SATELLITE ORBIT TRANSFER

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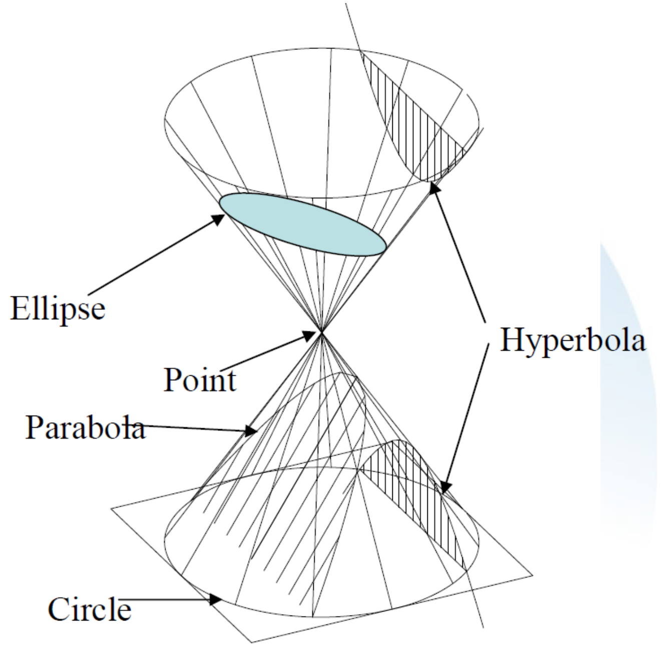
Two Body Problem and Solution

- $\ddot{\mathbf{r}} = - (\mu \mathbf{r})/r^3$
- This equation, called the two body equation of motion. A solution to this equation for a satellite orbiting Earth is the polar equation of a connic section.
- All orbits follow
  - Circle
  - Ellipse
  - Parabola
  - Hyperbola
  - Rectilinear



Basic Orbital Parameters

- Semi Latus Rectum :

$$p = a(1 - e^2)$$

- Radial Distance as a function of Orbital Pos

$$r = \frac{p}{1 + e\cos\theta}$$

- Periapsis and apoapsis distances

$$r_p = a(1 - e) \qquad r_a = a(1 + e)$$

- Angular Momentum

$$\vec{h} = \vec{r} \times \vec{v}$$

$$h = \sqrt{\mu p}$$

Orbit Properties

- K.E. is

$$K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}$$

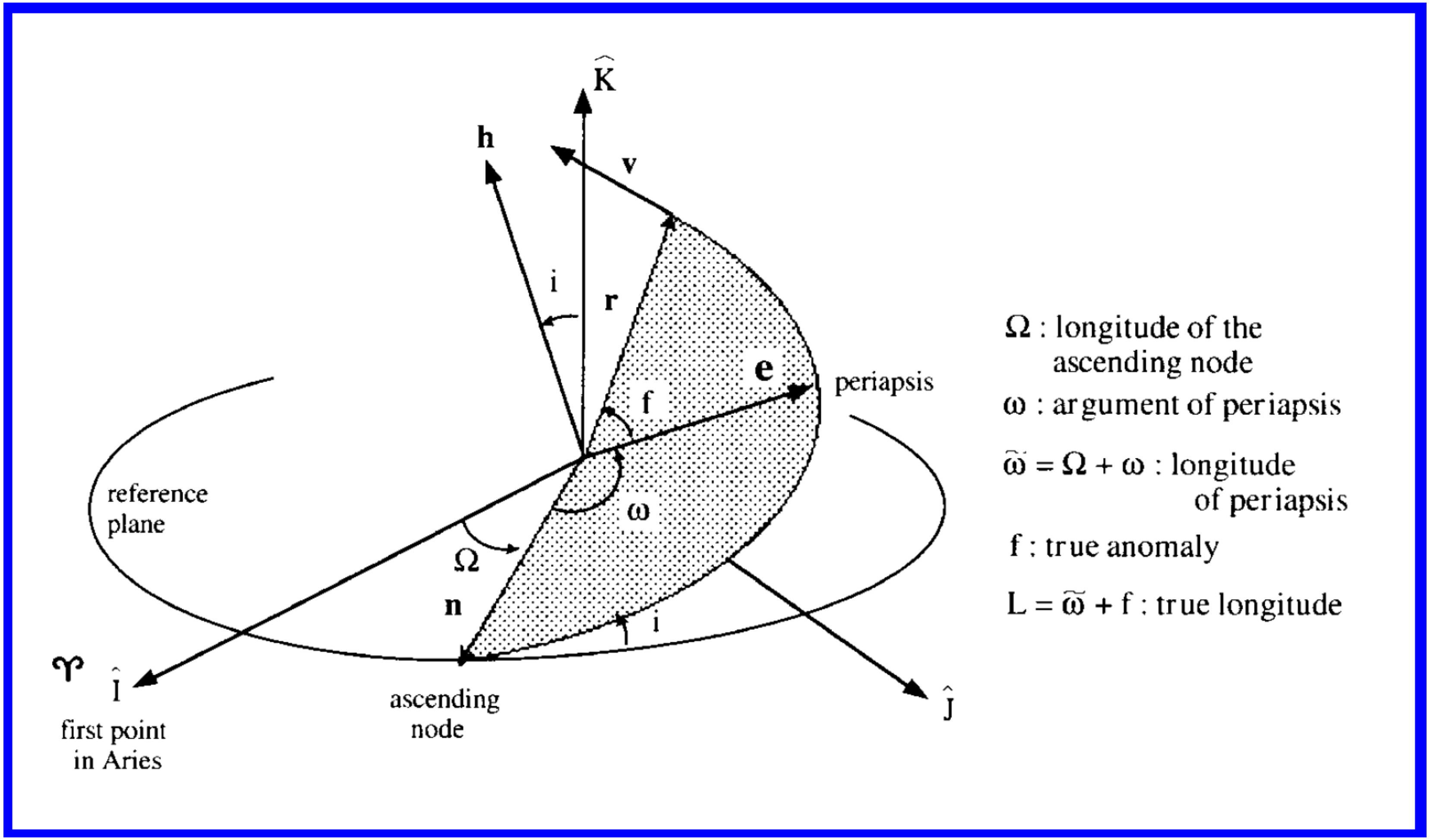
- P.E is

$$P.E. = -\frac{m\mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}$$

- Total Energy or Vis Viva Equation

$$Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

Classical Orbital Elements



Properties of Orbits

- Circular Orbit

$$v_{circular} = \sqrt{\frac{\mu}{r}}$$

- Parabolic escape orbit

$$v_{escape} = \sqrt{\frac{2\mu}{r}}$$

- Relationship between Circular and Parabolic orbits

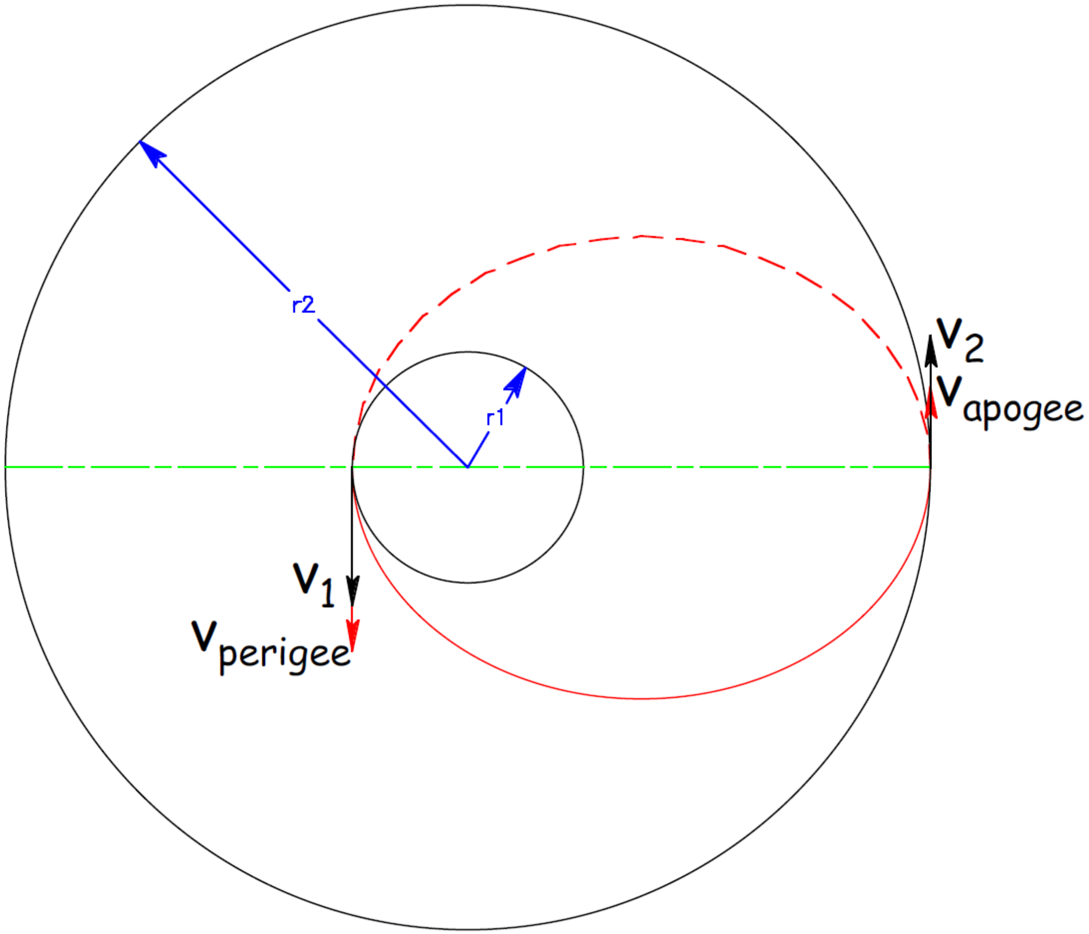
$$v_{escape} = \sqrt{2}v_{circular}$$

Orbit Maneuvers

- At some point during the lifetime of most space vehicles or satellites, we must change one or more of the orbital elements
- Most frequently, we must change the orbit altitude, plane, or both. To change the orbit of a space vehicle, we have to change its velocity vector in magnitude or direction.
- For this reason, any maneuver changing the orbit of a space vehicle must occur at a point where the old orbit intersects the new orbit.
- If the orbits do not intersect, we must use an intermediate orbit that intersects both. In this case, the total maneuver will require at least two propulsive burns.

Orbit Maneuvers - Hohmann Transfer

- Minimum Energy Transfer



First Maneuver Velocities

- Initial Vehicle velocity

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

- Final Velocity required

$$v_{perigee} = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$

- So Delta v is

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

Second Maneuver Velocities

- Intial Vehicle velocity

$$v_{apogee} = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$$

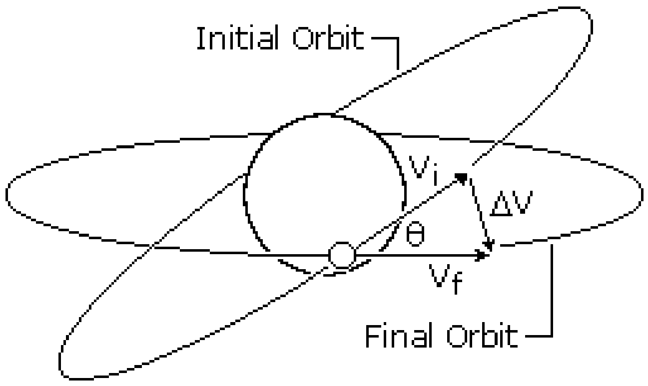
- Final Velocity required

$$v_2 = \sqrt{\frac{\mu}{r_2}}$$

- So Delta v is

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right)$$

Orbital Plane Changes



- To change the orientation of a satellite's orbital plane, typically the inclination, we must change the direction of the velocity vector.
- This maneuver requires a component of *V* to be perpendicular to the orbital plane and, therefore, perpendicular to the initial velocity vector.
- If the size of the orbit remains constant, the maneuver is called a *simple plane change*.
- We can find the required change in velocity by using the law of cosines. For the case in which V_f is equal to V_i, this expression reduces to

$$\Delta V = 2V_i \sin\left(\frac{\theta}{2}\right)$$

First Maneuver with Plane Change

- Intial Vehicle velocity

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

- Final Velocity required

$$v_{perigee} = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$

- So Delta v is

$$\Delta v_1 = \sqrt{v_1^2 + v_p^2 - 2v_1 v_p \cos(\Delta i_1)}$$

Second Maneuver with Plane change

- Intial Vehicle velocity

$$v_{apogee} = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$$

- Final Velocity required

$$v_2 = \sqrt{\frac{\mu}{r_2}}$$

- So Delta v is

$$\Delta v_2 = \sqrt{v_2^2 + v_a^2 - 2v_2 v_a \cos(\Delta i_2)}$$

PROBLEM

- A spacecraft is in a circular parking orbit with an altitude of 200 km. Calculate the velocity change required to perform a Hohmann transfer to a circular orbit at geosynchronous altitude.
- Solution

$\Delta V_A = 2455$ m/s

$\Delta V_B = 1478$ m/s

$\Delta V_T = \Delta V_A + \Delta V_B = 2455 + 1478$

$\Delta V_T = 3933$ m/s

Problem

- Calculate the velocity change required to transfer a satellite from a circular 600 km orbit with an inclination of 28 degrees to an orbit of equal size with an inclination of 20 degrees.
- Solution: Given $r = (6378.14 + 600) \times 1000 = 6978140$ m

$\Theta = 28 - 20 = 8$ degrees

$Vi = SQRT[\ GM / r\ ] = SQRT[\ 3.986005 \times 10^{14} / 6978140\ ]$

$Vi = 7558$ m/s

$\Delta V = 2 \times Vi \times \sin(\Theta / 2) = 2 \times 7558 \times \sin(8/2)$

$\Delta V = 1054$ m/s