MODULE 4

BALANCING OF ROTATING & RECIPROCATING MASSES

Balancing of Rotating Masses: Balancing of Several Masses Rotating in the Same Plane, Balancing of Several Masses Rotating in Different Planes (only Graphical Methods).

INTRODUCTION:

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If theseparts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimise pressure on the main bearings when an engine is running. Balancing of Rotating Masses We have already discussed, that whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal forces of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called ***balancing of rotating masses*.**

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.

2. Balancing of a single rotating mass by two masses rotating in different planes.

3. Balancing of different masses rotating in the same plane.

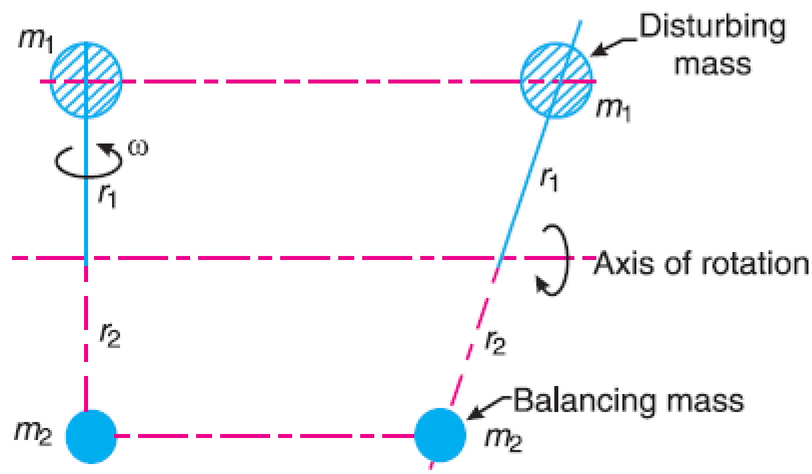
4. Balancing of different masses rotating in different planes.

We shall now discuss these cases, in detail, in the following pages.

BALANCING OF A SINGLE ROTATING MASS BY A SINGLE MASS ROTATING IN
THE SAME PLANE

Consider a disturbing mass *m*1 attached to a shaft rotating at ώ rad/s as shown in Fig. Let *r*1 be the radius of rotation of the mass *m*1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass *m*1). We know that the centrifugal force exerted by the mass *m*1 on the shaft This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (*m*2) may be attached in the same plane of rotation as that of disturbing mass (*m*1) such that the centrifugal forces due to the two masses are equal and opposite.

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$



$r2$ = Radius of rotation of the balancing mass $m2$ (*i.e*. distance between the axis of rotation of the shaft and the centre of gravity of mass $m2$)

Centrifugal force due to mass $m2$,

$$FC2= m2\ \dot{\omega}\ 2.r\ 2$$

$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

BALANCING OF A SINGLE ROTATING MASS BY TWO MASSES ROTATING IN

DIFFERENT PLANES:

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing*.**

2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero. The conditions **(1)** and **(2)** together give ***dynamic balancing*.**
The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses and

2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses. We shall now discuss both the above cases one by one.

1. *When the plane of the disturbing mass lies in between the planes of the two balancing Masses*

Consider a disturbing mass *m* lying in a plane *A* to be balanced by two rotating masses m_1 and m_2 lying in two different planes *L* and *M* as shown in Fig. 2. Let *r*, r_1 and r_2 be the radii of rotation of the masses in planes *A*, *L* and *M* respectively.

Let.

l_1 = Distance between the planes *A* and *L*,

l_2 = Distance between the planes *A* and *M*, and

l = Distance between the planes *L* and *M*.

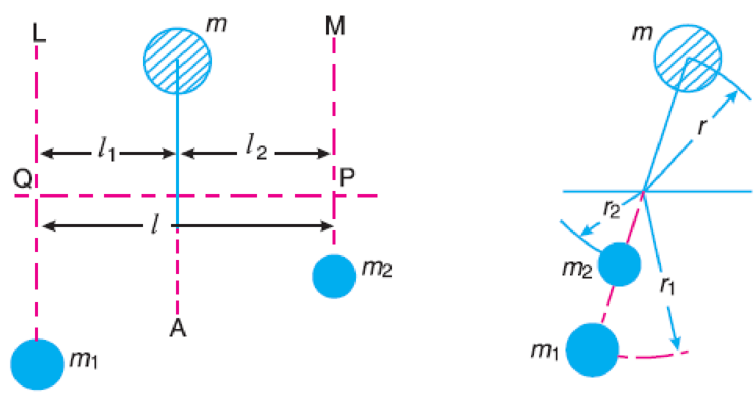


Fig 2. Balancing of a single rotating mass by two rotating masses in different planes when the plane of single rotating mass lies in between the planes of two balancing masses.

We know that the centrifugal force exerted by the mass *m* in the plane *A*,

$$F_C = m.\omega^2.r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane *L*,

$$F_{C1} = m_1.\omega^2.r_1$$

and, the centrifugal force exerted by the mass *m*2 in the plane *M*,

FC2= m2.ὠ2.r2

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1.\omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \quad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \quad \ldots (ii)$$

Similarly, in order to find the balancing force in plane *M* (or the dynamic force at the bearing *P* of a shaft), take moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \quad \ldots (iii)$$

It may be noted that equation **(*i*)** represents the condition for static balance, but in order to

achieve dynamic balance, equations **(*ii*)** or **(*iii*)** must also be satisfied.

2. *When the plane of the disturbing mass lies on one end of the planes of the balancing Masses*

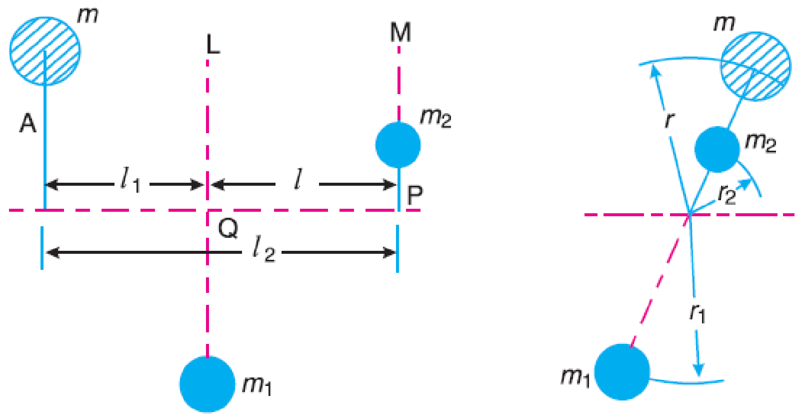


Fig. 3. Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of balancing masses.

In this case, the mass *m* lies in the plane *A* and the balancing masses lie in the planes *L* and *M*, as shown in Fig.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e*.

FC +FC2=F C1

Now, to find the balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation.

Therefore

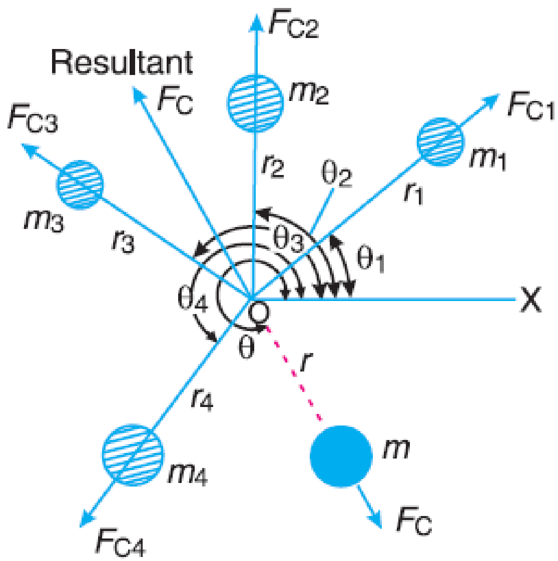
FC1×l=FC×l2

Similarly, to find the balancing force in the plane *M* (or the dynamic force at the bearing *P* of a shaft), take moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation. Therefore

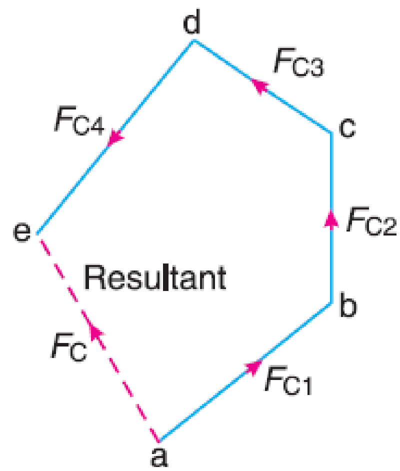
FC2×l=FC×l1

BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE:

Consider any number of masses (say four) of magnitude $m1$, $m2$, $m3$ and $m4$ at distances of $r1$, $r2$, $r3$ and $r4$ from the axis of the rotating shaft. Let masses with the horizontal line OX, as shown in Fig..4(a) Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ὠ rad/s. The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below



(*a*) Space diagram.



(*b*) Vector diagram.

Fig. 4. Balancing of several masses rotating in the same plane.

1.Analytical method:

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e*. Σ *H*and Σ*V*. We know that Sum of horizontal components of the centrifugal forces

$$\Sigma H = m1.r1.\cos\Theta1 + m2.r2.\cos\Theta2 + \ldots..$$

And sum of vertical components of the centrifugal forces.

$$\Sigma V = m1.r1.\sin\Theta1 + m2.r2.\sin\Theta2 + \ldots..$$

3. Magnitude of the resultant centrifugal force

$$FC = \sqrt{(\Sigma H) 2 + (\Sigma V) 2}$$

4. If Θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\Theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction*.**

6. Now find out the magnitude of the balancing mass, such that

FC= m.r

M=balancing mass, and

r = its radius of rotation.

2. *Graphical method:*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig..4 (*a*).

2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.

3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass *m*1 (or *m*1.*r*1) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses $m2$, $m3$ and $m4$ (or $m2.r2$, $m3.r3$ and $m4.r4$).

4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 4 (*b*).

5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***

6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

<u>BALANCING OF SEVERAL MASSES ROTATING IN DIFFERENT PLANES:</u>

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.

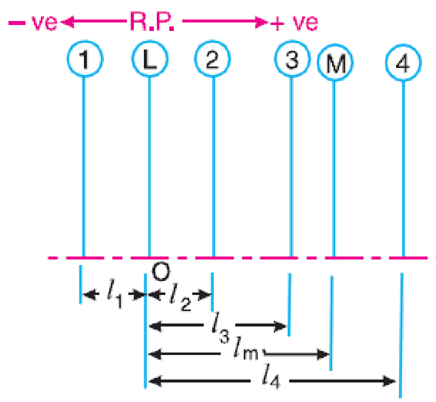
2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

` Let us now consider four masses $m1$, $m2$, $m3$ and $m4$ revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. 5(*a*). The relative angular positions of these masses are shown in the end view [Fig. 5 (*b*)]. The magnitude of the balancing masses m_L and m_M in planes L and M may be obtained as discussed below :

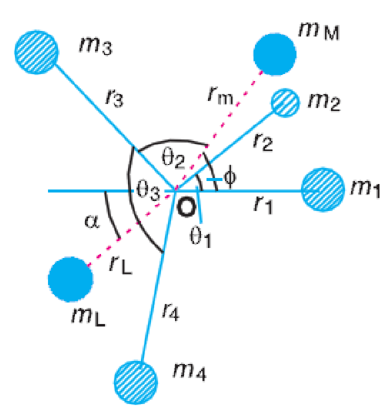
1. Take one of the planes, say L as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive***.

2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

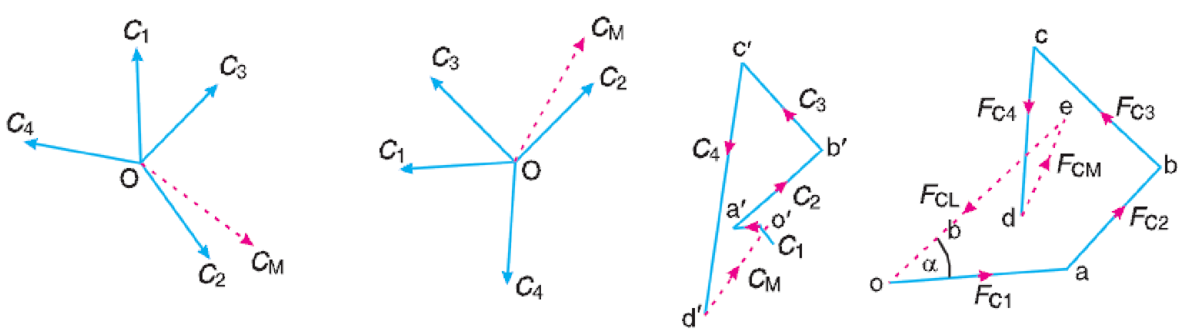
3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is proportional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper.



(*a*) Position of planes of the masses.



(*b*) Angular position of the masses.



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig. 5 Balancing of several masses rotating in different planes.

The vector representing this couple is drawn in the plane of the paper and perpendicular to $Om1$ as shown by $OC1$ in Fig.5 (*c*). Similarly, the vectors $OC2$, $OC3$ and $OC4$ are drawn perpendicular to $Om2$, $Om3$ and $Om4$ respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig.5(*d*). We see that their relative positions remains unaffected. Now the vectors $OC2$, $OC3$ and $OC4$ are parallel and in the same direction as $Om2$, $Om3$ and $Om4$, while the vector $OC1$ is parallel to $Om1$ but in *****opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane*.**

5. Now draw the couple polygon as shown in Fig. 5(*e*). The vector *d' o*' represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination Φ of this mass may be measured from Fig. 5 (*b*).

6. Now draw the force polygon as shown in Fig. 5 (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass mL in the plane L may be obtained and the angle of inclination α of this mass with the horizontal may be measured from fig 5(b).

BALANCING OF RECIPROCATING MASSES

Primary and Secondary Unbalanced Forces of Reciprocating Masses, Partial Balancing of Unbalanced Primary Force in a Reciprocating Engine, Balancing of Primary and secondary Forces of Multi-cylinder In-line Engines, Balancing of Radial Engines (only Graphical Methods)

INTRODUCTION

BALANCING OF RECIPROCATING MASSES:

The various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as ***unbalanced force*** or ***shaking force***. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.

Consider a horizontal reciprocating engine mechanism as shown in Fig.1.

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

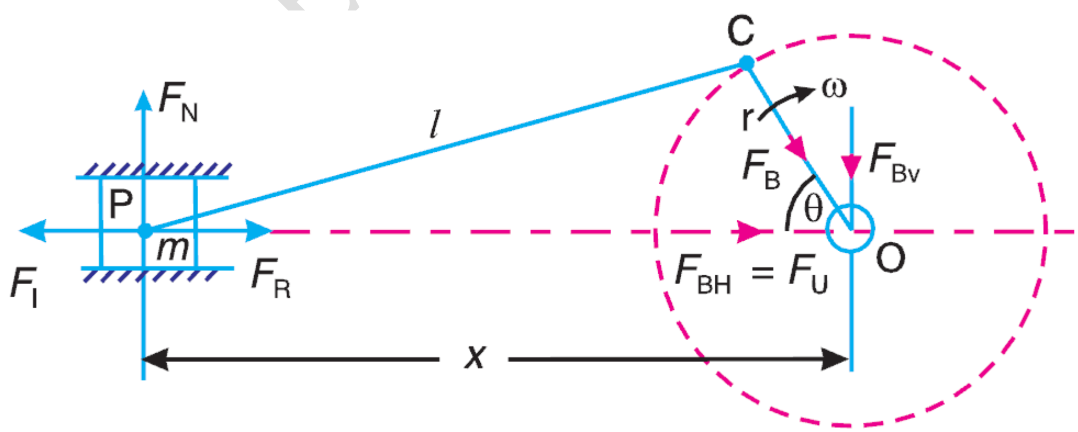


Fig.1. Reciprocating engine mechanism.

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (*i.e.* F_{BH}) acting along the line of reciprocation is also equal and opposite to F_I. This force $F_{BH} = F_U$ is an unbalanced force or shaking force and required to be properly balanced. The force on the sides of the cylinder walls (F_N) and the vertical component of F_B (*i.e.* F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$. From above we see that the effect of the reciprocating parts is to produce a shaking force and a shaking couple. Since the shaking force and a shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations. Thus the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

PRIMARY AND SECONDARY UNBALANCED FORCES OF RECIPROCATING MASSES:

Consider a reciprocating engine mechanism as shown in Fig

m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ= Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

We have already discussed in Art. that the acceleration of the reciprocating parts is approximately given by the expression

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$\therefore$ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

$\therefore$ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force***.

$\therefore$ Primary unbalanced force, $F_P = m \cdot \omega^2 \cdot r\cos\theta$

and secondary unbalanced force, $F_S = m \cdot \omega^2 \cdot r \times \dfrac{\cos 2\theta}{n}$

PARTIAL BALANCING OF UNBALANCED PRIMARY FORCE IN A RECIPROCATING ENGINE:

The primary unbalanced force (m ὼ2r cos ϴ) may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius r, as shown in Fig. 2 The primary force acts from O to P along the line of stroke. Hence, balancing of primary force is considered as equivalent to the balancing of mass m rotating at the crank radius r.

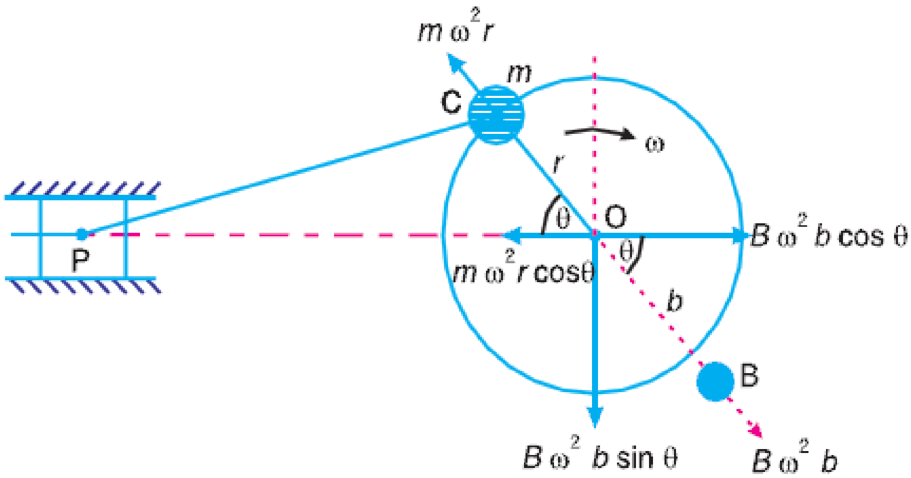


Fig. 2. Partial balancing of unbalanced primary force in a reciprocating engine.

This is balanced by having a mass *B* at a radius *b*, placed diametrically opposite to the crank pin *C*.

We know that centrifugal force due to mass *B*, =B.ὠ2.b and horizontal component of this force acting in opposite direction of primary force = B.ὠ2.bcos ϴ

The primary force is balanced, if

B.ὠ2.bcos ϴ= m.ὠ2.rcos ϴ

A little consideration will show, that the primary force is completely balanced if

B.*b* = *m*.*r*, but

the centrifugal force produced due to the revolving mass *B*, has also a vertical component (perpendicular to the line of stroke) of magnitude *B ὠ* 2*b*sinϴ. This force remains unbalanced. The maximum value of this force is equal to *B*2*b* when ϴ is 90° and 270°, which is same as the maximum value of the primary force *m*2*r* .

From the above discussion, we see that in the first case, the primary unbalanced force acts along the line of stroke whereas in the second case, the unbalanced force acts along the perpendicular to the line of stroke. The maximum value of the force remains same in both the cases. It is thus obvious, that the effect of the above method of balancing is to change the direction of the maximum unbalanced force from the line of stroke to the perpendicular of line of stroke. As a compromise let a fraction '*c*' of the reciprocating masses is balanced, such that

$$c.m.r = B.b$$

Unbalanced force along the line of stroke

$$= m \cdot \omega^2 \cdot r \cos\theta - B \cdot \omega^2 \cdot b \cos\theta$$

$$= m \cdot \omega^2 \cdot r \cos\theta - c \cdot m \cdot \omega^2 \cdot r \cos\theta \qquad \ldots (\because \ B.b = c.m.r)$$

$$= (1 - c) m \cdot \omega^2 \cdot r \cos\theta$$

and unbalanced force along the perpendicular to the line of stroke

$$= B \cdot \omega^2 \cdot b \sin\theta = c \cdot m \cdot \omega^2 \cdot r \sin\theta$$

$\therefore$ Resultant unbalanced force at any instant

$$= \sqrt{\left[(1-c) m \cdot \omega^2 \cdot r \cos\theta\right]^2 + \left[c \cdot m \cdot \omega^2 \cdot r \sin\theta\right]^2}$$

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

BALANCING OF PRIMARY FORCES OF MULTI-CYLINDER IN-LINE ENGINES:

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as ***In-line engines.*** The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multicylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must *close ; and

2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close. We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the ***primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses*****.**

Notes :

1. For a two cylinder engine with cranks at 180°, condition **(1)** may be satisfied, but this will result in an unbalanced couple. Thus the above method of primary balancing cannot be applied in this case.

2. For a three cylinder engine with cranks at 120° and if the reciprocating masses per cylinder aresame, then condition **(1)** will be satisfied because the forces may be represented by the sides of an equilateral triangle. However, by taking a reference plane through one of the cylinder centre lines, two couples with non parallel axes will remain and these cannot vanish vectorially. Hence the above method of balancing fails in this case also.

* The closing side of the primary force polygon gives the maximum unbalanced primary force and the closing side of the primary couple polygon gives the maximum unblanced primary couple

3. For a four cylinder engine, similar reasoning will show that complete primary balance is possible and it follows that **'For a multi-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four'.**

BALANCING OF SECONDARY FORCES OF MULTI-CYLINDER IN-LINE ENGINES:

When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises. We have discussed in Art. that the secondary force,

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

This expression may be written as

$$F_S = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

As in case of primary forces, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length r / $4n$ and revolving at twice the speed of the actual crank (*i.e.* $2\dot{\omega}$) as shown in Fig. 3

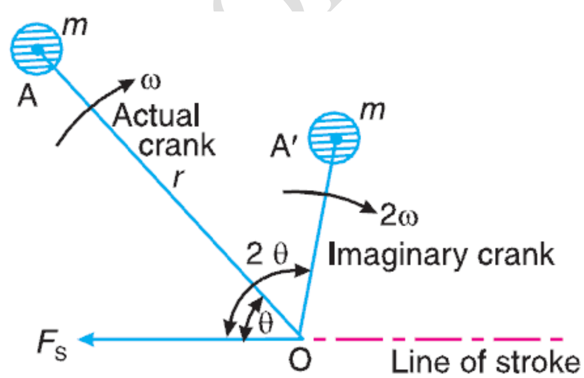


Fig. 3 Secondary force.

Thus, in multi-cylinder in-line engines, each imaginary secondary crank with a mass attached to the crankpin is inclined to the line of stroke at twice the angle of the actual crank. The values of the secondary forces and couples may be obtained by considering the revolving mass. This is done in the similar way as discussed for primary forces. The following two conditions must be satisfied in order to give a complete secondary balance of an engine :

1. The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and

2. The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

Note : The closing side of the secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

BALANCING OF V-ENGINES

Consider a symmetrical two cylinder *V*-engine as shown in Fig. 5, The common crank *OC* is driven by two connecting rods *PC* and *QC*. The lines of stroke *OP* and *OQ* are inclined to the vertical *OY*, at an angle α as shown in Fig 5

m = Mass of reciprocating parts per cylinder,

l = Length of connecting rod,

r = Radius of crank,

n = Ratio of length of connecting rod to crank radius = l / r

Ɵ= Inclination of crank to the vertical at any instant,

ὠ= Angular velocity of crank.

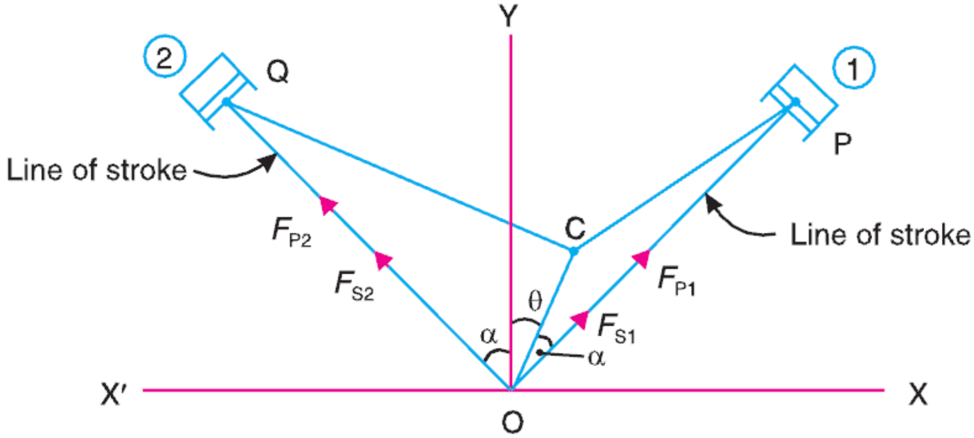


Fig. 5 Balancing of V-engines.

We know that inertia force due to reciprocating parts of cylinder 1, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha-\theta)}{n}\right]$$

and the inertia force due to reciprocating parts of cylinder 2, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha+\theta)}{n}\right]$$

The balancing of *V*-engines is only considered for primary and secondary forces* as discussed below :

Considering primary forces

We know that primary force acting along the line of stroke of cylinder 1

$$F_{P1} = m.\omega^2.r\cos(\alpha - \theta)$$

$\therefore$ Component of F_{P1} along the vertical line OY,

$$= F_{P1}\cos\alpha = m.\omega^2 r.\cos(\alpha - \theta)\cos\alpha \qquad \dots (i)$$

and component of F_{P1} along the horizontal line OX

$$= F_{P1}\sin\alpha = m.\omega^2 r\cos(\alpha - \theta)\sin\alpha \qquad \dots (ii)$$

Similarly, primary force acting along the line of stroke of cylinder 2,

$$F_{P2} = m.\omega^2.r\cos(\alpha + \theta)$$

$\therefore$ Component of F_{P2} along the vertical line OY

$$= F_{P2}\cos\alpha = m.\omega^2.r\cos(\alpha + \theta)\cos\alpha \qquad \dots (iii)$$

and component of F_{P2} along the horizontal line OX'

$$= F_{P2}\sin\alpha = m.\omega^2.r\cos(\alpha + \theta)\sin\alpha \qquad \dots (iv)$$

Total component of primary force along the vertical line OY

$$F_{PV} = (i) + (iii) = m.\omega^2.r\cos\alpha\,[\cos(\alpha - \theta) + \cos(\alpha + \theta)]$$

$$= m.\omega^2.r\cos\alpha \times 2\cos\alpha\cos\theta$$

$$\dots [\because \cos(\alpha - \theta) + \cos(\alpha + \theta) = 2\cos\alpha\cos\theta]$$

$$= 2\,m.\omega^2.r\cos^2\alpha.\cos\theta$$

<u>Considering secondary forces</u>

We know that secondary force acting along the line of stroke of cylinder 1

$$F_{S1} = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n}$$

$\therefore$ Component of F_{S1} along the vertical line OY

$$= F_{S1}\cos\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n} \times \cos\alpha \qquad \dots (ix)$$

and component of F_{S1} along the horizontal line OX

$$= F_{S1}\sin\alpha = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n} \times \sin\alpha \qquad \dots (x)$$

Similarly, secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2 r \times \frac{\cos 2(\alpha + \theta)}{n}$$

$\therefore$ Component of F_{S2} along the vertical line OY

$$= F_{S2} \cos \alpha = m.\omega^2 .r \times \frac{\cos 2(\alpha + \theta)}{n} \times \cos \alpha \qquad \ldots (xi)$$

and component of F_{S2} along the horizontal line OX'

$$= F_{S2} \sin \alpha = m.\omega^2 .r \times \frac{\cos 2(\alpha + \theta)}{n} \times \sin \alpha \qquad \ldots (xii)$$

Total component of secondary force along the vertical line OY,

$$F_{SV} = (ix) + (xi) = \frac{m}{n} \times \omega^2 .r \cos \alpha \, [\cos 2(\alpha - \theta) + \cos 2(\alpha + \theta)]$$

$$= \frac{m}{n} \times \omega^2 .r \cos \alpha \times 2 \cos 2\alpha \cos 2\theta = \frac{2m}{n} \times \omega^2 .r \cos \alpha . \cos 2\alpha \cos 2\theta$$

and total component of secondary force along the horizontal line OX,

$$F_{SH} = (x) - (xii) = \frac{m}{n} \times \omega^2 .r \sin \alpha \, [\cos 2(\alpha - \theta) - \cos 2(\alpha + \theta)]$$

$$= \frac{m}{n} \times \omega^2 .r \sin \alpha \times 2 \sin 2\alpha . \sin 2\theta$$

$$= \frac{2m}{n} \times \omega^2 .r \sin \alpha . \sin 2\alpha . \sin 2\theta$$

$\therefore$ Resultant secondary force,

$$F_S = \sqrt{(F_{SV})^2 + (F_{SH})^2}$$

$$= \frac{2m}{n} \times \omega^2 .r \sqrt{(\cos \alpha . \cos 2\alpha . \cos 2\theta)^2 + (\sin \alpha . \sin 2\alpha . \sin 2\theta)^2} \qquad \ldots (xiii)$$

BALANCING OF RADIAL ENGINES (DIRECT AND REVERSE CRANKS METHOD)

The method of direct and reverse cranks is used in balancing of radial or *V*-engines, in which the connecting rods are connected to a common crank. Since the plane of rotation of the various cranks (in radial or *V*-engines) is same, therefore there is no unbalanced primary or secondary couple.

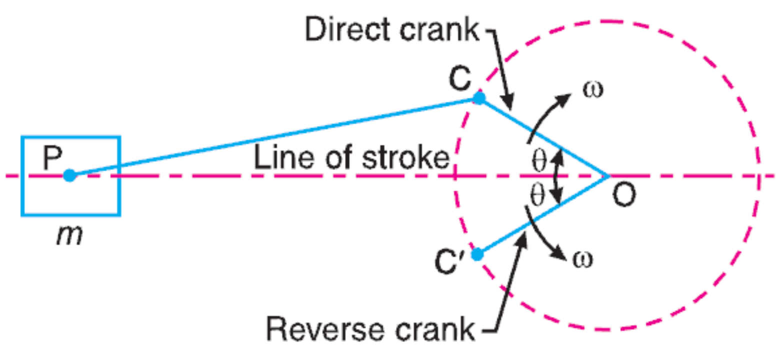


Fig. 4 Reciprocating engine mechanism.

Consider a reciprocating engine mechanism as shown in Fig. 5. Let the crank *OC* (known as the direct crank) rotates uniformly a ω radians per second in a clockwise direction. Let at any instant the crank makes an angle Θ with the line of stroke *OP*. The indirect or reverse crank *OC* □ is the image of the direct crank *OC*, when seen through the mirror placed at the line of stroke. A little consideration will show that when the direct crank revolves in a clockwise direction, the reverse crank will revolve in the anticlockwise direction. We shall now discuss the primary and secondary forces due to the mass (*m*) of the reciprocating parts at *P*.

Considering the primary forces

We have already discussed that primary force is m.ω2.r,cos Θ This force is equal to the component of the centrifugal force along the line of stroke, produced by a mass (*m*) placed at the crank pin *C*. Now let us suppose that the mass (*m*) of the reciprocating parts is divided into two parts, each equal to *m* / 2.

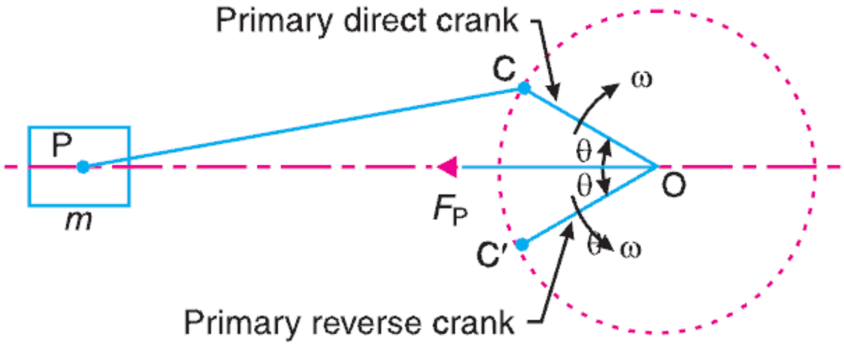


Fig. 5 Primary forces on reciprocating engine mechanism.

It is assumed that m / 2 is fixed at the ***direct crank*** (termed as ***primary direct crank***) pin C and m/ 2 at the ***reverse crank*** (termed as ***primary reverse crank***) pin C' , as shown in Fig. 5 We know that the centrifugal force acting on the primary direct and reverse crank

$$= \frac{m}{2} \times \omega^2 . r$$

∴ Component of the centrifugal force acting on the primary direct crank

$$= \frac{m}{2} \times \omega^2 . r \cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P)$$

and, the component of the centrifugal force acting on the primary reverse crank

$$= \frac{m}{2} \times \omega^2 . r \cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P)$$

∴ Total component of the centrifugal force along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2 . r \cos\theta = m.\omega^2 . r \cos\theta = \text{Primary force, } F_P$$

Hence, for primary effects the mass m of the reciprocating parts at P may be replaced by two masses at C and C' each of magnitude m/2. Considering secondary force *We know that secondary force*

$$= m(2\omega)^2 \frac{r}{4n} \times \cos 2\theta = m.\omega^2 r. \times \frac{\cos 2\theta}{n}$$

In the similar way as discussed above, it will be seen that for the secondary effects, the mass (m) of the reciprocating parts may be replaced by two masses (each m/2) placed at D and D' such that OD = OD□□ = r/4n. The crank OD is the ***secondary direct crank*** and rotates at 2ὼ rad/s in the clockwise direction, while the crank OD' is the ***secondary reverse crank*** and rotates at 2 ὼ rad/s in the anticlockwise direction as shown in Fig. 6

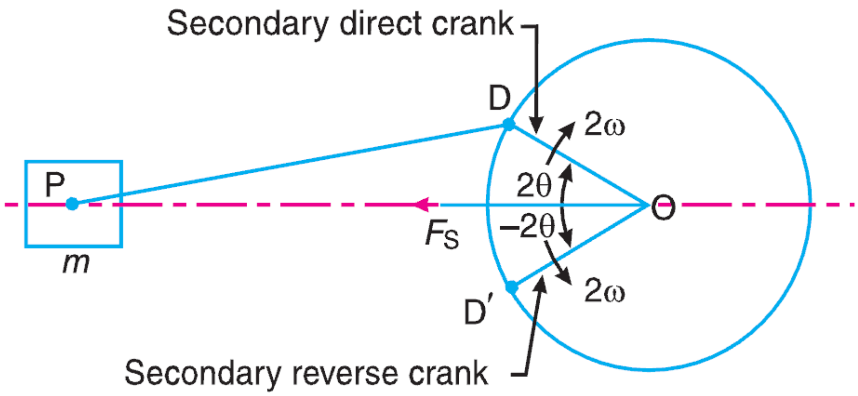


Fig. 6 Secondary force on reciprocating engine mechanism.

IMPORTANT PROBLEMS

Example 1 :Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

Given :

$m1$ = 200 kg ; $m2$ = 300 kg ; $m3$ = 240 kg ; $m4$ = 260 kg ; $r1$ = 0.2 m ;

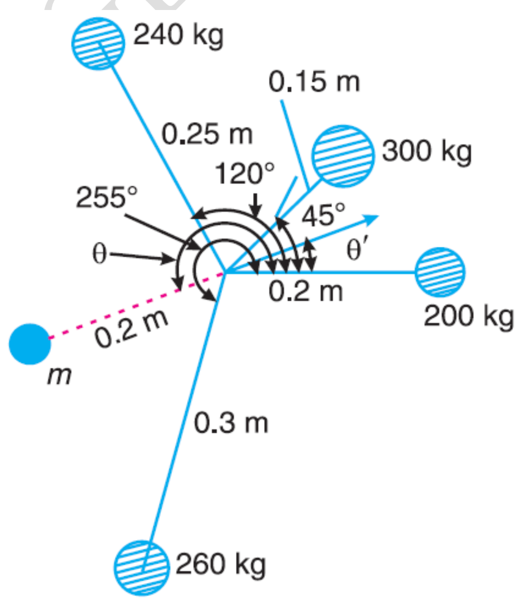
$r2$ = 0.15 m ; $r3$= 0.25 m ; $r4$ = 0.3 m ; ϴ1 = 0° ;ϴ2 = 45° ;ϴ3 = 45° + 75° = 120° ;

ϴ4 = 45° + 75°+ 135° = 255° ; r = 0.2 m

SOLUTION:

M*=balancing mass and*

ϴ= The angle which the balancing mass makes with m



Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius , therefore

- m1.r1=200 *0.2=40kg-m
- m2.r2=300 *0.15=45kg-m
- m3.r3=240 *0.25=60kg-m
- m4.r4=260 *0.3=78kg-m

The problem may, now, be solved either analytically or graphically. But we shall solve the problem by both the methods one by one.

Analytical method :

The space diagram is shown in fig

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$

$$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$

$$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2/r = 23.2/0.2 = 116 \text{ kg}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5/21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48°$$

Graphical method:

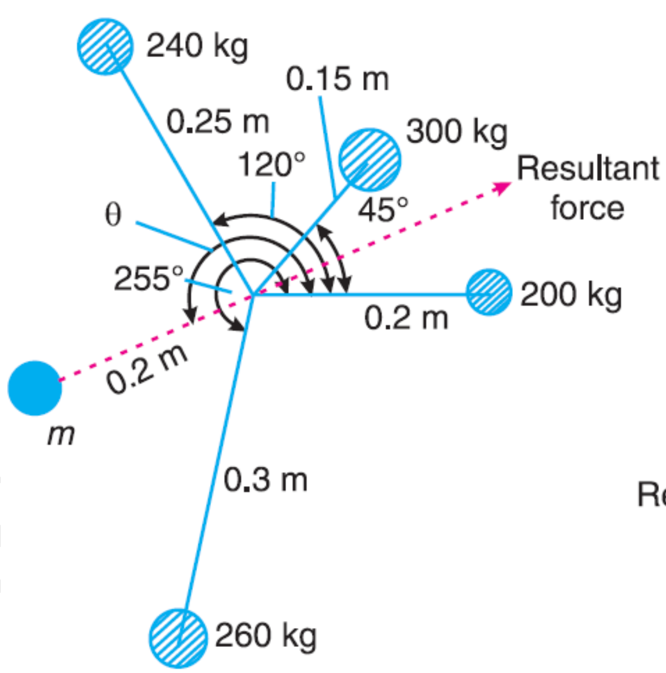
The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses as shown in Fig

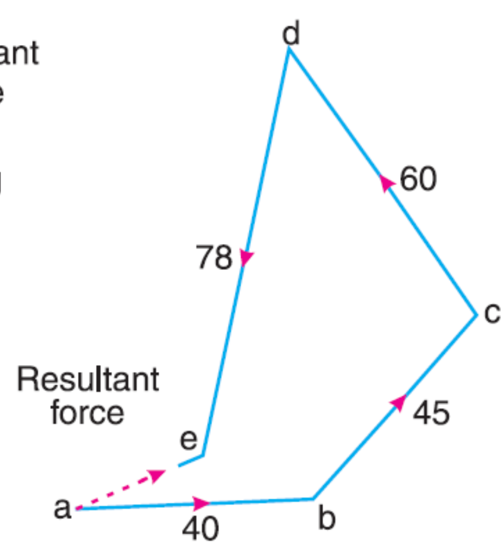
2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

- $m1.r1 = 200 \times 0.2 = 40$ kg-m
- $m2.r2 = 300 \times 0.15 = 45$ kg-m
- $m3.r3 = 240 \times 0.25 = 60$ kg-m
- $m4.r4 = 260 \times 0.3 = 78$ kg-m

3. Now draw the vector diagram with the above values, to some suitable scale, as shown in Fig. The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.



(*a*) Space diagram.



(*b*) Vector diagram

4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. Since the balancing force is proportional to *m.r*, therefore ,

$m \times 0.2$ = vector ea = 23 kg-m or m = 23/0.2 = **115 kg**

By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal mass of 200 kg,

$\theta = 201°$ Ans.

Example2. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; $r_X = r_Y$ = 100 mm = 0.1 m

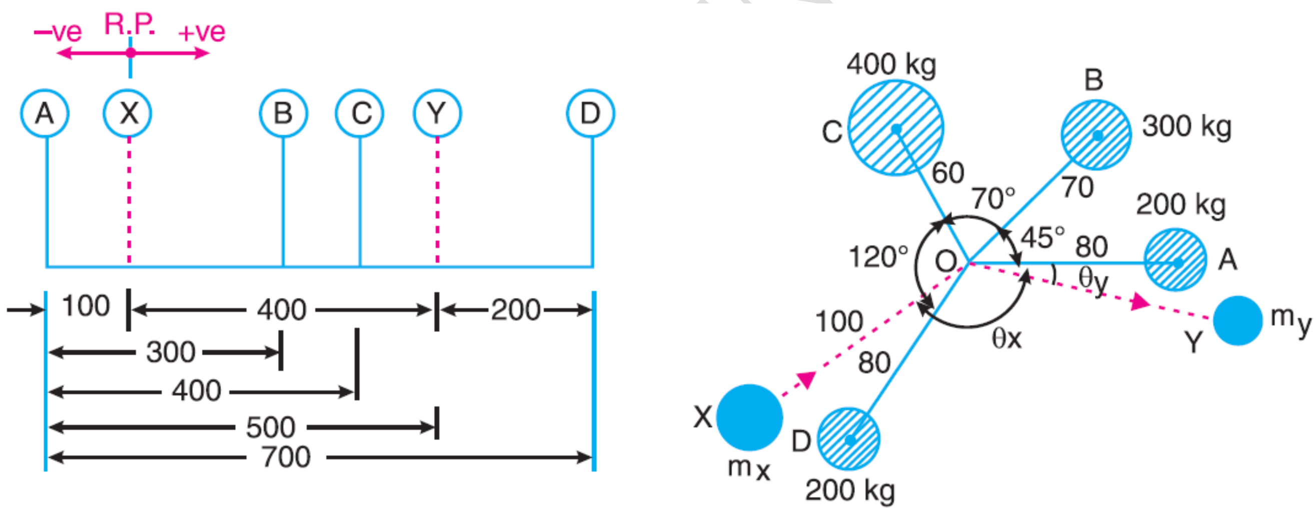
m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. *a*)and (*b*) respectively. Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force ÷ ω^2 (m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m²* (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

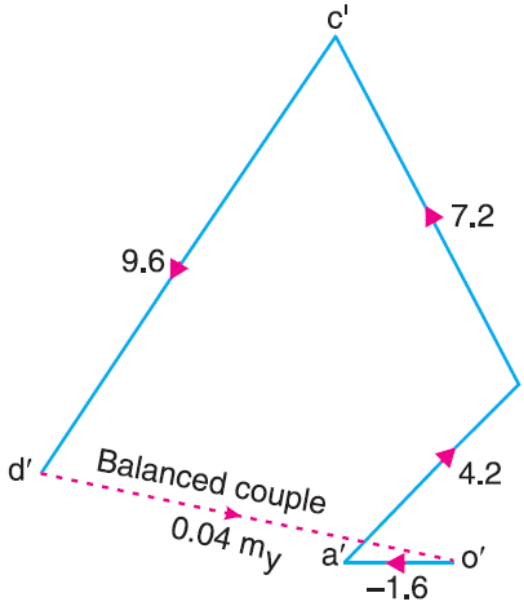
The balancing masses *m*X and *m*Y and their angular positions may be determined graphically as discussed below:



All dimensions in mm.

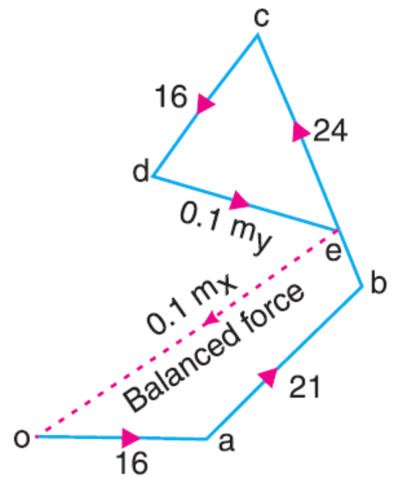
(*a*) Position of planes. (*b*) Angular position of masses.

1. First of all, draw the couple polygon from the data given in Table (column 6) as Shown in Fig.(*c*) to some suitable scale. The vector *d o*□□ □□ represents the balance Couple. Since the balanced couple is proportional to 0.04 *m*Y, therefore by measurement,



(*c*) Couple polygon.

Fig



(*d*) Force polygon.

2. Now draw the force polygon from the data given in Table (column 4) as shown inFig. (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 *m*X, therefore by measurement,

Ans *m*X = **355 kg.**

Example 3. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

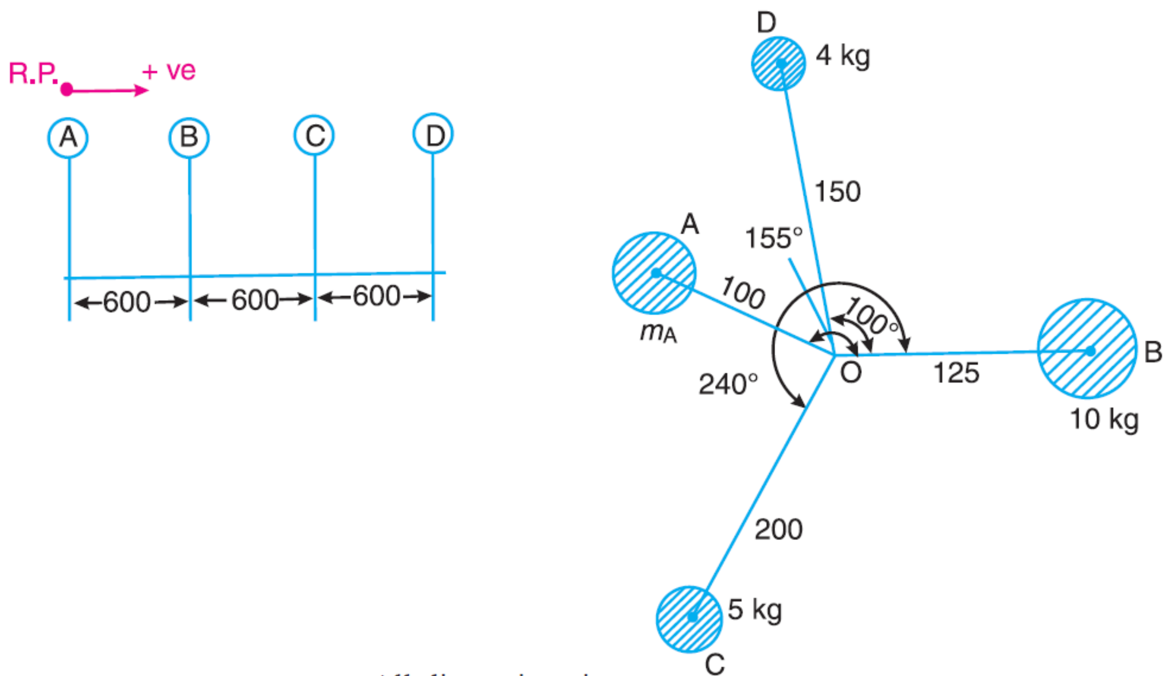
Given : *r*A = 100 mm = 0.1 m ; *r*B = 125 mm = 0.125 m ; *r*C = 200 mm = 0.2 m ; *r*D = 150 mm = 0.15 m ; *m*B = 10 kg ; *m*C = 5 kg ; *m*D = 4 kg

SOLUTION:

The position of planes is shown in Fig. (*a*). Assuming the plane of mass *A* as the Reference plane (*R.P*.), the data may be tabulated as below:

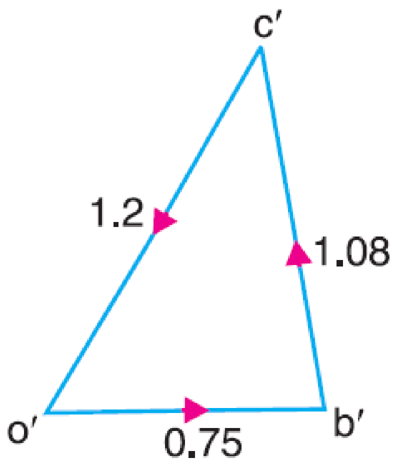
First of all, the angular setting of masses C and D is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass B in the Horizontal direction OB as shown in Fig. (b). Now the couple polygon as shown in Fig. (c) is drawn as discussed below :

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. Force* $\div \omega^2$ *(m.r)kg-m* (4)	*Distance from plane A (l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

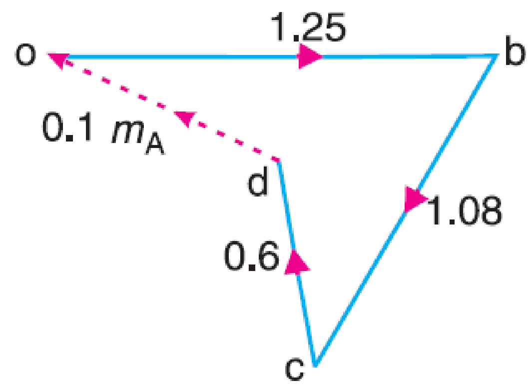


All dimensions in mm

(a) Position of planes. (b) Angular position of masses.



(*c*) Couple polygon.



(*d*) Force polygon.

Fig.

1. Draw vector *o*□ *b*□ in the horizontal direction (*i.e.* parallel to *OB*) and equal to 0.75 kg-m2, to some suitable scale.

2. From points *o*□ and *b*□, draw vectors o□ *c*□ and *b*□ *c*□ equal to 1.2 kg-m2 and 1.08 kg-m2 respectively. These vectors intersect at *c*□.

3. Now in Fig.(*b*), draw *OC* parallel to vector *o*□ *c*□ and *OD* parallel to vector *b*□ *c*□. By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.* □□*BOC* = 240° and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e.* □□*BOD* = 100° In order to find the required mass *A* (*m*A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig. (*d*), from the data given in Table (column 4). Since the closing side of the force polygon (vector *do*) is proportional to 0.1 *m*A, therefore by measurement,

0.1 *m*A = 0.7 kg-m2 or *m*A = 7 kg Ans.

Example 4.A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine : **1.** The magnitude of the masses at A and D ; **2.** the distance between planes A and D ; and **3.** The angular position of the mass at D.

Given : m_B = 18 kg ; m_C = 12.5 kg ; r_B = r_C= 60 mm = 0.06 m ; r_A = r_D = 80 mm = 0.08 m ; □□*BOC* = 100° ; □□*BOA* = 190°

SOLUTION:

1. *Magnitude of the masses at A and D*

The position of the planes and angular position of the masses is shown in Fig. (*a*) and (*b*) respectively. The position of mass *B* is assumed in the horizontal direction, *i.e*. along *OB*. Taking the plane of mass *A* as the reference plane, the data may be tabulated as below :

First of all, the direction of mass *D* is fixed by drawing the couple polygon to some suitable scale, as shown in Fig. (*c*), from the data given in Table (column 6). The closing side of the couple polygon (vector *c o*□□□) is proportional to 0.08 $m_D.x$. By measurement, we find that 0.08 $m_D.x$ = vector *c o* = 0.235 kg-m2

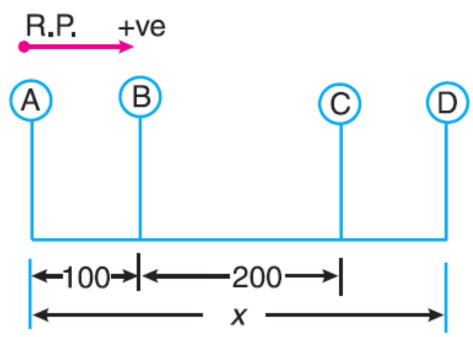
In Fig. (*b*), draw *OD* parallel to vector*c o* to fix the direction of mass *D*.

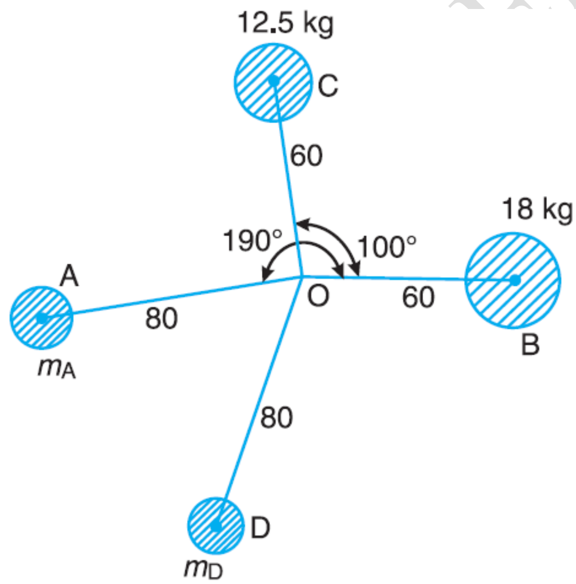
(*i*) Now draw the force polygon, to some suitable scale, as shown in Fig. (*d*), from the data given in Table (column 4), as discussed below :

1. Draw vector *ob* parallel to *OB* and equal to 1.08 kg-m.

2. From point *b*, draw vector *bc* parallel to *OC* and equal to 0.75 kg-m.

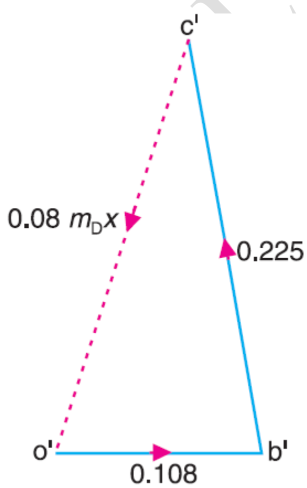
Plane (1)	*Mass* (m) kg (2)	*Eccentricity* (r) m (3)	*Cent. force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from plane A(l)m* (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A (R.P.)	m_A	0.08	0.08 m_A	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	0.08 m_D	x	0.08 m_D . x

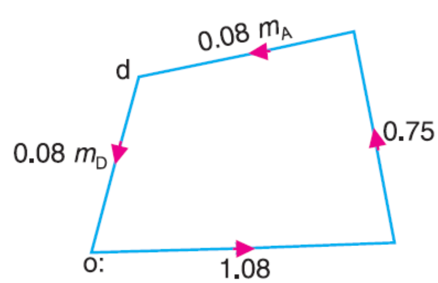




All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.





(c) Couple polygon. **Fig.** (*d*) Force polygon.

3. For the shaft to be in complete dynamic balance, the force polygon must be a closed figure. Therefore from point *c*, draw vector *cd* parallel to *OA* and from point *o* draw vector *od* parallel to *OD*. The vectors *cd* and *od* intersect at *d*. Since the vector *cd* is proportional to 0.08 mA, therefore by measurement 0.08 mA = vector *cd* = 0.77 kg-m or mA = 9.625 kg and vector *do* is proportional to 0.08 mD, therefore by measurement,

$$0.08\ m\mathrm{D} = \text{vector } do = 0.65 \text{ kg-m or } m\mathrm{D} = 8.125 \text{ kg}$$

2. Distance between planes A and D

From equation **(*i*),**

$0.08\ m\mathrm{D}.x = 0.235$ kg-m2

$0.08 \times 8.125 \times x = 0.235$ kg-m2 or $0.65\ x = 0.235$

=361.5 mm

Example 5 *A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If two-third of the reciprocating parts and all the revolving parts are to be balanced, find :* **1.** *The balance mass required at a radius of 400 mm, and* **2.** *The residual unbalanced force when the crank has rotated 60° from top dead centre.*

Solution. Given : N = 240 r.p.m. or $\omega = 2\pi \times 240/60$ = 25.14 rad/s ; Stroke = 300 mm = 0.3 m; m = 50 kg ; m_1 = 37 kg ; r = 150 mm = 0.15 m ; c = 2/3

1. Balance mass required

Let B = Balance mass required, and

b = Radius of rotation of the balance mass = 400 mm = 0.4 m . . . (Given)

We know that

$$B.b = (m_1 + c.m)\,r$$

$$B \times 0.4 = \left(37 + \frac{2}{3} \times 50\right) 0.15 = 10.55 \quad \text{or} \quad B = 26.38 \text{ kg}$$

2. Residual unbalanced force

Let θ = Crank angle from top dead centre = 60° . . . (Given)

We know that residual unbalanced force

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

$$= 50(25.14)^2\, 0.15 \sqrt{\left(1 - \frac{2}{3}\right)^2 \cos^2 60° + \left(\frac{2}{3}\right)^2 \sin^2 60°} \text{ N}$$

$$= 4740 \times 0.601 = 2849 \text{ N}$$

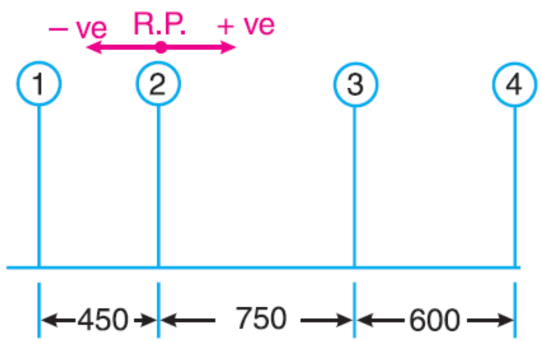
Example 6.A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks. If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force ?

Given : $m1$ = $m4$ = 400 kg ; r = 300 mm = 0.3 m ; l = 1.2 m ; N = 240 r.p.m. or = 25.14 rad/s

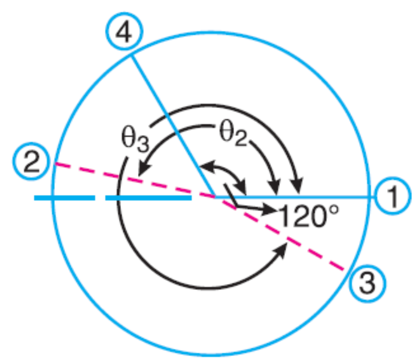
Reciprocating mass and the relative angular position for each of the inner cranks

Let m_2 and m_3 = Reciprocating mass for the inner cranks 2 and 3 respectively, and Θ_2 and Θ_3 = Angular positions of the cranks 2 and 3 with respect to crank 1 respectively. The position of the planes of rotation of the cranks and their angular setting are shown in Fig. (*a*) and (*b*) respectively. Taking the plane of crank 2 as the reference plane, the data may be tabulated as below:

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane (2) (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l.) kg-m²* (6)
1	400	0.3	120	– 0.45	– 54
2(*R.P.*)	m_2	0.3	0.3 m_2	0	0
3	m_3	0.3	0.3 m_3	0.75	0.225 m_3
4	400	0.3	120	1.35	162



(*a*) Positions of planes.

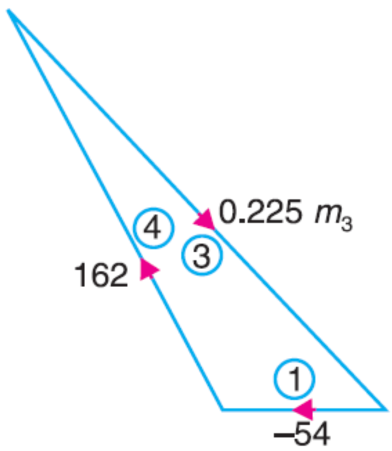


(*b*) Primary crank positions.

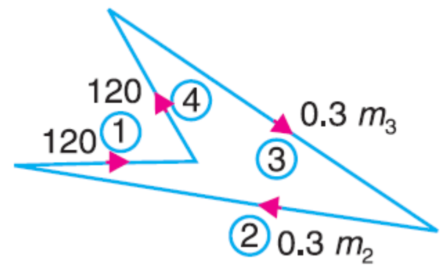
Since the engine is to be in complete primary balance, therefore the primary couple polygon and the primary force polygon must close. First of all, the primary couple polygon, as shown in Fig. (*c*), is drawn to some suitable scale from the data given in Table (column 6), in order to find the reciprocating mass for crank 3. Now by measurement, we find that

0.225 m_3 □ 196 kg-m or m_3= 871 kg

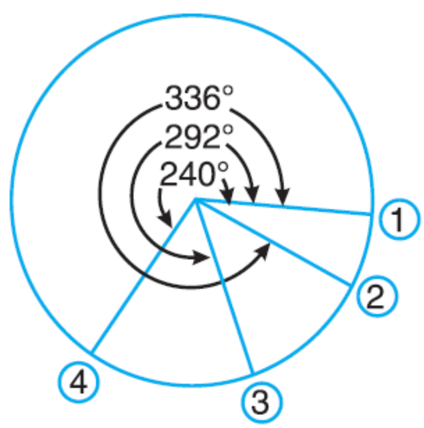
and its angular position with respect to crank 1 in the anticlockwise direction,



(c) Primary couple polygon.

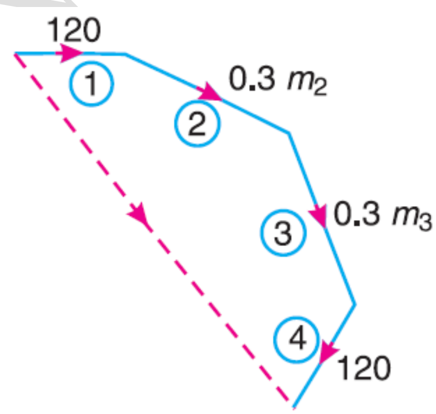


(*d*) Primary force polygon.



(*e*) Secondary crank positions.

Fig.



(*f*) Secondary force polygon.

$$\Theta 3 = 326°$$

Now in order to find the reciprocating mass for crank 2, draw the primary force polygon, as shown in Fig. (*d*), to some suitable scale from the data given in Table (column 4). Now by measurement, we find that

$$0.3\, m2 = 284 \text{ kg-m or } m2 = 947 \text{ kg}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\Theta 2 = 168°$$

Maximum secondary unbalanced force

The secondary crank positions obtained by rotating the primary cranks at twice the angle, is shown in Fig. (*e*). Now draw the secondary force polygon, as shown in Fig. (*f*), to some suitable scale, from the data given in Table (column 4). The closing side of the polygon shown dotted in Fig. (*f*) represents the maximum secondary unbalanced force. By measurement,

we find that

the maximum secondary unbalanced force is proportional to 582 kg-m.

Maximum secondary unbalanced force 91 960N = 91.96 kN

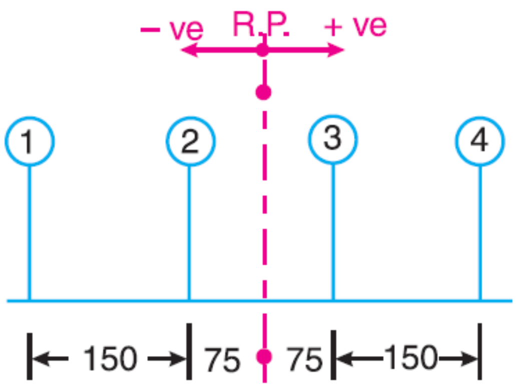
Example 7. The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg. Determine : **1.** Unbalanced primary and secondary forces, if any, and 2.Unbalanced primary and secondary couples with reference to central plane of the engine.

Given : N = 1800 r.p.m. or 2 × 1800/60 = 188.52 rad/s ; r = 60 mm = 0.6 m ; l = 240 mm = 0.24 m ; m = 1.5 kg

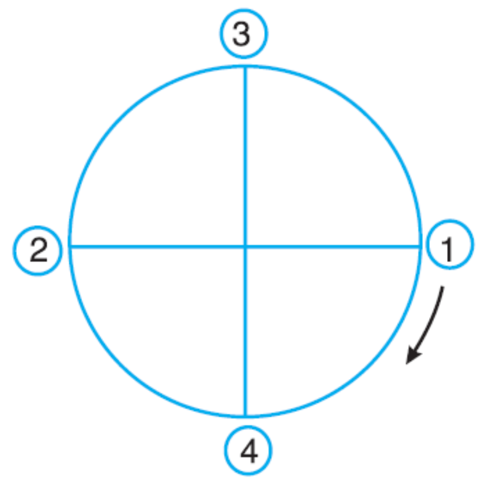
1. ***Unbalanced primary and secondary forces***

The position of the cylinder planes and cranks is shown in Fig. (*a*) and (*b*) respectively. With reference to central plane of the engine, the data may be tabulated as below :

Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent. force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from ref plane 3 (l) m* (5)	*Couple* $\div \omega^2$ (m.r.l.) kg-m^2 (6)
1	1.5	0.6	0.9	– 0.225	– 0.2025
2	1.5	0.6	0.9	– 0.075	– 0.0675
3	1.5	0.6	0.9	+ 0.075	+ 0.0675
4	1.5	0.6	0.9	+ 0.225	+ 0.2025



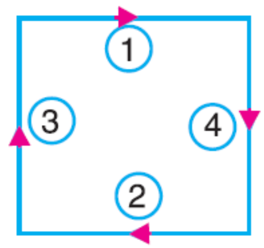
(*a*) Cylinder plane positions.



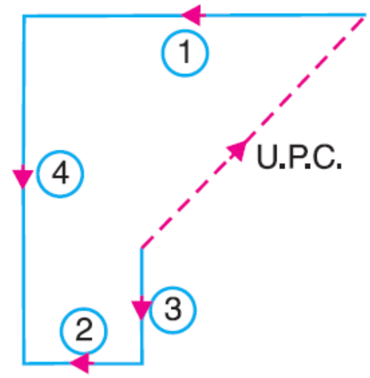
(*b*) Primary crank positions.

The primary force polygon from the data given in Table (column 4) is drawn as shown in Fig. (*c*). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces.

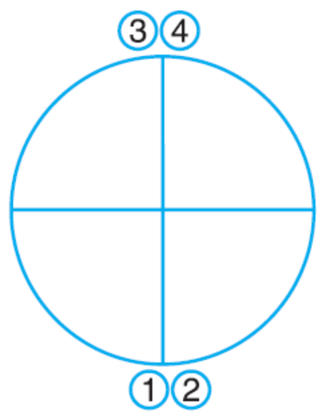
The secondary crank positions', taking crank 3 as the reference crank, is shown in Fig. (*e*). From the secondary force polygon as shown in Fig. (*f*), we see that it is a closed Figure. Therefore there are no unbalanced secondary forces.



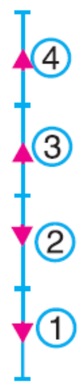
(*c*) Primary force polygon.



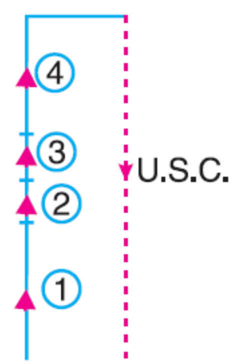
(*d*) Primary couple polygon.



(*e*) Secondary crank positions.



(*f*) Secondary force polygon.



(*g*) Secondary couple polygon.

Fig

2. *Unbalanced primary and secondary couples*

The primary couple polygon from the data given in Table (column 6) is drawn as shown in Fig. (*d*). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, we find the unbalanced primary couple is proportional to 0.19 kg-m2.

Unbalanced primary couple,

U.P.C = 0.19 × □2 = 0.19 (188.52)2 = 6752 N-m

The secondary couple polygon is shown in Fig. 22.1 (*g*). The unbalanced secondary couple is shown by dotted line. By measurement, we find that

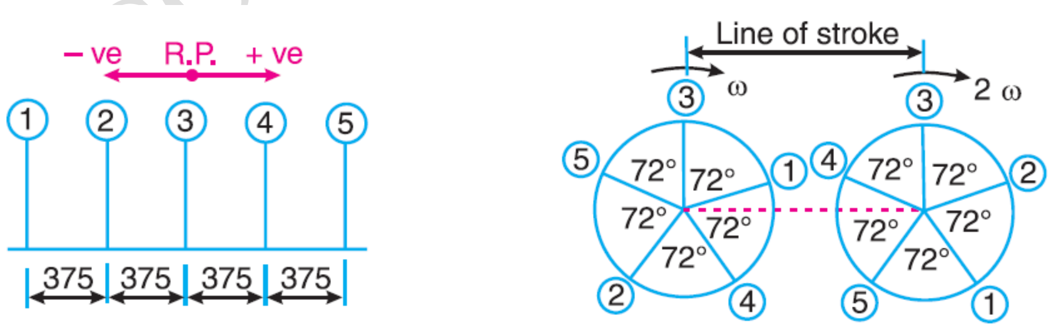
unbalanced secondary couple is proportional to 0.54 kg-m2.

Unbalanced secondary couple, = 4798 N-m

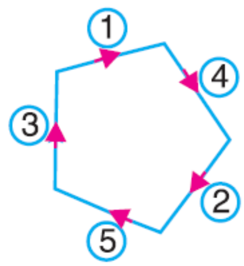
Example 8 A five cylinder in-line engine running at 750 r.p.m. has successive cranks 144° apart, the distance between the cylinder centre lines being 375 mm. The piston stroke is 225 mm and the ratio of the connecting rod to the crank is 4. Examine the engine for balance of primary and secondary forces and couples. Find the maximum values of these and the position of the central crank at which these maximum values occur. The reciprocating mass for each cylinder is 15 kg.

SOLUTION: Assuming the engine to be a vertical engine, the positions of the cylinders and the cranks are shown in Fig. (*a*), (*b*) and (*c*). The plane 3 may be taken as the reference plane and the crank 3 as the reference crank. The data may be tabulated as given in the following table

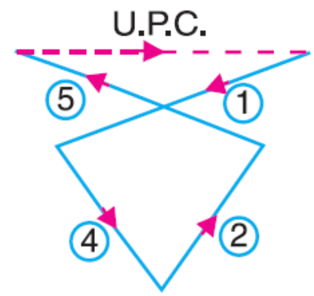
Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from ref. Plane 3 (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
1	15	0.1125	1.6875	– 0.75	– 1.265
2	15	0.1125	1.6875	– 0.375	– 0.6328
3(*R.P.*)	15	0.1125	1.6875	0	0
4	15	0.1125	1.6875	+ 0.375	+ 0.6328
5	15	0.1125	1.6875	+ 0.75	+ 1.265



(*a*) Position of planes. (*b*) Primary crank positions. (*c*) Secondary crank positions.

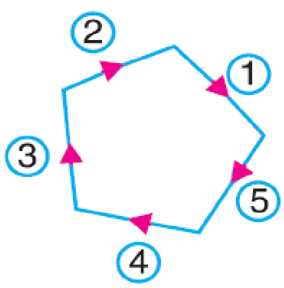


(*d*) Primary force polygon.

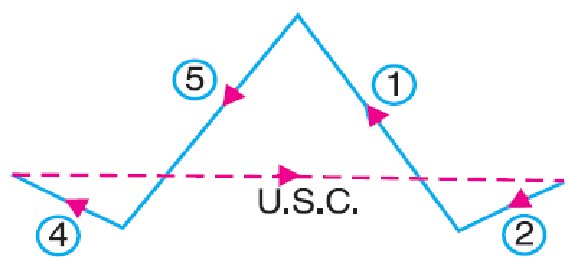


(*e*) Primary couple polygon.

Now, draw the force and couple polygons for primary and secondary cranks as shown in Fig. (*d*), (*e*), (*f*), and (*g*). Since the primary and secondary force polygons are close, therefore the engine is balanced for primary and secondary forces



(*f*) Secondary force polygon.



(*g*) Secondary couple polygon.

Maximum unbalanced primary couple

We know that the closing side of the primary couple polygon [shown dotted in Fig. (*e*)] gives the maximum unbalanced primary couple. By measurement, we find that maximum unbalanced primary couple is proportional to 1.62 kg-m2.

Maximum unbalanced primary couple,

$U.P.C. = 1.62 \times \square 2 = 1.62\ (78.55)2 = 9996$ N-m

We see from Fig. (*e*) [shown by dotted line] that the maximum unbalanced primary couple occurs when crank 3 is at 90° from the line of stroke.

Maximum unbalanced secondary couple

We know that the closing side of the secondary couple polygon [shown dotted in Fig. (*g*)] gives the maximum unbalanced secondary couple. By measurement, we find that

maximum unbalanced secondary couple is proportional to 2.7 kg-m2.

Maximum unbalanced secondary couple. = 4165 N-m

We see from Fig. (*g*) that if the vector representing the unbalanced secondary couple (shown by dotted line) is rotated through 90°, it will coincide with the line of stroke. Hence the original crank will be rotated through 45°. Therefore, the maximum unbalanced secondary couple occurs when crank 3 is at 45° and at successive intervals of 90° (*i.e*. 135°, 225° and 315°) from the line of stroke.

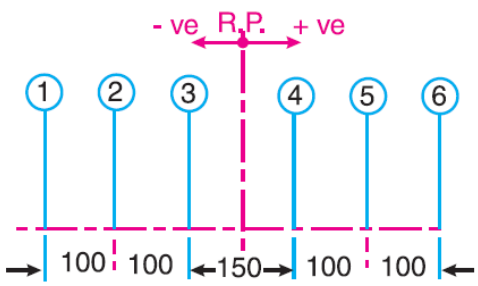
Example 9: The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m. Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.

Given : L = 100 mm or $r = L / 2$ = 50 mm = 0.05 m ; l = 200 mm ; m = 1 kg ; N = 3000 r.p.m.

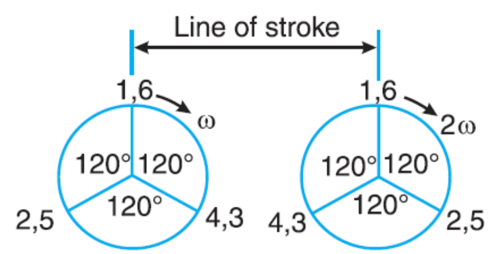
SOLUITON:

The position of the cylinders and the cranks are shown in Fig. (*a*), (*b*) and (*c*). With the reference plane midway between the cylinders 3 and 4, the data may be tabulated as given in the following table :

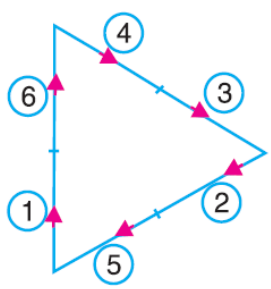
Plane *(1)*	*Mass* *(m) kg* *(2)*	*Radius* *(r) m* *(3)*	*Cent. force* $\div \omega^2$ *(m.r) kg-m* *(4)*	*Distance from* *plane 3 (l)m* *(5)*	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* *(6)*
1	1	0.05	0.05	– 0.275	– 0.01375
2	1	0.05	0.05	– 0.175	– 0.00875
3	1	0.05	0.05	– 0.075	– 0.00375
4	1	0.05	0.05	+ 0.075	+ 0.00375
5	1	0.05	0.05	+ 0.175	+ 0.00875
6	1	0.05	0.05	+ 0.275	+ 0.01375



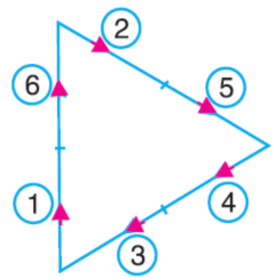
(*a*) Positions of planes.



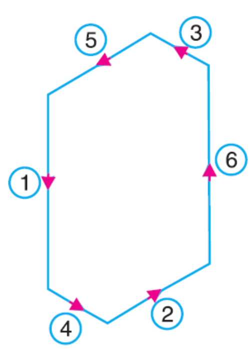
(*b*) Primary crank positions. (*c*) Secondary crank positions.



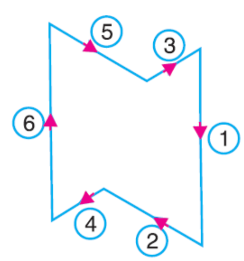
(*d*) Primary force polygon.



(*e*) Secondary force polygon.



(*f*) Primary couple polygon.



(g) Secondary couple polygon.

Now, draw the force and couple polygons for the primary and secondary cranks as shown in Fig.(*d*), (*e*), (*f*) and (*g*). From Fig. (*d*) and (*e*), we see that the primary and secondary force polygons are closed figures, therefore there are no out-of-balance primary and secondary forces. Thus the engine is balanced for primary and secondary forces. Also, the primary and secondary couple polygons, as shown in Fig. (*f*) and (*g*) are closed figures, therefore there are no out-of-balance primary and secondary couples. Thus the engine is balanced for primary and secondary couples.

Example10. A vee-twin engine has the cylinder axes at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius is 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be balanced for primary forces by means of a revolving balance mass. If the engine speed is 500 r.p.m. What is the value of maximum resultant secondary force ?

SOLUTION:

We know that resultant primary force,

$$F_P = 2m.\omega^2.r\sqrt{(\cos^2\alpha\cos\theta)^2 + (\sin^2\alpha\sin\theta)^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2 45°\cos\theta)^2 + (\sin^2 45°\sin\theta)^2}$$

$$= 2m.\omega^2.r\sqrt{\left[\frac{\cos\theta}{2}\right]^2 + \left[\frac{\sin\theta}{2}\right]^2} = m.\omega^2.r$$

Since the resultant primary force $m.\omega^2.r$ is the centrifugal force of a mass m at the crank radius r when rotating at ω rad / s, therefore, the engine may be balanced by a rotating balance mass.

Maximum resultant secondary force

We know that resultant secondary force,

$$F_S = \sqrt{2} \times \frac{m}{n} \times \omega^2 . r \sin 2\theta \qquad \text{. . . (When } 2\alpha = 90°)$$

This is maximum, when $\sin 2\theta$ is maximum *i.e.* when $\sin 2\theta = \pm 1$ or $\theta = 45°$ or $135°$.

$\therefore$ Maximum resultant secondary force,

$$F_{S_{max}} = \sqrt{2} \times \frac{m}{n} \times \omega^2 . r \qquad \text{. . . (Substituting } \theta = 45°)$$

$$= \sqrt{2} \times \frac{11.5}{0.3/0.075} (52.37)^2 0.075 = 836 \text{ N} \qquad \text{. . . } (\because n = l/r)$$

Example 11 The reciprocating mass per cylinder in a 60° V-twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 r.p.m., determine the maximum and minimum values of the primary and secondary forces. Also find out the crank position corresponding these values.

Maximum and minimum values of primary forces

We know that the resultant primary force,

$$F_P = 2m.\omega^2 .r\sqrt{(\cos^2 \alpha \cdot \cos\theta)^2 + (\sin^2 \alpha \cdot \sin\theta)^2}$$

$$= 2m.\omega^2 .r\sqrt{(\cos^2 30° \cos\theta)^2 + (\cos^2 30° \sin\theta)^2}$$

$$= 2m\omega^2 r\sqrt{\left(\frac{3}{4}\cos\theta\right)^2 + \left(\frac{1}{4}\sin\theta\right)^2}$$

$$= \frac{m}{2} \times \omega^2 r\sqrt{9\cos^2\theta + \sin^2\theta} \qquad \text{...}(i)$$

The primary force is maximum, when $\theta = 0°$. Therefore substituting $\theta = 0°$ in equation (*i*), we have maximum primary force,

$$F_{P(max)} = \frac{m}{2} \times \omega^2 r \times 3 = \frac{1.5}{2}(261.8)^2 0.05 \times 3 = 7710.7 \text{ N} \quad \textbf{Ans.}$$

The primary force is minimum, when $\theta = 90°$. Therefore substituting $\theta = 90°$ in equation (*i*), we have minimum primary force,

$$F_{P(min)} = \frac{m}{2} \times \omega^2 r = \frac{1.5}{2} (261.8)^2 \, 0.05 = 2570.2 \text{ N} \quad \textbf{Ans.}$$

Maximum and minimum values of secondary forces

We know that resultant secondary force.

$$F_S = \frac{2m}{n} \times \omega^2 \sqrt{(\cos\alpha \cos 2\alpha \cos 2\theta)^2 + (\sin\alpha \sin 2\alpha \sin 2\theta)^2}$$

$$= \frac{2m}{n} \times \omega^2 r \sqrt{(\cos 30° \cos 60° \cos 2\theta)^2 + (\sin 30° \sin 60° \sin 2\theta)^2}$$

$$= \frac{2m}{n} \times \omega^2 r \sqrt{\left(\frac{\sqrt{3}}{2} \times \frac{1}{2} \cos 2\theta\right)^2 + \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} \sin 2\theta\right)^2}$$

$$= \frac{\sqrt{3}}{2} \times \frac{m}{n} \times \omega^2 r$$

$$= \frac{\sqrt{3}}{2} \times \frac{1.5}{0.25/0.05} (261.8)^2 \, 0.05 \qquad \ldots (\because n = l/r)$$

$$= 890.3 \text{ N}$$