# MODULE-2 -15EC81

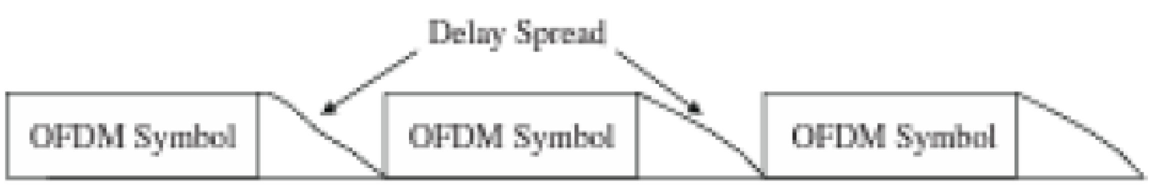
## OFDM Basics

In order to overcome the daunting requirement for LRF radios in both the transmitter and receiver, OFDM employs an efficient computational technique known as the Discrete Fourier Transform(DFT), which lends itself to a highly efficient implementation Commonly known as the Fast Fourier Transform (FFT). In this section, we will learn how the FFT (and its inverse, the IFFT) are able to create a multitude of orthogonal subcarriers using just a single radio.

## Block Transmission with Guard Intervals

We begin by grouping I data symbols into a block known as an *OFD*M sym*bol.* An OFDM symbol lasts for 1 duration of *T* seconds, where *I* = *I*T.. In order to keep each OFDM symbol independent of the others after going through a wireless channel, it is necessary to introduce a guard time in between each. OFDM symbol, as shown here:


This Way, after receiving a series of OFDM symbols, 18 long as the guard time *T.* is larger than the delay spread of the channel T, each OFDM symbol will only interfere with itself.




OFDM transmissions allow ISI within an OFDM symbol, but by including as sufficiently large quad band, it is possible to guarantee that there is no interference *b*etween subsequent OFDM symbols.

## Circula**r Convolution and the DFT**

Now that subsequent OFDM symbols have been rendered orthogonal with a guard in terval, the next task is to attempt to remove the ISI wit*h*in each OFDM symbol. As described in Chapter 2, when 101 input data stream x[n] is sent through a linear time invariant FIR channel in the output is the linear convolution of the input and the channel, that is, y [n] = x[n]* h[n]. However, let's imagine for dormant that it was possible to compute in terms of a circular convolution, that is

$$y[n] = x[n] \circledast h[n] = h[n] \circledast x[n],$$

**Where**

$$x[n] \circledast h[n] = h[n] \circledast x[n] \triangleq \sum_{k=0}^{L-1} h[k]x[n-k]_L$$

and the circular function $x[n]_L = x[n \bmod L]$ is a *periodic* version of $x[n]$ with period $L$. In other words, each value of $y[n] = h[n] \circledast x[n]$ is the sum of the product of $L$ terms.[5]

In this case of circular convolution, it would then be possible to take the DFT of the channel output $y[n]$ to get:

$$\mathrm{DFT}\{y[n]\} = \mathrm{DFT}\{h[n] \circledast x[n]\}$$

which yields in the frequency domain

$$Y[m] = H[m]X[m].$$

Note that the duality between circular convolution in the time domain and simple multiplication in the frequency domain is a property unique to the DFT.

The L point DFT is defined as

$$\mathrm{DFT}\{x[n]\} = X[m] \triangleq \frac{1}{\sqrt{L}} \sum_{n=0}^{L-1} x[n] e^{-j\frac{2\pi nm}{L}},$$

while its inverse, the IDFT is defined as

$$\mathrm{IDFT}\{X[m]\} = x[n] \triangleq \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} X[m] e^{j\frac{2\pi nm}{L}}.$$

Referring to this innocent formula actually describes an ISI-free channel in the frequency domain, where each input symbol X[m ]is simply scaled by a complex-value *H[*m]. So, given knowledge of the channel frequency response *H[m]* at the receiver, it is trivial to recover the input symbol by simply computing

$$\hat{X}[m] = \frac{Y[m]}{H[m]},$$

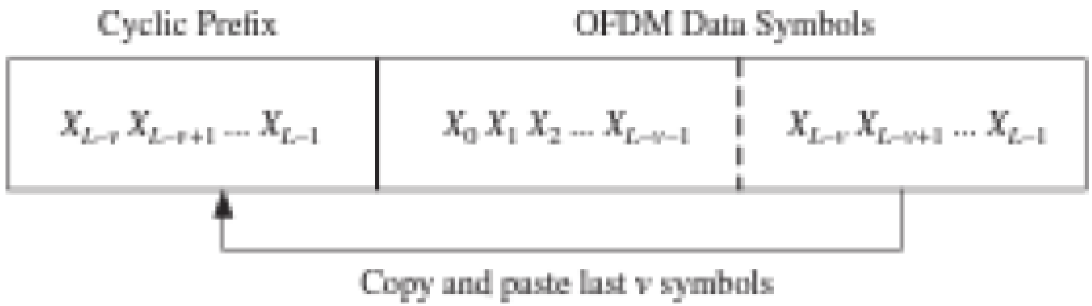



Figure 1:The OFDM last v symbols

where the estimate X[m] will generally be imperfect due to additive noise, co-channel interference, imperfect channel estimation, and other imperfections that will be discussed later. Nevertheless, in principle, the ISI—which is the most serious forms of interference in 1 wideband channel-has been mitigated.

**An OFDM Block Diagram :**

Let us now briefly review the key steps in 1 OFDM communication system, each of which can be observed in

1. The first step in OFDM is to break a wideband signal of bandwidth B into L narrowband signals (subcarriers) each of bandwidth *B/L.* This way, the aggregate symbol rate is maintained, but each subcarrier experiences flat fading, or ISI-free communication, 2 long as a cyclic prefix that exceeds the delay spread is used. The I subcarriers for a given OFDM symbol are represented by a vector X, which contains the I current symbols.

2. In order to use a single wideband radio instead of L independent narrow band radios, the subcarriers te created digitally using an IFFT operation.

3. In order for the IFFT/FFT to decompose the ISI channel into orthogonal subcarriers, 1 cyclic prefix of length u must be appended after the IFFT operation. The resulting a symbols are then sent in serial through the wideband channel.

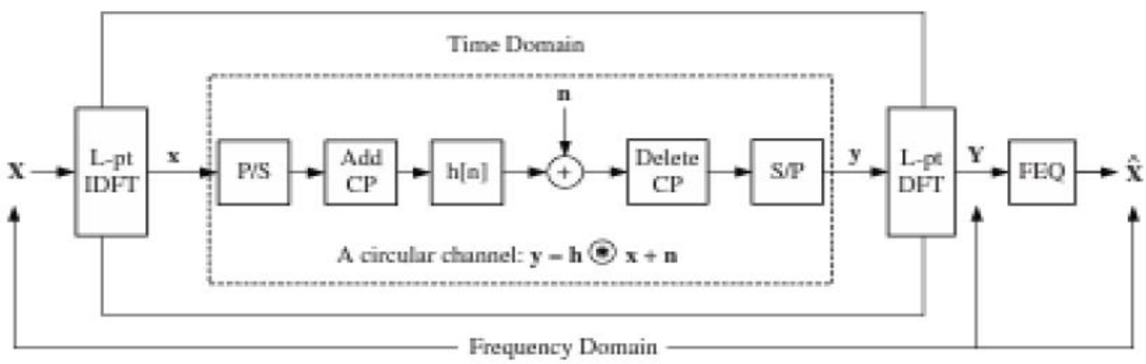



**Figure 2**: An OFDM system in vector notation in OFDM, where X,Y and X contain the L transmitted.

## OFDM in LTE:

To gain an appreciation for the time and frequency domain interpretations of OFDM, LTE systems can be used as 101 example. Although simple in concept, the subtleties of OFDM can be confusing if each signal processing step is not understood. To ground the discussion, we will consider a pass band OFDM system, and then give specific values for the important system parameters.

The inputs to this figure are *L* independent QAN symbols (the vector X), and these I symbols are treated as separate subcarriers. These *L* data-bearing symbols can be created from 1 bit stream by a symbol mapping und serial-to-parallel convertor (S/P). The L-point IFFT then creates a time domain L-vector x that is cyclic extended to have length (1 +G), where G is the fractional overhead.

In LTE G = 0.07 for the normal cyclic prefix and grows to G=0.25 for the extended cyclic prefix. This longer vector is then parallel-to serial (P/S) converted into a wideband digital signal that can be amplitude modulated with a single radio at a carrier frequency of

$f_e$= $w_c$/2¶.

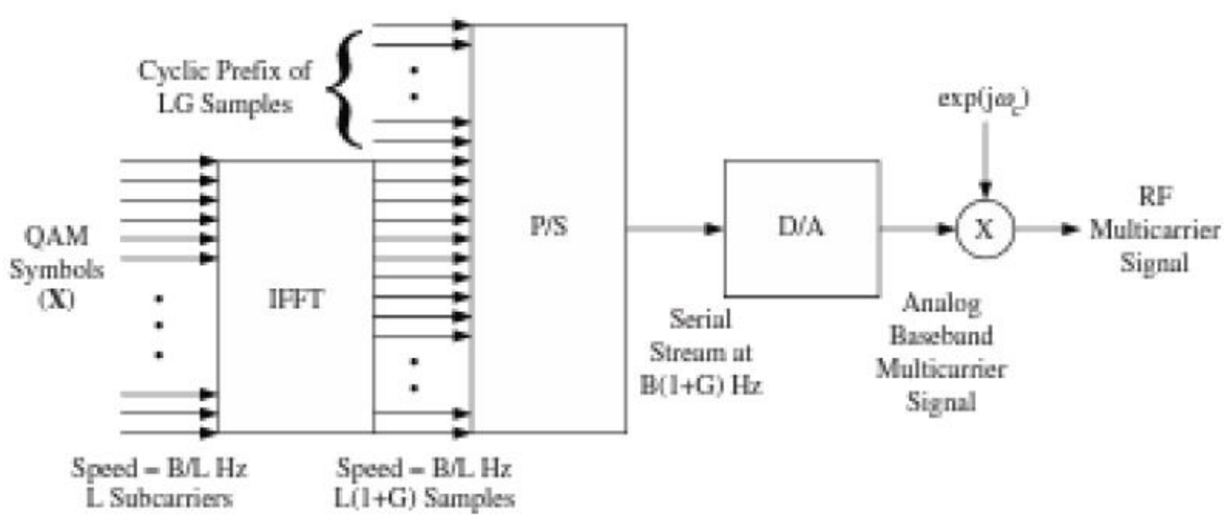



**Figure 3** : A close-up of the OFDM baseband to pass band transmitter.

**Table**: summary of Key OFDM parameters in LTE and example values for 10mHz

| Symbol | Description | Relation | Example LTE value |
|---|---|---|---|
| $B$ | Nominal bandwidth | $B = 1/2f_s$ | 7.68MHz |
| $B_{chan}$ | Transmission bandwidth | Channel spacing | 10MHz |
| $L$ | No. of subcarriers | Size of IFFT/FFT | 1024 |
| $G$ | Guard fraction | % of $L$ for CP | 0.07 |
| $L_d$ | Data subcarriers | $L-$ pilot/null subcarriers | 600 |
| $\Delta f$ | Subcarrier spacing | Independent of $L$ | 15KHz |
| $T_s$ | Sample time | $T_s = 1/\max(B) = 1/\Delta f \cdot 2048$ | $1/15\text{KHz} \cdot 2048 = 32.55$ nsec |
| $N_g$ | Guard symbols | $N_g = GL$ | 72 |
| $T_g$ | Guard time | $T_g = 144T_s$ or $160T_s$ | 4.7 or 5.2 $\mu$sec |
| $T$ | OFDM symbol time | $T = (L + N_g)/B$ | 142.7 $\mu$sec |

OFDM parameters are summarized in along with some potential numerical values for these parameters. As an example, if 16QAM modulation was used (M = 16) with the normal cyclic prefix, the aw (neglecting coding) data rate of this LTE system

$$R = \frac{B}{L}\frac{L_d \log_2(M)}{1+G}$$

$$= \frac{10^7\,\text{MHz}}{1024}\frac{600\log_2(16)}{1.07} = 21.9\text{ Mbps}.$$

In other words, there are L = 600 data-carrying subcarriers of bandwidth *B/L,* each carrying log (M) bits of data. An additional overhead penalty of (1+G) must be paid for the Cyclic prefix, since it consists of redundant information and sacrifices the transmission of actual data symbols.

### Timing and Frequency Synchronization:

In order to demodulate 10 OFDM signal, there are two important synchronization tasks that need to be performed by the receiver. First, the timing offset of the symbol and the optimal timing instants need to be determined. This is referred to as timin*g a*synchroniz*at*ion Second, the receiver must align its carrier frequency as closely is possible with the transmitted carrier frequency: This is referred to as *f*requency *sy*nchroniza*ti*on. Compared to single-currier systems, the timing synchronization requirements for OFDM are in fact somewhat relaxed, since the OFDM symbol structure naturally ac accommodates a reasonable degree of synchronization error. On the other hand, frequency synchronization requirements are significantly more stringent, since the orthogonality of the data symbols is reliant on their being individually discernible in the frequency domain.

A representation of an OFDM symbol in time (top) und frequency (bottom). In the time domain, the IFFT effectively modulates each data symbol onto a unique carrier frequency:

only two of the carriers are shown—the actual transmitted signal is the superposition of all the individual curriers. Since the time window is T = 1sec and a rectangular window is used, the frequency response of each subcarrier becomes a "Sine" function with zero Crossings every 1/T = IMHz. This can be confirmed using the Fourier Transform F{.} since

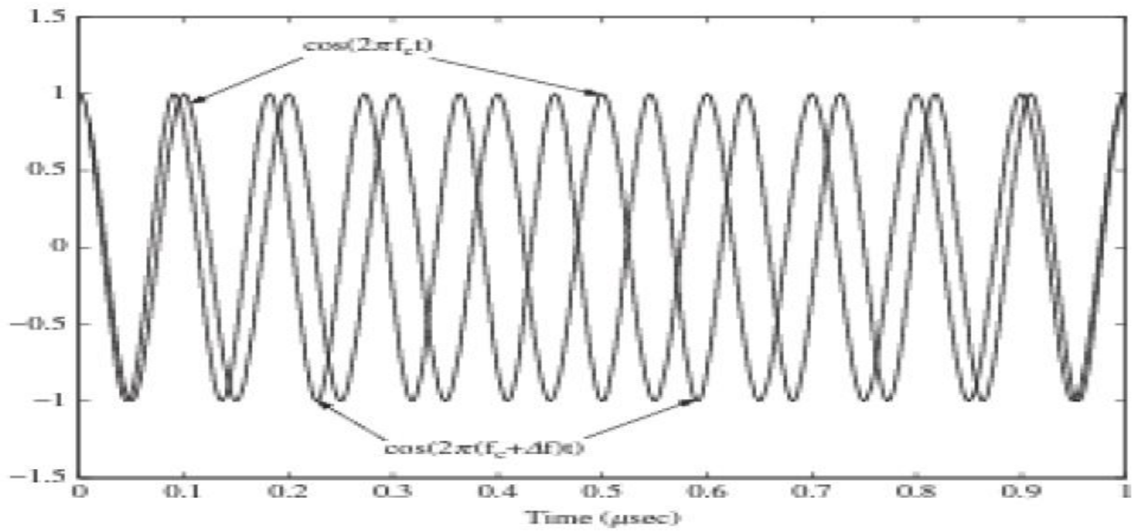
$$\begin{aligned}\mathcal{F}\{\cos(2\pi f_c t)\cdot \mathrm{rect}(t/T)\} &= \mathcal{F}\{\cos(2\pi f_c t)\} * \mathcal{F}\{\mathrm{rect}(2t/T)\}\\ &= \mathrm{sinc}\,(T(f-f_c)),\end{aligned}$$

Where rect (Z) = 1. 1€ (-0.5, 0.5), and zero elsewhere. This frequency response is shown for L=8 subcarriers in the bottom part The challenge of timing and frequency synchronization can be appreciated by inspecting these two figures. If the timing window is slid to the left or right, i unique phase change will be introduced to each of the subcarriers. In the frequency domain, if the carrier frequency synchronization is perfect, the receiver samples at the peak of each subcarrier, where the desired subcarrier amplitude is maximized and the inter-carrier interference (ICI) is zero.

# Timing Synchronization:

The effect of timing errors in symbol synchronization is somewhat relaxed in OFDM due to the presence of a cyclic prefix.  we assumed that only the L time domain samples after the cyclic prefix were utilized by the receiver. Indeed, this corresponds to "perfect timing synchronization, and in this case even if the cyclic prefix length N. is equivalent to the length of the channel impulse response successive OFDM symbols can be decoded ISI free.

In the case that perfect synchronization is not maintained, it is still possible to tolerate a timing offset of $\tau$ seconds without any degradation in performance as long as $0 \le \tau \le T_g - T_m$, where as usual $T_g$ is the guard time (cyclic prefix duration) and $T_m$ is the maximum channel delay spread. Here, $\tau < 0$ corresponds to sampling earlier than at the ideal instant, whereas $\tau > 0$ is later than the ideal instant. As long as $0 \le \tau \le T_g - T_m$, the timing offset simply results in a phase shift per subcarrier of $\exp(-j\Delta f\tau)$, which



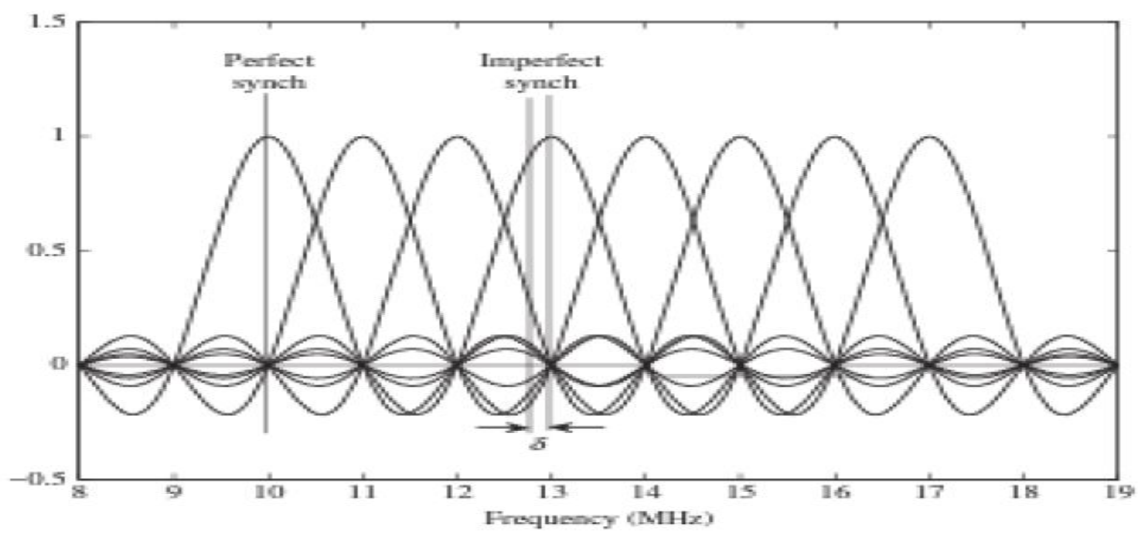




**Figure 4**:OFDM syschronization in time(top) and frequency (bottom)

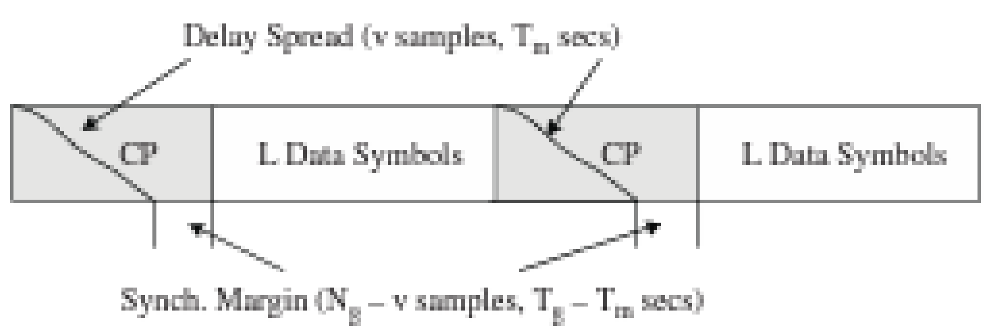



**Figure 5**: Timing synchronization margin

For both of these scenarios, the SNR loss can be approximated by

$$\Delta SNR(\tau) \approx -2\left(\frac{\tau}{LT_s}\right)^2,$$

SNR decreases quadratically with the timing offset.

• Longer OFDM symbols are increasingly immune from timing offset, that is, more sub carriers helps.

• Since in general T & IT timing synchronization errors not that critical as long as the induced phase change is corrected.

**Frequency Synchronization**

OFDM achieves a high degree of bandwidth efficiency compared to other wideband systems. The subcarrier packing is extremely tight compared to conventional modulation techniques, which require a guard band on the order of 50% Or more, in addition to special transmitter architectures such as the Weaver architecture or single-sideband modulation that suppress the redundant negative-frequency portion of the passband signal. The price to be paid for this bandwidth efficiency is that the multicarrier signal shown in Figure 3.8 is very sensitive to frequency offsets due to the fact that the subcarriers overlap, rather than having each subcarrier truly spectrally isolated.

We'll now analyze this inter-carrier interference (ICI) in order to better understand its effect on OFDM performance.

The matched Alter receiver corresponding to subcarrier $I$ can be simply expressed for the case of rectangular windows (neglecting the carrier frequency) :

$$x_l(t) = X_l e^{j\frac{2\pi l t}{LT_s}},$$

where $1/LT_s = \Delta f$, and again $LT_s$ is the duration of the data portion of the OFDM symbol, that is, $T = T_g + LT_s$. An interfering subcarrier $m$ can be written as

$$x_{l+m}(t) = X_m e^{j\frac{2\pi(l+m)t}{LT_s}}.$$

If the signal is demodulated with a fractional frequency offset of $\delta$, $|\delta| \leq \frac{1}{2}$

$$\hat{x}_{l+m}(t) = X_m e^{j\frac{2\pi(l+m+\delta)t}{LT_s}}.$$

The ICI between subcarriers $l$ and $l+m$ using a matched filter (i.e., the FFT) is simply the inner product between them:

$$I_m = \int_0^{LT_s} x_l(t)\tilde{x}_{l+m}(t)dt = \frac{LT_s X_m\left(1 - e^{-j2\pi(\delta+m)}\right)}{j2\pi(m+\delta)}.$$

It can be seen that in the above expression, $\delta = 0 \Rightarrow I_m = 0$, and $m = 0 \Rightarrow I_m = 0$, as expected. The total average ICI energy per symbol on subcarrier $l$ is then

$$ICI_l = E\left[\sum_{m \neq l} |I_m|^2\right] \approx C_0(LT_s\delta)^2\mathcal{E}_x,$$

The SNR loss induced by frequency offset is given by

$$\begin{aligned}\Delta SNR &= \frac{\mathcal{E}_x/N_o}{\mathcal{E}_x/\left(N_o + C_0(LT_s\delta)^2\mathcal{E}_x\right)} \\ &= 1 + C_0(LT_s\delta)^2 SNR\end{aligned}$$

- SNR decreases quadratically with the frequency offset.
- SNR decreases quadratically with the number of subcarriers.
- The loss in SNR is also proportional to the SNR itself.

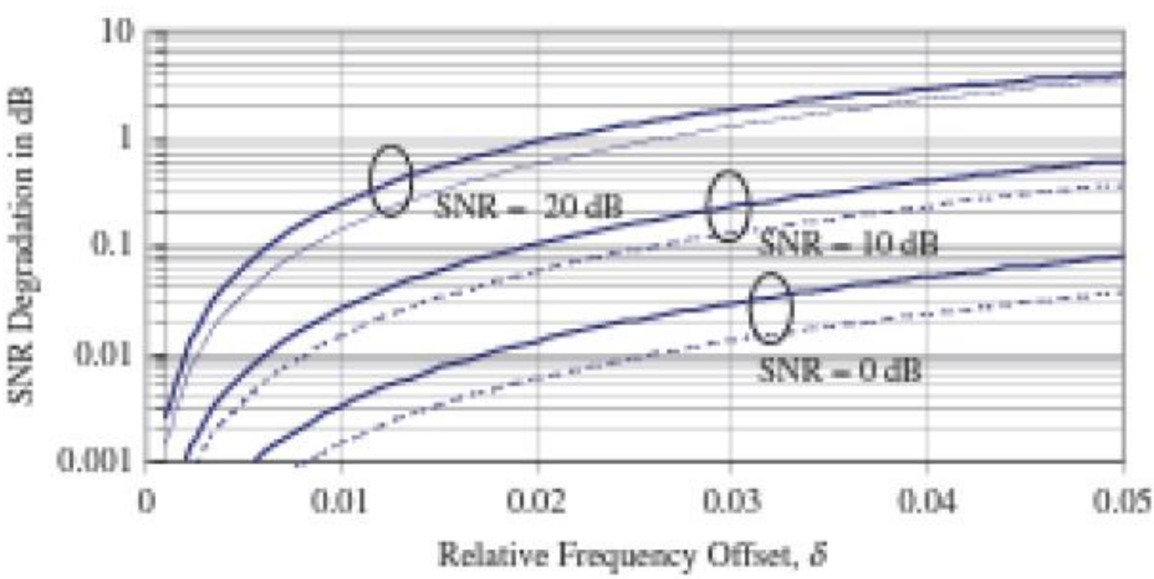



**Figure 6:** SNR loss a function of frequency offset

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### The Peak-to-Average Ratio (PAR)

OFDM signals have a higher peak-to-average ratio (PAR)often Called a peak-to-average power ratio (PAPR) than do single-carrier signals. The reason for this is that in the time domain, a multicarrier signal is the sum of many narrowband signals. At some times, this sum is large at other times it is small, which means that the peak value of the signal is substantially larger than the average value.

### The PAR Problem

When a high-peak signal is transmitted through a nonlinear device such as a high-power amplifier (HPA) or digital-to-analog converter (DAC), it generates out-of-band energy (spectral regrowth) and in-band distortion (Constellation tilting and scattering). These degradations may affect the system performance severely. The nonlinear behaviour of HPA ah be characterized by amplitude modulation,'amplitude nodulation (AM/AM) und amplitude modulation/phase modulation (AM/PM) responses.

The input backoff is defined as

$$IBO = 10\log_{10}\frac{P_{inSat}}{\bar{P}_{in}},$$

where $P_{inSat}$ is the saturation power (above which is the nonlinear region) and $\bar{P}_{in}$ is the average input power. The amount of backoff is usually greater than or equal to the PAR of the signal.

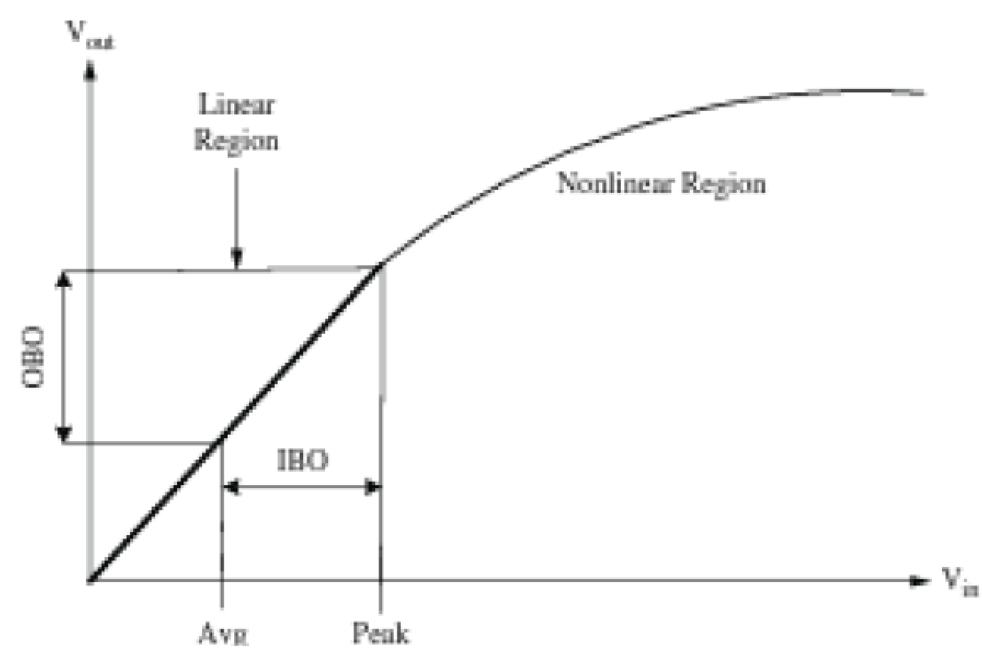

Figure 6: A typical power amplifier response

The power efficiency of an HPA can be increased by reducing the PAR of the trans mitted signal. For example, the efficiency of class A amplifier is halved when the input PAR is doubled or the operating point (average power).

In addition to the large burden placed on the HPA, a high PAR requires high resolution for both the transmitter's digital-to-analog convertor (DAC) and the receiver's

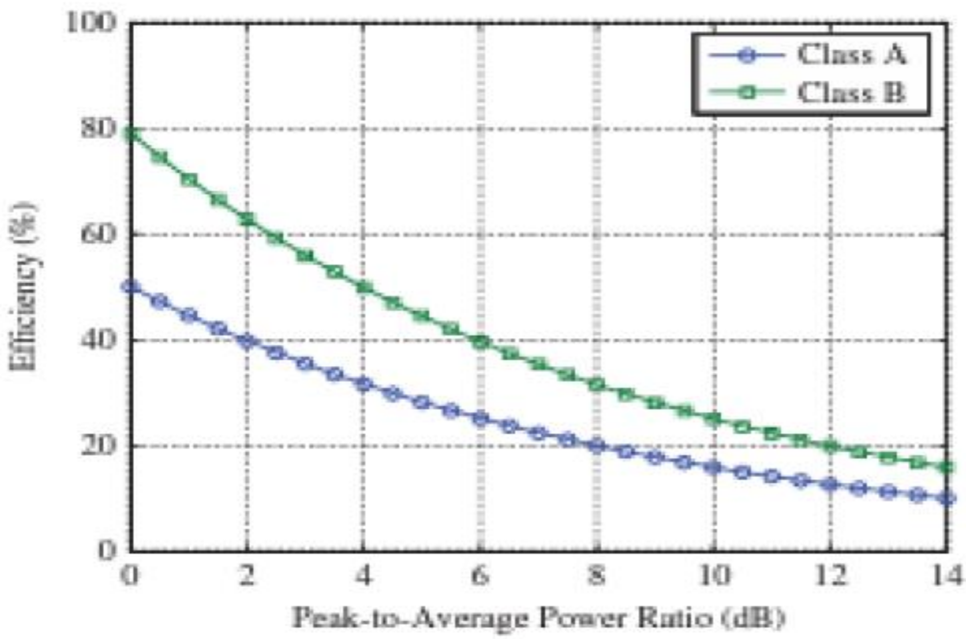


**Figure 7**: Theoritical efficiency limits of linera amplifiers

## Cli**pping and Other PAR Reduction Techniques**

In order to avoid operating the PA in the nonlinear region, the input power can be reduced up to 10 amount about equal to the PAR. This, of course, is very inefficient 101d will reduce the range and/or SINR of the system by the same amount. However, two important facts related to this IBO amount can be observed from Figure First, sing the highest PAR values are uncommon, it might be possible to simply "clip" off the highest peaks, at the cost of some hopefully minimal distortion of the signal. Second and Conversely, it can be seen that even for a conservative choice of IBO, say 10 dB, there is still a distinct possibility that given. OFDM symbol will have a PAR that exceeds the IBO and causes clipping.
Clipping, sometimes called "soft limiting," truncates the amplitude of signals that exceed the clipping level as

$$\bar{x}[n] = \begin{cases} Ae^{j\angle x[n]}, & \text{if } |x[n]| > A \\ x[n], & \text{if } |x[n]| \leq A, \end{cases}$$

The clipping ratio can be used as a metric and is defined as

$$\gamma \triangleq \frac{A}{\sqrt{E\{|x[n]|^2\}}} = \frac{A}{\sqrt{\mathcal{E}_x}}$$

Obviously, clipping reduces the PAR at the expense of distorting the desired signal The two primary drawbacks from clipping are (1) spectral regrowth (frequency domain leakage), which causes unacceptable interference to users in neighbouring RF channels,

$$\bar{X}_k = X_k + C_k, \quad k = 0, \ldots, L-1,$$

where $c_k$ represents the clipped off signal in the frequency domain. In Figure, the power spectral density of the original (X), clipped (X), und clipped-off (C) signals are plotted for different clipping ratios y of 3, 5, and 7 dB. The following deleterious effects are observed.

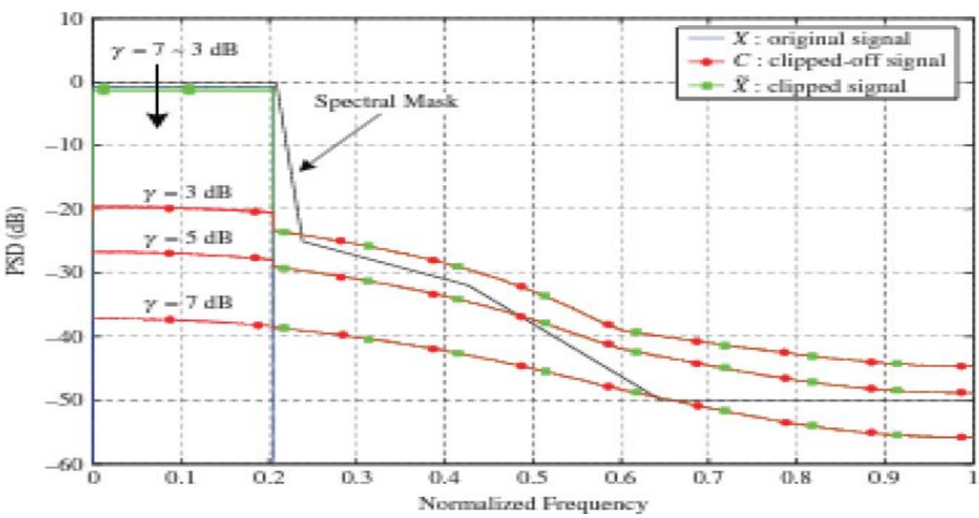


**Figure 8**: Power spectral density

Attenuation of the desired signal,

$$\bar{x}[n] = \alpha x[n] + d[n], \quad \text{for } n = 0, 1, \ldots, L-1.$$

Now, d[n]is uncorrelated with the signal #[n] and the attenuation factor a is obtained

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}\gamma}{2}\operatorname{erfc}(\gamma).$$

The attenuation factor a is plotted in Figure is a function of the clipping ratio . The attenuation factor a is negligible when the clipping ratio a is greater than 8 dB, so for high clipping ratios,

Gaussian input x[n] as

$$\sigma_d^2 = \mathcal{E}_x(1 - \exp(-\gamma^2) - \alpha^2).$$

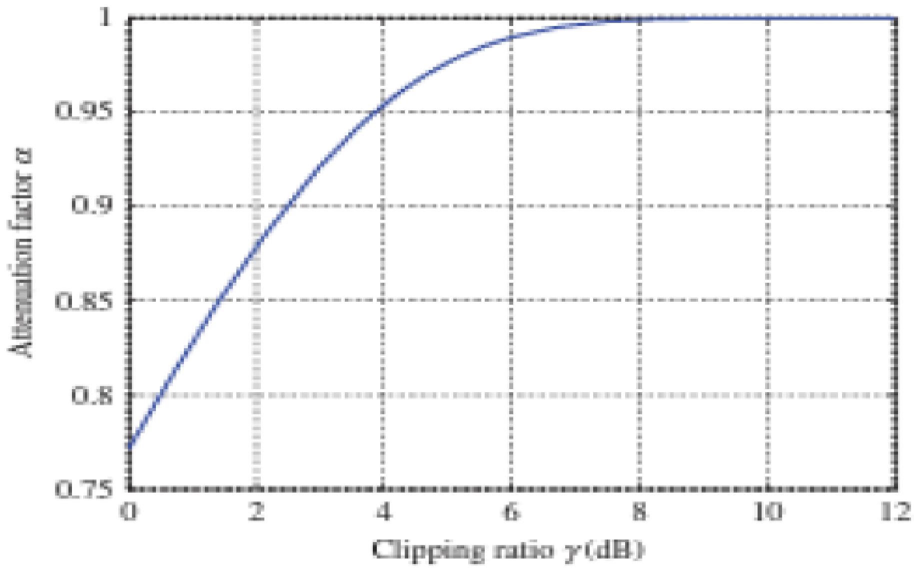


Figure 9: Attenuation Factor

input and channel noise (that has variance $N_0/2$):

$$\mathrm{SNDR} = \frac{\alpha^2 \mathcal{E}_x}{\sigma_d^2 + N_0/2}.$$

The bit-error probability (BEP) can be evaluated for different modulation types using the SNDR [12]. In the case of M-QAM and average power & the BEP can then be approximated as

$$P_b \approx \frac{4}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\mathcal{E}_x \alpha^2}{(\sigma_d^2 + N_0/2)(M-1)}}\right).$$

LTE's **Approach to PAR in the Uplink**

LTE has taken a pioneering new approach to PAR. In the downlink. PAR is less important because the base stations are fewer in number and generally higher in cost, and so we

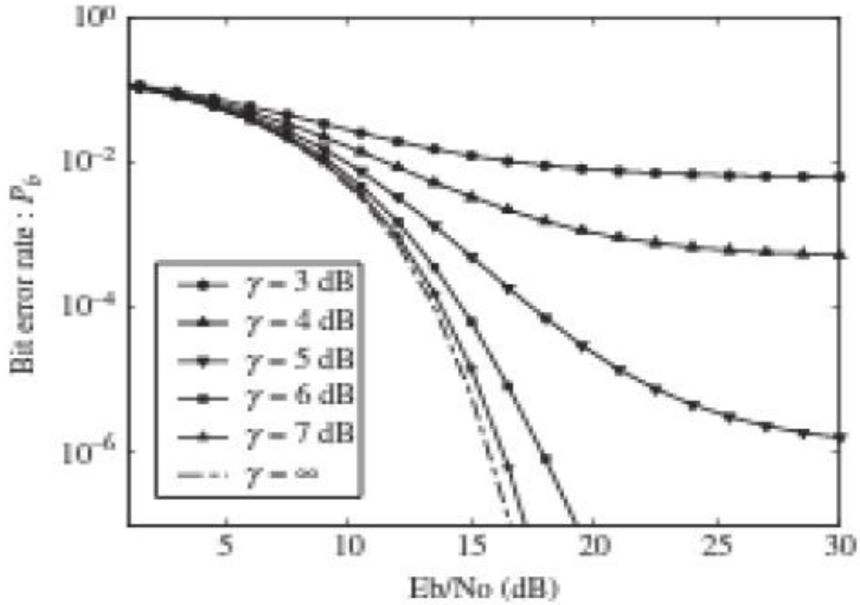


**Figure 10**: Bit error rate probibility for a clipped OFDM signal

Typically, the high PAR is basically tolerated and sufficient input power back off is undertaken in order to keep the in-band distortion and spectral regrowth at an acceptable level.

## Single-Carrier Frequency Domain Equalization (SC-FDE)

An alternative approach to OFDM is the less popular but conceptually similar single carrier frequency domain equalization (SC-FDE) approach to ISI suppression . SC-FDE maintains OFDM's three most important benefits low complexity even severe multipath channels excellent BER performance, close to theoretical bounds; and a decoupling of ISI from other types of interference, notably spatial interference, which is very useful when using multiple antenna transmission. By utilizing single-carrier transmission, the peak-to-average ratio is also reduced significantly (by several dB) relative to multicarrier modulation.

### SC-FDE System Description

Frequency domain equalization is used in both OFDM and SC-FDE systems primarily in order to reduce the complexity inherent to time-domain equalization, is discussed. The black diagrams for OFDM and SC-FDE are compared . which we can see that the only apparent difference between the two systems is that the

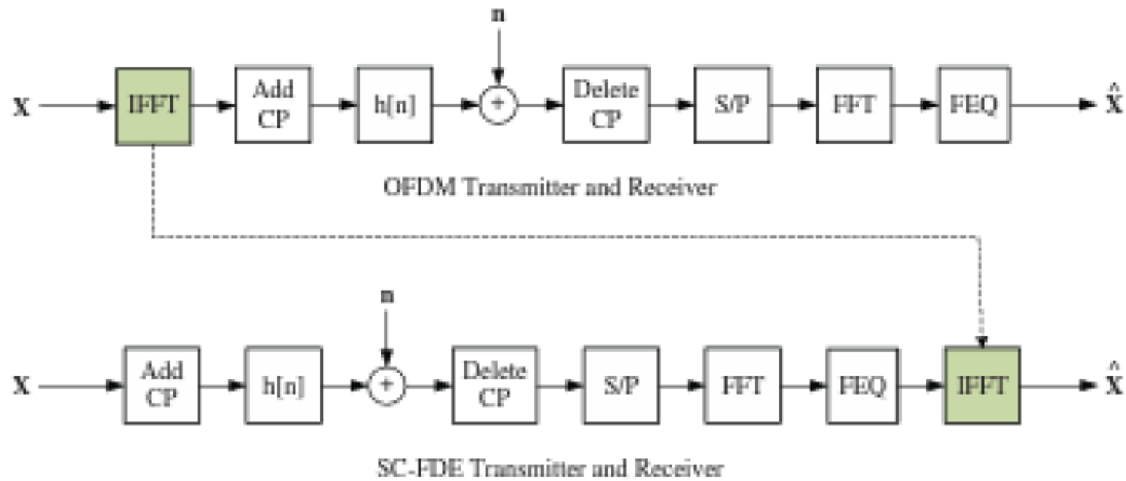


Figure 11: comparison between an OFDM system and an SC-FDE system.

IFFT is moved to the end of the receive chain rather than operating at the transmitter, to create a multicarrier waveform is in OFDM.

An SC-FDE system still utilizes al cyclic prefix at least as long as the channel delay spread, but now the transmitted signal is simply a sequence of QAM symbols, which have low PAR, on the order of 4-5 dB depending on the constellation size. Considering that an un modulated sine wave his a PAR of 3 dB, it is clear that the PAR cannot be lowered much below that of an SC-FDE system.

$$\mathrm{FFT}\{y[n]\} \triangleq Y[m] = H[m]X[m] + W[m]$$

just is in OFDM, with the important distinction being that now the frequency domain version X m is not precisely the data symbols, but rather the FFT of the data symbols ZI. Analogously, recall that in OFDM system the transmitted time-domain signal zn WALS not the actual data symbols, but rather the IFFT of the actual data symbols.

$$\hat{X}[m] = \frac{Y[m]}{H[m]}.$$

The resulting signal can then be converted back into the time domain using an IFFT operation to give in x[n], which are estimates of the desired data symbols. Naturally, in practice *H[m ]*must be estimated at the receiver using pilot signals or other standard methods

## SC-FDE Performance vs. OFDM

The primary difference in terms of performance between SC-FDE and OFDM comes from the way they treat noise. In both OFDM and SC-FDE receivers, the FEQ typically inverts each frequency bir, that is, the FEQ consists of I complex taps each of value $1/H.$ As noted earlier for OFDM in this does not result in damaging noise enhancement since the SNR of each data symbol is unchanged by multiplying by factor. High SNR symbols remain at high SNR, and low SNR symbols remain at low SNR. The discrepancies between the SNR on each carrier can be handled by either per-subcarrier adaptive modulation or coding and interleaving. In LTE, short scale variations in SNR would generally be addressed by coding and interleaving, which would allow 1 considerable number of degraded (low-SNR) symbols to be corrected.
In SC-FDE, however, the FEQ does not operate of data symbols themselves but rather on the frequency domain dual of the data symbols. Therefore, just is in OFDM's FEQ, lo SNR parts of the spectrum have their power increased by a factor of $1/H.2$ while the noise power is increased by a factor of $1/H;$. Unlike in OFDM, however, in SC-FDE when the ensuing IFFT is applied to move the signal back into the time domain for detection, the amplified noise is spread by the IFFT operation over all the data symbols. Therefore, although the total noise amplification is the same in OFDM und SC-FDE, the noise amplification is not isolated to il single symbol in SC-FDE, but instead affects all the symbols prior to decoding and detection.

## Design Considerations for SC-FDE and OFDM

Since the performance difference between SC-FDE and OFDM is not that significant, other considerations are more important in determining which is the appropriate method to use for a given application. An obvious difference is that SC-FDE has a lower complexity transmitter but a higher-complexity receiver, compared to OFDM. Since the receiver was already considerably more complex than the transmitter in a typical OFDM system due to channel estimation, synchronization, and the error correction decoder, this further skews the symmetry:

In a cellular system like LTE, this asymmetry can in fact be a favourable feature, since the uplink could utilize SC-FDE and the downlink could utilize OFDM. In such a situation, the base station would therefore perform 3 IFFT, FFT operations and the mobile, which is more power- and cost-sensitive, would perform only a single FFT operation (to receive its OFDM waveform from the base station). Adding in SC-FDE's benefits of reduced PAR and the commensurate cost and power savings, it appears that the cause for using SC-FDE in the uplink of a wideband data system is favourable indeed.
Channel estimation and synchronization are a bit different in practice for an SC FDE System VS. 111 OFDM system. In a typical wireless OFDM System—including LTE, WiMAX, und 802.11a/b/'n channel estimation and synchronization are accomplished via a preamble of known data symbols, and then pilot tones, which are inserted at known positions in all subsequent OFDM symbols. Although SC-FDE systems would typically also include 1 preamble, this perusable is in the time domain so it is not as straightforward to estimate the frequency domain values H. Similarly, it is not possible to insert pilot tones on a per-frame basis. As we will see, however, SC-FDMA Overcomes these potential problems for LTE by using both a DFT and an IFFT at the transmitter.

**Spatial Diversity Overview**

Diversity is indispensable for reliability in wireless systems. The primary advantage of spatial diversity relative to most forms of time and frequency diversity is that no additional bandwidth or power is needed in order to take advantage of spatial diversity. Instead, spatial diversity is exploited through two or more antennas, which are separated by enough distance so that the fading is approximately decor related between them. The cost of and space consumed by each additional antenna, its RF transmit and/or receive chain, and the associated signal processing required to modulate or demodulate multiple spatial streams may not be negligible. However, for as small number of antennas, the gains are significant enough to warrant the space and expense in most modern wireless systems.

**Array Gain**

When multiple antennas are used, there are two sons of gain available, which we will refer to is diversity gain and any gain. Diversity gain, which will be treated shortly. results from the creation of multiple independent channels between the transmitter and receiver, and is a product of the statistical richness of the channels. Array gain, on the other hand, does not rely on statistical diversity between the different channels. Instead it achieves its performance enhancement by coherently combining the energy of each of the antennas to gain an advantage versus the noise signal on each antenna.

For a $N_t \times N_r$ system, the array gain is $N_r$, which can be seen for a $1 \times N_r$ as follows. In correlated flat fading, each antenna $i \in (1, N_r)$ receives a signal that can be characterized as:

$$y_i = h_i x + n_i = hx + n_i,$$

Where, hi = h for all the antennas since they are perfectly correlated. Hence, the SNR on a single antenna is

$$\gamma_i = \frac{|h|^2}{\sigma^2},$$

where the noise power is $\sigma^2$ and we assume unit signal energy ($\mathcal{E}_x = E|x|^2 = 1$). If all the receive antenna paths are added, the resulting signal is

$$y = \sum_{i=1}^{N_r} y_i = N_r hx + \sum_{i=1}^{N_r} n_i,$$

**Diversity Gain** Traditionally, the main objective of spatial diversity has been to improve the communication reliability by decreasing the sensitivity to finding. The physical layer reliability is typically measured by the outage probability or average bit error rate. In additive noise, the bit error probability (BEP) can be written for virtually any modulation scheme as

$$P_b \approx c_1 e^{-c_2\gamma},$$

This simple inverse relationship between SNR and BEP is much, much weaker than a decaying exponential, which, results in terrible reliability for unmitigated finding channels.

If $N_t$ transmit antennas and $N_r$ receive antennas that are sufficiently spaced[3] are added to the system, it is said that the *diversity order* is $N_d = N_r N_t$, since that is the number of uncorrelated channel paths between the transmitter and receiver. Since the probability of all the $N_d$ uncorrelated channels having low SNR is very small, the diversity order has a dramatic effect on the system reliability. With diversity, the average BEP improves to:

$$\bar{P}_b \approx c_4 \gamma^{-N_d},$$

On the other hand, if only all 11 y pain was possible (for example, if the antennas 1re not sufficiently spaced or the channel is LOS), the average BEP would only decrease from,

$$\bar{P}_b \approx c_5 (N_d \gamma)^{-1},$$

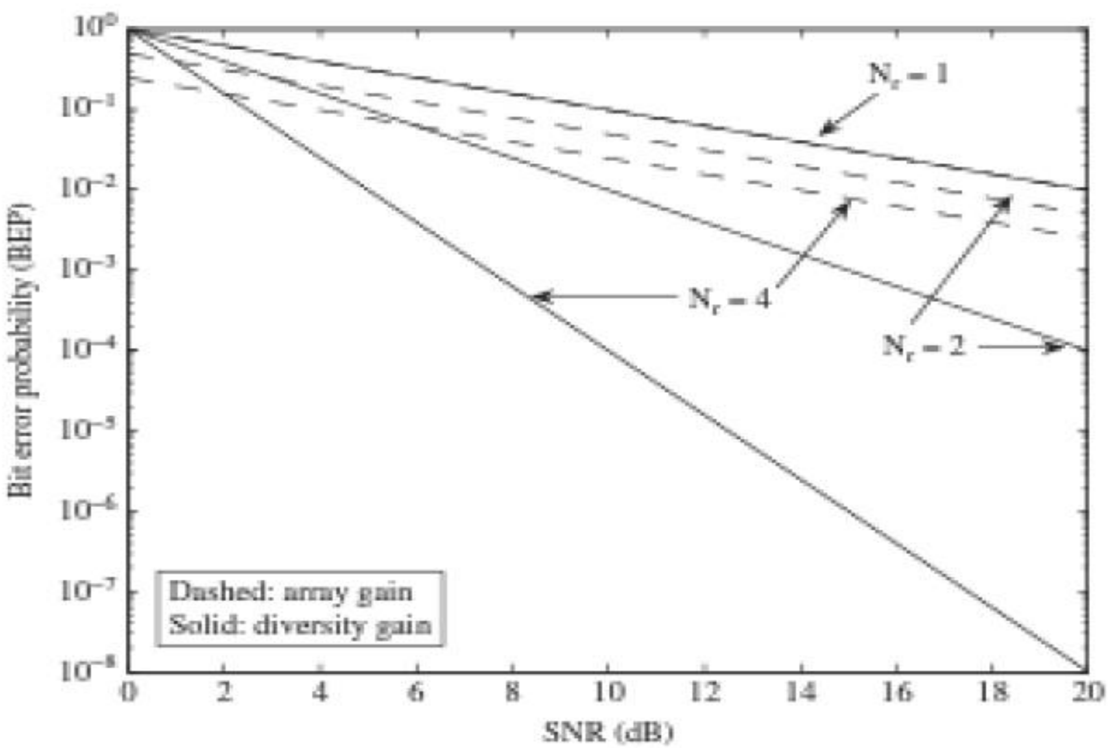


Figure 12: Relative bit error responsibility

## Increasing the Data Rate with Spatial Diversity

As just discussed, diversity techniques are very effective at averaging out fides in the channel and thus increasing the **system** reliability. Receive diversity techniques also in- crease the average received SNR at best linearly due to the ray gain. The Shannon capacity formula gives the maximum achievable data rate of a single communication link in additive white Gaussian noise (AWGN) as:

$$C = B \log_2(1+\gamma),$$

Where C is the capacity." or maximum error-free data mte, B is the bandwidth of the channel, and is again the SNR (ON SINR). Due to advances in coding, and with sufficient diversity, it may be possible to approach the Shannon limit in some wireless channels.

### Receive Diversity

The most prevalent form of spatial diversity is receive diversity, often with just two antennas. This type of diversity is nearly ubiquitous N. = 2 being by far the most common on cellular base stations and wireless LAN access points, and will be mandatory for LTE base stations fid handsets. Receive diversity of its own places to particular requirements on the transmitter, but requires a receiver that processes the N received streams and combines them in some fashion.

Because receive diversity places no requirements on the transmitter, these techniques are not specified in the LTE standard. Nevertheless, they most certainly will be used in nearly all LTE handsets in base stations.

In this section, we will overview two of the widely used combining algorithms, selection combining (SC) and maximal ratio combining (MRC)

### Selection Combining

Selection combining is the simplest type of "combiner," in that it simply estimates the instantaneous strengths of each of the N. streams, and selects the highest one. Since SC ignores the useful EDTEY on the other streams, it is clearly suboptimal, but its simplicity and reduced hardware and power requirements make it attractive for narrowband

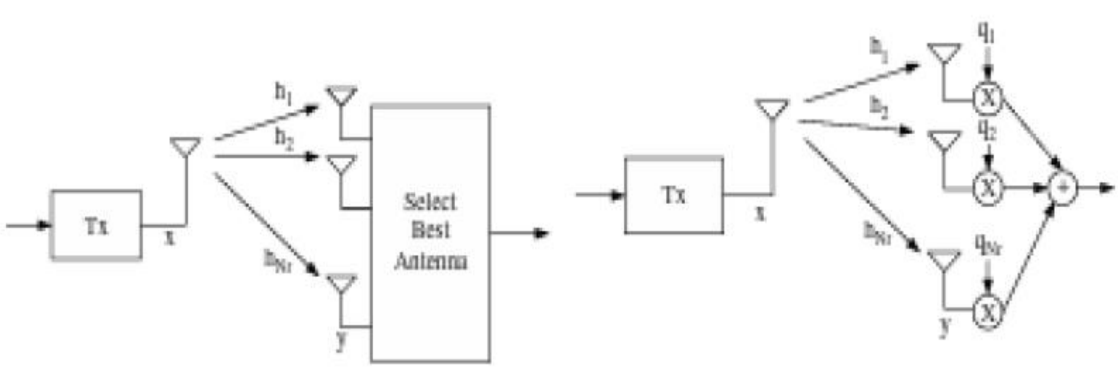


**Figure 13**: Receive diversity :selection combining (left) and maximal combining (right)

The diversity gain from employing selection combining can be confirmed quite quickly by considering the outage probability, defined as the probability that the received SNR drops below some required threshold,

$$\begin{aligned} P_{out} &= P[\gamma_1 < \gamma_o, \gamma_2 < \gamma_o, \ldots, \gamma_M < \gamma_o], \\ &= P[\gamma_1 < \gamma_o]P[\gamma_2 < \gamma_o] \ldots P[\gamma_M < \gamma_o], \\ &= p^{N_r}. \end{aligned}$$

For a Rayleigh fading channel:

$$p = 1 - e^{-\gamma_o/\bar{\gamma}},$$

where is the average received SNR at that location (for example, due to path loss).

Thus, selection combining decreases the outage probability to:

$$P_{out} = (1 - e^{-\gamma_o/\bar{\gamma}})^{N_r}.$$

The average received SNR for N.-branch SC can be derived in Rayleigh fading to be

$$\begin{aligned} \gamma_{sc} &= \gamma \sum_{i=1}^{N_r} \frac{1}{i}, \\ &= \gamma(1 + \frac{1}{2} + \frac{1}{3} + \ \ldots \ + \frac{1}{N_r}). \end{aligned}$$

Hence, although each added (uncorrelated) antenna docs increase the average SNR, it does so with rapidly diminishing returns.

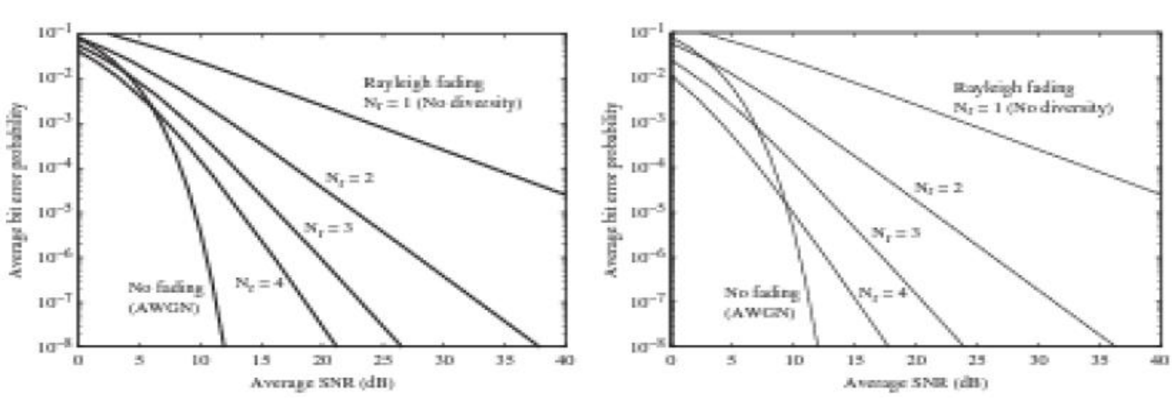


**Figure 14: Average bit error probability for selection combining (left) and maximal ratio combining (right) using coherent BPSK.**