MODULE 4

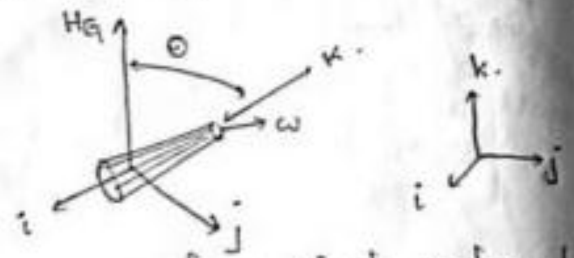
Satellite Attitude Dynamics:

Torque free Axi-symmetric rigid body, Attitude Control for Spinning Spacecraft, Attitude Control for Non-spinning Spacecraft, The Yo-Yo Mechanism, Gravity – Gradient Satellite, Dual Spin Spacecraft, Attitude Determination.

MODULE - 4.

Satellite Attitude dynamics.

Torque free - Axisymmetric Rigid body.



Hg : Angular momentum about centre of mass does not depend on time.

$\dot{H}_G = 0$ — (1)

k = axis of ~~inert~~ internal frame.
θ = Euler angle.

WKT $\cos\theta = \frac{H_G}{|H_G|} \cdot k$.

diff w.r.t time.

$\frac{d\cos\theta}{dt} = \frac{H_G}{|H_G|} \cdot \frac{dk}{dt}$ $\boxed{\text{but } \frac{dk}{dt} = \omega \times k}$

$\frac{d\cos\theta}{dt} = \frac{H_G[\omega \times k]}{|H_G|}$

now $\omega \times k = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ 0 & 0 & 1 \end{vmatrix}$ — (2)

$\omega \times k = \omega_y i - \omega_x j$

WKT

$H_G = A\omega_x i + B\omega_y j + C\omega_z k$

Hg related to the angular velocity in to body frame.

$H_G[\omega \times k] = (A\omega_x i + B\omega_y j + C\omega_z k) \times [\omega_y i - \omega_x j]$

$= (A - B)\omega_x \omega_y$

From eq (2)

$\theta = \omega_n = \dfrac{(A-B)\omega_x\omega_y}{|H_G|\sin\theta}$

WKT
Relative velocity to the body frame.

$H_{G\,relative} + \omega \cdot H_G = 0$

Euler equation

$$\left.\begin{aligned} A\dot{\omega}_x + (C-B)\omega_z\omega_y = 0 \\ B\dot{\omega}_y + (A-C)\omega_x\omega_z = 0 \\ C\dot{\omega}_z + (B-A)\omega_y\omega_x = 0 \end{aligned}\right\} 3 \; (1)(2)(3)$$

z axis is a rotational symmetrical.
so $A = B$.

$$\left.\begin{aligned} A\dot{\omega}_x + (C-A)\omega_y\omega_z = 0 \\ A\dot{\omega}_y + (A-C)\omega_x\omega_z = 0 \\ C\dot{\omega}_z = 0 \end{aligned}\right\} 4 \; (1)(2)(3)$$

The above eqn body to frame z component of a ang velocity is constant

$\omega_z = \omega_0(\text{constant}) \; — (5)$.

The assumption of rotational symmetry therefore reduces the (3) diff eq (3) to (2) sub (5) in to eq $4_{(1)}$ & $4_{(2)}$

$A\dot{\omega}_x - (A-C)\omega_0\omega_y = 0$

$(\div A)$

$\dot{\omega}_x - \dfrac{(A-C)}{A}\omega_0\omega_y = 0 \; — (6_{(1)})$

III ly $\boxed{\dot{\omega}_x - \lambda\omega_y = 0}$ $\left[\text{where } \lambda = \dfrac{(A-C)\omega_0}{A}\right]$

$\dot{\omega}_y + \dfrac{(A-C)}{A}\omega_0\omega_x = 0 \; — (6_{(2)})$

$\boxed{\dot{\omega}_y + A\omega_x = 0}$

diff w.r.t time eq (6)

$\ddot{\omega}_x - A\dot{\omega}_y = 0 \; — (7)$

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we solve 6(2) from ω_y & sub in eq (7)

$$\dot{\omega}_y = \lambda \omega_x$$

$$\omega_x + \lambda \omega_x = 0$$

$$\boxed{\omega''_x + \lambda^2 \omega_x = 0} \longrightarrow (8)$$

~~The solutn well dif known diffe~~

$$\omega_x = \omega_{xy} \sin\lambda t \quad \text{—} (9)$$

ω_{xy} [$\omega_{xy} \neq 0$] → const amplitude

from eq (6①)

$$\omega_y = \frac{1}{\lambda}\frac{d\omega_x}{dt}$$

$$= \frac{1}{\lambda}\frac{d}{dt}[\omega_{xy}\sin\lambda t]$$

$$\boxed{\omega_y = \omega_{xy}\cos\lambda t} \quad \text{—} (10)$$

eq (5), (9) & (10) give the component of absolute angular velocity ω along the 3 principal body axes.

$$\omega = \omega_{xy}\sin\lambda t\,i + \omega_{xy}\cos\lambda t\,j + \omega_0 k.$$

$$\omega = \omega_1 + \omega_0 k.$$

where $\omega_1 = \omega_{xy}[\sin\lambda t\,i + \cos\lambda t\,j]$

where $\omega_p = \dot\phi = \frac{1}{\sin\theta}[\omega_x \sin\psi + \omega_y \cos\psi]$

$\omega_n = \dot\theta = \omega_x \cos\psi - \omega_y \sin\psi$

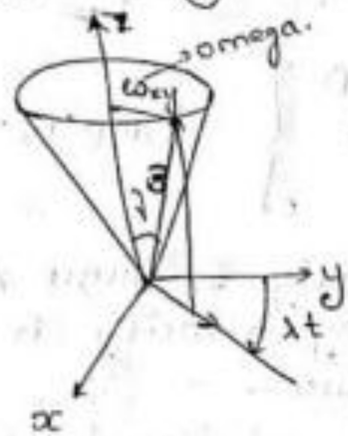
$\omega_s = \dot\psi - \frac{1}{\tan\theta}[\omega_x \sin\psi + \omega_y \cos\psi]$

sub eqtn 5, 9, 10 into 3 equation yield.

$$\left.\begin{aligned} \omega_p &= \frac{\omega_{xy}}{\sin\theta}\cos[\lambda t - \psi] \quad 11.1 \\ \omega_n &= \omega_{xy}\sin[\lambda t - \psi] \quad 11.2 \\ \omega_s &= \omega_0 - \frac{\omega_{xy}}{\tan\theta}\cos[\lambda t - \psi] \quad 11.3 \end{aligned}\right\} \text{eq}$$

since $A = B$ $\omega_n = 0$

$\psi = \lambda t$ — (12)



$\lambda t - \psi = n\pi$ $n = 0, 1, 2, 3.$

$n = 0 \rightarrow$ without loss of generality

eq (12) in eq (11). (1&3)

$\omega_p = \frac{\omega_{xy}}{\sin\theta}$ — (13)

&

$\cos' = \omega_0 = -\frac{\omega_{xy}}{\tan\theta}$ — (14)

differentiate (12)

$\lambda = \dot\psi = \omega_s$

sub $\lambda_{spinrate}$ in eq (5) (1)

$\omega_s = \frac{A - C}{A}\omega_0$

ω_s & ω_0 opposite side.

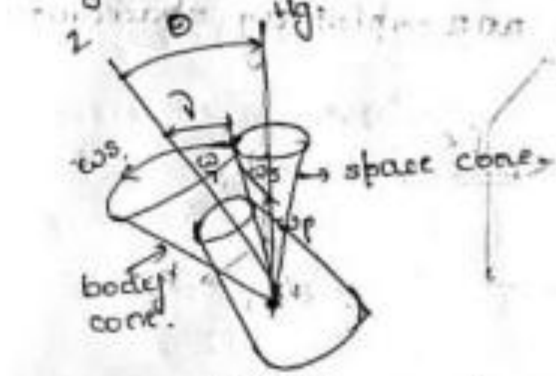
$\boxed{\text{if } A < C}$ it this condition occurs ω_s & ω_0 will be in opp side.

$$\omega_{xy} = \frac{C}{A}\omega_0 \tan\theta$$

$$\omega_0 = \frac{A}{C}\omega_p \cos\theta$$

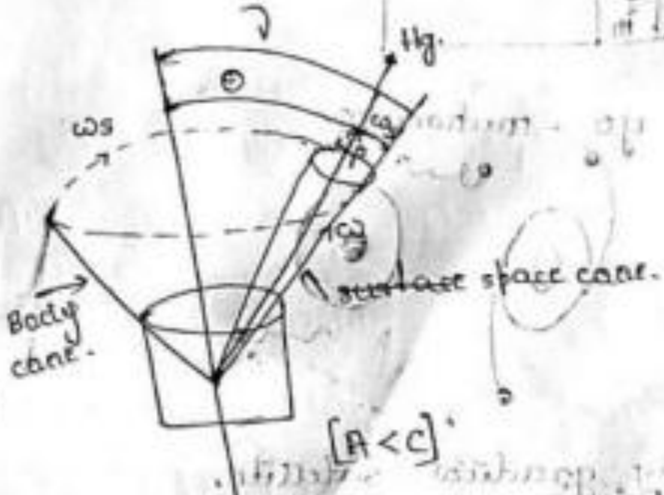
$$\boxed{\omega_p = \frac{C}{A \cdot C} - \frac{\omega}{\cos\theta}}$$

Prograde Precession:-



[A>C]

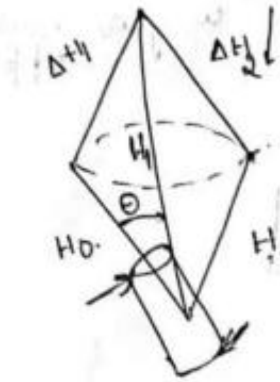
Retrograde precession:-



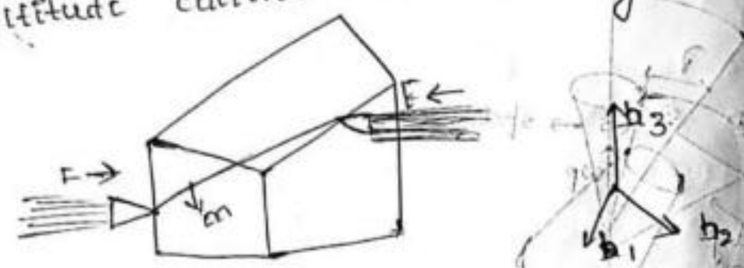
[A<C]

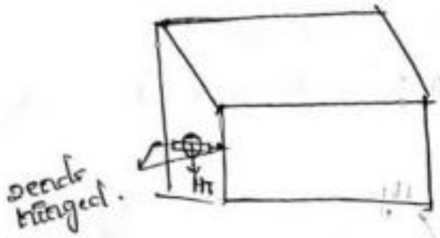
Attitude control spinning spacecraft.

mod 4 no prob



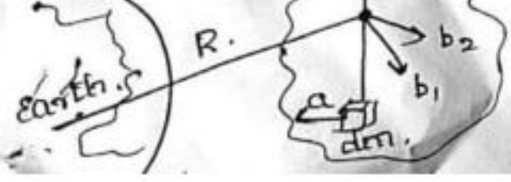
Attitude control non-spinning spacecraft



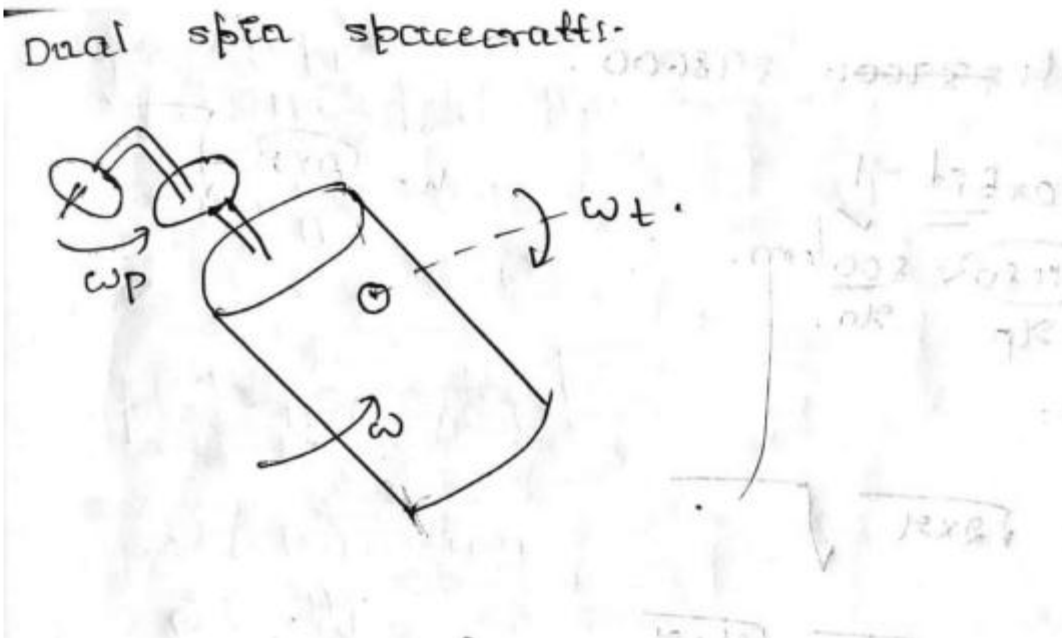


The yo-yo mechanism.

The gravity gradient satellite.



a=accelaration.



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