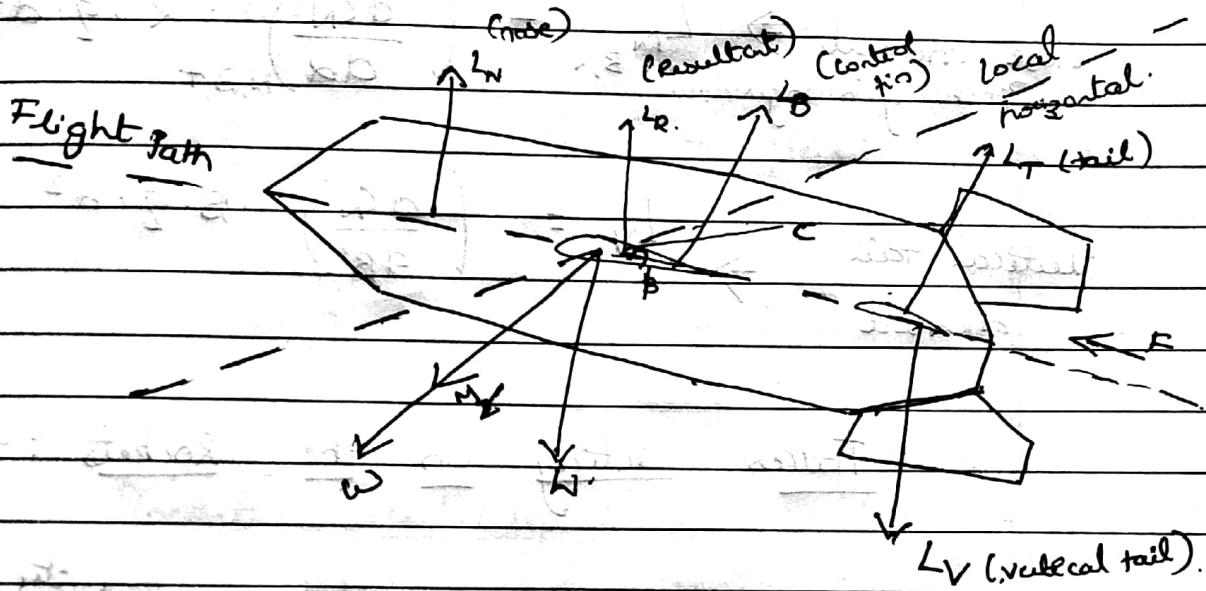


Module - 3

Aerodynamics of Rockets & Missiles

Forces acting on a missile while passing through atmosphere :-



M_x & M_y = load factor in longitudinal & transverse direction.

$C \rightarrow$ drag.

$W \rightarrow$ weight

$B \rightarrow$ angle α/ω c/r & body axis

L_N = nose force vector (aerodynamic shape)

L_V = normal component of thrust

L_T = tail force vector

$F \rightarrow$ thrust force

$L_B \rightarrow$ Body force

L_R = Resultant force

$$L_R = L_N + L_T + L_V + L_B$$

(load) factor

$$\therefore F = n_x \cdot w + c$$

↓ ↓ ↓ ↓
horizontal wt drag
forces.

if M_O is moment about center of gravity

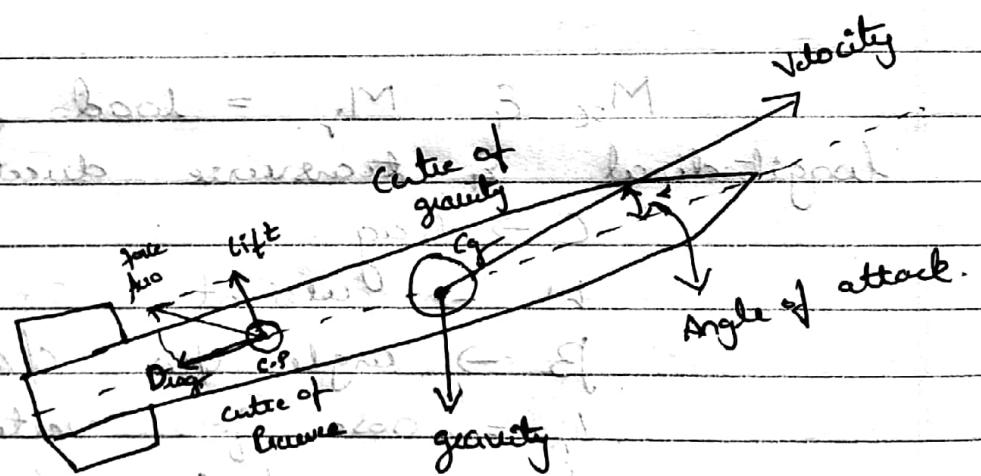
algebraic sum

$$M_O = M_N + M_B + M_V + M_V$$

nose moment $\Rightarrow L_N = \left(\frac{dC_N}{d\alpha} \right)_{N,B,T} \cdot \alpha \cdot g \cdot d^2$
Pitching is longitudinal

vertical tail moment $\Rightarrow L_V = \left(\frac{dC_N}{d\beta} \right)_{N,B,T} \cdot \beta \cdot g \cdot d^2$

Forces acting on the Rocket :-



Magnitude of aerodynamic forces depends on shape, size & velocity of the rocket & properties of air through which it flies.

Aerodynamic forces are mechanical forces which are generated by the interaction & contact of a solid body with fluid / air.

For the lift & drag to be generated, the rocket must be in contact with air.

Aerodynamic forces are generated by difference in velocity b/w air & rocket. If there is no relative motion, then there is no lift & drag.

Lift component is less in rocket, but thrust is more in order to push away from 'c' effect.

Rocket Stability :-

Fins placed on the rocket provides the stability during its flight. It allows the rocket to maintain the orientation & intend the flight path.

Problem of not having fins is C_p should be ahead of C_g .

$\Rightarrow C_g$ is where overall mass of the rocket is acting at that point and it spins about that axes.

$\Rightarrow C_p$ is where the aerodynamic force acts on the rocket. [where it perfectly balances the airflow, when it is placed fwd to air].

For stable rocket ' C_p ' should be aft the ' C_g '.

Stable rocket :-

[C_g is ahead of C_p]

Consider the free body diagram of rocket:

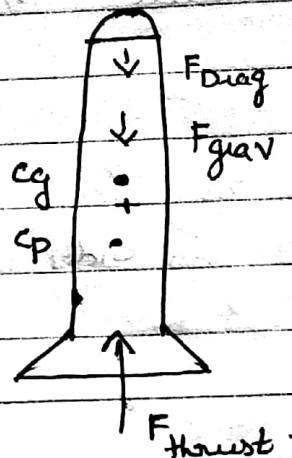


Fig:- 1

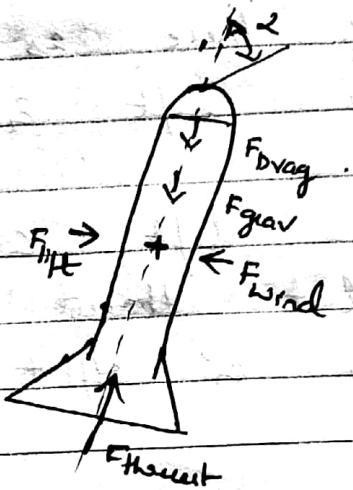


Fig:- 2

Fig 1 :- Shows the stable rocket with ~~obviously~~ C_g aft C_p in ideal state [all forces acting]

Fig 2 :- Perturbing force is introduced where the forces are caused due to gust of the wind. Resultant forces causing the rocket to rotate about the C_g changing its angle of attack ' α '

This change in ' α ' AoA causes the lift force acting through the C.P. This lift force causes the balance the force due to wind & rocket remains stable

Unstable rockets :- [CP ahead of CG]

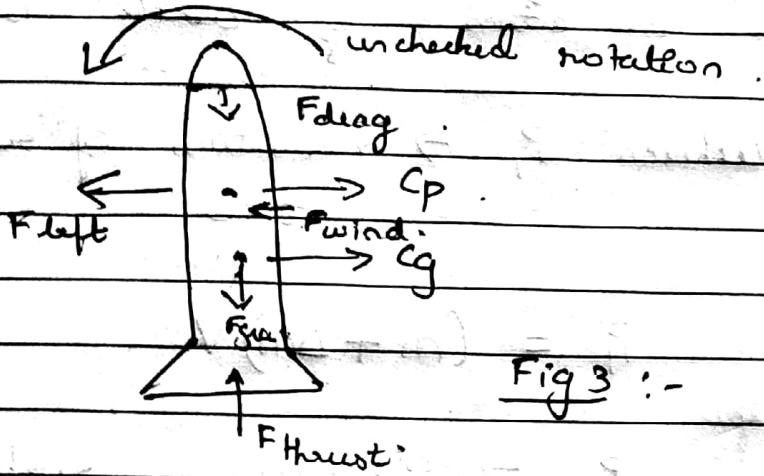


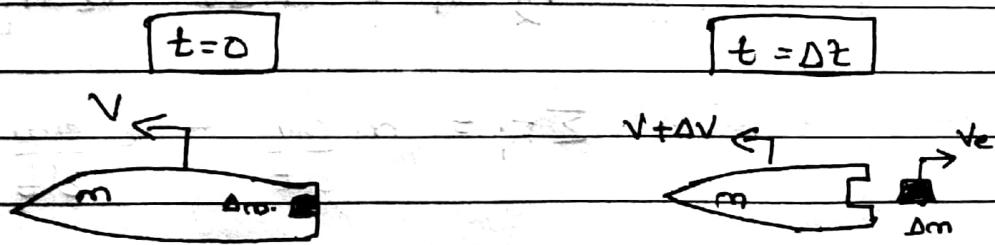
Fig 3 :-

blind face acts on ideal rocket as
in fig (1)

∴ lift force acts in same
direction of the wind force which
unstabilizes the rocket by rotating it
[as rocket rotates at CG location near
the tail part of the rocket]

Tsiolkovsky rocket equation :-

It describes the motion of the
vehicle / basic principle of rocket by
expelling the mass with high velocity.



Newton's 2nd law of motion :-

$$\sum F_i = \lim_{\Delta t \rightarrow 0} \frac{P_2 - P_1}{\Delta t}$$

"External force (F_i) to change in linear momentum of whole system".

where $P_1 \rightarrow$ momentum of rocket at $t=0$

$$P_1 = (m + \Delta m) V.$$

where $P_2 \rightarrow$ momentum of rocket at time $t = \Delta t$

$$P_2 = m(V + \Delta V) + \Delta m V_e$$

$V \rightarrow$ velocity at $t=0$.

$V + \Delta V =$ velocity at $t = \Delta t$.

$V_e =$ mass exhaust velocity.

$m + \Delta m =$ mass of rocket at $t = \Delta t$

$m = @. t = 0$

$$V_e = V - v_e$$

(negative exhaust direction).

$$\therefore P_2 - P_1 = m \Delta V - V_e \Delta m$$

→ decrease in mass

$$\sum F_i = m \frac{dV}{dt} + V_e \frac{dm}{dt}.$$

$$\therefore m \frac{dV}{dt} = -V_e \frac{dm}{dt}$$

$$\boxed{\Delta V = V_e \frac{m_0}{m_t}}$$

(OR)

Tsiolkovsky's equation states that a rocket vehicle of mass 'M', expelling combustion products at a rate 'm' with a constant exhaust velocity 'V_e'. The mass of vehicle is decreasing at rate 'm' due to thrust 'F' developed by exhaust, rocket is accelerating.

Rocket equation provides the achieved velocity at any time in terms of initial & current mass of rocket.

Thrust developed by exhaust is

$$F = V_e m \quad \rightarrow \textcircled{1}$$

where $m = \frac{dm}{dt}$

\because where $F = V_e m \Rightarrow$ $V_e = \text{exhaust velocity}$
 $\qquad\qquad\qquad \downarrow \textcircled{1} \qquad\qquad m = \text{decreasing mass wrt time}$
 $\qquad\qquad\qquad \frac{dm}{dt} \leftarrow \qquad\qquad\qquad \boxed{\qquad}$

From Newton's 3rd law

Acceleration of rocket $\Rightarrow \frac{dv}{dt} = \frac{F}{M}$ $\qquad \qquad \qquad [\because F = M \times a]$
is given by

Substituting eqn ① in above eqn.

$$\frac{dv}{dt} = V_e \frac{dm}{dt} \frac{1}{M}$$

Cancelling dt, & re-arranging

$$dv = V_e \frac{dm}{M}$$

integrating velocity b/w 0 to v, &
for mass M_0 to M , we get

$$\int_0^V dv = V_e \int_{M_0}^M \frac{dm}{m}$$

Solution is

$$V = V_e \log_e \left(\frac{M_0}{M} \right)$$

where $M_0 \rightarrow$ initial mass of rocket
 $M \rightarrow$ current mass of rocket
 this gives velocity of rocket

V_e = exhaust velocity.

V = velocity of rocket.

$\left(\frac{M_0}{M} \right)$ = mass ratio

(canceling terms)
 e.g. $\frac{M_0}{M} = \frac{1}{2}$

Aerodynamic forces & moments of rockets & missiles :-

let us assume vehicle's velocity vector is coinciding with one of symmetry planes through a pair of fins. Angle of in the trajectory plane between Velocity vector & longitudinal axis of vehicle is (AoA) angle of attack.

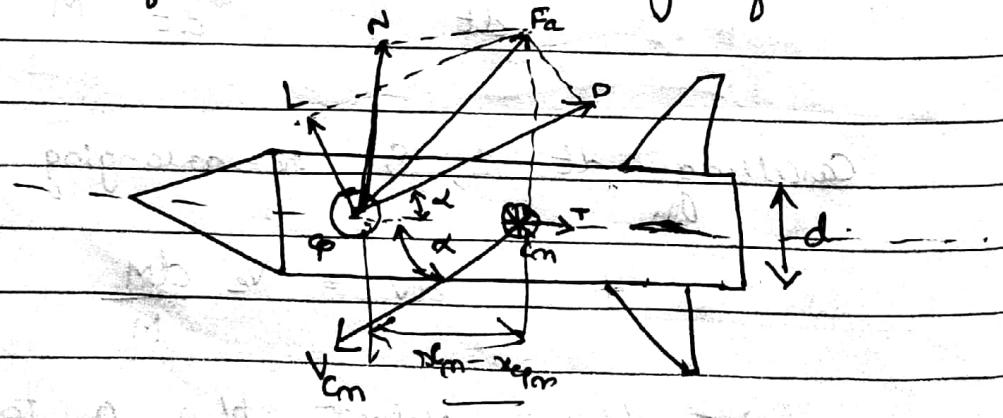


Fig 4

total aerodynamic force F_a is assumed to lie in trajectory plane.

The point where the line of force of F_a intersects the rocket's centre-line is called c_p Centre of pressure. No resultant aerodynamic moment is. c_p coincides with c_m (Centre of moment mass).

$$\text{moment is } \Rightarrow M' = F_a \times l$$

$$\text{where } l = x_{cm} - x_{cp} \quad [\text{dist bw } c_m \& c_p = l]$$

F_a is aerodynamic force which can be L/D .

let us assume longitudinal moment in x -axis by M' , Yawing moment in y -axis by L' & rolling moment in z -axis as N'

Net forces is written as:

$$\begin{bmatrix} M' \\ L' \\ N' \end{bmatrix} \rightarrow \text{forces}$$

$$L = C_L q S \quad (\text{or}) \quad C_L = \frac{L}{qS} \quad \begin{array}{l} \text{per unit} \\ \text{dy. cont} \end{array}$$

where q = dynamic pressure

$$S = \text{area of rocket/middle} \quad S = (\pi/4) d^2$$

$$\therefore M' = C_m q \underline{\cancel{S}} d \quad d = \text{base dia.}$$

$$\begin{array}{c} \leftarrow \quad \rightarrow \\ \text{moment} \quad \downarrow \quad \text{dry mass} \end{array} \quad Sd = \frac{\text{Vol}}{\text{..}}$$

where

$$C_m = \frac{M'}{q \cdot S \cdot d}$$

Moment coefficient are:-

$$C_N = C_L \cos \alpha + C_D \sin \alpha$$

from normal force

tangential moment coeff is C_T

$$C_T = -C_L \sin \alpha + C_D \cos \alpha$$

Pitching moment coeff is C_m

$$C_m = -C_N \cdot l/d$$

normal force dia
body dia

∴ overall forces acting on missile is given by

$$F_T = F_B + F_W + F_i$$

F_T = total force

F_B = Body force

F_W = Wing / fin force

F_i = body interface force

longitudinal moment of a rocket :-

longitudinal moment / Pitching moment is noted by C_m ; Ref fig 4

R.M of fin :-

$$C_{mW} = C_{NW} \left(\frac{x_m - x_{FW}}{d} \right)$$

where C_{NW} = of normal force on the fin wing (by lift)

x_m = centre of mass [c.g]

P.M = Pitching / longitudinal moment

x_{PA} = Aerodynamic centre of wing

d = nose diameter

C_{Mw} = Pitching moment of fin.

P.M of body in absence of fin :-

$$C_{Mb} = \frac{2\alpha}{S_r} \left[\int_0^L \frac{d\delta}{dx} (x_m - x) dx + \frac{\alpha^2}{d_{sr}} \int_0^L C_{dc} d(x) \right]$$

$(x_m - x) dx$

α = AOA of vehicle / mesh

L = vehicle length

δ = wing area

C_{dc} = Cross flow drag coeff

347] Drag - co-efficient :-

Drag of a rocket vehicle can be split into foll comp

→ Wave drag - due to presence of shock waves & on depending on Mach no

→ Viscous drag - due to friction

→ Induced drag - due to generation of lift

→ Base drag - due to wake behind vehicle

→ interference drag - due to interaction of various flow fields

→ Roughness drag - due to surface roughness like grits & welds

Wave drag :- Connected with shock waves at supersonic speed at higher M_a

$$C_D = \left(0.083 + \frac{0.096}{M_a^2} \right) (5.730)^{1.67} ; (M_a > 1)$$

where

$$M_a > 1$$

θ - half cone angle

Viscous drag :-

$$C_D = \frac{1.328}{\sqrt{Re}}$$

Induced drag :- Result of induced lift (C_L)

$$C_{D,i} = \frac{C_1^2}{\pi b^2 C_L^2} ; M_a < 1$$

$C_1 = \text{const}$.

Base drag :- due to presence of jet

$$C_D = a + b M_b^6 ; M_a < 1$$

$$C_D = a' + \frac{b'}{M_a^2} ; M_a \geq 1$$

Roll

Air

Lateral aerodynamic coefft :-

(125)

To lateral direction (z -axis) its
Rolling moment (C_B) that acts on the vehicle.

dimensions of C_B

$$C_B = \frac{l}{q S_w d}$$

$\Rightarrow C_B = \frac{\text{lateral moment}}{\text{dynamic moment}}$

To study of lateral motion of missile
'Damping' term arises primarily from
lift & moment from wing surfaces.

Rolling velocity induces AoA (α) which varies
linearly along the span

$$\Delta\alpha = \frac{V_t}{V} = \frac{p y}{V}$$

where :- $\Delta\alpha$ = induced angle of attack at pt y along span

V_t = tangential velocity

p = rolling velocity

V = missile forward speed.

damping-in-roll :-

$$L = -Ny = \int_{-b/2}^{b/2} (C_{NA})_z \Delta\alpha y z c \cdot dy$$

L = rolling moment

N = normal force

\bar{y} = resultant of roll moment arm

$(C_{NA})_z$ = section normal force coeff

c = local chord at span y

Rocket dispersion :-

R.P.U.

Chin
Pg (19)

Dispersion of rocket is a measure for the deviation of rocket's trajectory from the nominal trajectory [i.e. from some standard / defined trajectory]

Type of rocket dispersion :-

- 1] In plane dispersion
- 2] lateral dispersion

→ In-plane dispersion :- deflection error during the launcher phase (Boost Phase) like

→ Vertical launcher

→ deflection of thrust nozzle

→ Simulation / ground error

→ lateral dispersion :-

moving away from defined trajectory due to atmospheric effect / wind (or) technical error in guidance system

R.P.U.

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Re-entry body design consideration :-

Error due to rocket dispersion due to Velocity & missile attitude.

In design consideration :-

We consider aerodynamic heating concept, & vehicle should have minimum structural

out & reliability. It is desirable to avoid use of guidance system in much as a problem in dispersion. esp under gravity of vehicle.

Rectangular body configured must be carefully designed in order to free-fight dispersion resulting from atmospheric disturbance i.e. wind shear, cross winds etc. within specific limits.

Jackson
Nelson
Fig-VII

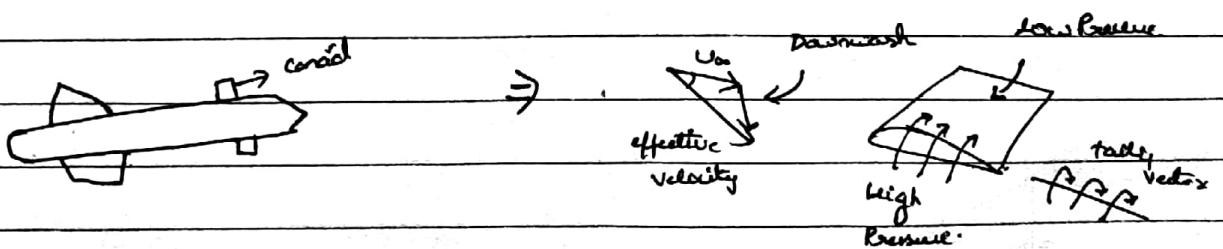
\leftarrow Downwash in missile :-

$$\text{down wash } \epsilon = -\arctan \frac{\bar{w}}{\bar{u}} = -\frac{\bar{w}}{\bar{v}}$$

Let the component of streamline velocity v wrt missile be $\bar{u}, \bar{v}, \bar{w}$ along positive axis of x, y & z respectively.

In downwash, local streamline velocity are compared with tangent definition of $\alpha(\theta)$.

Due to effect of canard in the missile, downwash effect is more



Lift forces back calculated from rolling moment are much lower again. Vortices interact with aft fin resulting in opposite

well moment.

... to be continued