INTRODUCTION TO BALLISTIC MISSILE TRAJECTORIES

By
P.Soma
ACS College of Engineering - 2020

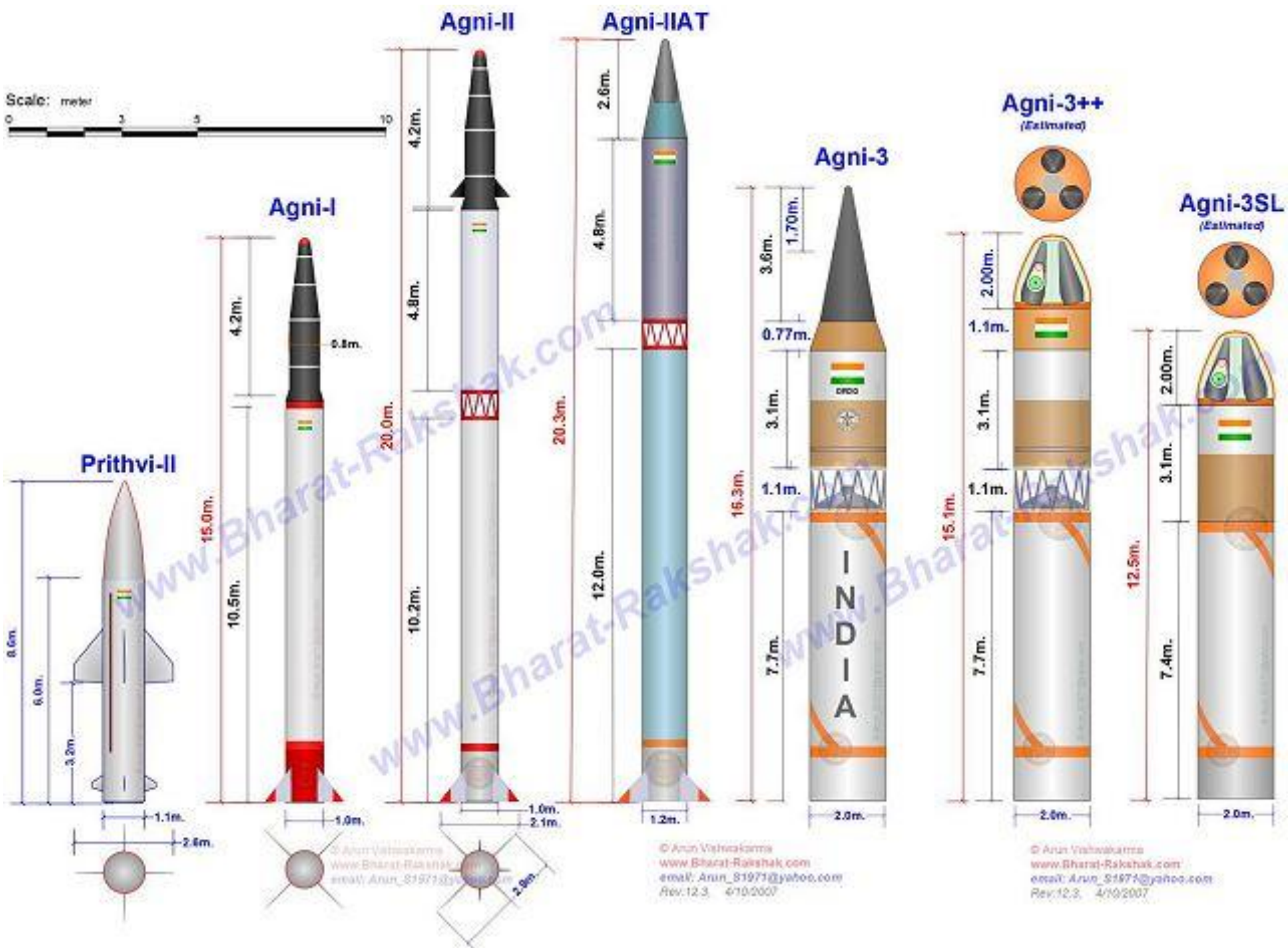
Introduction

- Ballistic missiles are used for transportation of payload from one point on the Earth, the launch site, to another point on the surface of the Earth, the impact point or target.
- It is characteristic of these ballistic missile that during a relatively short period they are accelerated to a high velocity. Then a re-entry vehicle containing the warhead, is released and this vehicle then simply coasts in a ballistic or free-fall trajectory to the final impact point.
- DRDO is the nodal agency for the development of Indian ballistic missiles.

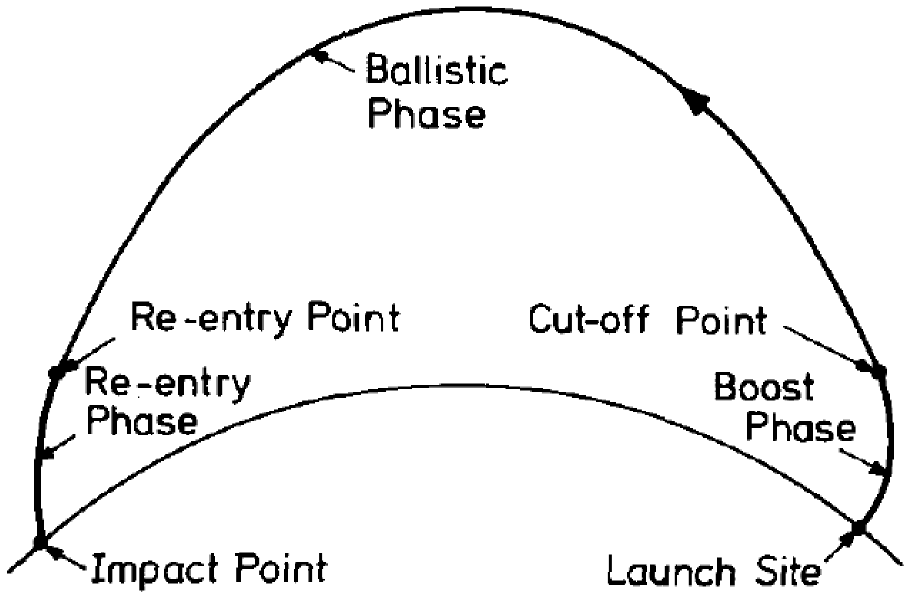
Types

- Air-launched ballistic missile (ALBM)
- Tactical ballistic missile: Range between about 150 km and 300 km
- Theatre ballistic missile (TBM): Range between 300 km and 3,500 km
  - Short-range ballistic missile (SRBM): Range between 300 km and 1,000 km
  - Medium-range ballistic missile (MRBM): Range between 1,000 km and 3,500 km
- Intermediate-range ballistic missile (IRBM) or long-range ballistic missile (LRBM): Range between 3,500 km and 5,500 km
- Intercontinental ballistic missile (ICBM): Range greater than 5,500 km
- Submarine-launched ballistic missile (SLBM): Launched from ballistic missile submarines (SSBNs)

Indian Ballistic missiles

- 

Typical Ballistic Missile Trajectory



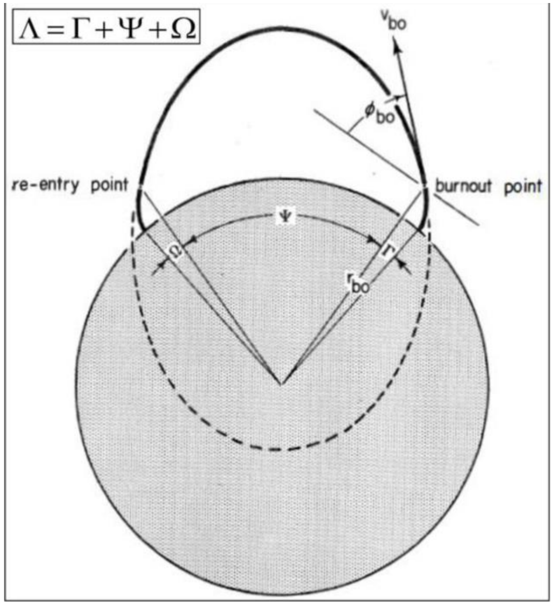
1. The boost phase in which the vehicle is boosted by one or more rocket stages through the atmosphere until final shut-down or cut-off. Concerning guidance, which we will not discuss, the boost phase can be subdivided into an *open-loop phase* and a *closed-loop*, or *guidance phase*. The open-loop phase is pre-programmed and consists of a vertical lift-off, during which the rocket is rolled such that the thrust vector plane coincides with the desired plane of motion, followed by a pitch-over and gravity turn. The subsequent

Typical Ballistic Missile Trajectory

guidance phase is characterized by the computation of steering commands from the vehicle's actual location and velocity and the coordinates of the desired impact point. If at shut-down the atmospheric drag cannot be neglected with respect to the gravitational force, a subsequent atmospheric ascent phase follows the boost phase.

2. The ballistic phase covering the major part of the range and at the end of which the vehicle enters the atmosphere.

3. The re-entry phase is the subsequent passage downwards through the atmosphere until impact at the surface of the Earth.



Boost Phase

As the total flight of an ICBM is of relatively short duration, about 30 minutes, the motion of the Earth about the Sun can be neglected and a geocentric non-rotating reference system will constitute a quasi-inertial frame for describing the motion. The main reference system we will use throughout this chapter will be the *geocentric equatorial mean-of-launch-date reference system*, with the X-axis pointing towards the *mean vernal equinox of date* (Section 2.3.6). With respect to this frame, the position of the launch site at launch time t_l is determined by the radius, r_l, the declination, δ_l, and the right ascension, α_l (Fig. 13.2). If Λ_l and Φ_l are known geographic longitude and geocentric latitude of the launch site respectively, and h_l is the altitude of the launch site, then we have approximately

$$r_l = R + h_l, \qquad (13.1\text{-}1)$$

Boost Phase

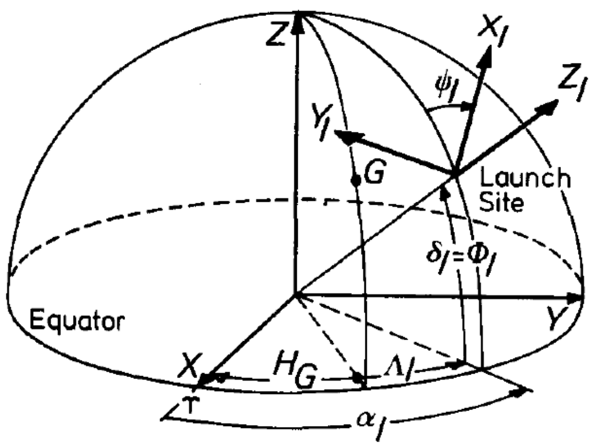


Fig. 13.2 The launch site coordinate system

where R is the local radius of the Standard Ellipsoid, discussed in Section 2.5.1, and, according to Section 2.3.4,

$$\alpha_l = H_{G_0} + \omega_e t_l + \Lambda_l, \tag{13.1-2a}$$

$$\delta_l = \Phi_l, \tag{13.1-2b}$$

where H_{G_0} is the Greenwich hour angle of the vernal equinox at 0^h U.T. of the day of launch and ω_e is the angular velocity of the Earth. The value of H_{G_0} can be found in Reference [1].

Boost Phase

For a description of the boost phase we introduce an *inertial launch-site coordinate system* as shown in Fig. 13.2. The origin of this system coincides with the launch site at the time of launch. The positive Z_l-axis points in outward radial direction while the X_lY_l-plane is defined as the plane normal to the local radius vector. The X_l-axis is pointing along the downrange heading, thus making an angle ψ_l (the launch azimuth or heading) with the local north direction. The Y_l-axis, finally, completes the right-handed orthogonal frame. This launch site reference system is obtained from the geocentric equatorial system by an Euler rotation about respectively the Z-, the X- and the Z-axis, the Euler angles being $90° + \alpha_l$, $90° - \delta_l$ and $90° - \psi_l$. Let $\boldsymbol{U}$ be a vector with components U_X, U_Y and U_Z with respect to the geocentric equatorial frame, and components U_{X_l}, U_{Y_l} and U_{Z_l} with respect to the launch site frame, then the rotation between both sets of components is given by

$$(U_X, U_Y, U_Z) = (U_{X_l}, U_{Y_l}, U_{Z_l})\mathbf{A}_l, \tag{13.1-3a}$$

where $\mathbf{A}_l$ is the Euler rotation matrix:

$$\mathbf{A}_l = \begin{bmatrix} -S\alpha_l S\psi_l - C\alpha_l S\,\delta_l C\psi_l & C\alpha_l S\psi_l - S\alpha_l S\,\delta_l C\psi_l & C\,\delta_l C\psi_l \\ S\alpha_l C\psi_l - C\alpha_l S\,\delta_l S\psi_l & -C\alpha_l C\psi_l - S\alpha_l S\,\delta_l S\psi_l & C\,\delta_l S\psi_l \\ C\alpha_l C\,\delta_l & S\alpha_l C\,\delta_l & S\,\delta_l \end{bmatrix}, \tag{13.1-3b}$$

and $S\alpha_l = \sin\alpha_l$, $C\alpha_l = \cos\alpha_l$, etc.

Boost Phase

compared to the radius of the Earth, the Earth may be considered flat during this phase. Neglecting for a moment the rotation of the Earth, the motion will be two-dimensional and the trajectory will lie in the X_lZ_l-plane of the launch site coordinate system. In Chapters 11 and 12 methods are discussed to determine two-dimensional powered flight trajectories in a homogeneous gravitational field, so we can use these theories to determine the boost phase.

Owing to the rotation of the Earth the launch site has an eastward velocity $\omega_e r_l \cos \delta_l$ with respect to inertial space. This eastward velocity can be resolved into an initial down-range velocity and an initial cross-range velocity along the X_l- and Y_l-axes respectively:

$$V_{X_l}(t_l) = \omega_e r_l \cos \delta_l \sin \psi_l, \qquad (13.1\text{-}4a)$$

$$V_{Y_l}(t_l) = -\omega_e r_l \cos \delta_l \cos \psi_l. \qquad (13.1\text{-}4b)$$

The initial down-range rate does not affect the two-dimensionality of the motion and therefore can easily be incorporated into the two-dimensional theories of powered flight trajectories by adjusting the initial conditions. Owing to the initial cross-range rate $V_{Y_l}(t_l)$, the trajectory cannot be planar anymore for launches neither due east nor due west. The main effect of this initial cross-range rate is a change of the effective firing azimuth. Therefore, injection azimuth, longitude and latitude will differ from the corresponding quantities as determined with the assumption that the Earth is not rotating.

Boost Phase

A simple analytic method to include the initial cross-range velocity in the analysis of the powered phase is given by Kohlhase [2]. Once the two-dimensional powered flight trajectory is determined, his method enables us to determine the necessary corrections in the shut-down conditions to account for the initial cross-range velocity. Thus, with this method it is possible to determine the shut-down or *injection state* at time t_i given by $(X_{l_i}, Y_{l_i}, Z_{l_i})$ and $(V_{X_{li}}, V_{Y_{li}}, V_{Z_{li}})$ with respect to the launch site reference system. The injection state with respect to the geocentric equatorial system may be found by applying Eq. (13.1-3a):

$$(V_{X_i}, V_{Y_i}, V_{Z_i}) = (V_{X_{li}}, V_{Y_{li}}, V_{Z_{li}})\mathbf{A}_l, \tag{13.1-5a}$$

$$(X_i, Y_i, Z_i) = r_l(\cos\delta_l \cos\alpha_l, \cos\delta_l \sin\alpha_l \sin\delta_l) + (X_{l_i}, Y_{l_i}, Z_{l_i})\mathbf{A}_l. \tag{13.1-5b}$$

As it is customary to express the injection conditions in radius, right ascension, declination, speed, azimuth and flight path angle, we will derive expressions for these quantities. Clearly the injection radius is given by

$$r_i = \sqrt{X_i^2 + Y_i^2 + Z_i^2}, \tag{13.1-6a}$$

Boost Phase

while declination and right ascension follow from

$$\sin \delta_i = Z_i / r_i; \qquad -90° \leqslant \delta_i \leqslant 90°, \tag{13.1-6b}$$

$$\left.\begin{aligned} \cos \alpha_i &= X_i / (r_i \cos \delta_i); \\ \sin \alpha_i &= Y_i / (r_i \cos \delta_i); \end{aligned}\right\} \quad 0° \leqslant \alpha_i \leqslant 360°. \tag{13.1-6c}$$

the injection velocity is

$$V_i = \sqrt{V_{X_i}^2 + V_{Y_i}^2 + V_{Z_i}^2}. \tag{13.1-7}$$

To determine azimuth and flight path angle we introduce the vectors $\boldsymbol{e}_{V_i}$ and $\boldsymbol{e}_{r_i}$ in the direction of injection velocity and radius respectively:

$$\boldsymbol{e}_{V_i} = (V_{X_i}\boldsymbol{e}_X + V_{Y_i}\boldsymbol{e}_Y + V_{Z_i}\boldsymbol{e}_Z)/V_i, \tag{13.1-8a}$$

$$\boldsymbol{e}_{r_i} = (X_i\boldsymbol{e}_X + Y_i\boldsymbol{e}_Y + Z_i\boldsymbol{e}_Z)/r_i, \tag{13.1-8b}$$

where $\boldsymbol{e}_X$, $\boldsymbol{e}_Y$ and $\boldsymbol{e}_Z$ are the unit vectors along the X, Y and Z-axis respectively. Then the flight path angle is determined by

$$\sin \gamma_i = \boldsymbol{e}_{V_i} \cdot \boldsymbol{e}_{r_i}; \qquad -90° \leqslant \gamma_i \leqslant 90°. \tag{13.1-9}$$

Boost Phase

The unit vector lying in the horizontal plane at injection and pointing along the downrange heading is

$$\boldsymbol{e}_{VH_i} = (\boldsymbol{e}_{V_i} - \boldsymbol{e}_{r_i} \sin \gamma_i)/\cos \gamma_i, \tag{13.1-10}$$

and the injection azimuth is found from

$$\left.\begin{aligned} \cos \psi_i &= \boldsymbol{e}_{VH_i} \cdot \boldsymbol{e}_{N_i}; \\ \sin \psi_i &= \boldsymbol{e}_{VH_i} \cdot \boldsymbol{e}_{E_i}; \end{aligned}\right\} \quad 0 \leqslant \psi_i \leqslant 360°, \tag{13.1-11}$$

where $\boldsymbol{e}_{N_i}$ and $\boldsymbol{e}_{E_i}$ are unit vectors lying in the horizontal plane at injection, pointing due north and due east respectively. From Eqs. (13.1-3) it can easily be derived that

$$\boldsymbol{e}_{N_i} = -\sin \delta_i \cos \alpha_i \boldsymbol{e}_X - \sin \delta_i \sin \alpha_i \boldsymbol{e}_Y + \cos \delta_i \boldsymbol{e}_Z, \tag{13.1-12a}$$

$$\boldsymbol{e}_{E_i} = -\sin \alpha_i \boldsymbol{e}_X + \cos \alpha_i \boldsymbol{e}_Y. \tag{13.1-12b}$$

We now have determined completely the shut-down conditions, which are the injection conditions for the ballistic phase which we will discuss next.

The Ballistic Phase

- To determine the ballistic phase, it is assumed that the earth is Spherical and has a central inverse square force field.
- It is also assumed that the only force acting during the ballistic phase is the gravitational force of the earth, neglecting the effects of Sun and moon and aerodynamic forces.
- As the motion is assumed to take place in a central inverse square force field, the trajectory will be a Keplerian trajectory, i.e. it will lie in a single plane determined by declination, right ascension and azimuth at injection and it will be a conic section