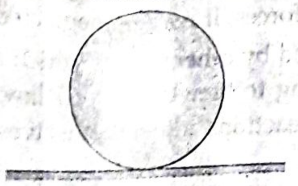
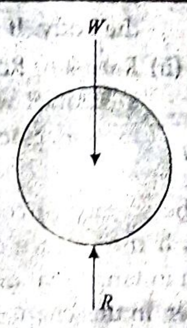
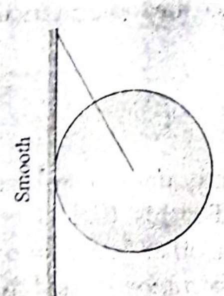
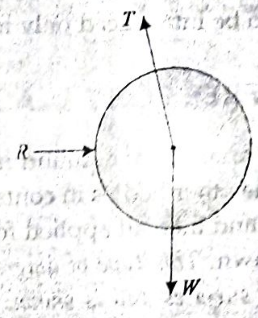
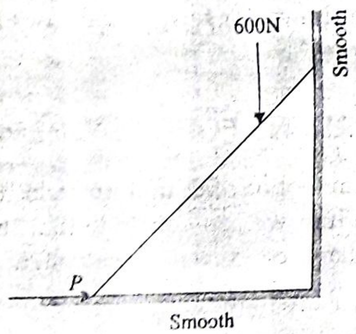
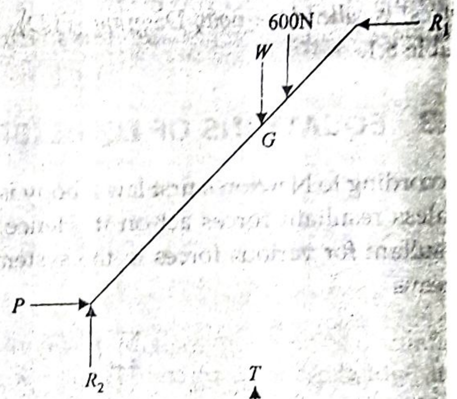
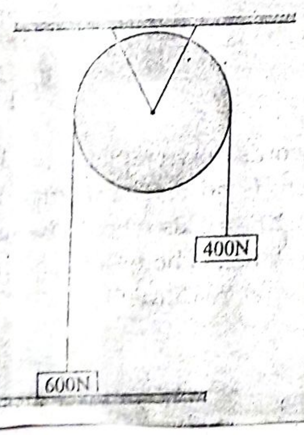
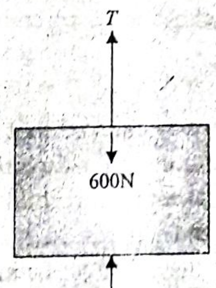
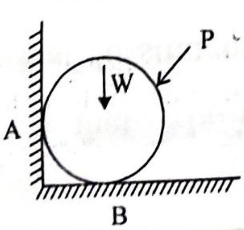
MODULE - 2

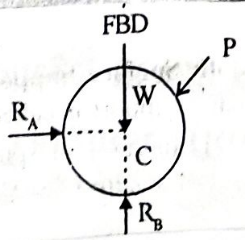
EQUILIBRIUM OF FORCES

2.1) FREE BODY DIAGRAMS

This type of diagram of the body in which the body under consideration is freed from all contact surfaces and is shown with all the forces on it (including self weight, reactions and applied forces) is called Free Body Diagram (FBD). The F.B.D is in equilibrium.

Reacting bodies	FBD required for	FBD
	Ball	
	Ball	
	Ladder	
	Block weighing 600 N	





NOTE - strings, cables, ropes are subjected to tension
Rod can be subjected to either tension/compression

SIGN CONVENTIONS

1) TENSION

→ ←

2) COMPRESSION

⟵⟶

2.2) LAMI'S THEOREM

If a body is in equilibrium, under the action of coplanar concurrent forces, it may be analysed using equations of equilibrium (eqn. 8.2). However, if the body is in equilibrium under the action of only three forces, Lami's theorem can be used conveniently.

Lami's theorem states that *if a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces.* Thus for the system of forces shown in Fig. 8.1(a),

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} \qquad \text{Eqn. (8.3)}$$

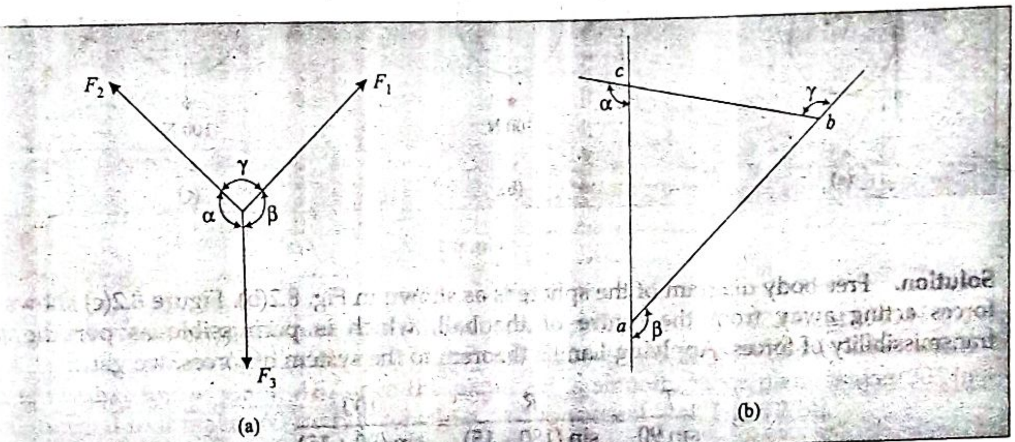


Fig. 8.1

Proof. Draw the three forces F_1, F_2 and F_3 one after the other in direction and magnitude starting from point '*a*'. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with '*a*'. Thus, it results in a triangle of forces *abc* as shown in Fig. 8.1(b). Now the external angles at *a*, *b* and *c* are equal to β, γ and α, since *ab*, *bc* and *ca* are parallel to F_1, F_2 and F_3 respectively. In the triangle of forces *abc*,

$$ab = F_1$$

$$bc = F_2$$

and

$$ca = F_3$$

Applying sine rule for the triangle *abc*,

$$\frac{ab}{\sin(180^\circ-\alpha)} = \frac{bc}{\sin(180^\circ-\beta)} = \frac{ca}{\sin(180^\circ-\gamma)}$$

i.e.,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

It is possible to apply Lami's theorem if 3 forces are acting on a particle or at a point.

Equilibrium of Forces

Any system of forces acting on a body are said to be in equilibrium when the resultant of all forces is zero and algebraic sum of moments of all the forces is zero.

2.3) Equations of Equilibrium

A system of forces is in equilibrium when $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 0$

and $\Sigma M = 0$

i.e., when (1) $\Sigma F_x = 0$

(2) $\Sigma F_y = 0$ ⟶ remember

also (3) $\Sigma M = 0$

Where ΣF_x = Algebraic sum of horizontal component of forces

ΣF_y = Algebraic sum of vertical component of forces

ΣM = Algebraic sum of moments of forces about any point.

EQUILIBRANT

Equilibrant is defined as a force or a moment required to keep an object in equilibrium.

For a concurrent force system, equilibrant is a force which has same magnitude as resultant force but opposite direction.

For a non-concurrent force system which has a non-zero resultant force, the equilibrant is a force which has same magnitude as resultant force, opposite direction on the same line of action as that of the resultant force. In case the resultant force becomes zero and the force system can be reduced to a single resultant couple moment, the equilibrant is a couple moment having same magnitude as resultant couple moment but opposite sense of rotation.

2.4) Conditions of equilibrium for different force systems OR Equilibrium of concurrent and non concurrent coplanar force systems

1. Coplanar concurrent force system

$\Sigma F_x = 0,\ \Sigma F_y = 0$ (moment is already zero). See Figure 5.1.

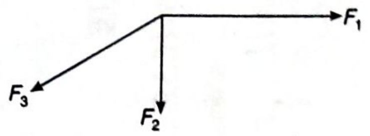


Figure 5.1 Coplanar concurrent force system.

2. Coplanar non-concurrent force system

$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0$. See Figure 5.2.

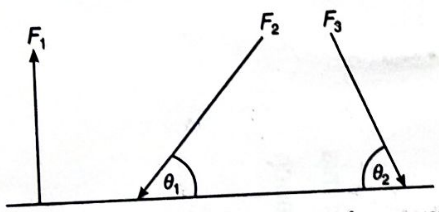


Figure 5.2 Coplanar non-concurrent force system.

3. Parallel force system

$\Sigma F = 0,\ \Sigma M = 0$

4. Non-coplanar force system

$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0,\ \Sigma M = 0$

FRICTION

Definition of Friction

When one body tends to move in contact over other body a resistance to its movement is set-up. This resistance to movement is called *Friction* or Force of Friction or Frictional Force.

The Force of Friction always acts in the direction opposite to the '*motion trend*' as shown in figure 2.

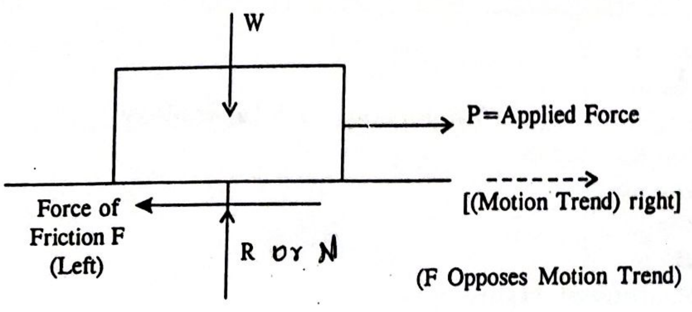


Fig. 2

2.5) Types of Friction

The various types of friction are :

1) **Static Friction :** The friction acting on a body which is at rest is called static friction.

2) **Limiting Friction :** The friction acting on a body which is just on the point or verge of sliding is called limiting friction.

3) **Dynamical Friction :** The friction acting on a body which is actually in motion is called Dynamical friction or Kinetic friction.

4) **Dry Friction :** The friction acting on a body when the contact surfaces are dry (i.e., unlubricated) and there is tendency of relative motion is called *Dry Friction* or Coulomb friction.

Dry Friction is further divided into two types :

a) **Solid Friction :** The friction acting on a body when two surfaces have tendency to slide relative to each other is called Solid friction (Figure 2).

b) **Rolling Friction :** The friction acting on a body due to rolling of one surface over another is called Rolling friction.

5) **Fluid Friction :** The friction acting on a body when the contact surfaces are lubricated is called *Fluid friction.*

Fluid friction is further divided into :

a) **Skin or greasy or Non-Viscous friction :** The friction acting on a body when the contact surfaces are lubricated with extremely thin layer of lubricant is called Skin or greasy or Non-viscous Friction also called *Boundary Friction.*

b) **Viscous or Film Friction :** The friction acting on a body when the contact surfaces are completely separated by lubricant is called *Viscous or Film friction.*

Motion Trend of a Block on Surface

Let a body of weight 'W' be subjected to pull 'P' which tend to move towards right as shown in figure 4.a. The forces induced are the Reaction 'R' acting perpendicular to the support and the force of Friction 'F' acting opposite to the motion trend.

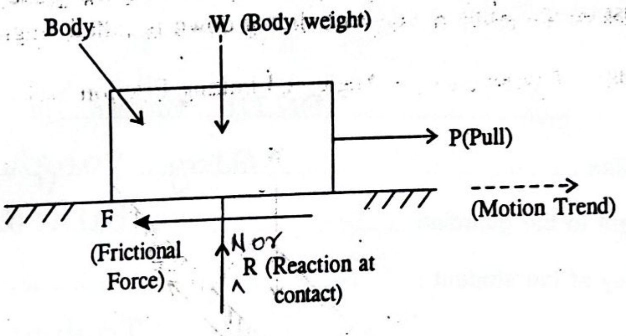


Fig. 4.a.

From figure 4.(b) you can see that the normal reaction and friction acting perpendicular to each other can be replaced by a single Resultant Reaction 'R_1' making angle 'φ'. So that,

$$\text{Tan}\varphi = \frac{F}{R} = \frac{F}{N} \quad \text{............(1)}$$

R or N

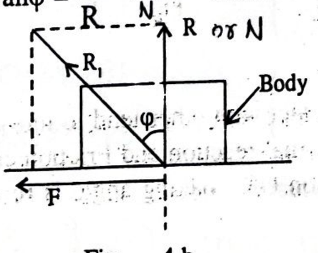


Fig. 4.b

Angle of Friction

The angle which the Resultant Reaction R_1 due to normal Reaction R and Friction F makes with the normal to the surface is called Angle of Friction (φ). In figure .4.b.

$$\boxed{\operatorname{Tan}\varphi = \frac{F}{R}} = \frac{F}{N} \quad \text{............(2)}$$

Co-efficient of Friction (μ)

It is the ratio of the limiting friction F to the normal reaction R between two surfaces. This is also equal to the Tangent of angle of friction.

$$\mu = \frac{F}{R} = \frac{F}{N}$$

$$\boxed{\therefore \operatorname{Tan}\varphi = \mu = \frac{F}{R}} = \frac{F}{N} \text{.........remember} \quad \text{............(3)}$$

Angle of Repose

In figure .7, if a body is placed on an inclined plane, then the angle at which the body is just on the point or verge of sliding down is called Angle of Repose.

Angle of repose (α) = Angle of Limiting Friction (φ).

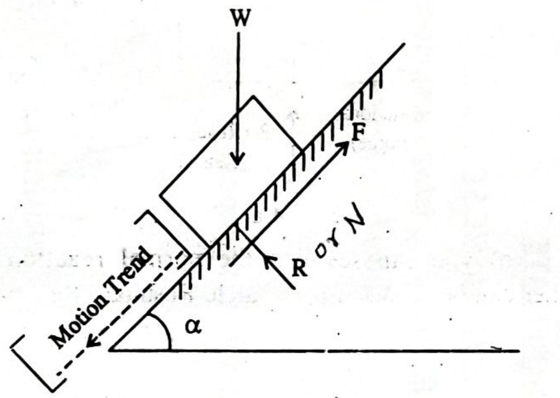
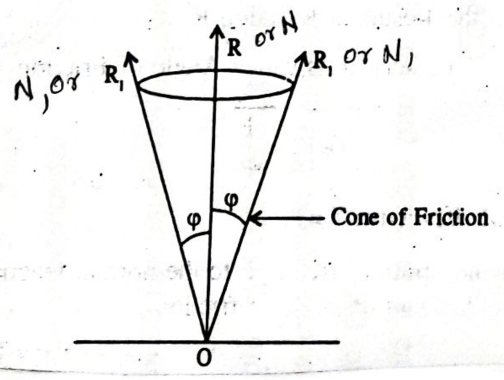


Fig. .7

Cone of Friction

Whenever a body in contact with other tend to move, then the normal reaction OR and Friction come into play. The normal reaction and Friction can be replaced by resultant reaction OR_1. When this resultant reaction OR_1 making angle is revolved around point O, will form a right circular cone.

This cone having the point of contact as the vertex O, the normal OR at the point of contact as its axis and φ as the semi-vertex angle is called the Cone of Friction (Fig. .8).



2.6) Laws of Dry Friction

1. The Force of Friction always acts in the direction opposite to that in which the body tends to move.

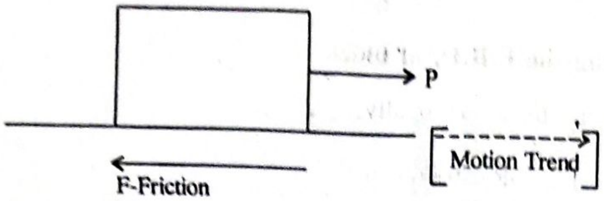


Fig. .9

2. The magnitude of limiting friction (F) bears a constant ratio to the normal reaction (R) between the two surfaces i.e., $\frac{F}{R} = \mu$ (constant).
3. The magnitude of Force of Friction is exactly equal to the force, which tends the body to move, as long as the body is at rest (i.e., P=F).
4. The force of friction is independent of the area of contact between two surfaces.
5. The force of friction depends upon the roughness of the surfaces in contact.

2.7) LIMITING FRICTION

The Friction acting on a body which is just on the point or verge of sliding

2.8) CONCEPT OF STATIC AND DYNAMIC FRICTION

Static Friction - Friction acting on a body which is at rest

Dynamic Friction - Friction acting on a body which is actually in motion is called Dynamic or kinetic Friction

Suppose a block of weight 'W' is kept on a rough horizontal surface and a horizo force 'P' is applied to it as shown in Fig. 7.2.1. When force 'P' is small, the block does not move. This is because of the frictional force which balances P. The frictional force is largely due to interlocking of irregularities in the two surfaces in contact as shown in Fig. 7.2.2. The frictional force is also, to a small extent, due to the molecular forces of attraction between the two surfaces in contact. If force P is increased, F_r increases and remains equal to P as long as the object is static. When P is increased beyond a certain value $(F_r)_{max}$, the object starts moving, the frictional force decreases and then remains nearly constant. This variation in magnitude of frictional force with the applied force P is shown in Fig. 7.2.3.

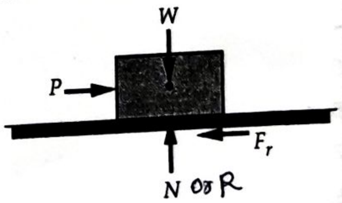


Fig. 7.2.1

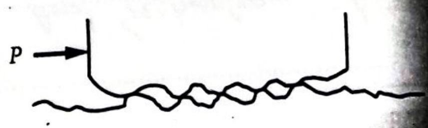
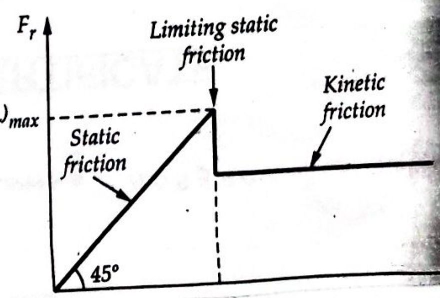


Fig. 7.2.2



From the above discussion, we conclude the following points which are important while solving problems :

1) As long as object is static, the frictional force has same magnitude as the net force trying to move the object and has opposite direction.

2) Motion impends when net force trying to move the object becomes equal to the maximum frictional force $(F_r)_{max}$ known as the limiting static friction force.

3) When object starts moving, the frictional force is constant, independent of the net applied force.

Experimental evidence shows that the maximum value of frictional force (the limiting static friction force) is proportional to the normal component of reaction N. or R i.e.,

$$(F_r)_{max} = \mu_s N = \mu_s R$$

Where μ_s is a constant known as coefficient of static friction. Similarly, the magnitude of the kinetic friction force is expressed as

$$F_K = \mu_k N = \mu_K R$$

Where μ_k is a constant known as coefficient of kinetic friction.

The coefficients μ_s and μ_k do not depend on the surface area in contact but depend on the nature of the surfaces in contact.

STATIC FRICTION

•) Under static conditions, the friction force opposes tendency for relative motion between the 2 surfaces in contact and acts tangential to the surfaces

•) limiting static friction force, which is the maximum value of friction force is directly proportional to the normal reaction between the 2 surfaces in contact

$(F_r)_{max} \propto N$ $\boxed{N \text{ or } R}$

$(F_r)_{max} = \mu_s N$

μ_s = coefficient of static friction

KINETIC FRICTION

•) The force of kinetic friction opposes the relative motion between 2 surfaces in contact

•) The force of kinetic friction is directly proportional to the normal reaction between the 2 surfaces in contact.

$F_K \propto N$

$F_K = \mu_K N$ $\boxed{N \text{ or } R}$

μ_K = coefficient of kinetic friction

7.5 Angle of Friction and Resultant Reaction

VTU : Aug.-07, 08, 10, 11 Feb.-08.

The normal reaction N and the frictional force F_r can be combined to give a resultant R called the resultant reaction as shown in Fig. 7.5.1.

The angle made by this resultant R with normal reaction N is called the angle of friction ϕ.

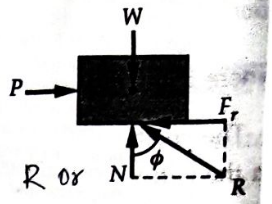


Fig. 7.5.1

$$\tan\phi = \frac{F_r}{N} \quad \text{... (7.5.1)}$$

For impending motion, $F_r = (F_r)_{max} = \mu_s N$

Then,

$$\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$$

$$\therefore \quad \tan\phi_s = \mu_s \quad \text{... (7.5.2)}$$

where ϕ_s is called the angle of static friction.

If motion takes place,

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\therefore \quad \tan\phi_k = \mu_k \quad \text{... (7.5.3)}$$

where ϕ_k is called angle of kinetic friction.

If direction of applied force P is changed, keeping its angle with the surface of contact same throughout, the resultant R will take different positions in space but making the same angle ϕ with the normal reaction N. In such a case, R lies on the surface of a cone known as cone of friction as shown in Fig. 7.5.2.

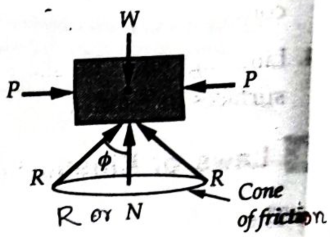


Fig. 7.5.2

If an object is kept on an inclined plane and the angle of inclination θ is increased, motion impends for a certain value of θ known as **angle repose**. The angle of repose is equal to the angle of friction. This can be proved as follows :

Consider F.B.D. of an object kept on an inclined plane of angle θ equal to the angle of repose as shown in Fig. 7.5.3.

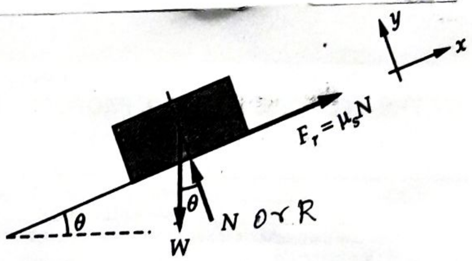


Fig. 7.5.3

$$\sum F_y = 0$$

$$N - W\cos\theta = 0$$

$$\therefore \quad N = W\cos\theta$$

$$\sum F_x = 0$$

$$\mu_s N - W\sin\theta = 0$$

$$\mu_s (W\cos\theta) = W\sin\theta$$

$$\therefore \quad \mu_s = \tan\theta$$

But $\quad \mu_s = \tan\phi_s$

$$\therefore \quad \tan\phi_s = \tan\theta$$

$$\therefore \quad \boxed{\phi_s = \theta}$$

Problems involving dry friction can be solved using either N and F_r or using R and ϕ_s.

7.6 Problems Involving Dry Friction

Problems involving dry friction can be broadly classified into two types. In the first type of problems all forces acting on the object and coefficients of friction μ_s and μ_k are known and we have to determine whether the object will move or not. In such problems, the frictional force is also unknown and has to be determined. The following procedure can be adopted for such problems :

1) Draw F.B.D. of object without frictional force.

2) Choose x-axis parallel to the direction in which object can move and y-axis perpendicular to it.

3) Use $\sum F_y = 0$ to find normal reaction N.

4) Find limiting static friction force $(F_r)_{max} = \mu_s N$

5) Find $\sum F_x$, excluding frictional force. This will be net force trying to move the object.

6) If $\left|\sum F_x\right| \le (F_r)_{max}$, object does not slide. In such a case, the actual value of frictional force will have same magnitude as $\sum F_x$ but opposite direction.

i.e. $F_r = \sum F_x$ in opposite direction.

7) If $\left|\sum F_x\right| > (F_r)_{max}$, object slides and $F_r = \mu_k N$ in a direction opposite to $\sum F_x$.

In problems where dimensions of object are given, the object may either slide or overturn. Consider an object as shown in Fig. 7.6.1 (a), where applied force P is zero. The normal reaction N has same line of action as weight W but opposite direction. There will be no frictional force. When applied force P is increased, the point of application of N shifts towards right as shown in Fig. 7.6.1 (b). If P is increased further, when the object is just about to overturn, N will act at the right end B as shown in Fig. 7.6.1 (c).

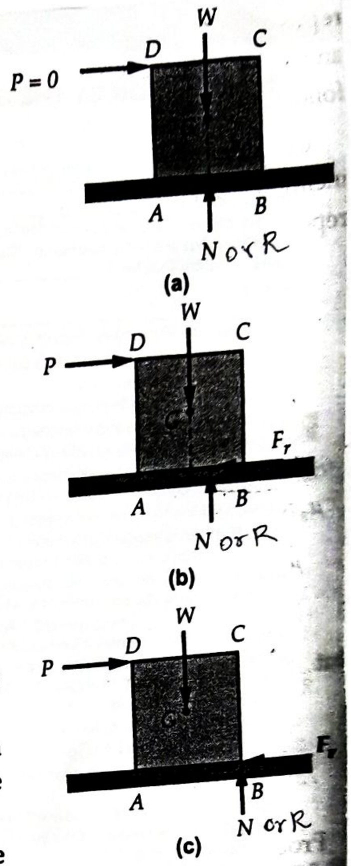


Fig. 7.6.1

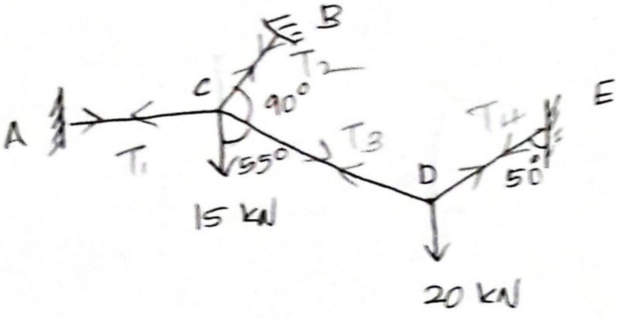
We can find the value of force P required to overturn the object by taking moment about the right edge B of the object.

In the second type of problems, it is known that there is impending motion. In such problems draw FBD, use $F_r = \mu_s N$ and use condition for equilibrium.

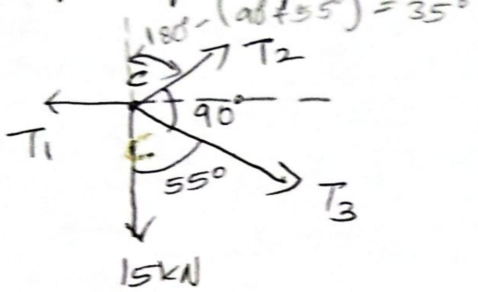
MODULE - 2

Q) Find tension in string if the system is in equilibrium shown in figure

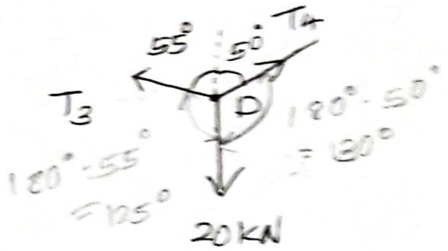
(dec 2019 | Jan 2020)



Free body diagram of point C



free body diagram of point D



Applying Lami's theorem to the point D

$$\frac{T_3}{\sin(130^\circ)} = \frac{T_4}{\sin(125^\circ)} = \frac{20}{\sin(50^\circ+55^\circ)}$$

$$\frac{T_3}{\sin 130^\circ} = \frac{T_4}{\sin 125^\circ} = \frac{20}{\sin(105^\circ)}$$

$$\frac{T_3}{\sin 130^\circ} = \frac{20}{\sin(105^\circ)}$$

By cross multiplication

$$T_3 \times \sin(105^\circ) = 20 \times \sin(130^\circ)$$

$$T_3 = \frac{20 \times \sin(130^\circ)}{\sin(105^\circ)} = 15.86\,kN$$

$$\frac{T_4}{\sin 125^\circ} = \frac{20}{\sin 105^\circ}$$

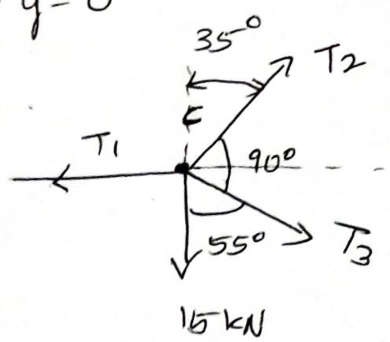
By cross multiplication

$$T_4 \times \sin 105^\circ = 20 \sin 125^\circ$$

$$T_4 = \frac{20 \sin 125^\circ}{\sin 105^\circ} = 16.96\,kN$$

Now consider the system of forces acting at C

$\Sigma F_y = 0$



$$-15 - T_3 \cos 55° + T_2 \cos 35° - T_1 = 0$$

$$-15 - 15.86 \cos 55° + T_2 \cos 35° = 0$$

$$T_2 \cos 35° = 15 + 15.86 \cos 55°$$

$$T_2 = \frac{15 + 15.86 \cos 55°}{\cos 35°} = 29.42 \text{ kN}$$

$\Sigma F_x = 0$

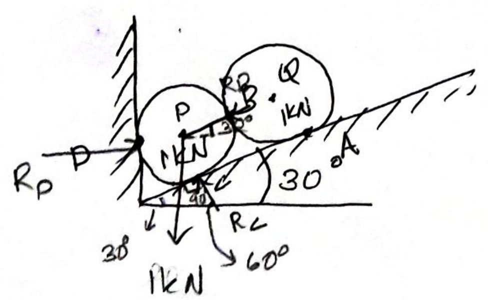
$$T_3 \sin 55° + T_2 \sin 35° - T_1 = 0$$

$$15.86 \sin 55° + 29.42 \sin 35° = T_1$$

$$29.87 \text{kN} = T_1$$

Q) Find contact pressure at surfaces of contact for the system shown in figure for 2 identical cylinders P and Q

(dec 2019/ Jan 2020)



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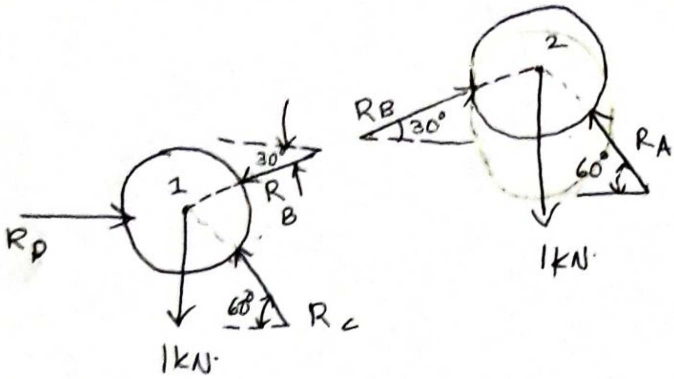
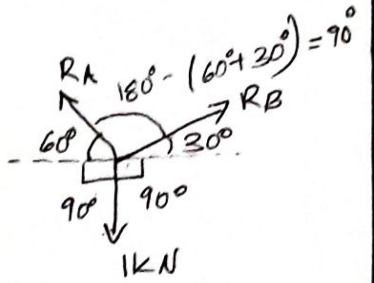




F.B.D of roller 2

Using Lami's theorem for roller 2

$$\frac{R_A}{\sin(90°+30°)} = \frac{R_B}{\sin(90°+60°)} = \frac{1}{\sin 90°}$$

$$\frac{R_A}{\sin 120°} = \frac{R_B}{\sin(15°)} = \frac{1}{\sin 90°}$$

$$\frac{R_A}{\sin 120°} = \frac{1}{\sin 90°}$$

By cross multiplication

$R_A \times \sin 90° = \sin 120° \times 1$

$R_A = \frac{\sin 120° \times 1}{\sin 90°} = 0.87 \text{ kN},$ 

$$\frac{R_B}{\sin 15°} = \frac{1}{\sin 90°}$$

By cross multiplication

$R_B \times \sin 90° = \sin 15° \times 1$

$R_B = \frac{\sin 15° \times 1}{\sin 90°} = 0.259 \text{ kN}$

For roller 1

$\Sigma F_y = 0$

$-1 + R_C \sin 60° - R_B \sin 30° = 0$

$-1 + R_C \sin 60° - 0.259 \sin 30° = 0$

$R_C \sin 60^\circ = 1 + 0.259 \sin 30^\circ$

$$R_C = \frac{1 + 0.259 \sin 30^\circ}{\sin 60^\circ} = 1.30\,kN, \; 60^\circ$$

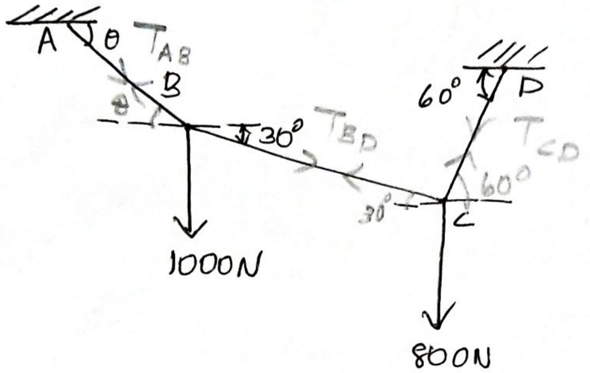
$\Sigma F_x = 0$

$R_D - R_B \cos 30^\circ - R_C \cos 60^\circ = 0$

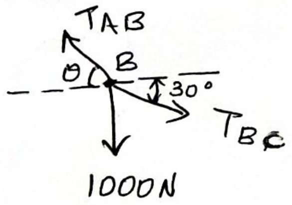
$R_D - 0.259 \cos 30^\circ - 1.30 \cos 60^\circ = 0$

$R_D = 1.30 \cos 60^\circ + 0.259 \cos 30^\circ = 0.87\,kN, \rightarrow$

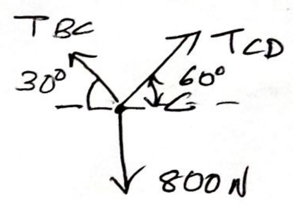
Q) Calculate the tension in the strings. Also calculate θ in figure (June/July 2019)



free body diagram for B and C



F.B.D of point B



FBD of point C

Using Lami's theorem for point C

$$\frac{T_{BC}}{\sin(90^\circ + 60^\circ)} = \frac{T_{CD}}{\sin(90^\circ + 30^\circ)} = \frac{800}{\sin(180^\circ - (30^\circ + 60^\circ))}$$

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{800}{\sin(90^\circ)}$$

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ}$$

By cross multiplication

$$T_{BC} \times \sin 90^\circ = 800 \times \sin 150^\circ$$

$$T_{BC} = \frac{800 \times \sin 150^\circ}{\sin 90^\circ} = 400\ N$$

$$\frac{T_{CD}}{\sin 120^\circ} = \frac{800}{\sin 90^\circ}$$

By cross multiplication

$$T_{CD} \times \sin 90^\circ = 800 \times \sin 120^\circ$$

$$T_{CD} = \frac{800 \times \sin 120^\circ}{\sin 90^\circ} = 692.82\ N$$

For point B

$$\Sigma F_x = 0$$

$$T_{BC} \cos 30^\circ - T_{AB} \cos\theta = 0$$

$$400 \cos 30^\circ - T_{AB} \cos\theta = 0$$

$$T_{AB} \cos\theta = 400 \cos 30^\circ = 346.41 \quad \text{— (1)}$$

$$\Sigma F_y = 0$$

$$-1000 - T_{BC} \sin 30^\circ + T_{AB} \sin\theta = 0$$

$$T_{AB} \sin\theta = 1000 + T_{BC} \sin 30^\circ$$

$$T_{AB} \sin\theta = 1000 + 400 \sin 30^\circ = 1200 \quad \text{— (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{T_{AB} \sin\theta}{T_{AB} \cos\theta} = \frac{1200}{346.41}$$

$$\tan\theta = \frac{1200}{346.41}$$

$$\theta = \tan^{-1}\left(\frac{1200}{346.41}\right) = 73.89^\circ$$

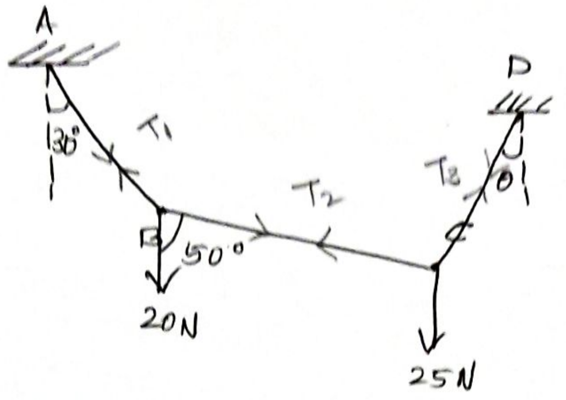
Substituting $\theta = 73.9^\circ$ in equation (2)

$$T_{AB}\sin\theta = 1200$$

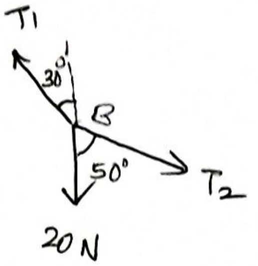
$$T_{AB} = \frac{1200}{\sin\theta} = \frac{1200}{\sin(73.9^\circ)} = 1248.98\,N \approx 1249\,N$$

Q) Compute the tensions in the strings AB, BC and CD as shown in figure

(dec 2018/ Jan 2019)



free body diagram of point B



Applying Lami's theorem for the system of forces at B

$$\frac{T_1}{\sin 50^\circ} = \frac{T_2}{\sin(180^\circ - 30^\circ)} = \frac{20}{\sin(180^\circ - 50^\circ + 30^\circ)}$$

$$\frac{T_1}{\sin 50^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{20}{\sin(160^\circ)}$$

$$\frac{T_1}{\sin 50^\circ} = \frac{20}{\sin(160^\circ)}$$

By cross multiplication

$$T_1 \sin(160^\circ) = 20 \times \sin 50^\circ$$

$$T_1 = \frac{20 \times \sin 50^\circ}{\sin(160^\circ)} = 44.79\,N$$

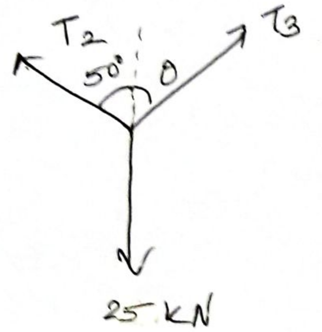
$$\frac{T_2}{\sin 150^\circ} = \frac{20}{\sin 160^\circ}$$

By cross multiplication

$$T_2 \times \sin 160^\circ = 20 \times \sin 150^\circ$$

$$T_2 = \frac{20 \times \sin 150^\circ}{\sin 160^\circ} = 29.23\ N$$

Free body diagram of point c



consider the equilibrium of forces at c

$$\Sigma F_x = 0$$

$$T_3 \sin\theta - T_2 \sin 50^\circ = 0$$

$$T_3 \sin\theta = T_2 \sin 50^\circ$$

$$T_3 \sin\theta = 29.23 \times \sin 50^\circ = 22.39\ N \quad \text{--- (1)}$$

$$\Sigma F_y = 0$$

$$T_3 \cos\theta + T_2 \cos 50^\circ \overset{-25}{\wedge} = 0$$

$$T_3 \cos\theta + 29.23 \cos 50^\circ = 25$$

$$T_3 \cos\theta = \overset{25}{\wedge} - 29.23 \cos 50^\circ = +6.21 \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{T_3 \sin\theta}{T_3 \cos\theta} = \frac{22.39}{6.21} = 3.605$$

$\tan\theta = 3.605$

$\theta = \tan^{-1}(3.605) = 74.5^\circ$

Substituting the value in equation (1)

$T_3 \sin\theta = 22.39$

$T_3 \sin(74.5^\circ) = 22.39$

$T_3 = \dfrac{22.39}{\sin(74.5^\circ)} = 23.23\ N$

Example 5.10.18 *Three cylinders weighing 500 N each 24 units in diameter are placed in channel as shown in Fig. 5.10.18. Determine reactions at all contact points. Take cylinders are smooth.*

VTU : Aug-09, Marks 10

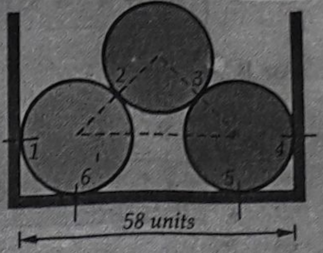


Fig. 5.10.18

Solution : The triangle formed by joining the centres of the three cylinders is an isosceles triangle as shown in Fig. 5.10.18 (a).

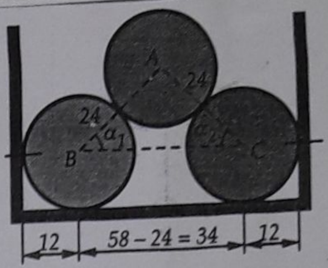


Fig. 5.10.18 (a)

$$\alpha_1 = \alpha_2 = \cos^{-1}\left(\frac{17}{24}\right)$$

$$\therefore \quad \alpha_1 = \alpha_2 = 44.9^\circ$$

The three free body diagrams are shown in Fig. 5.10.18 (b).

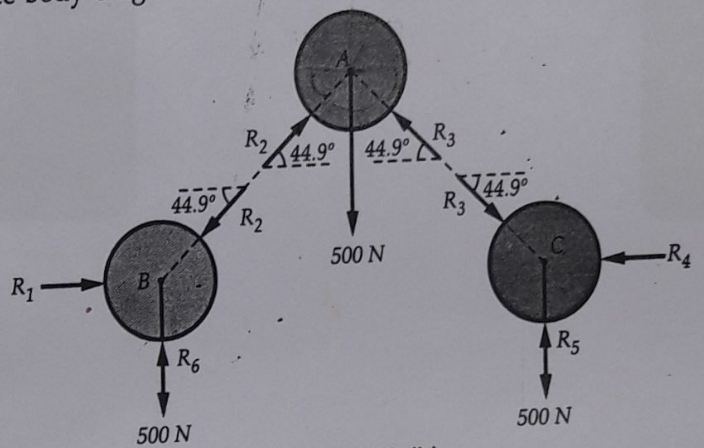


Fig. 5.10.18 (b)

For A, $\sum F_x = 0$:

$$R_2 \cos 44.9 - R_3 \cos 44.9 = 0$$

$$\therefore \quad R_2 = R_3$$

$\sum F_y = 0$:

$$R_2 \sin 44.9 + R_3 \sin 44.9 - 500 = 0$$

$$\therefore \quad \boxed{R_2 = R_3 = 354.172 \text{ N}}$$

For B, $\sum F_x = 0$:

$$R_1 - R_2 \cos 44.9 = 0$$

$$\therefore \quad \boxed{R_1 = 250.874 \text{ N}}$$

$\sum F_y = 0$:

$$R_6 - 500 - R_2 \sin 44.9 = 0$$

$\therefore$ $\boxed{R_6 = 750 \text{ N}}$

For C, $\sum F_x = 0$:

$$R_3 \cos 44.9 - R_4 = 0$$

$\therefore$ $\boxed{R_4 = 250.874 \text{ N}}$

$\sum F_y = 0$:

$$R_5 - 500 - R_3 \sin 44.9 = 0$$

$\therefore$ $\boxed{R_5 = 750 \text{ N}}$

Example 5.10.19 *Find the tension in the string and reaction at the contact surface for the cylinder of Wt = 1000 N placed as shown in Fig. 5.10.19. Solve by Lami's theorem.*

VTU : Feb.-10, Marks 6

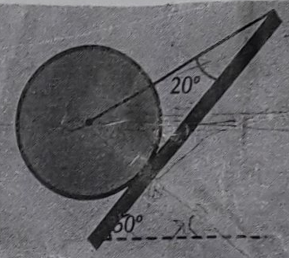
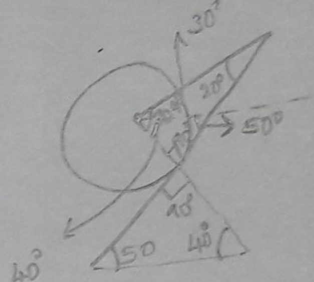


Fig. 5.10.19



Solution : The F.B.D. of cylinder is shown in Fig. 5.10.19 (a).

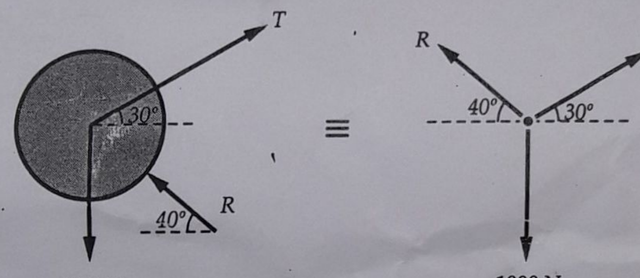


Fig. 5.10.19 (a)

Using Lami's theorem,

$$\frac{R}{\sin 120} = \frac{T}{\sin 130} = \frac{1000}{\sin 110}$$

$\therefore$ $\boxed{R = 921.6 \text{ N}}$

$\therefore$ $\boxed{T = 815.2 \text{ N}}$

Example 5.10.25 *Three identical spheres P, Q, R of weight 'W' are arranged on smooth inclined surfaces as shown in Fig. 5.10.25. Determine the angle 'α' which will prevent the arrangement from collapsing.*

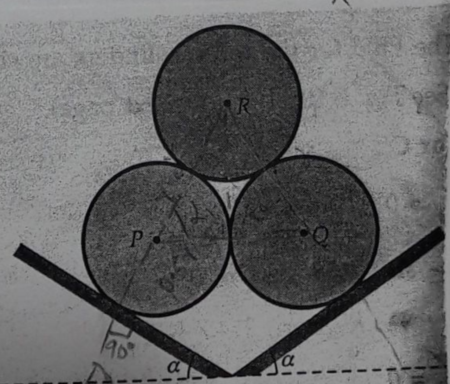


Fig. 5.10.25

$\cos \alpha_1 = \frac{adj}{hyp} = \frac{0.5r}{r}$

$\alpha_1 = \cos^{-1}(0.5) = 60°$

$180° - (90° + \alpha) = 90° - \alpha$

Solution : The free body diagrams of the three spheres are as shown in Fig. 5.10.25 (a).

When the arrangement is about to collapse, P and Q tend to move away from each other. As a result, reaction between P and Q tends to zero.

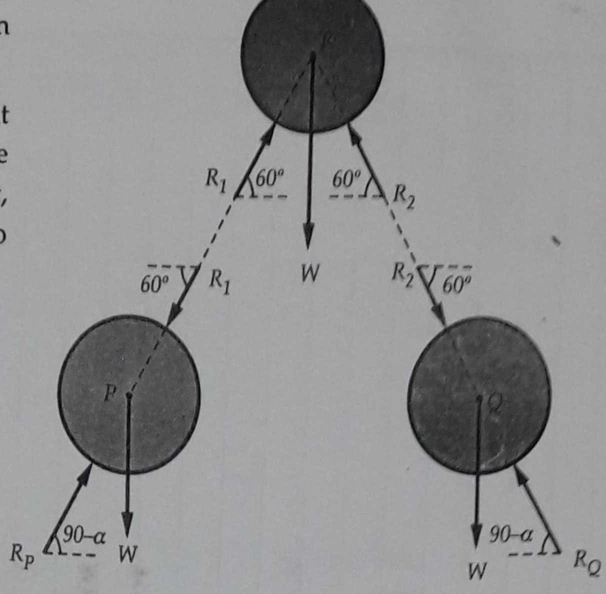


Fig. 5.10.25 (a)

From FBD of R :

$$\sum F_x = 0$$

$$R_1 \cos 60 - R_2 \cos 60 = 0$$

$$\therefore \quad R_1 = R_2$$

$$\sum F_y = 0$$

$$R_1 \sin 60 + R_2 \sin 60 - W = 0$$

$$R_1 \frac{\sqrt{3}}{2} + R_1 \frac{\sqrt{3}}{2} = W$$

$$R_1 = \frac{W}{\sqrt{3}}$$

From FBD of P :

$$\sum F_x = 0 : R_P \cos(90-\alpha) - R_1 \cos 60 = 0$$

$$R_P \sin\alpha = \frac{W}{\sqrt{3}} \cdot \frac{1}{2} \qquad \dots (1)$$

$$\sum F_y = 0 : R_P \sin(90-\alpha) - R_1 \sin 60 - W = 0$$

$$R_P \cos\alpha = \frac{W}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + W$$

$$R_P \cos\alpha = \frac{3}{2} W \qquad \dots (2)$$

Dividing equation (1) by equation (2), $\tan\alpha = \dfrac{\frac{W}{2\sqrt{3}}}{\frac{3}{2}W}$

$$\tan\alpha = \frac{1}{3\sqrt{3}}$$

$$\boxed{\alpha = 10.9^\circ}$$

Example 5.10.15 *Two cylinders A and B rest in a channel as shown in Fig. 5.10.15. A has a diameter of 100 mm and weighs 20 kN, B has diameter of 180 mm and weighs 50 kN. The channel is 180 mm wide at bottom with one side vertical and the other side at 60° inclinations. Find the reactions at contact points.*

VTU : Aug.-08, Marks 10

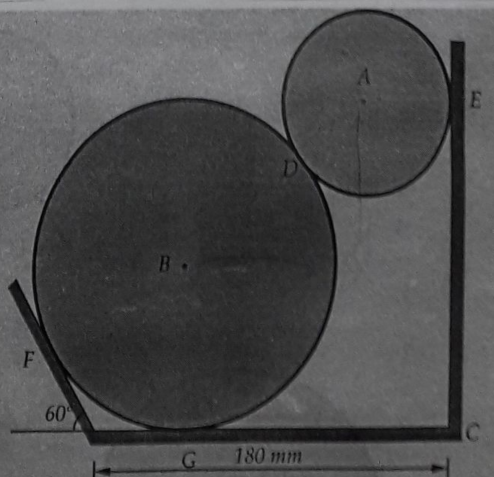
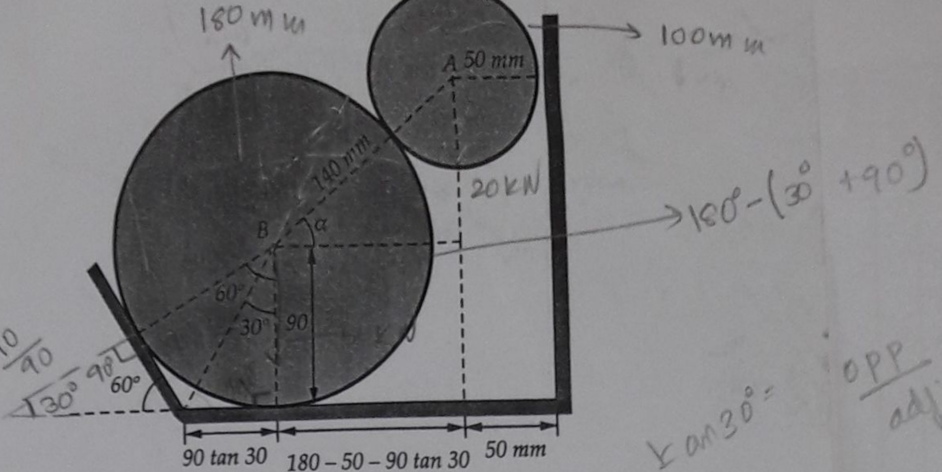


Fig. 5.10.15

Solution : To find the angle made by the reaction between A and B with horizontal, use constructions as shown in Fig. 5.10.15 (a).

12



$\cos\theta = \frac{adj}{hyp} = \frac{90}{90}$

$\tan 30° = \frac{opp}{adj} = \frac{opp}{90°}$

Fig. 5.10.15 (a)

$$\cos\alpha = \frac{180 - 50 - 90 \tan 30}{140} = \frac{adj}{hyp}$$

$$\therefore \quad \alpha = 56.12°$$

The free body diagram of the two cylinders are shown in Fig. 5.10.15 (b).

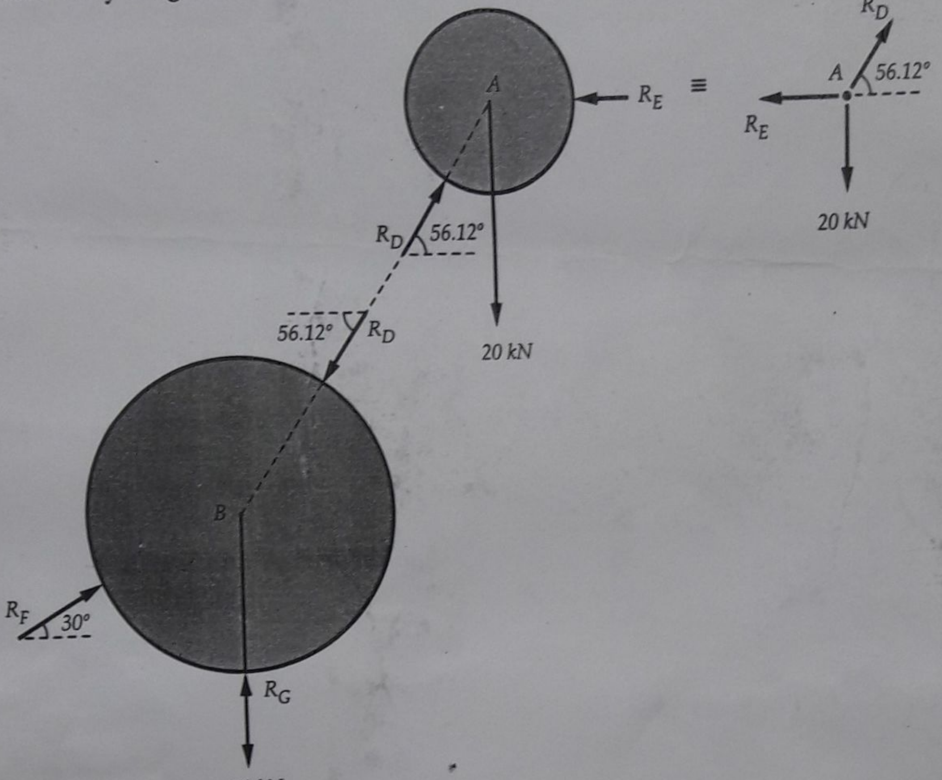


Fig. 5.10.15 (b)

Using Lami's theorem for A,

$$\frac{R_D}{\sin 90} = \frac{R_E}{\sin 146.12} = \frac{20}{\sin(180 - 56.12)}$$

$$\therefore \quad \boxed{R_D = 24.1 \text{ kN}}$$

$$\therefore \quad \boxed{R_E = 13.43 \text{ kN}}$$

For B, $\sum F_x = 0$:

$$R_F \cos 30 - R_D \cos 56.12 = 0$$

$$\therefore \quad \boxed{R_F = 15.51 \text{ kN}}$$

$\sum F_y = 0$:

$$R_F \sin 30 + R_G - 50 - R_D \sin 56.12 = 0$$

$$\therefore \quad \boxed{R_G = 62.25 \text{ kN}}$$

Example 5.10.20 *Find the forces in cables AB and CB shown in Fig. 5.10.20. The remaining two cables pass over frictionless pulleys E and F and support masses 1200 kg and 1000 kg respectively.*

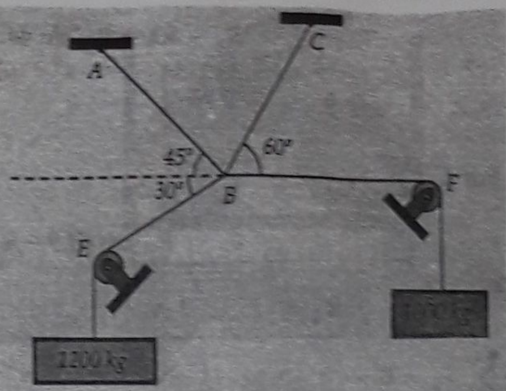


Fig. 5.10.20

Solution : All the forces are concurrent at B. The FBD of B is shown in Fig. 5.10.20 (a).

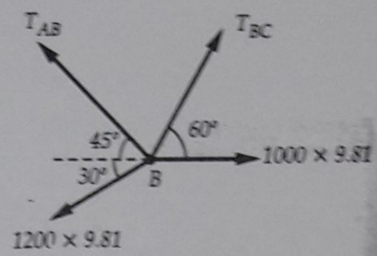


Fig. 5.10.20 (a)

$$\sum F_x = 0$$

$$T_{BC} \cos 60 - T_{AB} \cos 45 + 1000 \times 9.81 - 1200 \times 9.81 \cos 30 = 0 \quad \text{... (1)}$$

$$\sum F_y = 0$$

$$T_{BC} \sin 60 + T_{AB} \sin 45 - 1200 \times 9.81 \sin 30 = 0 \quad \text{... (2)}$$

From equations (1) and (2),

$$\boxed{T_{AB} = 2701.77 \text{ N}}$$

$$\boxed{T_{BC} = 4590.58 \text{ N}}$$

Example 5.10.21 *Under the action of five forces, following system is in equilibrium. Determine fifth force. Refer Fig. 5.10.21.*

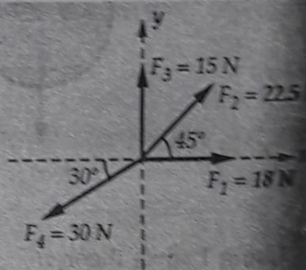


Fig. 5.10.21

Solution : Let the fifth force 'F' be in first quadrant with components F_x and F_y.

$$\sum F_x = 0 \ : \ F_x + 18 + 22.5 \cos 45 - 30 \cos 30 = 0$$

$$\therefore \quad F_x = -7.93 \text{ N}$$

$$\sum F_y = 0 \ : \ F_y + 22.5 \sin 45 + 15 - 30 \sin 30 = 0$$

coplanar concurrent force system

$$\therefore \quad F_y = -15.91$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-7.93)^2 + (-15.91)^2}$$

$\Sigma F_x = 0, \Sigma F_y = 0$

$$\therefore \quad \boxed{F = 17.78 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{|F_y|}{|F_x|}\right) = \tan^{-1}\left(\frac{15.91}{7.93}\right)$$

$$\therefore \quad \boxed{\theta = 63.51^\circ}$$

Example 5.10.22 *Determine a) the value of α for which the tension in rope BC is as small as possible and b) the corresponding value of the tension. Refer Fig. 5.10.22.*

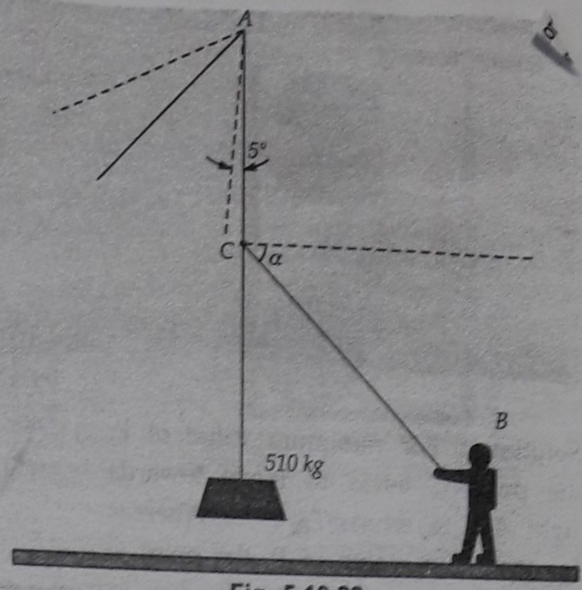


Fig. 5.10.22

Solution : The FBD of point 'C' is shown in Fig. 5.10.22 (a).

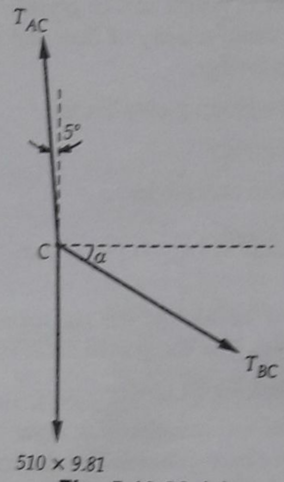


Fig. 5.10.22 (a)

Using Lami's theorem,

$$\frac{T_{AC}}{\sin(90-\alpha)} = \frac{T_{BC}}{\sin(180-5)} = \frac{510 \times 9.81}{\sin(90+5+\alpha)}$$

$$\sin(90+5+\alpha) = \cos(5+\alpha) \qquad \boxed{\sin(90^\circ+\theta) = \cos\theta}$$

$$T_{BC} = \frac{510 \times 9.81 \sin 175}{\cos(5+\alpha)}$$

For T_{BC} to be minimum, $\cos(5+\alpha)$ must be maximum.

$$\cos(5+\alpha) = 1$$

$$\therefore \quad 5+\alpha = 0^\circ$$

$$\therefore \quad \alpha = -5^\circ$$

$$\therefore \quad \boxed{\alpha = 5^\circ \angle}$$

$$\therefore \quad (T_{BC})_{min} = 510 \times 9.81 \sin 175$$

$$\therefore \quad \boxed{(T_{BC})_{min} = 436.05 \text{ N}}$$

Problem 25 : A ladder weighing 200N is to be kept in position as shown resting on a smooth floor and leaning against a smooth wall. Determine the horizontal force required to prevent it from slipping when a man weighing 700N is at a height of 2m above the floor level. (Jan/Feb. 2006 VTU)

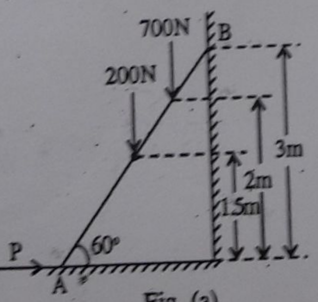
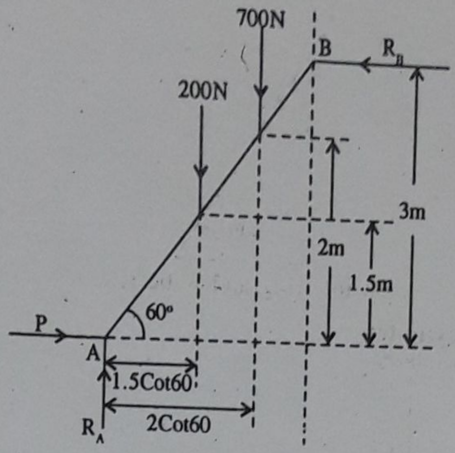


Fig. (a)

Solution : The wall and floor a smooth surface. Hence there will be only surface reaction acting $\perp^{ar}$ to surface.

Consider F.B.D. of ladder.



cot 60° = adj/opp

Fig. (b)

coplanar non concurrent force system

$\Sigma F_x = 0, \Sigma F_y = 0,$

$\Sigma M = 0$

The ladder system is in equilibrium, $\therefore \ \Sigma H = \Sigma V = \Sigma M = 0$

Resolving forces Vertically, $\Sigma V = 0$

$$R_A - 200 - 700 = 0$$

$$R_A = 900N(\uparrow) \qquad \text{............(1)}$$

Resolving forces Horizontally, $\Sigma H = 0$

$$P - R_B = 0$$

$$P = R_B \qquad \text{............(2)}$$

Taking moment about A, $\Sigma M_A = 0$

$$-R_B \times 3 + 200 \times 1.5\text{Cot}60 + 700 \times 2\text{Cot}60 = 0$$

$$R_B = 327.17N \qquad \text{............(3)}$$

From (2) and (3)

$$P = 327.17N \qquad \text{............(4)}$$

Example 5.10.28 *Two cylinders of masses $m_1 = 100$ kg and $m_2 = 50$ kg are connected by a rigid bar of negligible weight hinged at each cylinder. The cylinders are resting on smooth inclined planes and are in equilibrium in the position shown under a force P. Determine the magnitude of force P. Refer Fig. 5.10.28.*

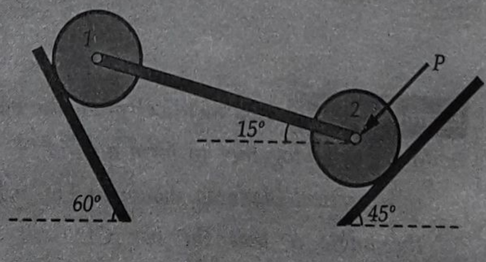


Fig. 5.10.28

Solution : The FBD of each cylinder is shown in Fig. 5.10.28 (a). The rigid bar is subjected to forces only at the ends and hence the force in it will be axial. As the length of bar tends to decrease, there will be compressive force F_C in it which is directed towards the cylinders as shown.

For cylinder 1 by Lami's theorem,

$$\frac{F_C}{\sin 120} = \frac{100 \times 9.81}{\sin 135}$$

$$\therefore \quad F_C = 1201.475 \text{ N}$$

For cylinder 2 : $\sum F_x = 0$

$$-P \cos 45 - R_2 \cos 45 + F_C \cos 15 = 0$$

$$\therefore \quad P \cos 45 + R_2 \cos 45 = 1201.475 \cos 15 \qquad \ldots(1)$$

$\sum F_y = 0$

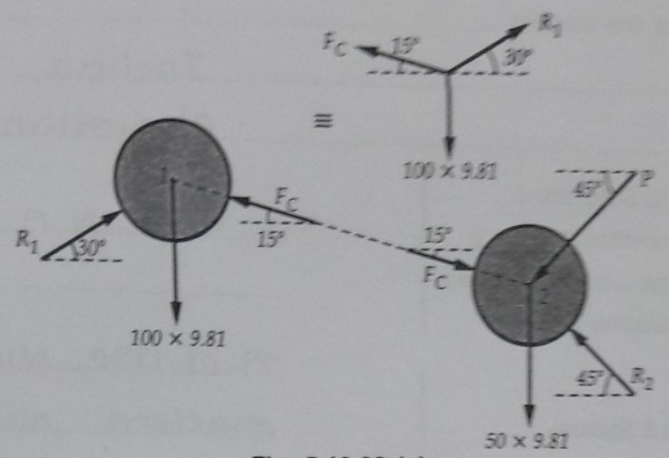


Fig. 5.10.28 (a)

$$-P \sin 45 + R_2 \sin 45 - F_C \sin 15 - 50 \times 9.81 = 0$$

$$-P \sin 45 + R_2 \sin 45 = 801.465$$

From equations (1) and (2),

$$\boxed{P = 253.9 \text{ N}}$$

Example 5.10.32 *A block placed under the head of the claw hammer as shown in Fig. 5.10.32 greatly facilitates the extraction of the nail. If the 200 N pull on the handle is required to pull the nail, calculate the tension T in the nail and the magnitude of the force exerted by the hammer head on the block. The contacting surfaces at A are sufficiently rough to prevent slipping.*

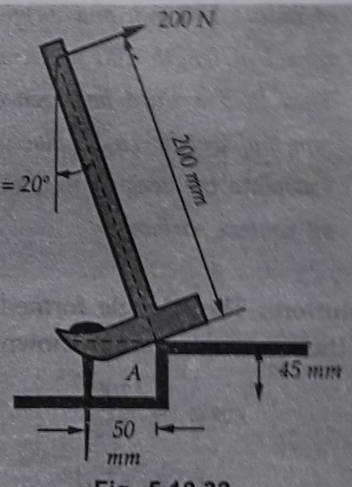


Fig. 5.10.32

Solution : The FBD of hammer is shown in Fig. 5.10.32 (a).

$$\sum M_A = 0 \ : \ -(200)(200) + (T)(50) = 0$$

$$\therefore \quad \boxed{T = 800 \text{ N}}$$

$$\sum F_x = 0 \ : \ 200 \cos 20 + A_x = 0$$

$$A_x = -187.94$$

$$A_x = 187.94 \text{ N} \leftarrow$$

$$\sum F_y = 0 \ : \ A_y - T + 200 \sin 20 = 0$$

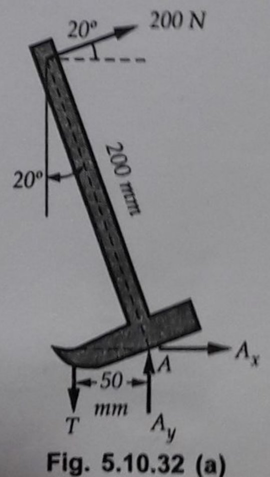


Fig. 5.10.32 (a)

$$A_y = 800 - 200 \sin 20$$

$$\therefore \quad A_y = 731.6 \text{ N} \uparrow$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{187.94^2 + 731.6^2}$$

$$\therefore \quad \boxed{R_A = 755.35 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) = \tan^{-1}\left(\frac{731.6}{187.94}\right)$$

$$\therefore \quad \boxed{\theta = 75.59^\circ}$$

Example 5.10.39 *Two cables tied together at C are loaded by a force P. If the maximum allowable tension in each cable is 800 N. Find a) The largest 'P' that may be applied at 'C', and b) The corresponding α. Refer Fig. 5.10.39.*

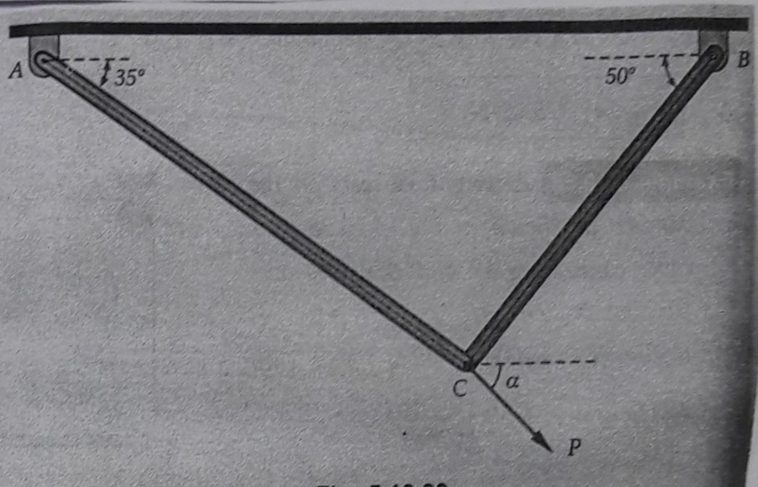


Fig. 5.10.39

Solution : FBD of point 'C' is shown in Fig. 5.10.39 (a).

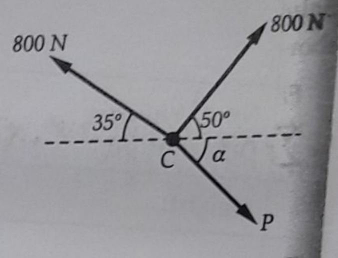


Fig. 5.10.39 (a)

Using Lami's theorem,

$$\frac{800}{\sin(50+\alpha)} = \frac{800}{\sin(180-\alpha+35)} = \frac{P}{\sin(180-50-35)}$$

$$\therefore \quad 50+\alpha = 180-\alpha+35$$

$$\therefore \quad 2\alpha = 165^\circ$$

$$\boxed{\alpha = 82.5^\circ}$$

$$\therefore \quad P = \frac{800 \sin 95}{\sin(50+82.5)}$$

$$\therefore \quad \boxed{P = 1080.94 \text{ N}}$$

Example 5.10.45 *Cylinder 'A' of diameter 200 mm and cylinder B of diameter 300 mm are placed in a trough shown in Fig. 5.10.45. If cylinder A weighs 800 N and cylinder B weighs 1200 N, determine the reactions developed at contact surfaces P, Q, R and S. Assume all contact surfaces are smooth.* **VTU : Aug.-11, Marks 10**

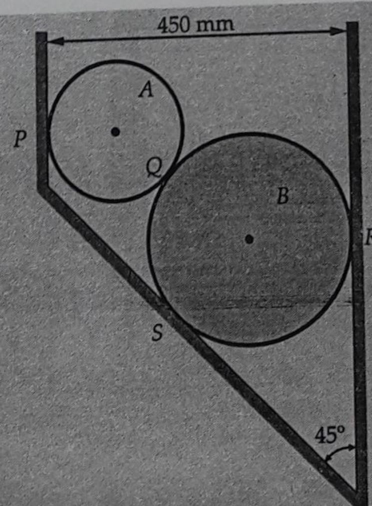


Fig. 5.10.45

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Solution : From Fig. 5.10.45 (a)

$$\cos\alpha = \frac{200}{250}$$

$\therefore$ $$\alpha = 36.87°$$

The free body diagrams of the two cylinders are shown in Fig. 5.10.45 (b).

Using Lami's theorem for A,

$$\frac{R_P}{\sin 126.87} = \frac{R_Q}{\sin 90} = \frac{800°}{\sin(180-36.87)}$$

$\therefore$ $$\boxed{R_P = 1066.66 \text{ N}}$$

$$\boxed{R_Q = 1333.33 \text{ N}}$$

For B,

$\sum F_y = 0:$

$$R_S \sin 45 - 1200 - R_Q \sin 36.87 = 0$$

$\therefore$ $$\boxed{R_S = 2828.43 \text{ N}}$$

$\sum F_x = 0:$

$$R_Q \cos 36.87 + R_S \cos 45 - R_R = 0$$

$\therefore$ $$\boxed{R_R = 3066.66 \text{ N}}$$

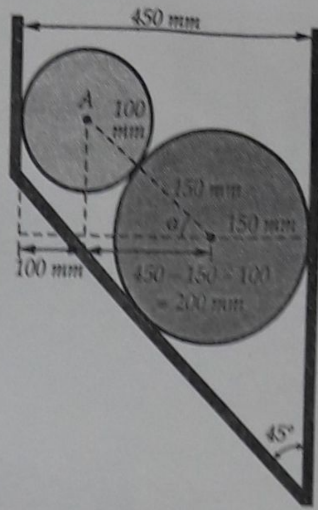


Fig. 5.10.45 (a)

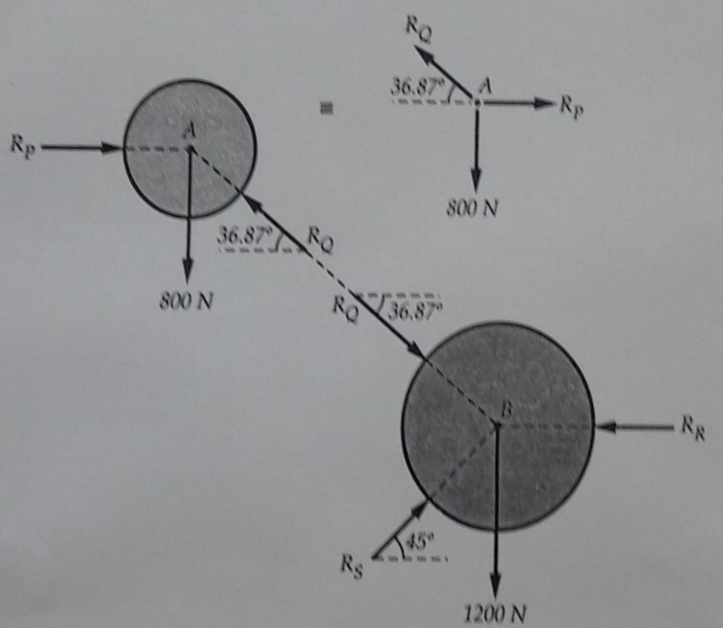


Fig. 5.10.45 (b)

Example 5.10.48 *Find the reaction at the contact surface for two identical cylinders weighing 1000 N each as shown in Fig. 5.10.48.*

VTU : Jan-13, Marks 10

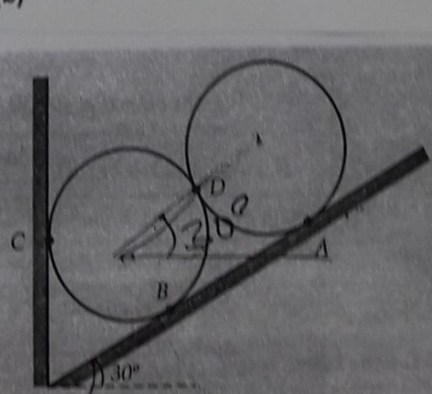


Fig. 5.10.48

Solution : The free body diagrams of the two cylinders are shown in Fig. 5.10.48 (a).

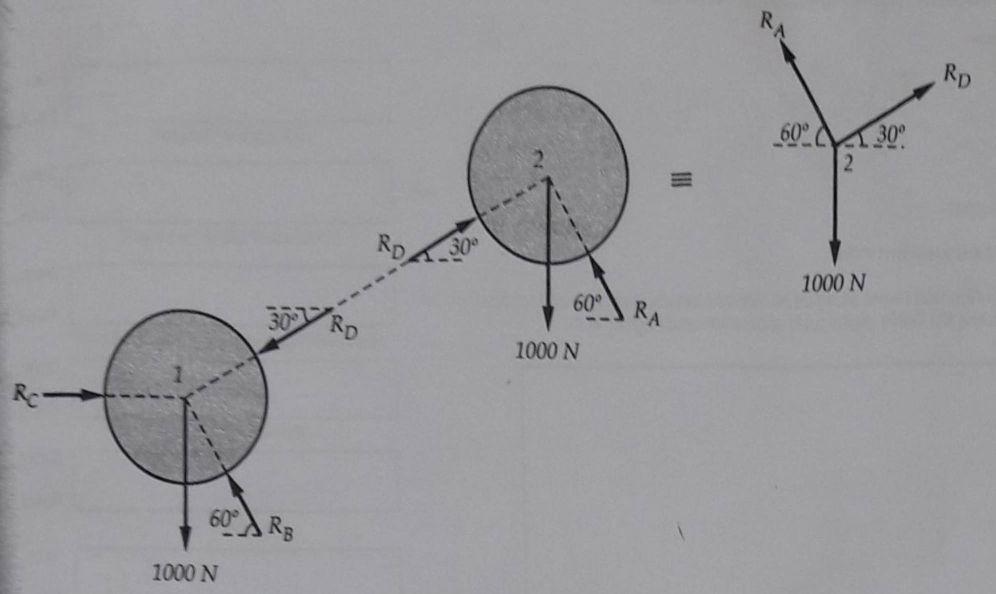


Fig. 5.10.48 (a)

Example 5.10.49 *Determine the reactions at the point of contact for the sphere shown in Fig. 5.10.49.*

VTU : June-13, Marks 6

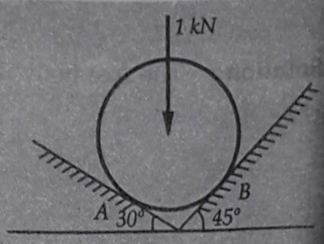
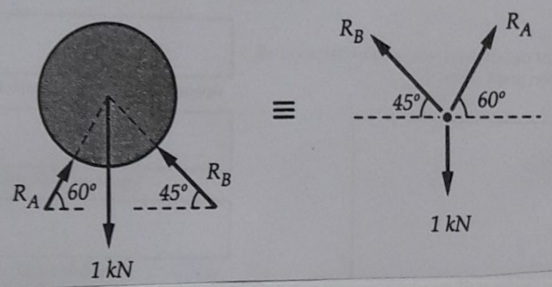


Fig. 5.10.49

Solution. : The F.B.D. of sphere is shown in Fig. 5.10.49 (a).



Using Lami's theorem,

$$\frac{R_A}{\sin 135} = \frac{R_B}{\sin 150} = \frac{1}{\sin 75}$$

$$\boxed{R_A = 0.732 \text{ kN}}$$

$$\boxed{R_B = 0.518 \text{ kN}}$$

Example 7.6.2 *Determine whether the 50 kg block shown in Fig. 7.6.3 is in equilibrium and find the magnitude and direction of the frictional force when a) $\theta = 10°$ and b) $\theta = 40°$. Take $\mu_s = 0.3$ and $\mu_k = 0.2$*

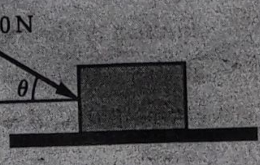
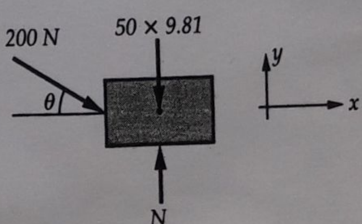


Fig. 7.6.3

Solution : The F.B.D. of block is shown in Fig. 7.6.3 (a) without frictional force.



$$\sum F_y = 0$$

$$N - 50 \times 9.81 - 200 \sin\theta = 0$$

$$N = 0.3\,(490.5 + 200 \sin\theta)$$

$$(F_r)_{max} = \mu_s N$$

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$$\therefore \quad (F_r)_{max} = 0.3\,(490.5 + 200 \sin\theta)$$

$$\sum F_x = 200 \cos\theta\ (\rightarrow)$$

a) For $\theta = 10°$,

$$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 10°) = 157.57 \text{ N}$$

$$\sum F_x = 200 \cos 10° = 196.96 \text{ N}(\rightarrow)$$

$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves towards right and

$$F_r = \mu_k N = 0.2 \times (490.5 + 200 \sin 10°)$$

$$\therefore \quad \boxed{F_r = 105.05 \text{ N}}$$

b) For $\theta = 40°$

$$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 40°) = 185.72 \text{ N}$$

$$\sum F_x = 200 \sin 40 = 153.21 \text{ N } (\rightarrow)$$

$$\sum F_x < (F_r)_{max} \quad \therefore \text{ Object doesn't move}$$

$$F_r = \sum F_x$$

$$\therefore \quad \boxed{F_r = 153.21 \text{ N} \leftarrow}$$

Example 7.6.3 *Determine whether the 50 kg block shown in Fig. 7.6.4 is in equilibrium and find the magnitude and direction of the frictional force when*
a) $P = 200$ N, $\theta = 20°$ b) $P = 200$ N, $\theta = 45°$
c) $P = 100$ N, $\theta = 20°$. $\mu_s = 0.3$ and $\mu_k = 0.2$

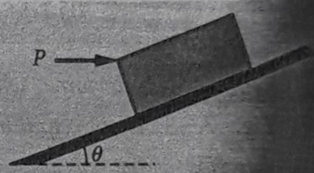
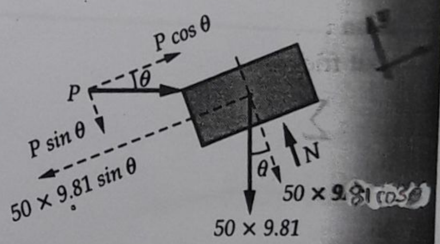


Fig. 7.6.4

Solution : The F.B.D. of block is shown in Fig. 7.6.4 (a) without frictional force



$$\sum F_y = 0$$

$$N - P \sin\theta - 50 \times 9.81 \cos\theta = 0$$

$$\therefore \quad N = P \sin\theta + 490.5 \cos\theta$$

$$(F_r)_{max} = \mu_s N$$

$$(F_r)_{max} = 0.3\,[P \sin\theta + 490.5 \cos\theta] \quad \text{... (1)}$$

$$\sum F_x = P \cos\theta - 50 \times 9.81 \sin\theta \quad \text{... (2)}$$

a) For $P = 200$ N and $\theta = 20°$,

$$(F_r)_{max} = 158.8 \text{ N and } \sum F_x = 20.18 \text{ N}$$

As $\sum F_x$ is positive, the net force is directed upward along the inclined plane.

$$\sum F_x < (F_r)_{max}$$

$\therefore$ Object does not move.

$$F_r = \sum F_x \text{ in opposite direction.}$$

$$\therefore \quad \boxed{F_r = 20.18 \text{ N}, \ \theta = 20°}$$

b) For $P = 200$ N, $\theta = 45°$

$$(F_r)_{max} = 146.48 \text{ N}, \sum F_x = -205.41 \text{ N}$$

$$\therefore \sum F_x = 205.41 \text{ N}, 45°$$

$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves down the incline

$$F_r = \mu_k N = 0.2\,(200 \sin 45 + 490.5 \cos 45)$$

$$\therefore \boxed{F_r = 97.65 \text{ N}, 45°}$$

c) For $P = 100$ N, $\theta = 20°$,

$$(F_r)_{max} = 148.54 \text{ N}, \sum F_x = -73.8$$

$$\therefore \sum F_x = 73.8 \text{ N}, 20°$$

$$\sum F_x < (F_r)_{max}$$

$\therefore$ Object does not move

$$F_r = \sum F_x \text{ in opposite direction}$$

$$\therefore \boxed{F_r = 73.8 \text{ N}, 20°}$$

Example 7.6.5 *For the block shown in Fig. 7.6.6, find the minimum value of 'P' which will just disturb the equilibrium of the system.*

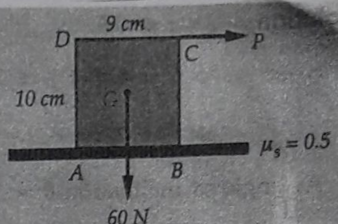


Fig. 7.6.6

Solution : The block will have sliding motion if $P > (F_r)_{max}$.

From F.B.D. of block shown in Fig. 7.6.6 (a) without frictional force,

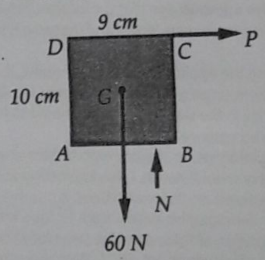


Fig. 7.6.6 (a)

$$\sum F_y = 0$$

$$N - 60 = 0$$

$$N = 60 \text{ N}$$

$$(F_r)_{max} = \mu_s N = 0.5 \times 60 = 30 \text{ N}$$

$\therefore$ Block will slide if

$$P > 30 \text{ N} \quad \text{... (1)}$$

Block will overturn if 'P' is greater than the value obtained from $\sum M_B = 0$ with N acting at B.

i.e. $-(P)(10) + (60)(4.5) = 0$

$$P = 27 \text{ N}$$

Overturning will take place if $P > 27$ N ... (2)

From equations (1) and (2), the smallest value of P for which equilibrium is disturbed is

$$\boxed{P = 27 \text{ N}}$$

(17)

Example 7.6.8 *Two blocks A and B having masses 50 kg and 100 kg respectively are connected by a string which passes over frictionless pulley as shown in Fig. 7.6.9. Coefficient of friction between block and surface is 0.2 for both A and B. Determine force P if,*

a) The system is prevented to move towards left.

b) The system is just on the point of moving towards right.

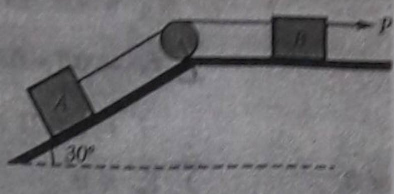


Fig. 7.6.9

Solution : a) For impending motion towards left, the free body diagram of A and B shown in Fig. 7.6.9 (a).

For A, $\sum F_y = 0$

$$-50 \times 9.81 \cos 30 = 0$$

$$\therefore \quad N_A = 424.785 \text{ N}$$

$$\sum F_x = 0$$

$$T + 0.2\, N_A - 50 \times 9.81 \sin 30 = 0$$

$$\therefore \quad T = 160.293 \text{ N}$$

For B, $\sum F_y = 0$

$$N_B - 100 \times 9.81 = 0$$

$$\therefore \quad N_B = 981 \text{ N}$$

$$\sum F_x = 0 \; : \; P + 0.2\, N_B - T = 0$$

$$\therefore \quad P = -35.27 \text{ N}$$

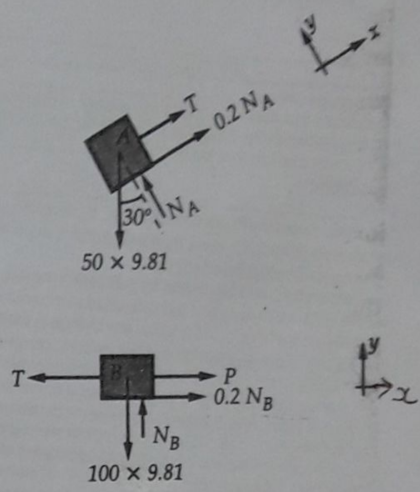


Fig. 7.6.9 (a)

Negative value of P indicates that the system will not be able to move towards left even if left to itself making P equal to zero.

$$\therefore \quad \boxed{P = 0}$$

b) For impending motion of the system towards right, the free body diagrams of A and B are shown in Fig. 7.6.9 (b).

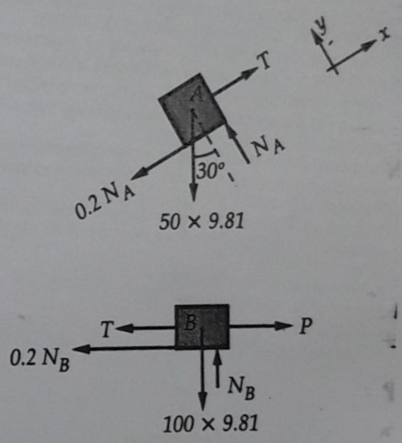


Fig. 7.6.9 (b)

For A, $\sum F_y = 0$

$$N_A - 50 \times 9.81 \cos 30 = 0$$

$$\therefore \quad N_A = 424.785 \text{ N}$$

$$\sum F_x = 0\; T - 0.2\, N_A - 50 \times 9.81 \sin 30 = 0$$

$$\therefore \quad T = 330.21 \text{ N}$$

For B, $\sum F_y = 0$

$$N_B - 100 \times 9.81 = 0$$

$$\therefore \quad N_B = 981 \text{ N}$$

$$\sum F_x = 0$$

$$P - T - 0.2\, N_B = 0$$

$$\therefore \quad \boxed{P = 526.41 \text{ N}}$$

Example 7.6.9 *The coefficients of friction are $\mu_s = 0.3$ and $\mu_k = 0.25$ between all surfaces of contact. Determine the smallest force P required to just start block D moving if*

a) Block C is restrained by cable AB as shown.

b) Cable AB is removed. Refer Fig. 7.6.10.

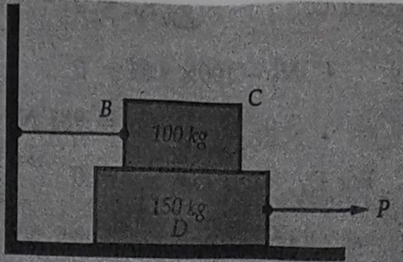


Fig. 7.6.10

Solution : As there is only impending motion (i.e. actual motion does not take place), we have to use μ_s.

a) F.B.D. of blocks are shown in Fig. 7.6.10 (a) when block C is restrained by cable AB.

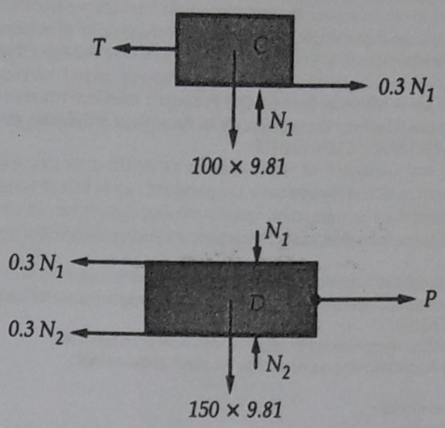


Fig. 7.6.10 (a)

For C, $\sum F_y = 0$

$$N_1 - 100 \times 9.81 = 0$$

$$\therefore \quad N_1 = 981 \text{ N}$$

$$\sum F_x = 0$$

$$-T + 0.3\, N_1 = 0$$

$$\therefore \quad T = 294.3 \text{ N}$$

For D, $\sum F_y = 0$

$$N_2 - N_1 - 150 \times 9.81 = 0$$

$$\therefore \quad N_2 = 2452.5 \text{ N}$$

$$\sum F_x = 0 \; : \; P - 0.3\, N_1 - 0.3\, N_2 = 0$$

$$\therefore \quad \boxed{P = 1030.05 \text{ N}}$$

b) The F.B.D. of blocks are shown in Fig. 7.6.10 (b) when block C is not restrained by cable AB. As there is no tendency for relative motion between C and D, there is no friction between them.

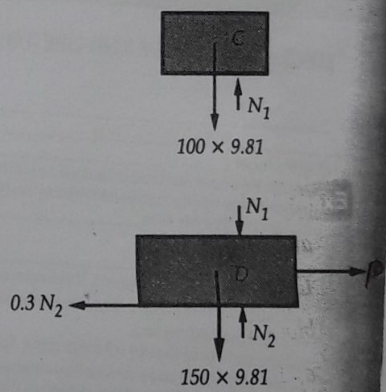


Fig. 7.6.10 (b)

For C, $\sum F_y = 0$

$$N_1 - 100 \times 9.81 = 0$$

$$\therefore \quad N_1 = 981 \text{ N}$$

For D, $\sum F_y = 0$

$$N_2 - 150 \times 9.81 - N_1 = 0$$

$$N_2 = 2452.5 \text{ N}$$

$$\sum F_x = 0$$

$$P - 0.3\, N_2 = 0$$

$$\therefore \quad \boxed{P = 735.75 \text{ N}}$$

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Example 7.9.2 *A block weighing 4000 N is resting on horizontal surface supports another block of 2000 N as shown in Fig. 7.9.2. Find the horizontal force F just to move the block to the left. Take coefficient of friction for all contact surfaces as 0.2.* **VTU : Aug.-03, Marks 15**

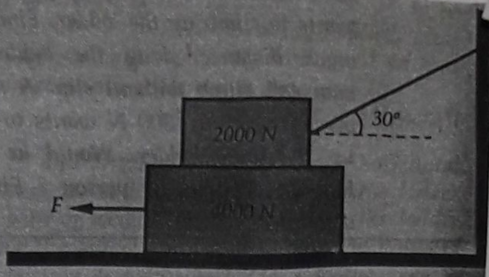


Fig. 7.9.2

Solution : The free body diagrams of the two blocks are shown in Fig. 7.9.2 (a).

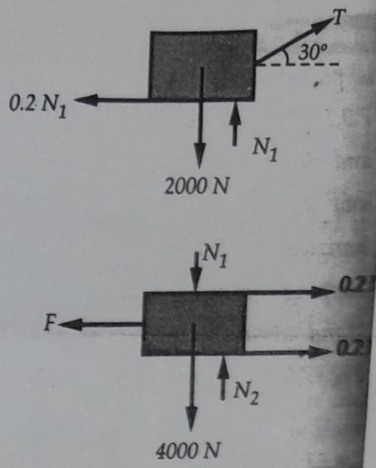


Fig. 7.9.2 (a)

For the 2000 N block,

$\sum F_x = 0:$

$$T \cos 30 - 0.2N_1 = 0$$

$$T = \frac{0.2N_1}{\cos 30} \quad \dots (1)$$

$\sum F_y = 0:$

$$T \sin 30 + N_1 - 2000 = 0$$

Substituting from equation (1),

$$\frac{0.2\,N_1}{\cos 30} \times \sin 30 + N_1 = 2000$$

$$\therefore \quad N_1 = 1792.97\ N$$

For the 4000 N block,

$\sum F_y = 0:$

$$N_2 - N_1 - 4000 = 0$$

$$\therefore \quad N_2 = 5792.97\ N$$

$\sum F_x = 0:$

$$0.2\,N_1 + 0.2\,N_2 - F = 0$$

$$\therefore \quad F = 0.2(N_1 + N_2)$$

$$\therefore \quad F = 0.2\,(1792.97 + 5792.97)$$

$$\therefore \quad \boxed{F = 1517.19\ N}$$

Example 7.9.3 *The coefficients of friction between 20 kg (mass) block and incline are $\mu_s = 0.4$ and $\mu_k = 0.3$. Determine whether the block is in equilibrium and find the magnitude and direction of friction force.* **VTU : Feb.-04, Marks 10**

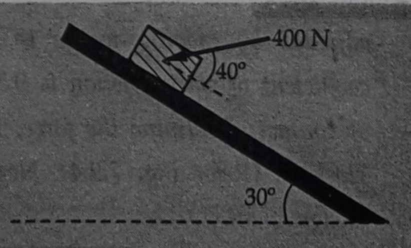


Fig. 7.9.3

Solution : The F.B.D. of block without frictional force is shown in Fig. 7.9.3 (a).

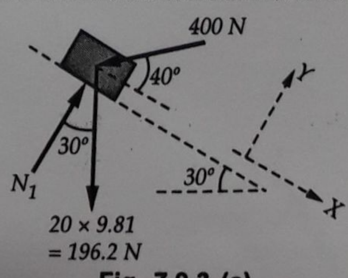


Fig. 7.9.3 (a)

As block cannot move perpendicular to the plane,

$\sum F_y = 0:$

$$N_1 - 400 \sin 40 - 196.2 \cos 30 = 0$$

$$\therefore \quad N_1 = 427.03\ N$$

The net force trying to move the block parallel to the plane is,

$$\sum F_x = -400 \cos 40 + 196.2 \sin 30$$

$$= -208.32 \text{ N}$$

$$\sum F_x = 208.32 \text{ N directed up the plane}$$

The maximum frictional force opposing the motion of the block is,

$$(F_r)_{max} = \mu_s N_1$$

$$= 0.4 \times 427.03$$

$$(F_r)_{max} = 170.81 \text{ N}$$

$$\sum F_x > (F_r)_{max}$$

∴ Block moves up the incline, i.e. the block is not in equilibrium.

The frictional force is

$$F_r = \mu_k N_1$$

$$= 0.3 \times 427.03 = 128 \cdot 11 N$$

The frictional force is directed downward along the inclined plane as the block is moving up the plane

Example 7.9.6 *A small block of weight 1000 N is placed on a 30° incline with a coefficienct of friction at 0.25 as shown in Fig. 7.9.6 . Determine the horizontal force to be applied for.*

i) The impending motion down the plane and ii) The impending motion up the plane.

VTU : March-05, Feb.-10, Marks 10

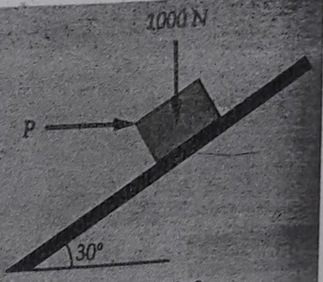


Fig. 7.9.6

Solution : The F.B.D. of block for the case of impending motion down the plane is shown in Fig. 7.9.6 (a).

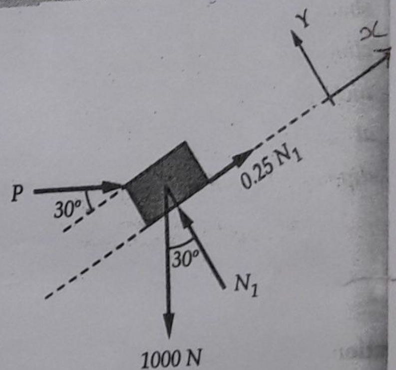


Fig. 7.9.6 (a)

$$\sum F_y = 0 :$$

$$N_1 - P \sin 30 - 1000 \cos 30 = 0$$

$$\therefore \quad N_1 = P \sin 30 + 1000 \cos 30$$

$$\sum F_x = 0 :$$

$$P \cos 30 - 1000 \sin 30 + 0.25\, N_1 = 0$$

$$\therefore$$

$$P \cos 30 - 1000 \sin 30 + 0.25\,(P \sin 30 + 1000 \cos 30) = 0$$

$$\cos 30 + 0.25 \sin 30] = 1000 \sin 30 - 0.25 \times 1000 \cos 30$$

$$\therefore \quad \boxed{P = 286.06 \text{ N}}$$

The F.B.D. of block for the case of impending motion up the plane is show Fig. 7.9.6 (b).

$$\sum F_y = 0 ;$$

$$N_1 - 1000 \cos 30 - P \sin 30 = 0$$

$\therefore \quad N_1 = P \sin 30 + 1000 \cos 30$

$\sum F_x = 0:$

$P \cos 30 - 1000 \sin 30 - 0.25\, N_1 = 0$

$\therefore P \cos 30 - 1000 \sin 30 - 0.25\,(P \sin 30 + 1000 \cos 30) = 0$

$\cos 30 - 0.25 \sin 30) = 1000 \sin 30 + 0.25 \times 1000 \cos 30$

$\therefore \quad \boxed{P = 966.91\ N}$

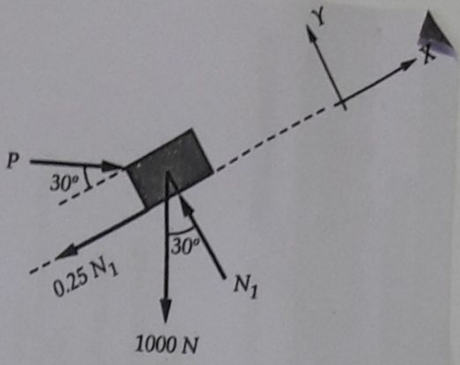


Fig. 7.9.6 (b)

Example 7.9.7 *Two blocks A and B weighing 2 kN and 1.5 kN are connected by a wire passing over a smooth frictionless pulley as shown in Fig. 7.9.7. Determine the magnitude of force P required to impend the motion, taking $\mu = 0.2$.*

VTU : Aug.-05, Marks 11

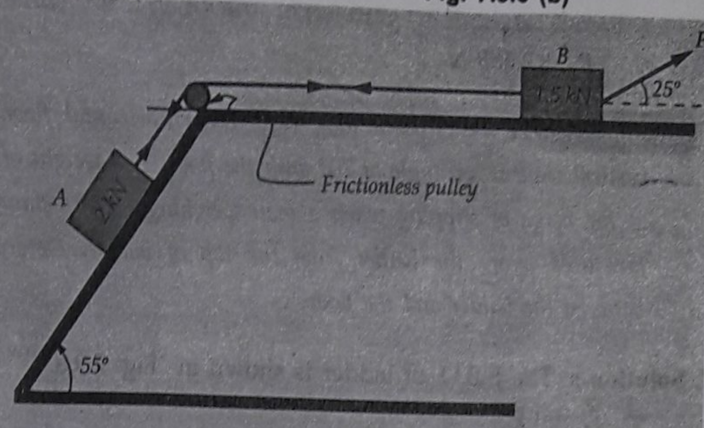


Fig. 7.9.7

Solution : The free body diagrams of the two blocks are shown in Fig. 7.9.7(a).

For F.B.D. of A,

$\sum F_y = 0:$

$N_A - 2 \cos 55 = 0$

$\therefore \quad N_A = 2 \cos 55$

$\sum F_x = 0:$

$T - 0.2\, N_A - 2 \sin 55 = 0$

$\therefore \quad T = 0.2 \times 2 \cos 55 + 2 \sin 55$

$T = 1.868\ kN$

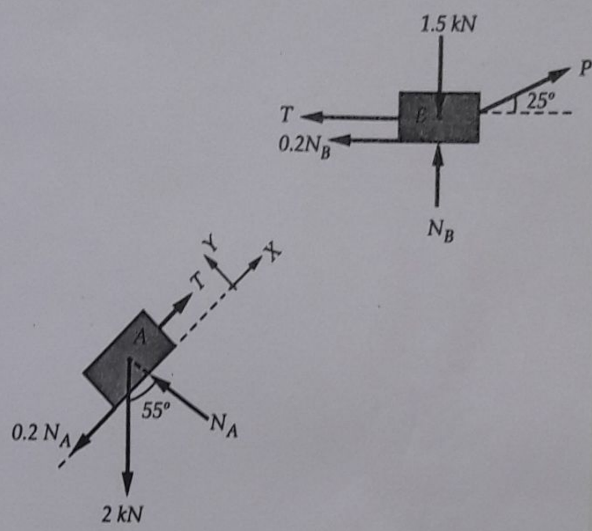


Fig. 7.9.7 (a)

For F.B.D. of B,

$\sum F_y = 0:$

$N_B - 1.5 + P \sin 25 = 0$

$N_B = 1.5 - P \sin 25$

$\sum F_x = 0:$

$P \cos 25 - T - 0.2\, N_B = 0$

$\therefore \quad P \cos 25 - 1.868 - 0.2\,(1.5 - P \sin 25) = 0$

$P\,(\cos 25 + 0.2 \sin 25) = 1.868 + 0.2 \times 1.5$

$\therefore \quad \boxed{P = 2.188\ N}$

Example 8.9 A block weighing 6 kN is attached to a string, which passes over a frictionless pulley and supports a weight of 3 kN, when the coefficient of friction between the block and the floor is 0.35 (Figure 8.26). Determine the value of force P when the

(i) motion is impending towards right.
(ii) motion is impending towards left.

Whenever a body is hanging free in air, there is no need to consider normal reaction.

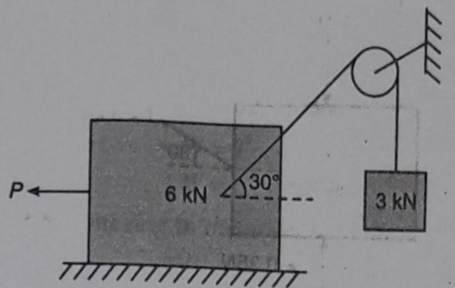
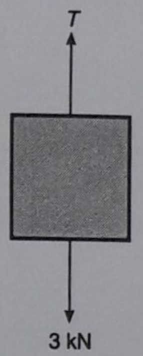


Figure 8.26 Example 8.9.

Solution Case 1: *When motion is impending towards right*

Consider the free body diagram of 3 kN block (Figure 8.27):



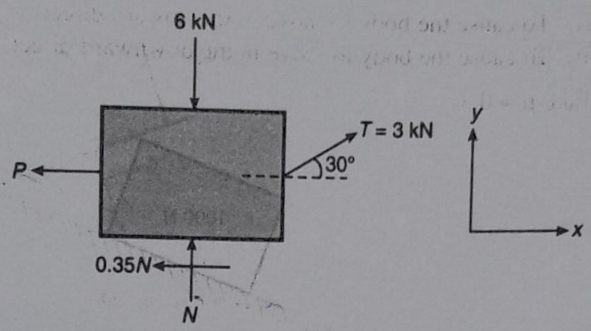


Figure 8.27 Example 8.9. **Figure 8.28** Example 8.9.

$$\Sigma F_y = 0$$

or $$3 - T = 0$$

or $$T = 3 \text{ kN}$$

Consider the free body diagram of 6 kN block (Figure 8.28):

$$\Sigma F_y = 0$$

$$3 \sin 30° - 6 + N = 0$$

or $$N = 4.5 \text{ kN}$$

Also, $$\Sigma F_x = 0$$

or $$-P + 3 \cos 30° - \mu N = 0$$

or $$P = 3 \cos 30° - 0.35 \times 4.5 = 1.023 \text{ kN}$$ **Ans.**

Case 2: *When motion is impending towards left* (Figure 8.29):

$$\Sigma F_y = 0$$

$$-3 \sin 30° - 6 + N = 0$$

or $$N = 7.5 \text{ kN}$$

Also, $$\Sigma F_x = 0$$

or $$P - 3 \cos 30° - \mu N = 0$$

or $$P = 5.223 \text{ kN}$$ **Ans.**

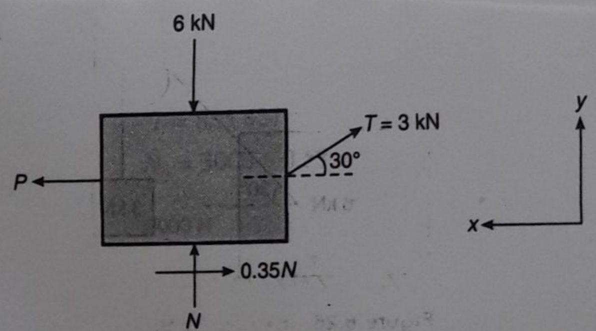


Figure 8.29 Example 8.9.

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Example 8.10 Calculate the value of W required in Figure 8.30:

(i) To cause the body to move in the upward direction.

(ii) To cause the body to move in the downward direction.

Take $\mu = 0.3$.

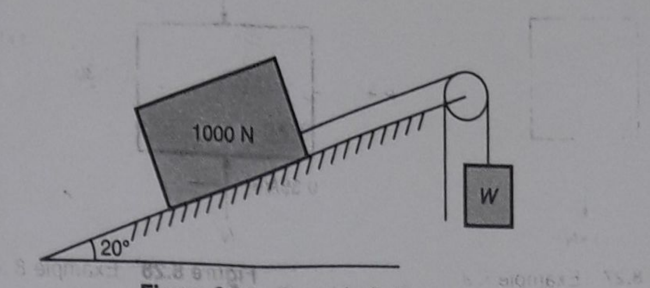


Figure 8.30 Example 8.10.

Solution Case 1: *When the 1000 N block is moving in the upward direction* (Figure 8.31):

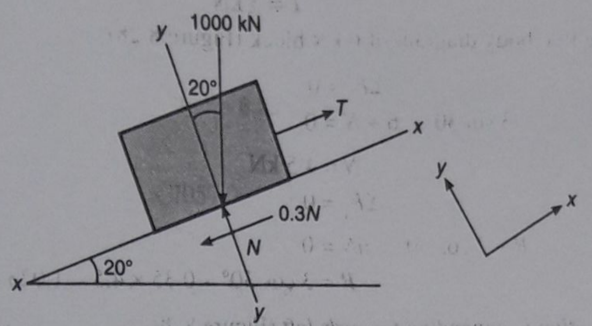


Figure 8.31 Example 8.10.

Consider the free body diagram of 1000 N block

$$\Sigma F_y = 0$$

or $$-1000 \cos 20° + N = 0$$

$$N = 939.693 \text{ N}$$

Also, $$\Sigma F_x = 0$$

or $$T - 0.3 \times 939.693 - 1000 \sin 20° = 0$$

or $$T = 623.928 \text{ N}$$

$$W = T$$

$$W = 623.928 \text{ N}$$ **Ans.**

Case 2: *When the 1000 N block is moving in the downward direction* (Figure 8.32):

$$\Sigma F_y = 0$$

or $$1000 \cos 20° = N$$

or $$N = 939.693 \text{ N}$$

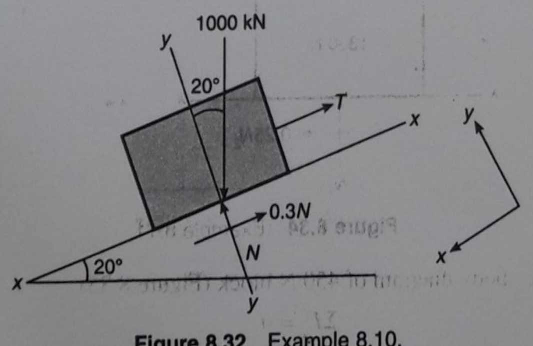


Figure 8.32 Example 8.10.

Also, $$\Sigma F_x = 0$$

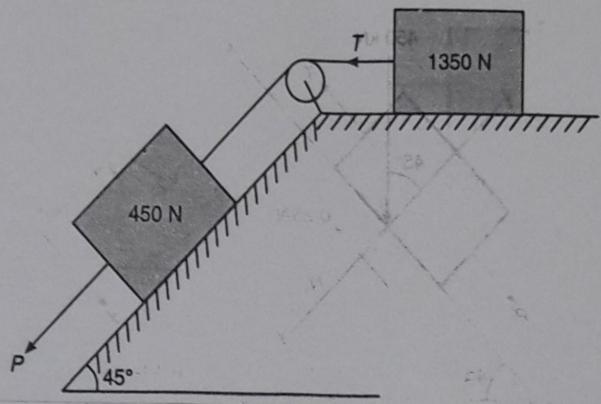
or $$T - \mu N + 1000 \sin 20° = 0$$

or $$T = 60.112 \text{ N}$$

$$W = T$$

$$W = 60.112 \text{ N}$$ **Ans.**

Example 8.11 Determine the necessary force P acting parallel to the plane, as shown in Figure 8.33, in order to cause motion to impend. Take $\mu = 0.25$. *VTU(July 2005)*



Solution Consider the free body diagram of 1350 N block (Figure 8.34):

$$\Sigma F_y = 0$$

or $$N_2 = 1350 \text{ N}$$

Also, $$\Sigma F_x = 0$$

or $$-\mu N_2 + T = 0$$

or $$T = 0.25 \times 1350 = 337.5 \text{ N}$$

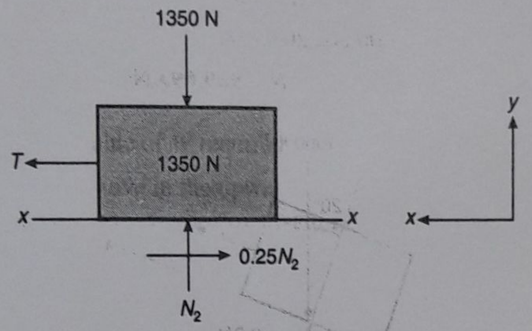


Figure 8.34 Example 8.11.

Consider the free body diagram of 450 N block (Figure 8.35):

$$\Sigma F_y = 0$$

or $$-450 \cos 45° + N_1 = 0$$

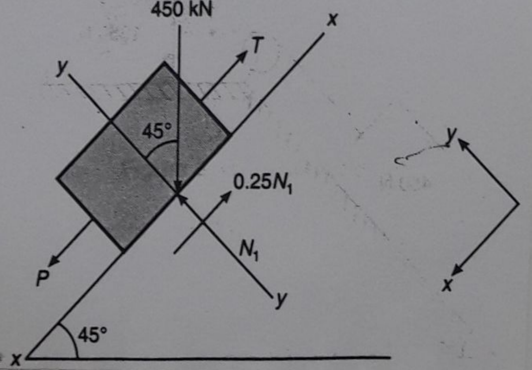
or $$N_1 = 450 \cos 45° = 318.198 \text{ N}$$

Also, $$\Sigma F_x = 0$$

or $$-\mu N_1 + P - T + 450 \times \sin 45°$$

or $$P = 0.25 \times 318.198 + 337.5 - 450 \sin 45° = 417.05 - 450 \sin 45°$$

or $$P = 98.852 \text{ N}$$



Example 7.9.9 *The crate shown in Fig. 7.9.9 has a mass of 580 kg. If P = 6000 N, find the magnitude and sense of the friction force which acts on the crate. What value of P will cause the crate to have impending motion up the plane? Find the minimum value of P required to keep the crate from sliding down the plane. For what range of values of P will the crate remain in the equilibrium position shown in Fig. 7.9.9?*

VTU : Aug.-06, Marks 12

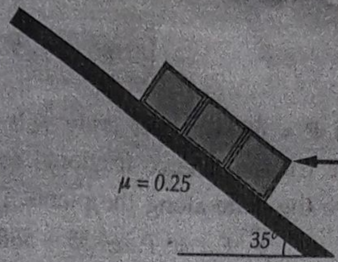


Fig. 7.9.9

Solution : The F.B.D. of crate for downward impending motion is shown in Fig. 7.9.9 (a).

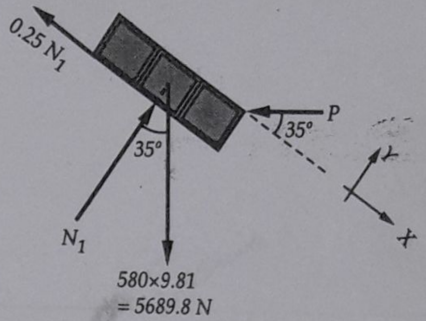


Fig. 7.9.9 (a)

$\sum F_y = 0$:

$$N_1 - 5689.8 \cos 35 - P \sin 35 = 0$$

$$N_1 = P \sin 35 + 5689.8 \cos 35$$

$\sum F_x = 0$:

$$-P \cos 35 + 5689.8 \sin 35 - 0.25\, N_1 = 0$$

$$-P \cos 35 + 5689.8 \sin 35 - 0.25\,(P \sin 35 + 5689.8 \cos 35) = 0$$

$$5689.8 \sin 35 - 0.25 \times 5689.8 \cos 35 = P\,(\cos 35 + 0.25 \sin 35)$$

$\therefore$ $\boxed{P = 2180 \text{ N}}$

For impending motion up the plane, the F.B.D. is shown in Fig. 7.9.9 (b).

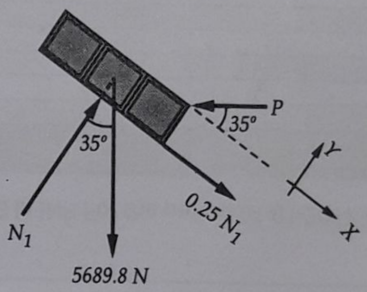


Fig. 7.9.9 (b)

$\sum F_y = 0$:

$$N_1 - 5689.8 \cos 35 - P \sin 35 = 0$$

$$N_1 = P \sin 35 + 5689.8 \cos 35$$

$\sum F_x = 0$:

$$-P \cos 35 + 5689.8 \sin 35 + 0.25\, N_1 = 0$$

$$-P \cos 35 + 5689.8 \sin 35 + 0.25\,(P \sin 35 + 5689.8 \cos 35) = 0$$

$$5689.8 \sin 35 + 0.25 \times 5689.8 \cos 35 = P(\cos 35 - 0.25 \sin 35)$$

$\therefore$ $\boxed{P = 6553.73 \text{ N}}$

Example 7.9.13 *Determine the force P required to cause motion of blocks to impend. Take the weight of A as 90 N and weight of B as 45 N. Take the coefficient of friction for all contact surfaces as 0.25 as shown in Fig. 7.9.13. Consider the pulley being frictionless.*

VTU : Feb.-08, Marks 8

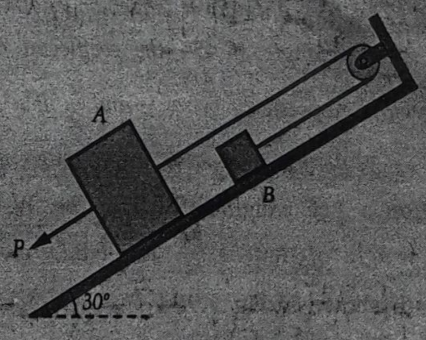


Fig. 7.9.13

Solution : As block *A* tends to move down the incline, *B* tends to move up the incline. The free body diagrams of the two blocks are shown in Fig. 7.9.13 (a).

For F.B.D. of B,

$\sum F_y = 0:$

$$N_B - 45 \cos 30 = 0$$

$\therefore N_B = 45 \cos 30$

$\sum F_x = 0:$

$$T - 45 \sin 30 - 0.25 N_B = 0$$

$\therefore T = 32.243$ N

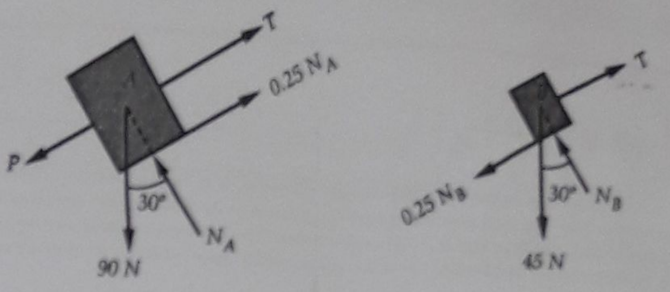


Fig. 7.9.13 (a)

For F.B.D. of A,

$\sum F_y = 0:$

$$N_A - 90 \cos 30 = 0$$

$\therefore N_A = 90 \cos 30$

$\sum F_x = 0:$

$$T - P + 0.25 N_A - 90 \sin 30 = 0$$

$\therefore P = T + 0.25 N_A - 90 \sin 30 = 32.243 + 0.25 \times 90 \cos 30 - 90 \sin 30$

$\therefore \boxed{P = 6.73 \text{ N}}$

Example 7.9.27 *Knowing that $W_A = 100$ N and $\theta = 30°$, determine the smallest and largest value of W_B for which the system is in equilibrium. Refer Fig. 7.9.27.*

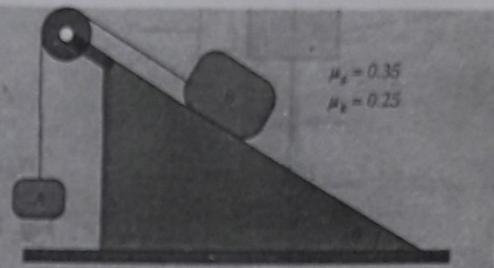


Fig. 7.9.27

Solution : For smallest value of W_A, A tends to move down and B tends to move upward along the incline. The F.B.D.s are shown in Fig. 7.9.27 (a).

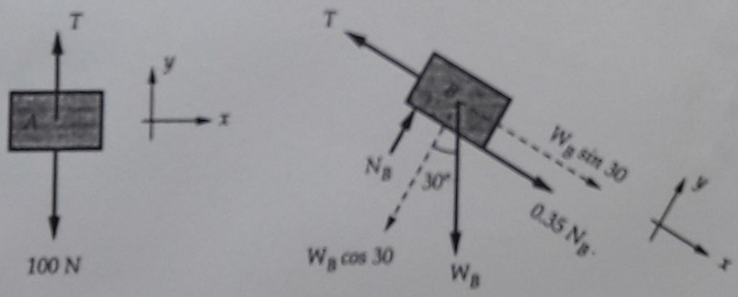


Fig. 7.9.27 (a)

From F.B.D. of A; $\sum F_y = 0;$

$$T - 100 = 0$$

$\therefore \quad T = 100$ N

From F.B.D. of B, $\sum F_y = 0;$

$$N_B - W_B \cos 30 = 0$$

$\therefore \quad N_B = W_B \cos 30$

$$\sum F_x = 0$$

$$W_B \sin 30 + 0.35 N_B - T = 0$$

$\therefore W_B \sin 30 + 0.35 \times W_B \cos 30 - 100 = 0$

$\therefore \quad \boxed{W_B = 124.52 \text{ N}}$

For largest value of W_B, A tends to move up and B tends to move down along the incline. The F.B.D.s are shown in Fig. 7.9.27 (b).

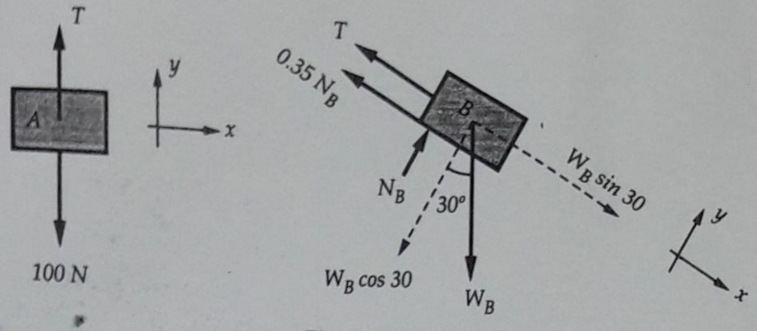


Fig. 7.9.27 (b)

From F.B.D. of A, $\sum F_y = 0$

$$T - 100 = 0$$

$$\therefore \quad T = 100 \text{ N}$$

From F.B.D. of B, $\sum F_y = 0$;

$$N_B - W_B \cos 30 = 0$$

$$\therefore \quad N_B = W_B \cos 30$$

$\sum F_x = 0$:

$$W_B \sin 30 - 0.35 N_B - T = 0$$

$$W_B \sin 30 - 0.35 \times W_B \cos 30 - 100 = 0$$

$$\therefore \quad \boxed{W_B = 507.9 \text{ N}}$$

Example 7.9.29 *Determine the value of "θ" for impending motion of the blocks. Take coefficient of friction (μ) for all contact surfaces as 0.25. [Refer Fig 7.9.29]*

VTU : Feb.-11, Marks 6

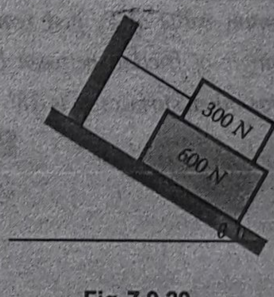


Fig 7.9.29

Solution : The free body diagrams of the two blocks are shown in Fig. 7.9.29 (a).

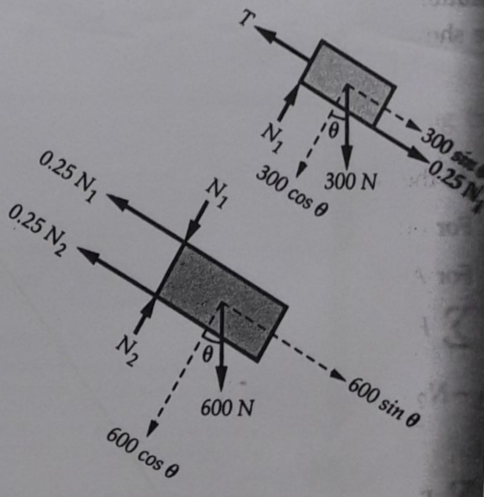


Fig. 7.9.29 (a)

For 300 N block,

$\sum F_y = 0$

$$N_1 - 300 \cos\theta = 0$$

$$\therefore \quad N_1 = 300 \cos\theta$$

For 600 N block,

$\sum F_y = 0$

$$N_2 - N_1 - 600 \cos\theta = 0$$

$$\therefore \quad N_2 = 900 \cos\theta$$

$\sum F_x = 0$

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$$600 \sin \theta - 0.25 N_1 - 0.25 N_2 = 0$$

$$600 \sin \theta - 0.25 \times 300 \cos \theta - 0.25 \times 900 \cos \theta = 0$$

$$600 \sin \theta = 300 \cos \theta$$

$$\tan \theta = \frac{1}{2}$$

$$\therefore \quad \boxed{\theta = 26.565^\circ}$$

9.10.3 Problems on Wedge Friction

A *wedge* is a piece of wood or metal which is usually triangular or trapezoidal in cross section. It is used for lifting heavy loads or used for slight adjustments in the position of body.

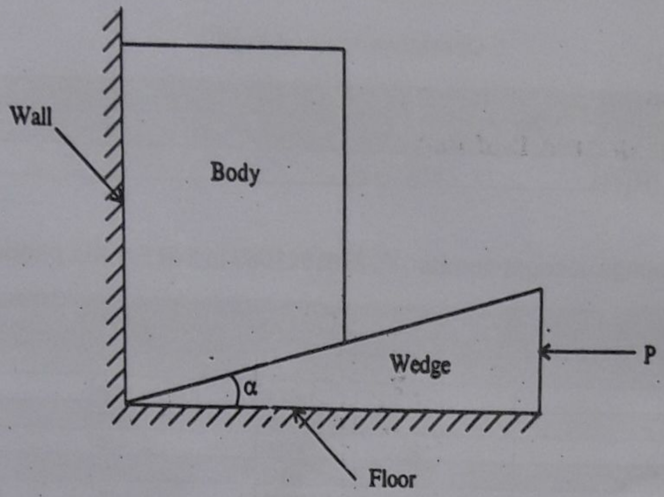
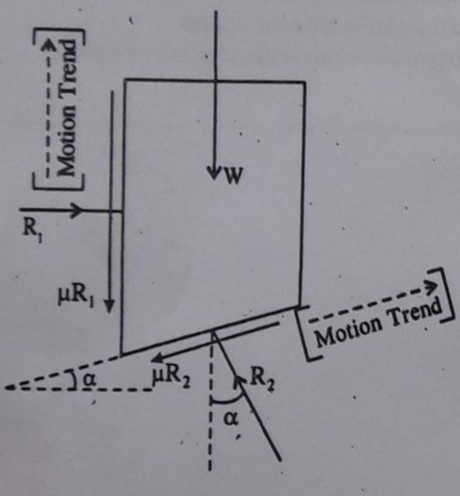
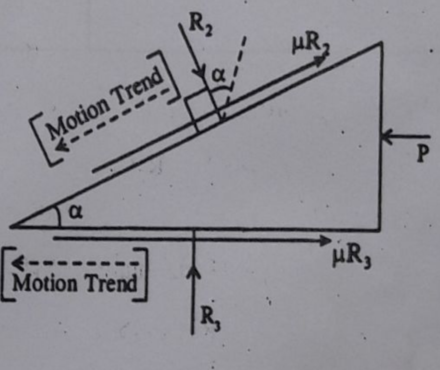


Figure shows a body being lifted by force P applied to the wedge. Let us consider F.B.D of Body and Wedge separately as below.



FBD of Body



FBD of Wedge

Example 7.7.1 *A block 'A' weighing 80 kN is to be moved towards left by light wedge B. Find necessary force 'P', if angle of friction at all rubbing surfaces is 15°. Refer Fig. 7.7.2.*

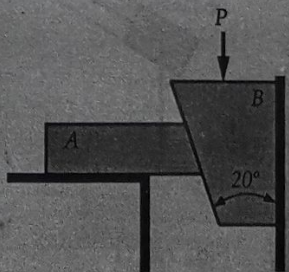


Fig. 7.7.2

Solution : We have $\mu = \tan\phi$ where ϕ = Angle of friction

$$\therefore \quad \mu = \tan 15° = 0.268$$

The free body diagrams of 'A' and 'B' are shown in Fig. 7.7.2 (a).

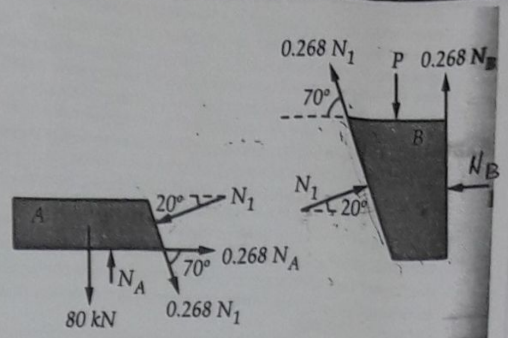


Fig. 7.7.2 (a)

For A, $\sum F_x = 0$

$$-N_1 \cos 20 + 0.268\, N_1 \cos 70 + 0.268\, N_A = 0 \quad \text{... (1)}$$

$$\sum F_y = 0$$

$$-N_1 \sin 20 - 0.268\, N_1 \sin 70 + N_A - 80 = 0$$

From equations (1) and (2),

$$N_1 = 31.123 \text{ kN}$$

For B, $\sum F_x = 0$

$$-0.268\, N_1 \cos 70 + N_1 \cos 20 - N_B = 0$$

$$\therefore \quad N_B = 26.393 \text{ kN}$$

$$\sum F_y = 0 : 0.268\, N_1 \sin 70 + N_1 \sin 20 + 0.268\, N_B - P = 0$$

$$\therefore \quad \boxed{P = 25.56 \text{ kN}}$$

7.4 *A block of 100 N is to be raised up by means of force P each acting on wedges as shown in Fig. 7.7.5. If angle of friction at all rubbing surfaces is 15°, determine P. Ignore weight of wedge.*

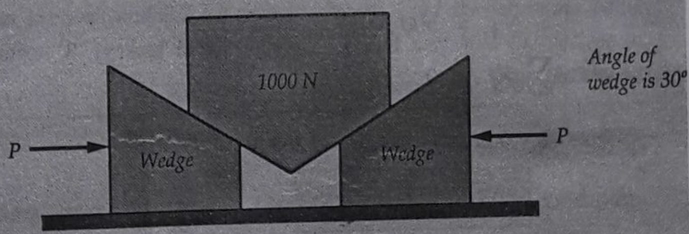


Fig. 7.7.5

Solution : $\mu_s = \tan\phi_s = \tan 15$

$$\therefore \quad \mu_s = 0.268$$

The three free body diagrams are shown in Fig. 7.7.5 (a)

For F.B.D. of 1000 N block 'A',

$$\sum F_x = 0 : N_1 \cos 60 + 0.268\, N_1 \cos 30 - N_2 \cos 60 - 0.268\, N_2 \cos 30 = 0$$

$$\therefore \quad N_1 = N_2$$

$$\sum F_y = 0 : N_1 \sin 60 + N_2 \sin 60 - 0.268\, N_1 \sin 30 - 0.268\, N_2 \sin 30 - 1000 = 0$$

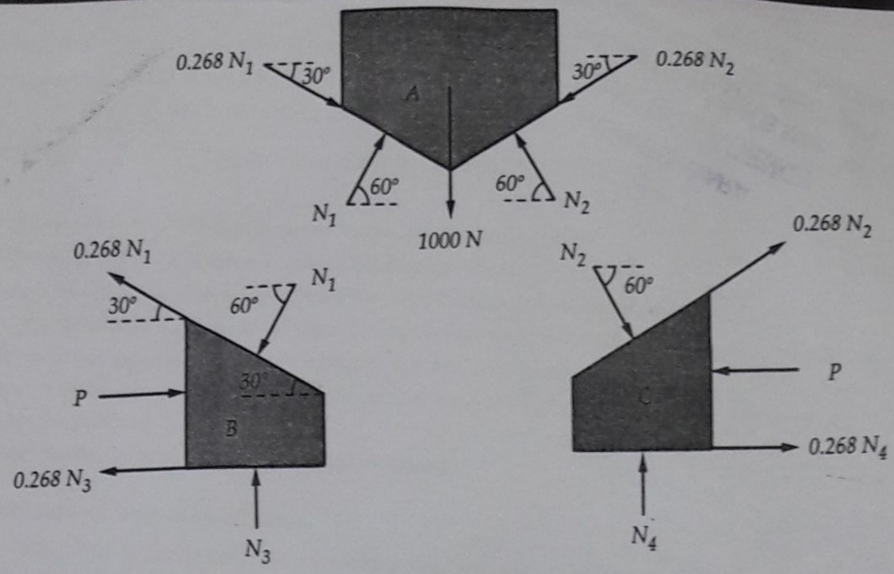


Fig. 7.7.5 (a)

$$\therefore \quad N_1 = N_2 = 683.036 \text{ N}$$

For B, $\sum F_y = 0$

$$N_3 - N_1 \sin 60 + 0.268\, N_1 \sin 30 = 0$$

$$\therefore \quad N_3 = 500 \text{ N}$$

$$\sum F_x = 0$$

$$-N_1 \cos 60 - 0.268\, N_1 \cos 30 - 0.268\, N_3 + P = 0$$

$$\therefore \quad \boxed{P = 634.047 \text{ N}}$$

Example 7.9.4 The position of the machine block is B is adjusted by moving wedge is A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force, P required to raise the block B. (Refer Fig. 7.9.4). Neglect the weight of the wedge.

VTU : Feb.-04, Dec.-11, Marks 10

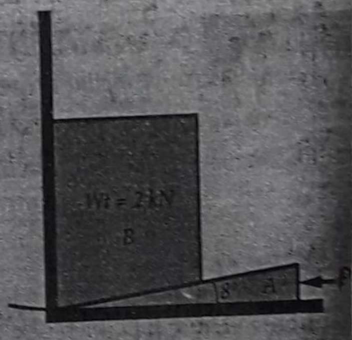


Fig. 7.9.4

Solution : The free body diagrams of the two blocks are shown in Fig. 7.9.4(a).

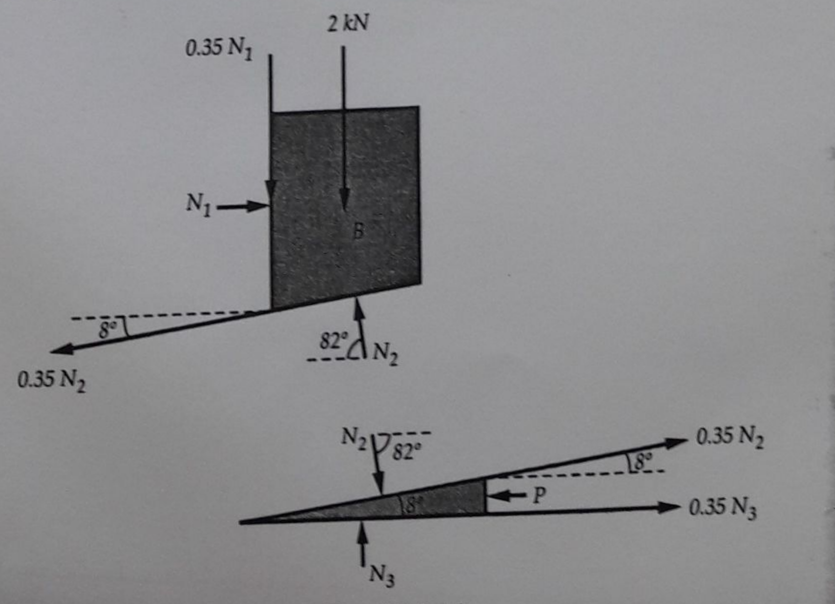


Fig. 7.9.4 (a)

For the F.B.D. of B,

$\sum F_x = 0$:

$$N_1 - N_2 \cos 82 - 0.35\, N_2 \cos 8 = 0 \quad \text{— (1)}$$

$\sum F_y = 0$:

$$-0.35\, N_1 + N_2 \sin 82 - 0.35\, N_2 \sin 8 - 2 = 0 \quad \text{... (2)}$$

From equation (1) and equation (2),

$$N_2 = 2.592 \text{ kN}$$

For F.B.D. of wedge,

$\sum F_y = 0$:

$$N_3 - N_2 \sin 82 + 0.35\, N_2 \sin 8 = 0$$

$$\therefore \quad N_3 = 2.44 \text{ kN}$$

$\sum F_x = 0$:

$$N_2 \cos 82 + 0.35\, N_2 \cos 8 - P + 0.35\, N_3 = 0$$

$$\therefore \quad \boxed{P = 2.113 \text{ kN} \leftarrow}$$

Example 7.9.28 *In the Fig. 7.9.28, determine the minimum value of P, just required to lift 3000 N up. The angle of friction between block and the wall is 15° and for other surfaces it is 18°.*

VTU : Aug.-10, Marks 12

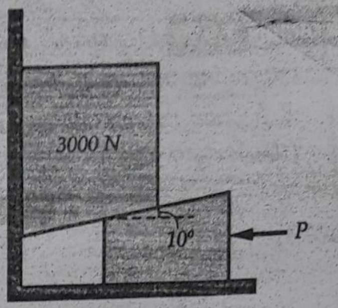


Fig. 7.9.28

Solution : The two free body diagrams are shown in Fig. 7.9.28 (a)

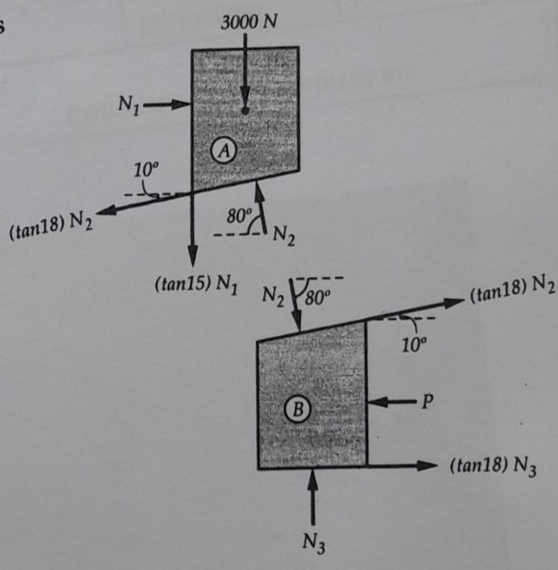


Fig. 7.9.28 (a)

At the wall, $\mu = \tan 15$

For other surfaces, $\mu = \tan 18$

For A,

$\sum F_x = 0$

$$N_1 - N_2 \cos 80 - N_2 \tan 18 \cos 10 = 0 \quad \text{... (1)}$$

$\sum F_y = 0$

$$-N_1 \tan 15 + N_2 \sin 80 - N_2 \tan 18 \sin 10 - 3000 = 0 \quad \text{... (2)}$$

From equation (1) and (2),

$$N_2 = 3768.3 \text{ N}$$

For B :

$\sum F_y = 0$

$$-N_2 \sin 80 + N_2 \tan 18 \sin 10 + N_3 = 0$$

$$\therefore \quad N_3 = 3498.44 \text{ N}$$

$$\sum F_x = 0$$

$$N_2 \cos 80 + N_2 \tan 18 \cos 10 + N_3 \tan 18 - P = 0$$

$\therefore$ $\boxed{P = 2996.86 \text{ N}}$

9.10.4 Problems on Ladder Friction

A ladder is a device used for climbing up or down. It has two contact points. One contact point with floor and another contact point with wall, which gives reactions. If the floor and wall are rough then friction will come into play. The forces which keep the ladder in equilibrium are shown in figure below Figure .11.

The conditions of equilibrium used to solve ladder problems are :

$$\Sigma V = 0, \ \Sigma H = 0 \text{ also } \Sigma M_A = 0$$

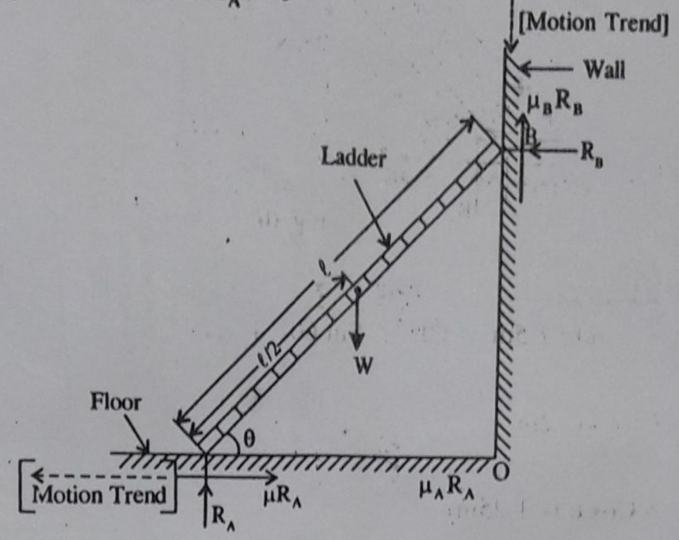


Fig. .11

Example 7.8.1 *A ladder AB weighing 196 N is resting against a rough wall and a rough floor, as shown in Fig. 7.8.1. Calculate the minimum horizontal force 'P' required to be applied at C in order to push the ladder towards the wall. Assume coefficient of friction at A = 0.3 and that B = 0.2.*

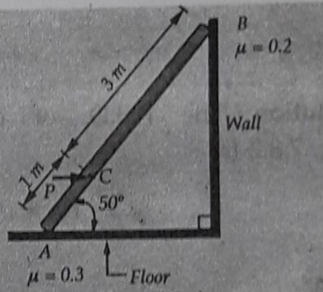


Fig. 7.8.1

Solution : The F.B.D. of ladder is shown in Fig. 7.8.1 (a).

There are three unknowns - P, N_A and N_B which can be obtained using the three equations $\sum F_x = 0$; $\sum F_y = 0$ and $\sum M = 0$

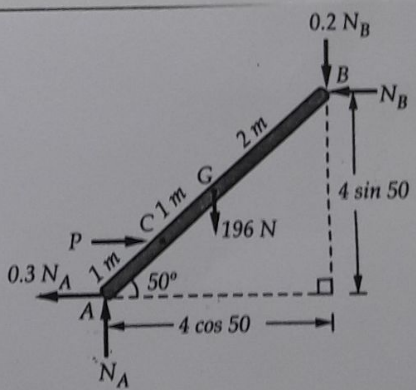


Fig. 7.8.1 (a)

$$\sum F_x = 0$$

$$-0.3\,N_A - N_B + P = 0 \qquad \text{... (1)}$$

$$\sum F_y = 0$$

$$N_A - 0.2\,N_B - 196 = 0 \qquad \text{... (2)}$$

$$\sum M_A = 0$$

$$(N_B)(4 \sin 50) - (0.2\,N_B)(4 \cos 50) - (P)(1 \sin 50) - (196)(2 \cos 50) = 0 \qquad \text{... (3)}$$

From equations (1), (2) and (3),

$\therefore$ $\boxed{P = 239.96 \text{ N}}$

Example 7.8.2 *A uniform ladder of length 15 m rests against a vertical wall making an angle of 60° with the horizontal. Coefficient of friction between wall and the ladder is 0.30 and between the ground and the ladder is 0.25. A man weighing 500 N ascends the ladder. How long will he be able to go before the ladder slips ? Find the weight that is necessary to be put at the bottom of the ladder so as to be just sufficient to permit the man to go to the top. Assume weight of the ladder to be 850 N.* **VTU : June-13**

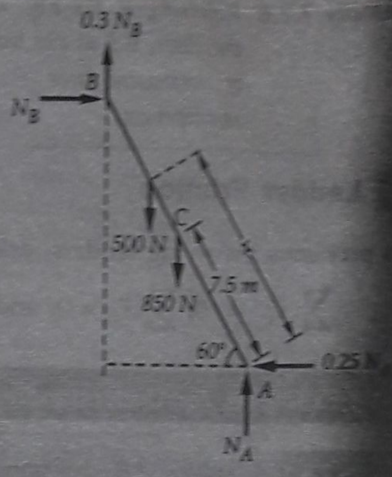


Fig. 7.8.2

Solution : The F.B.D. of ladder is shown in Fig. 7.8.2 (a).

$$\sum F_x = 0$$

$$N_B - 0.25\, N_A = 0$$

$$\therefore \quad N_A = 4\, N_B$$

$$\sum F_y = 0$$

$$N_A + 0.3 N_B - 850 - 500 = 0$$

$$4\, N_B + 0.3\, N_B = 1350$$

$$\therefore \quad N_B = 313.95 \text{ N}$$

$$\sum M_A = 0$$

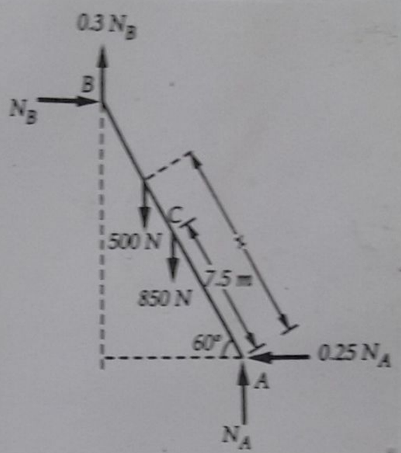


Fig. 7.8.2 (a)

$$(850)\,(7.5 \cos 60) + (500)\,(x \cos 60) - (N_B)\,(15 \sin 60) - (0.3\, N_B)\,(15 \cos 60) = 0$$

Substituting for N_B, we get

$$\boxed{x = 6.39 \text{ m}}$$

The F.B.D. of ladder when weight W is put at the bottom is shown in Fig. 7.8.2 (b).

$$\sum M_A = 0$$

$$(850)\,(7.5 \cos 60) + (500)\,(15 \cos 60)$$
$$- (N_B)(15 \sin 60) - (0.3\, N_B)\,(15 \cos 60) = 0$$

$$\therefore \quad N_B = 455.205 \text{ N}$$

$$\sum F_x = 0$$

$$N_B - 0.25\, N_A = 0$$

$$\therefore \quad N_A = 1820.82 \text{ N}$$

$$\sum F_y = 0$$

$$N_A + 0.3\, N_B - 500 - 850 - W = 0$$

$$\boxed{W = 607.38 \text{ N}}$$

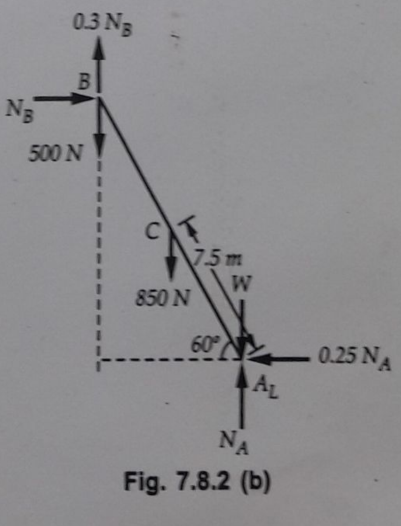


Fig. 7.8.2 (b)

Example 7.9.1 *A ladder of length 4 m, weighing 200 N is placed against a vertical wall as shown in Fig. 7.9.1. The coefficient of friction between the wall and the ladder is 0.25 and that between the ladder and the floor is 0.3. Determine the minimum horizontal force to be applied at A to prevent slipping when a man weighing 600 N wants to stand at a distance 3 m from A shown in the Fig. 7.9.1.* **VTU : Feb.-03, Marks 14**

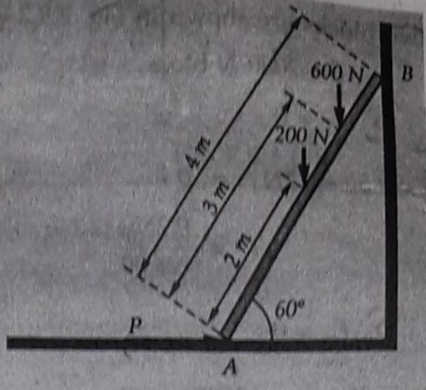


Fig. 7.9.1

Solution : The F.B.D. of ladder is shown in Fig. 7.9.1 (a).

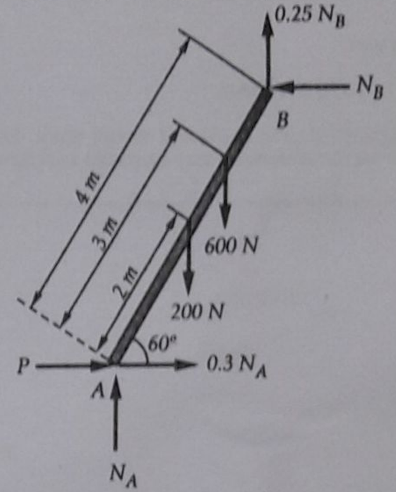


Fig. 7.9.1 (a)

$\sum M_A = 0$:

$$-(200)(2\cos 60) - (600)(3\cos 60) + (N_B)(4\sin 60) + (0.25\,N_B)(4\cos 60) = 0$$

$$N_B = 277.49 \text{ N}$$

$\sum F_y = 0$:

$$N_A - 200 - 600 + 0.25\,N_B = 0$$

$$N_A = 730.63 \text{ N}$$

$\sum F_x = 0$:

$$P + 0.3\,N_A - N_B = 0$$

$$P = 277.49 - 0.3 \times 730.63 = 58.3\text{N}$$

Example 7.9.5 *A ladder of 4 m weighing 200 N is supported by a horizontal floor and vertical wall shown in Fig. 7.9.5. If a man of weight 650 N climbs to the top of the ladder, determine the inclination of the ladder with reference to the floor at which the ladder is to be placed to prevent slipping. Take $\mu = 0.25$ for all contact surfaces.* **VTU : Aug.-04, Dec.-11, Jan.-13, Marks 14**

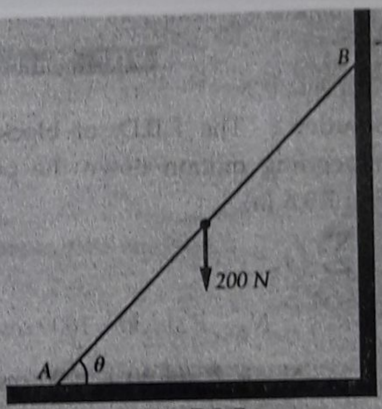
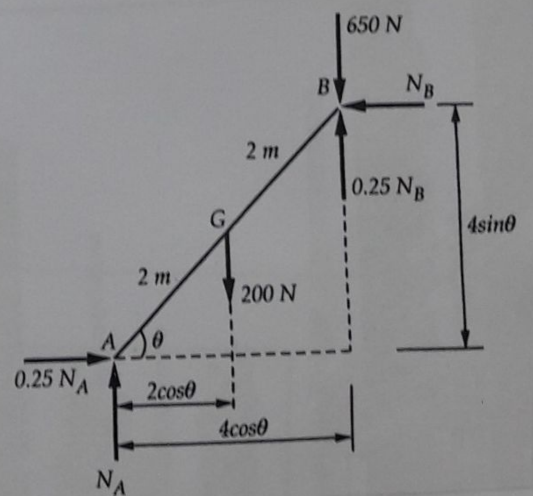


Fig. 7.9.5

Solution : The F.B.D. the ladder is shown in Fig. 7.9.5 (a).



$\sum F_x = 0$:

$$0.25\,N_A - N_B = 0 \quad \dots (1)$$

$\sum F_y = 0$:

$$N_A + 0.25\,N_B - 200 - 650 = 0$$

$$\therefore N_A + 0.25\,N_B = 850 \quad \dots (2)$$

From equation (1) and equation (2),

$$N_B = 200 \text{ N}$$

$\sum M_A = 0$:

$$-(200)(2\cos\theta)-(650)(4\cos\theta)+(N_B)(4\sin\theta)+(0.25N_B)(4\cos\theta) = 0$$

$$-400\cos\theta - 2600\cos\theta + 200\times 4\sin\theta + 200\cos\theta = 0$$

$$800\sin\theta = 2800\cos\theta$$

$$\tan\theta = \frac{2800}{800}$$

$$\therefore \quad \boxed{\theta = 74.05^\circ}$$

Example 7.9.12 *A ladder 5 m in length is resting against a smooth vertical wall and a rou[gh] horizontal floor. The ladder makes an angle of 60° with the horizontal. When a man [of] weight 800 N is at the top of the rung, what is the coefficient of friction required at [the] bottom of the ladder and the floor such that the ladder does not slip? Take the weight [of] ladder as 200 N.*

VTU : Feb.-08, Marks 5

Solution : The F.B.D. of ladder is shown in Fig. 7.9.12 .

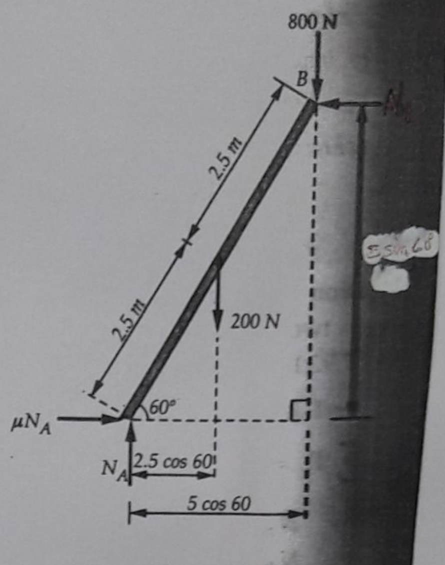


Fig. 7.9.12

$\sum M_A = 0:$

$$-(200)(2.5\cos 60)-(800)(5\cos 60) + (N_B)(5\sin 60) = 0$$

$$\therefore \quad N_B = 519.615 \text{ N}$$

$\sum F_y = 0:$

$$N_A - 200 - 800 = 0$$

$$\therefore \quad N_A = 1000 \text{ N}$$

$\sum F_x = 0:$

$$\mu N_A - N_B = 0$$

$$\therefore \quad \mu = \frac{N_B}{N_A}$$

$$= \frac{519.615}{1000} = 0.519615$$

$$\therefore \quad \boxed{\mu \cong 0.52}$$

Example 7.9.14 *A uniform ladder of length 20 m, rests against a vertical wall with which it makes an angle of 45°, the coefficient of friction between the ladder and the wall and ground respectively being $\frac{1}{3}$ and $\frac{1}{2}$. If a man, whose weight is one half that of the ladder, ascends the ladder, how high will he be, when the ladder slips?*

VTU : Aug.-08, Marks 10

Solution : Let weight of ladder be W. Then weight of man is $\frac{W}{2}$. The F.B.D. of ladder is shown in Fig. 7.9.14 .

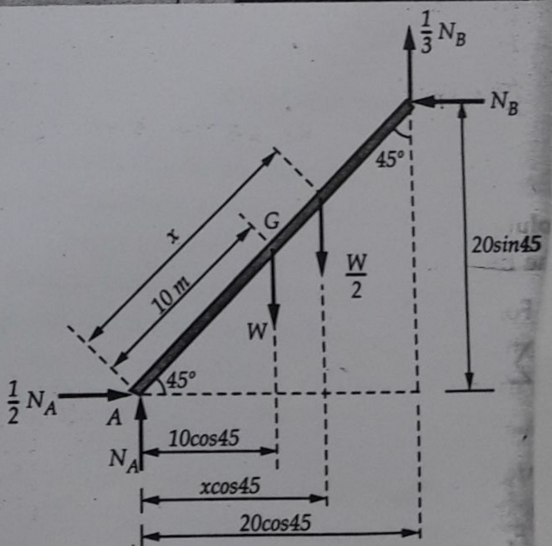


Fig. 7.9.14

$\sum F_x = 0:$

$$\frac{1}{2}N_A - N_B = 0$$

$$\therefore \quad N_A = 2N_B$$

$\sum F_y = 0:$

$$_A - W - \frac{W}{2} + \frac{1}{3}N_B = 0$$

$$\therefore \quad 2N_B + \frac{1}{3}N_B = \frac{3W}{2}$$

$$\frac{7}{3}N_B = \frac{3}{2}W$$

$$\therefore \quad N_B = \frac{9}{14}W$$

$$\sum M_A = 0:$$

$$-(W)(10\cos 45)-\left(\frac{W}{2}\right)(x\cos 45)+(N_B)(20\sin 45)+\left(\frac{1}{3}N_B\right)(20\cos 45) = 0$$

$$\text{As } \sin 45 = \cos 45 = \frac{1}{\sqrt{2}}$$

$$-W\times 10-\frac{W}{2}x+N_B\times 20+\frac{1}{3}N_B\times 20 = 0$$

$$\text{Substituting } N_B = \frac{9W}{14},$$

$$-W\times 10-\frac{W}{2}x+\frac{9W}{14}\times 20+\frac{1}{3}\times\frac{9W}{14}\times 20 = 0$$

$$\therefore \quad -10-\frac{x}{2}+\frac{90}{7}+\frac{30}{7} = 0$$

$$\therefore \quad \boxed{x = 14.29\ m}$$

This distance is along the ladder. The height from the floor will be 14.29 sin 45 = 10.1 m.

Example 7.9.15 *A ladder 7 m long weighing 300 N is resting against a wall at an angle of 60° to the horizontal ground. A man weighing 700 N climbs the ladder, at what position does he induce slipping. Take $\mu = 0.25$ for all contact surfaces.* **VTU : June-12, Marks 8**

Solution : The F.B.D. of ladder is shown in Fig. 7.9.15 .

$$\sum F_x = 0:$$

$$N_A - 0.25\,N_B = 0$$

$$\therefore \quad N_B = 4\,N_A$$

$$\sum F_y = 0:$$

$$0.25\,N_A + N_B - 700 - 300 = 0$$

$$0.25\,N_A + 4\,N_A = 1000$$

$$\therefore \quad N_A = 235.294\ N$$

$$\sum M_B = 0:$$

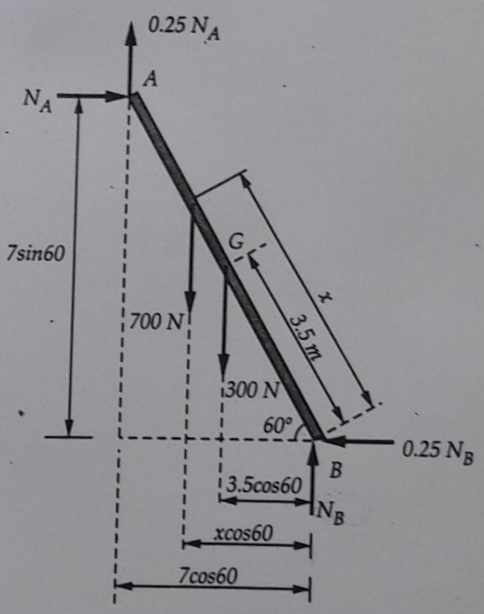


Fig. 7.9.15

$$(300)(3.5\cos 60)+(700)(x\cos 60)-(N_A)(7\sin 60)-(0.25\,N_A)(7\cos 60) = 0$$

$$1050\cos 60 + 700\,x\cos 60 - 235.294\times 7\sin 60 - 0.25\times 235.294\times 7\cos 60 = 0$$

$$\therefore \quad \boxed{x = 3.164\ m}$$

Example 7.9.30 *The ladder shown in Fig. 7.9.30, is 4 m long and is supported by a horizontal floor and vertical wall. The coefficient of friction at the wall is 0.25 and at the floor is 0.50. The weight of the ladder is 200 N, considered concentrated at "G". The ladder supports a vertical load of 1000 N at "C". Determine the reactions at A and B and compute the least value of "α" at which, the ladder may be placed without slipping.*

VTU : Feb.-11, Marks 8

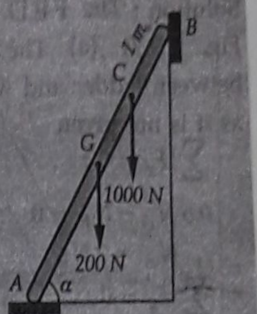


Fig. 7.9.30

Solution : The F.B.D. of ladder is shown in Fig. 7.9.30 (a).

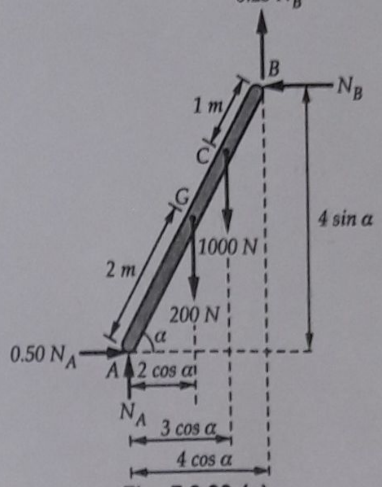


Fig. 7.9.30 (a)

$$\sum F_x = 0$$

$$0.5\,N_A - N_B = 0$$

$$\therefore\ N_A = 2N_B$$

$$\sum F_y = 0$$

$$N_A + 0.25\,N_B - 200 - 1000 = 0$$

$$\therefore\ 2N_B + 0.25\,N_B = 1200$$

$$\therefore\ N_B = 533.33\text{ N}$$

$$\sum M_A = 0$$

$$-(200)(2\cos\alpha) - (1000)(3\cos\alpha) + (N_B)(4\sin\alpha) + (0.25\,N_B)(4\cos\alpha) = 0$$

$$-400\cos\alpha - 3000\cos\alpha + 533.33\times 4\sin\alpha + 0.25\times 533.33\times 4\cos\alpha = 0$$

$$2133.32\sin\alpha = 2866.67\cos\alpha$$

$$\tan\alpha = \frac{2866.67}{2133.32}$$

$$\therefore\quad \boxed{\alpha = 53.34^\circ}$$

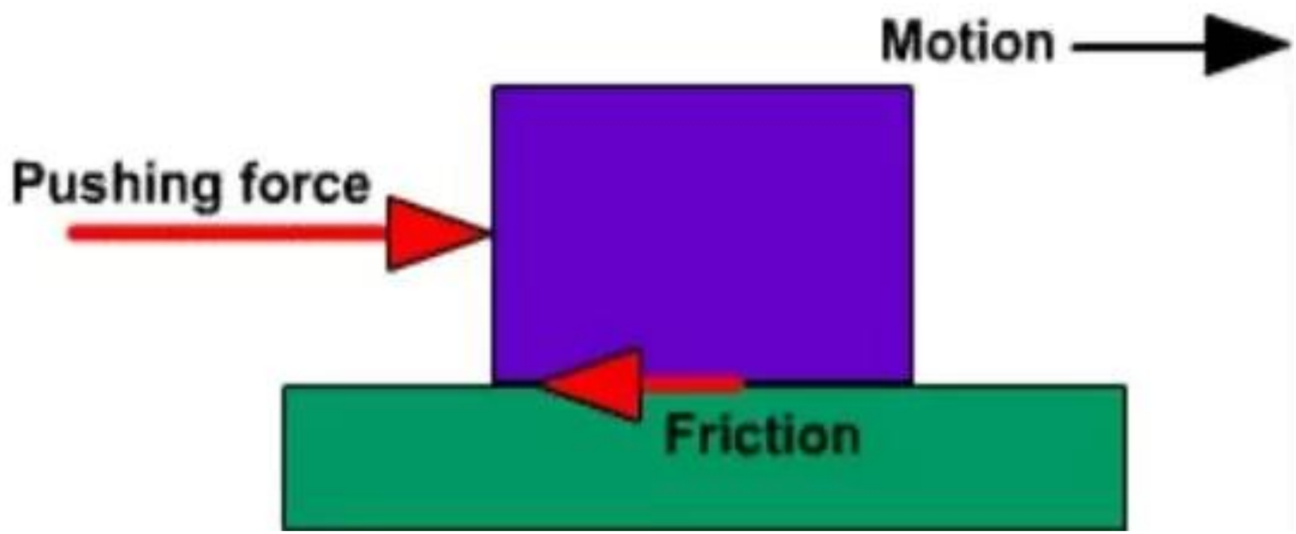
Friction

Module – 2

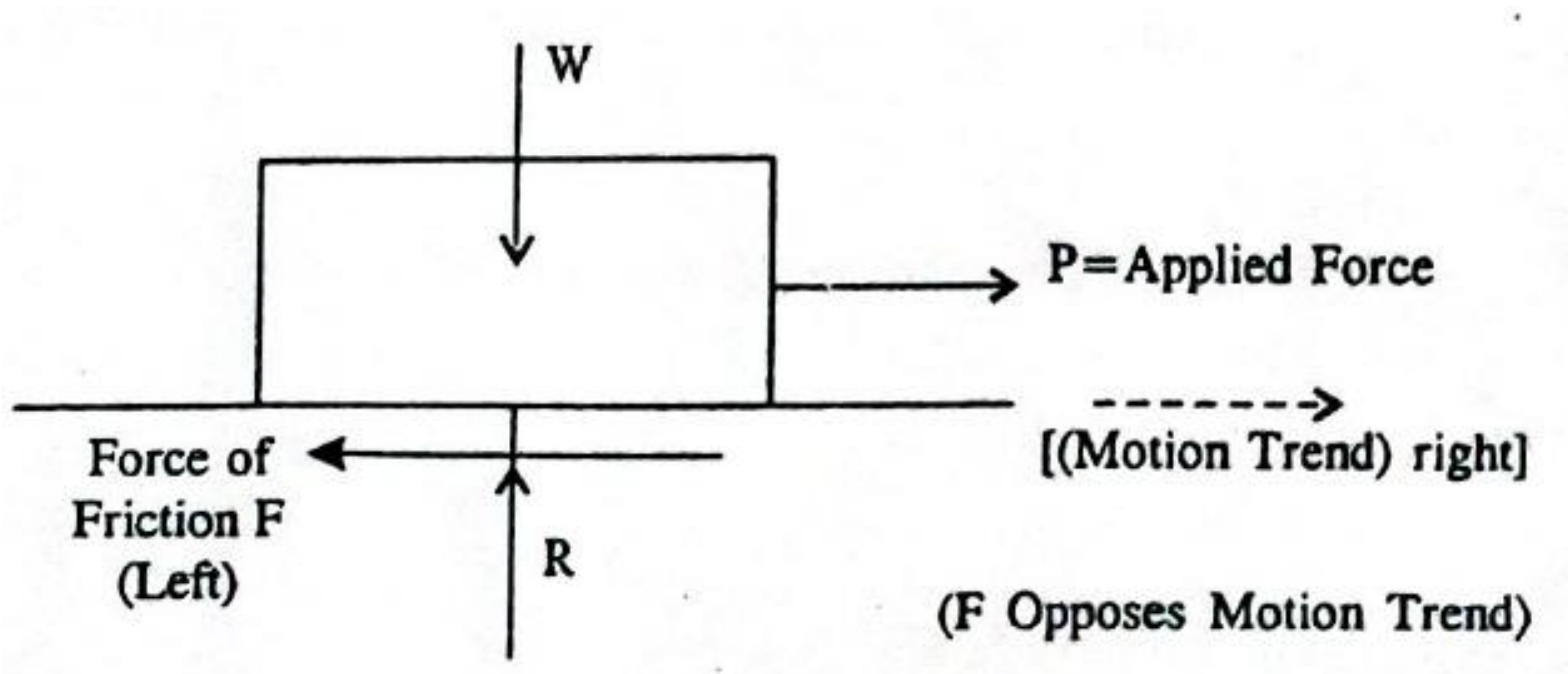
18CIV24

Definition:

When one body tends to move in contact with another body a resistance to its movement is setup. This resistance to the movement is called friction or force of friction or frictional force.



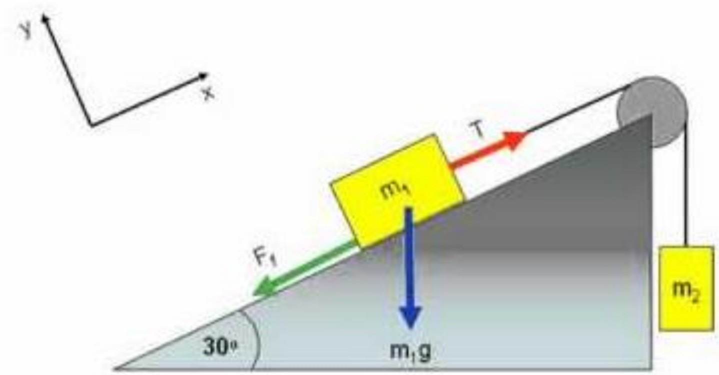
The force of friction always acts in the direction opposite to the motion trend as shown.



Types of Friction

1. **Static friction: friction acting on a body which is at rest**
2. **Limiting friction: friction acting on a body which is just on the point or edge of sliding**
3. **Dynamical friction: friction acting on a body which is actually in motion. Also called kinetic friction**

1



3

Types of Friction

4. Dry friction: friction acting on a body when the contact surfaces are dry and there is tendency to relative motion. Also called Coulomb friction.

2 types of dry friction

i. Solid friction: friction acting on a body when 2 surfaces have tendency to slide relative to each other
ii. Rolling friction: friction acting on a body due to rolling of one surface over another

Types of Friction

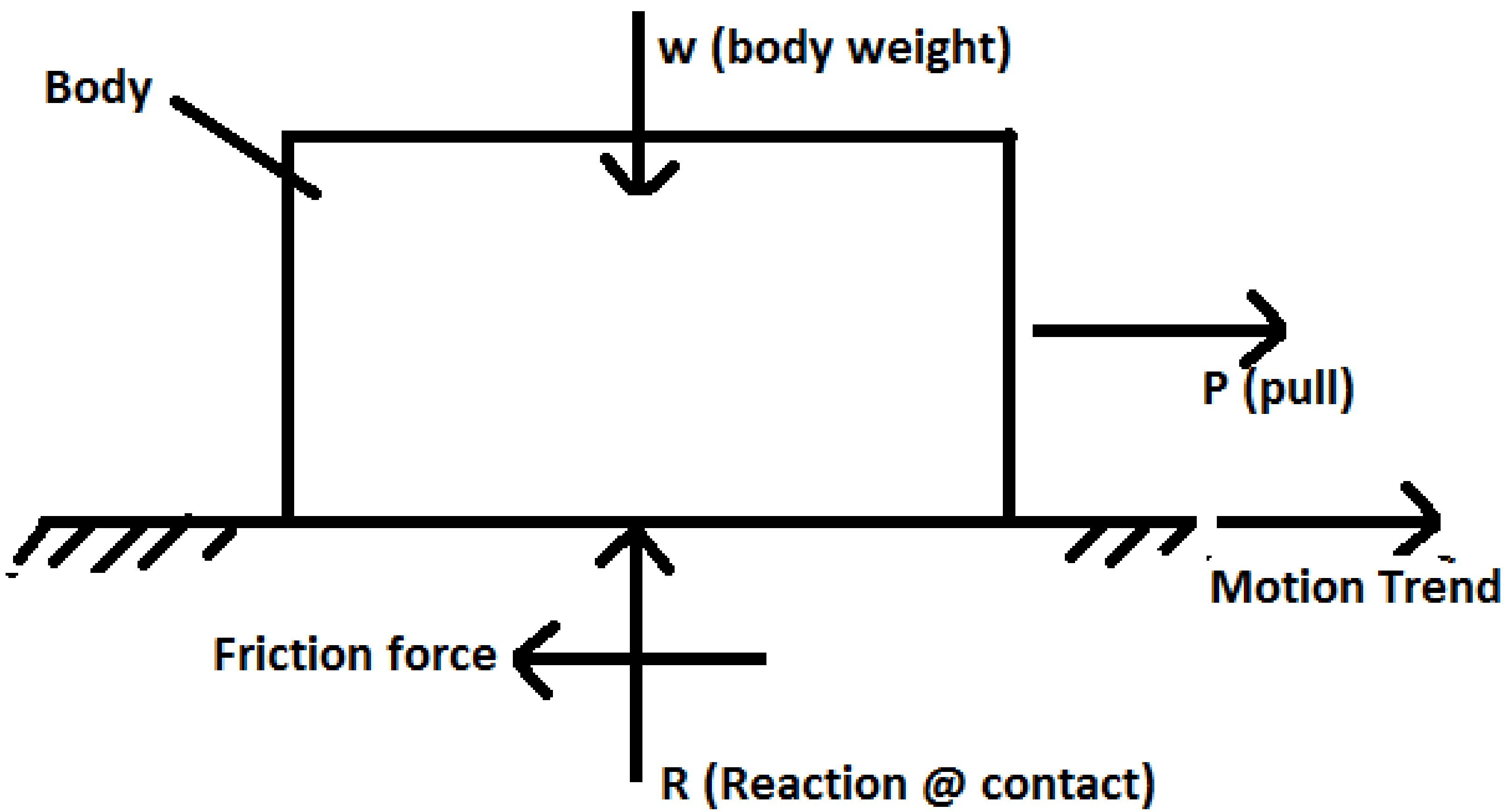
5. Fluid friction: friction acting on a body when the contact surfaces are lubricated

2 types of fluid friction

i. **Skin/ greasy/ Non viscous friction: friction acting on a body when the contact surfaces are lubricated with extremely thin layer of lubricant. Also called Boundary friction**
ii. **Viscous / film friction: friction acting on a body when the contact surfaces are completely separated by lubricant**

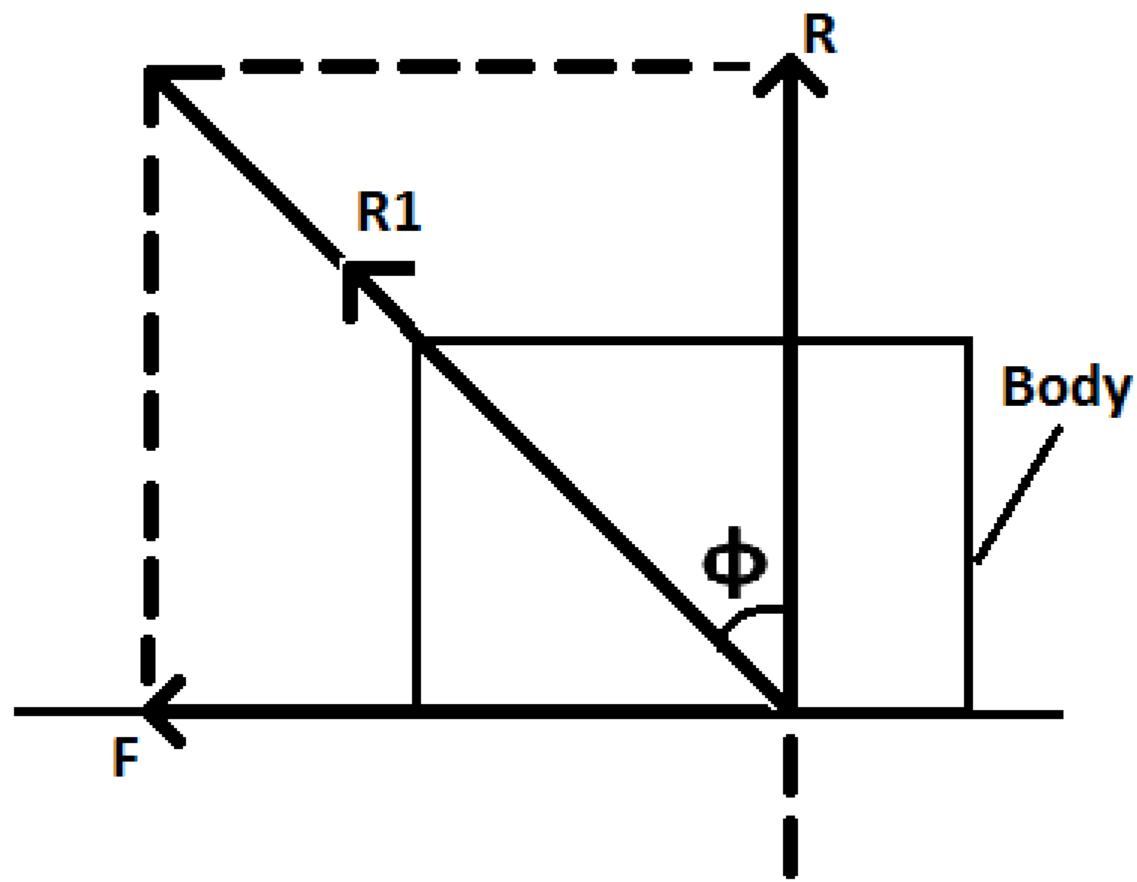
Motion trend of a block on a surface

A body of weight W is subjected to pull P which tends to move towards right as shown



From figure we can see the normal reaction and friction force are perpendicular to each other and can be replaced by a single resultant reaction R1 making angle ϕ

$\tan\phi = F/R$



Angle of friction: The angle which the resultant reaction R1 due to normal Reaction R and Friction F makes with the normal to the surface.

$$\tan\phi = F/R$$

Co-efficient of friction: (μ)

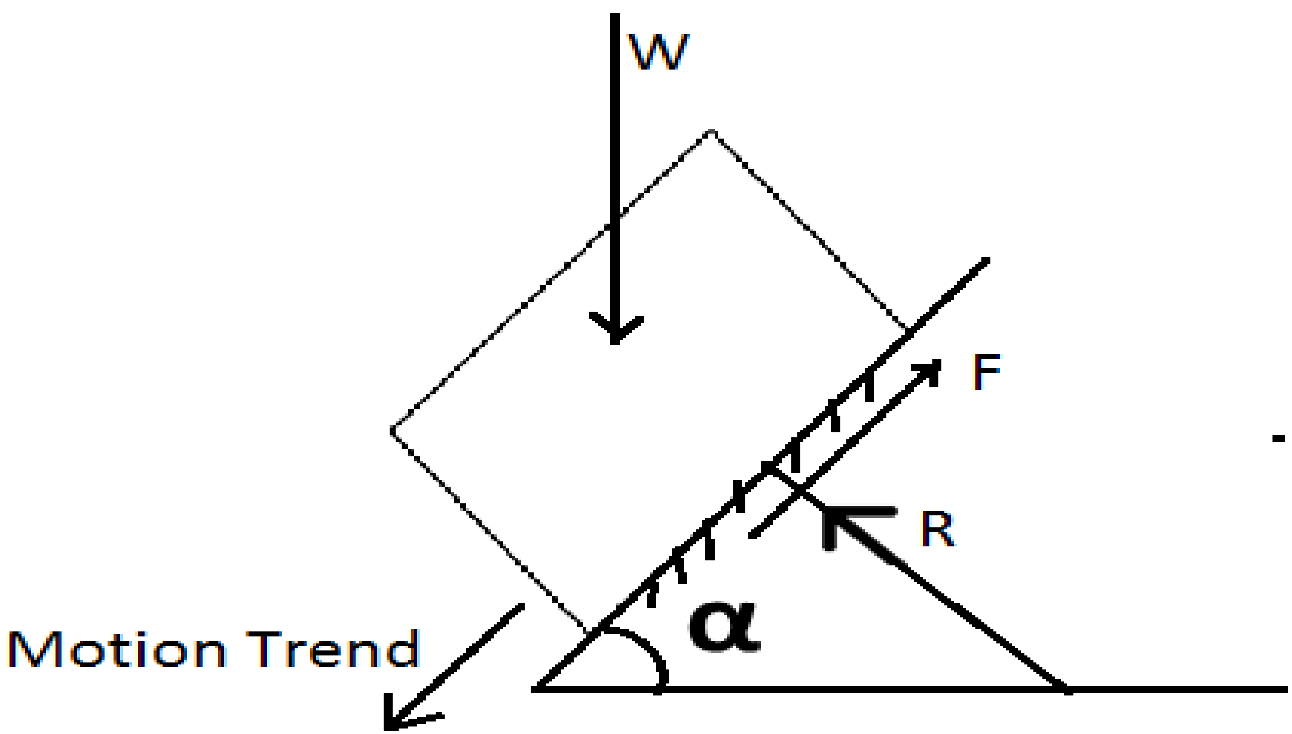
It is the ratio of limiting friction F to Normal reaction R between 2 surfaces. This is also equal to Tangent of angle of friction.

$$\mu = F/R$$

$$\tan\phi = \mu = F/R$$

Angle of Repose:

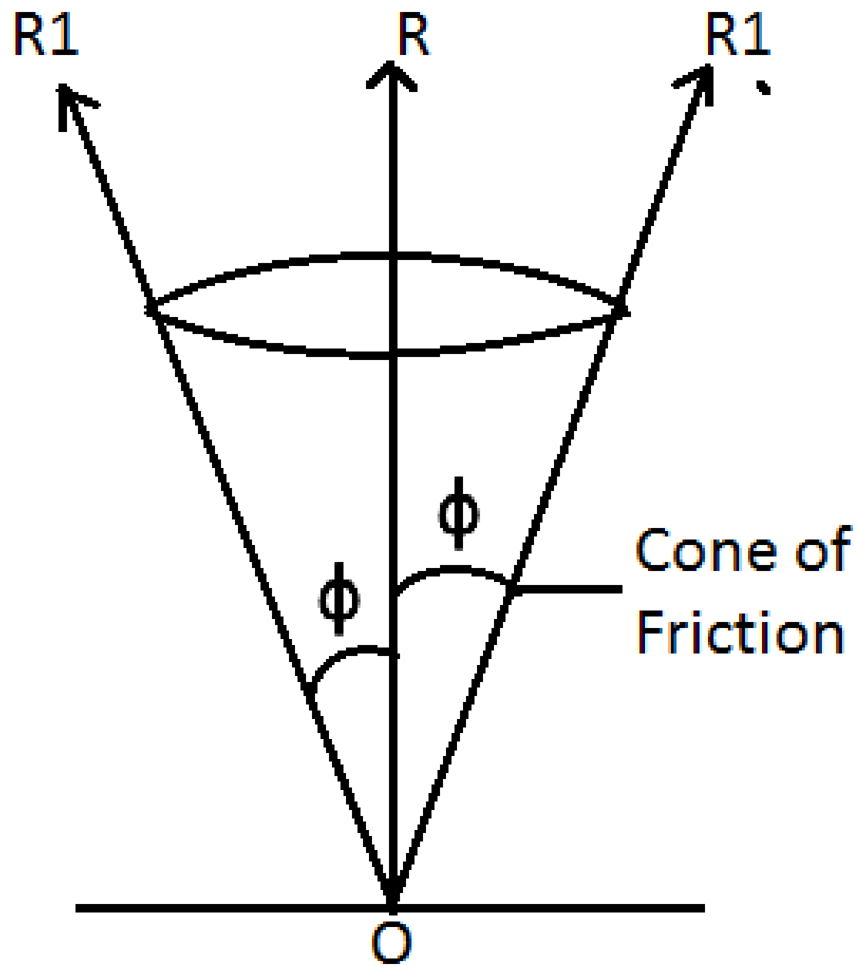
If a body is placed on an inclined plane, then the angle @ which the body is just on the point or verge of sliding down.



Cone of Friction:

- **Whenever a body is in contact with other tends to move, then the normal reaction OR and friction come into play.**
- **The normal reaction and friction can be replaced by resultant reaction OR1.**
- **When this resultant reaction OR1 making angle is revolved around point O will form a right circular cone.**
- **This cone having the contact point as the vertex O, the normal OR at point of contact as its axis and ϕ as the semi vertex angle is cone of friction.**

Cone of Friction:



Laws of Dry friction:

1. **The force of friction always acts in the direction opposite to that in which the body tends to move.**
2. **The magnitude of limiting friction F bears a constant ratio to the normal reaction R between the two surfaces i.e $F/R = \mu$**
3. **The magnitude of force of friction is exactly equal to force, which tends the body to move as long as the body is at rest**
4. **The force of friction is independent of area of contact between two surfaces.**
5. **The force of friction depends upon the roughness of the surfaces in contact.**

Static friction:

- **Under static conditions the friction force opposes the tendency for relative motion between 2 surfaces in contact and acts tangential to surfaces.**
- **Limiting static friction force which is maximum value of friction force is directly proportional to normal reaction between 2 surfaces in contact.**
- **$(Fr)_{max} \; \alpha \; N$**
- **$(Fr)_{max} = \mu_s N$**
- **μ_s = Coefficient of static friction**

Kinetic friction:

- **Force of kinetic friction opposes the relative motion between 2 surfaces in contact**
- **The force of kinetic friction is directly proportional to the normal reaction between 2 surfaces in contact**
- **$F_k \propto N$**
- **$F_k = \mu_k N$**
- **μ_k = Coefficient of kinetic friction**