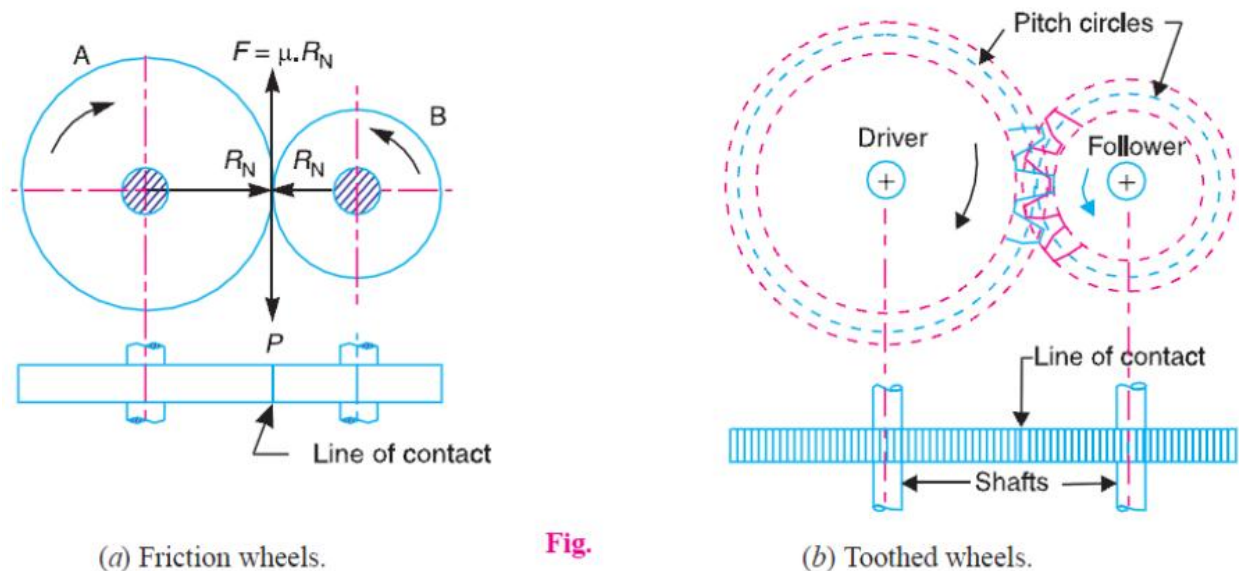


ACS COLLEGE OF ENGINEERING
MODULE 3
SPUR GEARS AND GEAR TRAINS

Spur Gears: Gear terminology, law of gearing, Path of contact, Arc of contact, Contact ratio of spur gear, Interference in involute gears, Methods of avoiding interference.

INTRODUCTION

Spur gears are used to transmit power between parallel shafts and to change rotation speeds. Speed ratios are precise, and gear systems can be designed for high power; however, shafts that carry gears must be located precisely. A gear drive is also provided, when the distance between the driver and the follower is very small. Friction Wheels The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be Transmitted by two toothed wheels, consider two plain circular wheels *A* and *B* mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in Fig Let the wheel *A* be keyed to the rotating shaft and the wheel *B* to the shaft, to be rotated. A little consideration will show, that when the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B* in the opposite direction as shown in Fig.(a)



The wheel *B* will be rotated (by the wheel *A*) so long as the tangential force exerted by the wheel *A* does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (*P*) exceeds the *frictional resistance (*F*), slipping will take place between the two wheels. Thus the friction drive is not a positive drive. In order to avoid the

ACS COLLEGE OF ENGINEERING

slipping, a number of projections (called teeth) as shown in Fig.(b), are provided on the periphery of the wheel *A*, which will fit into the corresponding recesses on the periphery of the wheel *B*. A friction wheel with the teeth cut on it is known as **toothed wheel or gear**. The usual connection to show the toothed wheels is by their **pitch circles**.

Note : Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers

ADVANTAGES AND DISADVANTAGES OF GEAR DRIVE

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

CLASSIFICATION OF TOOTHED WHEELS

The gears or toothed wheels may be classified as follows :

1. According to the position of axes of the shafts. The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 1. These gears are called **spur gears** and the arrangement is known as **spur gearing**. These gears have teeth parallel to the axis of the wheel as shown in Fig. 1. Another name given to the spur gearing is **helical gearing**, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 1 (a) and (b) respectively. The double helical gears are known as **herringbone gears**. A pair of spur gears are kinematically equivalent to a pair of

ACS COLLEGE OF ENGINEERING

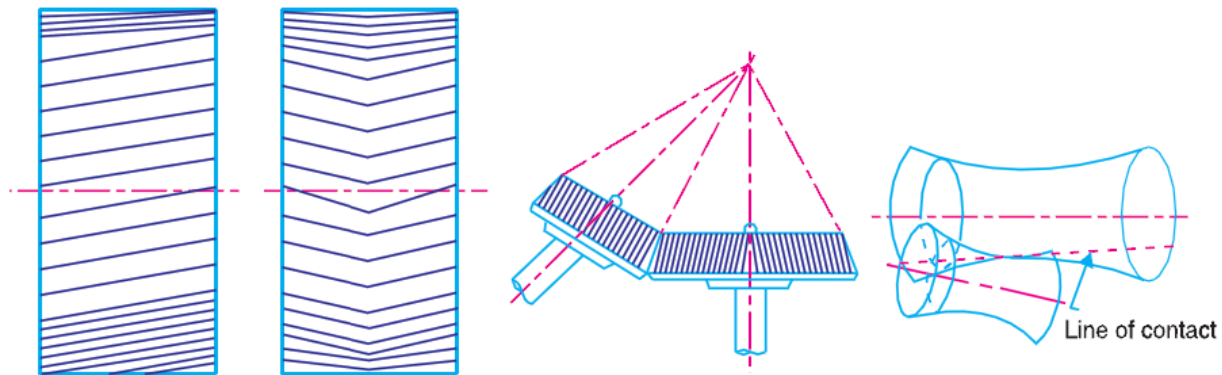
cylindrical discs, keyed to parallel shafts and having a line contact. The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 1(c). These gears are called **bevel gears** and the arrangement is known as **bevel gearing**. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as **helical bevel gears**. The two non-intersecting and non-parallel *i.e.* non-coplanar shaft connected by gears is shown in Fig. 1(d). These gears are called **skew bevel gears** or **spiral gears** and the arrangement is known as **skew bevel gearing** or **spiral gearing**. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as **hyperboloids**.

Notes :

(a) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as **mitres**.

(b) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(c) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.



(a) Single helical gear. (b) Double helical gear. **Fig. 1** (c) Bevel gear. (d) Spiral gear.

According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as :

(a) Low velocity, (b) Medium velocity, and (c) High velocity.

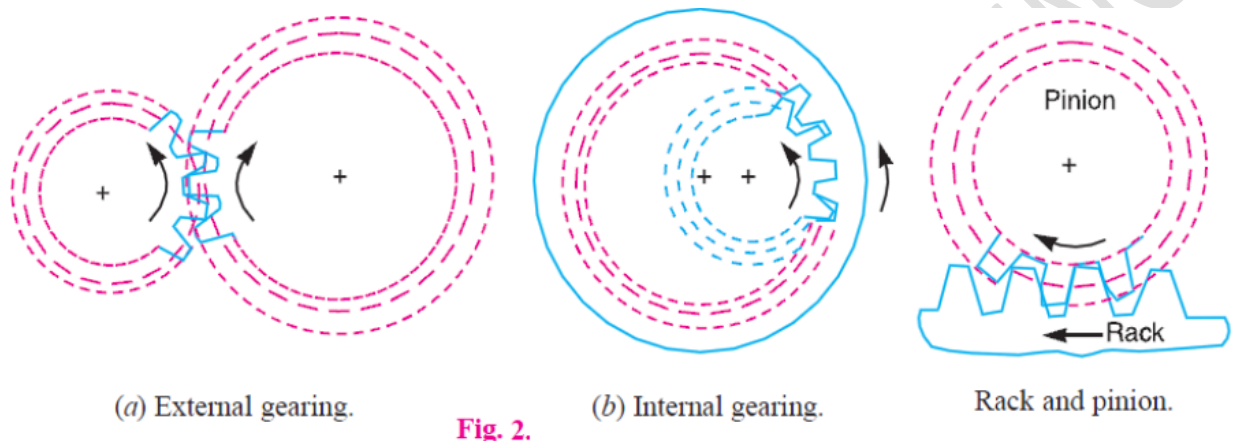
The gears having velocity less than 3 m/s are termed as **low velocity** gears and gears having velocity between 3 and 15 m/s are known as **medium velocity gears**. If the velocity of gears is more than 15 m/s, then these are called **high speed gears**.

ACS COLLEGE OF ENGINEERING

3. According to the type of gearing. The gears, according to the type of gearing may be classified as :

(a) External gearing, (b) Internal gearing, and (c) Rack and pinion.

In **external gearing**, the gears of the two shafts mesh externally with each other as shown in Fig.2(a). The larger of these two wheels is called **spur wheel** and the smaller wheel is called **pinion**. In an external gearing, the motion of the two wheels is always **unlike**, as shown in Fig. 2(a).



In **internal gearing**, the gears of the two shafts mesh **internally** with each other as shown in Fig 2 (b). The larger of these two wheels is called **annular wheel** and the smaller wheel is called **pinion**. In an internal gearing, the motion of the two wheels is always **like**, as shown in Fig. 2(b). Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. 2 Such type of gear is called **rack and pinion**. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and **vice-versa** as shown in Fig. 2

4. According to position of teeth on the gear surface.

The teeth on the gear surface may be

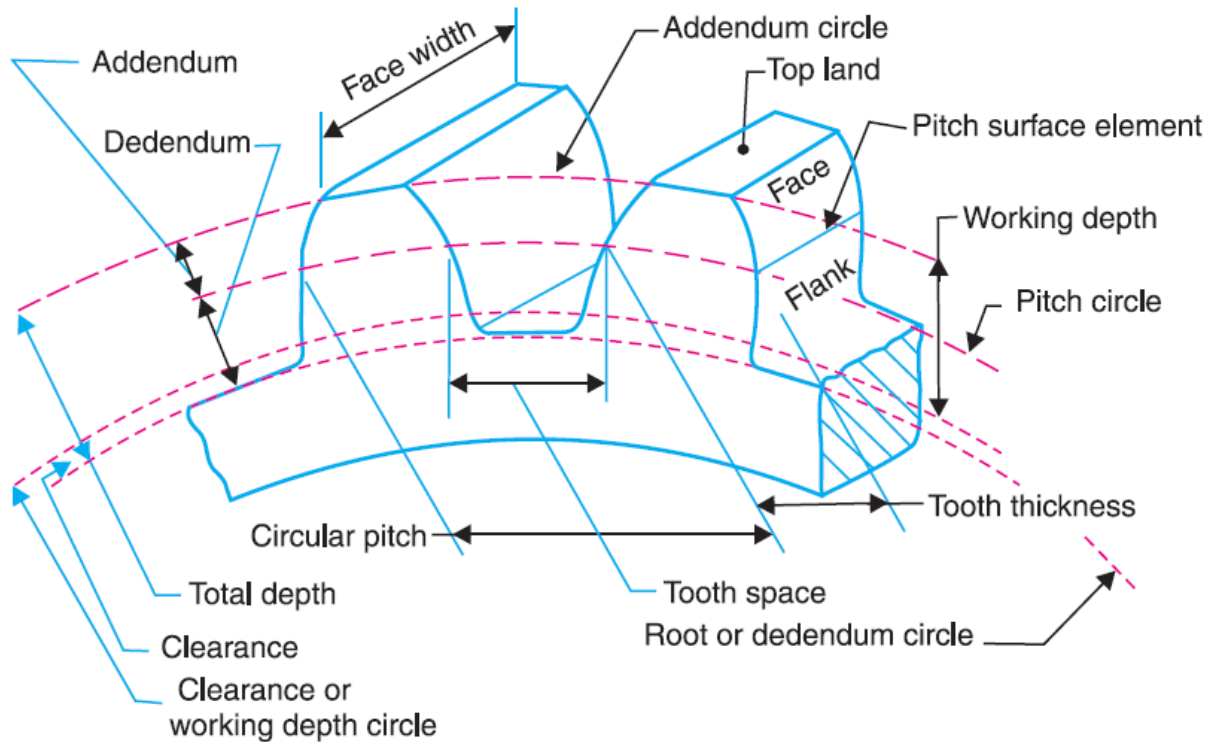
(a) straight, (b) inclined, and (c) curved.

We have discussed earlier that the spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

ACS COLLEGE OF ENGINEERING

NOMENCLATURE:

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 3



1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by Φ

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

ACS COLLEGE OF ENGINEERING

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

10. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by P_c . Circular pitch, $P_c = \pi D / T$. A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by P_d . $P_d = \pi D_1 / T_1$.

12. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m . Mathematically, Module, $m = D / T$

13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as **clearance circle**.

14. Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness. It is the width of the tooth measured along the pitch circle.

17. Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. Face of tooth. It is the surface of the gear tooth above the pitch surface.

20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.

21. Top land. It is the surface of the top of the tooth.

22. Face width. It is the width of the gear tooth measured parallel to its axis.

23. Profile. It is the curve formed by the face and flank of the tooth.

24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.

ACS COLLEGE OF ENGINEERING

25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. *Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

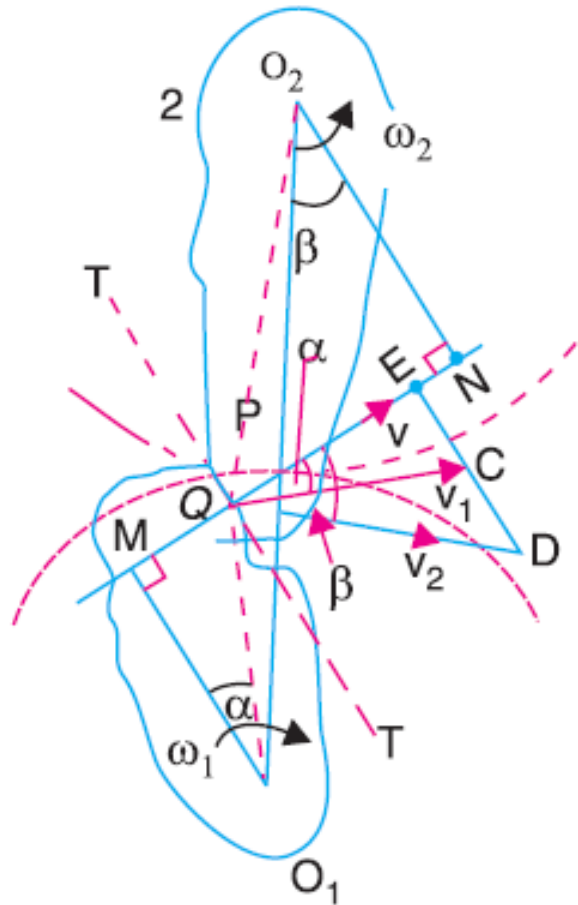
27. **Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.

(a) **Arc of approach.** It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(b) **Arc of recess.** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

LAW OF GEARING:

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. . Let the two teeth come in contact at point Q , and the wheels rotate in the directions as shown in the figure. Let TT be the common tangent and MN be the common normal to the curves at the point of contact Q . From the centres O_1 and O_2 , draw O_1M and O_2N perpendicular to MN . A little consideration will show that the point Q moves in the direction QC , when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2. Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.



ACS COLLEGE OF ENGINEERING

$$\therefore v_1 \cos \alpha = v_2 \cos \beta$$

or

$$(\omega_1 \times O_1 Q) \cos \alpha = (\omega_2 \times O_2 Q) \cos \beta$$

$$(\omega_1 \times O_1 Q) \frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q) \frac{O_2 N}{O_2 Q}$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} \quad \dots(i)$$

Also from similar triangles O_1MP and O_2NP ,

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} \quad \dots(ii)$$

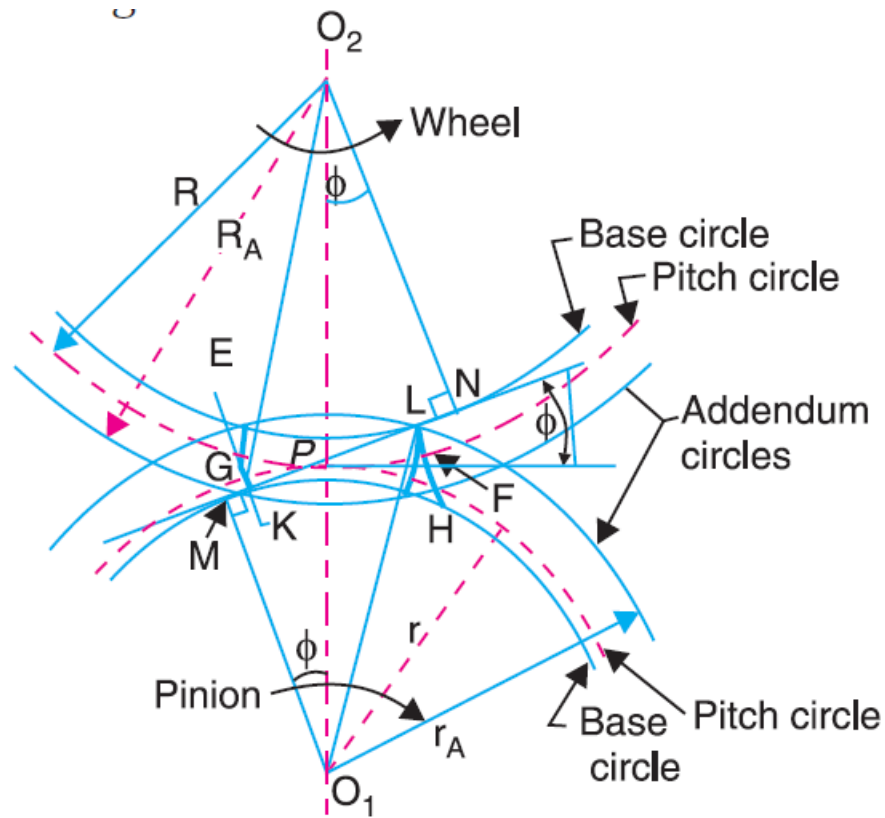
Combining equations (i) and (ii), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} \quad \dots(iii)$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O_1 and O_2 , or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities. Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*** This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as ***law of gearing.***

LENGTH OF PATH OF CONTACT:

Consider a pinion driving the wheel as shown in Fig. . When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and* ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). MN is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent. The point L is the intersection of the addendum circle of pinion and common tangent.



We have discussed in Art., that the length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL . The part of the path of contact KP is known as **path of approach** and the part of the path of contact PL is known as **path of recess**.

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig., we find that radius of the base circle of pinion,

ACS COLLEGE OF ENGINEERING

$$O_1M = O_1P \cos \phi = r \cos \phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P \cos \phi = R \cos \phi$$

Now from right angled triangle O_2KN ,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2 \cos^2 \phi}$$

and

$$PN = O_2P \sin \phi = R \sin \phi$$

\therefore Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

Similarly from right angled triangle O_1ML ,

and

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2 \phi}$$

$$MP = O_1P \sin \phi = r \sin \phi$$

\therefore Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

\therefore Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

LENGTH OF ARC OF CONTACT

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is EPF or GPH . Considering the arc of contact GPH , it is divided into two parts *i.e.* arc GP and arc PH . The arc GP is known as **arc of approach** and the arc PH is called **arc of recess**. The angles subtended by these arcs at O_1 are called **angle of approach** and **angle of recess** respectively.

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos \phi} = \frac{KP}{\cos \phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos \phi} = \frac{PL}{\cos \phi}$$

ACS COLLEGE OF ENGINEERING

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$\begin{aligned} &= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos \phi} + \frac{PL}{\cos \phi} = \frac{KL}{\cos \phi} \\ &= \frac{\text{Length of path of contact}}{\cos \phi} \end{aligned}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch**. Mathematically,

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

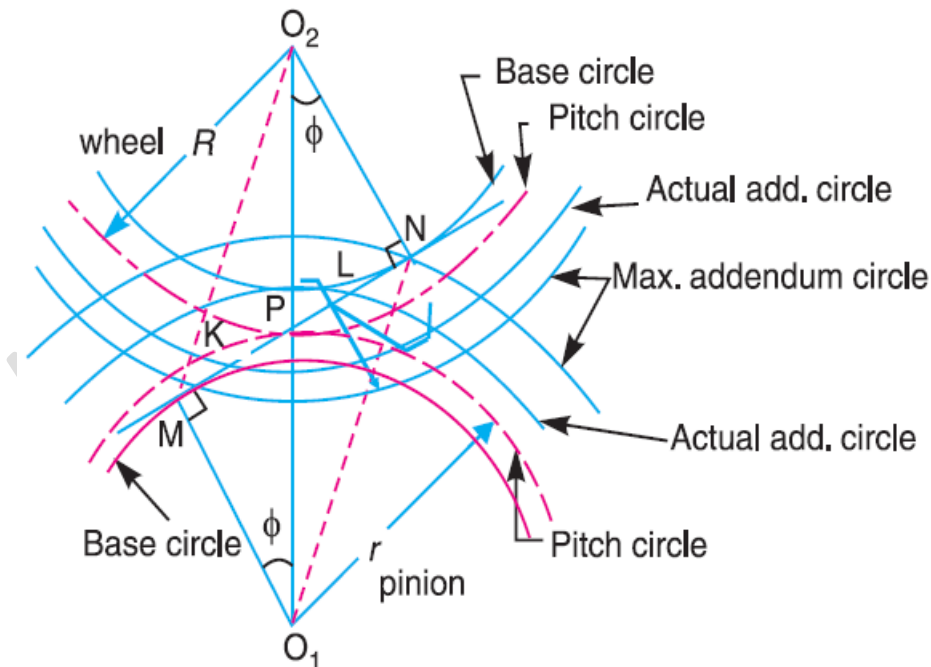
where

p_c = Circular pitch = πm , and

m = Module.

INTERFERENCE IN INVOLUTE GEARS:

Fig. shows a pinion with centre O_1 , in mesh with wheel or gear with centre O_2 . MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.



ACS COLLEGE OF ENGINEERING

A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N , the point of contact L will move from L to N . When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as **interference**, and occurs when the teeth are being cut. In brief, ***the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference***. Similarly, if the radius of the addendum circle of the wheel increases beyond O_2M , then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called **interference points**. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M . From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency***. When interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig. .

In such a case,

Maximum length of path of approach,

$$MP = r \sin \phi$$

and maximum length of path of recess,

$$PN = R \sin \phi$$

\therefore Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

ACS COLLEGE OF ENGINEERING

Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points N and M (see Fig.) respectively.

Let t = Number of teeth on the pinion,,
 T = Number of teeth on the wheel,
 m = Module of the teeth,
 r = Pitch circle radius of pinion $= m.t / 2$
 G = Gear ratio $= T / t = R / r$
 ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP ,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2 \phi - 2r.R \sin \phi \cos (90^\circ + \phi)$$

$$\dots (\because PN = O_2P \sin \phi = R \sin \phi)$$

$$= r^2 + R^2 \sin^2 \phi + 2r.R \sin^2 \phi$$

$$= r^2 \left[1 + \frac{R^2 \sin^2 \phi}{r^2} + \frac{2R \sin^2 \phi}{r} \right] = r^2 \left[1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi \right]$$

\therefore Limiting radius of the pinion addendum circle,

$$O_1N = r \sqrt{1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi} = \frac{m.t}{2} \sqrt{1 + \frac{T}{t} \left[\frac{T}{t} + 2 \right] \sin^2 \phi}$$

Let $A_p m$ = Addendum of the pinion, where A_p is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore A_p.m = \frac{m.t}{2} \sqrt{1 + \frac{T}{t} \left(\frac{T}{t} + 2 \right) \sin^2 \phi} - \frac{m.t}{2} \quad \dots (\because O_1P = r = mt/2)$$

$$= \frac{m.t}{2} \left[\sqrt{1 + \frac{T}{t} \left(\frac{T}{t} + 2 \right) \sin^2 \phi} - 1 \right]$$

ACS COLLEGE OF ENGINEERING

or

$$A_p = \frac{t}{2} \left[\sqrt{1 + \frac{T}{t} \left(\frac{T}{t} + 2 \right) \sin^2 \phi} - 1 \right]$$

\therefore

$$t = \frac{2 A_p}{\sqrt{1 + \frac{T}{t} \left(\frac{T}{t} + 2 \right) \sin^2 \phi} - 1} = \frac{2 A_p}{\sqrt{1 + G(G + 2) \sin^2 \phi} - 1}$$

This equation gives the minimum number of teeth required on the pinion in order to avoid interference.

Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

and A_{Wm} = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

Using the same notations as in Art. 12.20, we have from triangle O_2MP

$$\begin{aligned} (O_2M)^2 &= (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM \\ &= R^2 + r^2 \sin^2 \phi - 2 Rr \sin \phi \cos (90^\circ + \phi) \\ &= R^2 + r^2 \sin^2 \phi + 2 Rr \sin^2 \phi \quad \dots (\because PM = O_1P \sin \phi = r) \\ &= R^2 \left[1 + \frac{r^2 \sin^2 \phi}{R^2} + \frac{2r \sin^2 \phi}{R} \right] = R^2 \left[1 + \frac{r}{R} \left(\frac{r}{R} + 2 \right) \sin^2 \phi \right] \end{aligned}$$

\therefore Limiting radius of wheel addendum circle,

$$O_2M = R \sqrt{1 + \frac{r}{R} \left(\frac{r}{R} + 2 \right) \sin^2 \phi} = \frac{mT}{2} \sqrt{1 + \frac{t}{T} \left(\frac{t}{T} + 2 \right) \sin^2 \phi}$$

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore A_{Wm} = \frac{mT}{2} \sqrt{1 + \frac{t}{T} \left(\frac{t}{T} + 2 \right) \sin^2 \phi} - \frac{mT}{2} \quad \dots (\because O_2P = R = mT/2)$$

$$= \frac{mT}{2} \left[\sqrt{1 + \frac{t}{T} \left(\frac{t}{T} + 2 \right) \sin^2 \phi} - 1 \right]$$

or

$$A_W = \frac{T}{2} \left[\sqrt{1 + \frac{t}{T} \left(\frac{t}{T} + 2 \right) \sin^2 \phi} - 1 \right]$$

$$\therefore T = \frac{2 A_W}{\sqrt{1 + \frac{t}{T} \left(\frac{t}{T} + 2 \right) \sin^2 \phi} - 1} = \frac{2 A_W}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$$

Velocity of Sliding of Teeth

The sliding between a pair of teeth in contact at Q occurs along the common tangent TT to the tooth curves as shown in Fig. 12.6. *The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.*

The velocity of point Q , considered as a point on wheel 1, along the common tangent TT is represented by EC . From similar triangles QEC and O_1MQ ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1 \cdot MQ$$

Similarly, the velocity of point Q , considered as a point on wheel 2, along the common tangent TT is represented by ED . From similar triangles QCD and O_2NQ ,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 \cdot QN$$

Let v_s = Velocity of sliding at Q .

$$\begin{aligned} \therefore v_s &= ED - EC = \omega_2 \cdot QN - \omega_1 \cdot MQ \\ &= \omega_2 (QP + PN) - \omega_1 (MP - QP) \\ &= (\omega_1 + \omega_2) QP + \omega_2 \cdot PN - \omega_1 \cdot MP \end{aligned} \quad \dots(i)$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$ or $\omega_1 \cdot MP = \omega_2 \cdot PN$, therefore equation (i) becomes

$$v_s = (\omega_1 + \omega_2) QP \quad \dots(ii)$$

GEAR TRAINS:

Gear Trains: Simple gear trains, Compound gear trains, Reverted gear trains, Epicyclic gear trains, Analysis of epicyclic gear train (Algebraic and tabular methods), torques in epicyclic trains.

Introduction:

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called **gear train** or **train of toothed wheels**. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

TYPES OF GEAR TRAINS

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train, **2.** Compound gear train, **3.** Reverted gear train, and **4.** Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

SIMPLE GEAR TRAIN

When there is only one gear on each shaft, as shown in Fig.4, it is known as **simple gear train**. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 4 (a). Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower**.

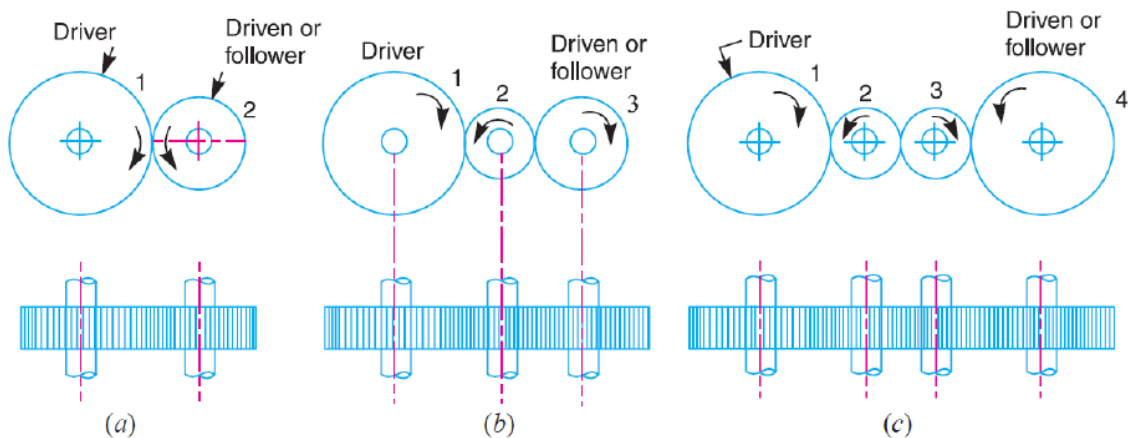


Fig. 4 Simple gear train.

ACS COLLEGE OF ENGINEERING

It may be noted that the motion of the driven gear is opposite to the motion of driving gear. Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as **train value** of the gear train. Mathematically From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are **odd**, the motion of both the gears (*i.e.* driver and driven or follower) is **like** as shown in Fig. 4 (b). But if the number of intermediate gears are **even**, the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig.4 (c). Now consider a simple train of gears with one intermediate gear as shown in Fig. 4 (b).

Let

N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \dots(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

ACS COLLEGE OF ENGINEERING

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \dots(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (b) is obtained by multiplying the equations (i) and (ii).

$$\therefore \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

$$i.e. \quad \text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{and} \quad \text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called **idle gears**, as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes :

1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise).

COMPOUND GEAR TRAIN

When there are more than one gear on a shaft, as shown in Fig. 5, it is called a **compound train of gear**. We have seen in Art. 5 that the idle gears, in a simple train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven. But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.5

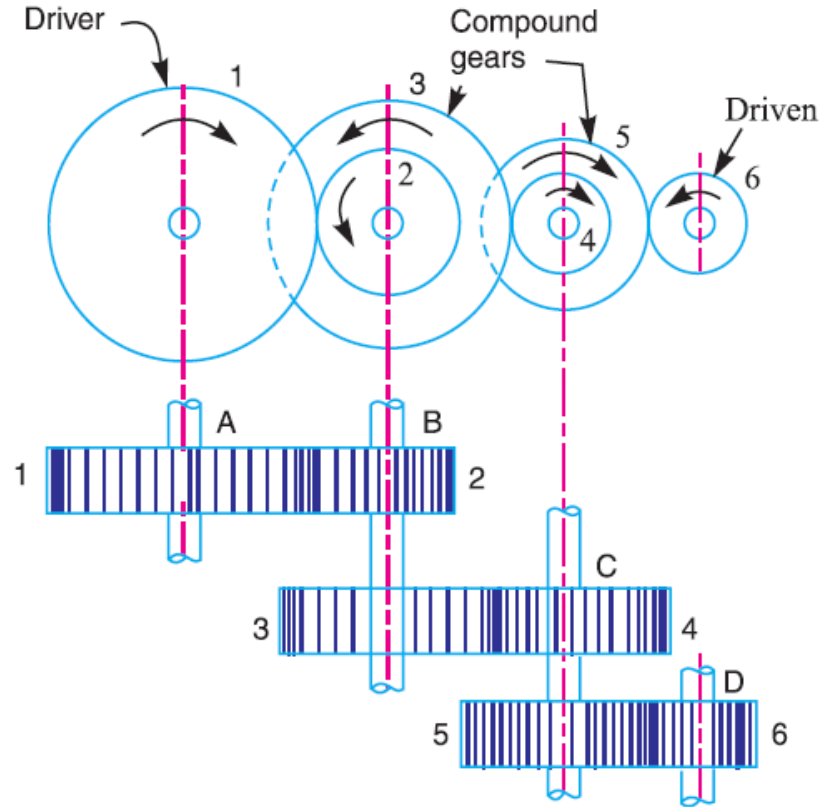


Fig. 5 Compound gear train.

In a compound train of gears, as shown in Fig. 5 the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

N_2, N_3, \dots, N_6 = Speed of respective gears in r.p.m., and

T_2, T_3, \dots, T_6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$N_1 / N_2 = T_2 / T_1$$

Similarly, for gears 3 and 4, speed ratio is

$$N_3 / N_4 = T_4 / T_3$$

and for gears 5 and 6, speed ratio is

$$N_5 / N_6 = T_6 / T_5$$

ACS COLLEGE OF ENGINEERING

The speed ratio of compound gear train is obtained by multiplying the equations (i), (ii) and (iii). The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.

$$\begin{aligned} \text{i.e. Speed ratio} &= \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} \\ &= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the driven}} \\ \text{and Train value} &= \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} \\ &= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the driven}} \end{aligned}$$

If a simple gear train is used to give a large speed reduction, the last gear has to be very large.

Usually for a speed reduction

in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed

EPICYCLIC GEAR TRAIN:

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 6, where a gear *A* and the arm *C* have a common axis at *O*₁ about which they can rotate. The gear *B* meshes with gear *A* and has its axis on the arm at *O*₂, about which the gear *B* can rotate. If the arm is fixed, the gear train is simple and gear *A* can drive gear *B* or *vice-versa*, but if gear *A* is fixed and the arm is rotated about the axis of gear *A* (i.e. *O*₁), then the gear *B* is forced to rotate *upon* and *around* gear *A*. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as **epicyclic gear trains** (*epi.* means upon and *cyclic* means around). The epicyclic gear trains may be **simple** or **compound**. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of

ACS COLLEGE OF ENGINEERING

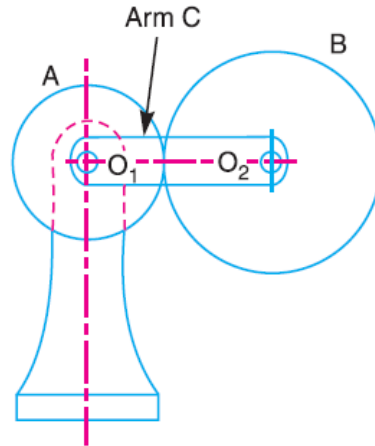


Fig. 6. Epicyclic gear train.

moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Velocity Ratio of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method, and
2. Algebraic method.

These methods are discussed, in detail, as follows :

1. Tabular method.

Consider an epicyclic gear train as shown in Fig.

Let T_A = Number of teeth on gear A , and

T_B = Number of teeth on gear B .

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed

relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make $*T_A / T_B$ revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes $+1$ revolution, then the gear B will make $(-T_A / T_B)$ revolutions. This statement of relative motion is entered in the first row of the table (see Table 13.1). Secondly, if the gear A makes $+x$ revolutions, then the gear B will make $-x \times T_A / T_B$ revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x . Thirdly, each element of an epicyclic train is given $+y$ revolutions and entered in the third row.

ACS COLLEGE OF ENGINEERING

Finally, the motion of each element of the gear train is added up and entered in the fourth row. A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution i.e. 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

2. Algebraic method.

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train viz. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train. Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C.

$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in *opposite* directions.

$$\therefore \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

ACS COLLEGE OF ENGINEERING
IMPORTANT QUESTIONS

1. Derive an expression to determine the length of path of contact between two spur gear of different size.

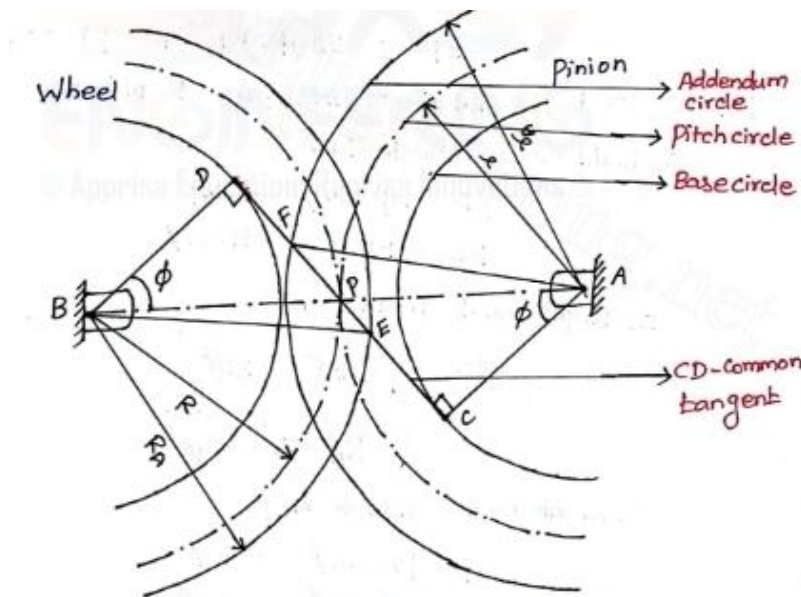


Figure: Line of Action Diagrams for Gears

The pinion is rotating in the clockwise direction and the wheel is driven. The two pitch circles are meeting at the point "P". The line CD is the common tangent of the two base circles. The line is also known as "Line of action".

The point "E" is the intersection of the tangent and the addendum circle of the wheel. The point "F" is the common tangent and the addendum circle of the pinion. The contact of two teeth begins, where the addendum circle of the wheel meets the common tangent (point E) and ends where the addendum circle of the pinion meets the common tangent (point F). The line EF gives the length of path of contact. The length EP is known as "path of approach" whereas the length PF is known as "path of recess".

$$\text{Length of path of contact} = \text{path of approach} + \text{path of recess}$$

ACS COLLEGE OF ENGINEERING

$$EF = EP + PF$$

$$EF = (ED - PD) + (CF - CP) \quad \text{--- (1)}$$

Let us find the values of ED, PD, CF and CD.

In Right Angle Triangle BPD

$$PD = BP \sin \phi = R \sin \phi$$

$$BD = BP \cos \phi = R \cos \phi$$

In Right Angle Triangle BED.

$$ED = \sqrt{(BE)^2 - (BD)^2}$$

$$= \sqrt{R_A^2 - R^2 \cos^2 \phi}$$

In Right Angle Triangle ACP,

$$CP = PA \sin \phi = r \sin \phi$$

$$CA = PA \cos \phi = r \cos \phi$$

In Right Angle Triangle FCA,

$$CF = \sqrt{(AF)^2 - (CA)^2}$$

$$= \sqrt{r_A^2 - r^2 \cos^2 \phi}$$

$$\therefore \text{Path of Approach} = EP = ED - PD$$

$$= \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$\text{Path of Recess} = PF = CF - CP$$

$$= \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$\text{Length of Path of Contact} = EF = EP + PF$$

$$= \sqrt{R_A^2 - R^2 \cos^2 \phi} + \sqrt{r_A^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

The above equation gives the length of path of contact between two spur gears of different sizes.

... with sketches:

ACS COLLEGE OF ENGINEERING

2. Calculate:

- Length of path of contact
- Arc of contact and
- The contact ratio when a pinion having 23 teeth drives a gear having teeth 57. The Profile of the gears is involute with pressure angle 20° , module 8mm and addendum equal to one module.

Given data:

$$t = 23, T = 57, \phi = 20^\circ, m = 8 \text{ mm}$$

$$\text{Addendum} = 1 \text{ module} = 8 \text{ mm}$$

To find:

- Length of path of contact (K_L).
- Arc of contact.
- Contact ratio.

Solution:

(i) Length of path of contact:

$$\begin{aligned} \text{Pitch circle radius of pinion } r &= \frac{m t}{2} \\ &= \frac{8 \times 23}{2} \\ &= 92 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Pitch circle radius of wheel } R &= \frac{m T}{2} \\ &= \frac{8 \times 57}{2} \\ &= 228 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Radius of Addendum circle of pinion } r_A &= r + \text{Addendum} \\ &= 92 + 8 \\ &= 100 \text{ mm} \end{aligned}$$

ACS COLLEGE OF ENGINEERING

Radius of Addendum circle of wheel $R_A = R + \text{Addendum}$
 $= 228 + 8$
 $R_A = 236 \text{ mm}$

Length of path of approach (KP):

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$KP = \sqrt{(236)^2 - (228)^2 (\cos 20^\circ)^2} - 228 \times \sin 20^\circ$$

$$KP = \sqrt{55696 - 45903} - 77.98$$

$$KP = 98.95 - 77.98$$

$$KP = 20.96 \text{ mm}$$

Length of path of Recess (PL):

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$PL = \sqrt{(100)^2 - (92)^2 (\cos 20^\circ)^2} - 92 \sin 20^\circ$$

$$PL = \sqrt{10000 - 747.39} - 31.47$$

$$PL = 50.26 - 31.47$$

$$PL = 18.79 \text{ mm}$$

Length of path of contact $KL = KP + PL$
 $= 20.96 + 18.79$

$$KL = 39.75 \text{ mm}$$

(ii) Length of Arc of contact:

$$\text{Length of Arc of contact} = \frac{\text{Length of path of contact}}{\cos \phi}$$

ACS COLLEGE OF ENGINEERING

$$= \frac{39.75}{\cos 20^\circ}$$

$$\text{Length of Arc of contact} = 42.30 \text{ mm}$$

ii) Contact Ratio:

$$\text{Circular Pitch } p_c = \pi \times m = \pi \times 8 = 25.12 \text{ mm}$$

$$\text{Contact Ratio} = \frac{\text{Length of Arc of contact}}{\text{circular pitch}}$$

$$= \frac{42.30}{25.12}$$

$$= 1.68 \approx 2$$

$$\text{Contact Ratio} = 2$$

2) Two gear wheels mesh externally to give a velocity ratio of 3 to 1. The involute teeth has 6 mm module and 20° pressure angle. Addendum is equal to one module. The pinion rotates at 90 rpm. Determine

(i) Number of teeth on pinion to avoid interference and the corresponding number on the wheel

(ii) The length of path and arc of contact

(iii) Contact ratio and

Given data:

$$\text{Velocity ratio, } G = \frac{T}{E} = 3, \text{ module } m = 6 \text{ mm}$$

$$A_p = A_w = 1 \text{ module} = 6 \text{ mm}, \phi = 20^\circ, N_1 = 90 \text{ rpm}$$

ACS COLLEGE OF ENGINEERING

Solution:

$$\omega_1 = \frac{2\pi N_1}{60}$$

$$\omega_1 = \frac{2 \times \pi \times 90}{60} = 9.43 \text{ rad/s}$$

(i) Number of teeth on the pinion and wheel:

No. of teeth on pinion to avoid interference

$$t = \frac{2 A_p}{\sqrt{1 + G(G+2) \sin^2 \phi} - 1}$$

$$= \frac{2 \times 6}{\sqrt{1 + 3(3+2) \sin^2 20^\circ} - 1}$$

$$= 18.2 \approx 19$$

$$t = 19$$

No. of teeth on the wheel

$$T = G \cdot t$$

$$= 3 \times 19$$

$$T = 57$$

(ii) Length of path and Arc of contact:

pitch circle radius of pinion $r = \frac{m t}{2}$

$$= \frac{6 \times 19}{2}$$

$$r = 57 \text{ mm}$$

pitch circle radius of wheel $R = \frac{m T}{2}$

$$= \frac{6 \times 57}{2}$$

$$R = 171 \text{ mm}$$

ACS COLLEGE OF ENGINEERING

Radius of Addendum circle of Pinion, $r_A = r + A_p$
 $= 57 + 6$
 $r_A = 63 \text{ mm}$

Radius of Addendum circle of wheel, $R_A = R + A_w$
 $= 171 + 6$
 $R_A = 177 \text{ mm}$

Length of path of Approach (kp):

$$k_p = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi$$
$$= \sqrt{(177)^2 - (171)^2 \cos^2 20^\circ} - 171 \sin 20^\circ$$
$$= 74.2 - 58.15$$
$$k_p = 15.7 \text{ mm}$$

Length of path of Recess (PL):

$$P_L = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi$$
$$= \sqrt{(63)^2 - (57)^2 \cos^2 20^\circ} - 57 \sin 20^\circ$$
$$= 33.175 - 19.5$$
$$P_L = 13.67 \text{ mm}$$

Length of path of contact $K_L = k_p + P_L$
 $= 15.7 + 13.67$
 $K_L = 29.37 \text{ mm}$

Length of Arc of contact $= \frac{\text{Length of path of contact}}{\cos \phi}$
 $= \frac{29.37}{\cos 20^\circ}$
 $= 31.25 \text{ mm}$

ACS COLLEGE OF ENGINEERING

(iii) Number of pairs of teeth in contact (or) Contact ratio:

$$\text{Circular pitch} = \pi \times m = 18.852 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Length of Arc of contact}}{\text{circular pitch}}$$

$$= \frac{31.25}{18.852}$$

$$= 1.66 \approx 2$$

$$\text{Contact ratio} = 2$$

(iv) Maximum Velocity of sliding (V_s):

Let, ω_1 - Angular speed of Pinion

ω_2 - Angular speed of Wheel

$$\frac{\omega_1}{\omega_2} = \frac{T}{t} \quad , \quad \omega_2 = \omega_1 \times \frac{t}{T}$$

$$\omega_2 = 9.43 \times \frac{19}{57}$$

$$\omega_2 = 3.14 \text{ rad/s}$$

Maximum velocity of sliding, $V_s = (\omega_1 + \omega_2) \cdot KP$

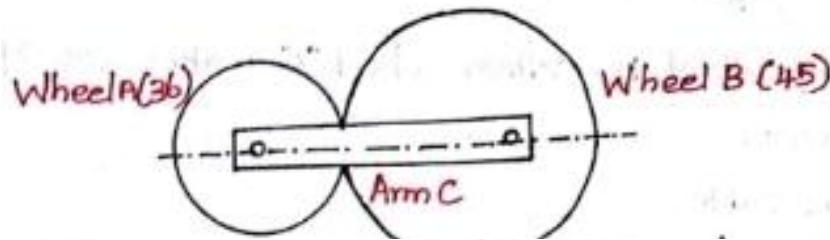
$$= (9.43 + 3.14) \times 15.7$$

$$V_s = 197.35 \text{ mm/s}$$

3) The arm of an epicyclic gear train rotates at 100 rpm in the anticlockwise direction. The arm carries two wheels two wheels A and B having 36 and 45 teeth respectively. The wheel A is fixed and the arm rotates about the centre of wheel A. Find

ACS COLLEGE OF ENGINEERING

the speed of Wheel B. What will be the speed of B, if the wheel A instead of being fixed, makes 200 rpm clockwise.



Given data:

$$T_A = 36, T_B = 45, N_C = 100 \text{ rpm [Anticlockwise]}$$

Table of motion.

Step No	Operations	Revolution of Elements (N)		
		Arm C	Gear A $T_A = 36$	Gear B $T_B = 45$
1.	Fix the arm C and give Gear A +1 revolution [Anticlockwise]	0	+1	$-\frac{T_A}{T_B}$
2.	Multiply by x	0	+x	$-x \times \frac{T_A}{T_B}$
3.	Add +y revolution to all elements	+y	+y	+y
4.	Total motion	y	x+y	$y - x \times \frac{T_A}{T_B}$

(i) Speed of Gear B, when wheel A is fixed:

$$\text{Speed of the Arm} = 100 \text{ rpm [Anticlockwise]}$$

$$\therefore y = + 100 \text{ rpm}$$

$$\text{Gear A is fixed, } x + y = 0$$

$$x = -y$$

$$x = -100 \text{ rpm}$$

$$\text{Speed of Gear B} = y - x \times \frac{T_A}{T_B}$$

ACS COLLEGE OF ENGINEERING

$$= 100 + 100 \times \frac{36}{45}$$

Speed of wheel "B" = 180 rpm

(ii) Speed of wheel B, When wheel "A" makes 200 rpm clockwise:

From Table,

$$x + y = -200$$

$$x = -200 - y$$

$$x = -200 - 100$$

$$x = -300 \text{ rpm (clockwise)}$$

$$\begin{aligned} \text{Speed of wheel B, } N_B &= y - x \times \frac{T_A}{T_B} \\ &= y - (-300) \times \frac{36}{45} \end{aligned}$$

$$= 100 + 240$$

$$N_B = 340 \text{ rpm (Anticlockwise)}$$

4) a) An epicyclic gear train consists of three gears 1, 2, 3 as shown in Figure. The internal gear 1 has 72 teeth and gear 3 has 32 teeth. The gear 2 meshes with both gear 1 and gear 3 and is carried on an arm A, which rotates about the centre O_2 at 20 rpm. If the gear 1 is fixed, determine the speed of gears 2 and 3. (12 Marks).

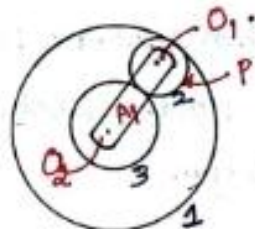


Figure.

(ii) Write short notes on speed ratio of a planetary

ACS COLLEGE OF ENGINEERING

Solution:

(1)

Table of motion

Step No	Operations	Revolution of Elements (N)			
		Arm A,	Gear 3 $T_3 = 32$	Gear 2 $T_2 = 20$	Gear 1 $T_1 = 72$
1.	Fix the arm A, and give gear 3 +1 revolution [Anticlockwise]	0	+1	$-\frac{T_3}{T_2}$	$-\frac{T_3}{T_1}$
2.	Multiply by x	0	+x	$-x \times \frac{T_3}{T_2}$	$-x \times \frac{T_3}{T_1}$
3.	Add +y revolution to all elements	+y	+y	+y	+y
4.	Total motion	y	x+y	$y - x \times \frac{T_3}{T_2}$	$y - x \times \frac{T_3}{T_1}$

Speed of Gear 3 (N_3):

Speed of the Arm A, is 20 rpm

$$y = 20 \text{ rpm}$$

Gear 1 is fixed,

$$y - x \times \frac{T_3}{T_1} = 0$$

$$20 - x \times \frac{32}{72} = 0$$

$$x = \frac{72}{32} \times 20 = 45$$

Speed of Gear 3 = x+y

$$= 45 + 20$$

$$N_3 = 65 \text{ rpm [Anticlockwise]}$$

Speed of Gear 2 (N_2):

Let d_1, d_2, d_3 be the pitch circle diameter of the gears 1, 2, 3 respectively.

ACS COLLEGE OF ENGINEERING

From the Figure

$$d_2 + \frac{d_3}{2} = \frac{d_1}{2}$$

$$2d_2 + d_3 = d_1$$

Since the no. of teeth are proportional to their pitch circle diameters. The above eqn can be written as

$$2T_2 + T_3 = T_1$$

$$2T_2 + 32 = T_2$$

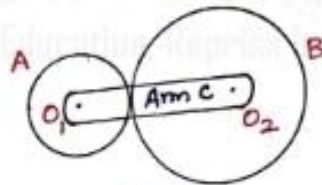
$$T_2 = 20$$

$$\begin{aligned} \text{Speed of Gear 2} &= y - x \times \frac{T_3}{T_2} \\ &= 20 - 45 \times \frac{32}{20} \end{aligned}$$

$$N_2 = -52 \text{ rpm [clockwise]}$$

(ii) Notes on speed ratio of Planetary Gear train:

In the Algebraic method to find the motion of each element of epicyclic gear train relative to the arm is set down in form of eqns



Figure

The Figure shows an epicyclic gear train with gears A & B and Arm C. Let the Arm C be fixed,

$$\therefore \text{Speed of Gear A relative to Arm C} = N_A - N_C$$

$$\text{Also, Speed of Gear B relative to Arm C} = N_B - N_C$$

Since the Gears A and B are meshing directly, therefore

ACS COLLEGE OF ENGINEERING

they will revolve in opposite direction

$$\therefore \frac{N_B - N_c}{N_A - N_c} = -\frac{T_A}{T_B}$$

Since $N_c = 0$ (\because Arm c is fixed)

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If Gear A is fixed, then $N_A = 0$

$$\frac{N_B}{N_c} = 1 + \frac{T_A}{T_B}$$

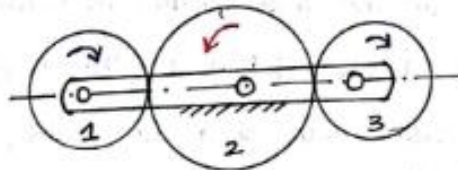
- 4.b. Briefly explain the classification of compound gear trains with neat sketch.

Contact

(ii) **classification of Gear Trains with sketches:**

there are Four types of Gear Trains. They are

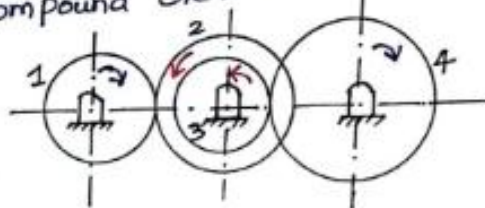
1) Simple Gear Train:



A Simple Gear Train is one in which there is

only one gear on each shaft

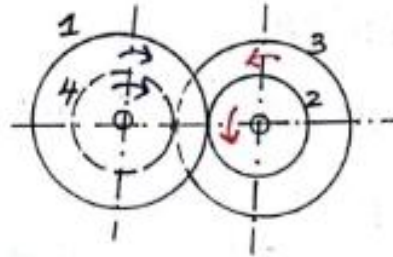
2) Compound Gear Train:



ACS COLLEGE OF ENGINEERING

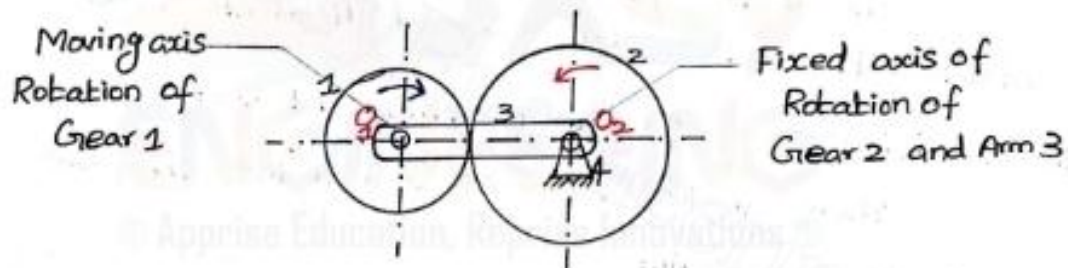
A pair of gears is compound, if they have a common axis are integral, then it is known as compound gear train.

3) Reverted Gear Train:



When the axes of the first gear [Driver] and the last gear (driven) are co-axial, then the gear train is known as Reverted Gear Train.

4) Epicyclic Gear Train:



These are the gear trains in which the axis of one (or) more gears relative to the frame. The gear at the centre is called sun and the gears, whose axes move are called planets.