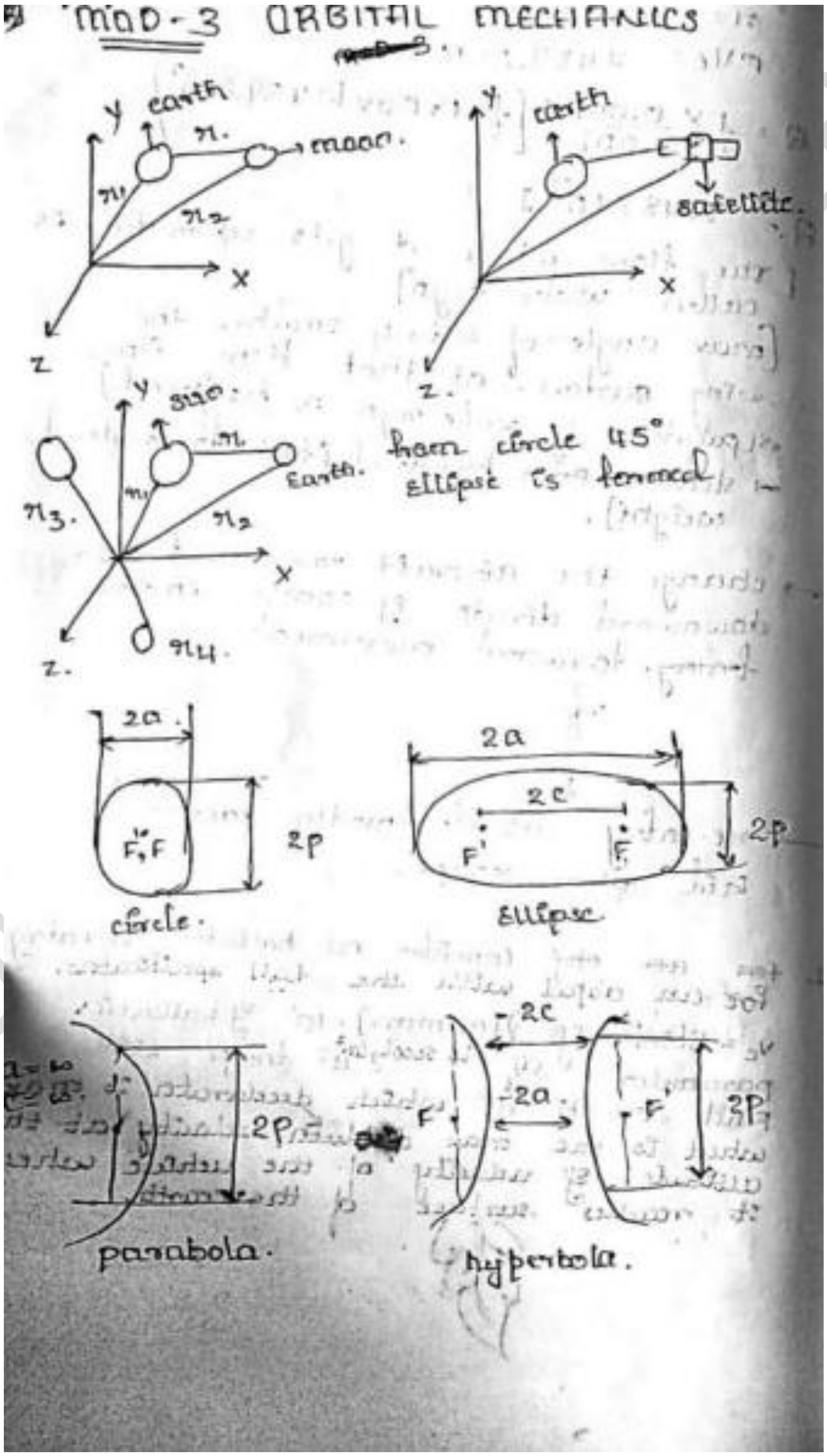
MODULE 3

Fundamentals of Orbit Mechanics, Orbit Maneuvers:

Two-body motion, Circular, elliptic, hyperbolic, and parabolic orbits-Basic Orbital Elements, Ground trace In-Plane Orbit changes, Hohmann Transfer, Bielliptical Transfer, Plane Changes, Combined Maneuvers, Propulsion for Maneuvers.

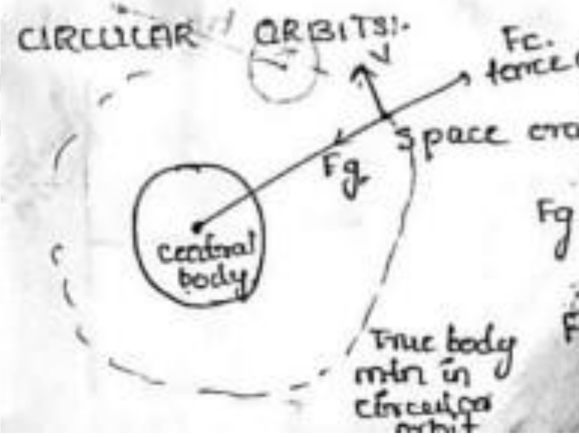


→ motion of spacecraft is governed by attractn to a single centered body.
→ The mass of the spacecraft is negligable compared to a central body.
→ The bodies are spherically symmetry and the mass concentrated at the centre.
→ No force act on the body except for gravitational & centrifugal force acting along the line of centres.
[These are the Assumptions of the 2 body problems].

Relation Acceleration over Earths orbital :-

Body.	Acceleration g. 200 km.
Earth.	0.9.
sun.	6×10^{-4}
mercury.	2.6×10^{-10}.
venus.	2×10^{-6}.
Jupiter.	3×10^{-6}.
mars.	3.2×10^{-8}
saturn	2.3×10^{-9}
uranus	8×10^{-11}
neptune	3.6×10^{-11}
pluto.	10^{-12}.

CIRCULAR ORBITS:-



$$Fg = \frac{M.m.G.}{r^2}$$

$$Fc = \frac{mv^2}{r}.$$

$$F_g = F_c$$

$$\text{or } \frac{M.m.G}{r^2} = \frac{mv^2}{r}$$

$$\left\{ v = \sqrt{\frac{M.G}{r}} \right\}$$

gravitational parameter $\mu = M.G.$

$$\left\{ v = \sqrt{\frac{\mu}{r}} \right\}$$

$\downarrow$

velocity of circular orbit in 2 body prob.

Period of circular orbit:-

$$P = \frac{\text{circumference}}{\text{velocity}}$$

$$= \frac{2\pi r}{\sqrt{\frac{\mu}{r}}}$$

$$= 2\pi r \times \sqrt{\frac{\mu}{r}}$$

$$= 2\pi \sqrt{\frac{r^3}{\mu}}$$

What is the velocity of the space shuttle in 150 nautical miles in a circular orbit. aeronautile miles

n-miles →

h = 150 n-miles.

= 150 × 1.852 km.

= 277.8 km.

r = ~~6378~~ R+h.

= 6378 + 277.8

= 6655.8 km.

$$v = \sqrt{\frac{MG}{r}}$$

$$= \sqrt{\frac{5.97\times10^{24}\times 6.67\times10^{-11}}{6655.8}}$$ convert to m.

$$= 7.79 \text{ m/s}$$

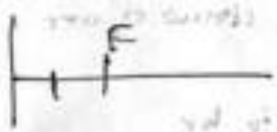
$$P = 2\pi\sqrt{\frac{(6655\times10^{3})^{3}}{(5.97\times10^{24})\times(6.67\times10^{-11})}}$$

$= 5404$ sec $= 1.5$ hrs $= 90$ min.

characteristics of conic orbits:

element	circle	Ellipse	Parabola	Hyperbola
eccentricity	0	<1	=1	>1
semimajor axis 'a'	= r	+ve		−ve
specific energy [E]	−ve	−ve	0	+ve

[not for syllabus]

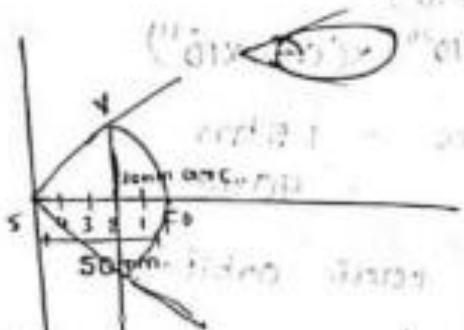


$e = \frac{VF}{VD}$ — eccentric, vortex, VF → focus distance, VD → directing

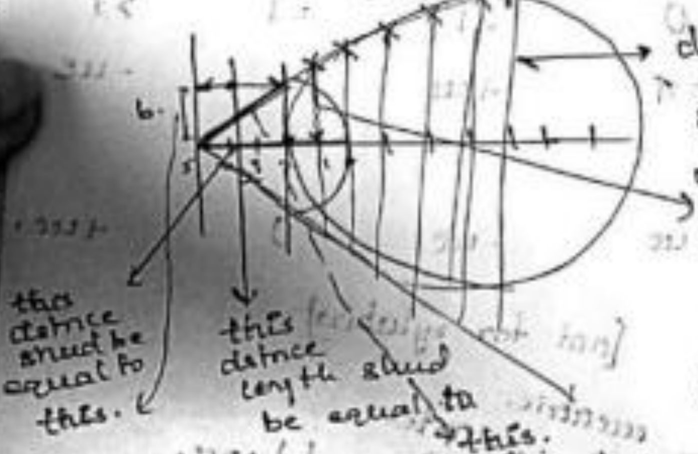
for eg: $\frac{2}{3} = 0.66$ (Ellipse)

focus distnce = 50mm.
so then.

we need to divide. the divisions! 50mm: 2+3=5. so the 5 divisions wid 10mm distnce.



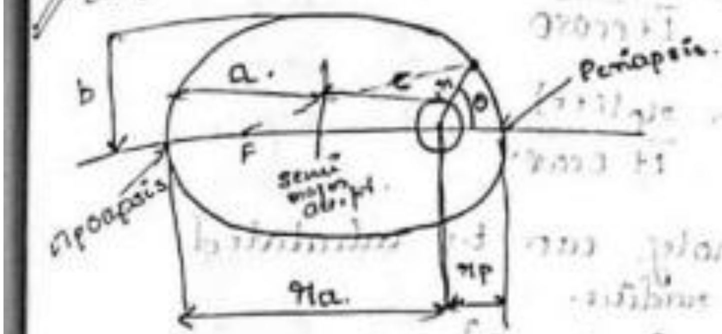
divide the no of divisions wid distnce of 10mm.



draw lines like this & the measure the distnce. of the lines & keeping vertex as centre draw a arc.

for parabola, only velocity to be considered. & for ellipse & hyperbola we shud find velocity, etc, all type of data.

Elliptical orbit :-



a = semimajor axis.
e = eccentricity.
r_a = apoapsis radius.
r_p = periapsis

semimajor axis $a = \frac{r_a + r_p}{2}$

classical orbital element.

semimajor axis ≠ the average radius.

$$2c = r_a - r_p$$

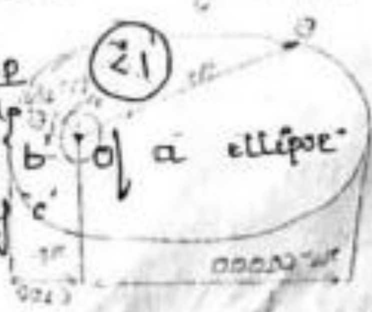
the distance b/w elliptical foci is $2c$

$$c = \frac{(r_a - r_p)}{2}$$

WKT $e = \frac{c}{a}$

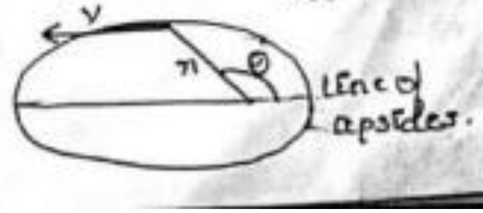
$$e = \frac{(r_a - r_p)/2}{(r_a + r_p)/2}$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

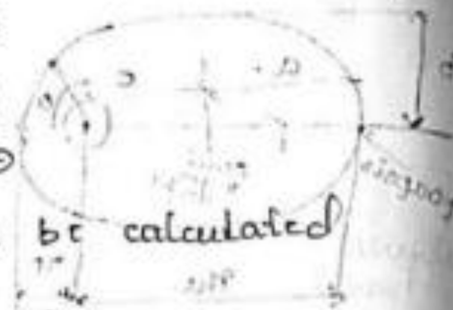


semiminor axis b of a ellipse is related to a & c

$$a^2 = b^2 + c^2$$



$$r = \frac{a[1-e^2]}{1+e\cos\theta}$$



shape for gng orbital orbit

$$r = \frac{r_p[1+e]}{1+e\cos\theta}$$

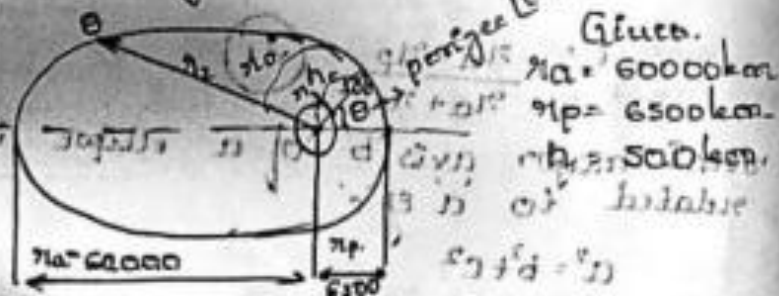
true anamoly can be calculated from the radius.

$$\cos\theta = \left[\frac{r_p[1+e]}{re}\right] - \frac{1}{e}$$

$$\cos\theta = \left[a[1-e^2]/re\right] - \frac{1}{e}$$

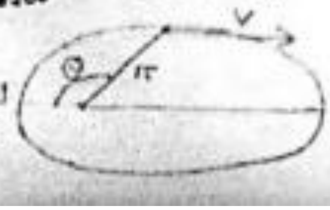
General → periapsis — Apoapsis.
sun → perihelion — Aphelion.
Earth → Perigee — Apogee.
Moon → Perilune — Apolune

1) Given an elliptical earth orbit with a perigee radius of 6500km and apogee radius of 69000km. Find the angle of the spacecraft position as it enters the van-allen belts at an altitude of abt 500km. Find the eccentricity.



Given.
r_a = 60000km
r_p = 6500km
r_{h} = 500km.

To find:-
e =
θ_1 =
θ_2 =



$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$= \left(\frac{60000 - 6500}{60000 + 6500}\right)$$

$e = 0.8045$

$r_0 = 6378.14 + 500$

$r_0 = 6878.14$ km.

$$\cos\theta = \left[\frac{r_p[1+e]}{r_c}\right] - \frac{1}{e}$$

$$= \left[\frac{6500[1+0.8045]}{(6878.14)(0.8045)}\right] - \frac{1}{0.8045}$$

by Graphically method we get this

$\cos\theta = 0.87$

$\theta = 28.76 \rightarrow$ perigee lte or $331.245°$

extra $r_1 = \dfrac{r_p[1+e]}{1+e\cos\theta_1} = \dfrac{6500[1+0.8045]}{1+0.8045\cos[28.76]} = 6878.25$

$r_2 = \dfrac{r_p[1+e]}{1+e\cos\theta_2}$ $\Leftarrow \theta = 331.245$

this reaches the space craft at the same radius.

* 3] design a transfer ellipse from Earth to heliocentric position of $r = 1$ A.U. of longitude of $41.26°$ to pluto at $r = 39.557$ AU & a longitude of 194.66 place a line of aspides at an angl longitude of $25°$. The true position of spacecraft at earth position is $41.26 - 25 = 16.26°$ at pluto position true lte is $194.66° - 25 = 169.66°$ degree

r_e = ~~6378 km~~. 6378.1 km. [radius taken from centre of the earth]

r_2 = 1AU or
= 1.4958×10^8 km.

we take this value becoz pluto is mentioned as a farer planet heliocentric.

$r_3 = 5.994 \times 10^9$ km

to find:- $\left.\begin{matrix} e = \\ r_p = \end{matrix}\right\}$?

Formula :-

$$e = \frac{r_3 - r_1}{r_1 \cos\theta_1 - r_2 \cos\theta_3}$$

$\theta_1 = 16.26$
$\theta_3 = 169.26$

$$r_p = \frac{r_1[1 + e\cos\theta]}{1 + e}$$

$e = 0.96$.

$r_p = 1.46 \times 10^8$ km

Relations defining an elliptical orbit.

Eccentricity 'e'

$e = c/a$ $\qquad$ $e = \frac{r_a}{a} - 1$

or $e = \frac{r_a - r_p}{r_a + r_p}$ $\qquad$ $e = 1 - \frac{r_p}{a}$.

Flight path angle [γ]

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta}$$

Mean motion [n]

$$n = \sqrt{\frac{\mu}{a^3}}$$

Period [P] =

$$P = \frac{2\pi}{n}.$$

$$P = 2\pi\sqrt{\frac{a^3}{h}}.$$

Radius [R] $\rightarrow$ General.

$$r = \frac{a[1-e^2]}{1+e\cos\theta}$$

$$r = \frac{r_p[1+e]}{1+e\cos\theta}$$

Radius of apoapsis (r_a)

$$r_a = a[1+e]$$

$$r_a = 2a - r_p$$

$$r_a = r_p\frac{[1+e]}{1-e}.$$

Radius of periapsis (r_p)

$$r_p = a[1-e]$$

$$r_p = r_a\frac{[1-e]}{[1+e]}$$

$$r_p = 2a - r_a.$$

$$r_p = \frac{r_1[1+e\cos\theta_1]}{1+e}.$$

semimajor axis [a]

$$a = \frac{(r_a + r_p)}{2}$$

$$a = \frac{hr}{2h - V^2 r}.$$

$$a = \frac{r_p}{1-e}.$$

$$a = \frac{r_a}{1+e}.$$

Time since periapsis 't'

$$t = [E - e\sin E] / n.$$

$$\cos E = \frac{e + \cos\theta}{1 + e\cos\theta}$$

E - eccentricity anomaly
e = eccentricity of orbit

True anomaly θ

$$\cos\theta = \frac{r_p[1+e]}{r e} - \frac{1}{e}$$

$$\cos\theta = \frac{a[1-e^2]}{r e} - \frac{1}{e}$$

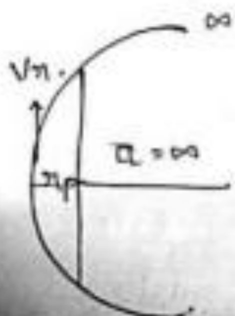
velocity :-

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

$$r_p V_p = r_a V_a .$$

~~Parabolic orbit :-~~ 11/10/18

PARABOLIC ORBIT :-



$E = 0,\ a = \infty$

$$v = \sqrt{\frac{2\mu}{r_e}}$$

earth mass = 5.97×10^{24}

$G = 6.67 \times 10^{-11}$

earth r = 6378

what is the escape velocity from the surface of the moon take μ for moon 4902.8 km^3/sec^2 & r = 1733km.

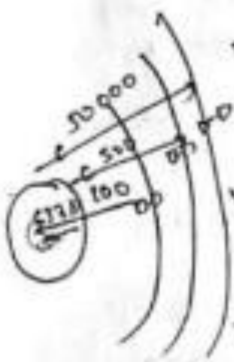
$$V = \sqrt{\frac{2\mu}{r}}$$

$= 2.37$ km/sec

v=of earth.

$$\sqrt{\frac{2\times 5.97\times10^{24}\times 6.67\times10^{-11}}{1738}}$$

$= 6.73\times10^{5}$ km/s



$$v = \sqrt{\frac{2\times5.97\times10^{24}\times6.67\times10^{-11}}{6378+100}}$$

$= 3.48\times10^{5}$ km/s.

for 500 $V = 3.38\times10^{5}$ km/s.

for 50000 $V = 1.1\times10^{5}$ km/s.

etc.

HYPERBOLIC ORBIT :-

Relations defining a hyperbolic orbit

angle of asymptote (β)

→ $\tan\beta = b/a$ $\quad\tan\beta = b\,V_{HE}^2/\mu$.

→ $\tan\beta = \frac{2br_p}{b^2-r_p^2}$ $\quad\cos\beta = \frac{1}{e}$.

Eccentricity (e)

$e = \frac{1}{\cos\beta}$ $\quad e = 1+(r_p/a)$

$e = \sqrt{1+(b^2/a^2)}$

Flight path angle [γ]

$$\tan\gamma = \frac{e\sin\theta}{1+e\cos\theta}$$

Mean motion [n]

$$n = \sqrt{\mu/a^3}$$

Radius [r]

general = $r = \frac{a[e^2-1]}{(1+e\cos\theta)}$

Radius of periapsis [r_p]

$$r_p = b\sqrt{(e-1)/(e+1)}$$

$$r_p = c-a.$$

$$r_p = a[e-1]$$

$$r_p = b\tan[\beta/2]$$

$$r_p = \frac{2\mu+\mu[e-1]}{V_p^2}$$

$$r_p = -\frac{\mu}{V_{HE}^2} + \sqrt{\left(\frac{\mu}{V_{HE}^2}\right)^2 + b^2}$$

$$r_p = -a + \sqrt{a^2+b^2}$$

semi major axis:-[a]

$$a = b\sqrt{e^2-1}$$

$$a = \mu / V_{HE}^2$$

$$a = r_p / [e-1]$$

$$a = (b^2 - r_p^2) / 2r_p$$

$$a = \frac{\mu - r_p}{r_p V_p^2 - 2\mu}.$$

semi major axis [b]

$b = r_p \sqrt{[e+1] / [e-1]}$

$b = a\sqrt{e^2-1}$

$b = r_p \sqrt{(2\mu / r_p - V_{HE}^2) + 1}$

Time since periapsis [t]

$t = [e\sinh F - F] / n.$

$\cosh F = \frac{(e+\cos\theta)}{(1+e\cos\theta)}$

$F = \ln[\cosh F + \sqrt{\cosh^2 F - 1}]$

$\sinh F = \frac{1}{2}[e^F - e^{-F}]$

True anomaly [θ]

$\cos\theta = \frac{a(e^2-1)}{re} - \frac{1}{e}$

True anomaly of asymptote [θa]

$\theta_a = 180^\circ \pm \beta$

$\cos\theta_a = \frac{-1}{e}.$

velocity [V] :-

$V_\infty = V_{HE} = \sqrt{\mu / a}.$

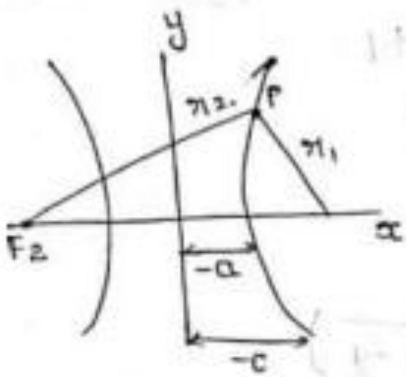
$V = \sqrt{(2\mu / r) + V_{HE}^2}$

$V = \sqrt{(2\mu / r) + \left(\frac{\mu}{a}\right)}$

$C3 = V_{HE}^2.$

31/9. HYPERBOLA.

Cartesian co-ord system:

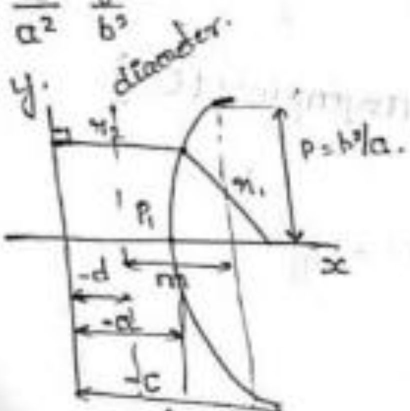


$r_2 - r_1 = -2a$.

$r_2 = \sqrt{(x+c)^2+y^2}$

$r_1 = \sqrt{(x-c)^2+y^2}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



at a pt P_1

$e = \frac{r_1}{r_2}$ @ $P_1 \rightarrow e = \frac{-c-[-a]}{-a-[-d]}$

$= \frac{-c+a}{-a+d}$.

$P_2 = e = \frac{P}{-c-[-d]} = \frac{-b/a}{-c+a}$

$d = \frac{a^2}{c}$

$e = \frac{c}{a}$.

$m = -c+d = P/a$.

$r_p = \frac{P}{1+e\cos\theta}\Big|_{\theta=0}$

$\frac{P}{1+e} = a(e-1)$

$a = \left.\frac{p}{1+e\cos\theta}\right|_{\theta=180^\circ}$

$\frac{p}{1-e} = a(e+1)$

Hyperbolic orbits.

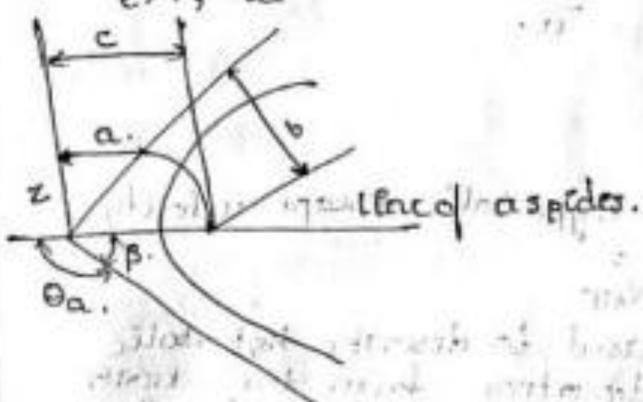
θ_a = ~~bur~~ true anamoly of the asymptote.

r_p = Periapsis radius.

a = semi major axis the distance from the centre of ~~to~~the periapsis.

b = semi minor axis the distance from the asymptote to a parallel passing through the central body.

$e>1$, c/a.



As with an ellipse

$c^2 = a^2+b^2$

The angle of the asymptote

$\cos\beta = 1/e$.

$r = \frac{a(e^2-1)}{1+e\cos\theta}$

$\cos\theta = \frac{a(e^2-1)}{re} - 1/e$ ①

The true anamoly of the asymptote θ_a.

$$\theta_a = 180deg \pm \beta.$$

$\theta_{a\,min}$ & $\theta_{a\,max}$ $\longrightarrow$ hyperbolic radius is infinite.

From eq① true anamoly of the asympote can also be expressed as

$$\cos\theta_a = -1/e.$$

Flight path angle.

$$\tan\gamma = \frac{e\sin\theta}{[1+e\cos\theta]}$$

The velocity at any pt on a hyperbola is

$$v = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}.$$

$$v_\infty = \sqrt{\frac{\mu}{a}}.$$

$v_\infty = V_{HE}$ = Hyperbolic escape velocity.

$$C3 = V_{HE}^2$$

C3 = used to describe hyperbolic departure from the earth
It is not used to describe arrival at a planet.

Hyperbolic earth departure:-

The elements of the departure hyperbola for the viking 1 mass lander if take a value = 18849.7 km if $e = 1.3482$. what c3 value was provided by the lander titan III E launch vehicle if take $\mu = 398600.4\ km^3/sec^2$.

Given:-

$a = 18849.7$ km;

$e = 1.3482$

$\mu = 398600.4\ km^3/s^2$

$e = 1.3482 \cdot \left(\frac{V.F}{V.O}\right)^2$

$V_{HE} = \sqrt{\frac{\mu}{a}}$.

To find:-

$C3 = V_{HE}^2$.

β :

Soln:- $C_3 = V_{HE}^2 = \frac{398600.4}{18849.7} = 21.14\ km^3/s^2$

$\cos\beta = \frac{1}{e}$

$\beta = \cos^{-1}\left[\frac{1}{1.3482}\right]$

$\boxed{\beta = 42.12^\circ}$

16/10/19

Time of flight :-

$t = (e\sinh F - F)/n$

$\cosh F = [e + \cos\theta] / [1 + e\cos\theta]$

where :-

t = time since periapsis passage.

F = hyperbolic eccentric anamoly.

e = eccentricity.

n = mean motion.

θ = true anamoly

$F = \ln[\cosh F + \sqrt{\cosh^2 F - 1}]$

$\sinh F = \frac{1}{2}[e^F - e^{-F}]$

On August 24. 1989 voyager-2 flew fast north pole of neptune elements of V-2 encounter hyperbola where a = 19985 km, eccentricity 2.45859. during departure of V2 passed the ~~trait~~ triton, one of the moons of neptune at a radius of 3,54,600 km. what was the time since periapsis for the encounter with the moon.

Given:-

$a = 19985 km$

$e = 2.45859$

$r = 354600 km.$

$t = ?$

$$t = (e\sinh F - F) / n.$$

$$F = \ln[\cosh F + \sqrt{\cosh^2 F - 1}]$$

$$\cosh F = \frac{e + \cos\theta}{1 + e\cos\theta}$$

$$\cos\theta = \frac{a(e^2-1)}{re} - \frac{1}{e}$$

$$n = \sqrt{\frac{\mu}{a^3}} \qquad \mu = mg. \rightarrow 6871307.8$$

$$\cos\theta = \cancel{0.11563} - 0.4067$$

$$\cancel{1.156 \times 10^{-4}} - 0.4067$$

$$= -0.2911$$

$$n = \sqrt{\frac{6871307.8}{(19985)^3}}$$

$$= 0.00092783^{-1}$$

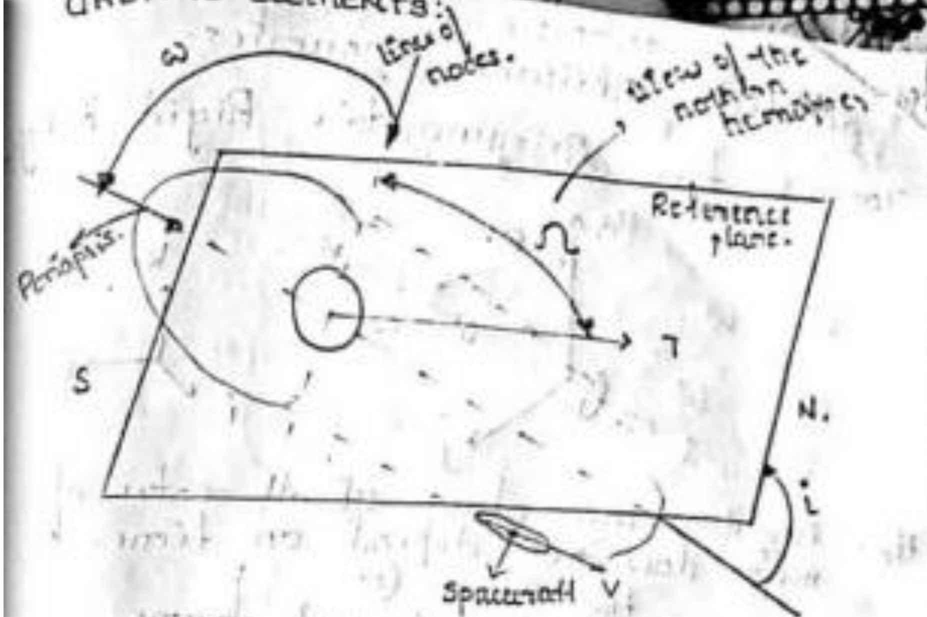
$$\cosh F = 7.623.$$

$$F = 2.719 \approx 2.72.$$

$$t = 17094 \text{ sec}$$

$$= 4.74 \text{ hrs.}$$

ORBITAL ELEMENTS:



a = semimajor axis

i = Inclination.

ω = Argument of periapsis.

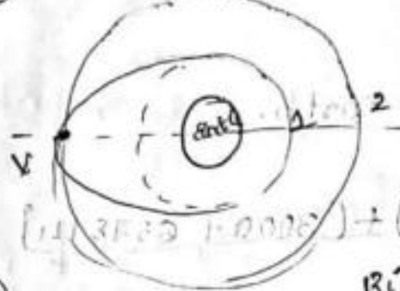
Ω = longitude of ascending node.

θ = true anomaly.

d. plane orbit change:

circular orbit + ΔV = ellipse.

(2) Hohmann transfer.



velocity needed is more.

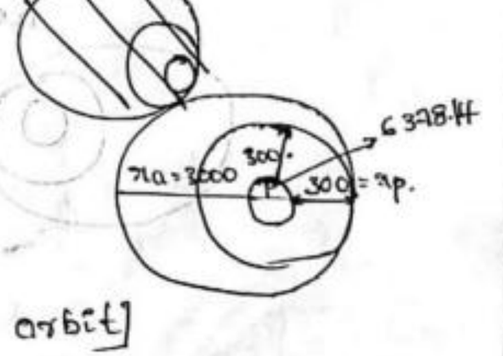
(3) Bi-elliptical transfer

velocity req is less.

8% of velocity is reduced.

Q In plane orbit change prob:

consider an initial circular lower earth orbit at a 300 km altitude. wt velocity increase would be required to produce an elliptical orbit with an 300 km altitude at perigee & 3000 km altitude at apogee.



r = 300 km.

r_a = 3000 km.

r_p = 300 km.

μ = MG = 398600.4

find:- velocity:

$V = \sqrt{\frac{\mu}{r}}$ [circular orbit]

major axis $a = \frac{r_a + r_p}{2}$

velocity at periapsis.

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

Soln:

$$V = \sqrt{\frac{398600.4}{(300 + \underbrace{6378.14}_{\text{radius of earth}})}}$$

$V = 7.726$ km/s.

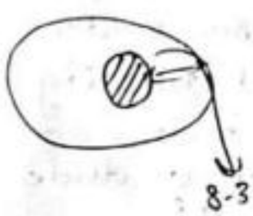
$$a = \frac{(300 + 6378.14) + (3000 + 6378.14)}{2}$$

$a = 8028.14$ km.

$$V = \sqrt{\frac{2 \times 398600.4}{6378.14 + 300} - \frac{398600.4}{8028.14}}$$

$$\sqrt{119.374 - 49.650}$$

$= 8.350$ km/s.



~~for~~ directly from earth to elliptical totally if that velocity is req.

but here.

Earth to circular orbit it is already velocity is 7.726 if then circular merges with elliptical so sub.

$8.350 - 7.726 = 0.624$ m/s. final velocity

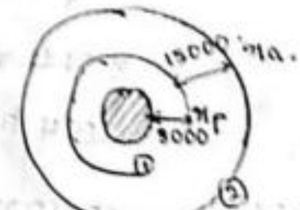
~~Design~~ design: ~~Define~~ a Holmann transfer from a circular mass of radius 8000km to a ~~of~~ " " orbit of radius 15000km. take $\mu = 42828.3\ km^3/sec^2$.

~~we need velocit~~

$r = 8000$ km.

$h = 15000$ km.

$\mu = 42828.3\ km^3/sec^2$



to find:-

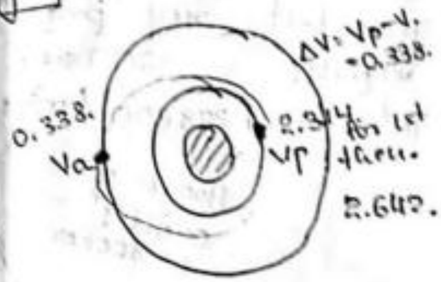
Velocity of period.

↓

design of Holmann transfer.

we need to find from orbit 1 to orbit 2 how the transformation of shape will be formed.

$$V = \sqrt{\frac{\mu}{r}}$$ initial orbit velocity



$$a = \frac{r+h}{2}$$

$$V_p = \sqrt{\frac{2\mu}{\underset{r = r_p = 8000}{r}} - \frac{\mu}{a}}$$

$$V_a = \sqrt{\frac{2\mu}{\underset{= r_a = 15000}{h}} - \frac{\mu}{a}}$$

$$\Delta V_1 = V_p - V.$$

$$p = 2\pi \sqrt{\frac{a^3}{\mu}}$$

Solution:-

$$V = \sqrt{\frac{42828.3}{8000}} = 2.314\ km/s.$$

$a = 11500$ km.

$$V_p = \sqrt{5.71044 - 3.7242}$$

$\boxed{V_a = 1.409\ km/s.}$

$\boxed{V_p = 2.642\ km/s}$

$\Delta V_1 = 2.642 - \cancel{2.2} \; 2.314$

$= \cancel{1.233} \; 0.338$ km/s.

$P = 2\pi \sqrt{\frac{(11500)^3}{42828.3}}$

$= \cancel{3.714 \times 10^4} \; 37442$ sec.

⑥ 18/10/19.

$= 10.4$ hrs.

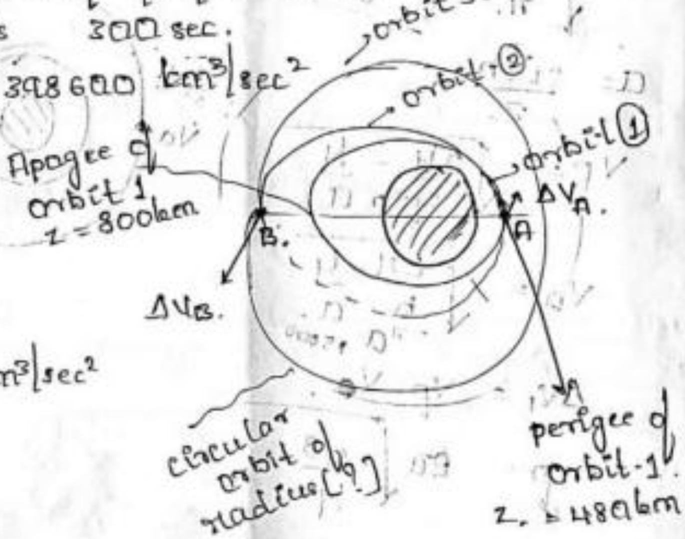
* A 2000kg spacecraft is in a 480x800km earth orbit shown in fig.

① the ΔV required at perigee A to ~~space~~, place a spacecraft in a 480x16000 km transform ellipse

② The ΔV apogee kick required at B of a transfer orbit to establish a circular orbit of 16000km of ~~or~~ altitude.

③ The total req propellant if the sp. impulse is 300 sec.

take $\mu = 398600$ km³/sec²



Given

$\mu = 398600$ km³/sec²

$I_{sp} = 300$ sec.

To find:-

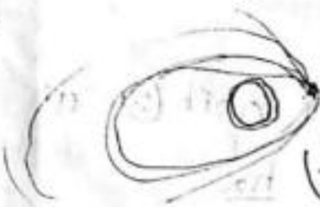
$\Delta V_A = V_{A_2} - V_{A_1}$

$\Delta V_T = \Delta V_A - \Delta V_B$.

$V_{A_1} = \frac{h_1}{r_A}$

$h_1 = \sqrt{2\mu} \sqrt{\frac{r_a r_p}{r_a + r_p}}$

$$\frac{\Delta m}{m} = 1 - e^{\left[\frac{-\Delta V_{total}}{I_{sp} \cdot g}\right]}$$



$$1 - e^{\left(\frac{-\Delta V_{tot}}{I_{sp} \cdot g}\right)}$$

solution:-

Orbit - 1.

480 x 800 km.

r_p r_a

$r_p = 480 + 6378 = 6858$ km

$r_a = 800 + 6378 = 7178$ km.

$h = \sqrt{2 \times 398600} \sqrt{\frac{6858 \times 7178}{6858 + 7178}}$

$h_1 = 52876.5$ km/sec^2

orbit - 2:-

(480 x 16000)
 r_p r_a

$480 + 6378 = 6858$ km $= r_p$

$16000 + 6378 = 22378$ km $= r_{a2}$

$h_2 = 64689.5$ km/sec^2

orbit - 3:-

$r_a = r_p = 16000 + 6378 = 22378$ km

$h_3 = 94445.06$ km/sec^2

$V_{B_3} - V_{B_2}$

$\frac{h_2}{r}$

case 1:-

speed of orbit 1 at a pt A :-

$$V_{A_1} = \frac{h_1}{r_A}$$

$$= \frac{52876.5}{6858}$$

$V_{A_1} = 7.710$ km/sec.

speed of orbit 2 at a pt A:

perigee lock @pt A.

$$V_{A_2} = \frac{h_2}{r_A}$$

$$= \frac{64689.5}{6858}$$

$= 9.432$ km/sec.

$$\boxed{\begin{aligned}\Delta V_A &= V_{A_2} - V_{A_1} \\ &= 1.722 \text{ km/s}\end{aligned}}$$

case (2):-

speed of orbit (2) at pt (B)

$$V_{B_2} = \frac{h_2}{r_B} = \frac{64689.5}{22378}$$

$$= 2.89 \text{ km/sec.}$$

speed of orbit (3) at pt (B)

$$V_{B_3} = \frac{h_3}{r_B} = 4.22 \text{ km/sec}$$

apogee kick req @ pt B.

$$\boxed{\Delta V_B = V_{B_3} - V_{B_2} = 1.33 \text{ km/s.}}$$

$$\Delta V_T = \Delta V_A + \Delta V_B$$

$$= 1.7228 + 1.33$$

$$\Delta V_T = 3.052 \text{ km/s} \rightarrow \text{m/sec.}$$

$$\frac{\Delta m}{m} = 1 - e^{-\left[\frac{3.052 \text{ km/sec}}{300 \times 9.8 \text{ m/sec}^2}\right]} \quad 3.052 \text{ km/sec} \rightarrow 3.052 \times 10^3 \text{ m/sec}$$

$$\left[\frac{\Delta m}{m} \quad \cancel{1 - 2.8238} = 0.6454\right].$$

m → initial mass.

$\Delta m = 0.6454 \times 2000$ → mass of propellant expended.

$$\boxed{\Delta m = 1290.8 \text{ kg.}}$$

~~mass left when it enters to the 3rd orbit.~~

$m - \Delta m = 709.2$ → Left out mass when it enters to the 3rd orbit.

..sign of hohmann transfer from a circular earth orbit of radius 8000 km to a " " " " 15000 km.

$r_p = 8000$ km

$r_a = 15000$ km.

$\mu = 398600$ km³/sec².

To find:- velocity change req to enter the transfer orbit

design of hohmann

$V = \sqrt{\dfrac{398600}{8000}} = 7.0586$ km/sec.

$a = \dfrac{8000 + 15000}{2} = 11500$.

$V_p = \sqrt{\dfrac{2(398600)}{8000} - \dfrac{398600}{11500}}$

$= 8.06$ km/sec.

$V_a = \sqrt{\dfrac{2 \times 398600}{15000} - \dfrac{398600}{11500}}$

$= 4.3$ km/s.

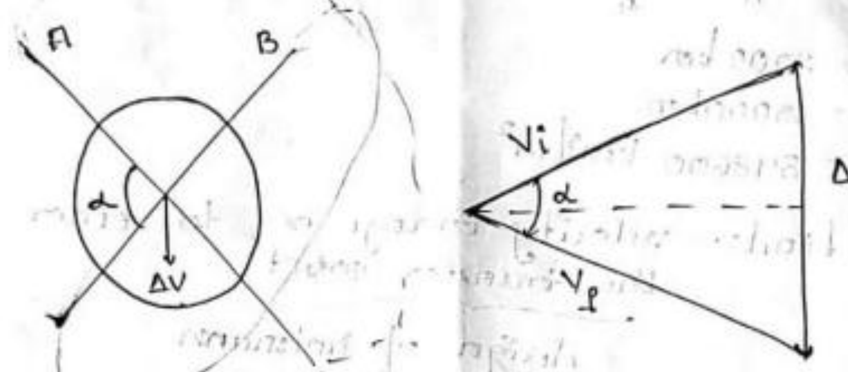
$\Delta V = V_p - V$

$= 8.06 - 7.0586$

$= 1.001$ km/sec.

change in velocity.

Plane changes :-

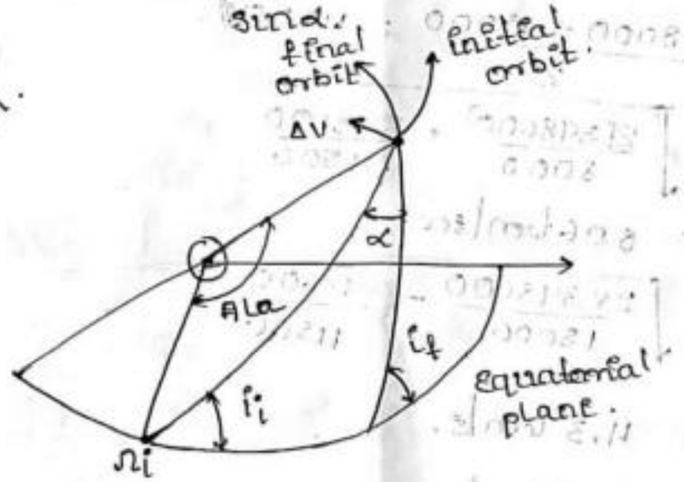


$\Delta V = 2V_i \sin[\alpha/2]$

$\cos\alpha = \cos i_i \cos i_f + \sin i_i \sin i_f \cos[\Delta\Omega]$

$\sin A_{la} = \dfrac{\sin i_f \sin[\Delta\Omega]}{\sin\alpha}$

$V = \sqrt{\mu/r}$.



where

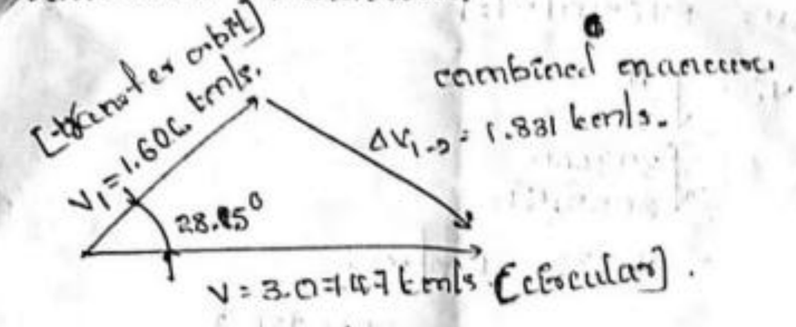
$\Delta\Omega$ = the change of longitude of the ascending node caused by the plane

$\Delta\Omega = [\Omega_f - \Omega_i]$

A_{la} = the argument of latitude which is the angle from the reference plane to the spacecraft position.

α = Angle of the plane change. [initial to final]

Combined maneuver:-



Separate maneuver:-

* plane change maneuver - $\Delta V = 0.791$ km/s.
* circularization maneuver - $\Delta V = 1.469$ km/s.

$\Delta V = 2.260$ km/s.

consider the ~~inertial~~ initial circular earth orbit with foll characteristics:-

$h = 275$ km., $i = 28.5^\circ$, $\Omega = 60^\circ$ W.

it is desired to make a plane change to a circular orbit with the foll final characteristics.

$h = 275$ km, $i = 10^\circ$, $\Omega = 100^\circ$ W.

take $\mu = 398600$ km^3/sec^2 & $r = 6655.94$ [includes earth radius].

design the plane change. $\boxed{r = R_e + h.}$

$$\cos\alpha = \cos[28.5]\cos[10] + \sin[28.5]\sin[10]\cos[100-60]$$

$$\cos\alpha = 0.9289$$

$$\alpha = 21.73.$$

$$\sin A_{ia} = \frac{\sin(10)\sin[100-60]}{\sin 21.73}$$

$$\sin A_{ia} = 0.3014.$$

$$A_{ia} = 17.54.$$

$$\Delta V = 2V_i \sin[\alpha/2]$$

$$V_i = \sqrt{\frac{\mu}{r}}$$

$$= \sqrt{\frac{398600}{6655.94}}$$

$$= 7.7386 \text{ km/sec.}$$

$$\Delta V = 2[7.7386] \sin[21.73/2]$$

$$= 2.91 \text{ km/sec.}$$