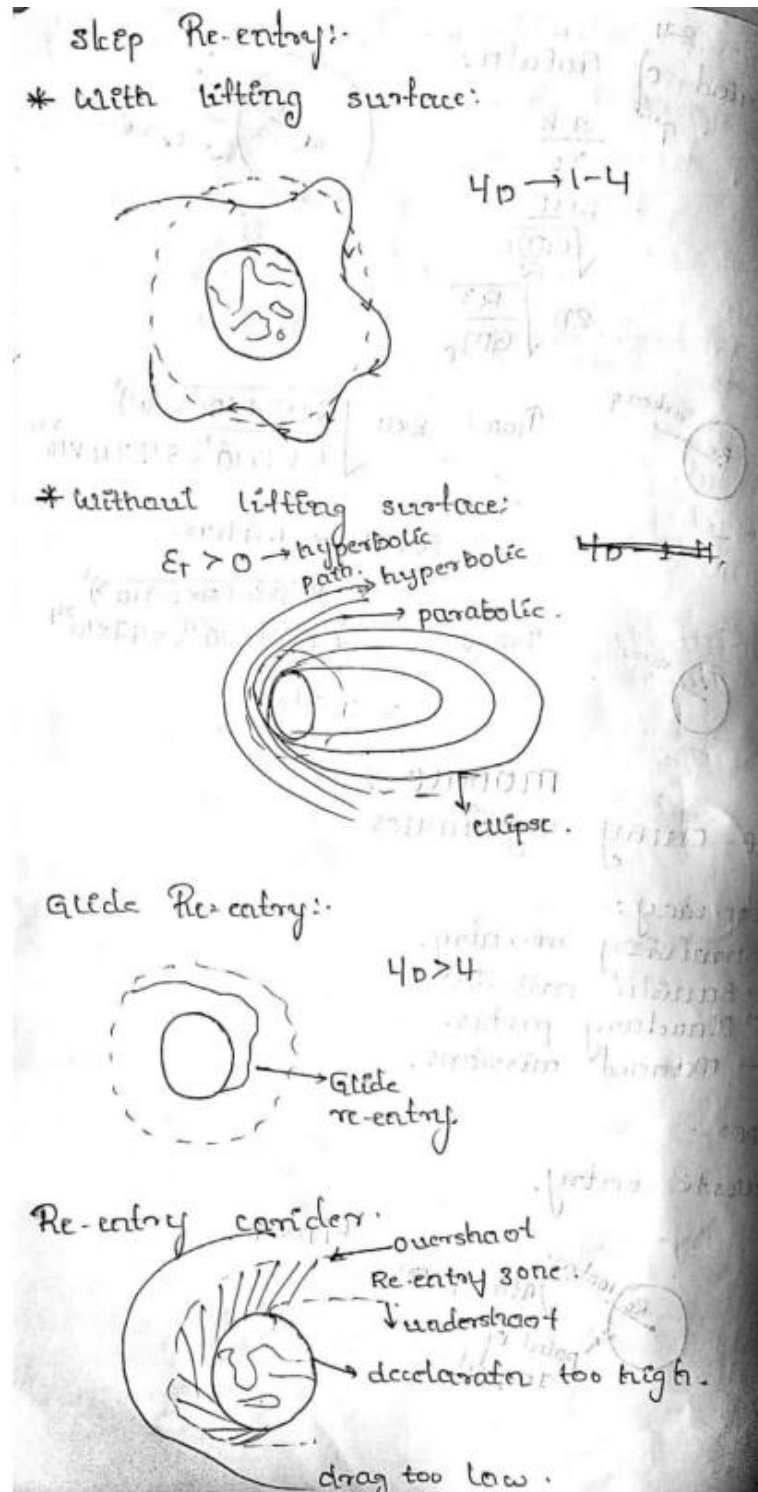


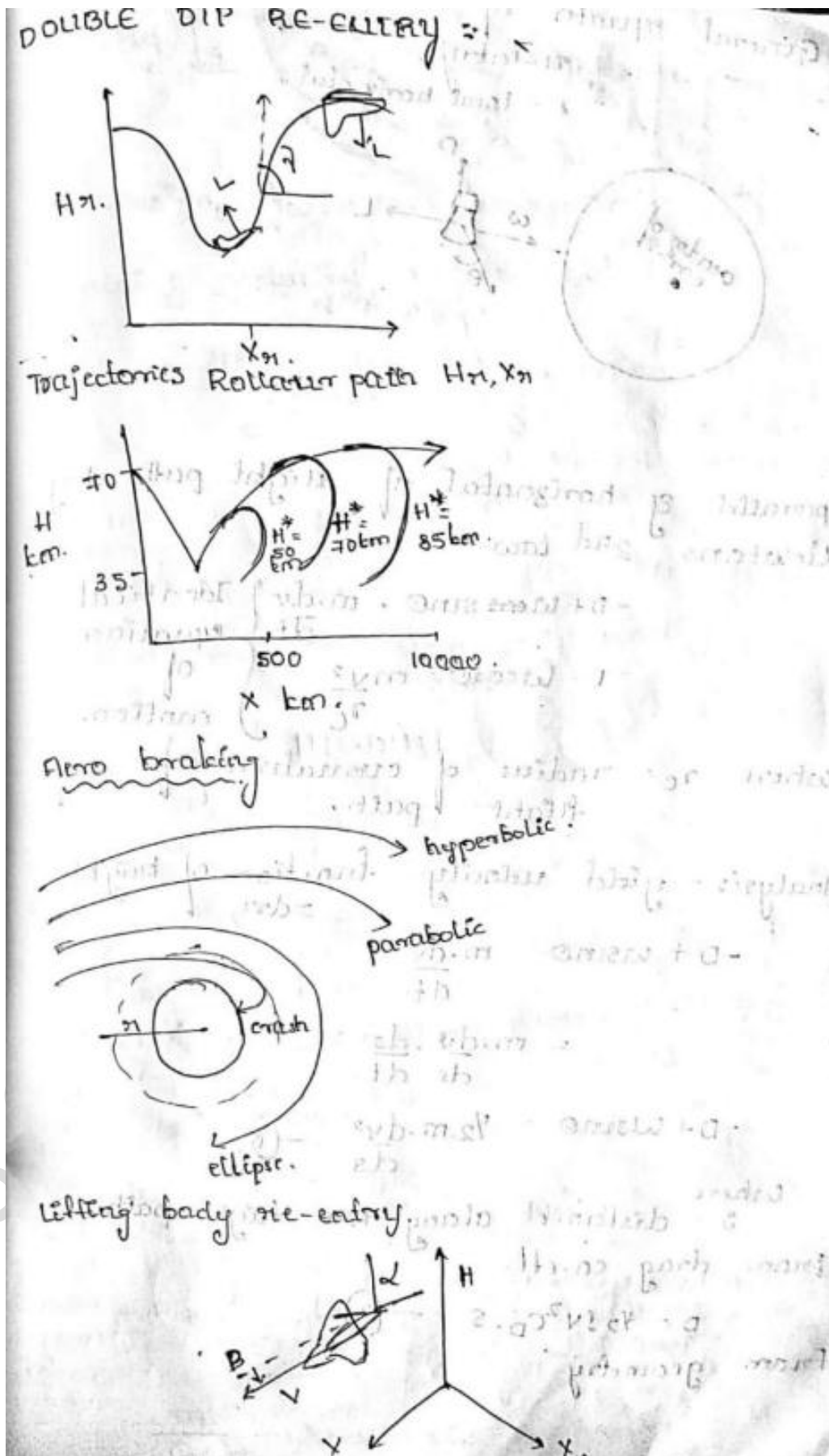
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MODULE 2

Atmospheric Reentry:

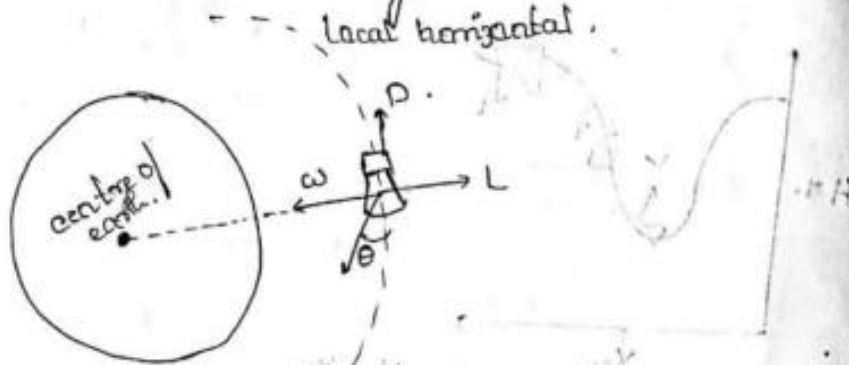
Introduction-Steep Ballistic Reentry, Ballistic Orbital Reentry, Skip Reentry, "Double-Dip" Reentry, Skip reentry, glide reentry, reentry corridor, reentry dynamics for ballistic reentry, reentry heating, Aero-braking, Lifting Body Reentry.





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General equation of motion of atmospheric re-entry.



parallel to horizontal of flight path by Newton's 2nd law.

$$\left. \begin{aligned} -D + L \sin \theta &= m \frac{dv}{dt} \\ L - L \cos \theta &= \frac{mv^2}{r_c} \end{aligned} \right\} \text{Identical equations of motion.}$$

where r_c = radius of curvature of flight path.

Analysis - yield velocity function of height = drag

$$\begin{aligned} -D + L \sin \theta &= m \frac{dv}{dt} \\ &= m \frac{dv}{ds} \cdot \frac{ds}{dt} \end{aligned}$$

$$-D + L \sin \theta = \frac{1}{2} m \frac{dv^2}{ds}$$

where

s = distance along the flight path.

From drag coeff:

$$D = \frac{1}{2} \rho v^2 C_D S$$

From geometry

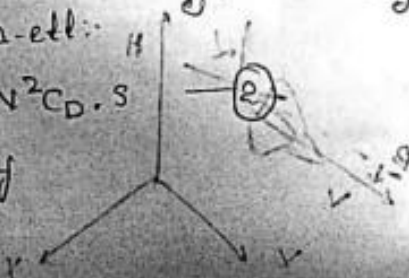


Diagram showing flight path at angle θ to horizontal. Path element ds , height element dh .

$$ds = -\frac{dh}{\sin \theta} \quad (3)$$

$$-\frac{1}{2} \rho v^2 \sin \theta + \rho g \sin \theta = -\frac{1}{2} \rho \sin \theta \frac{dv^2}{dh} \quad (4)$$

Obtaining velocity is function of h .

Let $\rho = f(h)$

$$\frac{\rho}{\rho_0} = e^{-\frac{g_0 h}{RT}}$$

$$= e^{-zh} \quad (5)$$

when $z = \frac{g_0}{RT}$

Relation b/w the velocity & density.

diff the above eqn.

$$\frac{\delta \rho}{\rho} = e^{-zh} [-z dh]$$

$$\frac{\delta \rho}{\rho} = \frac{\rho}{\rho_0} [-z dh]$$

$$dh = -\frac{\delta \rho}{z \rho} \quad (6)$$

eq (6) in (4)

$$-\frac{1}{2} \rho v^2 \sin \theta + \rho g \sin \theta = -\frac{1}{2} \rho \sin \theta \frac{dv^2}{d\rho} [-z \rho]$$

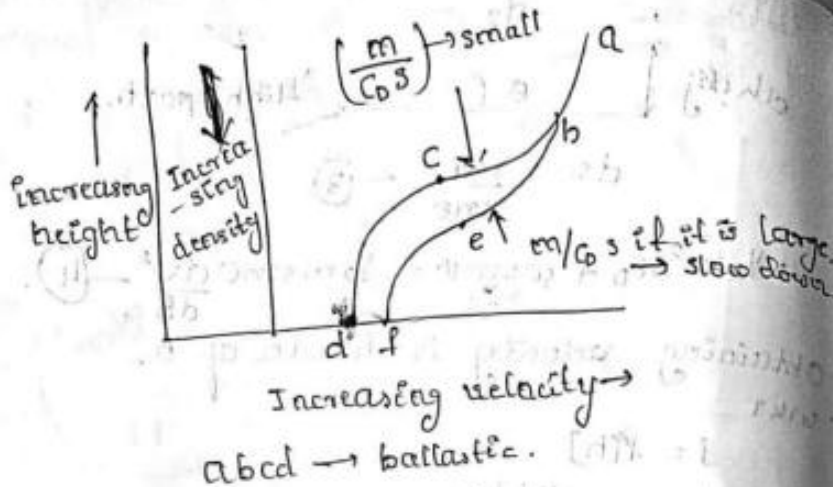
$$\frac{1}{2} \rho v^2 \sin \theta = \frac{2 \rho g}{z \sin \theta} = -\frac{dv^2}{d\rho}$$

$$\frac{dv^2}{d\rho} + \frac{1}{\rho} v^2 = \frac{2g}{z \sin \theta}$$

velocity & density

atmosphere.

$\frac{m}{C_{os}} \rightarrow$ ballistic parameter.



Increasing height

Increasing velocity \rightarrow

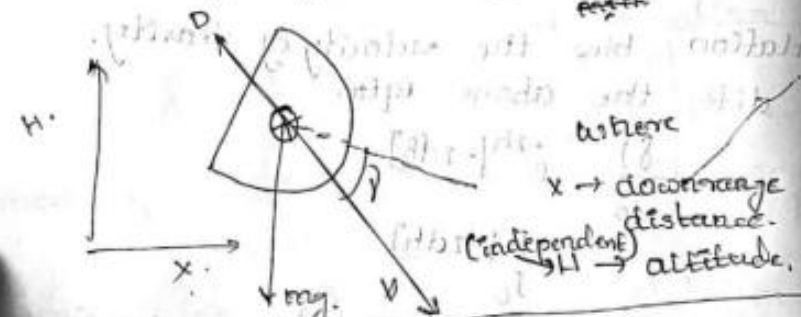
abcd \rightarrow ballistic.

$(\frac{m}{\cos}) \rightarrow$ small

$\frac{m}{\cos}$ if it is large \rightarrow slow down.

BALASTIC RE-ENTRY:

Ballistic vehicle without lift for the steep entry angle.



$u \rightarrow$ forward motion.

$x \rightarrow$ downrange distance.

$H \rightarrow$ altitude.

Eqn of motn:

$$\frac{dx}{dt} = V \cos \theta$$

$$\frac{dH}{dt} = V \sin \theta$$

$$\frac{dx}{dH} = \frac{V \cos \theta}{V \sin \theta} = \cot \theta$$

In-track acceleratn component:

$$m \cdot \frac{dv}{dt} = -D - mg \sin \theta$$

cross-track acceleratn comp

$$m \cdot V \cdot \frac{d\theta}{dt} = -mg \cos \theta$$

centripetal acceleratn $= g_c = \frac{V^2 (\cos^2 \theta)}{H}$

H as the independent variable

this eqn $\frac{dH}{dt} = V \sin \theta$

new eqn of motn.

$$\frac{dx}{dH} = \cot \theta \rightarrow \text{trajectory is straight line.}$$

$M \cdot V \cdot \frac{dv}{dH} = \frac{-D}{\sin \theta} - mg$

deceleratn of vehicle

$$\frac{dv}{dH} = \frac{-g \cot \theta}{V^2}$$

\rightarrow relatively changes (V)

for the case of steep entry

the cotangent is small

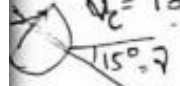
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Consider a solid mass in a shape of sphere entering a earth atm. at 13 km/sec. at an angle of 15° below the local horizontal. sphere diameter is 1m. the drag co-eff for the sphere at hypersonic speed is approx 1. The density of mass is 6963 kg/m^3 . Calc

- * altitude at which max deceleration occur.
- * value of max deceleration.
- * velocity at which the sphere would impact the earth surface.

Re-entry mach no > 27 } $= \frac{13000}{340} = 38 \text{ mach no.}$
 $\rightarrow \text{re-entry}$

$V_e = 13 \text{ km/sec} = 13000 \text{ m/s.}$



$\theta = 15^\circ$
 $D = 1 \text{ m.}$
 $C_d = 1.$
 $\rho = 6963 \text{ kg/m}^3.$

at sea level.
 $T = 288 \text{ K.}$
 $P = 1.01325 \times 10^5 \text{ N/m}^2.$
 $\rho_0 = 1.225 \text{ kg/m}^3.$

to find:

* max deceleration $\left[\frac{dv}{dt} \right]_{\text{max.}}$

* velocity-

formula:-

* $h = -\frac{1}{2} \ln \frac{\rho}{\rho_0}$

$z = \frac{g_0}{RT}$

$\beta = \frac{\rho}{\rho_0} \cdot z \sin \theta$

* $m = \frac{4}{3} \pi r^3 \rho$

$V = \frac{4}{3} \pi r^3$

$S = \pi r^2$

* $\left[\frac{dv}{dt} \right]_{\text{max.}} = \frac{V_e z \sin \theta}{\rho_0 S}$

* $\left(\frac{V}{V_e} \right) = e^{-\beta/2} \left[\frac{\rho}{\rho_0} \right] z \sin \theta$

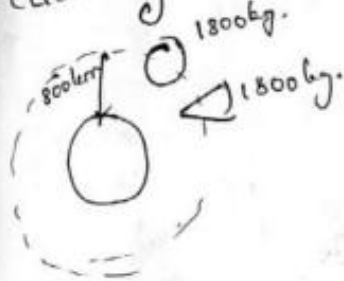
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$$\begin{aligned}
 & \rho_0 \ln \dots \\
 & m = 3645.81 \text{ kg} \\
 & \beta = \frac{3645.81 \times 2 \sin 15^\circ}{1 \times 0.785} \\
 & z = \frac{9.81}{287 \times 288} \\
 & = 0.000118 \text{ m}^{-1} \\
 & \beta = 0.1418 \text{ kg/m}^3 \\
 & h = -\frac{1}{\beta} \ln \frac{\beta}{\beta_0} \\
 & = -\frac{1}{0.000118} \ln \frac{0.1418}{1.225} \\
 & = 18273.4 \text{ m} \\
 & = 18.2 \text{ km} \\
 & \left(\frac{dv}{dt} \right)_{\max} = \frac{V_e^2 \sin \theta}{2r} \\
 & = \frac{(13000)^2 \times [0.000118] \times \sin 15^\circ}{2 \times 18273.4} \\
 & = 949.3 \text{ m/s}^2 \\
 & 949.3 = 96.81 \text{ g's} \\
 & \frac{V}{V_e} = e^{\left[\frac{g}{g_0} \right] \frac{V}{V_e}} \\
 & \frac{V}{13000} = e^{\frac{-1.225}{2} [4642] [0.000118] [3115]} \\
 & V = 998.8 \text{ m/s} \\
 & m = 2.9 \text{ kg}
 \end{aligned}$$

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Compare the heat transfer when entered to atmosphere of the earth. Each body is a cone with a total vertex angle of 10° . The other body is sphere, for the cone pressure drag co-eff at hypersonic mach no is 0.017. For sphere pressure drag co-eff is 0.01. Friction drag co-eff is 1. Friction drag co-eff 0.001. Calculate the total aerodynamic heating input to each body during atm entry.

→ heat transfer when entered to atmosphere?
→ cone of circular shape.



$$m_s = 1800 \text{ kg}$$

$$m_c = 1800 \text{ kg}$$

$$h_g = 800 \text{ km}$$

$$r_e = 6.4 \times 10^6 \text{ m}$$

$$\theta = 10^\circ$$

$$C_{Dp} = 0.017 \text{ } \left. \begin{array}{l} \text{cone} \\ C_{Df} = 0.01 \end{array} \right\}$$

$$C_{Dp} = 1 \text{ } \left. \begin{array}{l} \text{sphere} \\ C_{Df} = 0.001 \end{array} \right\}$$

To find:- aerodynamic heating into the body = ?

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Formulas:-

$$Q = \frac{1}{2} \frac{\rho}{C_D} \left[\frac{1}{2} m V_c^2 \right]$$

$$V = \sqrt{\frac{K}{\pi}}$$

$K = G.M \rightarrow$ mass of earth.

$$\pi = \pi_c + \pi_g$$

$$C_D = C_{Dp} + C_{Df} \rightarrow \text{skin friction coeff.}$$

drag
co-ef

pressure drag co-efficient

soln:- (leave)

$$C_D = C_{Dp} + C_{Df}$$

$$= 0.017 + 0.01$$

$$= 0.027$$

$$\pi = \pi_c + \pi_g$$

$$= (6.4 \times 10^6) + (800 \times 10^3)$$

$$= 7.2 \times 10^6 \text{ m}$$

$$V_c = \sqrt{\frac{(5.974 \times 10^{24} \times 6.67 \times 10^{-11})}{7.2 \times 10^6}}$$

$$= 7.484 \times 10^4 \text{ m/s} \quad 7484.25 \text{ m/s}$$

$$Q_{\text{min}} = \frac{1}{2} \times \frac{0.01}{0.027} \times \left[\frac{1}{2} \times 1800 \times (7484.25)^2 \right]$$

$$= 9.2 \times 10^9 \text{ J}$$

(sphere)

$$C_D = C_{Dp} + C_{Df}$$

$$= 1 + 0.001$$

$$= 1.001$$

$r = 2.5 \times 10^7 \text{ m}$
 $v_c = 1439.25 \text{ m/s}$

$$Q = \frac{1}{2} \times \frac{0.001}{1.001} \times \left[\frac{1}{2} \times 1800 \times (1439.25)^2 \right]$$

$$Q = 2.48 \times 10^7 \text{ J}$$

[The flow where it gets separated is called wake region]
 [max angle of attack reaches the wing surface. at that time flow separates & wake region is produced.]
 → stall is also produced [fall from dead weight].

→ change the aircraft movement in downward direction & move in the forward movement.

→ re-entry vehicle mostly preferred kind sphere shape.

for an obj consider a ballistic re-entry
 for an object with the following specifications:
 $v_e = 8 \text{ km/s}$ & $\gamma = 10^\circ$ of ballistic parameter
 is $300 \text{ km}^2 \text{ at temp } 288 \text{ K}$.
 Find the h at which deceleration is max
 what is the max deceleration, velocity at the altitude & velocity of the vehicle when it reaches surface of the earth.