 = 9.2376 \ m$

$\Rightarrow m = A/P = \frac{9.2376}{9.2376} = 1m$

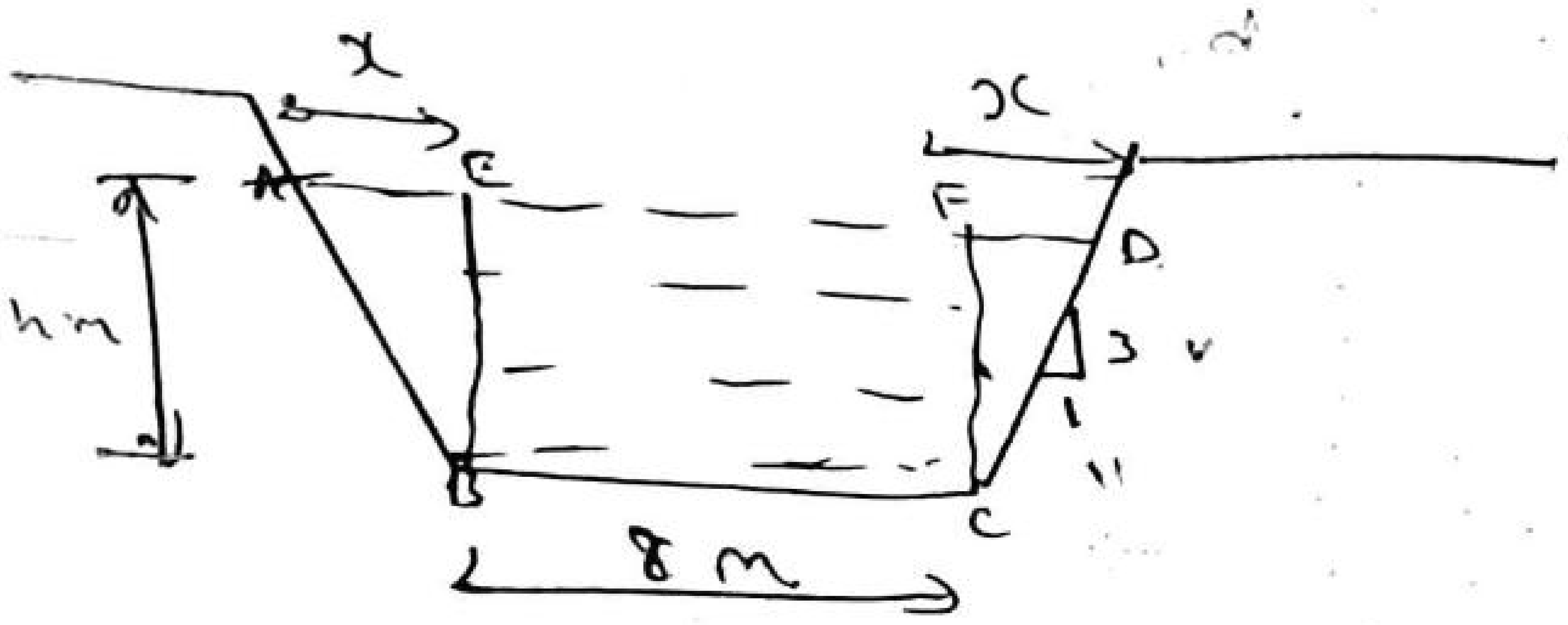
$\Rightarrow Q = AC\sqrt{mi}$

$= 9.2376 \times 55 \times \sqrt{1 \times 1/1000}$

$$\boxed{Q = 16.066 \ m^3/sec}$$

6) Find the discharge through a trapezoidal channel of width 8m & side slope of 1 horizontal to 3 verti the depth of flow of water is 2.4m & value of chezy's constant C = 50. the bed slope of the channel is given 1 in 4000.

Solun:



$b = 8m$
$d = 2.4m$
$S = 1H : 3V$
$i = 1/4000$
$c = 50$

$nd = d \times \frac{H}{V}$

$\therefore AE = FD = x = 2.4 \times \frac{1}{3} = 0.8m$

$\Rightarrow \therefore A = (bd) + \cancel{2}(\frac{1}{\cancel{2}} \times x \times BE)$

$A = (8 \times 2.4) + (0.8 \times 2.4)$

$$\boxed{A = 21.12 \ m^2}$$

$\Rightarrow P = AB + BC + CD$

$\therefore AB = \sqrt{(BE)^2 + (x)^2} = \sqrt{(2.4)^2 + (0.8)^2} = 2.53 \ m = CD$

$\Rightarrow P = 2.53 + 8 + 2.53 = 13.06 \ m$

$\Rightarrow m = A/P = \frac{21.12}{13.06} = 1.617 \ m$

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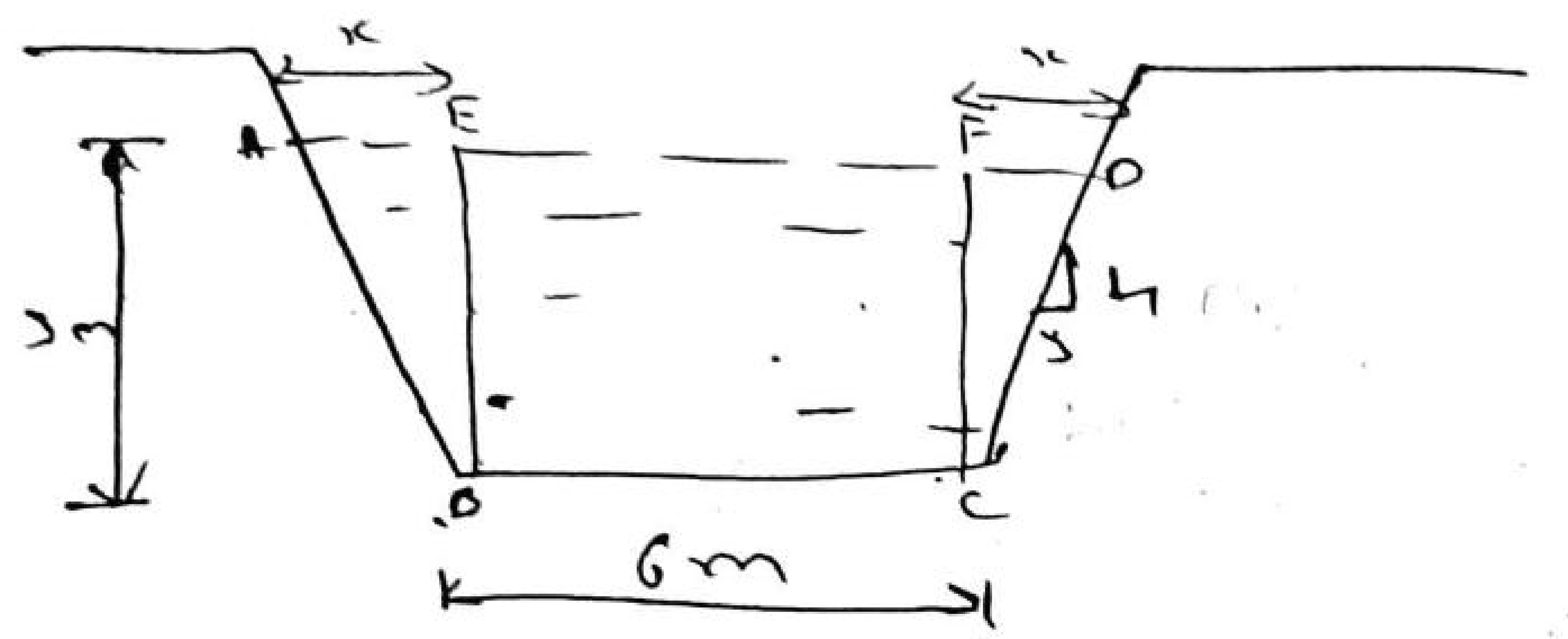
$\Rightarrow Q = AC\sqrt{mi}$

$Q = 21.12 \times 50 \times \sqrt{1.667 \times 1/1000}$

$\boxed{Q = 21.28 \ m^3/sec}$

7) Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical when the discharge through channel is 30 m^3/sec. Chezy's constant C=70

Soln:-



$b = 6m$

$d = 3m$

$s = 3H : 4V$

$i = ?$

$c = 70$

$Q = 30 \ m^3/sec$

$\therefore AE = FD = x = 3 \times \frac{3}{4} = 2.25 \ m$

$\Rightarrow A = (b \times d) + (2 \times \frac{1}{2} \times AE \times BE)$

$A = (6 \times 3) + (2.25 \times 3)$

$\boxed{A = 24.75 \ m^2}$

$\Rightarrow P = AB + BC + CD$

$\therefore AB = \sqrt{(AE)^2 + (BE)^2} = \sqrt{(2.25)^2 + (3)^2}$

$AB = 3.75 \ m = CD$

$\Rightarrow P = 3.75 + 6 + 3.75 = 13.5 \ m$

$\Rightarrow m = \frac{A}{P} = \frac{24.75}{13.5} = 1.833 \ m$

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$\Rightarrow \quad Q = AC\sqrt{mi}$

$30 = 24.75 \times 70 \times \sqrt{1.833 \times i}$

$\sqrt{i} = 0.0127$

$i = 1.635 \times 10^{-4} = 1 \text{ in } \underline{6113}$

Empirical formulae for the value of chezy's constant

(1) Ganguillet - kutter formula:-

The value of 'C' is given in MKS unit as

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}$$

where,

N = Roughness co-efficient (or) kutter's constant.

i = Slope of the bed

m = Hydraulic mean depth.

(2) Manning's Formula:- The value of 'C' A/c to the formula is given as.

$$C = \frac{1}{N} m^{1/6}$$

where, m = Hydraulic mean depth.

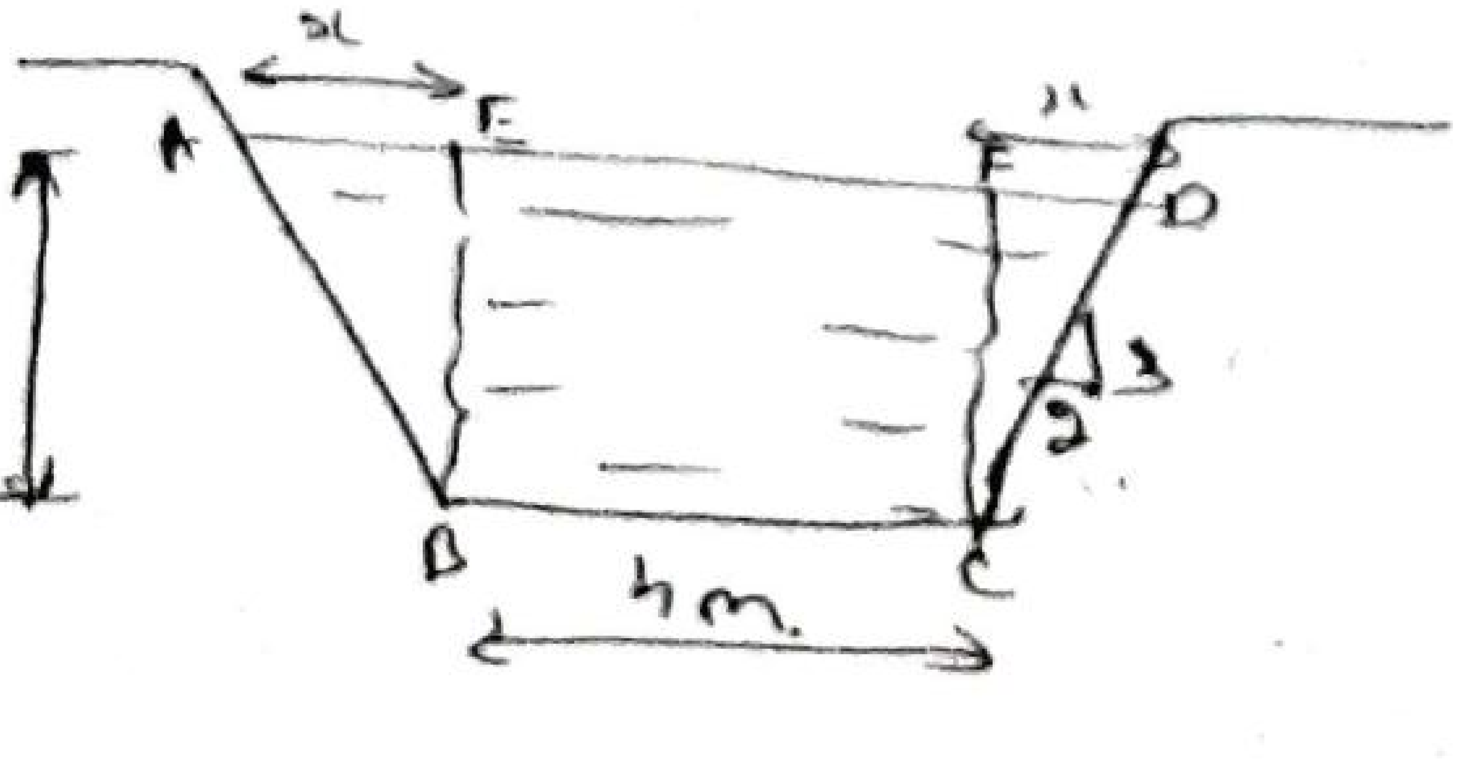
N = manning's constant.

Sl No	Nature of channel inside surface	Value of N
1)	very smooth surface of glass plastic (or) brass	0.010
2)	Smooth surface of concrete	0.012
3)	Rubble masonry (or) poor brick work	0.017
4)	Earthen channels neatly excavated	0.018
5)	Earthen channels of ordinary surface	0.027
6)	Earthen channels of rough surface	0.030
7)	Natural streams, clean & straight	0.030
8)	Natural streams, with weeds duppools etc	0.075 to 0.15

Problems

① Find the bed slope of trapezoidal channel of bed width 4m, depth of water 3m and side slope of 2 horizontal & 3 vertical when the discharge through the channel is 20 m³/sec. Take manning's N = 0.03 in manning's formula $C = \frac{1}{N} m^{1/6}$.

Sol:-



$Q = 20 \ m^3/sec$

$b = 4m$

$d = 3m$

$N = 0.03$

$S = 2H : 3V$

$AE = FD = x = 3 \times \frac{2}{3} = 2\,m \qquad \left\{ \because x = nd, \; n = \frac{H}{V} \right\}$

$\Rightarrow A = (bd) + (2 \times \frac{1}{2} \times x \times d)$

$A = (4 \times 3) + (2 \times 3) = 18\,m^2$

$\Rightarrow P = AB + BC + CD$

$AB = CD = \sqrt{(x)^2 + (BE)^2} = \sqrt{(2)^2 + (3)^2}$

$AB = CD = 3.605\,m$

$\therefore P = 3.605 + 4 + 3.605$

$\boxed{P = 11.21\,m}$

$\Rightarrow m = A/P = \frac{18}{11.21} = 1.605\,m$

$\Rightarrow C = \frac{1}{N} m^{1/6} = \frac{1}{0.03} \times (1.605)^{1/6} = 36.07$

$\Rightarrow Q = AC\sqrt{mi}$

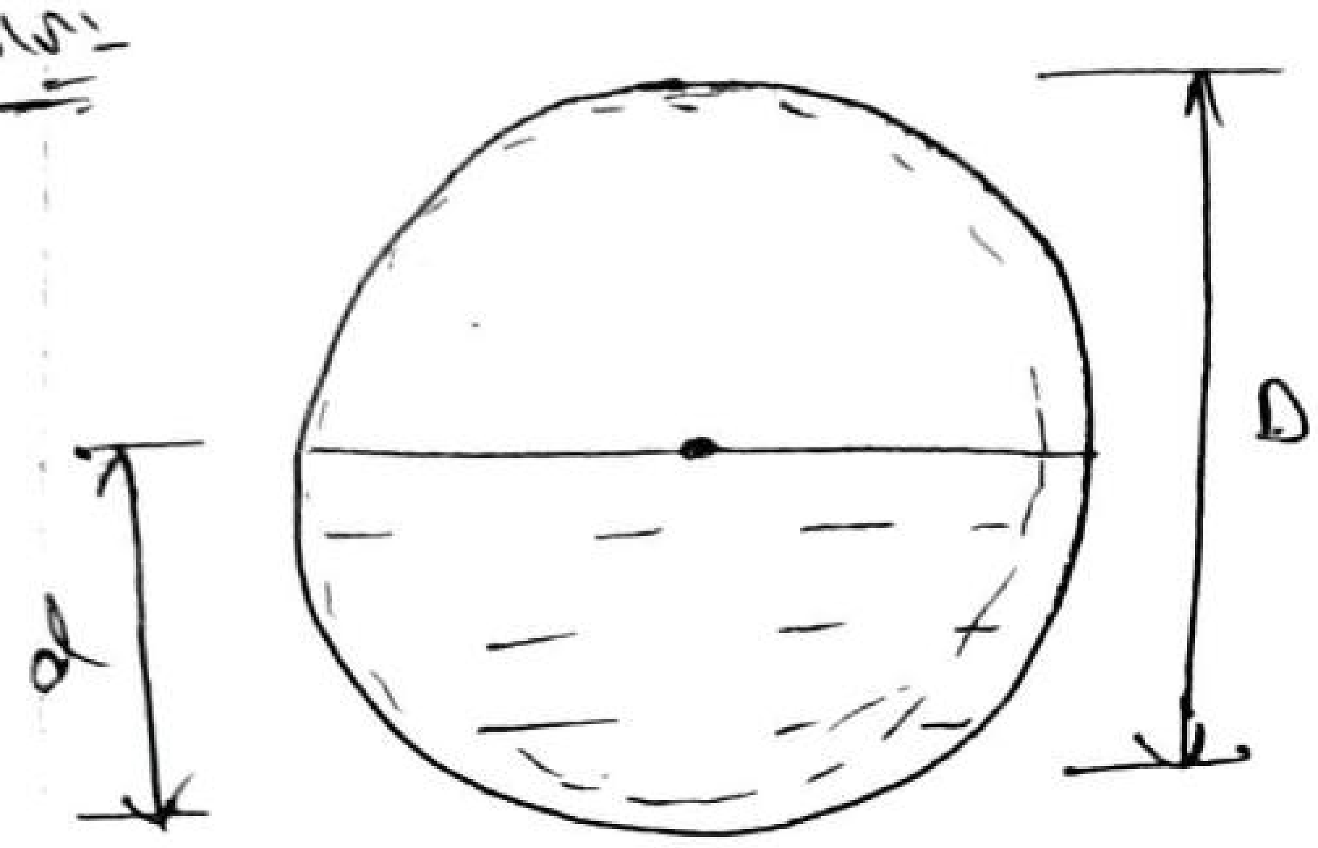
$20 = 18 \times 36.07 \times \sqrt{1.605 \times i}$

$\sqrt{i} = 0.0243$

$i = 5.912 \times 10^{-4} = 1 \text{ in } 1691$

2) Find the diameter of circular sewer pipe which is laid at a slope of 1 in 8000. & carries a discharge of 800 litres/sec when flowing half full. Take the value of manning's $N = 0.020$.

Soln:



$Q = 800 \text{ litre/sec} = 0.8 \text{ m}^3/\text{sec}$

$i = 1/8000$

$N = 0.020.$

$\Rightarrow$ Depth of flow (d) = $D/2$

$\Rightarrow$ Area of flow (A) = $\frac{\pi}{4} D^2 \times \frac{1}{2} = \frac{\pi D^2}{8}$

$\Rightarrow$ perimeter (P) = $\frac{\pi D}{2}$

$\Rightarrow$ Hydraulic mean depth (m) = $A/P = \dfrac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = D/4$

$\Rightarrow$ Using manning's formula. $(C) = \frac{1}{N} m^{1/6}$

$$C = \frac{1}{0.020} \times (D/4)^{1/6}$$

$\Rightarrow$ $Q = AC\sqrt{mi}$

$$0.8 = \frac{\pi D^2}{8} \times \frac{1}{0.020} \times \left(\frac{D}{4}\right)^{1/6} \times \sqrt{D/4 \times 1/8000}$$

$$9.182 = D^2 \cdot D^{1/6} \cdot D^{1/2}$$

$$D^{8/3} = 9.182$$

$\therefore D = (9.182)^{3/8}$

$\boxed{D = 2.296 \text{ m}}$

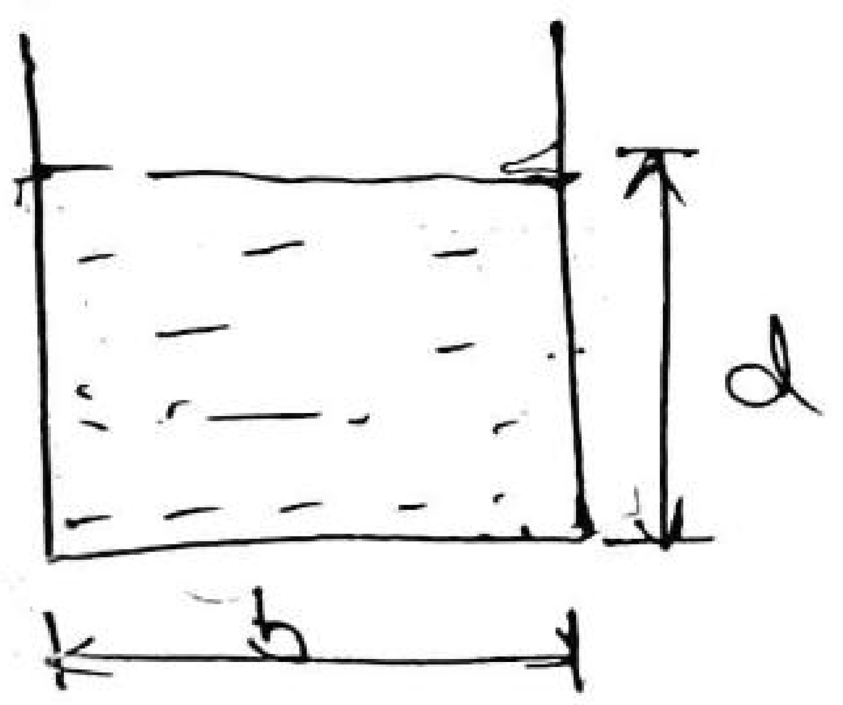
Ans

Most Economical Section of Channels :-

The most Economical section is one which gives the max discharge for a given amount of Excavation. From continuity eqn it is evident that discharge is maximum when velocity is maximum if cross section of channel is constant.

From Chezy's formula and manning's formula it can be see that for given value of slope & surface roughness. The velocity of flow will be maximum if hydraulic mean depth (or) hydraulic radius (m) is maximum. But $m = A/p$ therefore hydraulic mean depth is maximum. if wetted perimeter (P) is minimum. This condition is used to determine the dimensions of Economical Sections.

Most Economical rectangular channel Section :-



Consider a rectangular channel as shown in fig.

Let :, b = width of channel
d = depth of flow.
A = Area of flow.
P = wetted perimeter.

(i) $A = bd$

$b = \frac{A}{d} \longrightarrow (1)$

$P = b + 2d$

substitute eqn (1) in P

$\therefore P = \frac{A}{d} + 2d$

For most Economical section the wetted perimeter should be minimum. i.e., $\frac{d(P)}{d(d)} = 0$

$A\left(-\frac{1}{d^2}\right) + 2 = 0$

$\frac{A}{d^2} = 2$

$\therefore \boxed{A = 2d^2}$

$\therefore bd = 2d^2$

$\therefore \boxed{b = 2d}$

$\therefore$ Hydraulic mean depth (m)

$$m = A/P = \frac{bd}{b+2d} =$$

$$= \frac{2d \times d}{2d+2d} \quad (\because b = 2d)$$

$$= \frac{2d^2}{4d^2} = \frac{d}{2}$$

$$\therefore \boxed{m = d/2}$$

$\therefore$ The rectangular channel will be most Economical when.

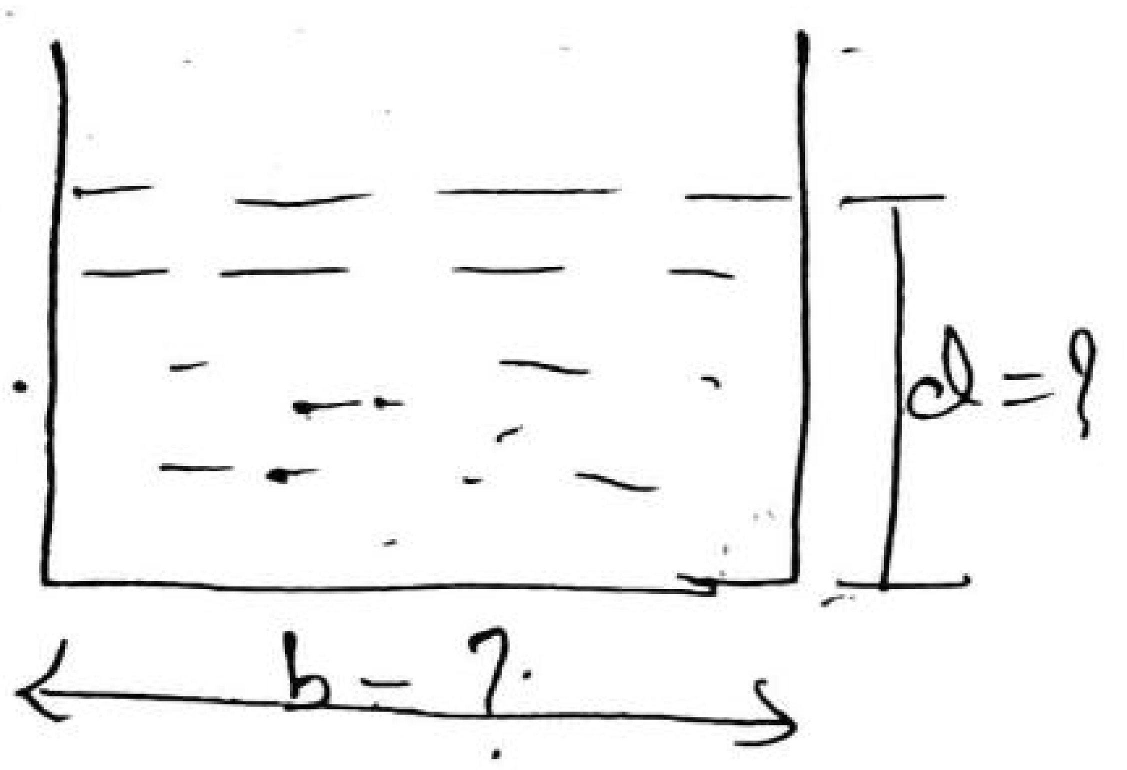
(i) Width of the channel is twice the depth of flow i.e, $\boxed{b = 2d}$

(ii) The hydraulic mean depth is half of the depth of flow i.e $\boxed{m = d/2}$

Problems

1) A rectangular channel carries water at the rate of 400 litre/sec. When bed slope is 1 in 2000 Find the most Economical dimension of the channel. If $C = 50$.

Solu^n:



$Q = 400 \text{ litre/s} = 0.4 \ m^3/sec$

$i = 1 \text{ in } 2000$

$C = 50$

For most Economical rectangular channel.

(i) $b = 2d$. (ii) $m = d/2$

$\therefore A = bd = 2d^2$ ~~2d~~

$\cancel{Q = AC\sqrt{mi}}$

$Q = AC\sqrt{mi}$

$0.4 = 2d^2 \times 50 \times \sqrt{d/2 \times 1/2000}$

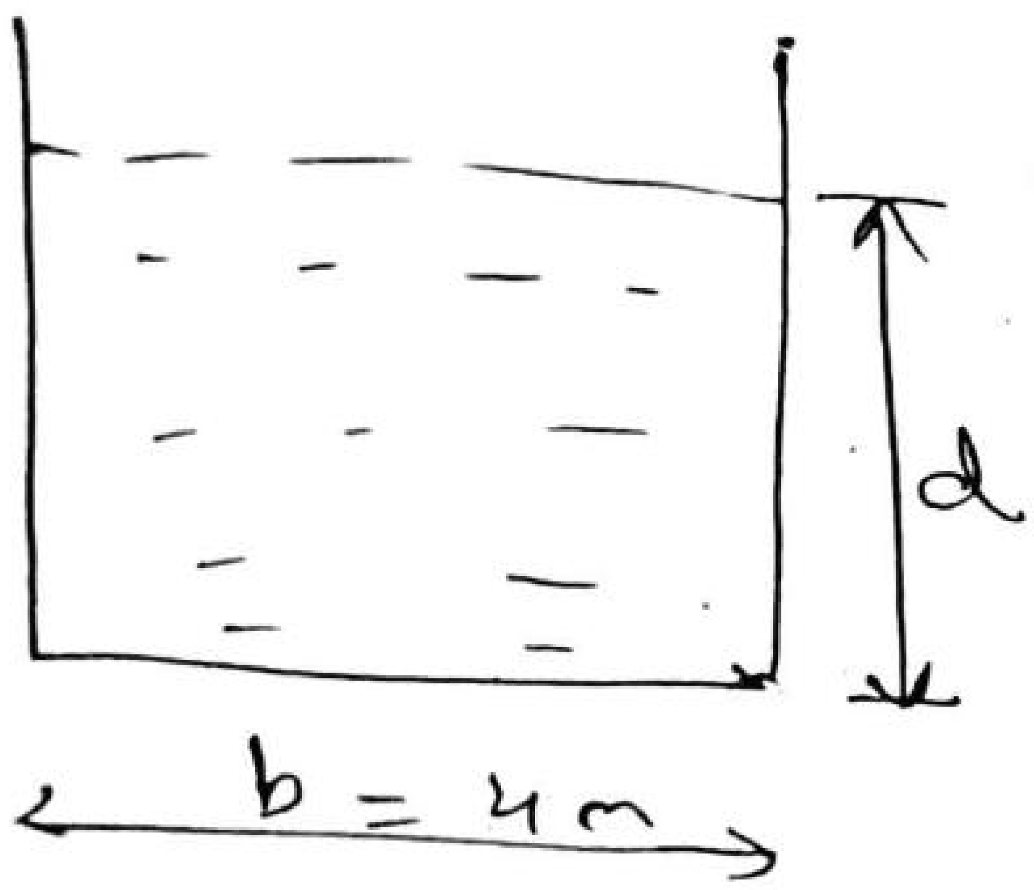
$0.253 = d^{5/2}$

$d = (0.253)^{2/5} = 0.577\ m$

$\therefore b = 2d = 2 \times 0.577 = 1.154\ m$

2) A rectangular channel of width 4m is having a bed slope of 1 in 1500. Find the max discharge through the channel. Take $C = 50$.

Soln:



$b = 4\ m$

$i = 1$ in 1500

$C = 50$

$Q = ?$

$d = ?$

The most Economical channel,

(i) $b = 2d$ & (ii) $m = d/2$

$\therefore b = 2d$

$\therefore d = b/2 = 4/2 = 2m$

$\therefore m = d/2 = 2/2 = 1m$

$\Rightarrow A = 2d^2 = 2(2)^2 = 8\ m^2$

$\Rightarrow Q = AC\sqrt{mi}$

$Q = 8 \times 50 \times \sqrt{1 \times 1/1500}$

$\boxed{Q = 10.327\ m^3/sec}$

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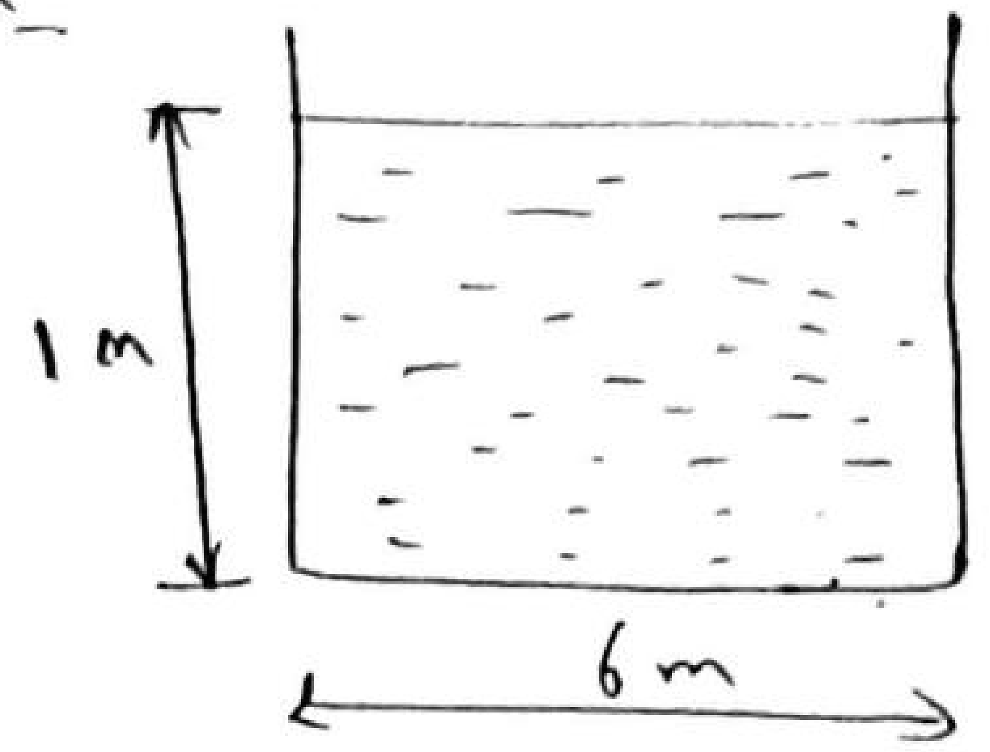
3) A rectangular Channel 6 m wide and 1 m deep as a slope of 1 in 900 and is lined with smooth Concrete plaster N = 0.012. It is required to increas the manning discharge to a maximum by changing the dimensions of the channel but keeping the amount of lining same. compute the new dimension and % increase in discharge.

Soln:-



$b = 6\,m$, $d = 1\,m$

$i = 1$ in 900

$N = 0.012$

$A = ?$

$Q_{max} = ?$

$\Rightarrow A = bd = 6 \times 1 = 6\,m^2$

$P = b + 2d = 6 + 2(1) = 8\,m$.

$m = A/P = \frac{6}{8} = 0.75\,m$

$C = \frac{1}{N} m^{1/6} = \frac{1}{0.012} \times (0.75)^{1/6} = 79.43$

$Q = AC\sqrt{mi} = 6 \times 79.43 \times \sqrt{0.75 \times 1/900}$

$\boxed{Q = 13.75\,m^3/sec}$

Let b_1 = New width of channel.

d_1 = New depth of channel.

A = Area of C/s is constant = 6m

$C = 79.43$

$i = 1/900$.

For max discharge

$b_1 = 2d_1$, $m = \frac{d_1}{2}$

$\Rightarrow A = b d_1$

$6 = 2d_1 \times d_1 = 2d_1^2$.

$d_1^2 = 6/2 = 3$

$d_1 = \sqrt{3} = 1.732m$, $b_1 = 2d_1 = 2 \times 1.732$

$\boxed{b_1 = 3.464m}$

$\Rightarrow m = \frac{A}{P_1}$

$\Rightarrow P_1 = b_1 + 2d_1 = 3.464 + (2 \times 1.732)$

$\boxed{P_1 = 6.928\ m}$

$\therefore m = \frac{A}{P_1} = \frac{6}{6.928} = 0.866m$

$\therefore Q_{max} = AC\sqrt{mi} = 6 \times 79.45 \times \sqrt{0.866 \times 1/400}$

$\boxed{Q_{max} = 14.787\ m^3/sec}$

Increase in discharge $= Q_{max} - Q$

$= 14.787 - 13.75$

$= 1.037\ m^3/sec$

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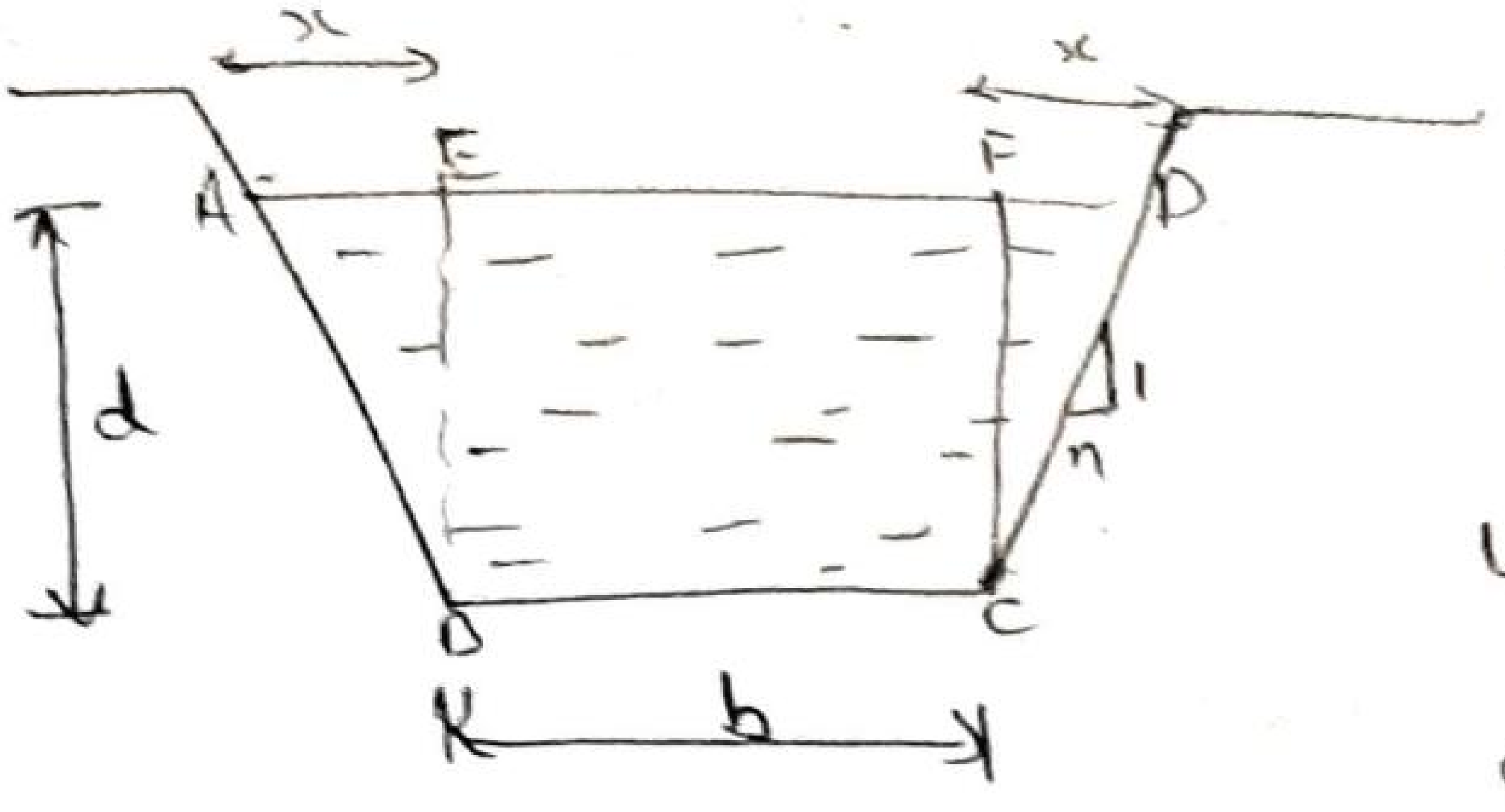
Most Economical trapezoidal Section



Consider a trapezoidal channel section as shown in fig.

Let, b = width of channel at bottom.

d = depth of flow.

The side slope as 1 vertical to the 'n' horizontal.

$\therefore x = nd$

Top width of the channel $\Rightarrow AD = nd + b + nd$

$$\boxed{AD = b + 2nd}$$

Area of trapezoidal $\Rightarrow A = (BC \times BE) + 2\left(\frac{1}{2} \times AE \times BE\right)$

$$A = (b \times d) + (nd \times d)$$

$$\boxed{A = d(b + nd)}$$

$$\therefore \boxed{b = \frac{A}{d} - nd}$$

One of the sloping side $= AB = CD = \sqrt{(AE)^2 + (BE)^2}$

$$\therefore AB = CD = \sqrt{(nd)^2 + (d)^2}$$

$$= \sqrt{d^2(n^2+1)}$$

$$CD = AB = d\sqrt{n^2+1}$$

Wetted perimeter $= P = AB + BC + CD$

$$= \left(d\sqrt{n^2+1}\right) + b + \left(d\sqrt{n^2+1}\right)$$

$$P = b + 2d\sqrt{n^2+1}$$

substituting 'b' value in 'P' eqn. we get,

$$\therefore P = \frac{A}{d} - nd + 2d\sqrt{n^2+1}$$

For most Economical section wetted perimeter should be minimum.

(i) ie, $\frac{dP}{dd} = 0$

$$A\left(\frac{-1}{d^2}\right) - n + 2\sqrt{n^2+1} = 0.$$

$$-\frac{A}{d^2} - n + 2\sqrt{n^2+1} = 0$$

$$2\sqrt{n^2+1} = \frac{A}{d^2} + n$$

$$2\sqrt{n^2+1} = \frac{(b+nd)\cancel{d}}{d^{\cancel{2}}} + n$$

$$2\sqrt{n^2+1} = \frac{b+nd}{d} + n$$

$$2\sqrt{n^2+1} = \frac{b+nd+nd}{d}$$

$$\therefore \boxed{d\sqrt{n^2+1} = \frac{b+2nd}{2}}$$

(ii) Hydraulic mean depth (m)

$$\therefore P = b + 2d\sqrt{n^2+1}$$

w.k.T $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

$$\therefore P = b + \cancel{2}\times\left(\frac{b+2nd}{\cancel{2}}\right)$$

$$P = b + b + 2nd$$

$$P = 2(b+nd)$$

$$\therefore A = (b+nd)d$$

$$\therefore m = \frac{A}{P} = \frac{(b+\cancel{nd})d}{2(b+\cancel{nd})} = \frac{d}{2} \quad \therefore \boxed{m = d/2}$$

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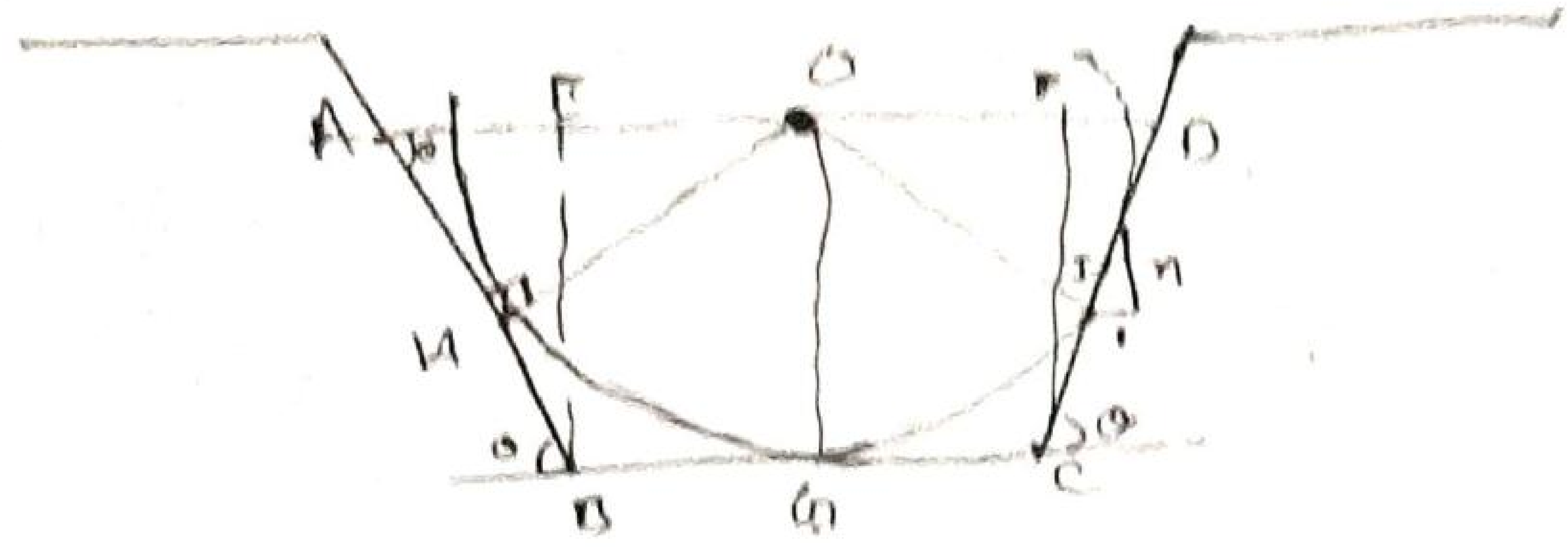
(iii)



Let θ = angle made by sloping side with horizontal

O = centre of top width

OH = $\perp^{ler}$ to sloping side

Δ^{le} AOH consider.

$\therefore \sin\theta = \frac{OH}{OA}$

$OH = OA \sin\theta$

Δ^{le} EAB

$\sin\theta = \frac{BE}{AB} = \frac{d}{d\sqrt{n^2+1}}$

$\sin\theta = \frac{1}{\sqrt{n^2+1}}$

$\therefore OH = OA \times \frac{1}{\sqrt{n^2+1}}$

AO = OD = Half of the top width

$\therefore OH = d\cancel{\sqrt{n^2+1}} \times \frac{1}{\cancel{\sqrt{n^2+1}}} = d$

$\therefore \boxed{OH = d}$

The conditions to be satisfied by a most Economical trapezoidal section.

(i) Half of the top width is equal to one of the sloping side ie, $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

(ii) Hydraulic mean depth is half of the flow of depth ie, $m = \frac{d}{2}$

(iii) semi circle drawn from 'O' with radius equal to depth of flow will touch the '3' sides of the channel.