MODULE-1

Definition of Operations Research:

Operations research is a scientific approach to problem solving for executive management.

OR

Operations research is the application of scientific method by interdisciplinary teams to problems involving the control of organized (man-machine) systems so as to provide solutions which best serve the purpose of the organization as a whole.

Introduction: The Origin of Operations Research

The beginning of the activity called OR has generally been recognized to the military services early in the world war II. Because of the war effort, there was an urgent need to allocate scare resources to the various military operations and to the activities within each operation in an efficient manner. The British and the U.S military management called upon a large number of scientists to apply a scientific approach to dealing with this and other strategic and tactical problems. In effect they were asked to do research on(Military) operations. These teams of scientist were the first operation research teams. By developing efficient methods of using the new tool of radar, these teams were instrumented in winning the Air Battle of Britain. Through their research on how to better manage convey and antisubmarine operations, they also played a major role in winning the Battle of the North Atlantic similar efforts assisted the Island campaign in the pacific.

The success of OR in the war effort spurred interest in applying OR outside the military as well. As the industrial boom following the war was running its course, the problems caused by the increasing complexity and specializations in organization were again coming to the forefront. By the early 1950s, the use of OR to a variety of organization in business, industry and government. The spread of OR soon followed,after the war, many of scientists who had participated on OR teams or who had heard about this work were motivated to pursue research work relevant to the field.

The simplex method for solving Linear programming problems developed by George Dantzig in 1947.

Many of the standard tools of OR, such as linear programming, dynamic programming, queing theory and inventory theory were relatively well deeloped before the end of the 1950s.

Another factor that gave great thrust to the growth of the field was the onslaught of the computer revolution. A large amount of computation is usually required to deal most effectively with the complex problems typically considered by OR. A boost came in the 1980s with the progress of increasingly powerful personal computers accompanied by good software packages for doing OR. This brought the use of OR within the easy reach of much larger number of people. Today literally millions of persons have ready access to OR software.

The Nature of Operations Research

Operations research involves “research on operations”. Thus, OR is applied to problems that concern how to conduct & coordinate the operations (i.e, the activities) within an organizations. The nature of the organization is immaterial and in fact, OR has been applied in areas such as manufacturing, transportation, construction, telecommunications, financial planning ,health care the military and public service.

The research part of the name means that operations research uses an approach that resembles the way research is conducted in recognized scientific fields. The scientific method is used to examine the problem of concern. The process begins by carefully observing and formulating the problem as well as gathering all relevant data. The next step is to construct a scientific model that attempts to abstract the essence of the real problem. It is then hypothesized and this model is a sufficiently precise representation of the essential features of the situation that the conclusion obtained from the model are also valid for the real problem. Suitable experiments are conducted to test this hypothesis and modify it as needed and eventually verify some form of the hypothesis. Thus in a certain sense, OR involves creative scientific research into the fundamental properties of operations. OR is also concerned with the practical management of the organization.

The characteristic of OR is its broad viewpoint. OR adopts an organizational point of view. OR frequently attempts to find best solution for the problem under consideration. Rather than simply improving the status quo, the goal is to identify a best possible course of action.

The Impact of Operations Research

Operations Research has had an remarkable impact on improving the effectiveness of numerous organizations around the world. In the process, OR has made a significant contribution to increase the productivity of the economics of various countries. There are member countries in the International Federation of OR Societies(IFORS), with each country have a national OR society both Europe & Asia have federations of OR societies to coordinate holding international conferences & publishing international journals in those continents. In addition, the Institute for OR and the management Sciences(INFORMS) is an international OR Society.

Phases of Operations Research Study

The different phases of OR study are

i) Defining the problem & gathering data
ii) Formulating a Mathametical Model
iii) Deriving solution from the model
iv) Testing the model
v) Preparing to apply the model
vi) Implementation

i) <u>Defining the problem & gathering data</u>

The first order of business is to study the relevant system and develop a well defined statement of the problem to be considered,

OR team normally works in an advisory capability. The team members are not just given a problem & told to solve it, however they see fit. Instead they advice management.The team performs a detailed technical analysis of the problem & then presents suggestion to management. Frequently, the report to management will identify a number of alternatives that are particularly attractive under different assumptions or over a different range of values of some policy parameters that can be evaluated only by management. Management evaluates the study and its recommendations takes into account.

ii) <u>Formulation of Mathematical Model</u>

This phase is concerned with restructuring of the problem in an appropriate form which is useful in analysis. The most suitable model is a mathematical model representing the problem under study. A mathematical model should include decision variables, objective function constraint. The benefit of a mathematical model is that it describes the problem more concisely which makes the overall structure of the problem more comprehensible and it also helps to reveal important cause and effect relationship.

iii) <u>Deriving solution from the model</u>

This phase is devoted to computation of values of decision variables which maximizes or minimizes the objective function, depending on their nature. It is important to determine an optimal or best solution for the problem.

IV) <u>Testing the Model</u>

The completed model is tested for errors if any. The principle of judging the validity of a model is whether or not it predicts the relative efforts of the alternative courses of action with sufficient accuracy to permit a sound decision. A good model should be applicable for a longer time and thus updates the model from time to time by taking into account the past, present and future specifications of the problem.

V) <u>Preparing to apply Model</u>

A solution which was felt most favourable today, may not be tomorrow, since the values of the parameters may change, new parameters may emerge and the structural relationship between the variables may undergo a change.

A solution derived from a model remains a solution only so long as the uncontrolled variables retain their values and the relationship between the variables does not change. The solution itself goes out of control if the values of one or more uncontrolled variables vary or relationship between variables undergoes a change. Therefore, controls must be established to indicate the limits within which the model and its solution can be considered as reliable. Also tools must be developed to indicate as to how and when the model or its solution will have to be modified to take the changes into account.

VI) <u>Implementation</u>

The implementation of the controlled solution involves the translation of the model's results into operating instructions. It is important in OR to ensure that the solution is accurately translated into an operating procedure to rectify faults in the solution.

<u>Some important definitions used in LP-models</u>

<u>Solution:</u> A set of variables $\{x_1, x_2, x_3\text{-------}x_n\}$is called a solution if it satisfies the constraints.

Feasible Solution: A set of variables {x_1, x_2, x_3-------x_n} is called a feasible solution if these variables satisfy constraints are non negative.

Basic solution: A solution obtained by setting 'n' variables (among m+n variables) to zero and solving for remaining m variables is called a basic solution. These m variables are basic variables and n variables are non-basic variables.

Basic Feasible Solution(BFS): A basic solution is called a basic feasible solution if all basic variables are ≥ 0.

Non-degenerate Basic Feasible Solution : It is a Basic Feasible Solution in which all m variables are positive and the remaining 'n' variables are zero each.

Degenerate Basic Feasible Solution : It is a Basic feasible solution in which one or more of the m basic variables are equal to zero.

Optimal Basic Feasible Solution: A Basic Feasible Solution is called optimal Basic Feasible Solution if it optimizes the objective function.

Unbounded Solution: If the value of the objective function can be increased or decreased indefinitely, then the solution is called an unbounded solution.

Feasible Region: It is a region in which all constraints and non-negativity conditions hold good.

Corner Point Feasible (CPF) Solution: It is a feasible solution that doesn't lie on any line segment connecting two other feasible solution.

Optimization: It is the technique of obtaining the best results under the given conditions.

Linear Programming: It is a decision making technique under the given constraints and the conditions that the relationship among the variables involved is linear.

Problems on Linear Programming Problems

1) Old hens can be bought at Rs. 50/- each but young ones cost Rs. 100/- each. The old hens lay 3 eggs/week and young hens 5 eggs/week. Each egg costs Rs. 2/-. A hen costs Rs. 5/- per week to fee. If a person has only Rs. 2000/- to spend for hens, formulate the problem to decide how many of each kind of hen should he buy ? Assume that he cannot house more than 40 hens.

Solution: Let x_1& x_2 be the number of old and young hens to be purchased

No. of eggs laid by old hens=3

No. of eggs laid by young hens=5

Total income from the eggs= (No. of eggs)*Selling price

$= (3x_1+5x_2)*2 = 6x_1+10x_2$

Feeding cost = $(x_1+x_2)*5 = 5x_1+5x_2$

Profit = Income-Feeding cost = $(6x_1+10x_2) -(5x_1+5x_2) = x_1+5x_2$

Thus,

$\mathbf{Z_{max}= x_1+5x_2}$ is the objective function

Subject to the constraints,

$50x_1 + 100x_2 \leq 2000$ (Budget Constraint)

$x_1 + x_2 \leq 40$ (Housing Constraint)

$x_1 \geq 0,\ x_2 \geq 0$

2) A computer company manufactures laptops & desktops that fetches profit of Rs. 700/- & 500/- unit respectively. Each unit of laptop takes 4 hours of assembly time & 2 hours of testing time while each unit of desktop requires 3 hours of assembly time & 1 hour for testing. In a given month the total number of hours available for assembly is 210 hours & for inspection is 90 hours. Formulate the problem as LPP in such a way that the total profit is maximum.

Solution: The objective function is

$Zmax=700x_1+500x_2$

Subject to the constraint:

$4x_1+3x_2 \leq 210$ (Assembly time constraint)

$2x_1+x_2 \leq 90$ (Inspection time constraint)

$x_1 \geq 0,\ x_2 \geq 0$

3) A toy company manufactures two types of dolls, a basic version-doll A and a deluxe version- doll B. Each doll of type B takes twice as long to produce as one of type A and the company would have time to make maximum of 2000 dolls per day. The supply of plastic is sufficient to produce 1500 dolls per day(Both A & B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs. 10/- & Rs. 18/- per doll on doll A & B respectively, then how many of each doll should be produced per day in order to maximize the total profit. Formulate the problem as LPP.

Solution: The objective function is

$Zmax=10x_1+18x_2$

Subject to the constraint:

$x_1+2x_2 \leq 2000$

$x_1+x_2 \leq 1,5000$

$x2 \leq 600$

$x_1 \geq 0, x_2 \geq 0$

4) The standard weight of a special purpose brick is 5Kg and it contains two ingredients B1 & B2. B1 cost Rs. 5/- per kg & B2 costs Rs. 8/- per kg. Strength considerations dictate that the brick contains not more than 4 kg of B1 & a minimum of 2 kg of B2, since the demand for the product is likely to be related to the price of the brick. Formulate the above problem as LP model.

Solution: The objective function is

$Zmin=5x_1+8x_2$

Subject to the constraint:

$x_1 \leq 4$

$x_2 \geq 2$

$x_1+x_2 =5$

$x_1 \geq 0, x_2 \geq 0$

5) A marketing manager wishes to allocate his annual advertising budget of Rs. 20,000 in two media group M & N. The unit cost of the message in the media 'M' is Rs. 200 & 'N' is Rs. 300. The media M is monthly magazine & not more than two insertions are desired in one issue. At least five messages should appear in the media N. The expected effective audience per unit message for media M is 4,000 & for N is 5,000. Formulate the problem as Linear Programming problem.

Solution: The objective function is

$Zmax=4000x_1+5000x_2$

Subject to the constraint:

$200x_1+300x_2 \leq 20,000$

$x_1 \leq 2$

$x_2 \geq 5$

$x_1 \geq 0, x_2 \geq 0$

6) A manufacturer produces two types of models M1 & M2. Each M1 model requires 4 hours of grinding & 2 hours of polishing, whereas each M2 model requires 2 hours of grinding & 5 hours of polishing. The manufacturer has 2 grinders & 3 polishers. Each

grinder works for 40 hours a week & each polisher works for 60 hours a week. Profit of M1 model is Rs. 3/- & on M2 model is Rs. 4/-. How should the manufacturer allocate his production capacity to the two types of models so as to make maximum profit in a week. Formulate the above problem as LPP.

Solution: The objective function is

$Z_{max}=3x_1+4x_2$

Subject to the constraint:

$4x_1+2x_2 \leq 80$ [Note: 40*2=80 hrs]

$2x_1+5x_2 \leq 180$ [Note: 60*3=180 hrs]

$x_1 \geq 0,\ x_2 \geq 0$

7) A company produces two types of products. Each product of first type requires twice as much time as the second type. The company can produce a total of 600 products a day. The market limits the daily sales of the first & second types of products of 175 & 250 respectively. If the profits per product are Rs. 9/- for the first & Rs. 6/- for second product. Formulate the problem as LPP.

Solution: The objective function is

$Z_{max}=9x_1+6x_2$

Subject to the constraint:

$2x_1+x_2 \leq 600$

$x_1 \leq 175$

$x_2 \leq 250$

$x_1 \geq 0,\ x_2 \geq 0$

8) A company produces two types of Hats. Each hat of the first type requires twice as much labour time as the second type. The company can produce a total of 500 hats a day. The market limits daily sales of the first & second type to 150 & 250 hats. Assuming that the profits/hat are Rs. 8/- for type A & Rs. 5 for type B. Formulate the problem as LP model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Solution: The objective function is

$Z_{max} = 8x_1+5x_2$

Subject to the constraint:

$2x_1+x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

$x_1 \geq 0,\ x_2 \geq 0$

9) An agriculturist has a farm with 126 acres. He produces Tomato, Mango and Potato. Whatever he raises is fully sold in the market. He gets Rs. 5/- for tomato/kg, Rs. 4/- for mango/kg and Rs. 5/- for potato/kg. The average yield is 1,500 kg of tomato/acre, 1800 kg of mango/acre and 1200 kg of potato/acre. To produce each 100 kg of tomato and mango and to produce each 80 kg of potato a sum of Rs. 12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man-days for tomato and potato each and 5 man-days for mango. A total of 500 man-days of labour at a rate of Rs. 40/- per man day are available. Formulate this as a LP model to maximize the agriculturist's total profit.

Solution:	Tomato	Mango	Potato
Selling Price	5*1500=7500	4*1800=7200	5*1200=6000
Manure Cost	12.50*(1500/100)=187.50	12.50*(1800/100)=225	12.50*1200/80=187.50
Labour cost	40*6=240	40*5=200	40*6=240
Profit	7500-(187.50+240) =7,072.50	7,200-(225+200) =6775	6000-(187.50+240) =5572.50

The objective function is

$$Zmax = 7072.5x_1+6775x_2+5572.5x_3$$

Subject to the constraint:

$x_1+x_2+x_3 \leq 126$

$6x_1+5x_2+6x_3 \leq 500$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

10) A company manufactures two products A & B. Theses products are processed in the same machine. It takes 10 minutes to process one unit of product A and 2 minutes for each unit of product B and the machine operates for a maximum of 35 hours in a week. Product A requires 1 kg and B 0.5 kg of raw material per unit the supply of which is

600 kg per week. Market constraint on product B is known to be 800 unit every week. Product A costs Rs. 5/- per unit and sold at Rs. 10/-. Product B costs Rs. 6/- per unit and can be sold in the market at a unit price of Rs. 8/-. Determine the number of units of A & B per week to maximize the profit.

Solution: Let x_1 & x_2 be the number of products A & B.

Cost of product A/unit is Rs.5 & sold at Rs.10/unit

Profit on one unit of product A=10-5=$5x_1$

Profit on one unit of product B=8-6=$2x_2$

The objective function is

$$Z_{max} = 5x_1+2x_2$$

Subject to the constraint:

$$10x_1+2x_2 \leq (35*60)$$

$$10x_1+2x_2 \leq 2100 \quad \text{[Time Constraint]}$$

$$x_1+0.5x_2 \leq 600 \quad \text{[Raw material constraint]}$$

$$x_2 \geq 800$$

$$x_1 \geq 0,\ x_2 \geq 0$$

11) A person requires 10,12 and 12 units chemicals A,B,C respectively for his garden. One unit of liquid product contains 5,2 and 1 units of A,B and C respectively. One unit of dry product contains 1,2 and 4 units of A,B,C. If the liquid product sells for Rs. 3/- and the dry product sells for Rs. 2/-, how many of each should be purchased, in order to minimize the cost and meet the requirements.

Solution: The objective function is

$$Z_{min}=3x_1+2x_2$$

Subject to the constraint:

$$5x_1+x_2 \geq 10$$

$$2x_1+2x_2 \geq 12$$

$$x_1+4x_2 \geq 12$$

$$x_1 \geq 0,\ x_2 \geq 0$$

12) A paper mill produces two grades of paper namely X and Y. Because of raw material restrictions, it cannot produce more than 400 tons of grade X and 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products X and Y respectively with corresponding profits of Rs. 200/-

and Rs. 500/- per ton. Formulate the above as a LPP to maximize profit and find the optimum product mix.

Solution: The objective function is

$$Zmax = 200x_1+500x_2$$

Subject to the constraint:

$x_1 \leq 400$

$x_2 \leq 300$

$0.2x_1+0.4x_2 \leq 160$

$x_1 \geq 0,\ x_2 \geq 0$

13) The owner of fancy goods shop is interested to determine how many advertisement to release in the selected three magazines A,B and C. His main purpose is to advertise in such a way that total exposure to principal buyers of his goods is maximized. Percentage of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements released multiplied by the number of principal buyers. The following data are available.

Particulars	Magazines		
	A	B	C
Readers	1.0 lakh	0.6 lakh	0.4 lakh
Principal Buyers	20 %	15 %	8 %
Cost per advertisement	8,000	6,000	5,000

The budgeted amount is at the most Rs. 1.0 lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets at least 8 advertisements. Formulate a linear programming model for this problem.

Solution: The total exposure of principal buyers of the magazine is

$$Z_{max}= (20\% \text{ of } 1,00,000)x_1 + (15\% \text{ of } 60,000)x_2+(8\% \text{ of } 40,000)x_3$$

The Objective function

$$Z_{max}= 20000x_1 +9000x_2+3,200x_3$$

Subject to the constraint

$8000x_1 +6000x_2+5000x_3 \leq 1,00,000$

$x_1 \leq 15,\ x_2 \geq 8,\ x_3 \geq 8$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

14) Farmer furniture makes chairs, arm-chairs and sofas, the profits are $ 50 per chair, $60 per arm-chair and $ 80 per sofa. The material used to manufacture these items are fabric and wood. A supplier can provide a maximum of 300 meters of fabric and 350 units of wood each week. Each item requires a certain amount of wood and fabric as well as certain assembly time.

These are given in the following table.

Item	Fabric	Wood	Ass. Time
Chair	2m	6 units	8 hours
Armchair	5m	4 units	4 hours
Sofa	8m	5 units	5 hours
Avail./Wk	300m	350 units	480 hours

How many chairs, armchairs and sofas that the company should make per week so that the total profit is maximized?

Solution: The objective function is

$$Zmax=50x_1+60x_2+80x_3$$

Subject to the constraint:

$$2x_1+5x_2+8x_3 \leq 300$$

$$6x_1+4x_2+5x_3 \leq 350$$

$$8x_1+4x_2+5x_3 \leq 480$$

$x_1 \geq 0,\ x_2 \geq 0, x_3 \geq 0$

Graphical Method

Linear programming problems involving two decision variables can easily be solved by graphical method, which provides a pictorial representation of the solution.

Steps in Graphical Method

- Formulate the given problem as LPP
- Draw a graph with one variable on the horizontal axis and one on the vertical axis.
- Plot each of the constraint as if they were equalities or equations.

- Identify the feasible region (Solution space) that is the area that satisfies all the constraints.
- Name the intersections of the constraints on the perimeter of the feasible region and get their co-ordinates,
- Substitute each of the co-ordinates into the objective function and solve for Z
- Select the solution that optimizes Z (based on objective) that is obtain Z_{min} or Z_{max}

Various Cases in Graphical Method

A Linear-programming problem may be having

- A unique optimal solution
- An infinite number of optimal solution (alternative opimal solution).
- An unbounded solution and
- No solution.

1) Solve the following Linear programming problem by graphical method

$Z_{max}=3x_1 +4x_2$

Subject to

$x_1+x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1 \geq 0,\ x_2 \geq 0$

Ans:

$x_1+x_2 \leq 450$ ------------------(1)

$2x_1 + x_2 \leq 600$ -------------------(2)

$x_1 \geq 0,\ x_2 \geq 0$

Writing the constraint (1) as equation

$x_1+x_2 = 450$

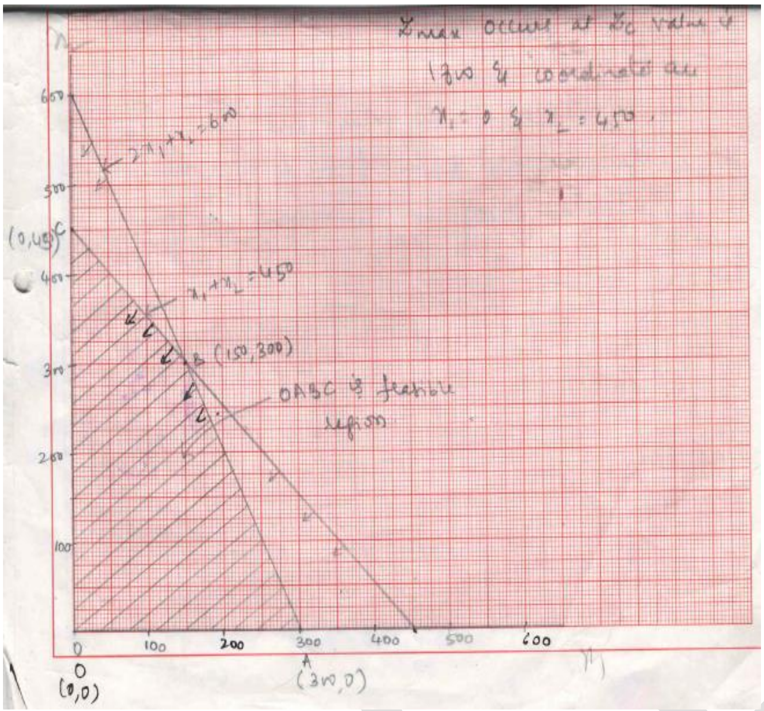
when $x_1 = 0$, $x_2 = 450$, The Co-ordinates are (0,450).

when $x_2 = 0$, $x_1 = 450$, The Co-ordinates are (450,0).

Similarly writing the constraint (2) as equation

when $x_1 = 0$, $x_2 = 600$, The Co-ordinates are (0,600).

when $x_2 = 0$, $x_1 = 300$, The Co-ordinates are (300,0).



Co-ordinates	**Z_{max}=3x_1+4x_2**
O(0,0)	Z_O=3(0)+4(0)=0
A(300,0)	Z_A=3(300)+4(0)=900
B(150,300)	Z_B=3(150)+4(300)=1650
C(0,450)	Z_C=3(0)+4(450)=1800

Z_{max} occurs at Z_c and the value is 1800 & co-ordinates are x_1=0 and x_2=450.

2) Solve the following Linear programming problem by graphical method

$Z_{min}=20x_1 +10x_2$

Subject to

$x_1+2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 +3 x_2 \geq 60$

$x_1 \geq 0$, $x_2 \geq 0$

Ans:

$x_1+2x_2 \leq 40$---------(1)

$3x_1 + x_2 \geq 30$----------(2)

$4x_1 + 3x_2 \geq 60$----------(3)

$x_1 \geq 0$, $x_2 \geq 0$

Writing the constraint (1) as equation

$x_1+2x_2 = 40$

when $x_1 = 0$, $x_2 = 20$, The Co-ordinates are (0,20).

when $x_2 = 0$, $x_1 = 40$, The Co-ordinates are (40,0).

Similarly writing the constraint (2) as equation

$3x_1 + x_2 = 30$

when $x_1 = 0$, $x_2 = 30$, The Co-ordinates are (0,30).

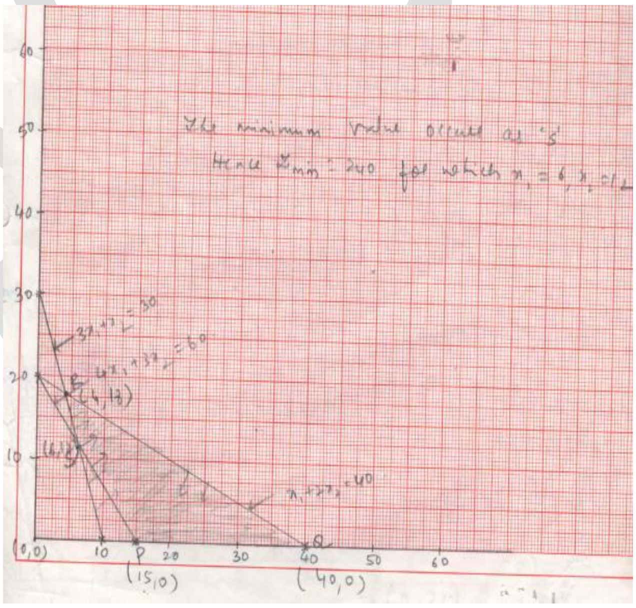
when $x_2 = 0$, $x_1 = 10$, The Co-ordinates are (10,0).

Similarly writing the constraint (3) as equation

$4x_1 + 3x_2 =60$

when $x_1 = 0$, $x_2 = 20$, The Co-ordinates are (0,20).

when $x_2 = 0$, $x_1 = 15$, The Co-ordinates are (15,0).



Co-ordinates	Z_{max}=$20x_1+10x_2$

P(15,0)	Z_p=20(15)+10(0)=300
Q(40,0)	Z_Q=20(40)+10(0)=800
R(4,18)	Z_R=20(4)+10(18)=260
S(6,12)	Z_S=20(6)+10(12)=240

The minimum value occurs at 'S'. Hence Z_{min}=240, and coordinates are x_1=6, x_2=12.

3) Solve the following Linear programming problem by graphical method

$Z_{max}=2x_1+3x_2$

Subject to

$x_1+2x_2 \leq 4$

$x_1 + x_2 = 3$

$x_1 \geq 0,\ x_2 \geq 0$

Ans:

$x_1+2x_2 \leq 4$ -----------------(1)

$x_1 + x_2 = 3$ ------------------(2)

$x_1 \geq 0,\ x_2 \geq 0$

Writing the constraint (1) as equation

$x_1+2x_2 = 4$

when $x_1 = 0$, $x_2 = 2$, The Co-ordinates are (0,2).

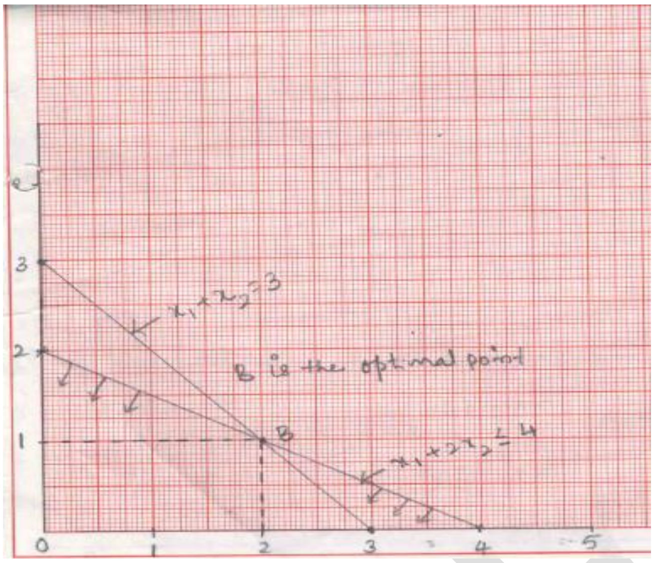
when $x_2 = 0$,$x_1 = 4$, The Co-ordinates are (4,0).

Similarly writing the constraint (2) as equation

$x_1 + x_2 = 3$

when $x_1 = 0$, $x_2 = 3$, The Co-ordinates are (0,3).

when $x_2 = 0$,$x_1 = 3$, The Co-ordinates are (3,0).



[Note: As $x_1 + x_2 = 3$ is an equation, optimal point is obtained at B]

B is the optimal point whose coordinates are (2,1) and $Z_{max} = 7$.

Thus $x_1=2$ and $x_2=1$.

4) Solve the following Linear programming problem by graphical method

$Z_{min}=x_1 + x_2$

Subject to

$5x_1+10x_2 \le 50$

$x_1 + x_2 \ge 1$

$x_2 \le 4$

$x_1 \ge 0,\ x_2 \ge 0$

Ans:

$5x_1+10x_2 \le 50$ -----------(1)

$x_1 + x_2 \ge 1$------------------(2)

$x_2 \le 4$----------------------(3)

$x_1 \ge 0,\ x_2 \ge 0$

Writing the constraint (1) as equation

$5x_1+10x_2 = 50$

when $x_1 = 0$, $x_2 = 5$, The Co-ordinates are (0,5).

when $x_2 = 0$, $x_1 = 10$, The Co-ordinates are (10,0).

Similarly writing the constraint (2) as equation

$x_1 + x_2 = 1$

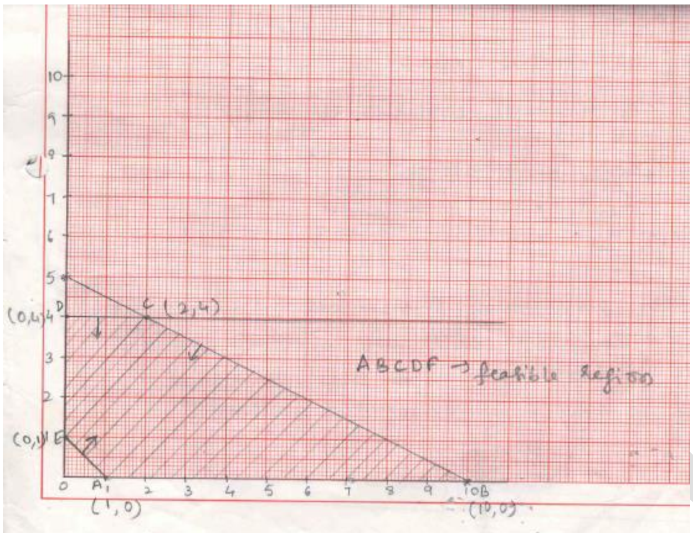
when $x_1 = 0$, $x_2 = 1$, The Co-ordinates are (0,1).

when $x_2 = 0, x_1 = 1$, The Co-ordinates are (1,0).

Similarly writing the constraint (3) as equation

$$x_2 = 4$$

when $x_2 = 4$, $x_1 = 0$ The Co-ordinates are (0,4).



Co-ordinates	**$Z_{min}=x_1+x_2$**
A(1,0)	$Z_A=1+0=1$
B(10,0)	$Z_B=10+0=10$
C(2,4)	$Z_C=2+4=6$
D(0,4)	$Z_D=0+4=4$
E(0,1)	$Z_E=0+1=1$

For this problem their lies an alternative optimal solution.

5) Solve the following Linear programming problem by graphical method

$$Z_{max}=3x_1+2x_2$$

Subject to

$$x_1 - x_2 \le 1$$

$$x_1 + x_2 \ge 3$$

$$x_1 \ge 0, \; x_2 \ge 0$$

Ans:

$x_1 - x_2 \le 1$-----------------(1)

$x_1 + x_2 \ge 3$------------------(2)

$x_1 \ge 0$, $x_2 \ge 0$

Writing the constraint (1) as equation

$$x_1 - x_2 = 1$$

when $x_1 = 0$, $x_2 = -1$, The Co-ordinates are (0,-1).

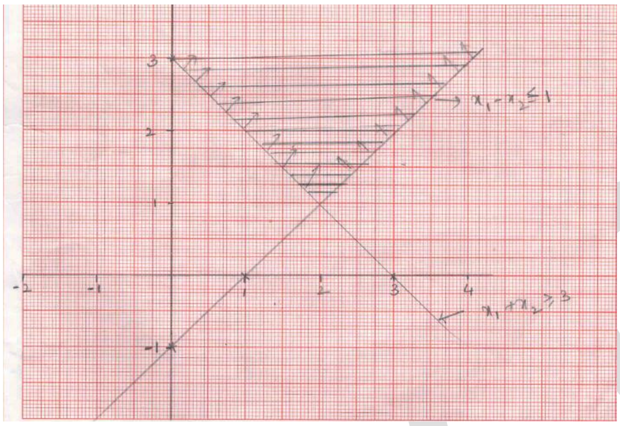
when $x_2 = 0, x_1 = 1$, The Co-ordinates are (1,0).

Similarly writing the constraint (2) as equation

$$x_1 + x_2 = 3$$

when $x_1 = 0$, $x_2 = 3$, The Co-ordinates are (0,3).

when $x_2 = 0, x_1 = 3$, The Co-ordinates are (3,0).



Note: The obtained feasible region is open or unbounded if Z is to be maximized then the solution is unbounded that is Z_{max} occurs at infinity in the given problem.

6) Solve the following Linear programming problem by graphical method

$$Z_{max} = x_1 + x_2$$

Subject to

$$x_1 + x_2 \leq 1$$

$$-3x_1 + x_2 \geq 3$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Ans:

$$x_1 + x_2 \leq 1 \text{-----------------(1)}$$

$$-3x_1 + x_2 \geq 3 \text{------------------(2)}$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Writing the constraint (1) as equation

$$x_1 + x_2 = 1$$

when $x_1 = 0$, $x_2 = 1$, The Co-ordinates are (0,1).

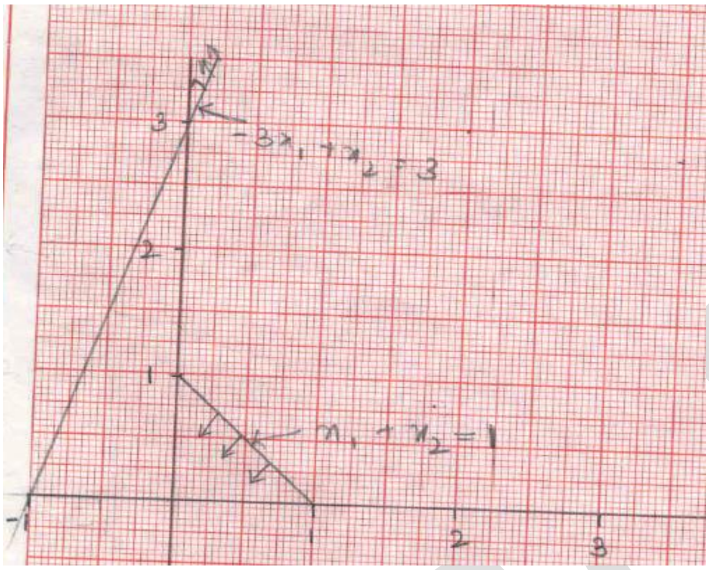
when $x_2 = 0, x_1 = 1$, The Co-ordinates are (1,0).

Similarly writing the constraint (2) as equation

$-3x_1 + x_2 = 3$

when $x_1 = 0$, $x_2 = 3$, The Co-ordinates are (0,3).

when $x_2 = 0, x_1 = -1$, The Co-ordinates are (-1,0).



It is observed that there is no common feasible region satisfying all the constraints. Hence the problem cannot be solved. In other words the given linear programming problem has no solution.