Stresses in Curved Beam

Consider a curved beam subjected to bending moment M_b as shown in the figure. The distribution of stress in curved flexural member is determined by using the following assumptions:

i) The material of the beam is perfectly homogeneous [i.e., same material throughout] and isotropic [i.e., equal elastic properties in all directions]

ii) The cross section has an axis of symmetry in a plane along the length of the beam.

iii) The material of the beam obeys Hooke's law.

iv) The transverse sections which are plane before bending remain plane after bending also.

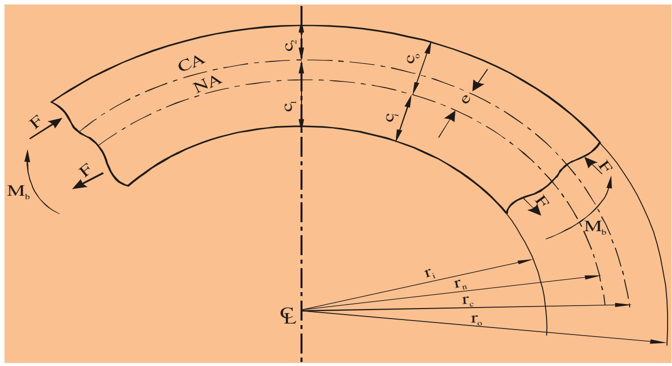
v) Each layer of the beam is free to expand or contract, independent of the layer above or below it.

vi) The Young's modulus is same both in tension and compression.

Derivation for stresses in curved beam

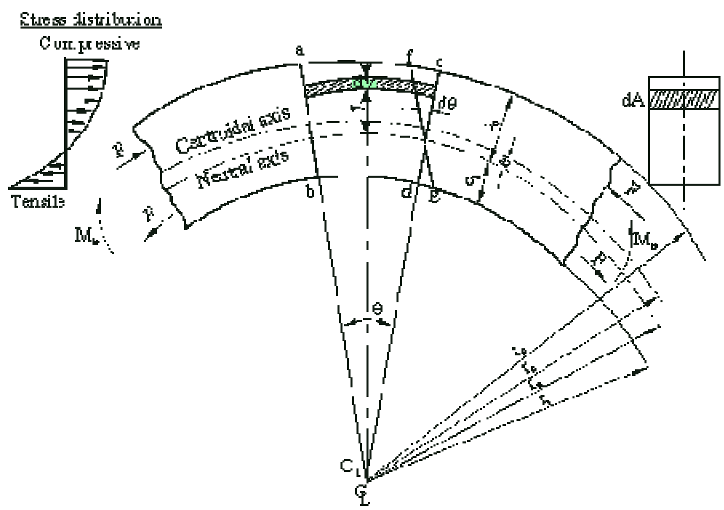
Nomenclature used in curved beam

C_i =Distance from neutral axis to inner radius of curved beam
C_o=Distance from neutral axis to outer radius of curved beam
C_1=Distance from centroidal axis to inner radius of curved beam
C_2= Distance from centroidal axis to outer radius of curved beam
F = Applied load or Force
A = Area of cross section
L = Distance from force to centroidal axis at critical section
σ_d= Direct stress
σ_{bi} = Bending stress at the inner fiber
σ_{bo} = Bending stress at the outer fiber
σ_{ri} = Combined stress at the inner fiber
σ_{ro} = Combined stress at the outer fiber



Stresses in curved beam

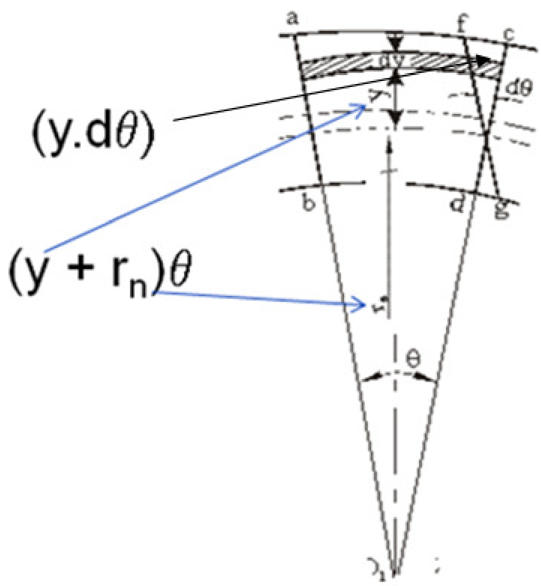
M_b = Applied Bending Moment
r_i = Inner radius of curved beam
r_o = Outer radius of curved beam
r_c = Radius of centroidal axis
r_n = Radius of neutral axis
C_L = Center of curvature



In the above figure the lines 'ab' and 'cd' represent two such planes before bending. i.e., when there are no stresses induced. When a bending moment 'M_b' is applied to the beam the plane cd rotates with respect to 'ab' through an angle 'dθ' to the position 'fg' and the outer fibers are shortened while the inner fibers are elongated. The original length of a strip at a distance 'y' from the neutral axis is $(y + r_n)\theta$. It is shortened by the amount $yd\theta$ and the stress in this fiber is,

$$\sigma = E.e$$

Where σ = stress, e = strain and E = Young's Modulus



We know, stress $\sigma = E.e$

We know, stress $e = \frac{\text{change in length}}{\text{original length}} = \frac{y d\theta}{(y+r_n)\theta}$

$$\text{i.e.,} \quad \sigma \quad = -E\,\frac{y d\theta}{(y+r_n)\theta} \qquad \text{..... (i)}$$

Since the fiber is shortened, the stress induced in this fiber is compressive stress and hence negative sign.

The load on the strip having thickness dy and cross sectional area dA is 'dF'

$$\text{i.e., } dF = \sigma dA = -\frac{E y d\theta}{(y+r_n)\theta} dA$$

From the condition of equilibrium, the summation of forces over the whole cross-section is zero and the summation of the moments due to these forces is equal to the applied bending moment.

Let

M_b = Applied Bending Moment

r_i = Inner radius of curved beam

r_o = Outer radius of curved beam

r_c = Radius of centroidal axis
r_n = Radius of neutral axis
C_L= Centre line of curvature

Summation of forces over the whole cross section

i.e. $$\int dF = 0$$

$$\therefore \quad \frac{Ed\theta}{\theta}\int \frac{ydA}{(y+r_n)}=0$$

As $\frac{Ed\theta}{\theta}$ is not equal to zero,

$$\therefore \quad \int \frac{ydA}{(y+r_n)} = 0 \qquad \text{..... (ii)}$$

The neutral axis radius 'r_n' can be determined from the above equation.

If the moments are taken about the neutral axis,

$$M_b = -\int ydF$$

Substituting the value of dF, we get

$$M_b = \frac{Ed\theta}{\theta}\int \frac{y^2}{(y+r_n)} dA$$

$$= \frac{Ed\theta}{\theta}\int \left(y - \frac{yr_n}{(y+r_n)}\right) dA$$

$$= \frac{Ed\theta}{\theta}\int ydA \qquad \left[\because \int \frac{ydA}{y+r_n} = 0\right]$$

Since $\int ydA$ represents the statical moment of area, it may be replaced by A.e., the product of total area A and the distance 'e' from the centroidal axis to the neutral axis.

$$\therefore \qquad M_b = \frac{Ed\theta}{\theta} A.e \qquad \text{..... (iii)}$$

From equation (i) $\frac{Ed\theta}{\theta} = -\frac{\sigma(y+r_n)}{y}$

Substituting in equation (iii)

$$M_b = -\frac{\sigma(y+r_n)}{y}. A. e.$$

$$\therefore \qquad \sigma = \frac{M_b y}{(y+r_n)Ae} \qquad \text{..... (iv)}$$

This is the general equation for the stress in a fiber at a distance 'y' from neutral axis.

At the outer fiber, $y = c_o$

$\therefore$ Bending stress at the outer fiber σ_{bo}

i.e., $\sigma_{bo} = -\frac{M_b C_o}{Aer_o}$ $(\because \ r_n + c_o = r_o)$ ..... (v)

Where c_o = Distance from neutral axis to outer fiber. It is compressive stress and hence negative sign. At the inner fiber, $y = -c_i$

$\therefore$ Bending stress at the inner fiber

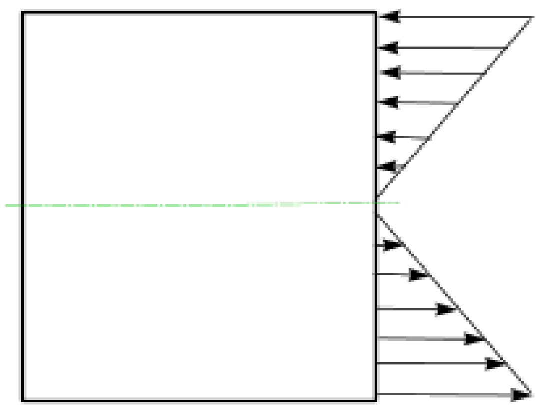
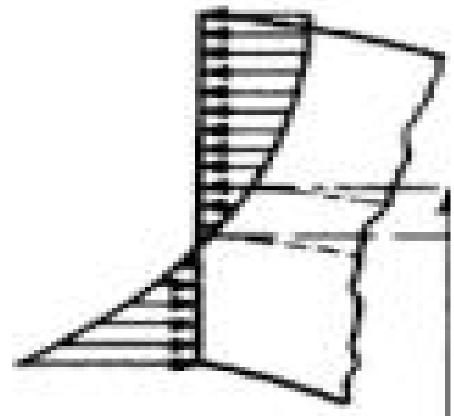
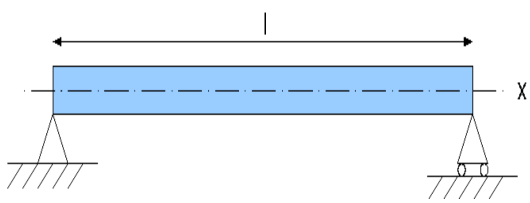
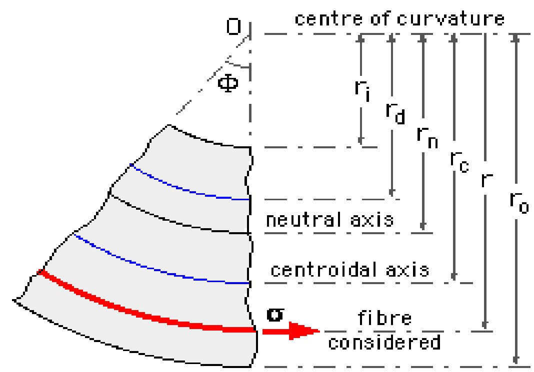
$$\sigma_{bi} = \frac{M_b C_i}{Ae(r_n - C_i)}$$

i.e., $\sigma_{bi} = \frac{M_b C_i}{Aer_i}$ $(\because r_n - c_i = r_i)$ ..... (vi)

Where c_i = Distance from neutral axis to inner fiber. It is tensile stress and hence positive sign.

Difference between a straight beam and a curved beam

Sl.no	straight beam	curved beam
1	In Straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear.	In case of curved beams the neutral axis of the section is shifted towards the center of curvature of the beam causing a non-linear stress distribution.

<table>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>3</td><td>Neutral axis and centroidal axis coincides
</td><td>Neutral axis is shifted towards the least centre of curvature
</td></tr>
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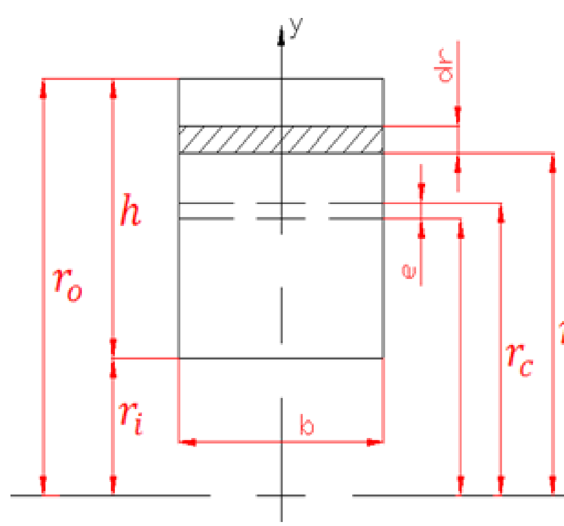
Location of the neutral axis By considering a rectangular cross section

$$\int \frac{y\,dA}{r_n + y} = 0$$

$$\int dA = \int \frac{r - r_n}{r} dA = 0$$

$$A - r_n \int \frac{dA}{r} = 0$$



$$r_n = \frac{A}{\int \frac{dA}{r}}$$

$$r_n = \frac{bh}{\int \frac{bdr}{r}}$$

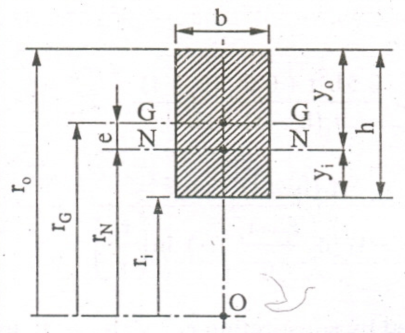
$$r_n = \frac{h}{\int_{r_i}^{r_0} \frac{dr}{r}}$$

$$r_n = \frac{h}{\int_{r_i}^{r_0} \frac{dr}{r}}$$

$$r_n = \frac{h}{(\ln r)_{r_i}^{r_0}}$$

$$r_n = \frac{h}{\ln r_o - \ln r_i} = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$$

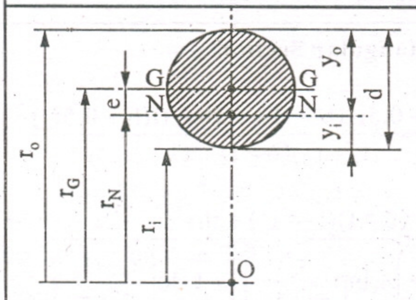
Centroidal and Neutral Axis of Typical Section of Curved Beams



Rectangular

$$r_G = r_i + \frac{h}{2}$$

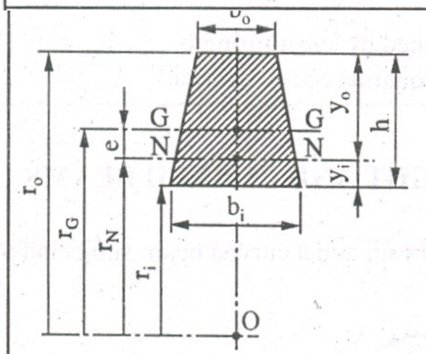
$$r_N = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)}$$



Circular

$$r_G = r_i + \frac{d}{2}$$

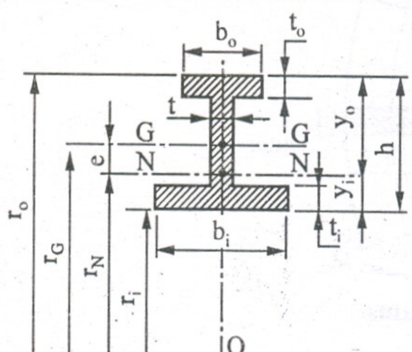
$$r_N = \frac{\left(\sqrt{r_o} + \sqrt{r_i}\right)^2}{4}$$



Trapezoidal

$$r_G = r_i + \frac{h}{3}\left[\frac{b_i + 2\,b_o}{b_i + b_o}\right]$$

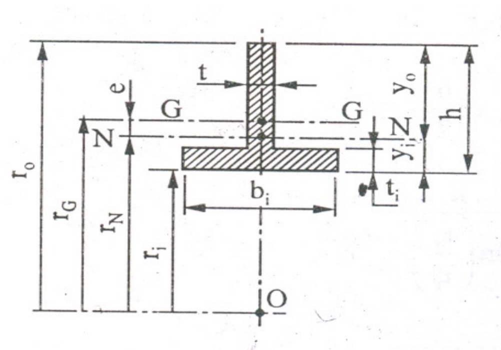
$$r_N = \frac{A}{\left[\frac{(b_i\,r_o - b_o\,r_i)}{h}\right]\ln\left(\frac{r_o}{r_i}\right) - (b_i - b_o)}$$



I - Section

$$r_G = r_i + \frac{0.5\,h^2 t + 0.5\,t_i^2 (b_i - t) + t_o(b_o - t)(h - 0.5\,t_o)}{t_i(b_i - t) + t_o(b_o - t) + t\,h}$$

$$r_N = \frac{t_i(b_i - t) + t_o(b_o - t) + th}{b_i\,\ln\left(\frac{r_i + t_i}{r_i}\right) + t\,\ln\left(\frac{r_o - t_o}{r_i + t_i}\right) + b_o\,\ln\left(\frac{r_o}{r_o - t_o}\right)}$$

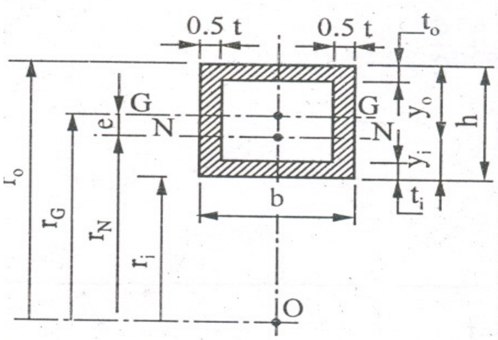


T - Section

$$r_G = r_i + \frac{0.5h^2t + 0.5t_i^2(b_i - t)}{t_i(b_i - t) + t\,h}$$

$$r_N = \frac{t_i(b_i - t) + t\,h}{(b_i - t)\ \ln\left(\frac{r_i + t_i}{r_i}\right) + t\ \ln\left(\frac{r_o}{r_i}\right)}$$

(can also be obtained by substituting $t_o = b_o = 0$ in the equation for I - Section)



Hollow Rectangular Section

$$r_G = r_i + \frac{0.5h^2t + 0.5t_i^2(b - t) + t_o(b - t)(h - 0.5t_o)}{(t_i + t_o)(b - t) + t\,h}$$

$$r_N = \frac{(b - t)(t_i + t_o) + th}{b\left[\ln\left(\frac{r_i + t_i}{r_i}\right) + \ln\left(\frac{r_o}{r_o - t_o}\right)\right] + t\ \ln\left(\frac{r_o - t_o}{r_i + t_i}\right)}$$

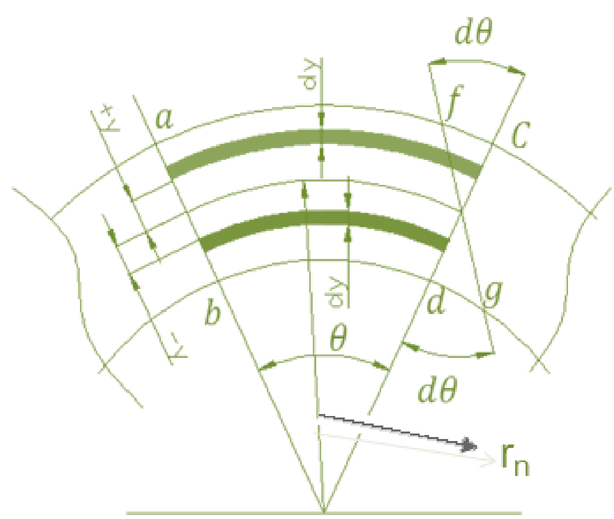
(can also be obtained by substituting $b_i = b_o = b$ in the equation for I - Section)

Why stress concentration occur at inner side or concave side of curved beam

Consider the elements of the curved beam lying between two axial planes 'ab' and 'cd' separated by angle θ. Let fg is the final position of the plane cd having rotated through an angle dθ about neutral axis. Consider two fibers symmetrically located on either side of the neutral axis. Deformation in both the fibers is same and equal to ydθ.

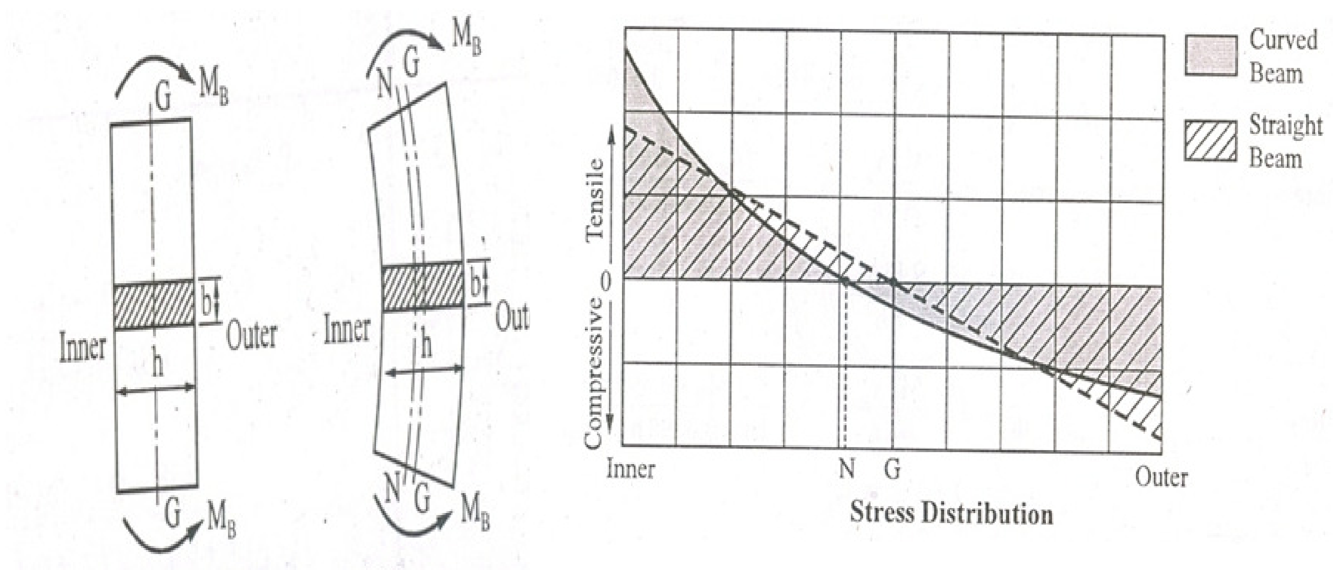
$$\sigma_{inner} = E\frac{(y.d\theta)}{(r_n - y)\theta}$$

$$\sigma_{outer} = E\frac{(y.d\theta)}{(r_n + y)\theta}$$



Since length of inner element is smaller than outer element, the strain induced and stress developed are higher for inner element than outer element as shown.
Thus stress concentration occur at inner side or concave side of curved beam

The actual magnitude of stress in the curved beam would be influenced by magnitude of curvature However, for a general comparison the stress distribution for the same section and same bending moment for the straight beam and the curved beam are shown in figure.



It is observed that the neutral axis shifts inwards for the curved beam. This results in stress to be zero at this position, rather than at the centre of gravity.
In cases where holes and discontinuities are provided in the beam, they should be preferably placed at the neutral axis, rather than that at the centroidal axis. This results in a better stress distribution.
Example:
For numerical analysis, consider the depth of the section ass twice the inner radius.

For a straight beam:

Inner most fiber: $$\sigma_{BSi} = \frac{M_b c_i}{I} = \frac{6M_B}{bh^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Outer most fiber: $$\sigma_{BSo} = \frac{M_b c_o}{I} = -\frac{6M_B}{bh^2} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

For curved beam: **h=2r$_i$**

$$r_c = \mathrm{r_i} + \frac{h}{2} = \frac{h}{2} + \frac{h}{2} = \mathrm{h}$$

$$\mathrm{r_o} = \mathrm{r_i} + \mathrm{h} = \frac{h}{2} + \mathrm{h} = \frac{3}{2}h$$

$$r_n = \frac{h}{ln\frac{r_o}{r_i}} = \frac{h}{\ln(3)} = 0.910h$$

e = r_c - r_n = h – 0.910h = 0.0898h

c_o = r_o - r_n= − h – 0.910h = 0590h

c_i = r_n - r_i = 0.910h - − = 0.410h

$$\sigma_{bci} = \frac{M_b c_i}{Aer_i} = \frac{M_b(0.410h)}{bh(0.0898h)(0.5h)}$$

$$= \frac{9.13\ X\ M_b}{bh^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\sigma_{bco} = \frac{M_b c_o}{Aer_o} = -\frac{M_b(0.590h)}{bh(0.0898h)(1.5h)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$= -\frac{4.38\ X\ M_b}{bh^2}$$

Comparing the stresses at the inner most fiber based on (1) and (3), we observe that the stress at the inner most fiber in this case is:

$$\sigma_{bci} = 1.522\sigma_{BSi}$$

Thus the stress at the inner most fiber for this case is 1.522 times greater than that for a straight beam.

From the stress distribution it is observed that the maximum stress in a curved beam is higher than the straight beam.

Comparing the stresses at the outer most fiber based on (2) and (4), we observe that the stress at the outer most fiber in this case is:

$$\sigma_{bco} = 1.522\sigma_{BSi}$$

Thus the stress at the inner most fiber for this case is 0.730 times that for a straight beam.

The curvatures thus introduce a non linear stress distribution.

This is due to the change in force flow lines, resulting in stress concentration on the inner side.

To achieve a better stress distribution, section where the centroidal axis is the shifted towards the insides must be chosen, this tends to equalize the stress variation on the inside and outside fibers for a curved beam. Such sections are trapeziums, non symmetrical I section, and T sections.

It should be noted that these sections should always be placed in a manner such that the centroidal axis is inwards.

Problem no.1

Plot the stress distribution about section A-B of the hook as shown in figure.

Given data:

$r_i = 50mm$

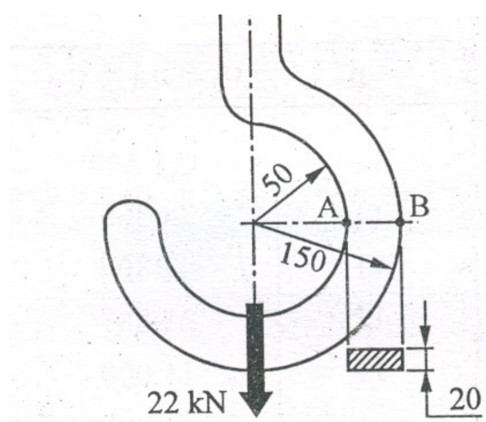
$r_o = 150mm$

$F = 22X10^3N$

$b = 20mm$

$h = 150-50 = 100mm$

$A = bh = 20X100 = 2000mm^2$



DESIGN OF MACHINE ELEMENTS - II

CHAPTER I

CYLINDERS, COMPOUND CYLINDERS, PRESS & SHRINK FITS

2.1 Introduction to pressure vessels

Pressure vessels in the form of cylinders and tanks are used for storing variety of liquids and gasses at different temperatures and pressures. Some of the substances stored can be lethal to human beings if exposed and some can be highly explosive. Bursting of pressure vessels due to improper design can prove fatal to human life and property. It is imperative that a designer should have a comprehensive understanding of the principles of designing such pressure vessels based on national and international standards.

2.2 Stresses in thin cylinders

If the **wall thickness is less than about 7% of the inner diameter** then the cylinder may be treated as a thin one. Thin walled cylinders are used as boiler shells, pressure tanks, pipes and in other low pressure processing equipments. A thin cylinder is also defined as one in which the thickness of the metal is less than 1/20 of the diameter of the cylinder. In thin cylinders, it can be assumed that the variation of stress within the metal is negligible, and that the mean diameter, d_m is approximately equal to the internal diameter, d_i.

In general three types of stresses are developed in pressure cylinders viz. circumferential or hoop stress, longitudinal stress in closed end cylinders and radial stresses. These stresses are demonstrated in figure 2.1.

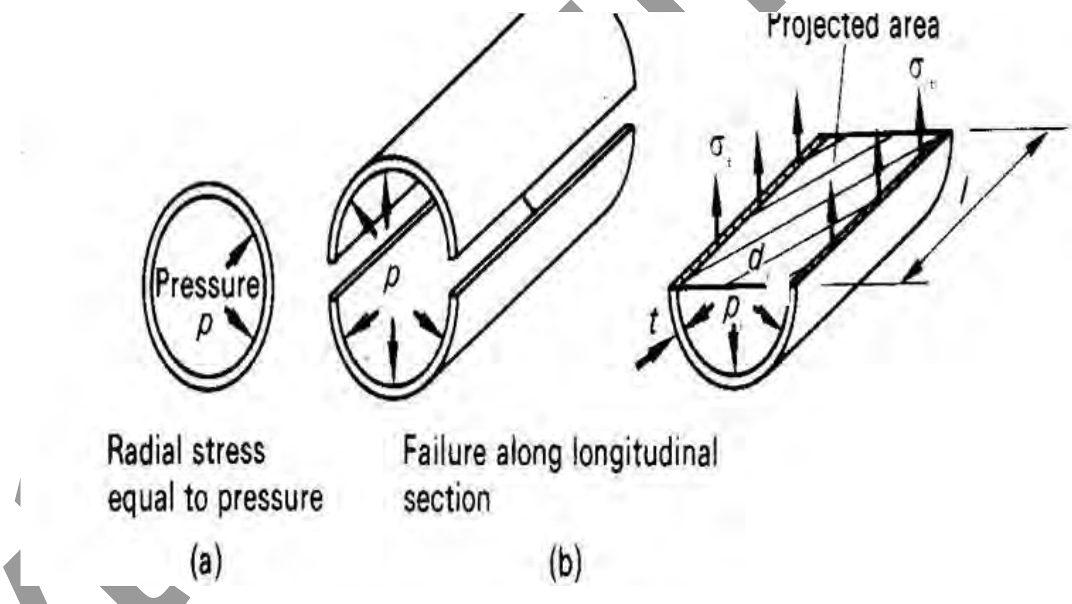


Figure 2.1

Radial stress in thin cylindrical shells can be neglected as the radial pressure is not generally high and that the radial pressure acts on a larger area.

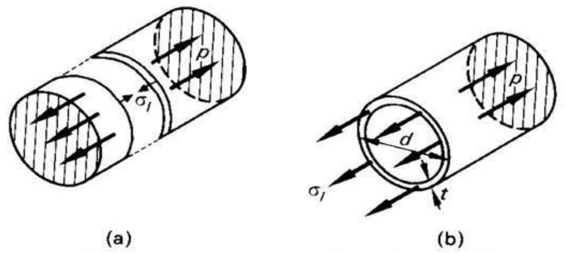
The internal pressure, p tends to increase the diameter of the cylinder and this produces a hoop or circumferential stress (tensile). If the stress becomes excessive, failure in the form of a longitudinal burst would occur.

Consider the half cylinder shown. Force due to internal pressure, p_i is balanced by the force due to hoop stress, σ_t i.e. hoop stress x area = pressure x projected area

$$\sigma_t \times 2 L t = p \times L \times d_i$$

$$\sigma_t = (p\, d_i) / 2t$$

Longitudinal stress in a cylinder:



Longitudinal stress in a thin cylindrical shell

Figure 1.2

The internal pressure p also produces a tensile stress in the longitudinal direction as shown in figure 1.2.

The force P acting on an area ($\pi d_i^2 / 4$) is balanced by longitudinal stress σ'_t acting over an approximate area $\pi d_i t$.

$\sigma'_t \times \pi d_i t = p\, (\pi d_i^2 / 4)$

$\boldsymbol{\sigma'_t = p d_i / 4t}$

Since hoop stress is twice longitudinal stress, the cylinder would fail by tearing along a line parallel to the axis, rather than on a section perpendicular to the axis. The equation for hoop stress is therefore used to determine the cylinder thickness.

Pressure vessels are generally manufactured from curved sheets joined by welding. Mostly V– butt welded joints are used. The riveted joints may also be used. Since the plates are weakened at the joint due to the rivet holes, the plate thickness should be enhanced by taking into account the joint efficiency. Allowance is made for this by dividing the thickness obtained in hoop stress equation by efficiency (i.e. tearing and shearing efficiency) of the joint. A typical welded construction of a pressure vessel is shown in figure 2.3 and riveted construction is shown in figure 2.4.

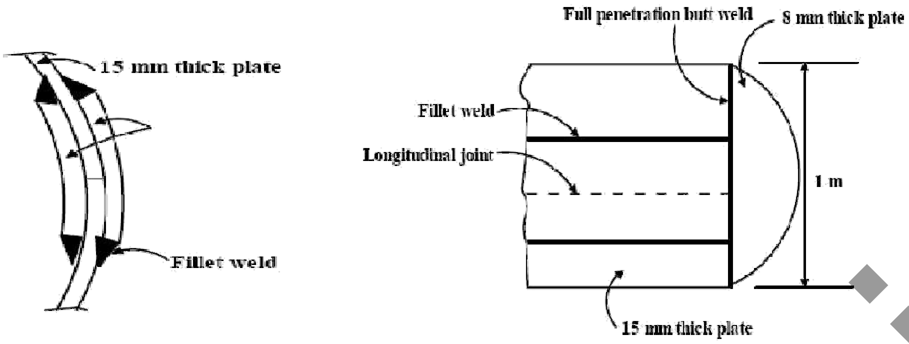


Figure 2.3. Welded construction of a pressure vessel

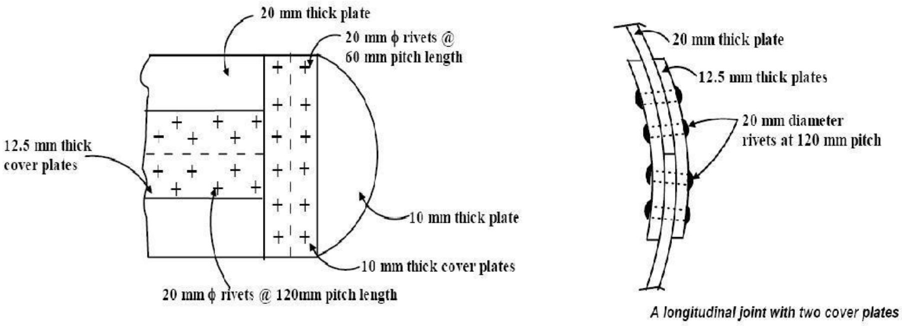
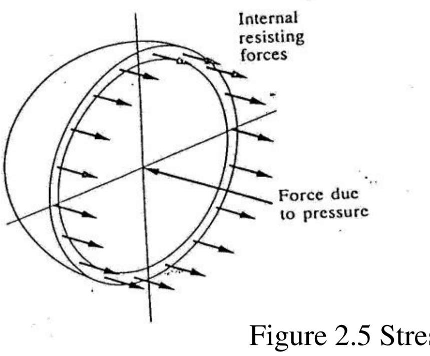


Figure 2.4. Rivetted construction of a pressure vessel

2.3 Stress induced in a spherical shell:

A sphere is the most favorably stressed shaped for a vessel requiring minimum wall thickness. It is used for extremely high pressure options. It is used in space vehicles and missiles for the storage of liquefied gasses at lower pressures but with light weight thin walls. Spheres also have the greatest buckling resistance. Spherical vessels are used as pressure carrying structures and as living space in most deep submerged vehicles for oceanography. The stress induced in a spherical vessel is as shown in figure 2.5 and is given by:

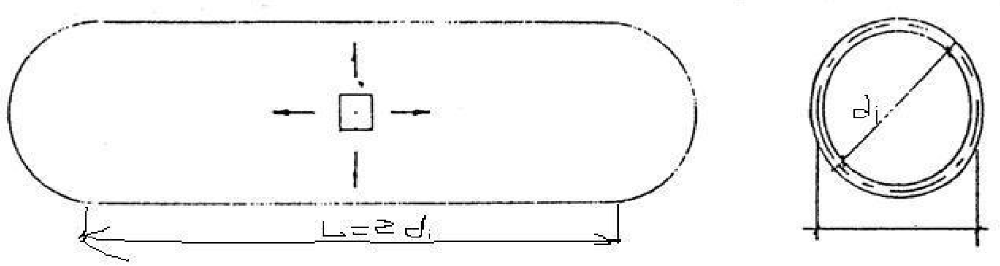


$$\sigma_t = pd_i / 4t$$

Figure 2.5 Stress in a spherical pressure vessel

2.4 EXAMPLE ON THIN CYLINDERS

E1. An air receiver consisting of a cylinder closed by hemispherical ends is shown in Figure below. It has a storage capacity of 0.25 m^3 and an operating internal pressure of 5 MPa. It is made of plain carbon steel 10C4 with an ultimate tensile strength of 340 MPa. Factor of safety to be used is 4. Neglecting the effect of welded joints, determine the dimensions of the receiver.



Volume of the vessel V= $\pi d_i^2 L/4 + \pi d_i^3/6$

Substituting L=$2d_i$ and simplifying;

$d_i = (3V/2\pi)^{1/3}$

= 0.492 m or 500 mm

L=2di = 500 X 2 =1000 mm.

Allowable stress= 340/4 =85 MPa.

Thickness of the cylinder = (neglecting the effect of welded joint)

t= $pd_i/2\sigma_t$

t= 14.7 mm or **15 mm.**

Thickness of the hemispherical Ends:

t= $pd_i/4\sigma_t$

t=7.35 mm or **7.5 mm**.

Problems for practice:

EP1. A seamless pipe 800 mm in diameter contains air at a pressure of 2 MPa. If the permissible stress of the pipe material is 100 MPa, find the minimum thickness of the pipe.

EP2. A cylindrical air receiver for a compressor is 2m in internal diameter and made of plate 15 mm thick. If the hoop stress is not to exceed 90 MPa and longitudinal stress is not to exceed 60 MPa find the safe air pressure.

EP3. A cylindrical shell of 2.2m internal diameter is constructed of mild steel plate. The shell is subjected to an internal pressure of 0.8 MPa. Determine the thickness of the shell plate by adopting a factor of safety of 6. The ultimate tensile strength of steel is 470 MPa. The efficiency of the longitudinal joint may be taken as 78%.

2.5 THICK CYLINDERS

If the wall thickness is more than about 7% of the inner diameter then the cylinder may be treated as a Thick Cylinder.

Difference in Treatment between Thin and Thick Cylinders

In thin cylinders the hope stress is assumed to be constant across the thickness of the cylinder wall. In thin cylinders there is no pressure gradient across the wall. In thin cylinders, radial stress is neglected (while it is of significant magnitude in thick cylinders). In thick cylinders none of these assumptions can be used and the variation of hoop and radial stress will be as shown in figure 2.6.

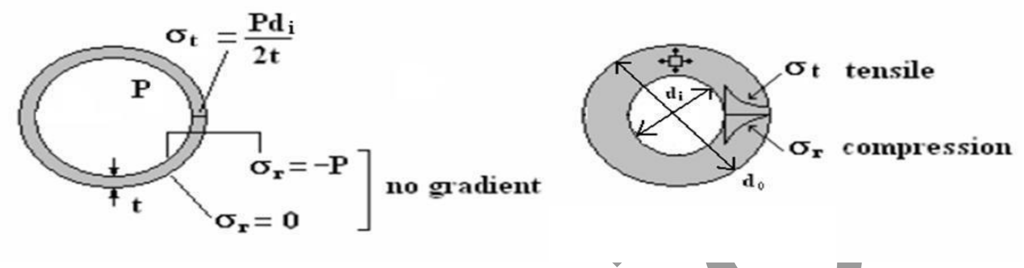


Figure 2.6 Variation of stresses in thin and thick cylinders

A thick cylinder subjected to both internal and external pressure is shown in figure 2.7.

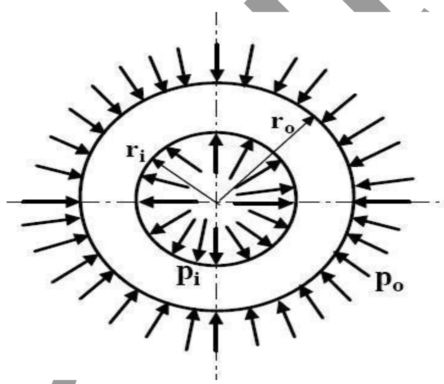


Figure 2.7 Thick cylinder subjected to both internal and external pressure

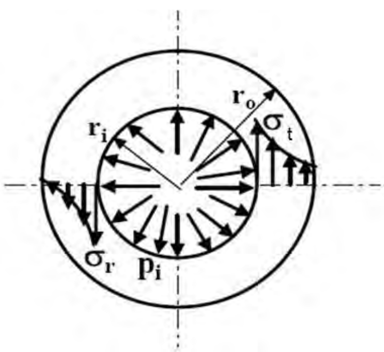


Figure 2.8 Radial and Tangential stress (hoop stress) distribution in a thick cylinder subjected to internal pressure only

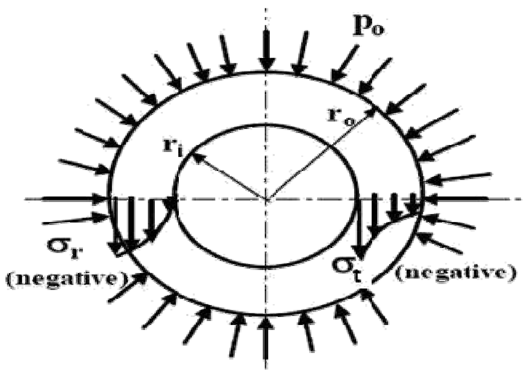


Figure 2.9 Radial and Tangential stress (hoop stress) distribution in a thick cylinder subjected to external pressure only

2.6 REVIEW OF LAMES EQUATION FOR THICK CYLINDERS:

When the material of the cylinder is brittle, such as Cast iron or Cast steel, Lame's equation is used to determine the wall thickness. It is based on the Maximum Principal stress theory, where maximum principal stress is equated to permissible stress of the material.

The three principal stresses at the inner surface of the cylinder are:

- Tangential or hoop stress σ_t
- Longitudinal stress σ^{1}_{t}
- Radial stress σ_r

Thick Cylinders

Lame's Equations:

The tangential stress in the cylinder wall at radius r

$$\sigma_t = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)}$$

$$= a + \frac{b}{r^2}$$

The radial stress in the cylinder wall at radius r

$$\sigma_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} - \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)}$$

$$= a - \frac{b}{r^2}$$

Lame's equation for internal pressure:

The tangential stress at radius r, $\sigma_t = \frac{p_i d_i^2}{4r^2} \frac{4r^2 + d_o^2}{d_o^2 - d_i^2}$

The radial stress at radius r, $\sigma_r = \frac{p_i d_i^2}{4r^2} \frac{4r^2 - d_o^2}{d_o^2 - d_i^2}$

The maximum tangential stress at the inner surface, $\sigma_{t\,max} = \frac{p_i (d_i^2 + d_o^2)}{d_o^2 - d_i^2}$

The maximum shear stress at the inner surface, $\tau_{max} = \frac{p_i d_o^2}{d_o^2 - d_i^2}$

The cylinder wall thickness for brittle materials based on the maximum normal stress theory

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t + p_i}{\sigma_t - p_i}\right)^{\frac{1}{2}} - 1\right]$$

The cylinder wall thickness for ductile materials based on the maximum shear theory

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t}{\sigma_t - 2p_i}\right)^{\frac{1}{2}} - 1\right]$$

Clavarino's equation is applicable to cylinders with closed ends and made of ductile materials. Clavarino's equation is based on maximum strain theory.

The thickness of a thick cylinrer based on Clavarino's equation is given by

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t' + (1 - 2\propto) p_i}{\sigma_t' - (1 + \propto) p_i}\right)^{\frac{1}{2}} - 1\right]$$

Birnie's equation for open ended cylinders made of ductile materials is given by

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t' + (1 - \propto) p_i}{\sigma_t' - (1 + \propto) p_i}\right)^{\frac{1}{2}} - 1\right]$$

2.7 EXAMPLE ON THICK CYLINDERS

E1: A pipe of 400 mm internal diameter and 100 mm thickness contains a fluid at 8 MPa. Find the maximum and minimum hoop stress across the section. Also sketch the hoop stress and radial stress distribution across the section.

Solution:

$\sigma_t = a + (b/r^2)$ **and** $\sigma_r == a - (b/r^2)$

At r = 200 mm, $\sigma_r = -8$ Mpa.

At r= 300 mm, $\sigma_r = 0$

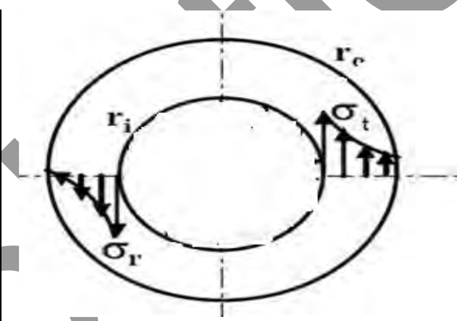
Determine the values of a & b.

a= 6.4 b=576000

$\sigma_t = 6.4 + 576000/r^2$ $\sigma_r = 6.4 - 576000/r^2$

Calculate the values of Hoop stress and Radial stress at 200,225,250,275 & 300 mm radius. Plot the variation on a graph sheet. Variation shown in the figure is not to scale.

r(mm)	σ_t (MPa)	σ_r (MPa)
200	20.8	-8.0
225	17.77	- 4.97
250	15.61	- 2.81
275	14.0	- 1.21
300	12.8	0



Problems for practice:

EP1: The piston rod of a hydraulic cylinder exerts an operating force of 10kN. The friction due to piston packings and stuffing box is equivalent to 10% of the operating force. The pressure in the cylinder is 10 MPa. The cylinder is made of cast iron FG 200 and the factor of safety is 5. Determine the diameter and thickness of the cylinder.

EP2: The inner diameter of a cylindrical tank for storing a liquefied gas is 250mm.The gas pressure is limited to 15 MPa. The tank is made of plain carbon steel 10C4 (S_{ut}= 340 MPa & μ = 0.27) and the factor of safety is 5. Calculate the thickness of the cylinder.

2.8 INTRODUCTION TO COMPOUND CYLINDERS

In thick walled cylinders subjected to internal pressure only, it can be seen from the equation of the hoop stress that the maximum stresses occur at the inside radius and this can be given by:

$$\sigma_{t\max} = \frac{p_i(d_o^2 + d_i^2)}{d_o^2 - d_i^2}$$

This means that as p_i increases σ_t may exceed yield stress even when $p_i < \sigma_{yield.}$ Furthermore, it can be shown that for large internal pressures in thick walled cylinders the wall thickness is required to be very large. This is shown schematically in figure 2-10.

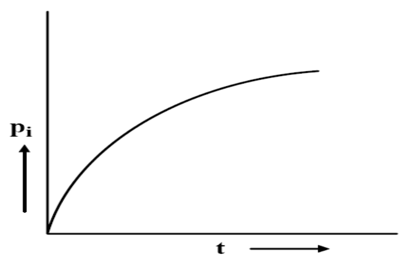


Figure 2.10 Variation of wall thickness with the internal pressure in thick cylinder

This means that the material near the outer edge of the cylinder is not effectively used since the stresses near the outer edge gradually reduce.

In order to make thick-walled cylinders that resist elastically large internal pressure and make effective use of material at the outer portion of the cylinder the following methods of pre-stressing are used:

- Shrinking a hollow cylinder over the main cylinder. (Compound cylinders)
- Multilayered or laminated cylinders.
- Autofrettage or self hooping.

Compound cylinders

An outer cylinder (jacket) with the internal diameter slightly smaller than the outer diameter of the main cylinder is heated and fitted onto the main cylinder. When the assembly cools down to room temperature, a compound cylinder is obtained. In this process the main cylinder is subjected to an external pressure leading to radial compressive stresses at the interface (P_c) as shown in figure 2.11.

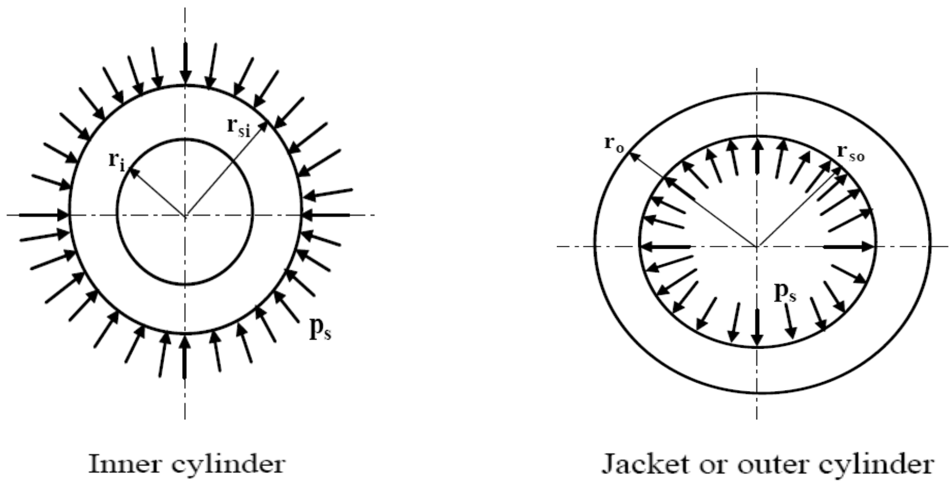


Figure 2.11 contact stress Pc in a compound cylinder

The outer cylinder is subjected to an internal pressure leading to tensile circumferential stresses at the interface (P_c) as shown in figure 2.11. Under these conditions as the internal pressure increases, the compression in the internal cylinder is first released and then only the cylinder begins to act in tension.

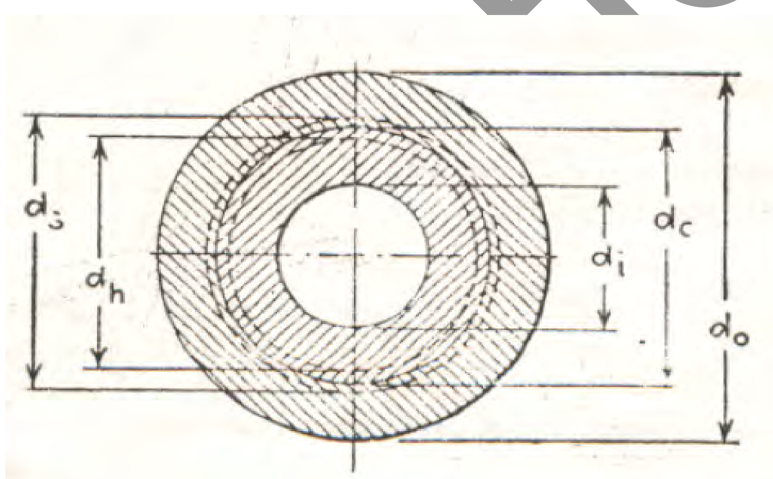


Figure. 2.11 Compound cylinders

Compound cylinders:

The tangential stress at any radius r for a cylinder open at both ends and subjected to internal pressure (Birnie's equation)

$$\sigma_t = (1-\mu)\frac{p_i d_i^2}{d_o^2 - d_i^2} + (1+\mu)\frac{p_i d_i^2 d_o^2}{4r^2(d_o^2 - d_i^2)}$$

The Radial stress at any radius r for a cylinder open at both ends and subjected to internal pressure (Birnie's equation)

$$\sigma_r = (1-\mu)\frac{p_i d_i^2}{d_o^2 - d_i^2} - (1+\mu)\frac{p_i d_i^2 d_o^2}{4r^2(d_o^2 - d_i^2)}$$

The tangential stress at the inner surface of the inner cylinder

$$\sigma_{t-i} = \frac{-2p_c d_c^2}{d_c^2 - d_i^2}$$

The tangential stress at the outer surface of the inner cylinder

$$\sigma_{t-c} = -p_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right)$$

The tangential stress at the inner surface of the outer cylinder

$$\sigma_{t-c} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right)$$

The tangential stress at the outer surface of the outer cylinder

$$\sigma_{t-o} = \frac{2p_c d_c^2}{d_o^2 - d_c^2}$$

Total shrinkage allowance when two cylinders are made of two different materials

$$B = p_c d_c\left[\frac{d_c^2 + d_i^2}{E_s(d_c^2 - d_i^2)} + \frac{d_o^2 + d_c^2}{E_h(d_o^2 - d_c^2)} - \frac{\mu_s}{E_s} + \frac{\mu_h}{E_h}\right]$$

When both the cylinders are made of same material the pressure between the cylinders is given by the equation

$$p_c = \frac{BE(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)}$$

Laminated cylinders

The laminated cylinders are made by stretching the shells in tension and then welding along a longitudinal seam. This is shown in figure 2.12.

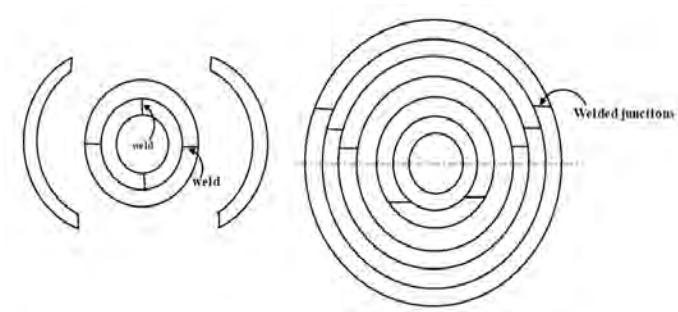


Figure.2.12 Laminated cylinder

Autofrettage

Pressure vessels are now widely used in nuclear power plants for steam and power generation. Other pressure vessel applications may involve pressures as high as 1380 MPa and temperatures of up to 300 °C, resulting in the pressure vessel material holding immense potential energy exerted by the working fluid. The process fluid may also be a source of hydrogen embrittlement and/or stress corrosion cracking. Such high-pressure vessels require proper understanding of the stress levels and their distributions in order to have fail-safe designs or even to minimize the probability of disruptive failures. Past pressure vessel catastrophic failures, arising from lack of understanding of stress levels, material properties and fluid/structure environmental interactions, particularly early in the last century, were very expensive in terms of losses in materials and human life, and they were the main impetus for the early studies of stresses in cylinders of various materials.

High-pressure vessels are now of great importance in many industries and their economic use often depends upon the occurrence of small, controlled, permanent deformations. Before commissioning, pressure vessels are normally pressure tested at an overstrain pressure of 1.25–1.5 times the design pressure in order to test for leakages. This process results in yielding of the bore and may also advantageously lead to catastrophic failure for poorly designed or fabricated vessels. Vessels with brittle characteristics may also fail at this stage. After overstraining, residual stresses are left in the cylinder and the nature of these residual stresses is now widely known. However, the residual stress levels are not documented for use in service or during de-rating after periodic inspections. In service, the vessels are able to carry a much higher load before re-yielding than would be the case without the leak test. Overstraining beyond the leak test pressure is usually carried out during manufacture and this technique is called **Autofrettage or self-hooping.**

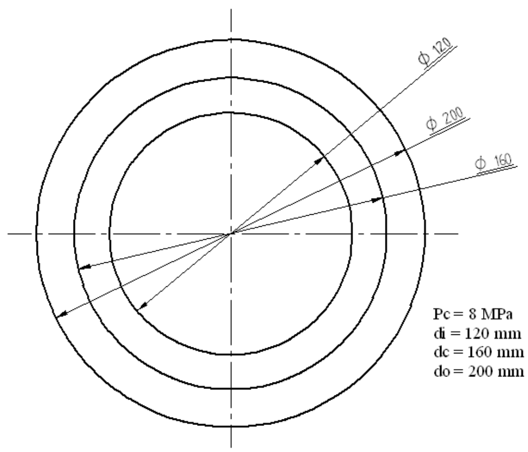
In some applications of thick cylinders such as gun barrels no inelastic deformation is permitted. But for some pressure vessel design satisfactory function can be maintained until the inelastic deformation that starts at inner bore spreads completely over the wall thickness. With the increase in fluid pressure yielding would start at the inner bore and then with further increase in fluid pressure yielding would spread outward. If now the pressure is released the outer elastic layer would regain its original size and exert a radial compression on the inner shell and tension on the outer region.

This gives the same effect as that obtained by shrinking a hoop over an inner cylinder. This is known as **Self- hooping** or **Autofrettage**. This allows the cylinder to operate at higher fluid pressure. For a given autofrettage fluid pressure a given amount of inelastic deformation is produced and therefore in service the same fluid pressure may be used without causing any additional inelastic deformation.

2.9 Examples on compound cylinders

E1: A shrink fit assembly formed by shrinking one tube over another tube is subjected to an internal pressure of 60 MPa. Before the fluid is admitted, the internal & external diameters of the assembly were 120 & 200mm and the diameter of the junction was 160 mm. After shrinking, the contact pressure at the junction was 8 MPa, plot the resultant stress distribution in each cylinder after the fluid has been admitted.

Solution:



<u>Hoop stresses due to shrink fit in both the cylinders</u>

The tangential stress at the inner surface of the inner cylinder

$$\sigma_{t-i} = \frac{-2p_c d_c^2}{d_c^2 - d_i^2} = -36.5 \text{ MPa}$$

The tangential stress at the outer surface of the inner cylinder

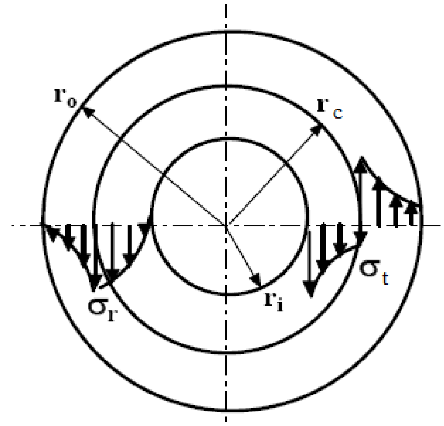
$$\sigma_{t-c} = -p_c\left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2}\right) = -28.57 \text{ MPa}$$

The tangential stress at the inner surface of the outer cylinder

$$\sigma_{t-c} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right) = 36.44 \text{ MPa}$$

The tangential stress at the outer surface of the outer cylinder

$$\sigma_{t-o} = \frac{2p_c d_c^2}{d_o^2 - d_c^2} = 28.44 \text{ MPa}$$

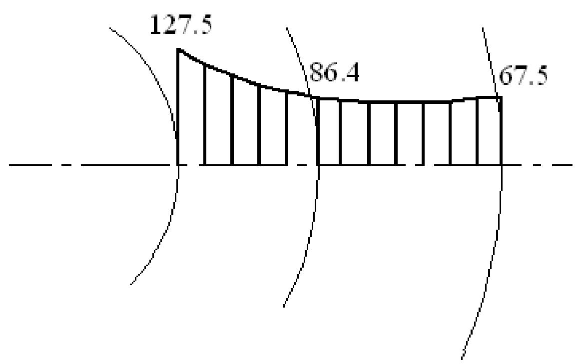


Variation of tangential stresses due to shrink fit alone

Stress distribution only due to internal pressure alone

Lame's equation for internal pressure:

The tangential stress at radius r, $\sigma_t = \dfrac{p_i d_i^2}{4r^2}\left(\dfrac{4r^2 + d_o^2}{d_o^2 - d_i^2}\right)$



At r= 60 mm, σ_t = +127.5 MPa (tensile)
At r= 80 mm σ_t = +86.4 MPa (tensile)
At r=100 mm σ_t = + 67.5 MPa (tensile)

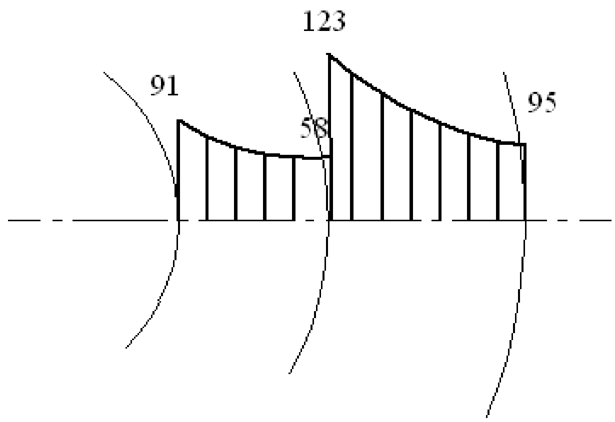
Resultant stress distribution

Resultant Stress at the inner surface of the inner cylinder= -36.57+127.5 = + 90.3 MPa
Resultant Stress at the outer surface of the inner cylinder= -28.57+86.41 = + 57.19 MPa

Resultant Stress at the inner surface of the outer cylinder=36.44+86.48

= + 122.92 MPa

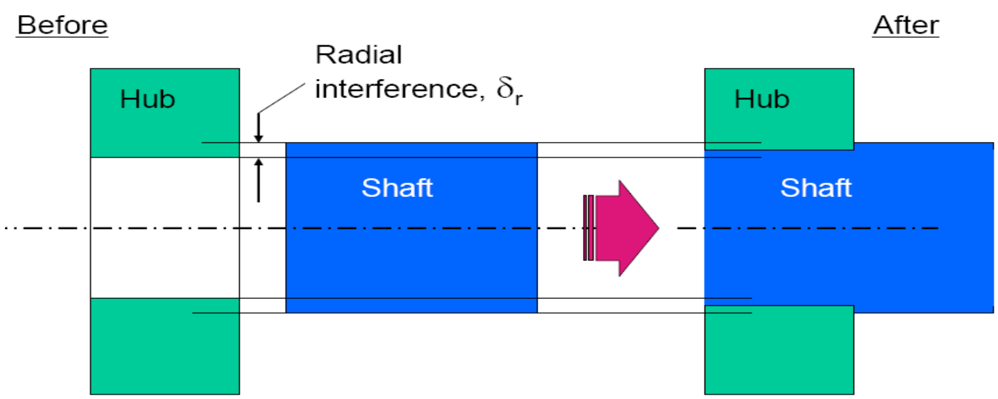
Resultant Stress at the outer surface of the outer cylinder= 28.44+67.5 = + 95.94 MPa



Resultant stress distribution

2.10 Introduction to press and shrink fits

Press fits, or interference fits, are similar to pressurized cylinders in that the placement of an oversized shaft in an undersized hub results in a radial pressure at the interface. In a press fit, the shaft is compressed and the hub is expanded. There are equal and opposite pressures at the mating surfaces. The relative amount of compression and expansion depends on the stiffness (elasticity and geometry) of the two pieces. The sum of the compression of the shaft and the expansion of the hub equals the interference introduced.



Commonly used Interference Fits

H7/p6 **Locational Interference:** Fit for parts requiring rigidity and alignment with prime accuracy of location, but without special bore pressure requirements.

H7/r6 **Medium Drive fit** for ferrous parts and light drive fit for non ferrous parts that can be dismantled.

H7/s6 Permanent or semi -permanent assemblies of steel and cast iron. Fit for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron. For light alloys this gives a press fit.

H7/u6 **Force fit:** Fit suitable for parts which can be highly stressed or for shrink fits where the heavy pressing forces required are impractical.

Analysis of Press Fits:

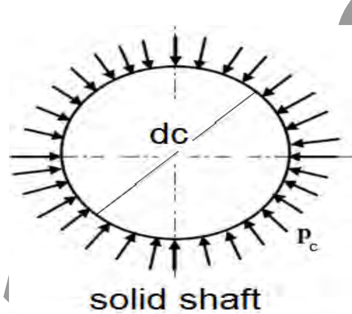
Start by finding the interface pressure.

$$p_c = \frac{BE(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)}$$

Where B is the shrinkage allowance for hub and shaft of the same material with modulus of elasticity E.

If the shaft is solid, $d_i = 0$ and

$$p_c = \frac{EB}{2d_c}\left[1 - \frac{d_c^2}{d_o^2}\right]$$



If the shaft and hub are of different materials

$$p_c = \frac{B}{d_c\left[\frac{1}{E_s} + \frac{d_o^2 + d_c^2}{E_h(d_o^2 - d_c^2)} - \frac{\mu_s}{E_s} + \frac{\mu_h}{E_h}\right]}$$

Once we have the pressure, we can use the cylinder equations to compute the hoop stresses at the interface. The hoop stress at the ID of the hub is tensile.

$$\sigma_{t-c} = p_c \left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} \right)$$

If shaft is solid,

$$\sigma_{t-c} = -p_c \left(\frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} \right) = -p_c$$

The hoop stress at the OD of the shaft is compressive.

Strain Analysis of Press Fits

The press fit has no axial pressure and it is a biaxial stress condition. The circumferential strain which equals the radial strain (because C = 2pr):

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu \sigma_r}{E}$$

Because the radial change, we get the increase in Inner diameter of the outer member (hub):

$$\Delta_h = p_c d_c \left[\frac{d_o^2 + d_c^2}{E_h (d_o^2 - d_c^2)} + \frac{\mu_h}{E_h} \right]$$

The decrease in Outer diameter of the inner member (shaft):

$$\Delta s = p_c d_c \left[\frac{d_c^2 + d_i^2}{E_s (d_c^2 - d_i^2)} - \frac{\mu_s}{E_s} \right]$$

The total shrinkage allowance

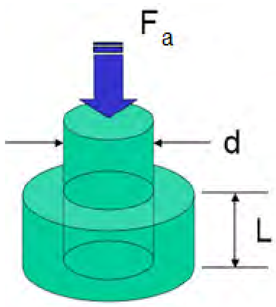
$$B = \Delta_h + \Delta s$$

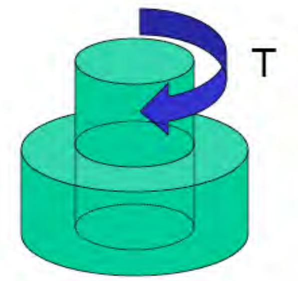
Forces resulting in interference fits:

The total axial force required to assemble a force fit (approximately): $F_a = \pi d L f p_c$

The torque transmitted: $T = \frac{F_a d}{2} = \frac{\pi d^2 L f p_c}{2}$

Where, f= coefficient of friction





SHRINK FITS

The temperature to which a piece to be shrunk must be heated for assembling:

$$t_2 \geq \left(\frac{B}{\alpha d_c} + t_1 \right)$$

t_2 is the final temperature to which hub must be heated
t_1 is the temperature of the shaft .
α is the thermal expansion coefficient of material of the hub.

Examples on Press and shrinkfit:

E1. A cast steel crank is to be shrunk on a 250 mm steel shaft. The outside diameter of the crank hub is 444.5 mm. The maximum tangential stress in the hub is limited to 150 MPa. Coefficient of friction between the hub and the shaft is 0.15. Determine:

- The required bore of the crank.
- Probable value of the normal pressure between the hub and the shaft.
- The torque that may be transmitted with out using a key, if the hub length is 250 mm.

Solution:

The hoop stress is maximum at the ID of the hub and is given by:

$$\sigma_{t-c} = p_c \left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} \right)$$

Limiting value of hoop stress is given to be 150 MPa
do=444.5mm , dc=250mm. Substitute and determine Pc.
Pc= 78 MPa.

Determine B, the total shrinkage allowance for hub and shaft of the same material, with modulus of elasticity, E.
If the shaft is solid, $d_i = 0$.

$$p_c = \frac{BE(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2 d_c^3 (d_o^2 - d_i^2)}$$

$E=200x10^3$ MPa

B = total shrinkage allowance = 0.28mm

Assuming that the assembly is done using selective assembly technique,
The required bore of the crank = basic size- shrinkage allowance
= 250-0.28
= 249.72 mm

The total axial force required to assemble a force fit (approximately): $F_a = \pi dLfp_c$

f = 0.15, Pc=78 MPa, L=75mm, d=250mm

Substitute and determine the force required to assemble

Fa= 2.29×10^6 N

If this force calculated can not be obtained with the **regular presses,** it becomes obvious that we should reduce the interference at the time of assembly. This can be achieved by heating the hub and expanding its internal diameter by a calculated amount and then slipping it over the shaft. This produces the required shrink fit once assembly cools and reaches the room temperature.

The torque transmitted, $T = \frac{F_a d}{2} = \frac{\pi d^2 Lfp_c}{2}$

Fa= 2.29×10^6 N, d= 250 mm. Substitute and determine T.

T= 287×10^6 N-mm.

The torque calculated is based on shrink fit only (without the presence of the key).

E2. A steel hub of 440 mm outside diameter, 250mm inside diameter & 300 mm length has an interference fit with a shaft of 250 mm diameter. The torque to be transmitted is 30×10^4 N-m. The permissible stress for the material of the shaft & hub is 120 MPa. The coefficient of friction is 0.18. Determine:

- The contact pressure
- The interference required
- The tangential stress at the inner and outer surface of the hub.
- Force required to assemble.
- Radial stress at the outer & inner diameter of the hub.

The contact pressure can be determined based on the torque transmitted.

The torque transmitted, $T = \frac{F_a d}{2} = \frac{\pi d^2 Lfp_c}{2}$

T= 30×10^4 N-m= 30×10^7 N-mm. d = 250mm,
L= 300 mm, f=0.18. Substituting the above values,
P_c = 56.6 MPa.

The hoop stress at the ID of the hub is tensile and is given by:

$$\sigma_{t-c} = p_c\left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2}\right)$$

Substitute do= 440mm, dc=250mm and obtain the value of hoop stress at the ID of the hub.

$$\sigma_{t-c} = 110.56 \text{ MPa } < 120 \text{ MPa}$$

The tangential stress at the outer surface of the outer cylinder, $\sigma_{t-o} = \frac{2p_c d_c^2}{d_o^2 - d_c^2}$

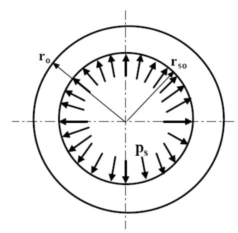
di =0, Pc = 56.6 MPa and determine B. **B=0.21 mm.**
The interference required is 0.21 mm.

The total axial force required to assemble a force fit (approximately):

$$F_a = \pi d L f p_c$$

$$= \mathbf{2.4 \times 10^6 \ N}$$

$$\sigma_r = \frac{p_i d_i^2}{4r^2}\left(\frac{4r^2 - d_o^2}{d_o^2 - d_i^2}\right)$$



Radial Stress at the inner surface of the hub= -Pc= -56 .6 MPa.
Radial Stress at the outer surface of the hub = 0

E3. A steel hub of 50 mm outside diameter, 28mm inside diameter & 75 mm length is to have an interference fit with a shaft of 28 mm diameter employing a heavy press fit not using selective assembly. Determine:

- The tolerances and interference of mating parts.
- The maximum radial contact pressure
- The actual maximum and minimum tangential stress at the contact surface of the hub.
- The maximum and minimum tangential stress based on maximum strain theory.
- Force required to assemble the hub onto the shaft. The coefficient of friction is 0.18.
- The maximum torque that can be transmitted by this assembly.

Choose 28 H7/s6 as the fit for the assembly.

Ø 28 s6= Upper limit=$28^{+0.048}$ =28.048 mm
= Lower limit=$28^{+0.035}$ =28.035 mm
Ø 28 H7 = upper limit=$28^{+0.021}$ =28.021 mm
= lower limit=$28^{+0.000}$ =28.000 mm.

Maximum Interference= Min.bore size- Max. shaft size
= 28.000-28.048= **- 0.048mm**

Minimum Interference= Max. bore size- Min. shaft size
= 28.021-28.035= - **0.014 mm**

The interference in the assemblies varies between the limits of a maximum of 0.048mm and a minimum of 0.014mm.

B_{max} =0.048mm B_{min} =0.014mm

The contact stress will be maximum when interference is maximum.

$$p_{c\max} = \frac{B_{\max} E(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)}$$

dc=28mm, do=50mm, E=210x10^3 MPa, B_{max} = 0.048 mm
& determine Pc max.
Pc max= 123.55 MPa.

$$p_{c\min} = \frac{B_{\min} E(d_c^2 - d_i^2)(d_o^2 - d_c^2)}{2d_c^3(d_o^2 - d_i^2)}$$

Pc min= 36.04 MPa.

The contact stress varies between the limits 123 MPa to 36 MPa.
The hoop stress at the ID of the hub is maximum and this also varies as the interference in the assemblies varies and the actual maximum and minimum at the ID is given by:

$$\sigma_{t_{\max}} = p_{c_{\max}} \left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} \right)$$

= + 236.44 MPa

$$\sigma_{t_{\min}} = p_{c_{\min}} \left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} \right)$$

= + 68.9 MPa

The hoop stress at the ID of the hub is maximum and this also varies as the interference in the assemblies varies and the equivalent maximum and minimum at the ID based on Strain theory is given by:

$$\sigma_{t-c} = p_c \left(\frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \mu \right)$$

Substituting poison's ratio=0.3 for steel and maximum and minimum radial contact stress Pc, determine $\sigma_{t_{\max}}$ & $\sigma_{t_{\min}}$ based on Max. strain theory.

$$\sigma_{t_{\max}} = 273MPa$$

$$\sigma_{t_{\min}} = +79.7 \text{ MPa}$$

Maximum torque can be transmitted when interference is maximum.
When interference is maximum, Pc is maximum.
The maximum torque can be calculated by:

$$T_{\max} = \frac{F_a d}{2} = \frac{\pi d^2 L f p_{c_{\max}}}{2}$$

T_{max} = 1.37x10^6 N-mm.

The maximum axial force required to assemble a force fit (approximately):

$$T_{\max} = \frac{F_{a_{\max}} d}{2}$$

$F_{a\,max}$= 97.8x10^3 N

UNFIRED PRESSURE VESSELS

An unfired pressure vessel is defined as a vessel or a pipe line for carrying, storing or receiving steam, gasses or liquids under pressure. I.S.2825 -1969 code for pressure vessels gives the design procedure for welded pressure vessels of ferrous materials, subjected to an internal pressure of 0.1 MPa to 20 MPa. This code does not cover steam boilers, nuclear pressure vessels or hot water storage tanks.

Classification of pressure vessels:

Pressure vessels are classified into 3 groups:

- Class 1
- Class 2
- Class 3

Class 1 pressure vessels are used to contain lethal and toxic substances. Substances like Hydrocyanic acid, carbonyl chloride or mustard gas which are dangerous to human life are contained by class 1 pressure vessels. Class 1 pressure vessels are used when the operating temperature is less than -20^0 C. Safety is of paramount importance in the design of these pressure vessels. Two types of welding are employed in this case:

- double welded butt joint with full penetration
- Single welded butt joint with backing strip.

Welded joints of class 1 pressure vessels are fully radiographed.

Class 3 pressure vessels are used for relatively light duties. They are not recommended for service when the operating temperature is less than 0^0 C or more than 250^0 C. The maximum pressure is limited to 1.75 MPa. The maximum shell thickness is limited to 16 mm. They are usually made from carbon and low alloy steels. Welded joints are not radiographed.

Class 2 pressure vessels are those which do not come under class 1 or class 3. The maximum shell thickness is limited to 38mm. Welded joints used are similar to class 1 but the joints are only spot radiographed.

THICKNESS OF SHELLS SUBJECTED TO INTERNAL PRESSURE

The minimum thickness of shell plates exclusive of corrosion allowance in mm

a) For cylindrical shells: $t = \frac{pD_i}{2fJ - p} = \frac{pD_o}{2fJ + p}$

b) For spherical shells: $t = \frac{pD_i}{4fJ - p} = \frac{pD_o}{4fJ + p}$

Where J is the joint efficiency, f is the allowable tensile stress & p is the internal pressure, Di & Do are the inside and outside diameter of the shell respectively.

Corrosion in the pressure vessels can be due to:

- Chemical attack where the metal is dissolved by a chemical reagent.
- Rusting due to air and moisture.
- Erosion due to flow of reagents over the surfaces of the walls at high velocities
- Scaling or oxidation at high temperatures.

Corrosion allowance (CA) is additional metal thickness over and above that required to withstand internal pressure. A minimum corrosion allowance of 1.5 mm is recommended unless a protective coating is employed.

END CLOSURES

Formed heads used as end closures for cylindrical pressure vessels are:

- Doomed heads
- Conical heads.

Doomed heads can further be classified into three groups:

- Hemispherical heads
- Semi ellipsoidal heads
- Torispherical heads

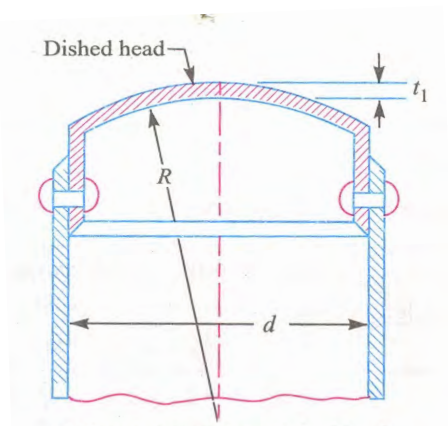
The section below gives various formulae connected with the calculations of the thickness of end closures.

THICKNESS OF SPHERICALLY DISHED COVERS

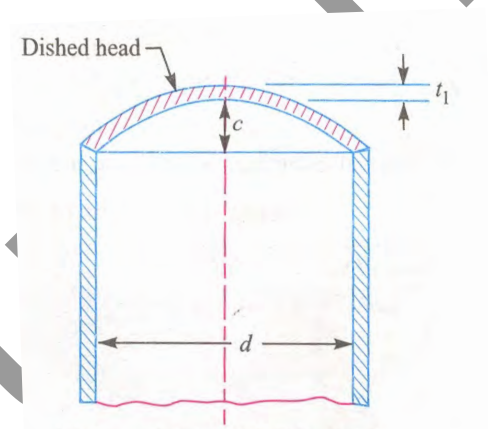
The thickness of spherically dished ends secured to the shell to a flange connection by means of bolts shall be calculated by the formula (provided that the inside crown radius R of the dished cover does not exceed 1.3 times the shell inside diameter Di and the value of 100t/R is not greater than 10)

$$t = \frac{3}{2}\frac{pD_i}{fJ}$$

Where J = joint factor (table 8.2)



(a) Riveted or welded dished head.



(b) Integral or welded dished head.

UNSTAYED FLAT HEADS AND COVERS

Thickness of flat heads and covers:
The thickness of flat unstayed circular heads and covers shall be calculated by the formula when the head or cover is attached by bolt's causing an edge movement

$$t = CD\sqrt{p/f}$$

For 'C' and 'D' refer table 8.1

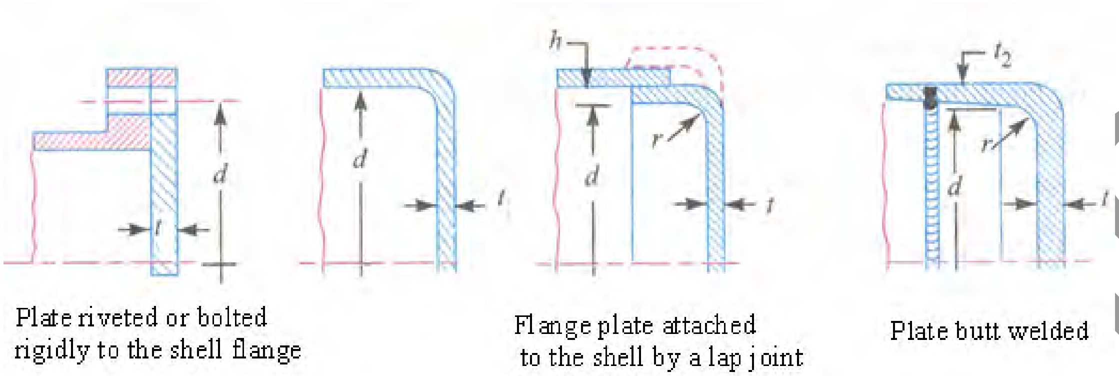
The thickness of flat unstayed non-circular heads and covers shall be calculated by the formula

$$t = CZa\sqrt{p/f}$$

Where Z= the factor for non-circular heads depending upon the ratio of short span to long span a/b

Unstayed flat plates with uniformly distributed loads

Integral flat head



Circular plate uniformly loaded:

The thickness of the plate with a diameter D supported at the circumference and subjected to a uniformly distributed pressure *p* over the total area.

$$t = C_1 D\sqrt{p/f}$$

The maximum deflection

$$y = \frac{c_2 D^4 p}{Et^3}$$

Where c_1 and c_2 are coefficients from table 8.5

Circular plate loaded centrally:

The thickness of a flat cast iron plate supported freely at the circumference with a diameter D and subjected to a load F distributed uniformly over an area $\pi D^2_o/4$

$$t = 1.2\sqrt{(1-0.670 D_o/D)\frac{F}{f}}$$

The deflection in mm, $y = \dfrac{0.12 D^2 F}{Et^3}$

Grashof's formula for the thickness of a plate rigidly fixed around the circumference with the above given type loading,

$$t = 0.65\left[\frac{F}{f}\log_e(D/D_o)\right]^{1/2}$$

The deflection in mm, $y = \dfrac{0.055 D^2 F}{Et^3}$

Rectangular plates:

The thickness of a rectangular plate subjected to uniform load (Grashof & Bach)

$$t = abc_3\left[\frac{p}{f(a^2+b^2)}\right]^{\frac{1}{2}}$$

The thickness of a rectangular plate on which a concentrated load F acts at the intersection of diagonals

$$t = c_4\left[\frac{abF}{f(a^2+b^2)}\right]^{\frac{1}{2}}$$

Where a = length of plate, mm
b = breadth of plate, mm
c_3, c_4 = coefficients from table 8.5

Examples on Cylinder heads

E1. A cast iron cylinder of inside diameter 160mm is subjected to a pressure of 15 N/mm^2. The permissible working stress may be taken as 25MPa for cast iron. If the cylinder is closed by means of a flat head cast integral with cylinder walls, find the thickness of the cylinder wall and the head.

Solution: Since the cylinder is made of cast iron, use the normal stress theory to calculate the wall thickness.

The cylinder wall thickness for brittle materials based on maximum normal stress theory

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t + p_i}{\sigma_t - p_i}\right)^{\frac{1}{2}} - 1\right]$$

d_i= 160 mm, σ_t =25 MPa, P_i= 15MPa
Substitute and find out the value of the thickness of the cylinder wall.

t= 80mm

Thickness of cylinder head
Circular plate uniformly loaded:
The thickness of the plate with a diameter D supported at the circumference and subjected to a uniformly distributed pressure *p* over the total area.

$$t = C_1 D\sqrt{p/f}$$

Since the head is cast integral with the wall, c_1 = 0.44 (from table 8.5)
D=160 mm, p=15 MPa, f=25 MPa,
Substitute and determine the value of t – thickness of the head.
t= 54.5mm or 55mm.

E2. The steam chest of a steam engine is covered by a rectangular plate 240x380 mm in size. The plate is made of cast iron and is subjected to a steam pressure of 1.2 N/mm^2 and the plate is assumed to be uniformly loaded and freely supported at the edges. Find the thickness of the plate if the allowable stress of the cast iron plate is 35 MPa.

Solution:

Rectangular plates:

The thickness of a rectangular plate subjected to uniform load (Grashof & Bach)

$$t = abc_3\left[\frac{p}{f(a^2+b^2)}\right]^{\frac{1}{2}}$$

Where a = length of plate, mm
b = breadth of plate, mm
c_3, c_4 = coefficients from table 8.5

a= 380mm, b= 240 mm, f= 35 MPa, p= 1.2 MPa,
c_3 = 0.75 for a rectangular plate with the edges freely supported
Determine the thickness t.

t= 28.2 mm or 30 mm.

Problems for practice

EP1. A steel cylinder is 160 mm ID and 320 mm OD. If it is subject to an internal pressure of 150 MPa, determine the radial and tangential stress distributions and show the results on a plot (using a spreadsheet). Determine the maximum shear stress in the cylinder. Assume it has closed ends.
(Ans: σ_t = 250 to 100 MPa, σ_r = 0 to ±150 MPa, max shear stress = 200 MPa.)

EP2. A cylinder is 150 mm ID and 450 mm OD. The internal pressure is 160 MPa and the external pressure is 80 MPa. Find the maximum radial and tangential stresses and the maximum shear stress. The ends are closed.
(Ans: σ_t= 20 to ±60 MPa, σ_r= ±80 to ±160 MPa, max shear stress = 90 MPa.)

EP3. A 400 mm OD steel cylinder with a nominal ID of 240 mm is shrunk onto another steel cylinder of 240 mm OD and 140 mm ID. The radial interference is 0.3 mm. Use Young's Modulus E = 200 GPa and Poisson's Ratio μ = 0.3. Find the interface pressure p_c and plot the radial and tangential stresses in both cylinders. Then find the maximum internal pressure which may be applied to the assembly if the maximum tangential stress in the inside cylinder is to be no more than 140 MPa.
(Ans : p_c= ±120.3 MPa. inner cylinder : σ_t= -365 to -244 MPa, σ_r= 0 to -120.3 MPa. outer cylinder : σ_t= 256 to 135 MPa,
σ_r= -120.3 to 0 MPa. Maximum internal pressure = 395 MPa.)

EP4. A cylinder with closed ends has outer diameter D and a wall thickness t = 0.1D. Determine the %age error involved in using thin wall cylinder theory to calculate the maximum value of tangential stress and the maximum shear stress in the cylinder.
(Ans: tangential stress ± 9.75% : max. shear stress ± 11.1%)

EP5. A gun barrel has an ID of 150 mm. It is made by shrink fitting an outer sleeve of ID 190 mm and OD of 210 mm over an inner sleeve of ID 150.0 mm and OD of 190.125 mm. The maximum pressure seen in the barrel is 50 MPa. The inner sleeve is made from steel with yield strength 320 MPa and the inner sleeve is made from steel with yield strength 670 MPa.
Determine the factor of safety using the Von Mises yield criterion.
Determine the minimum temperature difference needed between the sleeves to permit shrink fitting to take place.

DESIGN OF SPRINGS

Springs are flexible machine elements

Used to:

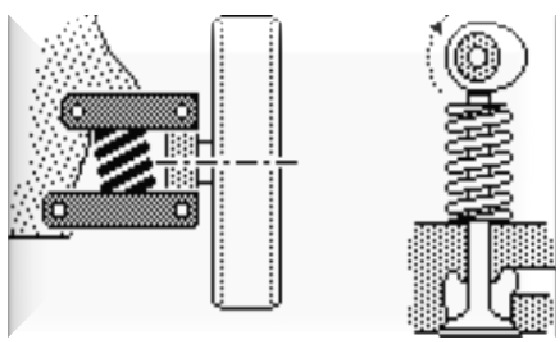
- Exert force
- Store energy

Definition of spring

A spring is an elastic object used to store mechanical energy. Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released. In other words it is also termed as a resilient member.

A spring is a flexible element used to exert a force or a torque and, at the same time, to store energy.

The force can be a linear push or pull, or it can be radial, acting similarly to a rubber band around a roll of drawings.



The torque can be used to cause a rotation, for example, to close a door on a cabinet or to provide a counterbalance force for a machine element pivoting on a hinge.

Objectives of spring

To provide Cushioning, to absorb, or to control the energy due to shock and vibration.

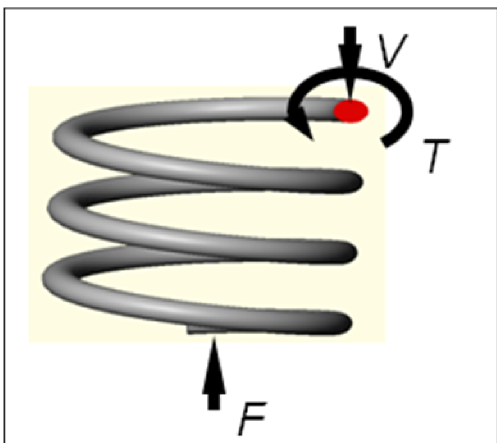
Car springs or railway buffers to control energy, springs-supports and vibration dampers

To Control motion

Maintaining contact between two elements (cam and its follower) Creation of the necessary pressure in a friction device (a brake or a clutch)

To Measure forces

Spring balances, gages



To Store energy

In clocks or starters

Commonly used spring materials

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire

This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 120^0C.

Oil-tempered wire

It is a cold drawn, quenched, tempered, and general purpose spring steel. It is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 180^0C.

Chrome Vanadium

This alloy spring steel is used for high stress conditions and at high temperature up to 220^0C. It is good for fatigue resistance and long endurance for shock and impact loads.

Chrome Silicon:

This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250^0C.

Music wire

This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. It cannot be used at subzero temperatures or at temperatures above 120^0C.

Stainless steel:

Widely used alloy spring materials.

Phosphor Bronze / Spring Brass:

It has good corrosion resistance and electrical conductivity. it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures

Springs can be classified according to the direction and the nature of the force exerted by the spring when it is deflected.

uses	Types of springs
Push	Helical compression spring, Belleville spring, Torsion spring, force acting at the end of torque arm. flat spring, such as a cantilever spring or leaf spring
Pull	Helical extension spring, Torsion spring, force acting at the end of torque arm. Flat spring, such as a cantilever spring or leaf spring, Draw bar spring (special case of the compression spring) constant – force spring.
Radial torque	Garter spring, elastomeric band, spring clamp, Torsion spring, Power spring

Push Types Springs

Helical compression spring

Belleville spring

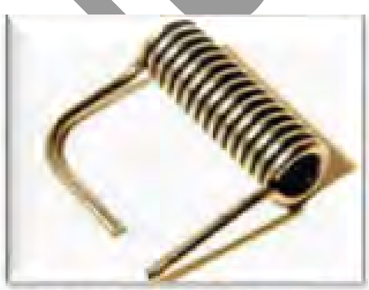
Torsion spring

Flat spring

leaf spring

Pull Types Springs

Helical extension spring



Torsion spring

Flat spring`

Draw bar spring

constant – force spring

Radial torque types springs

Garter spring

spring clamp

Torsion spring

Power spring

Spring manufacturing processes

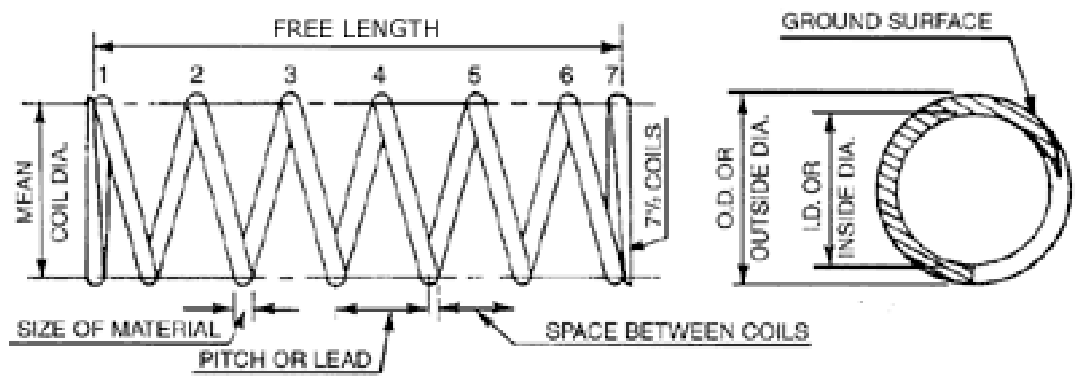
If springs are of very small diameter and the wire diameter is also small then the springs are normally manufactured by a cold drawn process through a mangle. However, for very large springs having also large coil diameter and wire diameter one has to go for manufacture by hot processes. First one has to heat the wire and then use a proper mangle to wind the coils.

Two types of springs which are mainly used are, helical springs and leaf springs. We shall consider in this course the design aspects of two types of springs.

HELICAL SPRING

Definition

It is made of wire coiled in the form of helix having circular, square or rectangular cross section.



Terminology of helical spring

The main dimensions of a helical spring subjected to compressive force are shown in the figure. They are as follows:

d = wire diameter of spring (mm)

D_i = inside diameter of spring coil (mm)

D_o =outside diameter of spring coil (mm)

D = mean coil diameter (mm)

Therefore

$$D = \frac{D_i + D_o}{2}$$

There is an important parameter in spring design called spring index. It is denoted by letter C. The spring index is defined as the ratio of mean coil diameter to wire diameter. Or

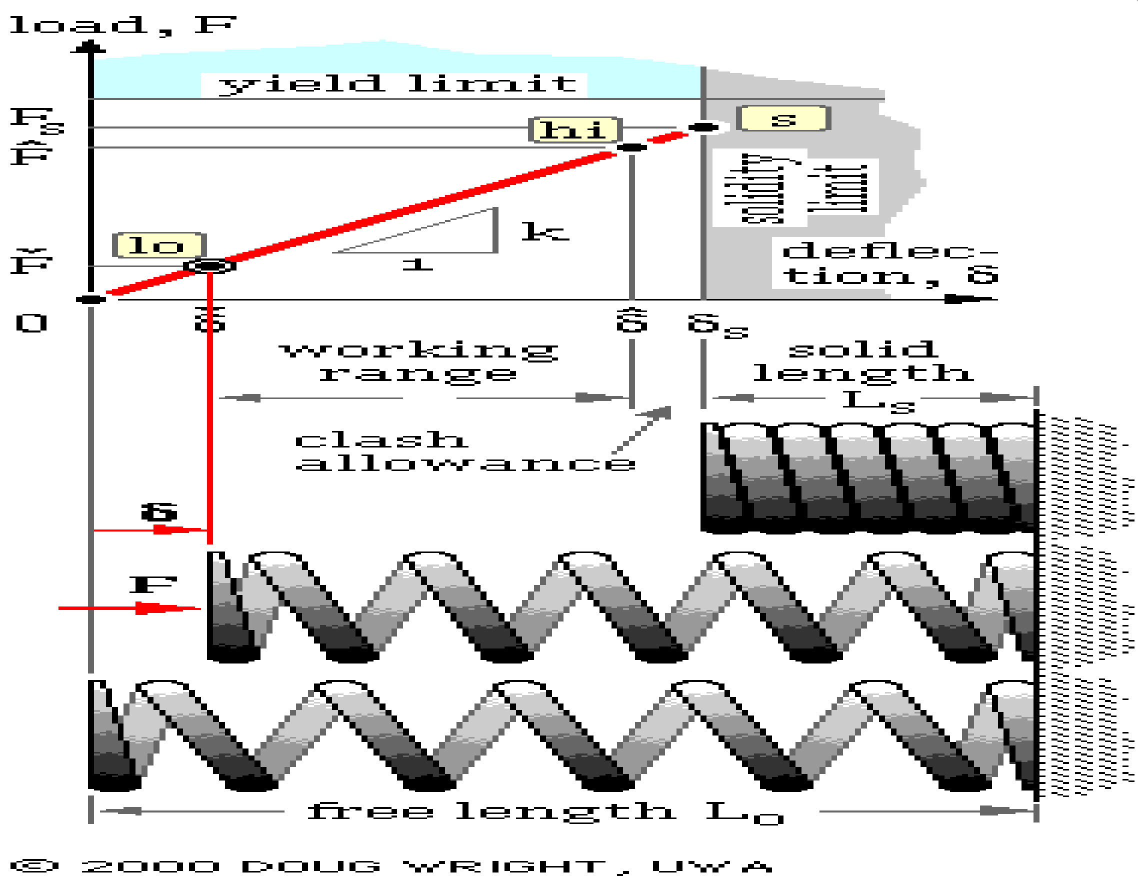
$C = D/d$

In design of helical springs, the designer should use good judgment in assuming the value of the spring index C. The spring index indicates the relative sharpness of the curvature of the coil.

A low spring index means high sharpness of curvature. When the spring index is low ($C < 3$), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high ($C > 15$), it results in large variation in coil diameter. Such a spring is prone to buckling and also tangles easily during handling. Spring index from 4 to 12 is considered better from manufacturing considerations.

Therefore, in practical applications, the spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

There are three terms - free length, compressed length and solid length that are illustrated in the figure. These terms are related to helical compression spring. These lengths are determined by following way



1) **Solid length:** solid length is defined as the axial length of the spring which is so compressed, that the adjacent coils touch each other. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by.

Solid length = N_t d

Where N_t = total number of coils

2) **Compressed length:** Compressed length is defined as the axial length of the spring that is subjected to maximum compressive force. In this case, the spring is subjected to maximum deflection $\boldsymbol{\delta}$. When the spring is subjected to maximum force, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils.

The clashing allowance or the total axial gap is usually taken as 15% of the maximum deflection. Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition. In this case, the total axial gap is given by,

Total gap = (N_t-1) x gap between adjacent coils

3) **Free length:** Free length is defined as the axial length of an unloaded helical compression spring. In this case, no external force acts on the spring. Free length is an important dimension in spring design and manufacture. It is the length of the spring in free condition prior to assembly. Free length is given by,

Free length = compressed length + y

= solid length + total axial gap + y

The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state of spring. It is denoted by p. It is given by,

$$p = \frac{free\ length}{N_t - 1}$$

The stiffness of the spring (k) is defined as the force required producing unit deflection

Therefore

$$k = \frac{p}{\delta}$$

Where k= stiffness of the spring (N/mm)

F = axial spring force (N)

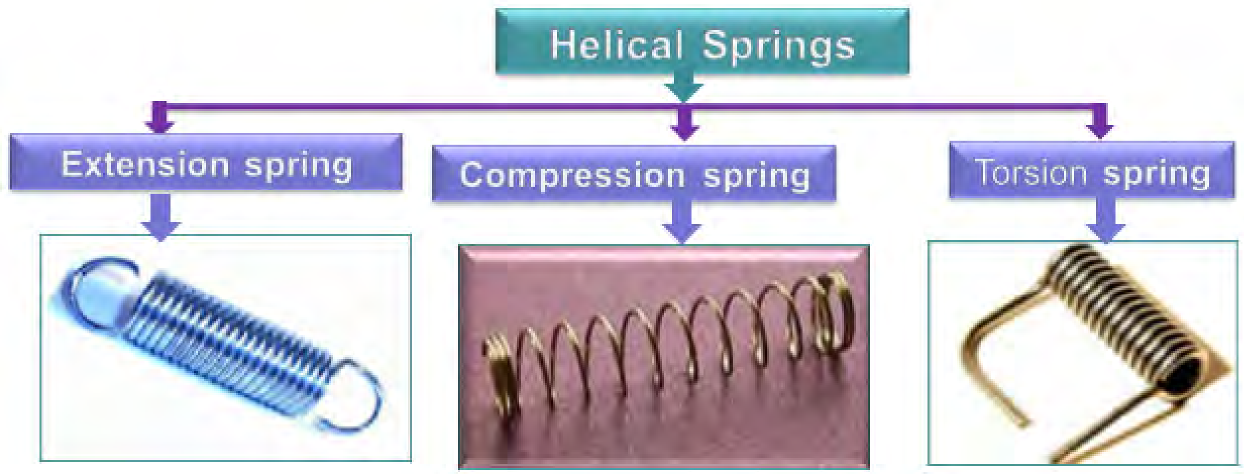
Y or $\boldsymbol{\delta}$ = axial deflection of the spring corresponding to force p (mm)

There are various names for stiffness of spring such as rate of spring, gradient of spring, scale of spring or simply spring constant. The stiffness of spring represents the slope of load deflection line. There are two terms are related to the spring coils, viz. active coils and inactive coils.

Active coils are the coils in the spring, which contribute to spring action, support the external force and deflect under the action of force. A portion of the end coils, which is in contact with the seat, does not contribute to spring action and called inactive coils. These coils do not support the load and do not deflect under the action o external force. The number of inactive coils is given by,

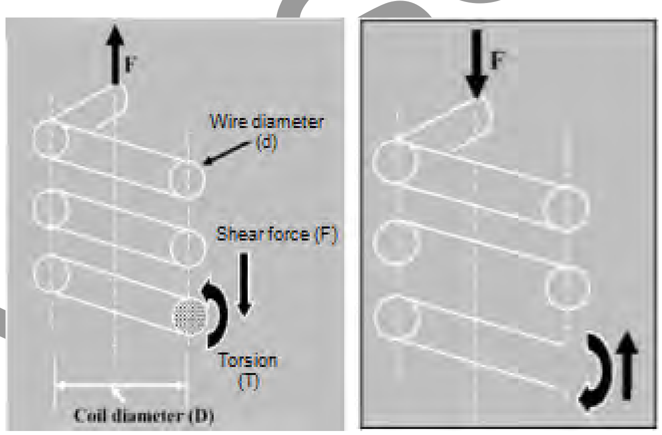
Inactive coils = $N_t - N$ where N = number of active coils

Classification of helical springs



Helical spring

The figures below show the schematic representation of a helical spring acted upon by a tensile load F and compressive load F. The circles denote the cross section of the spring wire.

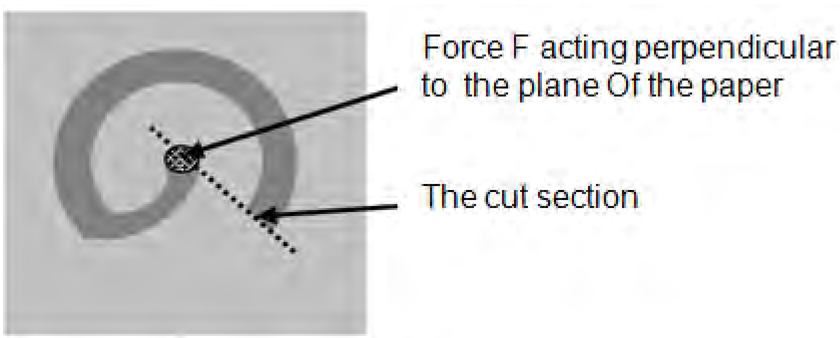


The cut section, i.e. from the entire coil somewhere we make a cut, is indicated as a circle with shade.

If we look at the free body diagram of the shaded region only (the cut section) then we shall see that at the cut section, vertical equilibrium of forces will give us force, F as indicated in the figure. This F is the shear force. The torque T, at the cut section and its direction is also marked in the figure.

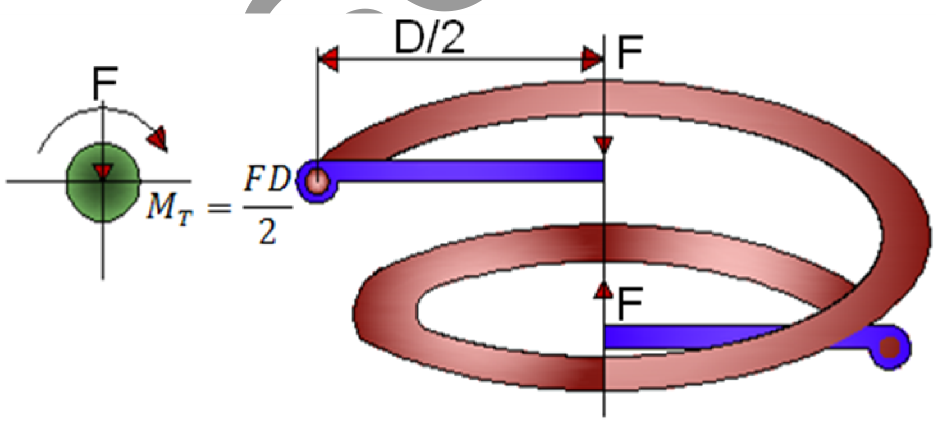
There is no horizontal force coming into the picture because externally there is no horizontal force present. So from the fundamental understanding of the free body diagram one can see that any section of the spring is experiencing a torque and a force. Shear force will always be associated with a bending moment.

However, in an ideal situation, when force is acting at the centre of the circular spring and the coils of spring are almost parallel to each other, no bending moment would result at any section of the spring (no moment arm), except torsion and shear force.

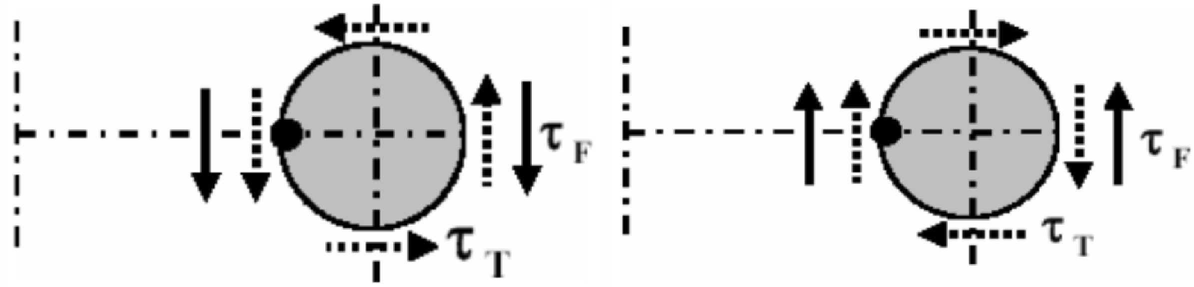


Stresses in the helical spring wire

From the free body diagram, we have found out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil.



The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the figure.



The broken arrows show the shear stresses (τ_T) arising due to the torsion T and solid arrows show the shear stresses (τ_F) due to the force F.

It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress ($\tau_T + \tau_F$) always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crake, is always initiated from the inner radius of the spring.

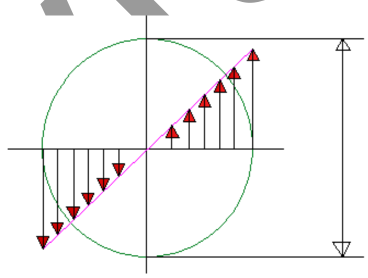
The radius of the spring is given by D/2. Note that D is the mean diameter of the spring. The torque T acting on the spring is

$$T = F x \frac{D}{2} \quad \text{…….. (1)}$$

If d is the diameter of the coil wire and polar moment of inertia,

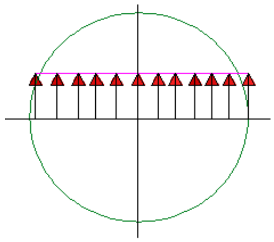
$$I_p = \frac{\pi d^4}{32}$$

The shear stress in the spring wire due to torsion is



$$\tau_T = \frac{Tr}{I_p} = \frac{F x \frac{D}{2} x \frac{d}{2}}{\frac{\pi d^4}{32}} \quad \text{…….. (2)}$$

Average shear stress in the spring wire due to force F is



$$\tau_F = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2} \qquad \text{........ (3)}$$

Therefore, maximum shear stress in the spring wire is

$$\tau_T + \tau_F = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

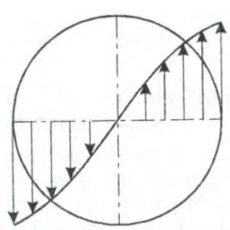
$$\text{or} \quad \tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{\frac{2D}{d}}\right) \quad \text{or} \quad \tau_{max} = \frac{8FD}{\pi d^3}\underbrace{\left(1 + \frac{1}{2C}\right)}_{K_s}$$

Where, C=D/d is called the spring index

$$\text{finaly} \quad \tau_{max} = \frac{8FD}{\pi d^3} K_s \qquad \text{......... (A)}$$

$$K_s = \left(1 + \frac{1}{2C}\right) \qquad \text{......... (4)}$$

The above equation gives maximum shear stress occurring in a spring. K_s are the shear stress correction factor. The resultant diagram of torsional shear stress and direct shear stress is shown



From the above equation it can be observed that the effect of direct shear stress i.e.,

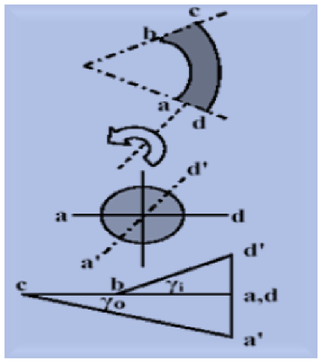
$$\tau = \frac{8FD}{\pi d^3}\frac{1}{2c}$$

Is appreciable for springs of small spring index 'C' Also the effect of wire curvature is neglected in equation (A)

Stresses in helical spring with curvature effect

What is curvature effect?

Let us look at a small section of a circular spring, as shown in the figure. Suppose we hold the section b-c fixed and give a rotation to the section a-d in the anti clockwise direction as indicated in the figure, then it is observed that line a-d rotates and it takes up another position, say a'-d'.



The inner length a-b being smaller compared to the outer length c-d, the shear strain y_i at the inside of the spring will be more than the shear strain y_o at the outside of the spring. Hence, for a given wire diameter, a spring with smaller diameter will experience more difference of shear strain between outside surface and inside surface compared to its larger counterpart. This phenomenon is termed as curvature effect.

So more is the spring index (C = D /d) the lesser will be the curvature effect. For example, the suspensions in the railway carriages use helical springs. These springs have large wire diameter compared to the diameter of the spring itself. In this case curvature effect will be predominantly high.

To take care of the curvature effect, the earlier equation for maximum shear stress in the spring wire is modified as,

$$\tau_{max} = (K_w)\frac{8FD}{\pi d^3}$$

Where, K_W is Wahl correction factor, which takes care of both curvature effect and shear stress correction factor and is expressed as,

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad \text{........ (6)}$$

Deflection of helical spring of circular cross section wire

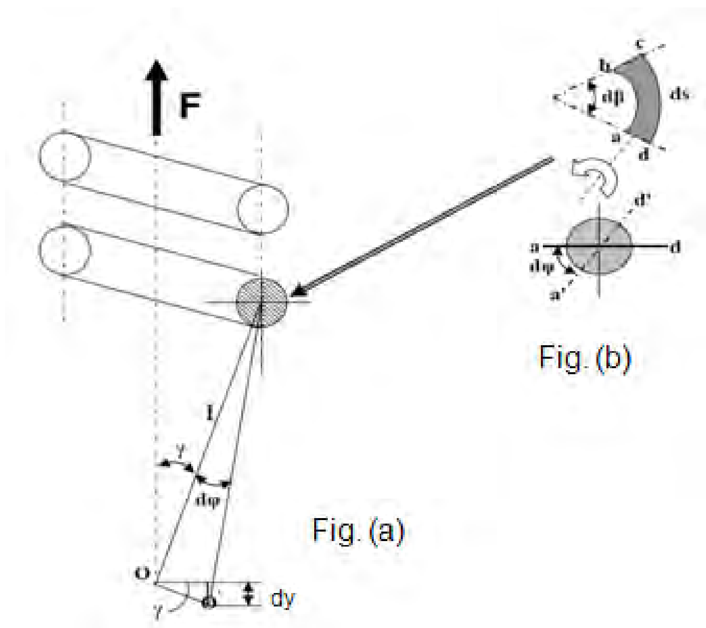
Total length of wire = length of one coil x number of active coils

$$\therefore\ l = \pi D i$$

$\therefore$ Axial deflection of spring $\quad y = \theta \frac{D}{2} \quad$ where θ=angular deflection

We know,

$$\frac{T}{J} = \frac{\tau}{\frac{d}{2}} = \frac{G\theta}{l}$$



The Fig. (a) And (b) shows a schematic view of a spring, a cross section of the spring wire and a small spring segment of length dl. It is acted upon by a force F. From simple geometry we will see that the deflection, in a helical spring is given by the formula,

$$\therefore y = \frac{8FD^3.i}{d^4.G}$$

$$\therefore\ \theta = \frac{Tl}{GJ} = \frac{\left(F\frac{D}{2}\right)(\pi Di)}{G\frac{\pi}{32}d^4}$$

$$\therefore Angular\ deflection\ \theta = \frac{16FD^2i}{Gd^4}$$

Hence axial deflection

$$y = \theta \frac{D}{2} = \frac{16FD^2 i}{Gd^4} . \frac{D}{2}$$

$$\therefore y = \frac{8FD^3 . i}{d^4 . G}$$

$$stiffness\ F_o = \frac{F}{y} = \frac{F}{\frac{8FD^3 . i}{d^4 . G}}$$

$$F_o = \frac{d^4 . G}{8D^3 . i}$$

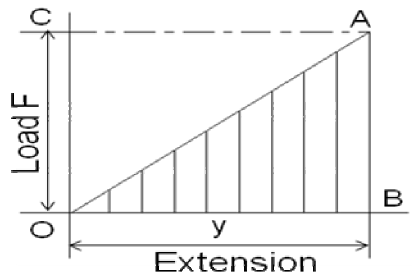
Here we conclude on the discussion for important design features, namely, stress, deflection and spring rate of a helical spring.

$$\tau_{max} = (K_w) \frac{8FD}{\pi d^3}$$

$$\therefore y = \frac{8FD^3 . i}{d^4 . G}$$

$$F_o = \frac{d^4 . G}{8D^3 . i}$$

Expression for strain energy in a body when the load is applied gradually



The strain energy stored in a body is equal to the work done by the applied load in stretching the body. Figure shows load extension diagram of a body under tensile load up to elastic limit.

The tensile load F increase gradually from zero to the value of F, And the extension of the body increase from zero to the value of y. The load F performs work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load F is removed.

Let

F = gradually applied load

Y = Extension of the body (spring)

A = Cross section area

l = Length of body

V= Volume of the body

E = Young's modulus

U = Strain energy stored in the body

σ = Stress induced in the body

Now, work done by the load = Area of load extension curve

= Area of Δ^{le} OAB = ½ F_y ……………. (1)

Load, F = stress x area = σ_A

Extension, y = strain x length

$$= \frac{stress}{E}.l = \frac{\sigma}{E}.l \left(\because \frac{stress}{strain} = E\right)$$

Substituting the values of F and y in equation (1)

Work done by the load =

$$\frac{1}{2}\sigma A \times \frac{\sigma}{E} \times l = \frac{1}{2}\frac{\sigma^2}{E}.A.l$$

$$= \frac{1}{2}\frac{\sigma^2}{E}.V \ \ (Volume\ V = Al)$$

Since work done by the load in stretching body is equal to the strain energy stored in the body,

∴ Strain energy stored in the body U = ½ Fy

$$U = \frac{\sigma^2}{2E} xV \qquad \text{………………..}\ (2)$$

Proof Resilience

The maximum energy stored in the body without permanent deformation [i.e., upto elastic limit] is known as proof resilience. Hence in equation (2) if σ is taken at elastic limit, then we will get proof resilience.

$$\therefore \text{ proof resilience } = \frac{\sigma^2}{2E} xVolume$$

Where σ = stress at elastic limit.

Modulus of resilience = strain energy per unit volume

$$= \frac{total\ strain\ energy}{Volume}$$

$$= \frac{\frac{\sigma^2}{2E} xV}{V} = \frac{\sigma^2}{2E}$$

Symbols Used In Helical Spring

l_o = free length of spring

d = Diameter of spring wire

D = Mean diameter of coil

D_o = Outer diameter of coil

D_i = Inner diameter of coil

p = Pitch

i = Number of active coils

i' = Total number of coils

F = load on the spring or Axial force

τ = Permissible stress or design shear stress

y = Deflection

G = Modulus of Rigidity

c = Spring index

k = Curvature factor or Wahl's stress factor

K_o or F_o =Stiffness of spring or Rate of spring

a = Clearance, 25% of maximum deflection.

t_y = Torsional yield shear strength (stress)

F.O.S = Factor of safety

F_1 = Minimum load

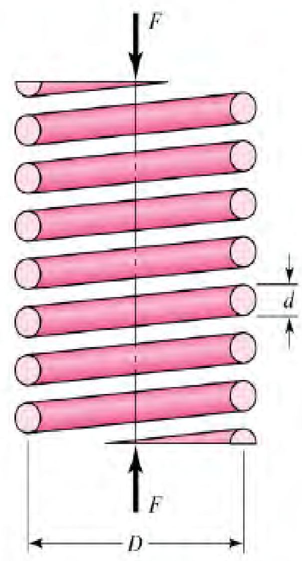
F_2 = Maximum load

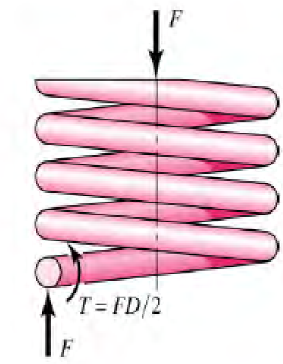
Y_2 = Maximum deflection

Y' = Deflection for the load range

n = Number of additional coils

g = Acceleration due to gravity





V = Volume

m = Mass of the spring

ρ = Mass density of the spring

y_1 = Initial deflection or initial compression

Design of Helical Springs

The design of a helical compression spring involves the following considerations:

- Modes of loading – i.e., whether the spring is subjected to static or infrequently varying load or alternating load.
- The force deflection characteristic requirement for the given application.
- Is there any space restriction.
- Required life for springs subjected to alternating loads.
- Environmental conditions such as corrosive atmosphere and temperature.
- Economy desired.

Considering these factors the designer select the material and specify the wire size, spring diameter, number of turns spring rate, type of ends, free length and the surface condition.

A helical compression spring, that is too long compared to the mean coil diameter, acts as a flexible column and may buckle at comparatively low axial force.

Springs which cannot be designed buckle- proof must be guided in a sleeve or over an arbor.

This is undesirable because the friction between the spring and the guide may damage the spring in the long run.

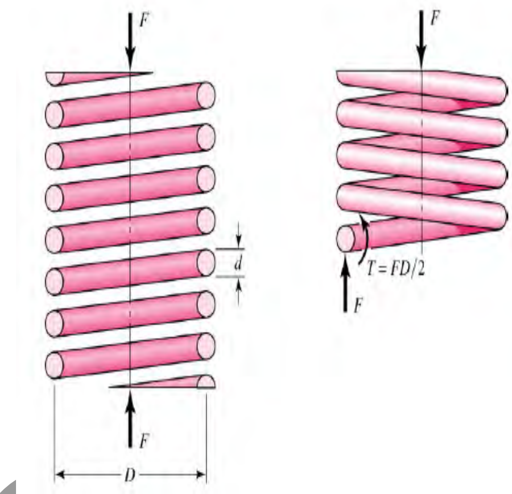
It is therefore preferable, if possible, to divide the spring into buckle proof component springs separated by intermediate platens which are guided over a arbor or in a sleeve.

If,

$$\frac{Free\ length}{Mean\ coil\ diameter} \leq 2.6[Guid\ not\ necessary]$$

$$\frac{Free\ length}{Mean\ coil\ diameter} \geq 2.6[Guid\ required]$$

Design procedure for helical compression spring of circular cross section



1) Diameter of wire:

Shear stress $$\tau = \frac{8FDk}{\pi d^3}$$

Wahl's stress factor $$k = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

Also refer for k value from DHB Figure

Spring index $$c = \frac{D}{d}$$

Where d = diameter of spring wire

'c' generally varies from 4 to 12 for general use

From data hand book select standard diameter for the spring wire.

2. Mean Diameter of Coil

Mean coil diameter $D = cd$

Outer diameter of coil $D_o = D + d$

Inner diameter of coil $D_i = D - d$

3. Number of coil or turns

Axial Deflection $$y = \frac{8FD^3 i}{Gd^4}$$

where i = Number of active turns or coils

4. Free length

$$l_o \geq (i + n)d + y + a$$

Where,

y = Maximum deflection

Clearance ‘a’ = 25% of maximum deflection or a = xdi, for x value refer figure in DHB

Assume squared and ground end

∴ Number of additional coil n = 2

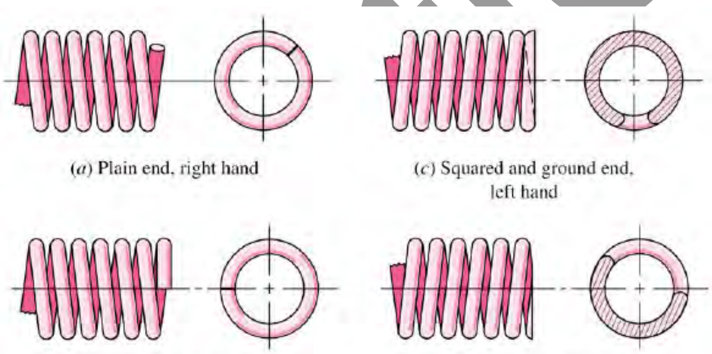
5. Stiffness or Rate of spring

$$F_o = \frac{F}{y}$$

6. Pitch

$$p = \frac{l_o - 2d}{i}$$

Four end types of compression springs



(a) Plain end, right hand

(c) Squared and ground end, left hand

(b) Squared or closed end, right hand

(d) Plain end, ground, left hand

Figure	Actual no. of coils	Solid length	Free length
a Plain end	I	(i+1)d	ip+d
b Square end	i+2	(i+3)d	ip+3d
c Square & Ground	i+2	(i+2)d	ip+2d
d Ground	i	id	ip

Problem 1

A helical spring of wire diameter 6mm and spring index 6 is acted by an initial load of 800N. After compressing it further by 10mm the stress in the wire is 500MPa. Find the number of active coils. G = 84000MPa.

D=spring index(C) x d=36 mm

$$\tau_{max} = (K_w)\frac{8FD}{\pi d^3}$$

$$or, 500 = 1.2525 \times \frac{8F \times 36}{\pi \times 6^3}$$

$$\therefore F = 940.6\ N$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

(Note that in case of static load one can also use K_S instead of K_W.)

$$K = \frac{F}{\delta} = \frac{940.6-800}{10} = 14\ N/mm$$

$$K = \frac{Gd^4}{8D^3N}$$

$$or, N = \frac{Gd^4}{K8D^3N} = \frac{84000 \times 6^4}{14 \times 8 \times 36^3} \approx 21\ turns$$

Questions and answers

Q1. What are the objectives of a spring?

A1. The objectives of a spring are to cushion, absorb, or controlling of energy arising due to shock and vibration. It is also used for control of motion, storing of energy and for the purpose of measuring forces.

Q2. What is the curvature effect in a helical spring? How does it vary with spring index?

A2. For springs where the wire diameter is comparable with the coil diameter, in a given segment of the spring, the inside length of the spring segment is relatively shorter than the outside length. Hence, for a given magnitude of torsion, shearing strain is more in the inner segment than the outer segment. This unequal shearing strain is called the curvature effect. Curvature effect decreases with the increase in spring index.

Q3. What are the major stresses in a helical spring?

A3. The major stresses in a helical spring are of two types, shear stress due to torsion and direct shear due to applied load.

Problem 1

Design a helical compression spring to support an axial load of 3000 N. The deflection under load is limited to 60mm. The spring index is 6.The spring is made of chrome vanadium steel and factor of safety is equal to 2

Data

F = 3000N, y = 60mm, c = 6, FOS = 2

Solution

From DHB for chrome-vanadium steel refer standard table

τ_y = 690MPa=690N/mm^2 (0.69GPa)

G=79340MPa=79340N/mm^2 (79.34GPa)

$$\tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345\ N/mm^2$$

Diameter of wire

Shear stress $\tau = \frac{8FDk}{\pi d^3}$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$

Spring index $c = \frac{D}{d}$

$6 = \frac{D}{d}$

$\therefore$ D = 6d

$$345 = \frac{8x3000x6dx1.2525}{\pi d^3}$$

$\therefore$ d = 12.89

Select standard diameter of wire from table

$\therefore$ d = 13 mm

Diameter of coil

$$c = \frac{D}{d}$$

$$6 = \frac{D}{13}$$

Mean diameter of coil= D = 78 mm

Outer diameter of coil =D_o = D+d = 78+13= 91 mm

Inner diameter of coil = D_i = D-d = 78-13 = 65 mm

3. Number of coil or turns

Deflection $y = \frac{8FD^3 i}{Gd^4}$

$$60 = \frac{8x300078^3 \times i}{79340x13^4}$$

$$i = 11.93$$

∴ Number active turns i = 12

4. Free length

$l \geq (i+n)\, d+y+a$

Clearance a = 25% of maximum deflection $= \frac{25}{100} x60 = 15mm$

Assume squared and ground end

∴ n = 2

∴Total number of turns i'= i+n=12+2=14

∴ $l_o \geq (12+2)13+60+15 \geq 257$ mm

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{257 - 2x13}{12} = 19.25\ mm$$

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{3000}{60} = 50\ N/mm$$

7. Spring specification

Material Chrome vanadium steel
Wire diameter d =13 mm
Mean diameter D = 78 mm
Free length l_o = 257 mm
Total number of terms i'= 14
Style of end-square and ground
Pitch p = 19.25 mm
Rate of spring F_o = 50 N/mm

Problem 2

A helical valve spring is to be designed for an operating load range of approximately 90to135N. The deflection of the spring for the Load range is 7.5mm.Assume a spring index of 10 and factor safety = 2. Design the spring.

Data: Maximum load F_2= 135N, minimum load F_1 =90N; y'=7.5mm, c=10, FOS=2

Solution:

From DHB for chrome-vanadium steel

(Refer table physical properties of spring materials)

τ_y = 690MPa = 690N/mm^2 (0.69GPa)

G=79340MPa = 79340N/mm^2 (79.34GPa)

$$\tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345\ N/mm^2$$

Maximum deflection $y_2 = \frac{y' F_2}{F_2 - F_1}$

$$= \frac{7.5x135}{135-90} = 22.5\text{mm}$$

Design the spring for Maximum load and deflection

Diameter of wire

Shear stress $\tau = \frac{8F_2Dk}{\pi d^3}$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x10-1}{4x10-4} + \frac{0.615}{6}$ =1.1448

Spring index $c = \frac{D}{d}$

$10 = \frac{D}{d}$

∴ D = 10 d

345 = $\frac{8x135x10dx1.1448}{\pi d^3}$

∴ d = 3.37 mm

Select standard diameter of wire

∴ d = 3.4 mm

Diameter of coil

$c = \frac{D}{d}$

$10 = \frac{D}{3.4}$

∴ Mean diameter of coil = D = 34mm

Outer diameter of coil = D_o= D+d = 34+3.4 = 37.4mm

Inner diameter of coil = D_i = D-d = 34-3.4 = 30.6

Number of coil or turns

Deflection $y = \frac{8F_2D^3 i}{Gd^4}$

$$22.5 = \frac{8x135x34^3xi}{79340x3.4^4}$$

$$i = 5.62$$

$\therefore$ Number active turns $i = 6$

Free length

$$l_o \geq (i+n)d+y+a$$

Clearance a = 25% of maximum deflection $= \frac{25}{100}x22.5 = 5.625$ mm

Assume squared and ground end

$\therefore$ Number of additional coil n = 2

$\therefore$Total number of turns $i' = i + n = 12+2 = 14$

$\therefore l_o \geq (6+2)3.4+22.5+5.625$

≥ 55.325 mm

Pitch

$$p = \frac{l_o - 2d}{i} = \frac{55.325 - 2x3.4}{6} = 8.0875\text{mm}$$

Stiffness or Rate of spring

$$F_o = \frac{F_2}{y_2} = \frac{135}{22.5} = 6\ \text{N/mm}$$

Total length of wire

$l = \pi Di'$ where $i' = i+n=6+2=8$

$= \pi x34x8 = 854.513$mm

Problem 3

Design a valve spring for an automobile engine, when the valve is closed, the spring produces a force of 45N and when it opens, produces a force of 55N. The spring must fit over the valve bush which has an outside diameter of 20 mm and must go inside a space of 35 mm. The lift of the valve is 6 mm. The spring index is 12. The allowable stress may be taken as 330 MPa. Modulus of rigidity 80GPa.

Data:

Maximum load F_2= 55N, Minimum load F_1 = 45N;

y'= 6mm, c =12

Solution

For chrome-vanadium steel

τ_y= 330MPa = 330N/mm^2

G = 80000MPa = 80000N/mm^2 (80GPa)

$$\tau = \frac{\tau_y}{FOS} = \frac{330}{2} = 165\ N/mm^2$$

Maximum deflection

$$y_2 = \frac{y' F_2}{F_2 - F_1}$$

$$= \frac{55x6}{55-45} = 33\text{mm}$$

Diameter of wire

Shear stress $\tau = \frac{8F_2Dk}{\pi d^3}$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x12-1}{4x12-4} + \frac{0.615}{6}$ =1.119

Spring index $c = \frac{D}{d}$

$$12 = \frac{D}{d}$$

$\therefore$ D = 12d

$$330 = \frac{8x55x12dx1.119}{\pi d^3}$$

∴ d = 2.387mm

Select standard diameter of wire ∴ d = 2.5mm

Diameter of coil

$$c = \frac{D}{d}$$

$$12 = \frac{D}{2.5}$$

∴ Mean diameter of coil = D = 30mm

Outer diameter of coil = D_o= D+d =30+2.5 = 32.5mm

Inner diameter of coil = D_i = D-d = 30-2.5 = 27.5mm

Check

Do = 32.5mm > 35mm

D_i = 27.5mm < 20mm

Design is safe

Number of coil or turns

Deflection $y = \frac{8F_2 D^3 i}{Gd^4}$

$$33 = \frac{8x55x33^3xi}{80000x2.5^4}$$

i = 8.68

∴ Number active turns i=9

Free length

$l_o \geq (i+n)d+y+a$

Clearance a = 25% of maximum deflection = $\frac{25}{100}x33$ = 8.25mm

Assume squared and ground end

∴ Number of additional coil n = 2

∴ Total number of turns $i' = i + n = 9 + 2 = 11$

∴ $l_o \geq (9+2)2.5+33+8.25$

≥ 68.75mm

Pitch

$$p = \frac{l_0 - 2d}{i} = \frac{68.75 - 2x2.5}{9} = 7.083\text{mm}$$

Stiffness or Rate of spring

$$F_o = \frac{F_2}{y_2} = \frac{55}{33} = 1.667 \text{ N/mm}$$

Total length of wire

$l = \pi Di'$ where $i' = i + n = 9 + 2 = 11$ $= \pi x30x11 = 1036.725$mm

Problem 4

Round wire cylindrical compression spring has an outside diameter of 75 mm. It is made of 12.5mm diameter steel wire. The spring supports an axial load of 5000N, Determine

(i) Maximum shear stress,

(ii) Total deflection if the spring has 8 coils with squared-ground end and is made of SAE 9260 steel.

(iii) Find also the pitch of coils and

(iv) The natural frequency of vibration of the spring if the one is at rest.

Data: D_o=75mm; i'=8; D=12.5; F=5000N; Material –SAE 9260

Solution:

From table for SAE 9260

G = 79340MPa = 79340N/mm^2 (79.34GPa)

Maximum Shear stress

Shear stress $\tau = \frac{8F_2 Dk}{\pi d^3}$

$D_o = D+d$

$75 = D+12.5$

$\therefore$ $D = 62.5$ mm

$$c = \frac{D}{d} = \frac{62.5}{12.5} = 5$$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x5-1}{4x5-4} + \frac{0.615}{6} = 1.3105$$

$$\therefore \ \tau = \frac{85000x62.5x1.3015}{\pi 12.5^3} = 533.95 \text{N/mm}^2$$

Total deflection

For squared and ground end n=2

$\therefore i' = i+n$

$8 = i+2$

$\therefore i' = 6$

$$y = \frac{8F_2 D^3 i}{Gd^4} = \frac{8x500062.5^3x6}{79340x62.5^4} = 30.25\text{mm}$$

Pitch

$$p = \frac{l_o - 2d}{i}$$

$l_o \geq (i+n)d+y+a$

Clearance a = 25% of maximum deflection = $\frac{25}{100} x 30.25$ = 7.5625mm

$\therefore$Total number of turns $i' = i+n = 6+2 = 8$

$\therefore l_o \geq (6+2)12.5+30.25+7.5625$

≥ 137.8125mm

$$p = \frac{137.8125 - 2x12.5}{6} = 18.8\text{mm}$$

Natural frequency

Natural frequency of vibration one end is at rest

$$f = \frac{1}{2\pi}\sqrt{\frac{2F_o}{m}}$$

$$F_o = \frac{F}{y} = \frac{5000}{30.25} = 165.28925 \text{ N/mm} = 165289.25 \text{ N/m}$$

Mass m = Volume x density

Volume $V = \pi D i \left[\frac{\pi d^2}{4}\right]$

Density ρ =7.81gm/cc = 7.81x10^{-6}kg/mm^3 for steel

$\therefore m = \pi x62.5x6x\pi x\frac{12.5^2}{4}x7.81x10^{-6} = 1.129$ kg

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2x165289.25}{1.129}} = 86.1 \text{ Hz}$$

Problem 5

The spring loaded safety valve for a boiler is required to blow off at a pressure of 1.3MPa. The diameter of the valve is 65 mm and maximum lift of the valve is 17.5mm. Design a suitable compression spring for a valve assuming spring index to be 6 and providing initial compression of 30mm take **τ** = 0.45GPa and G= 84 Gpa.

Data: p_1 = 1.3MPa; Diameter of a valve = 65mm, y' = 17.5mm, c = 6, y_1 = 30mm

τ = 450MPa = 450N/mm^2 (0.45GPa)

G = 84000MPa = 84000N/mm^2 (84GPa)

Solution:

Maximum deflection $y_2 = y_1 + y'$ = 30+17.5 = 47.5 mm

Minimum load $F_1 = P_1$xArea of valve = $1.3 x \frac{\pi}{4} x 65^2 = 4313.8N$

Maximum deflection $y_2 = \frac{y' F_2}{F_2 - F_1}$

$$47.5 = \frac{F_2 x 17.5}{F_2 - 4313.8}$$

F_2-4318.8 = 0.3684x F_2

F_2 = 6838.1 N = maximum load

Design of spring for maximum load and maximum deflection.

Diameter of wire

Shear stress $\tau = \frac{8 F_2 D k}{\pi d^3}$

Wahl's stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6}$ =1.2525

Spring index $c = \frac{D}{d}$

$$6 = \frac{D}{d}$$

∴ D = 6 d

$$450 = \frac{8x6838.1x6dx1.2525}{\pi d^3}$$

∴ d = 17.053 mm

Select standard diameter of wire

∴ d = 8mm

Diameter of coil

$$c = \frac{D}{d}$$

$$6 = \frac{D}{18}$$

∴ D= 108 mm = mean diameter of coil

D_o= D+d = 108+18 = 126 mm = Outer diameter of coil

D_i= D-d = 108-18 = 90 mm = Inner diameter of coil

Number of coil or turns

Deflection $y_2 = \frac{8F_2D^3i}{Gd^4}$

$$47.5 = \frac{8x6838.1x108^3xi}{84000x18^4}$$

i=6.078

∴Number active turns i=7

Free length

$l_o \geq (i+n)d+y+a$

Clearance a = 25% of maximum deflection =$\frac{25}{100}x47.5$=11.875 mm

Assume squared and ground end

∴ Number of additional coil n=2

∴Total number of turns i'= i+n=7+2=9

∴ $l_o \geq$ (7+2)18+47.5+11.875

$\geq$ 221.375mm

Pitch

$$p = \frac{l_o - 2d}{i} = \frac{221.375 - 2x18}{7} = 26.48 \text{ mm}$$

Stiffness or Rate of spring

$$F_o = \frac{F_2}{y_2} = \frac{6838.1}{47.5} = 143.96 \text{ N/mm}$$

Total length of wire

l = πDi' where i'=i+n=7+2=9

= πx108x9=3053.63mm

Problem 6

The valve spring of a gasoline engine is 40mm long when the valve is open and 48mm long when the valve is closed. The spring loads are 250N when the valve is closed and 400N when the valve is open. The inside diameter of the spring is not to be less then 25mm and factor of safety is 2.Design the spring

Data: F_1= 250N F_2= 400N D_i = 65mm, y'=48-40=8mm, FOS=2

Solution:

Maximum deflection $$y_2 = \frac{y' F_2}{F_2 - F_1}$$

$$= \frac{400x8}{400-250} = 21.33 \text{ mm}$$

Design of spring for maximum load and maximum deflection.

Assume chrome vanadium alloy steel from table 20.14

τ= 690MPa=690N/mm^2 (0.69 GPa)

.

G=79.34x10^{3}MPa=79340N/mm^2 (79.34GPa)

Diameter of wire

Shear stress $$\tau = \frac{8F_2 Dk}{\pi d^3}$$

Assume k=1.25 since 'c'is not given

D_i = D –d i.e., D – d =25

$\therefore$ D = d+25

$$\therefore \quad 345 = \frac{8x400x(25+d)1.25}{\pi d^3}$$

i.e., 0.271d^3 - d – 25 = 0

By hit **a**nd trial method d = 4.791mm.

Select standard diameter of wire from

$\therefore$ d=5mm

Diameter of coil

∴ D=d+25 = 25+5 = 30mm Mean diameter of coil

D_o=D+d=30+5=35mm=Outer diameter of coil

D_i=D-d=30 -5 =25mm=Inner diameter of coil

Check

Spring index

$$c = \frac{D}{d} = \frac{30}{5} = 6$$

Wahl's stress factor

$$\therefore \quad k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$$

$$\therefore \tau_{cal} = \frac{8x400x30x1.2525}{\pi x5^3}$$

τ_{cal} = 306.2 N/mm^2 < τ_{allow} (i.e., 345N/mm^2)

Therefore design is safe.

Number of coil or turns

Deflection $y_2 = \frac{8F_2D^3i}{Gd^4}$

$$21.33 = \frac{8x400x30^3xi}{79340x5^4}$$

i=12.24

∴ number active turns $i = 13$

Free length

$l_o \geq$ (i+n)d+y+a

Clearance a = 25% of maximum deflection

$$= \frac{25}{100}x21.33 = 11.875 \text{ mm}$$

Assume squared and ground end

Number of additional coil

$\therefore$ n =2

$\therefore l_o \geq (13+2)5+21.33+5.3325$

$\geq$ 101.6625mm

Pitch

$$p = \frac{l_o - 2d}{i} = \frac{101.6625 - 2x5}{13} = 7.05 \text{ mm}$$

Stiffness or Rate of spring

$$F_o = \frac{F_2}{y_2} = \frac{F}{y} = \frac{400}{21.33} = 18.75$$

Total length of wire

$l = \pi Di'$ where $i = i + n = 13 + 2 = 15$

$= \pi$ x 30 x 15 = 1413.72mm

Problem 7

A single plate friction clutch transmits 20kw at 1000 rpm. There are 2 pairs of friction surfaces having a mean radius of 150 mm. The axial pressure is provided by six springs. If the springs are compressed by 5 mm during declutching, design the spring, take c=6, G = 80GPa and $\boldsymbol{\mu}$ = 0.3, $\boldsymbol{\tau}$ =0.42GPa. G=80 Gpa and $\mu = 0.3$

Given data

N = 20kw c = 6

n = 1000rpm $\boldsymbol{\tau}$ = 0.42GPa = 420Mpa

i = number of active surfaces

R_m=150 mm $\therefore D_m$ = 300 mm

Number of springs = 6 y = 5mm

Solution:

Clutch

Torque $M_t = 9550x1000x\frac{N}{n} = 9550x1000x\frac{20}{1000} = 191000\ Nmm$

Also, $M_t = \frac{1}{2}\mu F_a D_m i$ for disc clutch

$$\therefore 191000 = \frac{1}{2}x0.3xF_a x300x2$$

$F_a = 2122.22$ N = axial force

$$\therefore \text{Axial force on each spring } F = \frac{F_a}{\text{number of spring}} = \frac{2122.22}{6} = 353.7\ N$$

1. Diameter of the wire

Shear stress $\tau = \frac{8FDk}{\pi d^3}$

Stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$

Spring index $c = \frac{D}{d} = 6 \quad \therefore D = 6d$

$$\therefore 420 = \frac{8x353.7x6dx1.2525}{\pi d^3}$$

Diameter of wire = d=4mm

2. Diameter of coil

Mean diameter of coil D=6d= 6x4= 24 mm

Outer diameter of coil $= D_o = D + d = 24 + 4 = 28$mm

Inner diameter of coil $D_i = D - d = 24 - 4 = 20$ mm

3. Number of turns

Deflection $y=\frac{8FiD^3}{Gd^4}$

i.e. $5=\frac{8x353.7xix24^3}{80000x4^4}$

$\therefore i = 2.61$

$\therefore Number\ of\ active\ terms\ or\ coil\ i = 3$

Problem 8

A single plate friction clutch is to be designed for a vehicle. Both sides of plate are to be effective. The clutch transmits 30KW at a speed of 3000rpm and should cater for an overload of 20%. The intensity of pressure on the friction surfaces should not exceed 0.085N/mm^2 and the surface speed at the mean radius should be limited to 2300m/min. The outside diameter may be assumed as 1.3times inside diameter and μ=0.3. If the axial thrust is to be provided by six springs of about 25mm coil diameter design the springs selecting the wire from following table.

Design the spring selecting the wire from the following gauges. Safe shear stress is limited to 0.42 GPa and modulus of rigidity 84Gpa

SWG.	4	5	6	7	8	9	10
Dia	5.893	5.385	4.877	4.470	4.064	3.658	3.251

Data:

Number of active surfaces for clutch, i = 2

N = 30Kw; Number of springs = 6

n = 3000rpm; D = 25mm;

Over load = 20%; G = 84000Mpa; p = 0.085N/mm^2;

v = 2300m/min; D_2= 1.3D_1; μ =0.3

Solution:

Clutch

$$M_t = 9550 \times 1000 \times 1.2 \times \frac{N}{n}\ (\because 20\%\ over\ load)$$

$$9550 \times 1000 \times \frac{30}{3000} \times 1.2 = 114600\ Nmm$$

For Disc clutch

$$M_t = \frac{1}{2}\mu F_a D_m i$$

Assume uniform wear

$$\therefore D_m = \frac{1}{2}(D_2 + D_1)$$

$$= \frac{1}{2}(1.3D_1 + D_1) = 1.15D_1$$

$$F_a = \frac{1}{2}$$

$$= \frac{1}{2}\pi \times 0.003 \times D_1(1.3D_1 - D_1)$$

$$= 0.04D_1^2$$

$$\therefore 114600 = \frac{1}{2} \times 0.3 \times 0.04D_1^2 \times 1.15D_1 \times 2$$

Inner diameter of friction surface $D_1 = 202.5mm$

$\therefore$ Outer diameter of friction surface $D_2 = 263.25mm$

Check

$$V_m = \frac{\pi D_m n}{60000} = \frac{\pi \times 1.15 \times 202.5 \times 3000}{60000}$$

$$V_m = 36.58m/sec$$

$$= 2195m/\min < 2300m/min$$

$$\therefore Safe$$

$$Axial\ force$$

$$F_a = 0.04 \times D_1^2 = 0.04 \times 202.5^2 = 1640.25N$$

$$\therefore Load\ on\ each\ spring = \frac{F_a}{No\ of\ springs}$$

$$= \frac{1640.25}{6} = 273.375N = \text{F}$$

Spring

(i) Diameter of wire

$$Shear\ Stress\ \tau = \frac{8FDk}{\pi d^3}$$

Assume k =1.25

$$420 = \frac{8 \times 273.375 \times 25 \times 1.25}{\pi \times d^3}$$

$$\therefore d = 3.72mm$$

From the table SWG 8 wire

$$\therefore Dia.of\ wire\ d = 4.064mm$$

Spring index $c = \frac{D}{d} = \frac{25}{4.064} = 6.1515$

Stress factor $k = \frac{4 \times 6.1515 - 1}{4 \times 6.1515 - 4} + \frac{0.615}{6.1515} = 1.2456$

$$\therefore \tau_{cal} = \frac{8 \times 273.375 \times 25 \times 1.2456}{\pi \times 4.064^3}$$

i.e., $\tau_{cal} = 322.96N/mm^2 < \tau_{allow(i.e.,420N/mm^2)}$

$$\therefore Safe$$

Number of coils

$$y = \frac{8FD^3 i}{d^4 G}$$

Assume allowable compression, y = 5mm

$$5 = \frac{8 \times 273.375 \times 25^3 \times i}{84000 \times 4.064^4}$$

$$i = 3.35$$

$\therefore$ Number of active turns i = 4

Free length

$l_o \geq (i+n)d+y+a$

Assume squared and ground end

Total number of turns $i' = i + n = 4 + 2 = 6$

$\therefore l_0 \geq (4 + 2)4.064 + 5 + 0.25 \geq$ 30.63mm

$$i = 3.35$$

Pitch

$$p = \frac{l_o - 2d}{i} = \frac{30.63 - 2 \times 4.064}{4} = 5.62mm$$

Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{273.375}{5} = 54.675\ N/mm$$

Total length of wire

l= πDi' where $\quad i' = i + \text{n} = 4 + 2 = 6$

= π x 25 x 6 = 471.23 mm

Problem 9

It is required to design a helical compression spring subjected to a maximum force of 1250 kN. The deflection of the spring corresponding to the maximum force should be approximately 30mm. The spring index can be taken as 6. The spring is made of patented and cold drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81370 N/mm^2 respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate;

i. Wire diameter;
ii. Mean coil diameter;
iii. Number of active coils;
iv. Total number of coils;
v. Free length of the spring; and
vi. Pitch of the coil.

Draw a neat sketch of the spring showing various dimensions.

Solution:

The permissible shear stress is given by,

$\tau = 0.5 \quad \sigma_{ut} = 0.5(1090) = 545$ N/mm^2

From Eq. (10.7),

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6}$$

$$= 1.2525$$

From Eq. (10.13),

$$\tau = K\left(\frac{8pc}{\pi d^2}\right) \text{ or } 545 = 1.2525\left(\frac{8x1250x6}{\pi d^2}\right)$$

d= 6.63 or 7mm (i)

D = cd =6x7 = 42mm (ii)

From Eq. (10.8),

Deflection

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$30 = \frac{8x1250x42^3xN}{81370x7^4}$$

N = 7.91

∴ Number active turns N=8 (iii)

It is assumed that spring has square and ground ends. The number of inactive coil is 2. Therefore

N_t = N+2 = 8+2 = 10 coils (iv)

The actual deflection of the spring is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8x1250x42^3x8}{81370x7^4} = 30.34\text{mm}$$

Solid length of spring = N_t d = 10x7 =70mm

It is assumed that there will be a gap of 1 mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 10.

The total axial gap between the coils will be (10-1) x1 = 9mm.

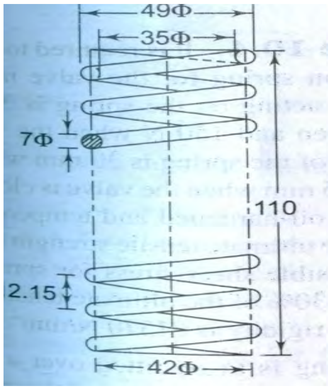
Free length = solid length + total axial gap + δ

= 70+9+30.34 = 109.34 or 110mm (v)

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t-1)} = \frac{109.34}{(10-1)}$$

$$= 12.15\text{mm} \qquad \text{(vi)}$$

The dimension of the spring as shown in figure



Problem 10

A helical compression spring, made of circular wire, is subjected to an axial force that varies from 2.5kN to 3.5kN. Over this range lf force, the deflection of the spring should be approximately 5mm. The spring index can be taken as 5. The spring has square and ground ends. The spring is made of patented and cold drawn steel wire with ultimate tensile strength of 1050 N/mm2 and modulus of rigidity of 81370 N/mm2. The permissible shear stress for the spring wire should. Design the spring and calculate:

i. Wire diameter;
ii. Mean coil diameter;
iii. Number of active coils;
iv. Total number of coils;
v. Solid strength of the spring;
vi. Free length of the spring;
vii. Required spring rate and
viii. Actual spring rate.

Solution:

The permissible shear stress is given by,

$$\tau = 0.5 \quad S_{ut} = 0.5(1090) = 545 \text{ N/mm2}$$

From Eq. (10.7),

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{5}$$

$= 1.3105$

From Eq. (10.13),

$$\tau = K\left(\frac{8pc}{\pi d^2}\right) \quad \text{or} \quad 525 = 1.3105\left(\frac{8x3500x5}{\pi d^2}\right)$$

d= 10.55 or 11mm (i)

D = cd =5x11 = 55mm (ii)

From Eq. (10.8),

Deflection

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$5 = \frac{8x(3500-2500)x55^3xN}{81370x11^4}$$

N = 4.48 or 5 coils

∴ Number active turns N=5

For square and ground ends. The number of inactive coil is 2. Therefore

$N_t = N+2 = 5+2 = 7$ coils (iv)

Solid length of spring =

$N_t\ d = 7x11 = 77mm$ (v)

The actual deflection of the spring under the maximum force of 3.5kN is given by.

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8x3500x55^3x5}{81370x11^4} = 19.55mm$$

It is assumed that there will be a gap of 0.5 mm between consecutive coils when the spring is subjected to the maximum force3.5kN. The total number of coils is 7.

The total axial gap between the coils will be (7-1) x 0.5 = 3mm.

Free length = solid length + total axial gap + δ

= 77+3+19.55 = 99.55 or 100mm (vi)

The required spring rate is given by,

$$k = \frac{P_1 - P_2}{\delta} = \frac{3500-2500}{5} = 200 N/mm \qquad \text{(vii)}$$

The actual spring rate is given by,

$$k = \frac{Gd^4}{8D^3N} = \frac{81370x11^4}{8x55^3x5} = 179.01 N/mm \qquad \text{(viii)}$$

Problem 10

It is required to design a helical compression spring subjected to a maximum force of 7.5kN. The mean coil diameter should be 150 mm from space consideration. The spring rate is 75 N/mm. The spring is made of oil hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm^2. The permissible shear stress for the spring wire is 30% of the ultimate tensile strength (G = 81370 N/mm^2). Calculate:

i. Wire diameter;
ii. Number of active coils;

Solution:

The permissible shear stress is given by,

$\tau = 0.3S_{ut} = 0.3(1250) = 375$ N/mm^2

$$c = \frac{D}{d} = \frac{150}{d} \quad \text{or} \quad d = \frac{150}{c} \qquad \text{...... (a)}$$

$$\tau = K\left(\frac{8pc}{\pi d^2}\right)$$

Substitute Eq. (a) in above expression.

$$\tau = K\left(\frac{8FC^3}{\pi x 150^2}\right)$$

$$Kc^3 = \left(\frac{\pi x 150^2 x 375}{8P}\right)$$

$Kc^3 = 441.79$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} \quad \text{…. (b)}$$

Equation (b) is too solved by trial and error method. The values are tabulated in the following way,

C	K	Kc^3
5	1.311	163.88
6	1.253	270.65
7	1.213	416.06
8	1.184	606.21
7.5	1.197	504.98
7.1	1.210	433.07
7.2	1.206	450.14
7.3	1.203	467.99

It observed from the above table that spring index should be between 7.1 and 7.2 to satisfy Eq.(b)

c= 7.2

$$d = \frac{150}{c} = \frac{150}{7.2} = 20.83 \text{ or } 21\text{mm}$$

$$k = \frac{Gd^4}{8D^3N} = 75 = \frac{81370x21^4}{8x150^3xN}$$

N= 7.81or 8 coils

Problem 11

It is required to design a helical compression spring for the valve mechanism. The axial force acting on the spring is 300N when the valve is open and 150N when the valve is closed. The length of the spring is 30mm when the valve is open and 35mmwhen the valve is closed. The spring is made of oil hardened and tempered valve spring wire and the ultimate tensile strength is $1370N.mm^2$. The permissible shear stress for spring wire should be taken as30% of the ultimate tensile strength. The modulus of rigidity is $81370N/mm^2$. The spring is to be fitted over a valve rod and the minimum inside diameter of the spring should be 20mm. Design the spring and calculate

i. Wire diameter;
ii. Mean coil diameter;
iii. Number of active coils;
iv. Total number of coils;
v. Free length of the spring; and
vi. Pitch of the coil.

Assume that the clearance between adjacent coils or clash allowance is 15% of the deflection under the maximum load.

Solution:

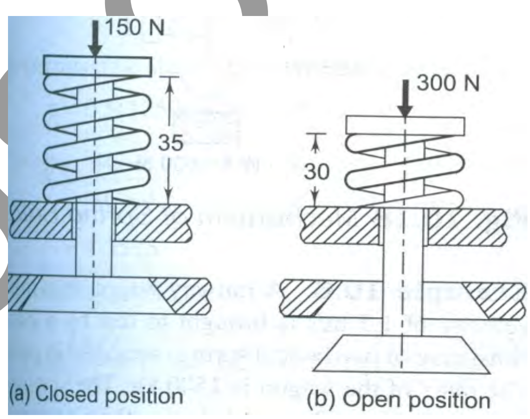
The spring forced and spring length corresponding to closed and open position of the valve is illustrated in fig. The permissible shear stress is given by,

The permissible shear stress is given by,

$\tau = 0.3 \quad \sigma_{ut} = S_{ut} = 0.3(1370) = 411 \text{ N/mm}^2$

$D_i = 20\text{mm}$

$D = D_i + d = (20+d)$ mm



(a) Closed position (b) Open position

$$\tau = K\left(\frac{8FD}{\pi d^3}\right) \text{ or } 411 = K\left(\frac{8x300(20+d)}{\pi d^3}\right) \qquad \text{(a)}$$

It is observed from the above expression that there are two unknowns, viz. K and d and one equation, therefore, it cannot be solved. As a first trial, let us neglect the effect of Whal's factor

K or substitute (K=1).At a later design stage, wire diameter d can be increased to account for K. Substituting (K=1) in eq. (a)

$$411 = \left(\frac{8x300(20+d)}{\pi d^3}\right)$$

$$or \quad \left(\frac{d^3}{(20+d)}\right) = 1.8587 \qquad \text{(b)}$$

.

The above equation is solved by trial and error method. The values are tabulated in the following way:

d	$d^3/(20+d)$
5	5
4	2.667
3	1.174

The value of d is between 3 to4 mm in order to satisfy Eq. (b). The higher value of d is selected to account for Wahl's correction factor.

Therefore,

d= 4mm

D = d_i+d = 20+4 = 24mm

C = D/d = 24/4 = 6

$$c = \frac{D}{d} = \frac{24}{4} = 6$$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$$

$$\tau = K\left(\frac{8FD}{\pi d^3}\right)$$

$$\tau = 1.2525\left(\frac{8x300x24}{\pi x4^3}\right) = 358.81N/mm^2$$

Therefore,

$$\tau < 411N/mm^2$$

Design is safe.

Deflection

$$\delta = \frac{8FD^3N}{Gd^4}$$

$$(35-30) = \frac{8x(300-150)x24^3xN}{81370x4^4}$$

N = 6.28 or75 coils

∴ Number active turns N=7

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$N_t = N+2 = 7+2 = 9$ coils

The deflection of the spring for the maximum force is given by,

$$\delta = \frac{8FD^3N}{Gd^4}$$

$$= \frac{8x300x24^3x7}{81370x4^4} = 11.15mm$$

The total gap between the adjacent coils is given by,

Gap = 15% of δ = 0.15x11.15 = 1.67mm

Solid length of spring = N_t d = 9x4 =36mm

Free length = solid length + total axial gap + δ

=36+1.67+11.15 = 48.82 or 50mm

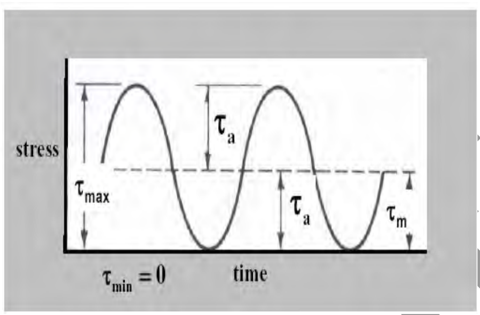
Pitch of coil = $\frac{\text{free length}}{(N_t-1)} = \frac{50}{(9-1)} = 6.25mm$

Design against fluctuating load

In many applications, the force acting on the spring is not constants but varies in magnitude with time. The valve springs of automotive engine subjected to millions of stress cycles during its life time.

On the other hand, the springs in the linkages and mechanisms are subjected to comparatively less number of stress cycles.

The spring subjected to fluctuating stresses are designed on the basis of two criteria- design for infinite life and design for finite life



Let us consider a spring subjected to external fluctuating force, that changes it's magnitude from to F_{max} to F_{min} in the load cycle. The mean force $_{and}$ the force amplitude F_a are given by

$$F_m = \frac{1}{2}(F_{max} + F_{min})$$

$$F_a = \frac{1}{2}(F_{max} - F_{min})$$

The mean stresses (τ_m) are calculated from mean force by using shear stress correction factor (k_s). It is given by,

$$\tau_m = \frac{8F_m D}{\pi d^3} K_s$$

Where,

$$K_s = (1 + \frac{0.5}{C})$$

K_s are the correction factor for direct shear stress and are applicable to mean stress only. For torsional stress amplitude (τ_a), it is necessary to also consider the effect of stress concentration due to curvature in addition to direct shear stress. Therefore,

$$\tau_a = K\left(\frac{8F_aD}{\pi d^3}\right)$$

Where, K is the Wahl factor, which takes into consideration the effect of direct shear stress as well as the stress concentration due to curvature.

There is a basic difference between the rotating-beam specimen and fatigue testing of spring wires. A spring is never subjected to completely reversed load, changing its magnitude from tension to compression passing through zero with respect to time. A helical compression spring is subjected to purely compressive force.

On the other hand, the helical extension spring is subjected to purely tensile force. In general, the spring wires are subjected to pulsating shear stresses which vary from zero to (τ_{se}), as shown in fig. τ_{se} is the endurance limit in shear for the stress variation from zero to some maximum value. The data regarding the experimental values of endurance strength of spring wires is not readily available. In absence of such values, the following relationships suggested by H.J.Elmendorf can be used,

For patented and cold-drawn steel wires (grades 1 to 4)

$\tau'_{se} = 0.21\ \sigma_{ut}$

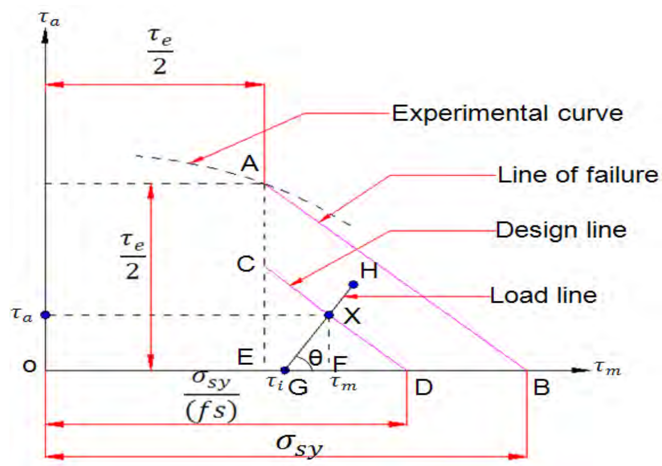
$\tau_{sy} = 0.42\ \sigma_{ut}$

For oil-hardened and tempered steel wires (SW and VW grade)

$\tau'_{se} = 0.22\ \sigma_{ut}$

$\tau_{sy} = 0.45\ \sigma_{ut}$

Where, σ_{ut} is the ultimate tensile strength.



The fatigue diagram for the spring shown in the above fig. the mean stress τ_m is plotted on abscissa, while the stress amplitude τ_a on the ordinate.

Point A with coordinates with $\left(\frac{1}{2}\tau_{se}, \frac{1}{2}\tau_{se}\right)$ indicates the failure point of the spring wire in fatigue test with pulsating stress cycle.

Point B on the abscissa indicates the failure under static condition, when the mean stress τ_m reaches the torsional yield strength (σ_{sy}). Therefore, the line $\overline{AB}$ is called the line of failure.

To consider the effect of factor of safety, a line is constructed from point D on the abscissa in such a way that,

$$\overline{OD} = \frac{\sigma_{sy}}{(fs)}$$

The line $\overline{DC}$ is parallel to line $\overline{BA}$. Any point on line $\overline{CD}$, such as X, represents a stress situation with the same factor of safety. Line $\overline{CD}$ is called *design line* because it is used to find out permissible stresses with a particular factor of safety.

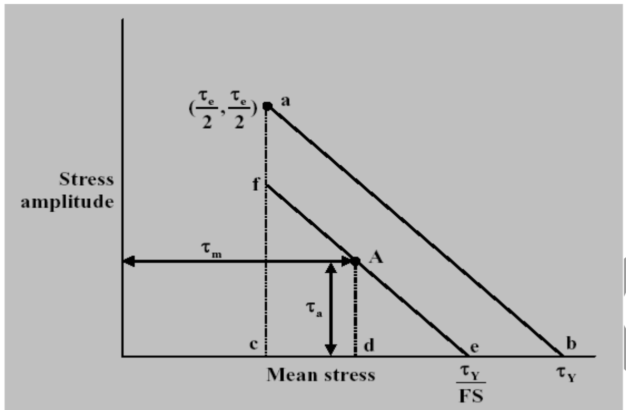
Considering the similar triangles XFD and AEB,

$$\frac{\overline{XF}}{\overline{FD}} = \frac{\overline{AE}}{\overline{EB}}$$

$$\frac{\tau_a}{\frac{\sigma_{sy}}{(fs)} - \tau_m} = \frac{\frac{1}{2}\tau_{se}}{\sigma_{sy} - \frac{1}{2}\tau_{se}}$$

$$\frac{1}{FS} = \frac{\tau_m}{\tau_Y} + \frac{\tau_a}{\tau_Y}\left(\frac{2\tau_Y}{\tau_e} - 1\right)$$

The above equation is used in the design of springs subjected to fluctuating stresses.



Soderberg failure criterion for springs

$$\frac{1}{FS} = \frac{\tau_m}{\tau_Y} + \frac{\tau_a}{\tau_Y}\left(\frac{2\tau_Y}{\tau_e} - 1\right)$$

Problem 12

A spring is subjected to a load varying from 400N to 1000N is to be made of tempered steel cold wound wire. Determine the diameter of wire and mean coil diameter of spring for a factor of safety of 1.5. Spring index 6. Torsional endurance limit is 400N/mm^2

Data

F_{max}=1000N, c=6,

$\boldsymbol{\tau}_{-1}$ = 4000N/mm^2

F_{min} =400N;

FOS n =1.5,

Solution

First method [suggested by wahl]

From DDHB for oil tempered carbon wire $\boldsymbol{\tau}_y$ = 550N/mm^2

Variable stress amplitude

$$\tau_a = K_w\left(\frac{8D}{\pi d^3}\right)\left(\frac{F_{max}-F_{min}}{2}\right) \quad K_\tau = 1+\frac{0.5}{6} = 1.0833$$

Where $K_w = K_\tau K_C$ From DDHB For C = 6

$$K_c = 1.15$$

$$K_\tau = 1+\frac{0.5}{c} = 1+\frac{0.5}{6} = 1.0833$$

$$\therefore K_w = 1.0833x1.15 = 1.2458 = k$$

$$c = \frac{D}{d} \qquad \therefore D = cd = 6d$$

$$\therefore \tau_a = 1.2458\left(\frac{8xd}{\pi d^3}\right)\left(\frac{1000-400}{2}\right)$$

$$= \frac{5710.32}{d^2}$$

Mean shear stress $\tau_m = K_\tau\left(\frac{8D}{\pi d^3}\right)\left(\frac{F_{max}+F_{min}}{2}\right)$

$$\tau_m = 1.0833\left(\frac{8x6d}{\pi d^3}\right)\left(\frac{1000+400}{2}\right)$$

$$= \frac{11586.12}{d^2}$$

$$n = \frac{\tau_y}{\tau_m - \tau_a + \frac{2\tau_a\tau_y}{\tau_{-1}}}$$

i,e., $1.5 = \dfrac{550}{\frac{11586.12}{d^2} - \frac{5710.32}{d^2} + \frac{2x5710.32x550}{d^2x400}}$

$$\therefore d = 7.67mm$$

From DDHB standard wire diameter d = 8mm

Second method [suggested by whal]

$$\tau_{max} - \tau_{min} = K_w\left(\frac{8D}{\pi d^3}\right)\left(\frac{F_{max}-F_{min}}{2}\right) = \frac{\tau\text{-}1}{n}$$

$$i.e., = \frac{1.2458x8x6d[1000-400]}{\pi xd^3} = \frac{550}{1.5}$$

$$i.e., = \frac{1.2458x8x6d[1000-400]}{\pi xd^3} = \frac{550}{1.5}$$

$\therefore$ d= 6.72mm,

Hence 8mm wire diameter is satisfactory

Mean diameter of coil D = 6d = 6×8 = 48mm

Problem 13

A helical compression spring of a cam –mechanical is subjected to an initial preload of 50 N. the maximum operating force during the load-cycle is 150N. The wire diameter is 3 mm, while the mean coil diameter coil diameter is 18mm. the spring is made of oil-hardened and tempered valve spring wire of grade –VW $(\sigma_{ut} = 1430N/mm^2)$. Determine the factor of safety used in the design on the basis of fluctuating stresses.

Solution

$$c = \frac{D}{d} = \frac{18}{3} = 6$$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

$$= \frac{4x6-1}{4x6-4} + \frac{0.615}{6} = 1.2525$$

$$K_s = 1 + \frac{0.5}{6} = 1.0833$$

$$F_m = \frac{1}{2}(F_{max} + F_{min})$$

$$= \frac{1}{2}(150 + 50) = 100\text{N}$$

$$F_a = \frac{1}{2}(F_{max} - F_{min})$$

$$F_a = \frac{1}{2}(150 - 50) = 50\text{N}$$

$$\tau_m = \frac{8F_m D}{\pi d^3} K_s$$

$$= \frac{8(100)(18)}{\pi x 3^3} 1.0833 = 183.91\ N/mm^2$$

$$\tau_a = \frac{8F_a D}{\pi d^3} K_s \quad = \frac{8(50)(18)}{\pi x 3^3} 1.2525 = 106.32\ N/mm^2$$

From the relationship for oil-hardened and tempered steel wire are as follows

$\boldsymbol{\tau}'_{se} = 0.22\ \boldsymbol{\sigma}_{ut} = 0.22(1430) = 314.6 N/mm^2$

$\boldsymbol{\tau}_{sy} = 0.45\ \boldsymbol{\sigma}_{ut} = 0.45(1430) = 643.5 N/mm^2$

$$\frac{106.32}{\frac{643.5}{(fs)} - 183.91} = \frac{\frac{1}{2}(314.6)}{643.5 - \frac{1}{2}(314.6)}$$

$$\frac{\tau_a}{\frac{\sigma_{sy}}{(fs)} - \tau_m} = \frac{\frac{1}{2}\tau_{se}}{\sigma_{sy} - \frac{1}{2}\tau_{se}}$$

$$\frac{106.32}{\frac{643.5}{(fs)} - 183.91} = \frac{157.3}{486.2}$$

$$\frac{643.5}{(fs)} - 183.91 = \frac{106.32(486.2)}{157.3}$$

$$\frac{643.5}{(fs)} = 183.91 + 328.63$$

$$(fs) = 1.26$$

Problem 14

An automotive single plate clutch with two pairs if friction surfaces, transmits a 300 N-m torque at 1500 rpm. The inner and outer diameters of the friction disk are 170 and 270mm respectively. The coefficient of friction is 0.35. The normal force on the friction surfaces is exerted by nine helical compression springs, so that the clutch is always engaged. The clutch is disengaged when the external force further compresses the springs. The spring index is 5 and the number of active coils is 6.the springs are made of patented and cold drawn steel wires of grade 2. (G =81370 N/mm^2).The permissible shear stress for the spring wire is 30% of the ultimate tensile strength. Design the springs and specify their dimension.

Solution:

There are two pairs of contacting surface and the torque transmitted by each pair is (300/2) or 150 N-m. Assuming uniform wear theory, the total normal force p_1 required to transmit the torque is given,

i.e. $$P_1 = \frac{4M_t}{\mu(D+d)} = \frac{4x150x10^3}{0.35x(270+170)} = 3896.1\ N$$

From Eq. (10.7)

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4x6-1}{4x6-4} + \frac{0.615}{5} = 1.3105$$

$$\tau = K\left(\frac{8pD}{\pi d^3}\right)$$

$$\tau = 1.3105\left(\frac{8x432.9x5}{\pi x d^2}\right)$$

The permissible shear stress is denoted by τ_d

In order to differentiate it from induced stress τ. It is given by,

$\tau_d = 0.3s_{ut}$

Equations (a) and (b) are solved by the trial and error method.

Trail 1

d = 3mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{3^2} = 802.59\ N/mm^2$$

From table 10.1

$s_{ut} = 1570$ N/mm^2

$\tau_d = 0.3xs_{ut} = 0.3x1570 = 471$ N/mm^2

Therefore,

$\tau > \tau_d$

The design is not safe.

Trail 2

d = 3.6mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{3.6^2} = 557.35\ N/mm^2$$

From table 10.1

$s_{ut} = 1510\ N/mm^2$

$\tau_d = 0.3xs_{ut} = 0.3x1510 = 453\ N/mm^2$

Therefore,

$\tau > \tau_d$

The design is not safe.

Trail 3

d = 4mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{4^2} = 451.46\ N/mm^2$$

From table 10.1

$s_{ut} = 1480\ N/mm^2$

$\tau_d = 0.3xs_{ut} = 0.3x1480 = 444\ N/mm^2$

Therefore,

$\tau > \tau_d$

The design is not safe.

Trail 4

d = 4.5mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{4.5^2} = 451.46\,N/mm^2$$

From table 10.1

s_{ut} = 1440 N/mm^2

$\tau_d = 0.3 x s_{ut} = 0.3x1440 = 432$ N/mm^2

Therefore,

$\tau < \tau_d$

The design is satisfactory and the wire diameter should be 4.5mm.

D = cd = 5x4.5 = 22.5mm

It is assumed that the springs have square and ground ends.

N_t = N+2 = 6+2 = 8coils

From Eq. (10.8)

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$= \frac{8x432.9x22.5^3x6}{81370x4.5^4} = 7.09mm$$

Solid length of spring = N_t d = 8x4.5 =36mm

It is assumed that there will be a gap of 1mm between consecutive coils when the spring is subjected to the maximum force. The total number of coils is 8.

The total axial gap between the coils will be

(8-1) x 1 = 7 mm.

Free length = solid length + total axial gap + δ

= 36+7+7.09

= 50.09 or 51mm

Spring specifications

i. Material = Patented and cold drawn steel wire of grade 2
ii. Wire diameter = 4.5mm
iii. Mean coil diameter = 22.5mm
iv. Total number of turns = 8
v. Free length of the spring = 51mm
vi. Style of ends = square and ground

Problem 15

A direct reading tension spring balance consists of a helical tension spring that is attached to a rigid support at one end and carries masses at the other free end. The pointer attached to the free end moves on a scale and indicates the mass. The length of the scale is 100mm that is divided into 50 equal divisions. Each division of the scale indicates 0.5 Kg. The maximum capacity of the spring is balance is 25kg. The spring index is 6. The spring made of an oil hardened and tempered steel wire of Grade SW (G =81370 N/mm^2). The permissible shear stress in the spring wire is recommended as 50% of the ultimate tensile strength. Design the spring and give its specifications.

Solution:

The maximum spring force is given by,

P = mg = 25x9.81 = 245.25N

Trail 1

d= 2mm

$$\tau = \frac{7223.28}{d^2} = \frac{7223.28}{2^2} = 1173.33\ N/mm^2$$

From table 10.2

s_{ut} = 1620 N/mm^2`

$\tau_d = 0.5xs_{ut} = 0.3x1620 = 810$ N/mm^2

Therefore,

$\tau > \tau_d$

The design is not safe.

Trail 4

d = 2.5mm

$$\tau = \frac{4693.3}{d^2} = \frac{4693.3}{2.5^2} = 750.93\ N/mm^2$$

From table 10.2

s_{ut} = 1570 N/mm^2

$\tau_d = 0.5 x s_{ut}$ = 0.5x1570 = 785 N/mm^2

Therefore,

$\tau < \tau_d$

The design is satisfactory and the wire diameter should be 2.5mm.

D = cd = 6x2.5 =15mm

$$Length\ of\ each\ division = \frac{length\ of\ scale}{number\ of\ divisions} = \frac{100}{50}$$

$$= 2N/mm$$

Each division indicates 0.5 kg. Therefore,

$$k = \frac{0.5x9.81}{2} = 2.4525\ N/mm$$

From Eq. (10.9)

$$N = \frac{Gd^4 N}{8D^3 k} = \frac{81370x2.5^4}{8x15^3x2.4525} = 48$$

For helical tension spring, all coils are active coils. Therefore,

$N_t = N = 48$

Solid length of spring = N_t d = 48x2.5 = 120mm

Spring specifications

i. Material = oil – hardened and tempered wire of Grade- SW
ii. Wire diameter = 2.5mm
iii. Mean coil diameter = 15mm
iv. Total number of turns = 48
v. Solid length = 120mm
vi. Style of ends = extended – hook

Concentric or Composite Springs

A concentric spring is used for one of the following purposes

1) To obtain greater spring force within a given space.

2) To insure the operation of a mechanism in the event of failure of one of the springs. Assume both the springs are made of same material, and then maximum shear stress induced in both the springs is approximately the same

$$\therefore \tau_1 = \tau_2$$

$$\frac{8F_1D_1k_1}{\pi d_1^3} = \frac{8F_2D_2k_2}{\pi d_2^3}$$

If $K_1=K_2$, then we have;

$$\frac{F_1D_1}{d_1^3} = \frac{F_2D_2}{d_2^3} \qquad \text{………………….. (1)}$$

Also $Y_1=Y_2$,

$$\therefore \frac{8F_1D_1^3i_1}{Gd_1^4} = \frac{8F_2D_2^3i_2}{Gd_2^4}$$

i.e.

$$\frac{F_1D_1^3i_1}{d_1^4} = \frac{F_2D_2^3i_2}{d_2^4}$$

Since the solid lengths are equal we have;

$$i_1d_1 = i_2d_2$$

$$\therefore \frac{F_1D_1^3(i_1d_1)}{d_1^4d_1} = \frac{F_2D_2^3(i_2d_2)}{d_2^4d_2}$$

$$\frac{F_1D_1^3}{d_1^5} = \frac{F_2D_2^3}{d_2^5} \qquad \text{……………………. (2)}$$

Dividing (2) by (1) we get;

$$\frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

$$\therefore \frac{D_1}{d_1} = \frac{D_2}{d_2} = c \qquad \text{.......................... (3)}$$

Therefore spring index for both the spring is same

From equation (1) and (2) we have

$$\therefore \frac{F_1}{F_2} = \frac{d_1^2}{d_2^2}$$

$$\frac{F_1 c}{d_1^2} = \frac{F_2 c}{d_2^2}$$

The radial clearance between the two springs,

$$2c = (D_1 - D_2) - \left(\frac{2d_1}{2} + \frac{2d_2}{2}\right)$$

$$C = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Take the standard radial clearance as $\frac{d_1 - d_2}{2}$

$$\frac{D_1}{2} \quad \frac{D_2}{2} - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

$$\therefore \frac{D_1 - D_2}{2} = \frac{d_1}{2} - \frac{d_2}{2} + \frac{d_1}{2} + \frac{d_2}{2}$$

i.e., = ...(4)

but $c = \frac{D_1}{d_1} = \frac{D_2}{d_2}$

$$\therefore D_1 = cd_1 \quad and \quad D_2 = cd_2$$

$sub.\, in\ equation(4)$

$$\therefore \frac{d_1 c - c d_2}{2} = d_1$$

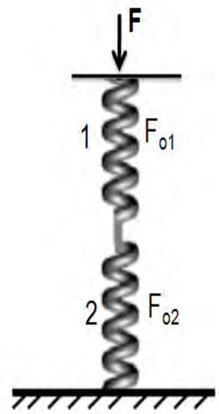
$$cd_1 - cd_2 = 2d_1$$

$$i.e., cd_1 - 2d_1 = cd_2$$

$$d_1(c-2) = cd_2$$

$$\therefore \frac{d_1}{d_2} = \frac{c}{c-2}$$

Spring in series



$$F_{01} = \frac{F_1}{y_1}$$

$$F_{02} = \frac{F_2}{y_2}$$

F= $F_1 = F_2$

y = $y_1 + y_2$

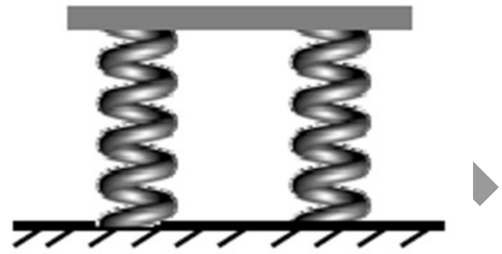
$$\therefore \frac{F}{F_0} = \frac{F_1}{F_{01}} + \frac{F_2}{F_{02}}$$

$$\therefore \frac{1}{F_0} = \frac{1}{F_{01}} + \frac{1}{F_{02}}$$

$$\therefore F = F_1 = F_2$$

$where\ F_0 = combined\ stiffness$

Springs in parallel



$$F = F_1 + F_2$$

$$y = y_1 + y_2$$

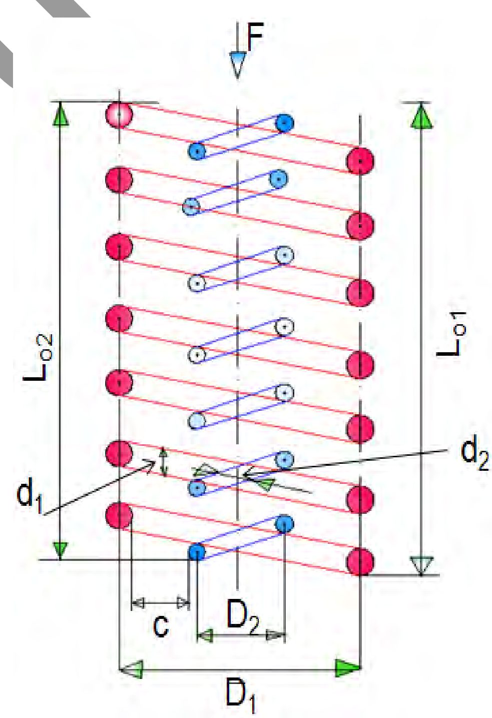
$$\therefore F_o\, y = F_{01} y_1 + F_{02} y_2$$

Therefore

$$F_o = F_{01} + F_{02}$$

Where $F_o = combined\ stiffness$

Concentric springs



D1 = Mean diameter of coil of outer spring

D2 = Mean diameter of coil of inner spring

d1 = Wire diameter of outer spring

d2 = Wire diameter of inner spring

Lo1 = Free length of outer spring

Lo2 = Free length of inner spring

F = Total load on the springs

F1 = Load on the outer spring

F2 = Load on the inner spring

Problem 16

Equal Free length

One helical spring is rested inside another. The dimensions are as tabulated both springs have same free length and carry a total load of 2500 N.

Take G = 80GPa Determine for each spring

1) Maximum load carried

2) Deflection

3) Maximum shear stress.

Given data;

Particulars	inner spring	outer spring
▪ Number of active coils	10	6
▪ Wire diameter (mm)	6.2	12.5
▪ Mean coil diameter (mm)	56.3	87.5

Solution

$i_1 = 6$ $\quad$ $i_2 = 10$

$d_1 = 12.5mm$ $\quad$ $d_2 = 6.2mm$

$D_1 = 87.5mm$ $\quad$ $D_2 = 56.3mm$

$F = 2500N$ $\quad$ $G = 80Gpa = 80000Mpa$

i) Maximum load on each spring

Since both springs have same equal free length

$F_1 + F_2 = 2500N$ ……………………..…. (1)

$$\frac{F_1}{F_2} = \left(\frac{D_2}{D_1}\right)^3 \left(\frac{d_1}{d_2}\right)^4 \left(\frac{i_2}{i_1}\right)\left(\frac{G_1}{G_2}\right)$$

$$= \left(\frac{56.3}{87.5}\right)^3 \left(\frac{12.5}{6.2}\right)^4 \left(\frac{10}{6}\right) \left(\frac{80000}{80000}\right)$$

$F_1 = 7.3354F_2$ ……………………….(2)

Sub in (1)

$7.3354\ F_2 + F_2 = 2500$

$\therefore F_2 = 300N$ and $F_1 = 2200N$

ii) Deflection

$$y_1 = y_2 \quad (\because \textit{Equal free length})$$

$$\therefore y_1 = \frac{8F_1D_1^3 i_1}{d_1^4 G} = \frac{8x2200x87.5^3x6}{12.5^4x80000}$$

$y_1 = 36.22mm = y_2$

iii) Maximum shear stress induced

Shear stress on outer spring

$$\tau_1 = \frac{8F_1D_1k_1}{\pi d_1^3}$$

$$C_1 = \frac{D_1}{d_1} = \frac{87.5}{12.5} = 7$$

$$\therefore k_1 = \frac{4c_1 - 1}{4c_1 - 4} + \frac{0.615}{c_1} = \frac{4x7 - 1}{4x7 - 4} + \frac{0.615}{7} = 1.2128$$

$$\therefore \tau_1 = \frac{8x2200x87.5x1.2128}{\pi x12.5^3} = 304.4N/mm^2$$

Shear stress on inner spring

$$\tau_2 = \frac{8F_2D_2k_2}{\pi d_2^3}$$

$$c_2 = \frac{D_2}{d_2} = \frac{56.3}{6.2} = 9.08$$

$$\therefore\ k_2 = \frac{4x9.08 - 1}{4x9.08 - 4} + \frac{0.615}{9.08} = 1.1605$$

$$\tau_2 = \frac{8x300x56.3x1.1605}{\pi x 6.2^3} = 209.4 N/mm^2$$

Problem 17

The table below gives the particulars of a concentric helical spring. If the spring is subjected to an axial load of 400N, determine for each spring,

(i) Change in length,

(ii) Amount of load carried,

(iii) Torsional shear stress induced.

Take G = 84 GPa = 84000MPa

Particulars	Inner spring	Outer spring
Mean coil diameter (mm)	30	40
Diameter of wire (mm)	4	4.875
Number of active turns	8	10
Free length (mm)	75	90

Given data:

D_2= 30mm; D_1=40mm; d_2=4mm; d_1=4.875mm

i_2=8; i_1=10; G=84000 N/mm^2; F=400N.

Solution

Unequal free length,

iv) Amount of load carried

The load required to compress the outer spring by 15mm is

$$y = \frac{8FD^3\, i}{d^4G}\ i.e., \qquad 15 = \frac{8xFx40^3x10}{4875^4x84000}$$

F=139N

$$\therefore \quad Remaining\ load\ F_R = 400 - 139 = 261\ N$$

This load will be shared by the springs for further equal deflection

i.e., $F_R = F_{R1} + F_{R2}$ ……….. (1)

$F_{R1} + F_{R2} = 261$

$$\frac{F_{R1}}{F_{R2}} = \left(\frac{D_2}{D_1}\right)^3 \left(\frac{d_1}{d_2}\right)^4 \left(\frac{i_2}{i_1}\right)\left(\frac{G_1}{G_2}\right)$$

$$= \left(\frac{30}{40}\right)^3 \left(\frac{4.875}{4}\right)^4 \left(\frac{8}{10}\right)\left(\frac{84000}{84000}\right)$$

$F_{R1} = 0.7446\, F_{R2}$ ………… (2)

Sub. In equation (1)

$0.7446\, F_{R2} + F_{R2} = 261$

$F_{R2} = 149.6N$

$F_{R1} = 111.4N$

Total load on the outer spring

$F_1 = F_{R1} + 139$

$= 111.4 + 139$

$F_1 = 250.4N$

Load on the inner spring

$F_2 = 149.6N$

ii) Change in length

$$y_2 = \frac{8F_2D_2^3 i_2}{d_2^4 G}$$

$$= \frac{8x149.6x30^3x8}{4^4x84000}$$

Change in length for inner spring

$Y_2 = 12.02$

Change in length outer spring

$Y_1 = 15 + 12.02$

$= 27.02mm$

iii) Torsional shear stress induced

Shear stress on the outer spring

$$\tau_1 = \frac{8F_1D_1k_1}{\pi d_1^3}$$

$$C_1 = \frac{D_1}{d_1} = \frac{40}{4.875} = 8.2$$

$$k_1 = \frac{4c_1 - 1}{4c_1 - 4} + \frac{0.615}{c_1} = \frac{4x8.2 - 1}{4x8.2 - 4} + \frac{0.618}{8.2}$$
$$= 1.179$$

$$\tau_1 = \frac{8x250.4x40x1.179}{\pi x4.875^3} = 259.55N/mm^2$$

Shear stress on the inner spring

$$\tau_2 = \frac{8F_2D_2k_2}{\pi d_2^3} \qquad C_1 = \frac{D_2}{d_2} = \frac{30}{4} = 7.5$$

$$k_2 = \frac{4x7.5 - 1}{4x7.5 - 4} + \frac{0.615}{7.5} = 1.197$$

$$\tau_2 = \frac{8x149.6x30x1.197}{\pi x4^3} = 213.75\ N/mm^2$$

HELICAL SPRINGS OF NON-CIRCULAR CROSS SECTIONS

Helical springs can be made from non-circular sections to provide greater resilience in a restricted space. They can also be used to provide for pre-determined altering of the stiffness of the spring, by using square or rectangular wires

Springs of non circular cross section are not as strong as the springs made of circular cross section wire.

A rectangular wire becomes trapezoidal when the coil is formed. When space is severely limited, the use of concentric springs should be considered.

Let

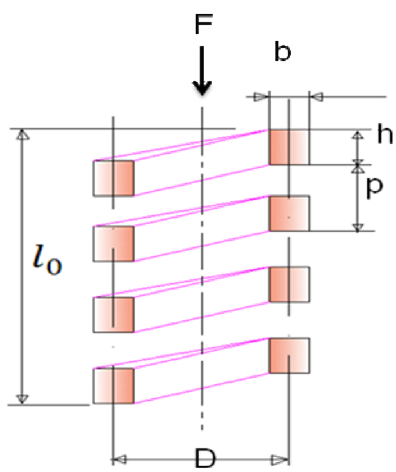
b = Width of rectangular c/s spring (side of rectangle perpendicular to the axis)

h = Axial height of rectangular c/s spring (side of rectangle parallel to the axis)

D = Mean diameter of coil;

C = spring index

The equation for the stress produced and deflection in a rectangular cross section



Shear stress

$$\tau' = \frac{kFD(1.5h + 0.9b)}{h^2h^2}$$

K = stress factor = $\dfrac{(4c-1)}{(4c-4)} + \dfrac{0.615}{c}$

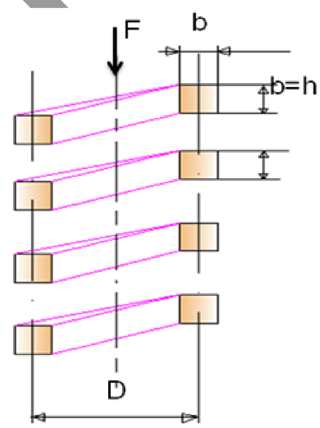
C = spring index $= \dfrac{D}{b}$ if b< h

$= \dfrac{D}{h}$ if h< b

Deflection in a rectangular cross section

Deflection $y = \dfrac{2.83 i F D^3 (b^2 + h^2)}{G b^3 h^3}$

For Square cross section spring, b = h



Shear stress $\tau' = \dfrac{2.4 k F D}{h^3}$

Deflection $y = \dfrac{5.66\, i\, F\, D^3}{G h^4}$

Problem 18

A rectangular section helical spring is mounted to a buffer to sustain a maximum load of 30 KN. The deflection under load is limited to 100mm. The spring is made of chrome-vanadium steel with reliability of 1.5. The longer side of rectangle is 2 times the shorter side and the spring is wound with longer side parallel to the axis. The spring index is 10.Design the spring and draw a conventional sketch.

Given data:

F = 30KN = 30000N,

h= 2b,

c=10

y = 100 mm,

FOS = 1.5;

Solution

For chromium-vanadium steel, from Data Hand book

G = 79.34 GPa = 79.34x103 N/mm^2

$$\tau = \frac{\tau_y}{FOS} = \frac{690}{1.5} = 460\ \frac{N}{mm^2} = \tau'$$

1. Cross Section of spring

Shear Stress $\tau' = \dfrac{kFD(1.5h + 0.9b)}{b^2h^2}$

Stress factor k $= \dfrac{(4c-1)}{(4c-4)} + \dfrac{0.615}{c}$

$$= \frac{(4\times 10-1)}{(4\times 10-4)} + \frac{0.615}{10} = 1.1448$$

Spring index $c = \dfrac{D}{b}$

$$\therefore D = cb = 10b$$

$$\therefore 460 = \frac{1.1448\times 30000\times 10b\times(1.5\times 2b+0.9b)}{b^2(2b)^2}$$

Width of spring b= 26.98mm = 27 mm

Height of spring h= 2b= 2x27= 54 mm

2. Diameter of coil

Mean Diameter D= 10b =10x27= 270mm

Outer diameter D_0 = D + b = 297 mm

Inner diameter D_i = D - b = 243 mm

3. Number of coils or turns

Deflection $$y = \frac{2.83iFD^3(b^2+h^2)}{Gb^3h^3}$$

i.e. $$100 = \frac{2.83i \times 30000 \times 270^3(27^2+54^2)}{79.34 \times 1000 \times 27^3 \times 54^3}$$

$$i = 4.037 \approx 5$$

Therefore the number of active turns i = 5

4. Free length

$$l_0 \geq (i+n)h + y + a$$

Assume squared and ground end

Number of additional coils n = 2

Total number of turns i' = i + n = 5+2 = 7

Clearance a = 25% of deflection

$$\therefore\ l_0 \geq (7 \times 54) + 100 + 25$$

$$l_0 \geq 503mm$$

5. Pitch

$$p = \frac{l_0 - 2h}{i} = \frac{503 - 2 \times 54}{5} = 79\ mm$$

6. Rate of spring

$$F_0 = \frac{F}{y} = \frac{30000}{100} = 300N/mm$$

7. Rate of spring

$$l = \pi D i' = \pi \times 270 \times 7 = 5937.6mm$$

$$F_0 = \frac{F}{y} = \frac{30000}{100} = 300N/mm$$

Spring specifications

1. Material: chrome-vanadium steel

2. Wire cross section

Width b = 27mm

Height h = 54mm

3. Mean coil diameter D = 270mm

4. Free length = 503mm

5. Total number of turns i' =7

6. Style of ends squared and ground

7. Pitch p =79mm

8. Rate of spring F_0=300N/mm

Problem 19

A diesel engine weighs 800KN is mounted on 16 springs in order to protect the building from vibration. The section of the spring wire is rectangular with side ratio 1.8 One spring has four effective coils. Spring index 6.

Determine

(i) Section of spring so that longer side is parallel to the axis.

(ii) Deflection under load when the engine is stationary

(iii) Maximum coil diameter and

(iv) Shear stress induced if shorter side is parallel to the axis.

Take $\boldsymbol{\tau} = 0.3$ GPa and **G** = 80 GPa

Data:

W = 800kN = 8×10^5 N,

Number of springs =16

i = 4,

c = 6,

$$\frac{h}{b} = 1.8$$

$$\tau = 0.3GPa = 300N/mm^2$$

$$G = 80GPa = 80 \times 10^3 N/mm^2$$

Solution

Axial load on each spring

$$F = \frac{Weight\ of\ engine}{Number\ of\ springs} = \frac{8 \times 10^5}{16}$$

$$= 50000N$$

i) Cross Section of Spring

Stress factor k $= \frac{(4c-1)}{(4c-4)} + \frac{0.615}{c}$

Spring index since b $c = \frac{D}{b}$ < h

$$\therefore D = cb = 6b$$

Shear Stress $\tau' = \frac{kFD(1.5h+0.9b)}{b^2h^2}$

$$i.e., 300 = \frac{1.2525 \times 50000 \times 6b[1.5 \times 1.8b + 0.9b]}{b^2(1.8b)^2}$$

i.e., Width of spring b = 37.3 37.5mm

Height of spring h = 1.8b = 67.5mm

ii) Maximum Coil diameter

Mean coil diameter

D = c.b =6x37.5=225mm

Outer diameter of the coil (Maximum Coil Diameter)

D_0 = D + b = 225+37.5 =262.5mm

iii) Deflection under the load

$$y = \frac{2.83 i F D^3 (b^2 + h^2)}{G b^3 h^3}$$

$$= \frac{2.83 \times 4 \times 50000 \times 225^3 (37.5^2 + 67.5^2)}{80 \times 10^3 \times 37.5^3 \times 67.5^3}$$

$$y = 29.6276mm$$

iv) Shear stress induced if shorter side is parallel

$$\therefore h = 37.5mm \text{ and } b = 67.5mm$$

Shear Stress

$$\tau' = \frac{1.2525 \times 50000 \times 225(1.5 \times 37.5 + 0.9 \times 67.5)}{67.5^2 \times 37.5^2}$$

$$\tau' = 257.3\ N/mm^2$$

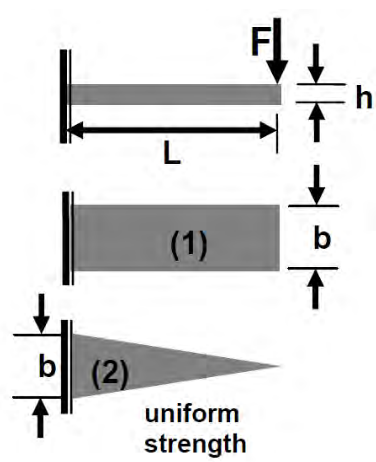
Leaf springs

Characteristics

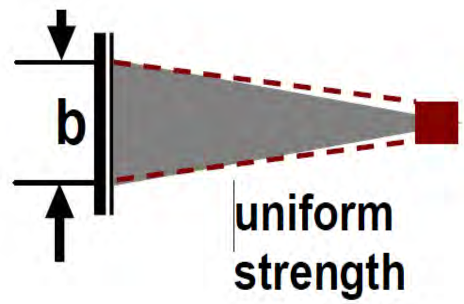
1. Sometimes it is also called as a semi-elliptical spring; as it takes the form of a slender arc shaped length of spring steel of rectangular cross section.
2. The center of the arc provides the location for the axle, while the tie holes are provided at either end for attaching to the vehicle body.
3. Supports the chassis weight
4. Controls chassis roll more efficiently-high rear moment center and wide spring base
5. Controls rear end wrap-up
6. Controls axle damping
7. Controls braking forces
8. Regulates wheelbase lengths (rear steer) under acceleration and braking

Leaf Springs

In the cantilever beam type leaf spring, for the same leaf thickness, h, leaf of uniform width, b (case 1) and, leaf of width, which is uniformly reducing from b (case 2) is considered. From the basic equations of bending stress and deflection, the maximum stress $\boldsymbol{\sigma}_{\mathbf{max}}$, and tip deflection $\boldsymbol{\delta}_{max}$, can be derived.



It is observed that instead of uniform width leaf, if a leaf of varying width is used, the bending stress at any cross section is same and equal to maximum stress σ_{max}. This is called as leaf of a uniform strength.



Moreover, the tip deflection being more, comparatively, it has greater resilience than its uniform width counterpart.

Resilience, as we know, is the capacity to absorb potential energy during deformation.

For case 1(uniform width)

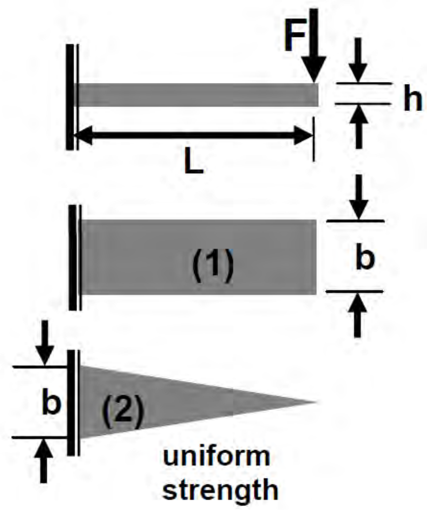
$$\sigma_{max} = \frac{6FL}{bh^2}$$

$$\delta_{max} = \frac{4FL^3}{Ebh^3}$$

For case 2(non uniform width)

$$\sigma_{max} = \frac{6FL}{bh^2}$$

$$\delta_{max} = \frac{6FL^3}{Ebh^3}$$



For case 1(uniform width)

$$\sigma_{max} = \frac{3FL}{bh^2}$$

$$\delta_{max} = \frac{2FL^3}{Ebh^3}$$

$$\sigma_{max} = \frac{3FL}{bh^2}$$

$$\delta_{max} = \frac{3FL^3}{Ebh^3}$$

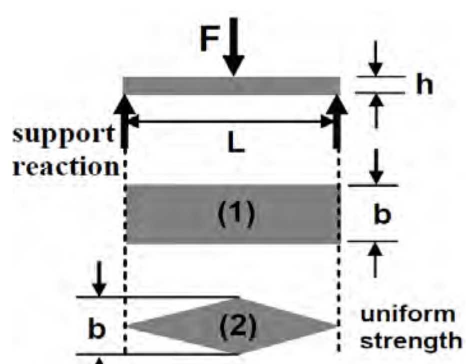


Fig. 7.3.3

One of the applications of leaf spring of simply supported beam type is seen in automobiles, where, the central location of the spring is fixed to the wheel axle. Therefore, the wheel exerts the force F on the spring and support reactions at the two ends of the spring come from the carriage.

Design theme of a leaf spring

Let us consider the simply supported leaf of Lozenge shape for which the maximum stress and maximum deflection are known.

From the stress and deflection equations the thickness of the spring plate, h, can be obtained as,

$$h = \frac{\sigma_{max} L^2}{E\delta_{max}} = \frac{\sigma_{des} L^2}{E\delta_{des}}$$

The $\boldsymbol{\sigma}_{max}$ is replaced by design stress $\boldsymbol{\sigma}_{des}$ similarly, $\boldsymbol{\delta}_{max}$ is replaced by $\boldsymbol{\delta}_{des}$. E is the material property and depends on the type of spring material chosen.

L is the characteristic length of the spring.

Therefore, once the design parameters, given on the left side of the above equation, are fixed the value of plate thickness, h can be calculated.

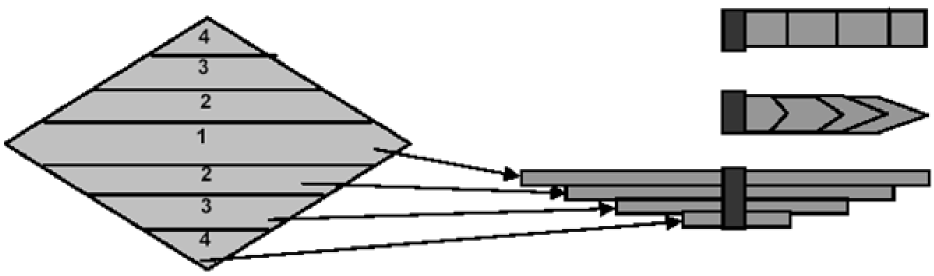
Substitution of h in the stress equation above will yield the value of plate width b.

$$b = \frac{3FL}{\sigma_{des} h^2}$$

In the similar manner h and b can be calculated for leaf springs of different support conditions and beam types.

Laminated springs

One of the difficulties of the uniform strength beam, say Lozenge shape, is that the value of width b sometimes is too large to accommodate in a machine assembly. One practice is that instead of keeping this large width one can make several slices and put the pieces together as a laminate. This is the concept of laminated spring. The Lozenge shaped plate is cut into several longitudinal strips, as indicated in the figure.



Laminated Spring

The central strip, marked 1 is the master leaf which is placed at the top. Then two pieces, marked 2 are put together, side by side to form another leaf and placed below the top leaf. In the similar manner other pairs of strips, marked 3 and 4 respectively are placed in the decreasing order of strip length to form a laminated spring. Here width of each strip, b_N is given as;

$$\mathbf{b_N = \frac{b}{N}} \quad \text{or} \quad b' = \frac{b}{i}$$

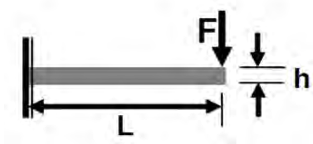
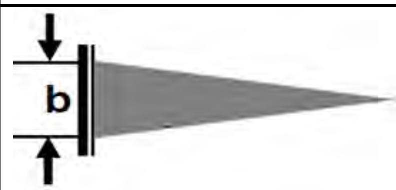
Where N is the number of strips

The stress and deflection equations for a laminated spring is,

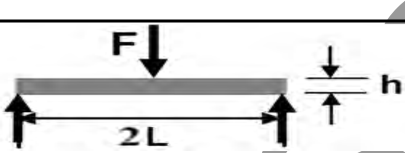
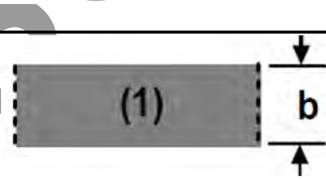
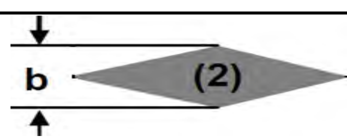
$$\sigma_{max} = \frac{C_1 FL}{ib'h^2} \quad \text{and} \quad \delta_{max} = \frac{C_2 FL^3}{Eib'h^3}$$

Where, constants C_1 and C_2 are different for different cases,

The values of the constants C_1 and C_2 for cantilever beam case

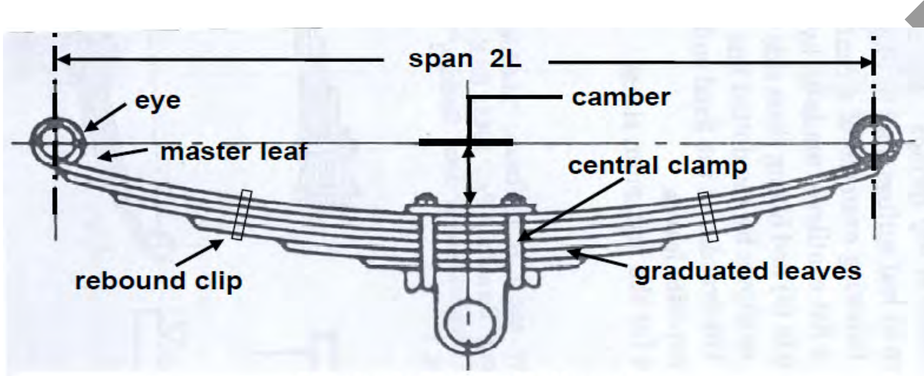
Cantilever Beam		Constants	
		C_1	C_2
Uniform Width		6	4
Non-Uniform Width		6	6

The values of the constants C_1 and C_2 for simply supported beam case

Simply Supported Beam		Constants	
		C_1	C_2
Uniform Width		3	2
Non-Uniform Width		3	3

Laminated semi-elliptic spring

The figure shows a laminated semi- elliptic spring. The top leaf is known as the master leaf. The eye is provided for attaching the spring with another machine member. The amount of bend that is given to the spring from the central line, passing through the eyes, is known as camber. The camber is provided so that even at the maximum load the deflected spring should not touch the machine member to which it is attached. The central clamp is required to hold the leaves of the spring.



Laminated semi-elliptic spring

To prove that stress developed in the full length leaves is 50% more than that in the graduated leaves.

Step1: Bending stress and displacement in the graduated leaves

For analysis half the spring can be considered as a cantilever. It is assumed that the individual leaves are separated and the master leaf placed at the center. Then the second leaf is cut longitudinally into two halves, each of width (b/2) and placed on each side of the master leaf. A similar procedure is repeated for rest of the leaves

The graduated leaves along with the master leaf thus can be treated as a triangular plate of thickness‘t’ as shown in figure 1.

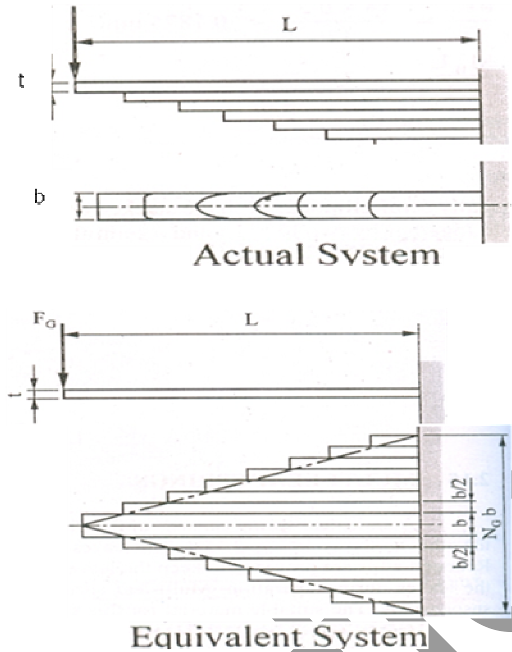


Fig 1

Let,

i_f = No. of extra full length leaves

i_g = No. of graduated leaves including the master leaf

b = Width of each leaf

t = Thickness of each leaf

L = Length of the cantilever or half the length of the spring

F = Total force applied at the end of the spring

F_f = Force absorbed by the full length leaves.

F_g = Force absorbed by the graduated leaves.

The bending stress developed in the graduated leaves will be:

$$\sigma_{bg} = \frac{M_b y}{I} = \frac{F_g L\left(\frac{t}{2}\right)}{\frac{1}{12}(i_g b)t^3} = \frac{6F_g L}{i_g b t^2}$$

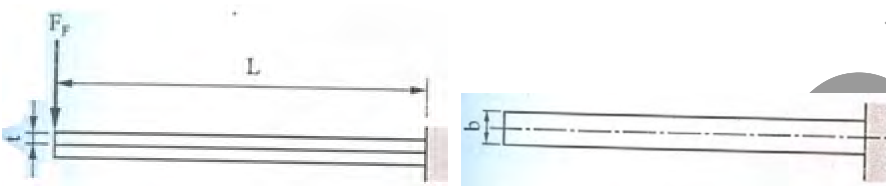
For cantilever triangular plate, the deflection at the point of application of force is given by:

$$\delta_g = \frac{F_g L^3}{2EI_{max}} = \frac{6F_g L^3}{E i_g b t^3}$$

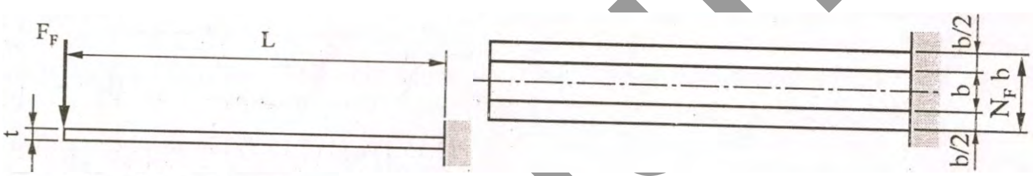
Step 2: Bending stress and displacements in full length leaves

It is assumed that the individual leaves are separated and the full length leaf is placed at the center. Then the second full length leaf is cut longitudinally into two halves, each of width (b/2) and placed on each side of the first leaf. A similar procedure is repeated for the rest of the leaves.

The resulting cantilever beam of thickness't' is shown in the figure 2.



Actual System



Equivalent System

Fig 2

The bending stress developed in the full length leaves will be:

$$\sigma_{bf} = \frac{M_b y}{I} = \frac{F_f L\left(\frac{t}{2}\right)}{\frac{1}{12}(i_f b)t^3} = \frac{6F_f L}{i_f b t^2}$$

For a cantilever rectangular plate, the deflection at the point of application of force is given by:

$$\delta_f = \frac{F_f L^3}{3EI_{max}} = \frac{4F_f L^3}{E i_f b t^3}$$

Step 3:

$$\delta = \delta_g = \delta_f$$

Since the graduated leaves and the full leaves are clamped together the deflection for both should be the same.

$$\therefore \qquad \delta = \delta_g = \delta_f$$

$$\frac{6F_g L^3}{Ei_g bt^3} = \frac{4F_f L^3}{Ei_f bt^3}$$

$$\frac{F_g}{F_f} = \frac{2i_g}{3i_f}$$

Also
$$F = F_g + F_f$$

$$F = F_f\left(1 + \frac{2i_g}{3i_f}\right) \qquad \because \frac{F_g}{F_f} = \frac{2i_g}{3i_f}$$

Solving we get;

$$F_f = \left(\frac{3Fi_f}{3i_f + 2i_g}\right) \quad \text{And} \quad F_g = \left(\frac{2Fi_g}{3i_f + 2i_g}\right)$$

Substituting the values of F_f and F_g in the equations of σ_{bf} and σ_{bg} we get;

$$\sigma_{bf} = \frac{18FL}{(3i_f + 2i_g)bt^2} \quad \text{And} \quad \sigma_{bg} = \frac{12FL}{(3i_f + 2i_g)bt^2}$$

Taking the ratios of both stresses and solving we get;

$$\frac{\sigma_{bf}}{\sigma_{bg}} = 1.5$$

Hence proved.

The maximum deflection of the leaf spring can be found as follows;

We have from previous equations

$$F_f = \left(\frac{3Fi_f}{3i_f + 2i_g}\right) \; ; \; F_g = \left(\frac{2Fi_g}{3i_f + 2i_g}\right)$$

And $$\delta_g = \frac{6F_g L^3}{Ei_g bt^3} \; ; \; \delta_f = \frac{4F_f L^3}{Ei_f bt^3}$$

Consider anyone of the deflection equation

i.e.; $$\delta_f = \frac{4F_f L^3}{Ei_f bt^3}$$

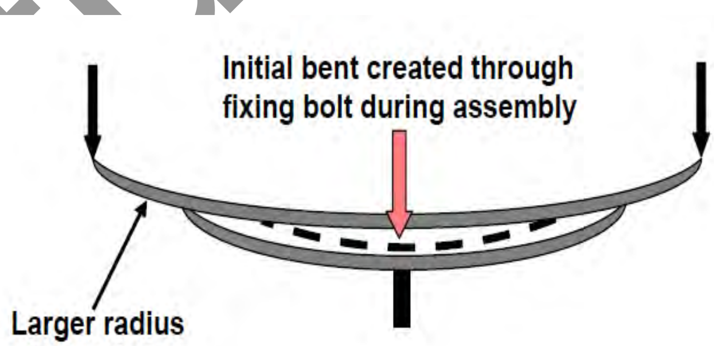
Substituting $$F_f = \left(\frac{3Fi_f}{3i_f + 2i_g}\right)$$

in the above equation and solving, we get

Maximum deflection

$$\delta = \frac{12FL^3}{(3i_f + 2i_g)Ebt^3}$$

Equalized stress in spring leaves (Nipping);



Nipping of leaf spring

- The stress in the full length leaves is 50% greater than the stress in the graduated leaves.
- To distribute this additional stress from the full length leaves, pre-stressing is done. This is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre bolt
- The full length leaves are given in greater radii of curvature than the adjacent one. Due to the different radii of curvature, when the full length leaves are staked with the graduated leaves, without bolting, a gap is observed between them. This gap is called Nip
- The nip eliminated by tightening of the center bolt due to these pre-stresses is induced in the leaves. This method of pre-stressing by giving different radii of curvature is called as nipping.
- By giving a greater radius of curvature to the full length leaves than graduated leaves before the leaves are assembled to form a spring.

Nip: C

The value of the initial Nip C is nothing but the difference in deflection between the full length and the graduated leaves

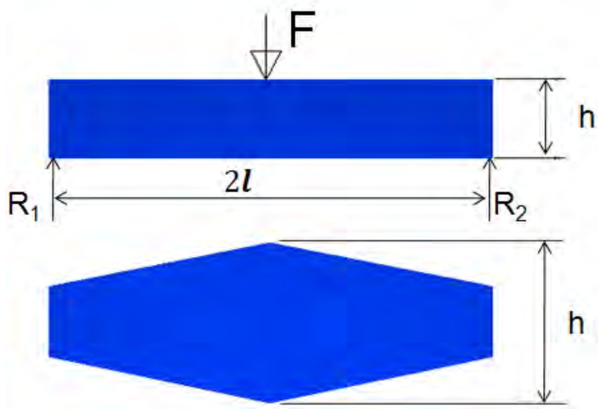
i.e.; $C = \delta_g - \delta_f$

Solving, we get;

$$C = \frac{2FL^3}{ib'h^3E}$$

Problem 20

Determine the width and thickness of a flat spring carrying a central load of 5000N.The deflection is limited to 100mm. The spring is supported at both ends at a distance of 800mm. The allowable stress is 300N/mm2 and modulus of elasticity 221GPa. The spring is of constant thickness and varying width.



Given data:

F = 5000N; y = 100mm;

$2l$ = 800mm $\therefore l$ = 400mm

σ = 300Nmm^2; E = 221GPa = 221x10^3 N/mm^2

Solution:

Since the spring is of constant thickness and varying width. It is as shown in figure and from table

c_1 =3; c_2 = 3

Maximum stress in the spring $\sigma = \frac{c_1 F l}{bh^2}$

i.e.

$$300 = \frac{3x5000x400}{bh^2}$$

$\therefore bh^2 = 20000$ …………… (1)

Maximum deflection $y = \frac{C_2 F l^3}{Ebh^3}$

$$i,e \quad 100 = \frac{3 \times 5000 \times 400^3}{221 \times 10^3 bh^3}$$

$$bh^3 = 43438.914 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

eqn. (2) divided by eqn. (1)

$$\frac{bh^3}{bh^2} = h = \frac{43438.914}{20000} = 2.172$$

Take thickness of spring h = 2.5mm

Width of spring at the centre,

From equation (1)

$$b = \frac{20000}{2.5^2} = 3200\text{mm}$$

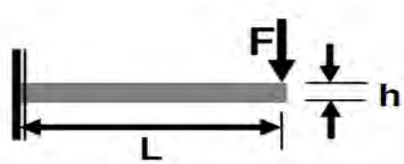
From equation (2)

$$b = \frac{43438.914}{2.5^3} = 2780mm$$

Select the bigger value as the permissible value

$$\therefore b = 3200mm$$

The values of the constants C_1 and C_2 for cantilever beam case

Cantilever Beam		Constants	
		C_1	C_2
Uniform Width		6	4
Non-Uniform Width		6	6

Problem 21

An automobile semi- elliptical leaf spring has 12 numbers of graduated leaves and 3 number of full length leaves. The spring is to sustain a load of 25kN at its center and the ratio of total depth to the width of the spring is 2.5. The material of the leaves has design normal stress of 450 MPa and a modulus of elasticity of 207 GPa. Determine

1. Width and thickness of leaves.
2. Initial gap between the full length and graduated leaves before assembly.
3. Bolt load
4. Central deflection.
5. Radius of curvature of first full length leaf.
6. The width of the central band is 100mm and the span of the leaves is 1200mm.

Given data;

i_g =12,

σ_f = 450MPa

i_f = 3

$$\frac{ih}{b'} = 2.5$$

2F=25kN

E = 207GPa

L_b =100mm

2L=1200mm

Solution:

Effective length $$l = \frac{2L - L_b}{2} = \frac{2(600) - 100}{2} = 550mm$$

$$\frac{ih}{b'} = 2.5$$

$i = i_g + i_f$

i= 12+3 = 15

b’ = 6h

The maximum stress in the spring with the full length leaf pre-stress

$$\sigma_f = \frac{3FL}{ib'h^2}$$

$$450 = \frac{3x12500x550}{15x6xh^3}$$

h = 7.98mm ≈ 8mm (std)

b’ = 6xh=6x8 = 48mm

The initial gap between the full length and graduated length

$$C = \frac{Fl^3}{ib'Eh^3}$$

$$C = \frac{12500x550^3}{15x48x207x10^3x8^3}$$

c= 27.25mm

The load on the clip bolts

$$F_b = \frac{i_g i_f F}{i(2i_g + 3i_f)}$$

$$F_b = \frac{12x3x12500}{15(2x12 + 3x3)}$$

F_b= 909.1MPa

Deflection of the spring

$$y = \frac{6Fl^3}{b'h^3E(2i_g + 3i_f)}$$

$$y = \frac{6x12500x550^3}{48x8^3x207(2x12 + 3x3)} = 74.33mm$$

Problem 22

A semi elliptical is to sustain a load of 25kN. The span of the spring is 1100mm with a central band of 100mm. The material selected for the leaves as a design normal stress of 400N/mm^2 and E = 207GPa. The ratio between total depth of the spring and width ‘2’ also determine the radius of curvature to which the first full length leaf is to bend such that the spring becomes flat with the full load

Solution

i_g =10, $\boldsymbol{\sigma}_f$ = 400MPa, i_f = 2

2F=25kN, E = 207GPa, L_b =100mm, 2L=1100mm

$$\frac{ih}{b'} = 2$$

Effective length $\quad l = \dfrac{2L - L_b}{2}$

$$\frac{2(550) - 100}{2} = 500mm$$

i =i_g+i_f

i= 10+2 = 12

ih/b' = 2

The maximum stress in the spring with the full length leaf pre-stress

$$\sigma_f = \frac{3FL}{ib'h^2}$$

$$\frac{12xh}{b'} = 2$$

$$b' = 6h$$

$$400 = \frac{3x12500x500}{12x6xh^3}$$

h = 8.65mm ≈ 10mm (std)

b' = 6xh=6x10 = 60mm

The load on the clip bolts

$$F_b = \frac{i_g i_f F}{i(2i_g + 3i_f)} \qquad F_b = \frac{2x10x12500}{12(2x10+3x2)}$$

$$F_b = 801.2N$$

Deflection of the spring

$$y = \frac{6Fl^3}{b'h^3E(2i_g + 3i_f)}$$

$$y = \frac{6x12500x500^3}{60x10^3x207(2x10+3x2)} = 29.03mm$$

Combination of springs

Problem 23

A 100mm outside diameter steel coil spring having 10 active coils of 12.5 diameter wire is in contact with a 600mm long steel cantilever spring having 5 graduated leaves 100mm wide and 10 mm thick as shown in figure.

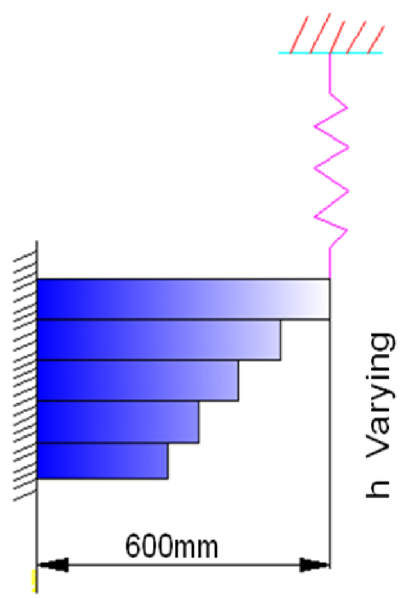
i) What force "F" is gradually applied to the top of the coil spring will cause the cantilever to deflect by 50mm

ii) What is the bending stress in cantilever beam?

iii) What is the shear stress in coil spring?

iv) What energies stored by each spring.

Take 210 GPA and G=84 GPa



Cantilever Coil

i = 5 i = 10

l = 600 mm d = 12.5mm

b' = 100 mm D_0= 100mm

$\therefore$ D = 100 – 12.5 = 87.5mm

y = 50mm G = 84x103N/mm^2

E = 210x103N/mm^2

h = 10 mm

Solution:

Spring Index

$$c = \frac{D}{d} = \frac{87.5}{12.5} = 7$$

Since the coil spring is on the top of the cantilever,

Load on cantilever = load on coil spring

i.e., $F_1 = F_2$

For constant width varying depth

$c_1 = 6$ and $c_2 = 8$

i) Cantilever Spring

Deflection

$$y = \frac{c_2 F l^3}{Ei\, b' h^3}$$

$$\text{i.e.,} \quad 50 = \frac{8 \times F_1 \times 600^3}{210 \times 10^3 \times 5 \times 100 \times 10^3}$$

∴ Load applied on the top of the coil spring

$F_1 = 3038.2$ N

ii) Bending Stress in cantilever spring

$$\sigma = \frac{C_1 F l}{i b' h^2} = \frac{6 \times 3038.2 \times 600}{5 \times 100 \times 10^2}$$

$\sigma = 218.75$ N/mm2

iii) Shear Stress in coil spring

$$\tau = \frac{8FDK}{\pi d^3}$$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times7-1}{4\times7-4} + \frac{0.615}{7}$$
$$= 1.2128$$
$$\tau = \frac{8\times3038.2\times87.5\times1.2128}{\pi\times12.5^3}$$
$$\tau = 420\ \mathrm{N/mm^2}$$

iv) Energy stored

Energy stored in the cantilever spring

$$U_1 = \frac{1}{2}F_1\,y_1$$
$$= \frac{1}{2}\times3038.2\times50$$
$$= 75955\ \text{N-mm}$$
$$U_1 \approx 76\ \mathrm{Nm}$$

v) Energy stored in coil spring

$$U_2 = \frac{1}{2}F_2\,y_2$$
$$y_2 = \frac{8FD^3 i}{Gd^4}$$
$$= \frac{8\times3038.2\times87.5^3\times10}{84\times10^3\times12.5^4} = 79.4\ \mathrm{mm}$$
$$\therefore U_2 = \frac{1}{2}\times3038.2\times79.4$$
$$= 120616.54\ \mathrm{Nmm}$$
$$U_2 \approx 120.62\ \mathrm{Nm}$$

Buckling of compression spring

Buckling is an instability that is normally shown up when a long bar or a column is applied with compressive type of load.

Similar situation arise if a spring is too slender and long then it sways sideways and the failure is known as buckling failure.

Buckling takes place for a compressive type of springs. Hence, the steps to be followed in design to avoid buckling are given below.

Free length (L) should be less than 4 times the coil diameter (D) to avoid buckling for most situations.

For slender springs central guide rod is necessary.

A guideline for free length (L) of a spring to avoid buckling is as follows,

$$L < \frac{\pi D}{C_e}\sqrt{\frac{2(E-G)}{2G+E}}$$

$$L < 2.57\frac{D}{C_e},$$ For steel, where, c_e is the end condition and its value is given below

c_e	End condition
2.0	Fixed and free end
1.0	hinged at both ends
0.707	hinged and fixed end
0.5	fixed at both ends

If the spring is placed between two rigid plates, then end condition may be taken as 0.5. If after calculation it is found that the spring is likely to buckle then one has to use a guide rod passing through the center of the spring axis along which the compression action of the spring takes place.

Spring surge (*critical frequency*)

If a load F act on a spring there is a downward movement of the spring and due to this movement a wave travels along the spring in downward direction and a to and fro motion continues.

This phenomenon can also be observed in closed water body where a disturbance moves toward the wall and then again returns back to the starting of the disturbance. This particular situation is called surge of spring.

If the frequency of surging becomes equal to the natural frequency of the spring the resonant frequency will occur which may cause failure of the spring.

Hence, one has to calculate natural frequency, known as the fundamental frequency of the spring and use a judgment to specify the operational frequency of the spring.

The fundamental frequency can be obtained from the relationship given below.

Fundamental frequency:

$$f = \frac{1}{2}\sqrt{\frac{Kg}{W_s}}$$ Both ends within flat plates

$$f = \frac{1}{4}\sqrt{\frac{Kg}{W_s}}$$ One end free and other end on flat

Where, **K : Spring rate**

W_s : Spring weight = $2.47\gamma d^2 DN$

Where K is the spring rate and W_s is the spring weight and d is the wire diameter, D is the coil diameter, N is the number of active coils and ã is the specific weight of spring material.

The operational frequency of the spring should be at least 15-20 times less than its fundamental frequency.

This will ensure that the spring surge will not occur and even other higher modes of frequency can also be taken care of.

Questions and answers

What are the forms of leaf spring?

Leaf springs are of two forms: cantilever and simply supported type.

What does the term "uniform strength" in the context of leaf spring mean?

If the leaf spring has a shape of uniformly varying width (say Lozenge shape) then the bending stress at all section remains uniform. The situation is also identical as before in case of varying thickness, the thickness should vary non-uniformly with length to make a beam of uniform strength (L/h_2 =constant). These leaves require lesser material; have more resilience compared to a constant width leaf. These types of springs are called leaf springs of uniform strength.

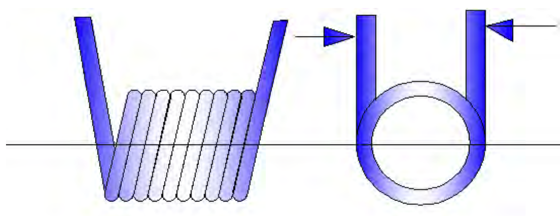
What is "nipping" in a laminated spring?

In general the differential curvature between the master leaf and the next leaves is provided in a laminated spring, where, radius of curvature being more for the master leaf. This construction reduces the stress in the master leaf as compared to the other leaves of the spring in a laminated spring. This type of constructional feature is termed as *nipping*.

SPECIAL SPRINGS

Helical Torsion Spring

A helical torsion spring is as shown in figure. It is used in door hinges, levelers, pawl ratchets and various other electrical devices where torque is required.
It is wound similar to extension or compression springs but have the ends shaped to transmit torque.



The helical torsion spring resists the bending moment which tends to wind up the spring. The primary stresses in this spring are flexural in contrast with torsional shear stresses in compression or extension springs. The torsional moment on a spring produces a bending stress in the wire, the bending moment on the wire being numerically equal to torsional moment. In addition to this stress, there is a direct tensile or compressive stress due to the force "F" that is tangential to the coil.
Each individual section of the torsion spring is considered as a curved beam.

Therefore using curved beam principle bending stress in torsion spring considering stress concentration factor is

$$\frac{K' M_b Y}{I} = \frac{K' M_t}{Z}$$

Maximum shear stress in torsion spring

$$\sigma = \frac{M_t}{Z} + \frac{F}{A}$$

The stress in torsion spring taking into consideration the correction factor K'

$$K' = \frac{K' M_t}{Z} + \frac{2M_t}{DA}$$

Stress in a round wire spring

$$\sigma = \frac{8\,M_t\,(4K'D + d)}{\pi D d^3}$$

Where, K' = K_1 from DDHB

Deflection in torsion spring

$$y = \frac{M_t L D}{2EI}$$

Where

M_t = Twisting moment

c = Spring Index

F = Force applied

i = Number of active coil

D = Mean diameter of coil

d= Wire diameter

L= Length of wire = πDi

A = Cross sectional area of wire = $\frac{\pi}{4}d^2$

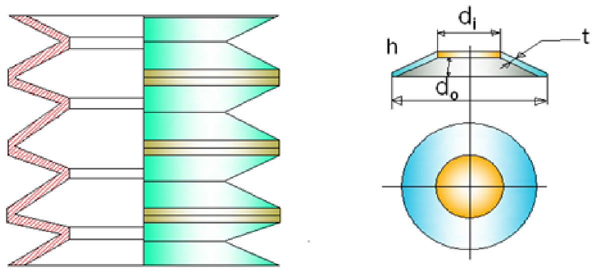
Z= Section modulus = $\frac{\pi d^3}{32}$

I = Moment of Inertia = $\frac{\pi d^4}{64}$

E = Modulus of Elasticity

Belleville Spring

Disc spring, also called Belleville spring are used where high capacity compression springs must fit into small spaces.
Each spring consists of several annular discs that are dished to a conical shape as shown in figure.



They are stacked up one on top of another as shown in above figure.
When the load is applied, the discs tend to flatten out, and this elastic deformation constitutes the spring action. In safety valve the disc springs are used.

The relation between the load "F" and the axial deflection "y" of each disc

$$F = \frac{4Ey}{(1 - v^2)Md_o^2}\left[(h - y)\left(h - \frac{y}{2}\right)\iota + \iota^3 \right]$$

Maximum stress induced at the inner edge

$$\sigma_i = \frac{4Ey}{(1 - v^2)Md_o^2}\left[C_1\left(h - \frac{y}{2}\right) + C_2 t\right]$$

Maximum stress at the outer edge

$$\sigma_o = \frac{4Ey}{(1 - v^2)Md_o^2}\left[C_1\left(h - \frac{y}{2}\right) - C_2 t\right]$$

Where

C_1 and C_2 are constraints

t = thickness

h = height of spring

M = constant

F = axial force

d_o= outer diameter

d_i = inner diameter

ϑ= Poisson's ratio

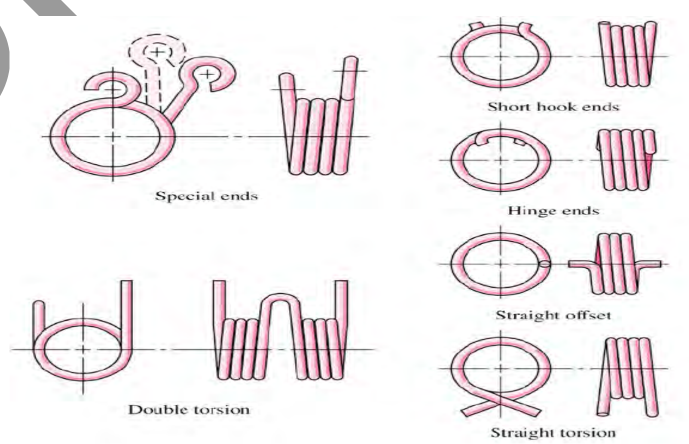
E = Modulus of elasticity

y = axial deflection

The ratio $\frac{d_0}{d_i}$ should be lie between 1.5 and 3

Torsion springs

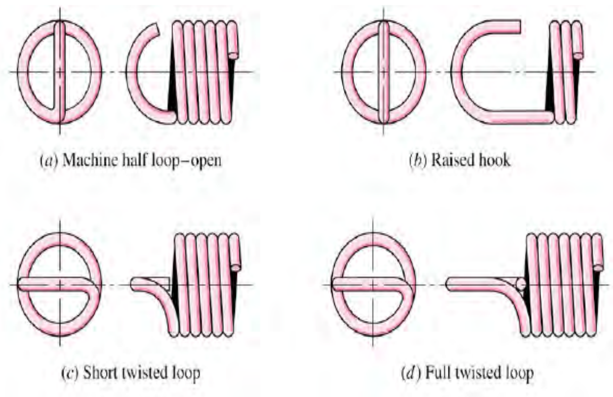
They are used in any application where torque is required, such as door hinges, automobile starters, etc



Types of ends used on extension springs

An extension spring is an open-coil helical spring that is designed to offer resistance to a tensile force applied axially. Unlike compression springs, they have specially designed hooks at both ends. They provide wide range of load-deflection curves.

Standard springs have constant diameter and pitch, thus providing a constant spring rate.



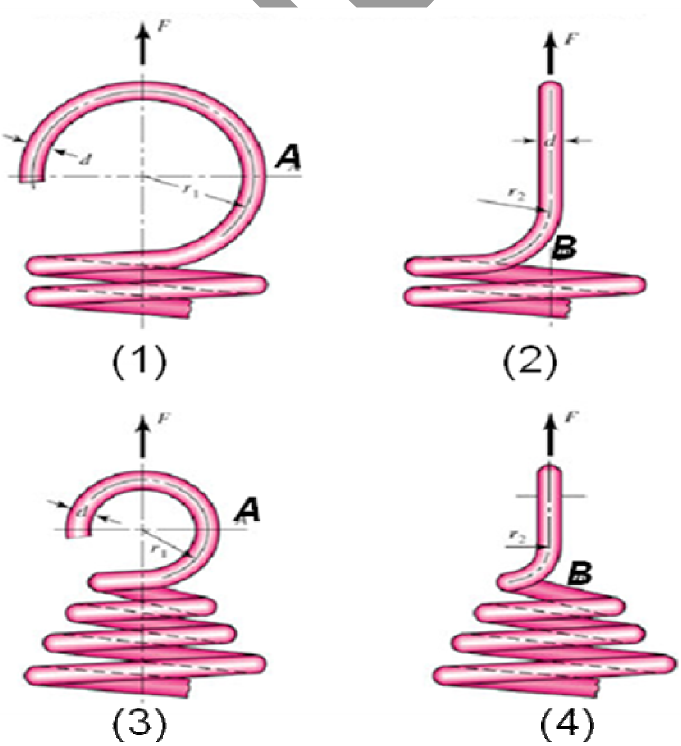
Ends for extension springs

(1) Usual design; stress at A is due to combined axial force and bending moment.

(2) Side view of part a; stress is mostly torsion at B.

(3) Improved design; stress at A is due to combined axial force and bending moment.

(4) Side view of part c; stress at B is mostly torsion



RUBBER SPRINGS

Rubber springs have good damping properties and hence it is used in vibration loading. Rubber is usually bonded to metal plates and can be in tension, compression and shear. In Shear rubber displays maximum elastic properties and in compression it displays maximum stiffness. At high temperature and in oil rubber cannot be used.

Problem 24

A Helical torsional spring of mean diameter 50mm is made of a round wire of 5mm diameter. If a torque of 5Nm is applied on this spring, find the bending stress, maximum stress and deflection of the spring in degrees. Modulus of elasticity =200GPa and number of effective turns 10.

Data

D = 50mm, d = 5mm, M_t= 5Nm = 5000N-mm,

E = 200GPa = 200X10^3 N/mm^2, i=10

Solution

Spring Index $c = \frac{D}{d} = \frac{50}{5} = 10$

When c=10, Stress factor $K' = K_1 = 1.08$

Bending Stress induced $\sigma_b = \frac{K' M_t}{Z} = \frac{1.08X5000}{\frac{\pi}{32}d^3} = 440.032$ N/mm^2

Maximum Stress $\sigma = \frac{K' M_t}{Z} + \frac{2M_t}{DA} = 440.032 + \frac{2X5000}{\frac{\pi}{4}5^3\ X50} = 450.2180$ N/mm^2

Axial deflection $y = \frac{M_t LD}{2EI} = \frac{5000\times(\pi\times50\times10)\times50}{2\times200\times10^3\times\frac{\pi}{64}\times5^4} = 32$ mm

Also $y = \theta\times\frac{D}{2}$

i.e. $32 = \theta \times\frac{50}{2}$

$\therefore$ $\theta = 1.28$ radians $= 1.28 \times\frac{180}{\pi} = 73.3386°$

Problem 24

A Belleville spring is made of 3 mm sheet steel with an outside diameter of 125mm and an inside diameter of 50mm. The spring is dished 5mm. The maximum stress is to be 500MPa. Determine

(i) Safe load carried by the spring
(ii) Deflection at this load
(iii) Stress produced at the outer edge
(iv) Load for flattening the spring

Assume poison's ratio, ν = 0.3 and E= 200 GPa

Data:

t=3mm, d_0=125 mm, d_i=50mm, h=5 mm, σ_{ri}=500MPa, ν=0.3, E=200GPa=200$\times 10^3$ N/mm^2

Solution:

For $\frac{d_0}{d_i} = \frac{125}{50} = 2.5$

M= 0.75, C_1=1.325, C_2=1.54

(i) Deflection

Stress at the inner edge

$$\sigma_{ri} = \frac{4Ey}{(1-\nu^2)Md_0^2}\left[C_1\left(h-\frac{y}{2}\right) + C_2 t\right]$$

i.e. $$500 = \frac{4\times200\times10^3\times y}{(1-0.3^2)0.75\times125^2}\left[1.325\left(5-\frac{y}{2}\right) + 1.54\times3\right]$$

6.665 = y (6.625 – 0.6625y +4.62)

= 11.245 y – 0.6625 y^2

i.e. 0.6625y2 -11.245 y +6.665 = 0

$$\therefore \quad y = \frac{11.245 \pm\sqrt{11.245^2 - 4\times0.6625\times6.665}}{2\times0.6625}$$

y= 16.36mm or 0.615m

∴ Deflection y = 0.615 mm (∵ y < h)

(ii) Safe Load

$$F = \frac{4Ey}{(1-v^2)Md_0^2}\left[(h-y)\left(h-\frac{y}{2}\right)t + t^3\right]$$

$$F = \frac{4\times200\times10^3\times0.615}{(1-0.3^2)\times0.75\times125^2}\left[(5-0.615)\left(5-\frac{0.615}{2}\right)3 + 3^3\right]$$

F = 4093.66 N

(iii) Stress at the outer edge

$$\sigma_o = \frac{4Ey}{(1-v^2)Md_0^2}\left[C_1\left(h-\frac{y}{2}\right) - C_2 t\right]$$

$$\sigma_o = \frac{4\times200\times10^3\times0.615}{(1-0.3^2)\times0.75\times125^2}\left[1.325\times\left(5-\frac{0.615}{2}\right) - 1.54\times3\right]$$

σ_o = 73.706 N/mm^2

(iv) Load for flattering the spring

When y= h, the spring will become flat

$$\therefore\ F = \frac{4Eht^3}{(1-v^2)Md_0^2} = \frac{(4\times200\times10^3\times5\times3^3}{(1-0.3^2)\times0.75\times125^2}$$

F= 10127.5 N

Problem 25

A multi leaf spring with camber is fitted to the chassis of an automobile over a span of 1.2m to absorb shocks due to a maximum load of 20kN. The spring material can sustain a maximum stress of 0.4GPa. All the leaves of the spring were to receive the same stress. The spring is required at least 2 full length leaves out of 8 leaves. The leaves are assembled with bolts over a span of 159mm width at the middle. Design the spring for a maximum deflection of 50mm.

Given data:

L = 1.2m = 1200mm; 2F = 20kN = 20000N

∴ F = 10000N; σ = 0.4Gpa = 400N/mm^2 (∵ equally stressed);

i_f = i' =2; ∴ i_g = 6; i=8; l_b = 150mm; y = 50mm.

Solution:

Effective length

$$l = \frac{L - l_b}{2} = \frac{1200 - 150}{2} = 525mm$$

Assume E = 206.92GPa = 206.92x10^3 N/mm^2

$$\alpha' = \beta = \frac{12}{12 + r}$$

$$= \frac{12}{2 + \frac{i'}{i}} = \frac{12}{2 + \frac{2}{8}} = 5.33$$

α' =6; β'= 2,

Width and thickness of spring leaves

Maximum equalized stress

$$\sigma = \frac{\alpha' F l}{i b' h^2}$$

$$400 = \frac{6x10000x525}{8xb'h^2}$$

∴ b'h^2 = 9843.75 ………(1)

Deflection

$$y = \frac{\beta F l^3}{E i b' h^3}$$

$$50 = \frac{5.333 x 10000 x 525^3}{206.92 x 10^3 x b' h^3}$$

∴ $b'h^3$ =93242.5575 ……… (2)

Equation (2) Divided by (1) gives

$$\frac{b'h^3}{b'h^2} = h = \frac{93242.5575}{9843.75} = 9.47mm$$

- Take thickness of spring h= 9.5mm
- Substituting in equation (1) b' = 109mm
- Substitution in equation (2) b' = 108.754mm
- Select bigger value as the permissible value

∴ Width of spring leaves b' = 109mm.

(ii) Initial space between full length and graduated leaves

Camber

$$c = \frac{\beta' F l^3}{i b' E h^3}$$

$$= \frac{2x10000x525^3}{8x109x206.92x10^3x9.5^3} = 18.7mm$$

Load on the central band

Load on the band

$$F_b = \frac{2F i_g\, i_f}{i[2i_g + 3i_f]}$$

$$= \frac{2x10000x6x2}{8[2x6 + 3x2]} = 1666.667N$$

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- “DESIGN OF MACHINE ELEMENTS” by V.B.Bhandari
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- “DESIGN OF SPRINGS” Version 2 ME, IIT Kharagpur, IITM

MECHANICAL DRIVES:

Two groups,

They are

1. Drives that transmit power by means of friction: eg: belt drives and rope drives.
2. Drives that transmit power by means of engagement: eg: chain drives and gear drives.

   However, the selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, velocity ratio, shifting arrangement, Maintenance and cost.

GEAR DRIVES

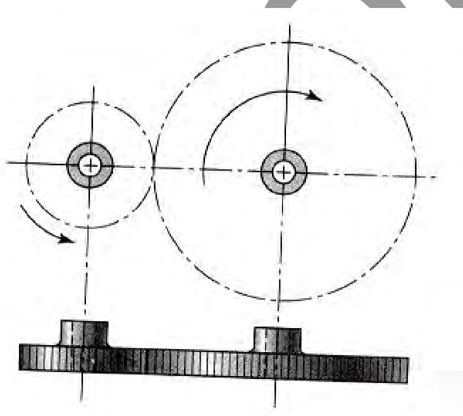
Gears are defined as toothed wheels, which transmit power and motion from one shaft to another by means of successive engagement of teeth

1. The centre distance between the shafts is relatively small.
2. It can transmit very large power
3. It is a positive, and the velocity ratio remains constant.
4. It can transmit motion at a very low velocity.

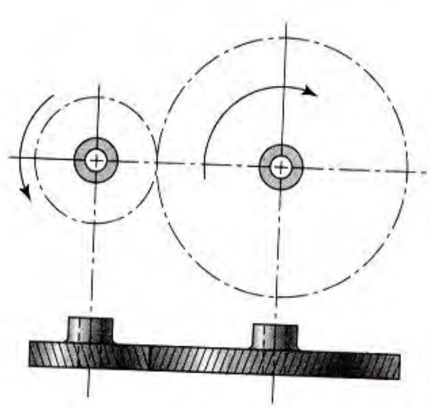
CLASSIFICATION OF GEARS:

Four groups:

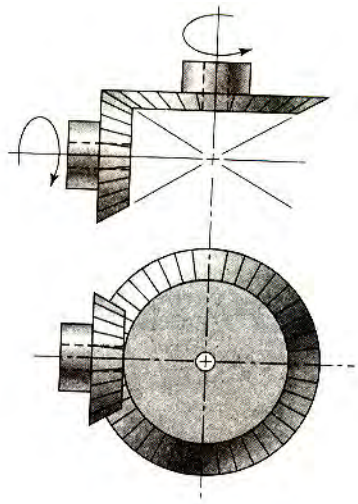
1) Spur Gears
2) Helical gears
3) Bevel gears and
4) Worm Gears



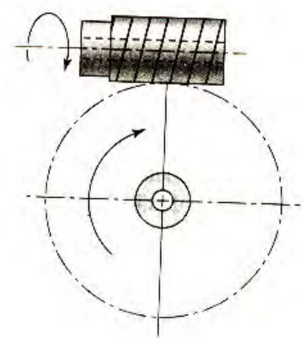
Spur Gear



Helical Gear



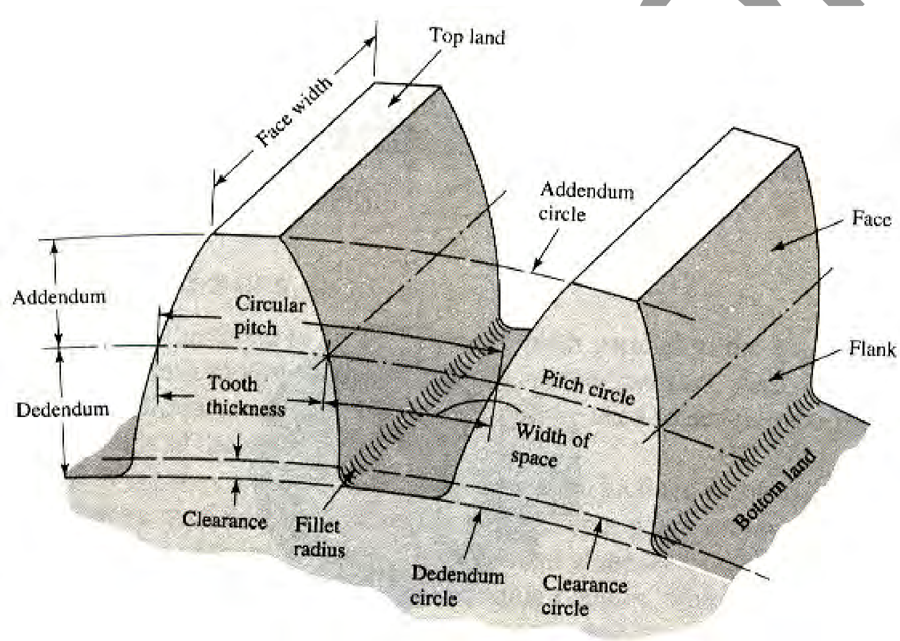
Bevel Gear



Worm Gear Set

NOMEN CLATURE
Spur gears are used to transmit rotary motion between parallel shafts. They are usually cylindrical is shape and the teeth are straight and parallel to the axis of rotation.

In a pair of gears, the larger is often called the GEAR and, the smaller one is called the PINION



Nomenclature of Spur Gear

1. **Pitch Surface:** The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together with out slipping.
2. **Pitch circle:** It is a theoretical circle upon which all calculations are usually based. It is an imaginary circle that rolls with out slipping with the pitch circle of a mating gear. Further, pitch circles of a mating gears are tangent to each other.

3. **Pitch circle diameter:** The pitch circle diameter is the diameter of pitch circle. Normally, the size of the gear is usually specified by pitch circle diameter. This is denoted by "d"
4. **Top land:** The top land is the surface of the top of the gear tooth
5. **Base circle:** The base circle is an imaginary circle from which the involute curve of the tooth profile is generated (the base circles of two mating gears are tangent to the pressure line)
6. **Addendum**: The Addendum is the radial distance between the pitch and addendum circles. Addendum indicates the height of tooth above the pitch circle.
7. **Dedendum**: The dedendum is the radial distance between pitch and the dedendum circles. Dedendum indicates the depth of the tooth below the pitch circle.
8. **Whole Depth**: The whole depth is the total depth of the tooth space that is the sum of addendum and Dedendum.
9. **Working depth:** The working depth is the depth of engagement of two gear teeth that is the sum of their addendums
10. **Clearance:** The clearance is the amount by which the Dedendum of a given gear exceeds the addendum of it's mating tooth.
11. **Face:** The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called face of the tooth.
12. **Flank:** The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.
13. **Face Width:** is the width of the tooth measured parallel to the axis.
14. **Fillet radius:** The radius that connects the root circle to the profile of the tooth is called fillet radius.
15. **Circular pitch:** is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.
16. **Circular tooth thickness:** The length of the arc on pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically circular tooth thickness is half of circular pitch.
17. **Width of space:** (tooth space) The width of the space between two adjacent teeth measured along the pitch circle. Theoretically, tooth space is equal to circular tooth thickness or half of circular pitch
18. **Working depth:** The working depth is the depth of engagement of two gear teeth, that is the sum of their addendums
19. **Whole depth:** The whole depth is the total depth of the tooth space, that is the sum of addendum and dedendum and (this is also equal to whole depth + clearance)
20. **Centre distance:** it is the distance between centres of pitch circles of mating gears. (it is also equal to the distance between centres of base circles of mating gears)
21. **Line of action:** The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give smooth operation. The force is transmitted from the driving gear to the driven gear on this line.
22. **Pressure angle:** It is the angle that the line of action makes with the common tangent to the pitch circles.

23. **Arc of contact:** Is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.
24. **Arc of approach:** it is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.
25. **Arc of recess:** It is the arc of the pitch circle through witch a tooth moves from the contact at the pitch point until the contact ends.

26. **Contact Ratio?**

Velocity ratio: if the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

27. **Module:** It is the ratio of pitch circle diameter in mith meters to the number of teeth. it is usually denoted by ‘m’ Mathematically

$$m = \frac{D}{Z}$$

28. **Back lash**: It is the difference between the tooth space and the tooth thickness as measured on the pitch circle.

29. **Velocity Ratio**: Is the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

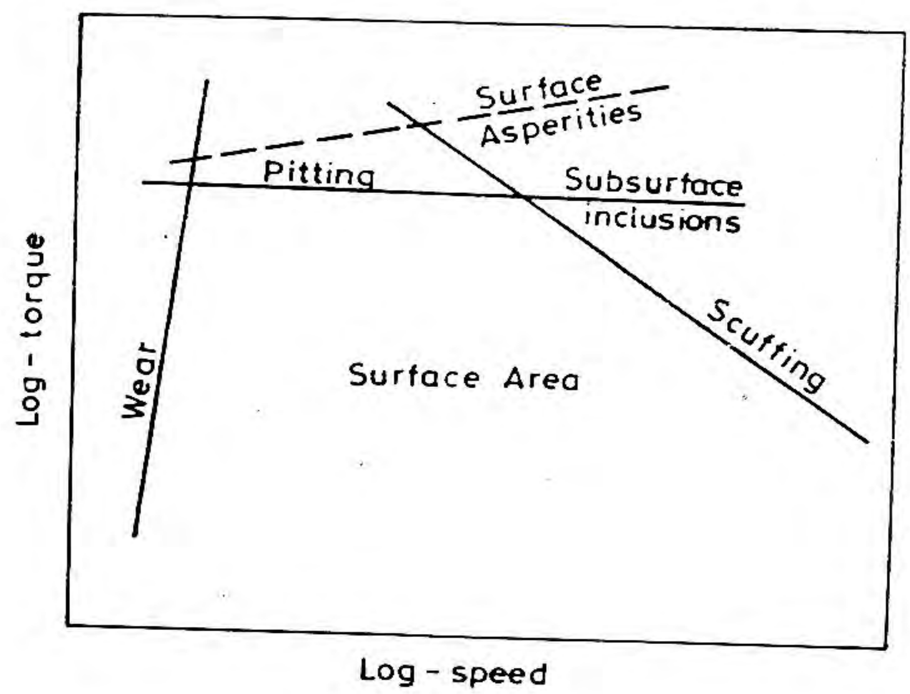
NOTATION

ENGLISH SYMBOLS

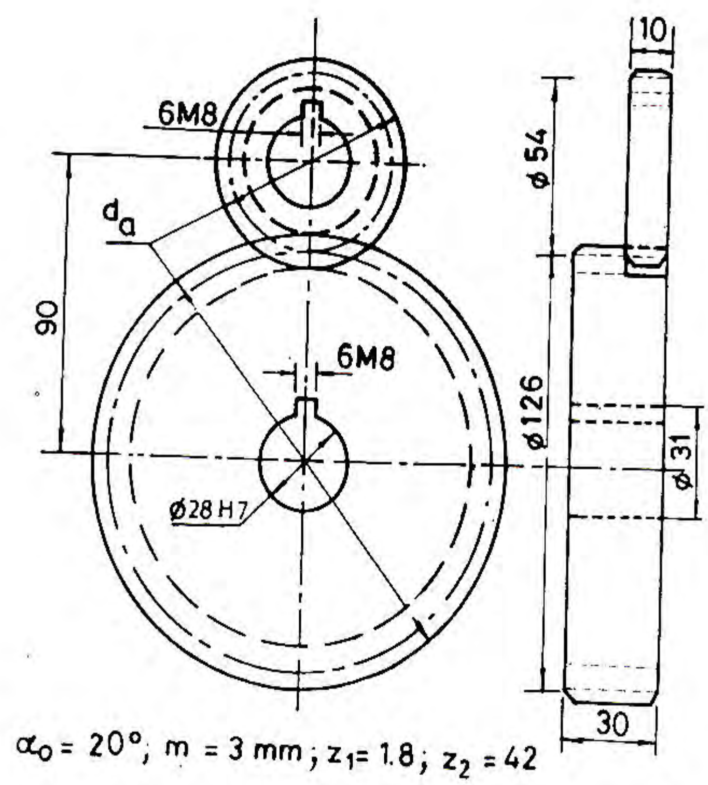
A_o	Centre distance
B	Face width
d_a	Addendem circle diameter
d_o	Pitch circke diameter
d_r	Root circle diameter
m	Module
r_a	Addendem circle radius
r_b	Base circle radius
r_o	Pitch circle radius
R	Radius of curvature of tooth profile
R_g	Gear ratio
Z	Number of teeth
α	Preasure angle
σ	Stress value
σ_b	Bending stress
σ_H	Hertz contact stress
σ_{HB}	Contact stress at the beginning of the engagement
σ_{HE}	Contact stress at the end of the engagement
σ_{HL}	Pitting limit stress
τ	Shear stress
ω	Angle velocity
Suffix 1	Pinion
Suffix 1	Gear

Nomenclature of Spur Gear

Failure Map of Involute Gears



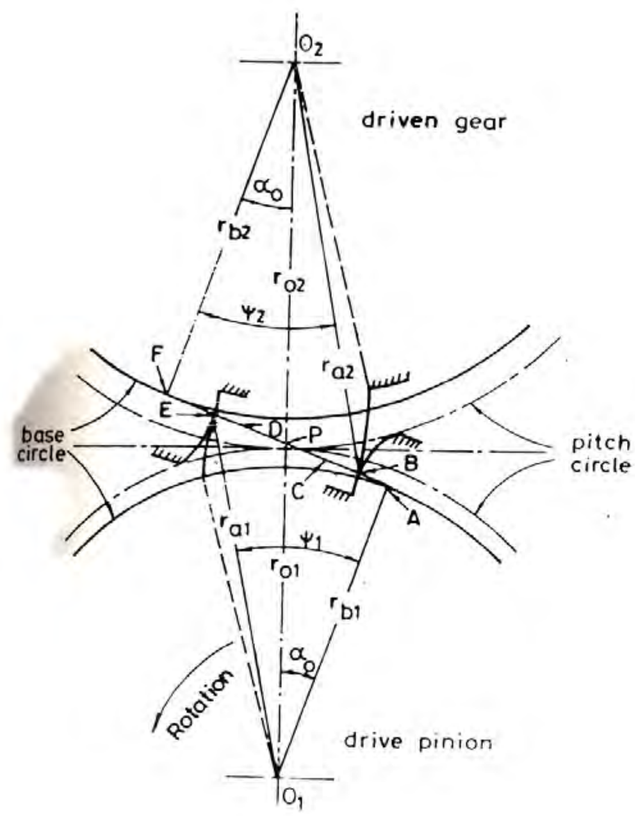
Failure Map of Involute Gears



Gear Set

Specification of Test Pinions and Gears

Variable	Symbol	Unit	Values of variables used in the experiments: Pinion		Gears
Module	m	(mm)		3.0	
Pressure angle	α_0	(deg)		20°	
Number of teeth	z	(--)	18		42
Pitch circle diameter	d	(mm)	54.0		126.0
Centre distance	a_o	(mm)		90.0	
Addendum circle diameter	d_a	(mm)	60.0		132.0
Root circle diameter	d_r	(mm)	46.5		118.5
Face width	B	(mm)	10.0		30.0



Different Phases of Gear Tooth Contact

Phase of contact	Position and number of pairs of teeth (J) in contact		Radius of curvature	
			R_1	R_2
Beginning of engagement	B	2	$C_3 - C_2$	C_2
Transition phase	C	2 to 1	$C_1 - C_6$	$C_3 - C_1 - C_6$
Pitch point	P	1	C_4	C_5
Transition phase	D	1 to 2	$C_3 - C_2 + C_6$	$C_2 - C_6$
End of engagement	E	2	C_1	$C_3 - C_1$

Expressions for the Calculation of Equivalent Radii of Curvature at Various Phases of Contact

Design consideration for a Gear drive

In the design of gear drive, the following data is usually given

i. The power to be transmitted
ii. The speed of the driving gear
iii. The speed of the driven gear or velocity ratio
iv. The centre distance

The following requirements must be met in the design of a gear drive

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions
(b) The gear teeth should have wear characteristics so that their life is satisfactory.
(c) The use of space and material should be recommended
(d) The alignment of the gears and deflections of the shaft must be considered because they effect on the performance of the gears
(e) The lubrication of the gears must be satisfactory

Selection of Gears:

The first step in the design of the gear drive is selection of a proper type of gear for a given application. The factors to be considered for deciding the type of the gear are

- ☞ General layout of shafts
- ☞ Speed ratio
- ☞ Power to be transmitted
- ☞ Input speed and
- ☞ Cost

1. Spur & Helical Gears – When the shaft are parallel

2. Bevel Gears – When the shafts intersect at right angles, and,

3. Worm & Worm Gears – When the axes of the shaft are perpendicular and not intersecting. As a special case, when the axes of the two shafts are neither intersecting nor perpendicular crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6: 1. On rare occasions this can be raised to 10: 1. When the velocity ratio increases, the size of the gear wheel increases. This results in an increase in the size of the gear box and the material cost increases. For high speed reduction two stage or three stage construction are used.

The normal velocity ratio for a pair of bend gears is 1: 1 which can be increased to 3: 1 under certain circumstances.

For high-speed reduction worm gears offers the best choice. The velocity ratio in their case is 60: 1, which can be increased to 100: 1. They are widely used in materials handling equipment due to this advantage.

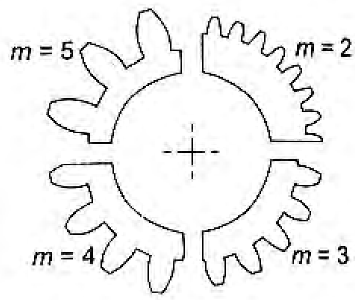
Further, spur gears generate noise in high-speed applications due to sudden contact over the entire face with between two meeting teeth. Where as, in helical gears the contact between the two meshing teeth begans with a point and gradually extends along the tooth, resulting in guite operations.

From considerations spurgears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

Low of Gearing:

The fundamental law of gearing states “The common normal to the both profile at the point of contact should always pass through a fixed point called the pitch point, in order to obtain a constant velocity ratio.

MODULE:
The module specifies the size of gear tooth. Figure shows the actual sizes of gear tooth with four different modules. It is observed that as the modules increases, the size of the gear tooth also increases. It can be said that module is the index of the size of gear tooth.



Standard values of module are as shown.

Recommended Series of Modules (mm)

Preferred (1)	Choice 2 (2)	Choice 3 (3)	Preferred (1)	Choice 2 (2)	Choice 3 (3)
1			8	7	(6.5)
1.25	1.125		10	9	
1.5	1.375		12	11	
2	1.75		16	14	
2.5	2.25		20	18	
3	2.75	(3.25)	25	22	
4	3.5		32	28	
5	4.5	(3.75)	40	36	
6	5.5		50	45	

Note: The modules given in the above table apply to spur and helical gears. In case of helical gears and double helical gears, the modules represent normal modules

The module given under choice 1, is always preferred. If that is not possible under certain circumstances module under choice 2, can be selected.

Standard proportions of gear tooth in terms of module m, for 20º full depth system.

Addendum = m
Dedendum = 1.25 m
Cl earance (c) = 0.25 m
Working depth = 2 m
Whole depth = 2.25 m
Tooth thickness = 1.5708 m $\left[\frac{\pi d}{2z} = \frac{\pi mz}{2z}\right] = 1.5708m$
Tooth space = 1.5708 m
Fillet radius = 0.4 m

Standard Tooth proportions of involute spur gear

Gear Terms	Proportions of Machine cut teeth		
	Circular pitch p	Diametral pitch P	Module m
Addendum	0.3183 p	1/P	m
Dedendum	0.3977 p	1.25/P	1.25 m
Tooth thickness	0.5 p	1.5708/P	1.5708 m
Tooth space	0.5 p	1.5708/P	1.5708 m
Working depth	0.6366 p	2/P	2 m
Whole depth	0.7160 p	2.25/P	2.25 m
Clearance	0.0794 p	0.25/P	0.25 m
Pitch diameter	zp/π	z/P	zm
Outside diameter	$(z+2)p/\pi$	(z+2)/P	(z+2) m
Root diameter	$(z-2.5)p/\pi$	(z − 2.5)/P	(z − 2.5) m
Fillet radius	0.1273p	0.4/P	0.4 m

<u>Selection of Material :</u>

- The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or yield strength of the material.
- When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the deciding factor.
- The gear material should have sufficient strength to resist failure due to breakage of tte tooth.
- In many cases, it is wear rating rather than strength rating which decides the dimensions of gear tooth.
- The resistance to wear depends upon alloying elements, grawn size, percentage of carbon and surface hardness.
- The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
- For high-speed power transmission, the sliding velocities are very high and the material should have a low co-efficient of friction to avoid failure due to scoring.
- The amount of thermal distortion or wrapping during the heat treatment process is a major problem on gear application.
- Due to wraping the load gets concentrated at one corner of the gear tooth.
- Alloy steels are superior to plain carbon steel in this respect (Thermal distortion)

Allowable Static Stresses σ_d to use in Lewis formulae

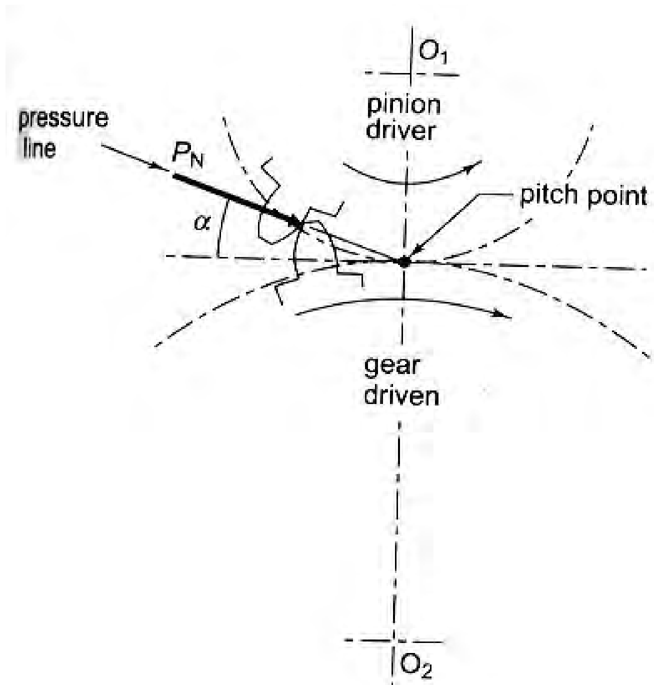
Material	Allowable static stress σ_d, MN/m² (kgf/mm²)	BHN
Cast Iron Grade 20	47.1 (4.80)	200
Cast Iron Grade 25	56.4 (5.75)	220
Cast Iron Grade 35	56.4 (5.75)	225
Cast Iron Grade 35 (Heat treated)	78.5 (8.00)	300
Cast steel, 0.20%C, untreated	138.3 (14.10)	180
Cast steel, 0.20%C, heat treated	193.2 (19.70)	250
Bronze	68.7 (7.00)	80
Phosphor gear bronze	82.4 (8 .40)	100
Manganese bronze	138.3 (14.10)	100
Aluminium bronze	152.0 (15.50)	180
Forged steel, about 0.30%C (untreated)	172.6 (17.60)	150
Forged steel, about 0.30%C (heat treated)	220.0 (22.40)	200
Steel, C30 (heat treated)	220.6 (22.50)	300
Steel, C40, untreated	207.0 (21.10)	150
Steel, C45, untreated	233.4 (23.80)	200
Alloy steel, case hardened	345.2 (35.20)	650
Cr-Ni Steel, about 0.45%C heat treated	462.0 (47.10)	400
Cr-Va steel, about 0.45%C, heat treated	516.8 (52.70)	450
Rawhide, Fabroil, etc.	41.2 (4.20)	—
Plastic	58.8 (6.00)	—
Laminated phenolic materials (Bakelite, Micarta, Celoron)	41.2 (4.20)	—
Laminated steel (silent material)	82.4 (8.40)	—

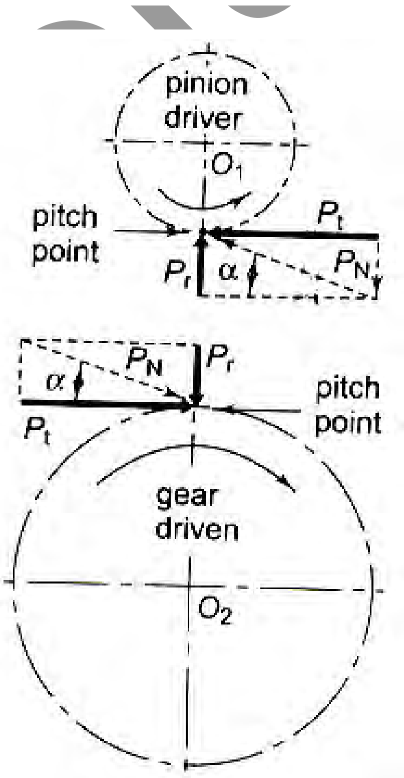
Force analysis – Spur gearing.

We know that, the reaction between the mating teeth occur along the pressure line, and the power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. (i.e. driving pinion exerting force P_N on the tooth of driven gear).

According to fundamental law of gear this resultant force P_N always acts along the pressure line.

This resultant force $P_{N,}$ can be resolved into two components – tangential component P_t and radial components P_r at the pitch point.





The tangential component P_t is a useful component (load) because it determines the magnitude of the torque and consequently the power, which is transmitted.

The radial component P_r services no useful purpose (it is a separating force) and it is always directed towards the centre of the gear.

The torque transmitted by the gear is given by

$$M_t = \frac{P \times 60}{2\,\pi N_1}\ N-m$$

Where,

M_t = Torque transmitted gears (N- m)
PkW = Power transmitted by gears
N_1 = Speed of rotation (rev / mn)

The tangential component F_t acts at the pitch circle radius.

$$\therefore \quad M_t = F_t\,\frac{d}{2}$$

OR

$$F_t = \frac{2M_t}{d}$$

Where,

M_t = Torque transmitted gears N- mm
d = Pitch Circle diameter, mm

Further, we know,

Power transmitted by gears $= \dfrac{2\pi\ N\ M_t}{60}$ (kW)

Where

$$F_r = F_t\ \tan\alpha$$

and

resultant force,

$$FN = \frac{F_t}{Cos\alpha}$$

The above analysis of gear tooth force is based on the following assumptions.

i) As the point of contact moves the magnitude of resultant force P_N changes. This effect is neglected.

ii) It is assumed that only one pair of teeth takes the entire load. At times, there are two pairs that are simultaneously in contact and share the load. This aspects is also neglected.

iii) This analysis is valid under static conditions for example, when the gears are running at very low velocities. In practice there are dynamic forces in addition to force due to power transmission.

For gear tooth forces, It is always required to find out the magnitude and direction of two components. The magnitudes are determined by using equations

$$M_t = \frac{P \times 60}{2\pi N_1}$$

$$F_t = \frac{2M_t}{d_1}$$

Further, the direction of two components F_t and F_r are decided by constructing the free body diagram.

?

How

Minimum Number of Teeth:

The minimum number of teeth on pinion to avoid interference is given by

$$Z_{\min} = \frac{2}{\sin^2 \alpha}$$

For 20°full depth involute system, it is always safe to assume the number of teeth as 18 or 20

Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio $i = \frac{Z_2}{Z_1}$

Face Width:

In designing gears, it is required to express the face width in terms of module.

In practice, the optimum range of face width is $9.5m \le b \le 12.5m$

Generally, face width is assumed as ten times module

$$\therefore \boxed{b = 12.5m}$$

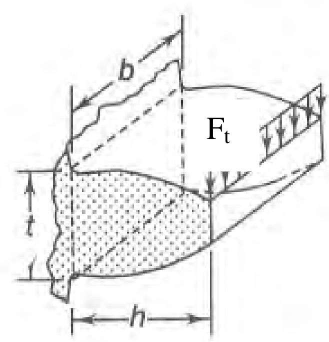
The LEWIS Bending Equation:

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth.

This equation announced in 1892 still remains the basis for most gear design today.

In the lewis analysis, the gear tooth is treated as a cantilever beam and the tangential component (F_t) causes the bending moment about the base of the tooth.

GEAR TOOTH AS CANTILEVER



The Lewis equation is based on the following assumption.

(i) The effect of radial component (F_r) which induces compressive stresses is neglected.

(ii) It is assumed that the tangential component (F_t) is uniformly distributed over the face width of the gear (this is possible when the gears are rigid and accurately machined.)

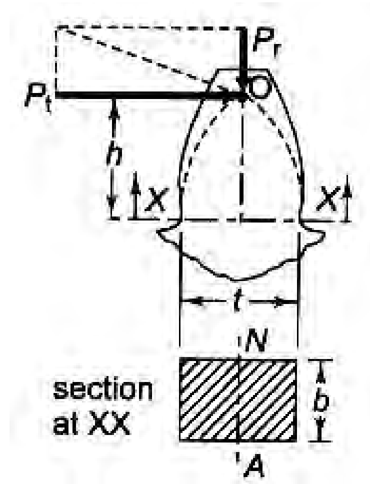
(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time only one pair of teeth is in contact and Takes total load

It is observed that the cross section of the tooth varies from free end to fixed end. Therefore, a parabola is constructed with in the tooth profile and shown in dotted lines.

Gear tooth as parabolic beam

The advantage of parabolic out lines is that it is a beam of uniform strength, and the stress at any cross section is uniform.



We know

$$M_b = F_t \times h$$

$$I = \frac{bt^3}{12} \; and \qquad y = \frac{t}{2}$$

$$\therefore \sigma_b = \frac{M_b y}{I} = \frac{M_b}{(I/y)} = \frac{M_b}{Z}$$ (Z= Section modulus)

$$= \frac{bt^3}{12} / \frac{t}{2} \qquad = \frac{bt^2}{6}$$

σ_b = Permissible bending stress (N/mm^2) $\frac{6 F_b \times h}{bt^2}$

$$\therefore F_t = \left[\frac{bt^2 \sigma_b}{6h} \right]$$

Multiplying the numerator and denominator of the right hand side by m, (m=Module)

$$F_t = mb\,\sigma_b \left(\frac{t^2}{6hm} \right)$$

The bracketed quantity depends on the from of the tooth and is termed as **lewis from stress factor Y**

Let $$y = \frac{t^2}{6hm}$$

Then the equation can be rewritten as $F_t = mb\sigma_b y$

This y is called as lewis form factor

When the stress reaches the permissible magnitude of bending stresses, the corresponding force (F_t) is called the beam strength (S_b)

$$\therefore\ S_b = mb\,\sigma_b\,y$$

Where,

S_b = beam strength of gear tooth (N)
σ_b = Permissible bending stress (N/mm^2)

The above equation is known as <u>LEWIS EQUATION</u>

The values of the lewis from factor y is given in table below,

Values of Tooth-Form Factor (Lewis), Load at Tip of Tooth

z	y					
	14½-deg form	14½-deg variable centre distance	20-deg full depth form	20-deg stub-tooth form	Internal Gears: Spur pinion	Internal Gears: Internal gear
12	0.067	0.125	0.078	0.099	0.104	
13	0.071	0.123	0.083	0.103	0.104	
14	0.075	0.121	0.088	0.108	0.105	
15	0.078	0.120	0.092	0.111	0.105	
16	0.081	0.120	0.094	0.115	0.106	
17	0.084	0.120	0.096	0.117	0.109	
18	0.086	0.120	0.098	0.120	0.111	*
19	0.088	0.119	0.100	0.123	0.114	
20	0.090	0.119	0.102	0.125	0.116	
21	0.092	0.119	0.104	0.127	0.118	
22	0.093	0.119	0.105	0.129	0.119	
24	0.095	0.118	0.107	0.132	0.122	
26	0.098	0.117	0.110	0.135	0.125	
28	0.100	0.115	0.112	0.137	0.127	0.220
30	0.101	0.114	0.114	0.139	0.129	0.216
34	0.104	0.112	0.118	0.142	0.132	0.210
38	0.106	0.110	0.122	0.145	0.135	0.205
43	0.108	0.108	0.126	0.147	0.137	0.200
50	0.110	0.110	0.130	0.151	0.139	0.195
60	0.113	0.113	0.134	0.154	0.142	0.190
75	0.115	0.115	0.138	0.158	0.144	0.185
100	0.117	0.117	0.142	0.161	0.147	0.180
150	0.119	0.119	0.146	0.165	0.149	0.175
300	0.122	0.122	0.150	0.170	0.152	0.170
Rack	0.124	0.124	0.154	0.175		

*Internal gears with less than 28 teeth must be designed specially for the particular application, and their values of y must be determined for each one individually.

In order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

In design of gears, It is required to decide the weaker between pinion and gear.

When the same material is used for pinion and gear, the pinion is always weaker than the gear-------------- **Why?**

- We know that $S_b = mb\sigma_b y$
- It can be observed that 'm' and 'b' are same for pinion and as well as for gear in a gear pair,
- When different materials are used, the product $\sigma_b . y$ decides the weaker between the pinion and gear
- The lewis form factor y is always less for pinion compared to gear
- Therefore, when the same material is used for pinion and gear, the pinion is always weaker than the gear.

Effective load-Calculation

Earlier we have seen how to determine the tangential component of the resultant force between two meshing teeth.

This component can be calculated by using

I. $$M_t = \frac{P \times 60}{2\pi N_1}$$

And

II. $$F_t = \frac{2M_t}{d_1}$$

The value of the tangential component, depends upon rated power and rated speed.

In gear design, the maximum force (due to maximum torque) is the criterion. This is accounted by means of a factor called service factor – (C_s)

This service factor (C_s) is defined as

$$C_s = \frac{Maximum\,Torque}{Rated\ Torque}$$

$$\therefore C_s = \frac{(M_t)_{\max}}{M_t} = \frac{(F_t)_{\max}}{F_t}$$

Where, (F_t) is the tangential force due to rated torque (M_t)

$$(F_t)_{\max} = C_s F_t$$

The values of service factors are given in table…

Service factor C_s for gears

Type of load	Type of service		
	Intermittent or 3 h per day	8 to 10 h per day	Continuous 24 h/day
Steady	0.80	1.0	1.25
Ligh shocks	1.00	1.25	1.50
Medium shocks	1.25	1.50	1.80
Heavy shocks	1.50	1.80	2.00

We know, that

σ_b is permissible static bending stress which is modified to $C_v \sigma_b$ where, C_v is the velocity factor used for taking into account the fatigue loading

This velocity factor C_v developed by. **Carl. G. Barth,** expressed in terms of pitch line velocity.

The values of velocity factor are as below

(i) $C_v = \frac{3}{3+V}$, for ordinary and commercially cut gears (made with form cutters) and V<10 m / Sec

(ii) $C_v = \frac{6}{6+V}$, For accurately hobbed and generated gears and V < 20 m/Sec.

(iii) $C_v = \frac{5.6}{5.6+\sqrt{v}}$, For precision gears with shaving grinding and lapping and V > 20 m/Sec

Where, v = pitch line Velocity (m/Sec)

$$= \frac{\pi dn}{60 \times 10^3}$$ d, mm

n, rev/min

(The velocity factor is an empirical relationship developed by past experience).

Dynamic effects (Dynamic Tooth Load)

When gears rotate at very low speed, the transmitted load P_t can be considered to be the actual force present between two meshing teeth

However in most of the cases the gears rotate at appreciable speed and it becomes necessary to consider the dynamic force resulting from impact between mating teeth.

The **Dynamic force** is induced due to the following factors

1. Inaccuracies of the tooth profile
2. Errors in tooth spacings
3. Misalignment between bearings
4. Elasticity of parts, and
5. Inertia of rotating masses.

There are two methods to account for Dynamics load.

I. Approxinate estimation by the velocity factor in the preliminary stages of gear design

II. Precise estimation by **Bucking Hams** equation in the final stages of gear design.

Note: Approximate estimation, Using velocity factor (C_v) developed by Barth discussed earlier.

In the final stages of gear desing when gear dimensions are known errors specified and quality of gears determined, the Dynamic load is calculated by equation derived by

Earle Buckingham

Where, F_d = Dynamic load

$= F_t + F_i$

Where, F_t = Tangential tooth load

F_i = Inevement load due to dynamic action

$$F_d = F_t + \frac{k_3 V(Cb + F_t)}{k_3 V + \sqrt{Cb + F_t}}$$

Where,

V = Pitch line Velocity (m/Sec)
C = Dynamic factor (N/mm^2) depending upon machining errors
e = measured error in action between gears in mm
b = face width of tooth (mm)
F_t = tangential force due to rated torque (N)
K_3 = 20.67 in SI units

The Dynamic factor C, depends upon modulus of elasticity of materials for pinion and gear and the form tooth or pressure angle and it is given by

$$C = \frac{e}{K_1 \left(\frac{1}{E_1} + \frac{1}{E_2} \right)}$$

Where

K = Constant depending upon the form of tooth – (take from DDH)
E_1 = Modulus of elasticity of pinion material (N/mm^2)
E_2 = Modulus of elasticity of gear material (N/mm^2)

The Values of K, for various tooth forms are given as.

The error, e, in the dynamic load equation is measured error in action between gears in mm

This error depends upon the quality of gear and manufacturing methods.

WEAR TOOTH LOAD

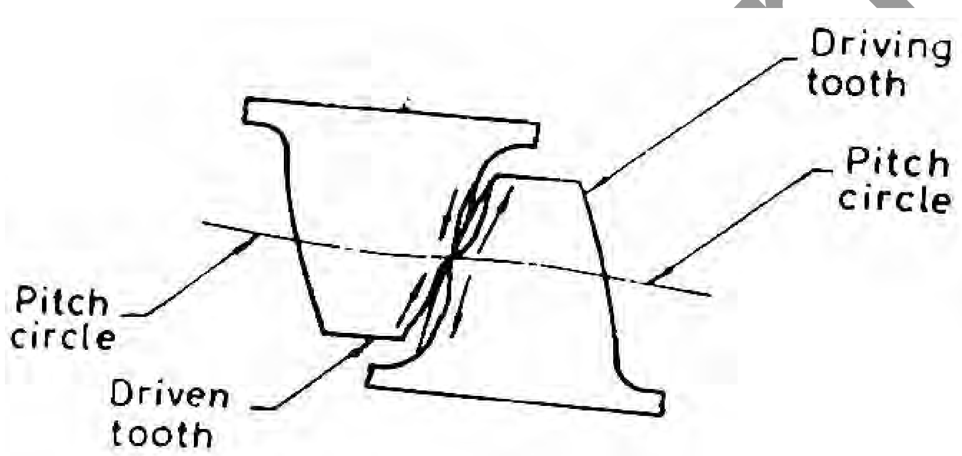
WEAR:
For gears wear is defined as loss of material from contacting surfaces of teeth.

It is further classified as

- Normal wear
- Moderate wear
- Destructive wear
- Abrasive wear
- Scratching and etc.

Generally, normal wear (Polishinging in) does not constitute failure because it involves loss of metal at a rate too slow to affect performance

- Moderate wear refers to loss of metal more rapid than normal wear.
- This need not necessarily be destructive and may develop on heavily loaded gear teeth.
- Destructive wear usually results from loading that is excessive for the lubricant employed.
- The effect of destructive wear on the tooth profile of an involute gear is depicted in the figure.



PITTING
Pitting is the principal mode of failure of rolling surfaces. The details of the process vary with the material and operating conditions, but in all cases it manifests itself by the initiation and propagation of cracks in the near surface layer until microscopic pieces detach themselves to form a pit or a spall.

In spur gears surface pitting has long been recognised as one of the failure modes. This is often refereed to as “Pitch line Pitting”

The main factors affecting pitting type of failure,

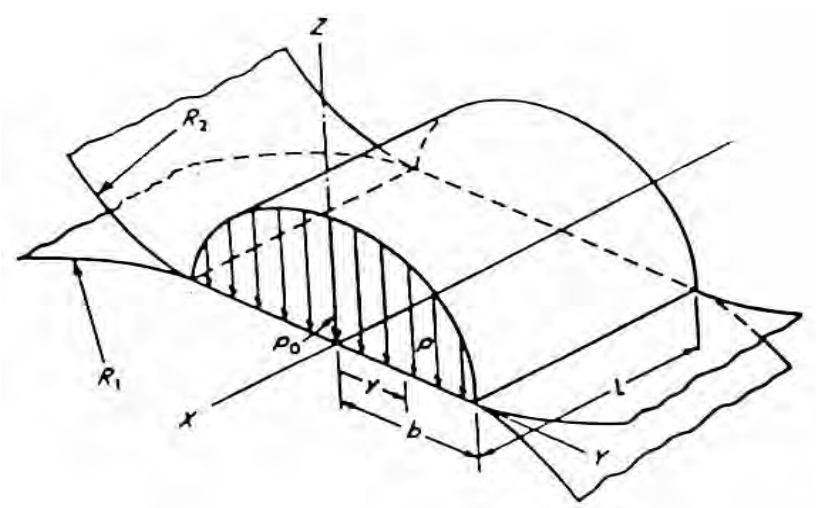
- Contact stress.
- Material pouring and hardness.
- Surface finish and lubrication

Contact stress was originally conceived By “HERTZ” (1896) in whose name it is often referred to as Hertz Contact Stress.

The failure of the gear tooth due to pitting occurs when the contact stress between two meshing teeth exceeds the surface endurance strength the material

In order to avoid this type of failure, the proportions of the gear tooth and surface properties such as surface hardness should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.

The Hertz stress is based on the assumptions of electric and isometric material behaviours, load is compressive and normal to the contacting surfaces which are stationary and the size of contacting area whose dimensions are relatively smaller compared with the curvature radius of the contacting bodies



The above figure,
Illustrates the contact are and corresponding stress distribution between two cylinders.

Here the area of contact stress which is theoretically rectangular with one dimension being the cylinder length L. (i.e. corresponding to face width of the gear)

The distribution of pressure is represented by a semi elliptical prism and the maximum contact pressure Po exists on the load axis,

The current gear design practice is to estimate the contact stress at the pitch point of the teeth by assuming line contact between two cylinders whose radil of contact depends on the gear geometry at the pitch point.

The analysis of wear strength was done by Earle Buckingham and was accepted by AGMA (American Gear Manufacturing Association) in 1926. This Buckingham equation gives the wear strength of the gear tooth based on Hertz theory of contact stress.

Hence, the maximum tooth load from wear consideration as evaluated from Hertz contact stress equation applied for pitch point contact is given by

$$\boxed{F_t = d_1 b Q K}$$

Where,

d_1 = Pitch circle diameter of pinion in mm.

b = Face width of the pinion in mm.

Q = Ratio factor $= \dfrac{2VR}{VR+1}$

$= \dfrac{2\,Z_2}{Z_2+Z_1}$

and

K = Load stress factor (also known as material combination factor in N / mm^2)

This load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle, and the modulus of elasticity of the materials of the gear.

According to Buckingham, this load stress factor is given by

$$K = \frac{(\sigma_{es})^2 \; Sin\;\alpha}{1.4} \quad \left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

Where, σ_{es} = Surface endurance limit of a gear pair. (N / mm^2)

= [2.75 (BHN) -70]

Where BHN = the average Brinall hardness number of gear and pinion for the steels

Design procedure for spur gears:

(i) Find design tangential tooth load, from power transmitted and pitch line velocity

$$= F_T = \frac{1000\,p\;C_s}{V}.$$

(ii) Apply lewis relationship i.e. $F_t \; = \sigma_d . P_C . b . y \;\; = \sigma_d \, C_v \; b\pi \; m \; y$

a) This lewis equation is applied only to the weaker of the two wheels

b) When both the pinion and gear are made of the same material, then pinion is weaker.

c) When the pinion and gear are made of different materials then the product of (σ_o x y) is the deciding factor. The lewis equation is used to that wheel for which (σ_o x y) is less.

d) The product of [(σ_o . C_v) .y] is called as strength factor of the gear

e) The face width may be taken as 9.5m to 12.5m for cut teeth and 6.5m to 9.5 m for cast teeth.

(iii) Calculate the dynamic load (F_d) on the tooth by using Backingham equation, i.e.,

$$F_D = F_t + F_i$$

by

(iv) Find the static tooth load (i.e.,. Beam strength or the endurance strength of the tooth) by using the relation,

$$F_s = \sigma_e \; b \; p_c \; y$$

$$= \sigma_e \; b \; \pi \; m \; y$$

for safety against breakage $F_S > F_d$

(v) Finally find the wear tooth load by using the relation

$$F_W = d_1 b \, Q \, K$$

The wear load (F_W) should not be less than the dynamic load (F_D)

Design a pair of spur gears to transmit 20kW of power while operating for 8 – 10 hrs/day sustaining medium shock, from shaft rotating at 1000 rev/min to a parallel shaft which is to rotate at 310 rev/min. Assume the number of teeth on pinion to be 31 and 20° full depth involute tooth profile. if load factor C = 522.464 N/mm and also for wear load taking load stress factor, K = 0.279 N/mm^2. Suggest suitable hardness. Boththe pinio gears are made of cast steel 0.2% corbon (untreated) whose σ_d = 137.34 N/mm
check the design for dynamic load if

Given: P = 20kW, =20x10^{3}W, Z_1 = 31, Z_2 = 100, V. R = 1:3.225, ∝ = 20°Full depth.
N_1 = 1000 rev/min, N_2 = 310 rev/min

Material: Cast steel 0.2% C, (untreated) σ_d = 137.34 N/mm^2

Type of load: Medium skock, with 8-10hrs/day.

C = dynamic factors depending up on machining errors 522.464 N/mm

K = load stress factor (wear) = 0.279 N/mm^2

Solution:

σ_{d1} = Allowable static stress = 207.0 N/mm2(Pinion)
σ_{d2} = 138.3N/mm2(Gear)

Let C_v = $\frac{3.05}{3.05+V}$ **(assume)**

V = pitch lin velocity $= V = \frac{\pi \, d_1 \, N_1}{60}$

$$= \frac{\pi \; m \; Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 31 \times 1000}{60} = 1623.15m \;\; mm/Sec$$

$$\boxed{V = 1.623m \quad mm/Sec}$$

For, Medium shock, with 08- 10 hrs/day the service factor C_s, for gear, C_s =1.5

The tangential tooth load $= F_t = \dfrac{1000 \; p}{V}.C_s$ P, in kW, V, m/ Sec

$$= \frac{20 \times 10^3}{1.623m}.\times 1.5$$

$$= \frac{18484}{m}.\text{ N}$$

Now $C_v = \dfrac{3.05}{3.05 + 1.623 \; m.}$

W.K.T, Tooth form factor for the pinion,

$$= \;\; 0.154 - \frac{0.912}{Z_1} \quad \text{(for 20° full depth)}$$

$$= 0.154 - \frac{0.912}{31}$$

$$= 0.154 - 0.0294$$

$$= 0.1246$$

and Tooth from factor, for the gear

$$= \;\; 0.154 - \frac{0.912}{Z_2}$$

$$= 0.154 - \frac{0.912}{100} \qquad (\because \; Z_2 \; = 100)$$

$$= 0.154 - 0.00912$$

$$= 0.1448$$

σ_{d1} x y_1 = 137.34 x 0.01246 = 17.11

σ_{d1} x y_2 = 137.34 x 0.1448 = 19.88

Since (σ_d .y_2)is less than (σ_{d2} .y_2), there fore PINION is WEAKER

Now, by using the lewis equation to the pinion, We have,

$$F_t = \sigma_d \, C_v \, b\pi \, m \, y_1$$

$$= \frac{18484}{m} = 207 \times \left(\frac{3.05}{3.05 + 1.623\, m} \right) \times \; 10m \times \pi \; m \times 0.1246$$

$$= \frac{18484}{m} = \frac{2471.37\, m^2}{3.05 + 1.623\, m}$$

By hit and trial methd,

m =	LHS	RHS
01	18484	528.86
02	9242	1570.12
03	6161	2808.75
04	4621	4143.98
05	3696	5533.74
Let m =	4107	4833.65

M 04.5 Hence, m = module = 4.5 is OK.

But the standard module is 5.0 mm

∴ Let us take $\boxed{m = 5.0 \text{ mm}}$

Face width = b = 10m (assumed)
= 10 x 5 =50mm

Pitch circle diameter of
i) Pinion , $d_1 = mz_1$
= 5 x 31 = 155mm

ii)Gear, $d_2 = mz_2$
= 5 x 100 = 500mm

$$= F_t = \frac{18484}{m} . = \frac{18484}{5} = 3696.8 \; N_s$$

V = 1.623m = 1.623 x 5
= 8.115 m/sec

Checking the gear for dynamic and wear loads

We know, that, the Dynamic load on gear tooth

$$F_d = F_t + F_i$$

$$= F_i = \frac{K_3 v\ (Cb + F_t)}{K_3 v + \sqrt{Cb + F_t}}.$$

$$\therefore F_d = 3696.8 + \frac{20.67 \times 8.115\ (522.464 \times 50 + 3696.9)}{20.67 \times 8.115 + \sqrt{522.464 \times 50 + 3696.8}}.$$

$$= 3696.8 + \frac{167.73\ (29820)}{167.73 + 129820}. = 3696.8 + \frac{5001708.6}{167.73 + 172.68}_s$$

$$= 3696.8 + \frac{5001708.6}{340.41}.$$

= 3696.8 + 14693.18

$$\boxed{\therefore\ F_d = 18389.98\ N}$$

Assuming:

$\sigma_{en} = 259.0\ N/mm^2$

State tooth load or endurance strength of the tooth $= F_{en} = \sigma_{en} .b\pi$

$F_{en} = 259 \times 50 \times \pi \times 5 \times 0.1246$

= 25345.89 N

For medium shock taking $F_{en} = 1.35\ F_d$

= 1.35 x 18389.98

= 24826.473

i.e., $\frac{F_{en}}{F_d} = \frac{25345.89}{18389.98} = 1.378$

Design is safe

Wear load
W.K.T,

$$Q = \text{Ratio factor} = \ = \frac{2VR}{VR+1}.$$

$$= \frac{2\times 1.3225}{1.3225+1}. = 1.138$$

F_w = d, b Q K

= 155 x 50 x 1.138 x 0.279

= 2460.6 N

Is the design is safe from the point of wear?

$\therefore$ find new k

$$= \frac{F_d}{155\times 50\times 0.138}$$

$$= \frac{18389}{8819.5}$$

$$\boxed{\therefore\ k = 2.08}$$

Heat treaded for 350 BHN

$\therefore$ $F_w > F_d$ design is safe

A pair of carefully cut spur gears with 20° stub involute profile is used to transmit a maximum power 22.5 kW at 200 rev/min. The velocity ratio is 1:2. The material used for both pinion and gear is medium cast iron, whose allowable, static stress may be taken as 60 Mpa. The approximate center distance may be taken as 600 mm, determine module and face width of the spur pinion and gear. Check the gear pair for dynamic and wear loads

The dynamic factor or deformations factor in Bucking hams dynamic load equation may be taken as 80, and material combination/load stress factor for the wear may be taken as 1.4

Given: VR = 2, N_1 = 200 rev/min, N_2 = 100 rev/ min, P = Power transmitted, 22.5 kW

Center distance = L = 600mm σ_{d_1} = σ_{d_2} = 60Mpa, C= 80, K= 1.4

Assumption:

i) b = face width = 10m

ii) Steady load condition and 8 – 10 hrs/day

$\therefore$ C_s = 1.0

Both the gear and pinion are mode of the same material. Therefore pinion is weaker and the design will be based on pinion.

W.K.T,
Centre distance between the shafts (L) = 600mm

$$= \frac{d_1 + d_2}{2}.$$

and $$= \frac{d_1 + 2d}{2}. = 600\ mm$$

$\therefore$ d_1 = 400mm = 0.4 m
d_2 = 800mm = 0.8m

V_1 = Pitch line velocity of pinion $= \frac{\pi\ d_1\ N_1}{60}.$

$$V = \frac{\pi \times 0.4 \times 200}{60}. = 4.2 m/\sec$$

Since V_i = pitch line velocity is less than 12 *m/sec* the velocity factor = C_v, may be taken as

$$= \frac{3.05}{3.05 + v}.$$

$$= \frac{3.05}{3.05 + v}. = \frac{3.05}{3.05 + 4.2}. = \frac{3.05}{7.25}.$$

$$= 0.421$$

Now, $$Z_1 = \frac{d_1}{m} = \frac{400}{m}.$$

$\therefore$ y_1 = tooth form factor = $0.175 - \frac{0.910}{Z_1}$ (for 20° stub systems)

$$0.175 - \frac{0.910}{400} m$$

$$= 0.175 - 0.002275\ m$$

W.K.T,

Design tangential tooth load $= F_t = \frac{P \times 10^3}{v\ .} \times C_s$

$$= \frac{22.5 \times 10^3}{4.2} \times 1.0$$

$$= 5357\ N$$

W.K.T,

$F_t = \sigma_d . C_v \, b\pi \, m \times y_1$

$= 60 \times 0.421 \times 10m \times \pi m \times (0.175 - 0.002275m)$

Solving for m, we get m = 6.5

$\therefore$ m = 8.0 (standard)

Face width = b = 10m = 10 x 8 = 80mm

$$Z_1 = \frac{d_1}{m} = \frac{400}{m}. = 50$$

$$Z_2 = \frac{d_2}{m} = \frac{800}{8}. = 100$$

Checking two gears for dynamic and wear load

W.K.T

(i) Dynamic load = $F_d = F_T + F_i$

$$g = F_T + \frac{20.67 \times 4.2\ (80 \times 80 + 53.57)}{20.67 \times 4.2 + \sqrt{(80 \times 80 + 53.57)}}$$

= 5357 + 5273

= 10630 N

W.K.T,

y_1 = Tooth form factor for pinion = 0.175 – 0.002275m

= [0.175 – 0.002275 x 8]

= 0.175 – 0.018200

= 0.1568

Let flexural endurance limit (σe) for cast iron may be taken as 85 Mpa = (85 N/mm)

$$\therefore Fen = \sigma_{en}. \, b \, \pi \, my$$

$$= 85 \times 80 \times \pi \times 8 \times 0.1568$$

= 26720 N

For steady loads $\quad Fen = 1.25 fd.$

1.25×10610

13287.5 N

W.K.T,

$$Q = \text{Ratio factor} = \frac{2VR}{VR + 1} = \frac{2 \times 2}{2 + 1}$$

$$= 1.33$$

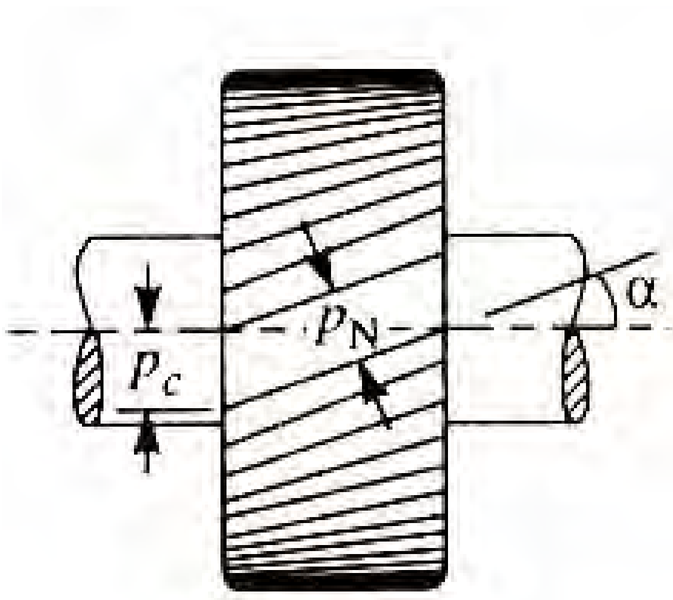
$F_w = d_1\ b\ Q\ K$

$= 400 \times 80 \times 1.33 \times 1.4 = 59584$ N

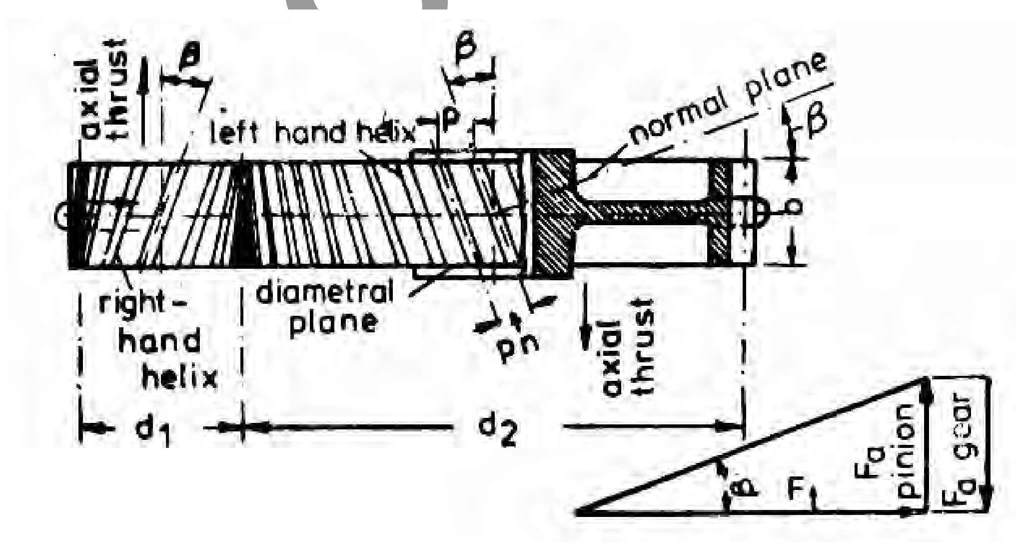
Since both F_{en} and F_w are greater than F_d, the design is safe

Helical Gears:

A Helical gear has teeth in the form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spurgearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contacts as in spurgearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with high efficiency of transmission.



The helical gears may be single helical type or double helical type. In case of single helical type there is some axial thrust between the teeth which is a disadvantage. In order to eliminate this axial trust double helical gears (i.e., herning bone gears) are used. It is equivalent to two single helical gears, In which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.



Terms used:

Helix angle: It is constant angle made by the helices with the axis of rotation

Axial pitch: It is the distance parallel to the axis between similar faces of adjacent teeth. It is same as circular pitch and is therefore denoted by P_C.

Normal pitch: It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by P_N.

$$P_N = P_C \cos \beta$$

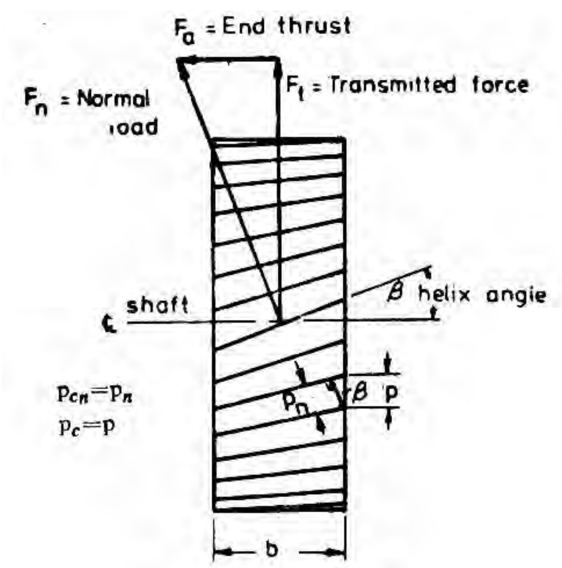
again

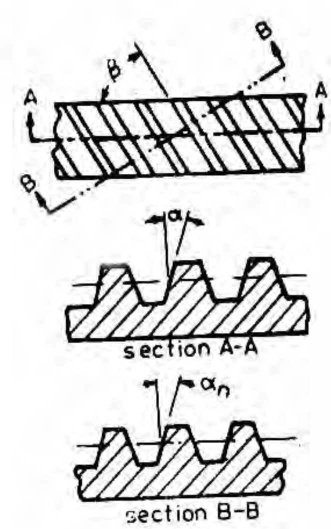
$$\boxed{\tan \propto_N = \tan \propto \cos\beta}$$

$\propto_N$ = Normal pressure angle
$\propto$ = Pr. angle

Face width: In order to have more than one pair of teeth in contact, the tooth displacement (i.e., the advancement of one end of tooth over the other end) or over lap should be atleast equal to the axial pitch such that, over lap P_C =b tan β--------- (i)

The normal tooth load (F_N) has two components, one is tangential component (F_t) and the other axial component (F_A) as shown in fig





The axial or end thrust is given by

$$F_A = F_N \sin \beta = F_t \tan \beta \text{ --------(ii)}$$

From the above equation (i), we see that as the helix angle increases then the tooth over lop increases. But at the same time the end thrust as given by the equation (ii) also increases which is not desirable. It is usually recommended that the over lop should be 15% of the circular pitch.

Over lop = b tan β = 1.11 P_C

$\therefore \quad b = \dfrac{1.11\, P_c}{\tan \beta} \qquad (\because = p_c = \pi_m)$ b = minimum face width

m = Module,

Note:

1. The maximum face width may be taken as 12.5 to 20.0m
2. In case of double helical or herring bone gears the minimum face width is given by

$$b = \frac{2.3 P_c}{\tan \beta} \qquad = b = \frac{2.3 \times \pi\ m}{\tan \beta} \qquad \geq = \frac{2.3 \times \pi\ m}{\sin \beta}$$

3. In a single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears it may be made up to 45°

$b = 12.5\, m_n$ To $20.m_n$.

Formative or equivalent number of teeth for helical gear:

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth an a helical gear

$Z_E = Z / \cos^3 . \beta$

Z = Actual number of teeth on a helical gear and

β = helix angle.

Proportion of Helical Gears:

AGMA Recommendations.

Pressure angle in the plane of rotation	$\propto$ = 15° to 25°
Helix angle,	β = 20 - 45°
Addendum	= 0.8 m (maximum)
Dedendum	= 1.0 m
Minimum total depth	= 1.8 m (maximum)
Minimum clearance	= 0.2 m
Thickness of tooth	= 1.5708 m

STRENGTH OF HELICAL GEARS: (P962 K/G)

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore, in order to find the strength of helical gear, a modified lewis equation is used.

It is given by, $F_T = \sigma_o . C_V\, b\, \pi\, m\, y'$.

Where

(i) F_T, σ_o, C_V, b, π, m, as usual , with same meanings,

And
y' = Tooth from factor or lewis factor corresponding to the **FORMATIVE OR VIRTUAL OR EQUIVALENT NUMBER OF TEETH.**

The values of C_V, velocity factor, from equation, (D.D.H)

Item	Equation
(a) For low-angle helical gears when v is less than 5 m/s	$C_v \frac{4.58}{4.58+v}$
(b) For all helical and herringbone gears when v is 5 to 10 m/t	$C_v \frac{6.1}{6.1+v}$
(c) For gears when v is 10 to 20 m/s (Barth's formula)	$C_v \frac{15.25}{15.25+v}$
(d) For precision gear with v greater than 20 m/s	$C_v \frac{5.55}{5.55+\sqrt{v}}$
(e) For non metallic gears	$C_v \frac{0.7625}{1.0167+v} + 0.25$

(ii) The dynamic tooth load, $F_d = F_t + F_i$

Where $$F_i = \frac{K_3 v\left(cb \cos^2\beta + F_t\right)\cos\beta}{K_3 v + \left(cb \cos^2\beta + F_t\right)^{1/2}}$$

K_S = 20.67 in SI units
= 6.60 in metric units,

(iii) The static tooth load or endurance strength of the tooth is given by

$$F_S = \sigma_e\, b\, \pi\, m y' \geq F_d$$

The maximum or limiting wear tooth load for helical gears is given by,

$$F_w = \frac{d_1\, b\, Q\, K}{\cos^2\beta} \geq F_d$$

Where d_1. b, Q and K have usual meanings as discussed in spur gears

In this case,
Where K = The load stress factor

$$K = \frac{(\sigma_{es})^2 \sin\alpha_N}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

Pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10,000 rev/min and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 Mpa. Determine the suitable module and face width from static strength considerations and check the gears for dynamic and wear loads. given σ_{es} = 618 MPa

Given: P = 15kW= 15x10^3 W, $\propto = 20°$, $\beta = 45°$, N_1 = 10,000 rev/min,
d_1 = 80mm = 0.08 m, d_2 = 320 mm = 0.32 m, $\sigma_{d1} = \sigma_{d2}$ = 100 MPa = 100 N/mm^2,
σ_{es} = 618 MPa = 618 N/mm^2

Since, both the pinion and gear are made of the same material (i.e., cast steel) the pinion is weaker. Thus the design is based on the pinion.

W K T,
Torque transmitted by the pinion

$$T = \frac{P \times 60}{2\pi N_1} = \frac{15 \times 10^3 \times 60}{2\pi \times 10,000} = 14.32 \text{ N-m}$$

$\therefore$ Tangential tooth load on the pinion $F_t = \frac{T}{d_1/2} = \frac{14.32}{0.08/2} = 358 \text{ N}$

W.K.T

Number of teeth on the pinion $= Z_1 = \frac{d_1}{m} = \frac{80}{m}$

And formative or equivalent number of teeth for pinion $= Z_{E1} = \frac{Z_1}{\cos^3\beta}$

$$= \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}.$$

$\therefore$ Tooth from factor for pinion for 20°stub teeth

$$y'_1 = 0.175 - \frac{0.841}{Z_{E1}}$$

$$= 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

W.K.T

$$V = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.08 \times 10,000}{60} = 42\ m/Sec$$

$\therefore$ $$C_v = \frac{5.55}{5.55 + \sqrt{V}} = \frac{5.55}{5.55 + \sqrt{42}}$$ $\because$ V is greater than 20 m/sec

$$C_v = \frac{5.55}{5.55 + 6.48} = \frac{5.55}{12.03} = 0.461$$

Since maximum face width,(b) for helical gear may be taken as 12.5 m to 20.0 m.
Let us take b = 12.5 m

W.K.T
tangential tooth load (F_t)

$= 358 = (\sigma_{d1}.\ C_V)\ b\ \pi\ m\ y_1^1$

$= (100 \text{ x } 0.461) \text{ x } 12.5m \text{ x } \pi \text{ x } m \text{ x } (0.175-0.003)$

$= 72m^2 - 1.5m^3$

By Trial and hit method,
Solution for m, =
m = 2.1 say 2.5 mm (standard)

and face width = b = 12.5 m = 12.5 x 2.5 = 31.25 mm say 32.0 mm

cheeking the gear for wear:

WKT. $\quad V.R = \frac{d_2}{d_1} = \frac{320}{80} = 4$

$$Q = \frac{2 \times VR}{VR + 1} = \frac{2 \times 4}{4 + 1} = \frac{8}{5} = 1.6$$

WKT. tan $\alpha_N = \tan \propto \cos \beta$
$= \tan 20° \cos 45°$
$= 0.2573$

$\therefore \quad \alpha_N = 14.4°$

Since, both the gears are made of same material (i.e., cast steel).

Therefore, let

$E_1 = E_2 = 200 \text{ x } 10^3 \text{ N/ mm}^2$

Load stress factor = $K = \frac{\sigma_{es}^2\ .\ Sin\ \alpha_N}{1.4}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$

$$= \frac{618^2 \times Sin\ 14.4}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right)$$

$$= 0.678\ N/mm^2$$

W.K.T,

$$= F_W = \frac{d_1 \; b \; Q \; K}{Cos^2 \; \beta} = \frac{80 \times \; 32 \times 1.6 \times 0.678}{Cos^2 \; 45} \; 5554 \text{ N}$$

Since maximum load for wear is much more than the tangential load on the tooth. Design is satisfactory for wear consideration.

Seminally try for F_d = dynamic load

$$F_d = F_t + F_i$$

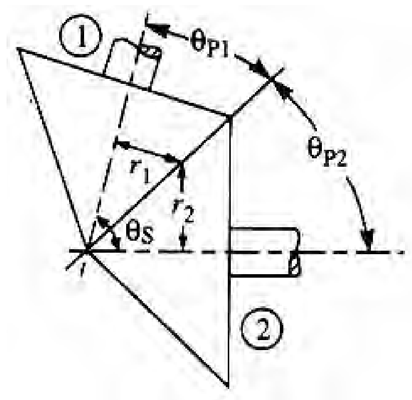
$$= F_t + \frac{k_3 \; v \; (C_b \; \cos^2 \beta) \cos \beta}{k_3 \; v \; + \sqrt{C_b \; Cos^2 \; \beta + F_t}}$$

$$\left.\begin{array}{l}\text{C= dynamic factor depending} \\ \quad \text{upon machine error} \\ \quad \text{(for an error of 0.04)}\end{array}\right\} = 712.0$$

$$= 358 + \frac{20.67 \times 42 \; (712 \times 32 \; Cos^2 \; 45 + 358) \; \cos 45}{(20.67 \times 42) + \sqrt{(712 \times 32 \; Cos^2 \; 45 + 358) \; \cos 45}}$$

$$= F_D = \; ?$$

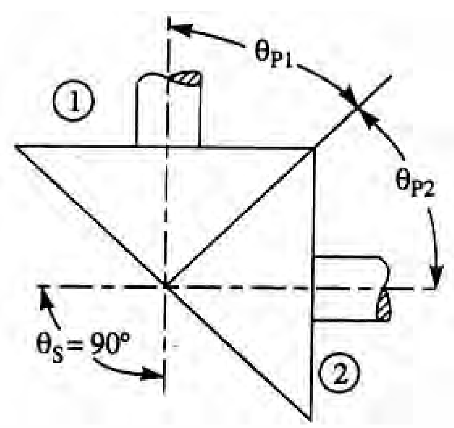
Bevel gears:
The bevel gears are used to transmit power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones.



CLASSIFICATION OF BEVEL GEARS:

Classified depending upon the angles between the shafts and the pitch surfaces.

(i) **Miter gears:** when equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angles as shown, then they are known as miter gear.



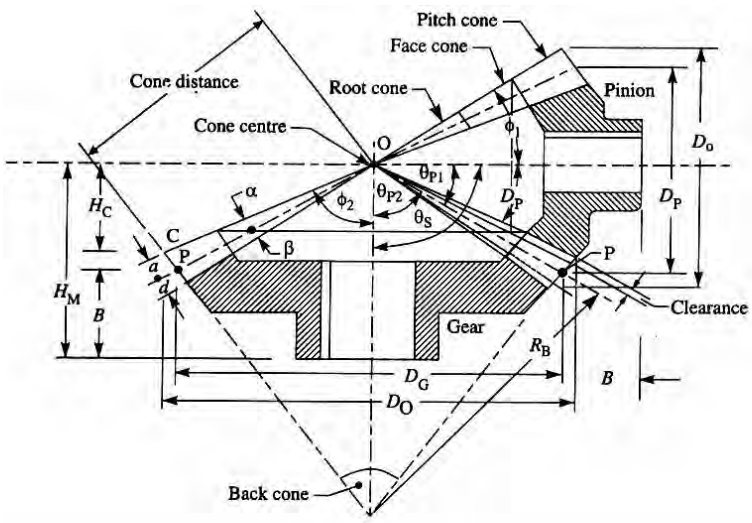
(ii) **Angular bevel gears:** when the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as angular bevel gears.

(iii) **Crown bevel gears:** when bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90° then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing as shown.

(iv) **Internal bevel gears:** when the teeth on the bevel gear are cut on the inside of the pitch cone then they are known ass inter bevel gears.

Note: The bevel gears may have straight or spiral teeth. It may be assumed, unless other wise stated that the bevel gear has straight teeth and and the axes of the shafts intersect at right angle.

TERMS USED IN BEVEL GEARS:



A sectional view of two bevel gears in mesh is as shown. The following terms are important from the subject point of view.

(i) **Pitch cone:** It is a cone containing the pitch elements of the teeth.

(ii) **Cone centre:** It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

(iii) **Pitch angle:** It is the angle made by the pitch line with the axis of the shaft. It is denoted by (i.e, δ_1 & δ_2)

(iv) **Cone distance:** It is the length of the pitch cone element. It is also called as a pitch cone radius. It is denoted by 'OP' Mathematically cone distance or pitch cone radius

$$= OP = \frac{\text{pitch radius}}{\sin O_p} = \frac{D_p/2}{\sin O_{p1}} = \frac{D_G/2}{\sin O_{p2}}$$

(v) **Addendum angle:** It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by θ_a. Mathematically addendum angle.

$$\tan \theta_a = \frac{2h_{a1}\ Sin\ \delta_1}{d_1}$$

$$= \frac{2h_{a2}\ Sin\ \delta_2}{d_2}$$

(vi) **Dedendum angle:** It is the angle subtended by the Dedendum of the tooth at the cone centre. It is denoted by θ_d. Mathematically,

$$\tan \theta_d = \frac{2h_{f1} \sin \delta_1}{d_1}$$

$$= \frac{2h_{f2} \sin \delta_2}{d_2}$$

Where, h_{a1}, h_{a2} = addendum of the pinion and gear respectively, mm

h_{f1}, h_{f2} = dedendum of pinion and gear respectively, mm

(vii) **Face angle:** It is the angle subtended by the face of the tooth at the cone centre. The face angle is equal to the pitch angle plus addendum angle.

(viii) **Root angle:** It is the angle subtended by the root of the tooth at the cone centre. It is equal to the pitch angle minus dedendum angle

(ix) **Back cone:** (Normal cone): It is the imaginary cone perpendicular to the pitch cone at the end of the tooth.

(x) **Crown height:** It is the distance of the crown point C, from the cone centre O, parallel to the axis of the gear. It is the denoted by C

(xi) **Mounting height:** It is the distance of the back of the boss from the cone centre. It is denoted by ‘ m’

(xii) **Pitch diameter:** It is the diameter of the largest pitch circle.

(xiii) **Outside or addendum cone diameter**: It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically outside dia ,

$$d_{O1} = d_1 + 2ha_1, \ \text{Cos } \delta_1$$

$$d_{O2} = d_2 + 2ha_2, \ \text{Cos } \delta_2$$

Proportions of Bevel gears:
The proportion for the bevel gear may be taken as

(i) Addendum:	a = 1.0 m
(ii) Dedendum:	d=1.2 m
(iii) Clearance	= 0.2 m
(iv) Working depth	= 2.0m
(v) Tooth thiknes	= 1.5708

Formative or Equivalent number of teeth for Bevel Gears: (Tredgold’s approximation)

$$Z_e = Z/\text{Cos}\delta$$

STRENGTH OF BEVEL GEARS:

The strength of a bevel gear tooth is obtained in a similar way as discussed in the previous articles. The modified form of the lewis equation for the tangential tooth load is given as follows

$$= F_t = (\sigma_d \times C_v)\ b\ \pi\ m\ y^1 \left(\frac{L - b}{L}\right)$$

y^1= lewis form factor based on formative or equivalent number of teeth

L = Slant height of pitch cone (or cone distance)

$$= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$$

Where d_1 and d_2 are the pitch circle diameters on the larger diameter of pinion and gears respectively

(i) The factor i.e, $\frac{L-b}{L}$ may be called as bevel factor

(ii) For satisfactory operation of bevel gears the face width should be from 6m to 10 m. Also ratio L/b should not exceed 3, (i.e., b ≤ L / 3)for this the number of teeth in the pinion must not be less than $\frac{48}{\sqrt{1+(vR)^2}}$

(iii) The dynamic loads for bevel gears may be obtained in the same similar manner as discussed for spur gears.

(iv) The static tooth load or endurance strength of the tooth for bevel gears is given by

$$= F_e = \sigma_e\ b\ \pi\ m\ y^1 \left(\frac{L - b}{L}\right)$$

The value of flexural endurance limit (σ_e) may be taken from table

(v) The maximum or limiting load for wear for bevel gears is given by

$$= F_w = \frac{D_1\, b\, Q_e\, k}{Cos\ \ \delta_1}$$

Where,
D_1, b, Q, k, have usual meanings as discussed in spur gears except that Q_e is based on formative or equivalent number of teeth, such that,

$$Q = \frac{2\, Ze_2}{Ze_2 + Ze_1}$$

A pair of bevel gears to connect two shafts at right angles and transmit 9 kW. The allowable static stress for pinion and gear materials may be taken respectively as 85 MPa and 55 MPa and brinill hardness of 200 and 160. The speed may be assumed as 1200/420 and number of teeth may be assumed as 21 for pinion and 60 for gear. Tooth profile may be taken as 20° full depth involute. Check the design for dynamic and wear loads.

Given: $\theta_s = 90°$, $P = 9kW = 9000W$, $Z_1 = 21$, $Z_2 = 60$, $\sigma_{d1} = 85$ MPa, $\sigma_{d2} = 55$ MPa, $N_1 = 1200$ rev/min, $N_2 = 420$ rev/min, $\alpha = 20°$(full depth involute)

Find, module, and check the design for dynamic and wear loads,

Since, the shafts are at right angles,
Therefore, pitch angle for pinion,

$$= \tan\delta_1 = \frac{d_1}{d_2} = \frac{Z_1}{Z_2} = \frac{1}{i}$$

$$\therefore \qquad \delta_1 = \tan^{-1}\left(\frac{21}{60}\right) = 19.3°$$

$$\boxed{\therefore \quad \delta_1 = 19.3°}$$

and the pitch angle for gear $= \delta_2 = 90 - \delta_1$

$$\boxed{\therefore \quad \delta_2 = 70.7°}$$

W.K.T,

formative number of teeth for pinion

$$Z_{e_1} = \frac{Z_1}{\cos\delta_1} = \frac{21}{\cos 19.3}.$$

$$= \frac{Z_1}{0.9438} = 22.25°$$

$$\boxed{Z_{e_1} = 22.25°}$$

and
$$Z_{e_2} = \frac{Z_2}{\cos\delta_2} = \frac{60}{\cos 70.7} = \frac{60}{0.3305}$$

$$\boxed{Z_{e_2} = 181.54°}$$

W.K.T.

for 20° full depth involute system tooth from factor

For pinion $= y_1^1 = 0.154 - \frac{0.912}{Z_{e_1}}$

$$= 0\,154 - \frac{0.912}{22\,.\,25} = (0\,154 - 0.0405) = 0.11350$$

and for gear $= y_2^1 = 0\,154 - \frac{0.912}{181.54}$

$$0.154 - 0.00502 = 0.14898$$

$$\sigma_{d_1} \times y_1^1 = 85 \times 0.11350 = 9.6475$$
$$\sigma_{d_2} \times y_2^1 = 55 \times 0.14898 = 8.1939$$

Since the product, $\sigma_{d_2} \times y_2^1$ is less than $\sigma_{d_1} \times y_1^1$ therefore Gear is weaker, and thus the design should be based on gear only.

W.K.T.

$$F_t = \frac{P \times 10^3}{v}$$

Here $v = \frac{\pi . d_2\ N_2}{60} \qquad = \frac{\pi . m Z_2\ N_2}{60}$

$$= \frac{\pi \times m \times 60 \times 420}{60} = 1320m \quad mm / Sec$$

$$\boxed{v = 1.320m \quad m / Sec}$$

Now

$$F_t = \frac{P \times 10^3}{v} = \frac{9 \times 10^3}{1.320\, m} = \frac{6818.18}{m}$$

$$\boxed{F_t = \frac{6818.18}{m} \text{ N}}$$

Taking velocity factor

$$= C_v = \frac{6.1}{6.1 + v}$$

(taking into consideration that gears are very accurately cut and ground gears having a pitch line velocity from 6 m/ sec to 20 m/sec)

$$\therefore = C_v = \frac{6.1}{6.1 + 1.32\, m}$$

W.K.T,

Length of pitch cone element

$$= L = \frac{d_2}{2\ Sin\ \delta_2} = \frac{m \times 60}{2\ Sin\ 70.7}$$

$$= \frac{m \times 60}{2 \times 0.9438} = 31.78\ m$$

$$\therefore \boxed{\text{L = 31.78 m}}$$

Assuming the face width b = 1/3rd of the length of the pitch cone element L,

$$\therefore\ b = \frac{L}{3} = \frac{31.78\ m}{3} = 10.60\, m$$

$$\therefore \boxed{\text{b = 10.60 m}}$$

W.K.T,

The tangential tooth load on gear

$$F_t = (\sigma_{d2} \times C_v)\ b\ \pi\ m\ y_2^1 \left(\frac{L-b}{L}\right)$$

$$= \frac{6818.18}{m} = 55 \times \frac{6.1}{6.1 + 1.32m} \times 10.60\ m$$

$$\times \pi \times m \times 1.4898 \times \left(\frac{31.78\ m - 10.60m}{31.78\ m}\right)$$

$$= \frac{6818.18}{m} = \frac{1109m^2}{6.1 + 1.32\ m}$$

$$\boxed{41590\ +\ 8999\ \text{m}\ = 1109\ \text{m}^3}$$

Solving this by hit and trial method we get, m = 4.58

$\boxed{\therefore\ \ \text{m} = 5.0}$ (Standard)

and b = 10.60 x m

= 10.60 x 5 = 53.0 mm

$\boxed{\therefore\ \text{b = face width = 53.0 mm}}$

Thus, d_2 = m x 60 = 5 x 60 = 300 mm

d_1 = m x 21 = 5 x 21 = 105 mm

& L = 31.78 m = 31.78 x 5 = 158.9

Check for dynamic load

W.K.T,

Pitch line velocity

V = 1.320 m m/sec

= 1.32 x 5

$$\boxed{v = 6.600 \text{ m / Sec}}$$

and tangential tooth load on the gear

$$= F_t = \frac{6818.18}{m} \quad N$$

$$= \frac{6818.18}{5}$$

$$\boxed{F_t = 1363.63 \text{ N}}$$

From table the tooth error in action for first class commercial gears having module 5 mm is

$$\boxed{e = 0.0555}$$

Take K_1 = 9.0 for 20° full depth teeth

and E1 = 210 x 10^3 N/mm^2 and

E_2 = 84 x 10^3 N/mm^2

C = dynamic factor depending upon machining errors

$$= \frac{e}{k_1(1/E_1 + 1/E_2)}$$

$$= \frac{0.0555}{9.0 \times \left[\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}\right]}$$

$$= \frac{6.166 \times 10^{-3}}{(4.76 \times 10^{-6} + 1.190 \times 10^{-5})}$$

$$= \frac{6.166 \times 10^{-3}}{(0.476 + 1.190)10^{-5}}$$

$$= \frac{6.166 \times 10^{-3} \times 10^{5}}{1.666}$$

$$= \frac{6.166 \times 10^{-2}}{1.666} = 370.1 \; N/m$$

$\therefore$ $\boxed{\text{C = dynamic factor = 370.1.N/m}}$

W.K.T

Dynamic load on the gear

$$= F_d = F_t + F_i$$

$$= F_t + \frac{k_3 v \, (cb + F_t)}{k_3 \; v + \sqrt{c_b + F_t}}$$

$$= 1363.63 + \frac{20.67 \times 6.6 \; (370.1 \times 53 + 1863.63)}{20.67 \times 6.6 + \sqrt{370.1 \times 53 + 1363.63}}$$

$$= 1363.63 + \frac{136.422 \; (19615.3 + 1363.63)}{136.422 + \sqrt{19615.3 + 1363.63}}$$

$$= 1363.63 + \frac{136.422 \times 20978.93}{136.422 + \sqrt{20978.93}}$$

$$= 1363.63 + \frac{2861987.588}{136.422 + 144.841}$$

$$= 1363.63 + \frac{2861987.588}{281.263}$$

$$= 1363.63 + 10175.485$$

$$\boxed{F_d = 11539.115 \text{ N}}$$

σ_{en} = for gear material of BHN = 160, is taken as 83.5 N/mm^2

Further, we know static tooth load or endurance strength of the tooth

$$= F_S = \sigma_{en}\ b\,\pi m y_2^1 \left(\frac{L-b}{L}\right)$$

$$= 83.5 \times 53 \times \pi \times 5 \times 0.14898$$

$$\times \left(\frac{158.9 - 53}{158.9}\right)$$

$$= 83.5 \times 53 \times \pi \times 5 \times 0.14898 \times 0.666$$

$$\boxed{F_S = 6902.116 \text{ N}}$$

Since $F_S < F_d$, the design is not satisfactory from the standpoint of dynamic load.

It is known that $F_S \geq 1.25\ F_d$ for steady loads

i.e., F_d – dynamic load on gear must be reduced

i.e, by assuming for a satisfactory design against dynamic load, let us take the precision gears (class III) having tooth error in action

$$e = 0.0150 \text{ mm}$$

$$\boxed{\therefore C = 100.02 \text{ N/mm}}$$

$$\therefore\ F_D = 1363.63 + \frac{20.67 \times 6.6\ (100.02 \times 53 + 1363.63)}{20.67 \times 6.6 + \sqrt{100.02 \times 53 + 1363.63}}$$

$$= 1363.63 + \frac{136.422\ (5300 + 1363.63)}{136.422 + \sqrt{5300 + 1363.63}}$$

$$= 1363.63 + \frac{136.422 \times 6663.63}{136.422 + \sqrt{6663.63}}$$

$$= 1363.63 + \frac{136.422 \times 6663.63}{136.422 + 81.63}$$

$$= 1363.63 + \frac{909065.7319}{281.052}$$

$$= 1363.63 + 4169.0318$$

$$\boxed{F_D = 5532.66 \text{ N}}$$

From the above we see that by taking precision gear, F_S is greater than F_D, therefore, the design is satisfactory from the standpoint of dynamic load

$$\therefore \text{ here } = \frac{F_s}{F_D} = \frac{6902.166}{5532.66} = 1.2475$$

(Hence, design is safe)

<u>Check for wear load</u>

For 180 BHN, σ_{es} may be 617.8 N /mm^2 normally steel for pinion and cost iron for gear of 200 & 160 , (Hence in the table take 180 & 180)

$$\therefore \text{ k = load stress factor } = \frac{(\sigma_{es})^2 \ Sin\phi}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$$= \frac{(617.8)^2 \ Sin 20}{1.4}\left[\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}\right]$$

$$= \frac{381676 \times 0.342}{1.4}\left[\frac{1 + 2.5}{210 \times 10^3}\right]$$

$$= \frac{130533.192}{1.4}\left[\frac{3.5}{210 \times 10^3}\right]$$

$$= 93237.994 \times \frac{0.01666}{10^3}$$

$$= 93.2379 \text{ x } 0.01666$$

$$1.553 \text{ N / mm}^2$$

$$\boxed{\therefore \text{ k} = 1.553 \text{ N / mm}^2}$$

and

Q_e = ratio factor

$$= \frac{2\ Ze_2}{Ze_2 + Ze} = \frac{2 \times 181.54}{22.25 + 181.54}$$

$$= \frac{363.08}{203.79} = 1.78$$

W.K.T

Maximum or limiting load for wear

$= F_W = d_1^{*}\ b\ Q_e\ K$ (*? For pinion, please explain)

$= 105 \times 53 \times 1.78 \times 1.553$

$$\boxed{F_w = 15397.70 \text{ N}}$$

Since, F_w is greater than F_D the design is satisfactory from the standpoint of wear also

CLUTCHES AND BRAKES

Clutches:

A Clutch is a mechanical device which is used to connect or disconnect the source of power from the remaining parts so the power transmission system at the will of the operator. The flow of mechanical power is controlled by the clutch.

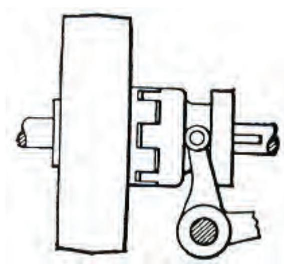
Types of Clutches

(i) Positive Clutches (ii) Friction clutches

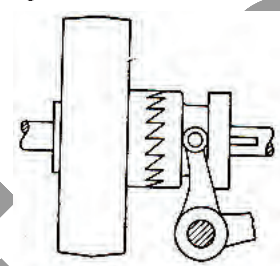
Positive Clutches: In this type of clutch, the engaging clutch surfaces interlock to produce rigid joint they are suitable for situations requiring simple and rapid disconnection, although they must be connected while shafts are stationery and unloaded, the engaging surfaces are usually of jaw type. The jaws may be square jaw type or spiral jaw type. They are designed empirically by considering compressive strength of the material used.

The merits of the positive clutches are

. (i) Simple (ii) No slip (iii) No heat generated compact and low cost.



Square-jaw clutch



Spiral-jaw clutch

Friction Clutches: Friction Clutches work on the basis of the frictional forces developed between the two or more surfaces in contact. Friction clutches are usually – over the jaw clutches due to their better performance. There is a slip in friction clutch. The merits are

(i) They friction surfaces can slip during engagement which enables the driver to pickup and accelerate the load with minimum shock.

(ii) They can be used at high engagement speeds since they do not have jaw or teeth

(iii) Smooth engagement due to the gradual increase in normal force.

The major types of friction clutches are

(i) Plate clutch (Single plate) (multiple plate)

(ii) Cone clutch

(iii) Centrifugal clutch

(iv) Dry

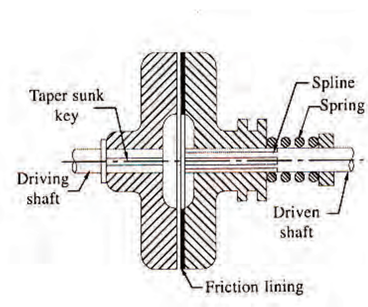
(v) Magnetic current clutches

(vi)Eddy current clutches

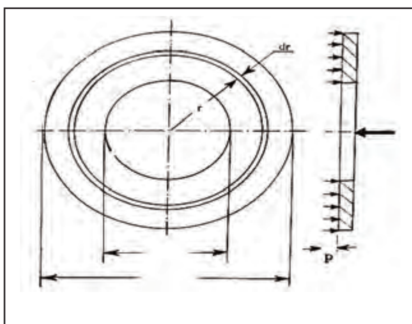
We will be studying about single plate multi-plate and cone clutches.

Single plate clutch:

A single plate friction clutch consisting of two flanges shown in fig 2. One flange is rigidly keyed in to the driving shaft, while the other is free to move along the driven shaft due to spliced connection. The actuating force is provided by a spring, which forces the driven flange to move towards the driving flange. The face of the drive flange is linked with friction material such as cork, leather or ferodo



Torque transmitted by plate or disc clutch



A friction disk of a single plate clutch is shown in above fig

The following notations are used in the derivation

D_o = Outer diameter of friction disc (mm)

D_i = Inna diameter of friction disc (mm)

P = pressure of intensity N/mm^2

F = Total operating force (N) (Axial force)

T = torque transmitted by friction (N-mm)

Consider an elemental ring of radius r and radial thickness dr

Area of elemental length = 2πr. dr

Axial force length = 2πr r. P

(μ or f) friction force = 2πr. drp μ

Friction torque = 2πr dr Pμ * r

Total axial force Fa = $F_a = \int_{D_i/2}^{D_0/2} 2\pi r\, drp$ ------------ (1)

Torque Transmitted by friction T = $\int_{D1/2}^{D_0/2} 2\pi r^2\, dr\ \mu p$ ------------ (2)

There are two criteria to obtain the torque capacity – uniform pressure and uniform wear

1. Uniform pressure Theory:

In case of new clutches, un playing assumed to be uniformly distributed over the entire surface area of the friction disc. With this assumption, P is regarded as constant.

Equation – 1 becomes

$$F_a = \int_{D_i/2}^{D_0/2} 2\pi r\, drp$$

$$F_a = 2\pi\, p \int_{D_i/2}^{D_0/2} r\, dr =$$

$$2\pi\, p \left[\frac{r^2}{2}\right]_{D_1/2}^{D_0/2}$$

$$F_a = \frac{2\pi}{2} p \left[\frac{D_0^2}{2} - \frac{D_1^2}{2}\right]$$

$$F_a = \frac{1}{4} \pi\, p \left(D_0^2 - D_1^2\right)$$

or $$P = \frac{4\,Fa}{\pi\left[D_0^2 - D_1^2\right]}$$

From Equation -2

$$T = \int_{D1/2}^{D_0/2} 2\pi\, \mu\, pr^2 dr$$

$$T = 2\pi\, \mu\, p \int_{D1/2}^{D_0/2} r^2 dr$$

$$T = 2\pi\,\mu\,p\left[\frac{r^3}{3}\right]_{D_1/2}^{D_0/2}$$

$$T = \frac{2}{3}\pi\mu\,p\left[\frac{Do^3}{2} - \frac{Di^3}{2}\right]$$

$$T = \frac{1}{3}\pi\,\mu\,p\left(Do^3 - D_1^{\,3}\right)$$

Substituting the value of p from equation 3

$$T = \frac{\pi\mu\left(D^3 - D_1^{\,3}\right)}{12} \qquad \frac{4Fa}{\pi\left(D_o^{\,3} - D_1^{\,3}\right)}$$

$$T = \frac{1}{3}\mu\,\mathrm{Fa}\,\frac{\left(D_0^3 - D_i^3\right)}{\left(D_0^2 - D_i^2\right)}$$

$$T = \frac{\mu\,\mathrm{Fa}\,Dm}{2}$$

Where $\frac{2}{3}\left[\frac{D_0^3 - D_i^3}{D_0^2 - D_i^2}\right] = Dm$ mean diameter

Torque transmitted by n- friction surfaces

$$T = \frac{n^1\mu FaDm}{2}$$

Axial force = $\pi P\left(R_2^2 - R_1^2\right) = \frac{\pi}{4}P\left(D_0^2 - D_i^2\right)$

Uniform Wear Theory:

According to this theory, it is assumed that the wear is uniformly distributed over the entire surface --- of the friction disc. This assumption is used for workout clutches. The axial wear of the friction disc is import ional to the frictional work. The work done by the frictional force (μ P) and subbing velocity (2πrN) where 'N' is speed in rpm. Assuming speed N and coefficient of friction 'μ' is constant for given configuration

Wear ∝ Pr

Pr = constant C

When clutch plate is new and rigid. The wear at the outer radius will be more, which will release the pressure at the outer edge due to the rigid pressure plate this will change the pressure distribution. During running condition, the pressure distribution is adjusted in such a manner that the product pressure is constant, C.

From equation - (1)

$$\text{Axial force Fa} = 2\pi \int_{D_1/2}^{D_0/2} \Pr dr$$

$$\text{Fa} = 2\pi\, c \int_{D_1/2}^{D_0/2} dr$$

$$\text{Fa} = 2\pi\ c\left[\, r\,\right]_{D_1/2}^{D_0/2}$$

$$\text{Fa} = 2\pi\, c\left[\frac{D_0}{2} - \frac{D_1}{2}\right]$$

$$\therefore\ \text{C} = \frac{Fa}{\dfrac{2\pi\ [D_0\ - D_1]}{2}} \qquad - \qquad (7)$$

From equation - (2)

$$\text{T} = \int_{D_1/2}^{D_0/2} 2\pi\,\mu \Pr^2 dr\ = 2\pi\ \ \mu c \int_{D_1/2}^{D_0/2} r\, dr$$

$$= 2\pi\,\mu c\left[\frac{r^2}{2}\right]_{D_1/2}^{D_0/2}$$

$$= 2\pi\,\mu c\left[\frac{D_0\ /\ 2}{8} - \frac{D_1\ /\ 2}{8}\right]$$

$$\text{T} = \frac{\pi\,\mu c}{8}\left[D_0^2\ - D_1^2\right]$$

Substitute the value of C from equation – (7)

$$\text{T} = \frac{\pi\,\mu}{4}\left[D_0^2\ - D_1^2\right]\frac{Fa}{\pi\left[D_0\ - D_1\right]}$$

$$\text{T} = \frac{\mu\ \text{Fa}}{2}\frac{\left[D_0\ \ - D_1\ \right]}{2}$$

$$\text{T} = \frac{\mu\, Fa\ \ Dm}{2} \qquad - \qquad (8)$$

$$\text{Where Dm} = \frac{D_0 + D_1}{2} \qquad - \qquad (9)$$

Torque transmitted by "n" friction plates

$$T = \frac{n^1 \ \mu Fa \ Dm}{2}$$

Axial force Fa =

Average pressure occurs at mean radius $(\because r_m = D_m / 2)$

$$\therefore \ Fa = \frac{\pi b \ Dm (D_0 - D_1)}{2} \qquad - \qquad (10)$$

Maximum pressure occurs at inner radius

$$\therefore \ Fa = \frac{\pi \ p \ D_i}{2}(D_0 - D_i) \qquad - \qquad (11)$$

Note: The major portion of the life of friction lining comes under the uniform wear friction lining comes under the uniform wear criterion in design of clutches uniform wear theory is justified.

Problems:

1. A single plate friction clutch of both sides effective has 300 mm outer diameter and 160 mm inner diameter. The coefficient of friction o.2 and it runs at 1000 rpm. Find the power transmitted for uniform wear and uniform pressure distributions cases if allowable maximum pressure is 0.08 Mpa.

Given:

N^1 = I = 2, D_0 = 300 mm D_1 = 160mm μ = 0.2

N = 1000 rpm p = 0.08 Mpa = 0.08 N /mm^2

Solution:

i. Uniform wear theory

$$\text{Mean Diameter } D_{m=} \frac{D_0 - D_1}{2} = \frac{300+160}{2} = 230\,mm$$

Axial Force

$$Fa = \frac{1}{2}\pi \ b \ D_1 (D_0 - D_1)$$

From DDH 13.32 or Equation - (11)

$$\therefore \ \text{Fa} = \frac{1}{2}\pi \times 0.08 \ \times 160 \ (300 - 160)$$

$$= 2814.87 \text{ N}$$

Torque transmitted

$$\text{T} = \frac{1}{2} \ \mu \ n^1 \ Fa\, Dm$$

$$\text{T} = \frac{1}{2} 0.2 \times 2 \ \times 2814.87 \times 230$$

$$\text{T} = 129484 \ \text{ N – mm}$$

Power transmitted

$$\text{P} = \frac{2\pi \ M\, T}{60 \times 10^6}$$

$$\text{P} = \frac{2\pi \times 1000 \times 129484}{60 \times 10^6} =$$

$$\text{P} = 13.56 \ \text{ kW}$$

ii. Uniform wear theory

Mean Diameter $D_{m=} \dfrac{2}{3} = \left(\dfrac{D_0^3 - D_1^3}{D_0^2 - D_1^2} \right)$

$$D_{m=} \frac{2}{3} = \left(\frac{300^3 - 160^3}{300^2 - 160^2} \right)$$

$$\text{D}_\text{m} = 237.1 \text{ mm}$$

Axial Force $\quad \text{Fa} = \dfrac{\pi \ p \ (D_0^2 - D_1^2)}{4}$

$$\text{Fa} = \frac{\pi \ 0.08 \ (300^2 - 160^2)}{4}$$

$$\text{Fa} = 4046.4 \text{ N}$$

Torque transmitted

$$\text{T} = n^1 \ \frac{1}{2} \ \mu \quad Fa\, Dm$$

$$T = 2\ \frac{1}{2} 0.2 \times 4046.4 \times 237.1$$

$$T = 191880.3\ \ N - mm$$

Power transmitted

$$P = \frac{2\pi\ nT}{60 \times 10^6}$$

$$P = \frac{2\ \times \pi \times 1000 \times 191880.3}{60 \times 10^6} =$$

$$P = 20.1\ \ kW$$

2. A car engine develops maximum power of 15 kW at 1000 rpm. The clutch used is single plate clutch both side effective having external diameter 1.25 times internal diameter $\mu = 0.3$. Mean axial pressure is not to exceed 0.085 N/ mm^2. Determine the dimension of the friction surface and the force necessary to engage the plates. Assume uniform pressure condition.

Given p = 15 kW, n – 1000rpm, I =2 both sides are effective $D_0 = 1.25\ D_1$, $\mu = 0.3$,
p = 0.085 N/mm^2

Torque transmitted

$$P = \frac{2\pi\ nT}{60 \times 10^6}$$

$$T = \frac{P \times 60 \times 10^6}{2\pi\ n}$$

$$T = \frac{15 \times 60 \times 10^6}{2\pi\ 1000}$$

$$= 143.239\ N - mm$$

Mean Diameter D_m

$$D_{m=}\ \frac{2}{3} = \left(\frac{D_0^3 - D_i^3}{D_0^2 - D_i^2}\right) = \frac{2}{3} = \left(\frac{(1.25D_i)^3 - D_i^3}{(1.25D_i)^2 - D_i^2}\right)$$

$= 1.13\ D_i$

Axial Force $\quad Fa = \frac{\pi}{4}\, p\ \ (D_0^2 - D_i^2)$

$$Fa = \frac{\pi}{4} 0.085\ \ (1.25D_i^2 - D_i^2)$$

$$Fa = 0037552\ \ D_i^2$$

Torque transmitted $\quad T = i\,\frac{1}{2}\,\mu \quad Fa\,Dm$

$$143239 = 2 \times \frac{1}{2} 0.3 \times 0.037552 \times D_i^2 \times 1.13\,D_i$$

$\therefore \quad D_i = 224$ mm

$D_0 = 280$ mm

$D_m = 253$ mm

$Fa = 1884.21$ N

Thickness of disc h = 2 mm

3. Design a single plate clutch consist of two pairs of contacting surfaces for a torque capacity of 200 N-m. Due to space limitation the outside diameter of the clutch is to be 250mm

Given:
Single plate clutch, Torque = 2 x 10^5 N-mm, D0 = 250mm I = 2 (since two pairs of contacting surfaces)

Solution:

Assume suitable friction material – leather μ = 0.3 to 0.5 P = varies from 0.07 to 0.29 Mpa select μ = 0.4, P = 0.135 Mpa – N /mm^2

1. Torque transmitted= 2 x 10^5 N-mm

2. Mean diameter

Assuming uniform wear theory

$$D_m = \frac{D_i + D_0}{2} = \frac{D_i + 250}{2}$$

3. Axial force :

For uniform, wear condition

$$\text{Fa} = \frac{1}{2}\pi\, p\, D_i\ \ (D_i - D_i) =$$

$$= \frac{1}{2}\pi\, 0.135\ \times D_i\,(250 - D_i)$$

$$= 0.212\ D_i\ (250 - D_i)$$

Torque transmitted

$$\text{T} = \frac{1}{2}\mu\ Fa\ D_m\, i$$

$$\text{i.e}\ \ 2 \times 10^5 = \frac{1}{2} 0.4 \times 0.212\ D_i\,(250 - D_i) = \frac{1}{2}(250 + D_i) \times 2$$

$$= 62500\ D_1 - D_1^3 - 4716981.132 = 0$$

By trial and error method

Inner dia D_i = 85.46 mm is 86 mm

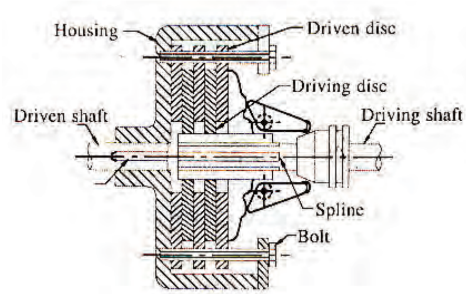
Outer dia D_0 = 250 mm given, mean m = dia of friction surface

Dm = 168 mm

Fa = 0.212 x 86 (250 – 86) = 2990 N

Multiple plate clutch

Fig. shows a multiple plate clutch. The driving discs are splined to the driving shaft so that they are free to slip along the shaft but must rotate with it. The driven discs drive the housing by means of bolts along which they are free to slide. The housing is keyed to the driven shaft by a sunk key. In the clutch shown there are five pairs of friction surfaces. The driving discs may be pressed against the driven discs by a suitable mechanism so that the torque may be transmitted by friction between the discs.



Multi disc Clutch:

Equations derived for torque transmitting velocity of single plate are modified to account for the number of pairs of contacting surfaces in the following way.

For uniform pressure T = $\frac{1}{2} i \ \mu_1 \ \ Fa \ Dm =$

$$\frac{\pi}{4} p\,(D_2^2 - D_i^2)$$

Uniform wear T = $(D_0^2 - D_i^2)$

Where I = number of pairs of contacting surfaces.

For uniform pressure theory

$$T = \frac{1}{2} i \ \mu_1 \ \ Fa \ Dm$$

$$D_m = \frac{2}{3} \frac{(D_0^3 - D_i^3)}{(D_0^2 - D_i^2)}$$

$$Fa = \frac{\pi}{4} p\,(D_0^2 - D_i^2)$$

Where I = number of friction surfaces

Uniform wear theory

$$T = \frac{1}{2} i \ \mu_1 \ \ Fa \ Dm$$

$$D_m = \frac{(D_2 + D_i)}{2}$$

$$Fa = \frac{1}{2} \pi b \ D_m (D_0 - D_i)$$

Maximum pressure occurs at inner radius

$$Fa = \frac{1}{2} \pi_1 \ \ p \ D_1 \ (D_0 - D_i)$$

Problem: A multi plate clutch having effective diameter 250mm and 150mm has to transmit 60 kW at 1200 rpm. The end thrust is 4.5 kN and coefficient of friction is 0.08 calculate the number of plates easuming (i) Uniform wear and (ii) uniform pressure distribution on the plates

Given: D_0 = 250 mm D_i = 150mm, P = 60 kW, N = 1200 rpm, Fa = 4.5 kN = 4500N, μ = 0.08

$$P = \frac{2\pi\ NT}{60 \times 10^6}$$

$$\text{Torque } T = \frac{P \times 60 \times 10^6}{2\ \pi\ 1200} = 477500\, N - mm$$

(i) Uniform wear theory

$$\text{Mean diameter Dm} = \frac{D_0\ +\ D_i}{2} = \frac{250\ +150}{2} = 200\,mm$$

$$T = \frac{1}{2} i\ \mu_1\ \ Fa\ Dm$$

$$477500 = \frac{1}{2} i\ 0.08 \times 4500 \times 200$$

$\therefore$ Number of friction plates, I = 13.26 $\cong$ 1.4 (even numbers)

Total number of plates i + 1 = 14 + 1 = 15

Uniform pressure

$$D_m = \frac{2}{3}\ \frac{(D_0^3\ -\ D_i^3)}{(D_0^2\ -\ D_i^2)} = 204.17 \text{ mm}$$

$$T = i\ \times \frac{1}{2}\ \ Fa \times \mu \times Dm$$

$$477500 = \ i\ \frac{1}{2} \times 4500 \times 0.08 \times 204.17$$

$\therefore$ i = 12.99 $\cong$ 14 (even number)

Total number of plates = 14 + 1 = 15

Problem 4:

A multi plate clutch of alternate bronze and steel plates is to transmit 6 kW power at 800 rpm. The inner radius is 38 mm and outer radius is 70 mm. The coefficient of friction is 0.1 and maximum allowable pressure is 350 kN /m^2 determine

(i) Axial force required
(ii) Total number of discs
(iii) Average pressure and
(iv) Actual maximum pressure

Given: P = 60 kW, N = 800 rpm, R_1 = 38mm, D_i= 76 mm R_0 = 70, D_0 = 140mm, μ = 0.1,
P = 350 kN / m^2 = 0.35 N / mm^2

1. Axial force

$$\text{Fa} = \frac{1}{2}\pi_1 \ \ p\ D_1\ (D_0 - D_1) \text{ - (13.32 DDH)}$$

$$\text{Fa} = \frac{1}{2}\pi\ 0.35 \times 76\ (140 - 76)$$

$$= 2674.12 \text{ N}$$

Torque to be transmitted

$$P = \frac{2\pi\ NT}{60 \times 10^6}$$

$$T = \frac{P \times 60 \times 10^6}{2\ \pi\ N} = \frac{6 \times 60 \times 10^6}{2 \times \pi \times 800} = 71625\ N - mm$$

Assuming uniform wear theory

Mean Diameter $D_m = \frac{(D_2\ +\ D_1)}{2} = \frac{140 + 76}{2} = 108\,mm$

Torque transmitted $T = \frac{1}{2} i\ \mu_1\ \ Fa\ Dm$

$$71625 = n\ \frac{1}{2} \times 0.1 \times 2674.12 \times 108$$

n = 4.96 ≅ 6 (even number)

Number of driving (steel) discs = $n_1 = \frac{n}{2} = \frac{6}{2} = 3$

Number of driven (bronze) discs n_2

= n_1 + 1 = 3 + 1 = 4

3. Average pressure occurs at mean diameter

Axial force $\text{Fa} = \frac{1}{2}\pi_1\ \ p\ D_m\ (D_0 - D_i)$

$$2674.12 = \frac{1}{2}\pi_1\ \ p\ 108\ (140 - 76) \text{ -}$$

∴ Average pressure p = 0.246 N/ mm^2

4. For 6 friction surface, torque transmitted

$$T = \frac{1}{2} i \; \mu_1 \;\; Fa \;\; Dm$$

$$71625 = 6 \; \frac{1}{2} \; 0.1_1 \;\; Fa \times 108 \text{-}$$

$\therefore$ Fa = 2210.6 N

Maximum pressure occur at inner radius

$$\text{Axial force} = \frac{1}{2} \; \pi_1 \;\; p \;\; D_1 \; (D_0 \; - \; D_i)$$

$$2210.6 = \frac{1}{2} \; \pi_1 \;\; p \;\; 76 \; (140 \; - \; 76) \text{-}$$

$\therefore$ Actual maximum pressure P = 0.289 N/ mm^2

Problem 5:
In a maultilate clutch radial width of the friction material is to be 0.2 of maximum radius. The coefficient of friction is 0.25. The clutch is 60KW at 3000 rpm. Its maximum diameter is 250mm and the axial force is limited is to 600N. Determine (i) Number of driving and driven plates (ii) mean unit pressure on each contact surface assume uniform wear

Given: Radial width = 0.2 Ro, μ = 0.25, P = 60KW, N = 3000rpm, D_0 = 250mm,
$\therefore$Ro = 125mm, Fa = 600N uniform wear condition.

Solution b = Ro- Ri
0.2 Ro =Ri
Ri= 0.8 Ro = 0.8x 125 = 100mm
$\therefore$ Inner diameter 2x100 = 200mm

i) Number of disc

Torque transmitted T = $\dfrac{P \times 60 \times 10^6}{2\pi \, N}$

$$\frac{60 \times 60 \times 10^6}{2\pi \, 3000} = 191000 N.mm$$

For uniform wear condition

Mean diameter $D_m = \frac{1}{2}\left(D_o + \; D_i\right)) = \frac{1}{2}\left(250 + 200\right) = 225mm$

Torque Transmitted

$$T \; = \frac{1}{2} \mu \, n \, FaDm$$

$$19100 = \frac{1}{2} 0.25 \times n, 600 \times 225$$

i.e n = 11.32

Number of active surfaces n =12 ($\therefore$ n must be even number)

Number of disc on the driver shaft

$$n_1 = \frac{n}{2} = \frac{12}{2} = 6$$

Number disc on the driven shaft

$$n_1 = \frac{n}{2} + 1 \quad = \quad \frac{12}{2} + 1 = 7$$

$\therefore$ Total number of plates : n1 + n2 = 617 = 13

ii) Mean unit pressure

$$\text{Fa} = \frac{1}{2} \pi \, p \; D_m \; (D_o - D_i)$$

$$600 = \frac{1}{2} \pi \, p \; 225 \; (250 - 200)$$

$\therefore$ P = 0.34 N/ mm^2

iii) for actual mean unit pressure

Actual axial force

$$\text{Fa} = \frac{2\,T}{\mu \, n \, D_m} \qquad \therefore T = \frac{1}{2} \mu \; fa \, D_m{}^n$$

$$= \frac{2 \times 191000}{0.25 \times 12 \times 225} = \pi 565.926 N / \backslash$$

$$565.926 = \frac{1}{2} \pi \, P D_m \; 225 \; (250 - 200)$$

Actual mean unit pressure P = $0.032 \, N \;/\; mm^2$

A Multiple plate clutch has steel on bronze is to transmit 8 KW at 1440 rpm. The inner diameter of the contact is 80mm and outer diameter of contact is140 mm. The clutch operates in oil with coefficient of friction of 0.1. The overage allowable pressure is 0.35Mpa. Assume uniform wear theory and determine the following.

a) Number of steel and bronze plates
b) Axial force required
c) Actual maximum pressure

Given P = 8KW, N = 1440 rpm, D1 = 80mm, Do = 140mm, μ = 0.1, P = 0.35 N / mm^2

Uniform Wear Theory.

Solution:

1) Number of steel and bronze plates
For uniform wear theory

Axial force $\text{Fa} = \frac{1}{2}\pi\, p\, (D_o - D_i)$

$$= \frac{1}{2}\pi\, 0.35\ 80(140 - 80)$$

$\text{Fa} = 2638.9\ \mu\ N$

Mean diameter $Dm = \frac{1}{2}(D_2 + D_i) = \frac{1}{2}(140 + 80) = 110mm$

Torque transmitted $T = \frac{P \times 60 \times 10^6}{2\pi N}$

$$T = \frac{8 \times 60 \times 10^6}{2\pi - 1440} = 53055556\, N - mm$$

Also $T = \frac{1}{2}\mu\ FaDmn$

$$53055.556 = \frac{1}{2} \times 2638.94 \times 110 \times n$$

$$\therefore n = 3.655$$

No. of Active surface is 4

Number discs on the driver shaft (Bronze)

$$n_1 = \frac{n}{2} = \frac{4}{2} = 2$$

Number disc on the driven shaft (Steel)

$$n_2 = \frac{n}{2} + 1 = \frac{4}{2} + 1 = 3$$

Total No. of disc $n_1\ n_2 = 2 + 3 = 5$

b) Axial force required

$$Fa = \frac{2T}{\mu\ n\ D\ m} = \frac{2 \times 53055.552}{0.1 \times 04\ 110} = 2411.62$$

c) Actual maximum pressure since maximum presser occur at inner diameter

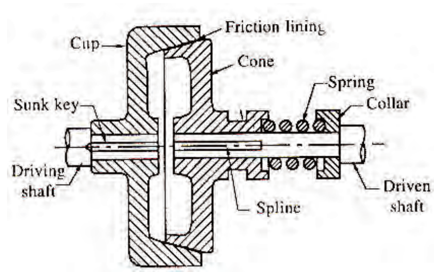
$$\text{Fa} = \frac{1}{2}\pi\ P\ D_i (D_o - D_{i1})$$

$$2411.62 = \frac{1}{2}\ P\ .80\ (140 - 80)$$

$$\therefore P = 0.32\ N\ / mm^2$$

Cone clutch

A simple form of a cone clutch is shown in fig. it consists of ;i driver or cup and the follower or cone. The cup is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits the outside conical surface of the cone. The slope of the cone face is made small enough to give a high normal force. The cone is fitted to the driven shaft by a feather key. The follower may be shifted along the shaft by a forked shifting lever in order to engage the clutch by bringing the two conical surfaces in contact.



Advantages and disadvantages of cone clutch:

Advantages:

1. This clutch is simple in design.
2. Less axial force is required to engage the clutch.

Disadvantages :

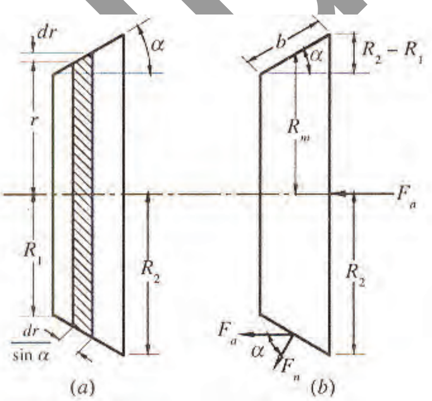
1. There is a tendency to grab.
2. There is some reluctance in disengagement.

Strict requirements made to the co-axiality of the shafts being connected

Torque transmitted by the cone clutch:-

Let Di = Inner diameter of cone, Ri = inner radius of cone,
Do = Outer diameter of cone, Ro = Outer radius of cone,
Rm = mean radius of cone, Dm = mean diameter of cone,
∝ = Semi cone angle or pitch cone angle or face angle,
P = Intensity of normal pressure at contact surface,
μ = Coefficient of friction,
Fa = Axial force

Fn=Normal force = $\frac{Fa}{\sin\alpha}$



Consider an elemental ring of radius 'r' and thickness 'dr' as shown in the figure

The sloping length $= \dfrac{dr}{\sin\alpha}$

Area of elementary ring $= 2\pi\ r\ \dfrac{dr}{\sin\alpha}$

Normal force on the ring $= P\ 2\pi\ r\ \dfrac{dr}{\sin\alpha}$

Axial component of the above force $\dfrac{p2\pi\ r.\ dr}{Sin\alpha} \times Sin\alpha$

$$= 2\pi\ pr\ dr$$

Total axial force Fa = $\int_{Ri}^{Ro} 2\pi\ pr\ dr$ ------------ (1)

Frictional force outer ring $= \mu\ p\ 2\pi r\ \dfrac{dr}{Sin\ \alpha}$

Moment of friction force about the axial

$$= \mu\ P2\pi\ r\ \frac{dr}{Sin\ \alpha} \times r$$

$$= 2\pi\ p\ \mu\ r^2\ \frac{dr}{Sin\ \alpha}$$

Total torque T = $\int_{Ri}^{Ro} 2\pi\ \mu \Pr^2 \dfrac{dr}{Sin\ \alpha}$ ---------- (2)

Uniform pressure theory: p constant

Equation (1) becomes

Fa = $\int_{Ri}^{Ro} 2\pi\ pr\ dr \quad = 2\pi\, p\ \int_{Ri}^{Ro} dr$

Fa = $\pi\ P\ \left(R_o^2 - R_i^2\right)$

p = $\dfrac{Fa}{\pi\left(R_o^2 - R_i^2\right)}$ ---------- (3)

Equation 2 becomes

T = $\int\limits_{Ri}^{Ro} 2\pi\ p\ \mu\ r^2\left(R_o^2 - r_i^2\right)\dfrac{dr}{Sin\alpha} = \dfrac{2\pi\ P\ \mu}{Sin\alpha}\int\limits_{Ri}^{Ro} r^2 dr$

$$T = \frac{2\pi\,\mu\,p}{Sin\alpha} = \frac{R_a^3 - R_i^3}{3}$$

Substitutes the value of P from equation ---------- (3)

$$T = \frac{2\pi\ \mu P}{Sin\alpha} = \frac{Fa}{\pi\left(R_o^2 - R_1^2\right)} \times \frac{R_a^3 - R_i^3}{3}$$

$$T = \frac{2\pi\ Fa}{3\ Sin\alpha} = \frac{Fa}{\pi\left(R_o^2 - R_1^2\right)} \times \frac{R_a^3 - R_i^3}{\left(R_o^2 - R_i^2\right)}$$

$$T = \frac{2\pi\ Fa}{2\ Sin\alpha} \times \frac{2}{3} \times \left(\frac{D_o^3 - D_i^3}{D_o^2 - D_{io}^2}\right) = \frac{\mu\ Fa}{2\ Sin\ \alpha}.\ Dm \quad \text{----------} \ (4)$$

Where $D\ m = \frac{2}{3}\left(\frac{D_o^3 - D_i^3}{D_o^2 - D_{io}^2}\right)$

Axial force $Fa = \pi\ P\ \left(D_o^2 - D_i^2\right)$ ---------- (5)

Uniform wear: For uniform wear condition Pr = C Constant

Equation (1) become Fa = $\int_{Ri}^{Ro} 2\pi\ P\ r\ d\ r \quad = 2\pi P \int_{Ri}^{Ro} dr$

$$\text{Fa} = 2\pi\ c\ \left(R_o - R_i\right) \quad \text{or}$$

$$C = \frac{Fa}{2\pi\left(R_o - R_i\right)}$$

Equation (2) become

$$T = \int_{Ri}^{Ro} 2\pi\ P\ \mu\ r^2\ \frac{dr}{Sin\alpha} = \frac{2\pi\ \mu\ C}{Sin\alpha}\int_{Ri}^{Ro} r\ d\ r$$

$$T = \frac{2\pi\ \mu\ c}{Sin\alpha} = \frac{\left(R_a^2 - R_i^2\right)}{2}$$

Substitute for C

$$T = \frac{2\pi\ \mu\ \left(R_a^2 - R_i^2\right)}{Sin\alpha} \times \frac{Fa}{2\pi\left(R_o - R_i\right)}$$

$$= \frac{\mu \, \text{Fa}}{2 \; Sin\alpha} \times \frac{\left(D_o + D_i\right)}{2} . \frac{\mu \; \text{Fa} \; D \; m}{2 \; Sin \; \alpha}$$

Where Dm = Mean diameter $\quad Dm = \frac{D_o \; + D_i}{2}$

If the clutch is engaged when one member is stationary and other rotating, then the cone faces will tend to slide on each other in the direction of an element of the cone. This will resist the engagement and then force

Axial load $\quad \text{Fa}^i = \text{Fa} \; \left(Sin\alpha + \cos\alpha\right)$

Force width $\quad b = \frac{D_o - D_i}{2 \; Sin \; \alpha}$

Outer diameter $\quad D_o = Dm + b \; Sin\alpha$

Inner diameter $\quad D_i = Dm - b \; Sin\alpha$

Problem:
A cone clutch is to transmit 7.5 KW at 600 rpm. The face width is 50mm, mean diameter is 300mm and the face angle 15°. Assuming co efficient of friction as 0.2, determine the axial force necessary to hold the clutch parts together and the normal pressure on the cone surface.

Given $\quad$ P = 7.5 KW, $\quad$ N = 600 rpm, $\quad$ b = 50mm,
Dm = 300mm, $\quad$ ∝ = 15° $\quad$ μ = 0.2

Solution:

$$\text{T} = \frac{P \times 60 \times 10^6}{2 \; \pi \; N} = \frac{7.5 \times 60 \div 10^6}{2 \; \pi \; 600} = 119375 \; N - mm$$

Torque transmitted $\quad \text{T} = \frac{\mu \, \text{Fa} \; Dm}{2 \; Sin \; \alpha}$

$$119375 = \frac{0.2 \; \text{Fa} \times 300}{2 \; Sin \; 15}$$

$$\therefore \; Fa = 1029.88 \; N$$

Also $\quad \text{Fa} = \pi \; Dm \; Pb \; Sin \; \alpha$ ---------- Equation 13.37 DDH

$$1029.88 = \pi \times 300 \, \text{P} \times 50 \; Sin \; 15$$

$$\text{P} = 0.0844 N \; / \; mm^2$$

A friction cone clutch has to transmit a torque of 200 N-m at 1440 rpm. The longer diameter of the cone is 350mm. The cone pitch angle is 6.25°the force width is 65mm. the coefficient of friction is 0.2. Determine i) the axial force required to transmit the torque. ii) The average normal pressure on the contact surface when maximum torque is transmitted.

Data T = 200 N-m, 2×10^5 N-mm N = 1440 rpm
Do = 350, $\propto$ = 6.25° b = 65mm, μ = 0.2

Solution

I) Axial force

Outer diameter $D_o = D_m + b\ Sin\ \alpha$

$$350 = D_m + 65\ Sin\ 6.25$$

$$\therefore D_m = 342.92 mm$$

Torque transmitted $T = \frac{1}{2}\ \frac{\mu \text{Fa}\ D_m}{Sin\ \alpha}$

$$2\times10^5 = \frac{1}{2}\times\frac{0.2\times\text{Fa}\ D_m}{Sin\ 6.25}$$

$\therefore$ Axial force required Fa = 634.934 N

ii) Average normal pressure

$$\text{Fa} = \pi\ D_m\ Pb\ Sin\ \alpha$$

$$634.934 = \pi.\ 342.92\times65 Sin\ 6.25°\ \text{P}$$

$\therefore$Average Normal pressure

$$\text{P} = 0.0833\ N/\ mm^2$$

An engine developing 30 KW at 1250 rpm is filted with a cone clutch. The cone face angle of 12.5°. The mean diameter is 400 rpm μ = 0.3 and the normal pressure is not to exceed 0.08 N / mm^2. Design the clutch

Date: P = 30KW, N = 1250 rpm, $\propto$ = 12.5°, Dm = 400mm, μ = 0.3, P = 0.08 N/ mm^2

Solution

i) Torque transmitted

$$\text{T} = \frac{p\times 60\ 10^6}{2\pi\ N} = \frac{30\times60\times10^6}{2\pi\ 1250}$$

$$\text{T} = 229200 \ \ N - mm$$

ii) Axial force Fa

$$\text{T} = \frac{1}{2} \ \frac{\mu \ Fa \ D_m}{Sin \ \alpha} \quad 229200, \quad \frac{1}{2} \times \frac{0.3 \ Fa \times 400}{Sin \ 12.5}$$

Fa= 826.8 N

Dimensions

$$\text{Fa} = \pi \ D_m \ Pb \ Sin \ \alpha$$

$$826.8 = \pi . \times 400 \times 0.08 \times b \times Sin \ 12.5$$
$$b = 38mm$$

Inner diameter $D_m = D_m - b \ Sin\alpha = 400 - 38 \ Sin \ 12.5 = 392mm$

Outer diameter $D_m = D_m + b \ Sin\alpha = 400 + 38 \ Sin \ 12.5 = 408mm$

BRAKES

A brake is defined as a machine element used to control the motion by absorbing kinetic energy of a moving body or by absorbing potential energy of the objects being lowered by hoists, elevators, etc. The obsorbed energy appears as heat energy which should be transferred to cooling fluid such as water or surrounding air. The difference between a clutch and a brake is that whereas in the former both the members to be engaged are in motion, the brake connects a moving member to a stationary member.

Block or shoe brake

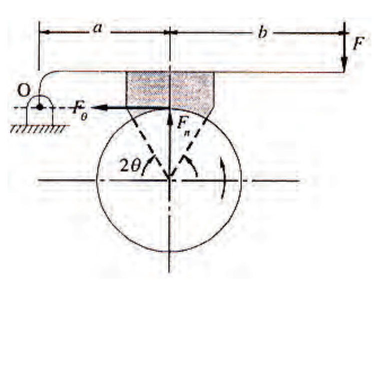
A single-block brake is shown in fig. It consists of a short shoe which may be rigidly mounted or pivoted to a lever. The block is pressed against the rotating wheel by an effort Fat one end of the lever. The other end of the lever is pivoted on a fixed fulcrum O. The friclional force produced by the block on the wheel will retard the rotation of the wheel. This type of brake is commonly used in railway trains. When the brake is applied, the lever with the block can be considered as a free body in equilibrium under the action of the following forces.

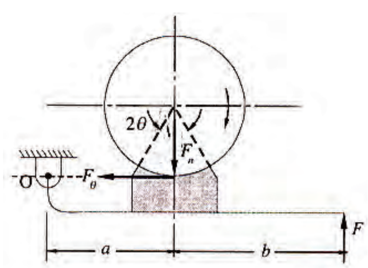
1. Applied force F at the end of the lever.
2. Normal reaction F_n between the shoe and the wheel.
3. Frictional or tangential braking force F_θ between the shoe and the wheel.
4. Pin reaction.

Let F = Operating force
M_1 = Torque on the wheel
r = Radius of the wheel
2θ = Angle of contact surface of the block
μ = Coefficient of friction
F_θ = Tangential braking force = $\frac{M_t}{r}$
F_n = Normal force = $\frac{F_\theta}{\mu}$
a = Distance between the fulcrum pin and the center of the shoe
b = Distance between the center of the shoe to the end of lever where the effort is applied
c = Distance between the fulcrum pin and the line of action of F_g

Consider the following three cases;

(i) Line of action of tangential force F_0 passes through fulcrum





Taking moments about O,

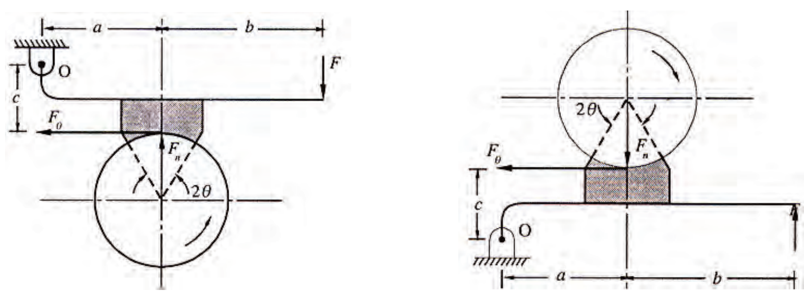
$$F\ (a+b) = F_n\, a = \frac{F_\theta}{\mu}\, a \qquad \left[\because F_n \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\ a}{\mu(a+b)}$$

In this case the actuating force is the same whether the direction of tangential force is towards or away from the fulcrum.

(ii) Line of action of tangential force F_θ is in between the center of the drum and the fulcrum

(a) Direction of F_θ is towards the fulcurm :



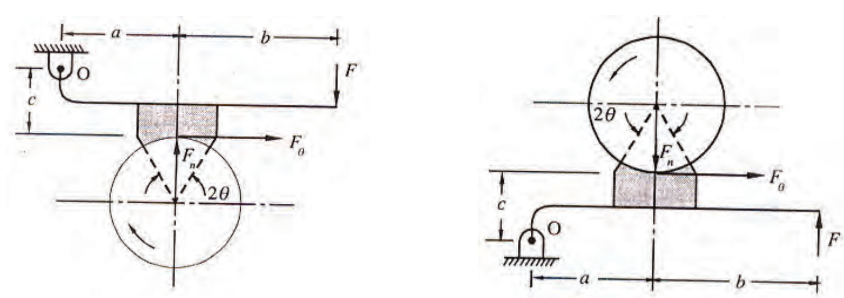
Taking moments about O,

$$F\ (a+b) = F_n\, c = F_n\, a$$

$$F\ (a+b) = F_n\, a - F_\theta\ c = \frac{F_\theta\, a}{\mu} - F_\theta\, c \qquad \left[\because F_n \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\ a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

(b) Direction of F_θ is away from the fulcrum :



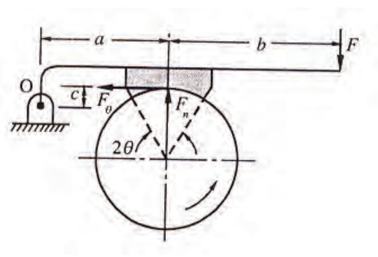
Taking moments about O,

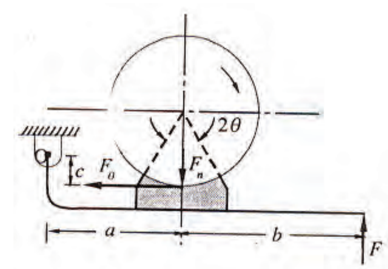
$$F\ (a+b) = F_n\, a - F_\theta\ c = \frac{F_\theta}{\mu} a\ + F_\theta\, c \qquad \left[\because F_n \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\ a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

(iii) Line of action of tangential force F_θ is above the center of the drum and the fulcrum:

(a) Direction of F_θ is towards the fulcrum :



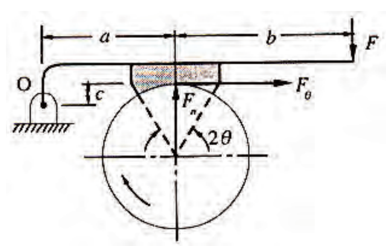


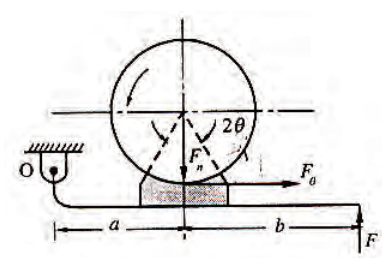
Taking moments about O,

$$F\ (a+b) = F_n\, a - F_\theta\ c = \frac{F_\theta\, a}{\mu} + F_\theta\, c \qquad \left[\because F_n \frac{F_\theta}{\mu}\right]$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\ a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

(b) Direction of F_θ is away from the fulcrum :





Taking moments about O,

$$F\ (a+b)\ + F_\theta\ c = F_\theta\, a = \frac{F_\theta\, a}{\mu}\ a \qquad \left[\because F_n \frac{F_\theta}{\mu}\right]$$

$$F\ (a+b) = \frac{F_\theta\, a}{\mu}\ - F_\theta\, c$$

$$\therefore \text{Actuating force } F = \frac{F_\theta\ a}{(a+b)}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

Note: If the direction of F_0 is towards the fulcrum, use the clockwise rotation formula and if the direction of $F_\&$ is away from the fulcrum, use counter clockwise formula from the data handbook.

When the angle of contact between the block and the wheel is less than 60°, we assume that the normal pressure is uniform between them. But when the angle of contact 2θ is more than 60°, we assume that the unit pressure normal to the surface of contact is less at the ends than at the center and the wear in the direction of applied force is uniform. In such case we employ the equivalent coefficient of friction μ', which is given by.

Equivalent coefficient of friction $\mu^1 = \mu \times \dfrac{4\sin\theta}{2\theta + \sin 2\theta}$

Where μ = Actual coefficient of friction

θ = Semi block angle

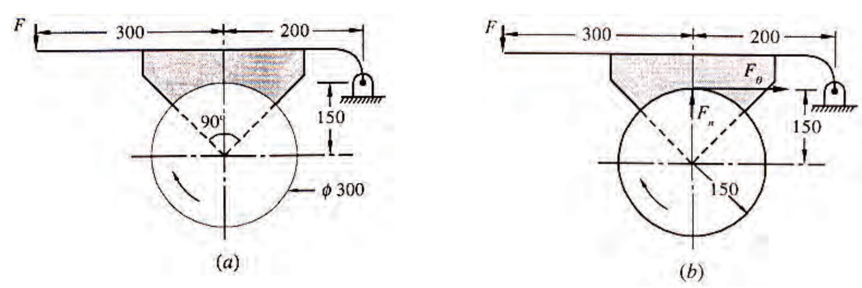
For the given value of θ. The value of $\dfrac{4\sin\theta}{2\theta + \sin 2\theta}$ can be found by using the chart given in

The brake is self -energizing when the friction force helps to apply the brake. If this effect is great enough to apply the brake with zero external force, the brake is called self-locking i.e., the brake is self locking when the applied force *F* is zero or negative.

Normal pressure on the shoe p $P = \dfrac{Normal\ load}{\Pr ojected\ area\ of\ shoe} = \dfrac{F_n}{2wr\ \sin\theta}$

Where *w* = Width of the shoe

Example: The block type hand brake shown in fig. 3.1 la has a face width of 45 mm. The friction material permits a maximum pressure of 0.6 MPa and a coefficient of friction of 0.24. Determine; 1. Effort F, 2. Maximum torque, 3. Heat generated if the speed of the drum is 100 rpm and the brake is applied for 5 sec. at full capacity to bring the shaft to stop.



Data: w = 45 mm, p = 0.6 MPa = 0.6 N/mm^2, μ = 0.24, 2θ = 90°, θ = 45^e, d = 300 mm, r = 150 mm, n = 100 rpm

Solution:

Since 2θ > 60°, equivalent coefficient of friction $\mu^1 = \mu \times \dfrac{4\sin\theta}{2\theta + \sin 2\theta}$

$$= 0.24 \times \frac{4\sin 45}{\frac{90 \times \pi}{180} + \sin 90} = 0.264$$

Allowable pressure $p = \dfrac{F_n}{2wr\sin\theta}$

$$\text{i.e., } 0.6 = \frac{F_n}{2 \times 45 \times 150 \sin 45}$$

$\therefore$ Normal force $F_n = 5727.56$ N

Tangential force $F_\theta = \mu' F_n = 0.264 \times 5727.56 = 1512.1$ N
The various forces acting on the shoe are shown in fig. 3.11b.
From the figure, a = 200 mm, b = 300 mm, c = 0
The tangential force F_θ, passes through the fulcurm.

$\therefore$ Effort $F = \dfrac{F_\theta\, a}{\mu'\,(a + b)}$

$$= \frac{1512.1 \times 200}{0.264\,(200 + 300)} = 2291.1\ N$$

Torque on the drum $M_1 = F_\theta\, r = 1512.1 \times 150 = 226815$ N-mm = 226.815 N-m

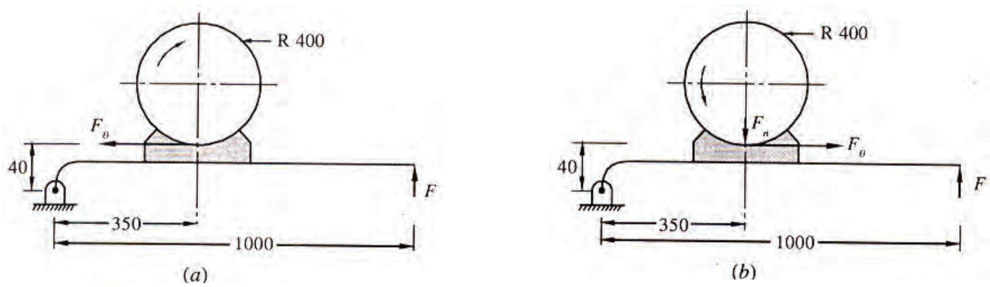
Power absorbed $N = \dfrac{M_1\, n}{9550} = \dfrac{226.815 \times 100}{9550} = 2.375\,kW = 2.375\ kJ/\sec$

Heat generated during 5 sec = 5 x 2.375 = 11.875 kJ

Example:

A 400 mm radius brake drum contacts a single shoe as shown in fjg.3.12a, and sustains 200 N-m torque at 500 rpm. For a coefficient of friction 0.25, determine:

1. Normal force on the shoe.
2. Required force F to apply the brake for clockwise rotation.
3. Required force F to apply the brake for counter clockwise rotation.
4. The dimension c required to make the brake self-locking, assuming the other dimensions remains the same.
5. Heat generated.



Data; r = 400 mm, M_1- 200 N-m = 200 x 10^3 N-mm, n = 500 rpm, μ=0.25,
a = 350 mm, i = 1000 mm, b = l - a = 1000 - 350 = 650 mm, c = 40 mm

Solution:

Tangential friction force $F_\theta = \frac{M_1}{r} = \frac{200\times 10^3}{400} = 500\ N$

Normal force on the $F_\theta = \frac{F_\theta}{\mu} = \frac{500}{0.25} = 2000\ N$

From the figure, the tangential force F_θ lies between the fulcrum and the center of drum.. When the force F_θ acts towards the fulcrum (clockwise rotation),

Actuating force $$F_\theta = \frac{F_0\, a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right]$$

$$= \frac{500\times 350}{350+650} = \left[\frac{1}{0.25} - \frac{40}{350}\right] = 680N$$

For anticlockwise rotation of the drum, the tangential force F_s acts away from the fulcrum as shown in figure.

∴ Actuating force $$F_\theta = \frac{F_0\, a}{a+b}\left[\frac{1}{\mu} + \frac{c}{a}\right]$$

$$= \frac{500\times 350}{350+650} = \left[\frac{1}{0.25} + \frac{40}{350}\right] = 720N$$

When the drum rotates in clockwise direction, self locking will occur. For self locking effort $F \leq 0$.

$$i.e., = \frac{F_0\, a}{a+b}\left[\frac{1}{\mu} - \frac{c}{a}\right] \le 0 \qquad \text{or} \qquad \frac{c}{a} \ge \frac{1}{\mu}, \quad c \ge \frac{a}{\mu}$$

$$\therefore \qquad c \ge \frac{350}{0.25} \ge 1200\,mm$$

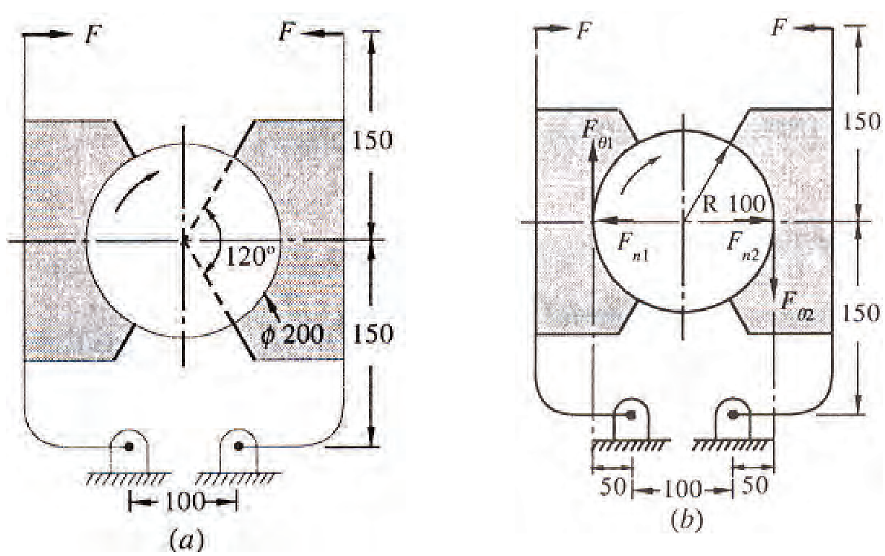
Heat generated $\qquad H_g = \mu \; p \; A_c \; v = F_n \; v$

$$= 0.25 \times 2000 \times \frac{\pi \times 800 \times 500}{60 \times 1000} = 1071.98\ W$$

$$= 10.472\ kW = 10.472\, kJ\ /\ s$$

Example: The layout of a brake to be rated at 250 N-m at 600 rpm is shown in figure. The drum diameter is 200 mm and the angle of contact of each shoe is 120°. The coefficient of friction may be assumed as 0.3 Determine.

1. Spring force F required to set the brake.
2. Width of the shoe if the value of pv is 2 N-m/mm^2-sec



Data: $\quad M_t$ = 250 N-m = 250 x 10^3 N-mm, $\quad$ n = 600 rpm, $\quad$ d = 200 mm, $\quad$ r = 100 mm,
2θ =120°, $\quad$ 0=60°, $\quad$ μ = 0.3, $\quad$ pv = 2 N-m/mm^2-sec

Solution:

Since 26 > 60°, equivalent coefficient of friction $\mu' = \mu \times \frac{4Sin\theta}{2\theta + Sin2\theta}$

$$= 0.3 \times \frac{Sin\ 60}{\frac{120 + \pi}{180} + \sin 120} = 0.351$$

The various forces acting on the shoes are shown in figure.

Left hand shoe:

For left hand shoe, a = 150 mm, b = 150 mm, c = 100 $- \frac{100}{2} = 50\ mm$

The tangential force $F_{\theta 1}$ lies above the center of the drum and fulcrum, and acting away from the fulcrum (counter clockwise).

$\therefore$ Spring force $F = \frac{F_{01}\ a}{a + b}\left[\frac{1}{\mu} - \frac{c}{a}\right]$

$$F = \frac{F_{01} \times 150}{150 + 150}\left[\frac{1}{0.351} - \frac{50}{150}\right]$$

$\therefore$Tangential force F_{gi}= 0.795 F

Right hand shoe:

The tangential force $F_{\theta 2}$ lies below the center of the drum and the fulcrum, and acting towards the fulcrum (clockwise).

$\therefore$ Spring force $F = \frac{F_{02}\ a}{a + b}\left[\frac{1}{\mu} - \frac{c}{a}\right]$

$$F = \frac{F_{02} \times 150}{150 + 150}\left[\frac{1}{0.351} - \frac{50}{150}\right]$$

$\therefore$Tangential force $F_{02} = 0.6285\ F$

Torque $M_1 = (F_{01} + F_{01})\ r$

i.e., $250 \times 10^3 = (0.795\ F + 0.6285\ F) \times 100$

$\therefore$Spring force F = 1756.2 N

The maximum load occurs on left hand shoe.

$\therefore$ Tangential force $F_{01} = 0.795 \times 1756.2 = 1396.2$ N

Normal force on left hand shoe $\quad F_{01} = \dfrac{F_{01}}{\mu} = \dfrac{1396.2}{0.351} = 3977.8 \ N$

Surface velocity of drum $v = \dfrac{\pi \ dn}{60 \times 1000} = \dfrac{\pi \times 200 \times 600}{60 \times 1000} = 6.283 \ m/\sec$

By data $\quad$ pv = 2 N-m/mm^2 – sec

$\therefore$ Normal pressure $p = \dfrac{2}{v} = \dfrac{2}{6.283} = 0.3183 \ N/mm^2$

Also pressure $p = \dfrac{F_{n1}}{2wr \ \sin \ \theta}$

1.e., $0.3183 = \dfrac{3977.8}{2wr \times 100 \sin \ 60}$

$\therefore$ Width of the shoe w = 72.15mm

Band brakes

A band brake consists of a band, generally made of metal, and embracing a part of the circumference of the drum. The braking action is obtained by tightenting the band. The difference in the tensions at each end of the band determines the torque capacity.

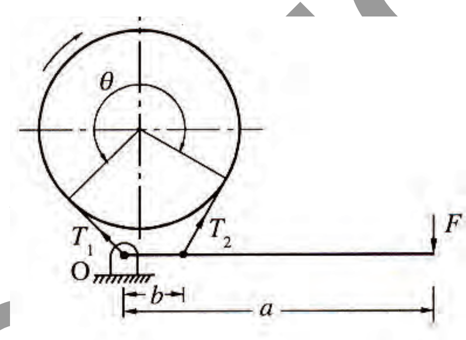
Simple band brakes: When one end of the band is connected to the fixed fulcrum, then the band brake is called simple band brake as shown in figure

Let T, = Tight side tension in N
T_2 = Slack side tension in N
θ = Angle of lap in radians
μ = Coefficient of friction
D = Diameter of brake drum in mm
M_1 = Torque on the drum in N-mm

Ratio of tensions $\dfrac{T_1}{T_2} = e^{\mu\theta}$

Braking force $F_\theta = T_1 - T_2 = \dfrac{2M_t}{D}$

Clockwise rotation of the drum:



Taking moments about the fulcrum O,

$$F \times a = T_2 \times b$$

$\therefore$ Force at the end of lever $F = \dfrac{T_2\, b}{a}$

Braking force $F_\theta = T_1 - T_2 = T_2\, e^{\mu\theta} - T_2 = T_2\, (e^{\mu\theta} - 1)$ $\left[\because \dfrac{T_1}{T_2} = e^{\mu\theta}\right]$

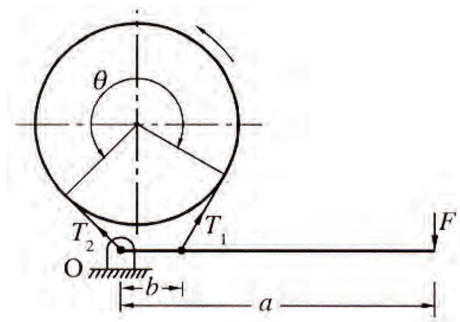
:. Slack side tension $T_2 = \dfrac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

$$F = \frac{T_2\ b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$$

Counter clockwise rotation of the drum:

For counter clockwise rotation of the drum, the tensions T_1 and T_2 will exchange their places and for the same braking torque, a larger force F is required to operate the brake.



Taking moments about the fulcrum O,

$$F \times a = T_1 \times b$$

$\therefore$ Force at the end of lever $F = \dfrac{T_1\ b}{a}$

Braking force $F_\theta = T_1 - T_2 = T_1 - \dfrac{T_1}{e^{\mu\theta}} = T_1\ (1 - \dfrac{1}{e^{\mu\theta}})$ $\left[\because \dfrac{T_1}{T_2} = e^{\mu\theta}\right]$

Tight side tension $T_2 = \dfrac{F_\theta\ e^{\mu\theta}}{e^{\mu\theta} - 1}$

Substitute the value of T_1, in equation (i), we get

$$F = \frac{F_\theta\ b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$$

This type of brake does not have any self-energizing and self-locking properties.

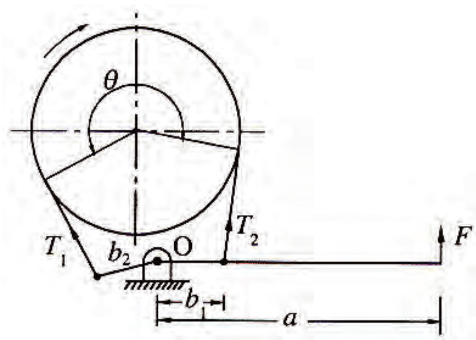
Thickness of the band h = 0.005 D

Width of band $w = \dfrac{T_1}{h\ \sigma_d}$

where σ_d is the allowable tensile stress in the band.

Differential band brakes:

In differential band brake the two ends of the band are attached to pins on the lever at a distance of b_l and b_2 from the pivot pin as shown in figure. It is to be noted that when $b_2 > b_1$, the force F must act upwards in order to apply the brake. When $b_2 < b_1$, the force F must act downwards to apply the brake.



Clockwise rotation of the drum:

Taking moments about the fulcrum O,

$$T_1 \times b_2 = T_2 \times b_1 + F \times a$$

$$T_2\ e^{\mu\theta}\ b_2 - T_2\ b_1 = Fa \qquad (\because T_1 = T_2 e^{\mu\theta})$$

Braking force $F_\theta = T_1 - T_2\ \ T_2\ e^{\mu\theta} - T_2 = T_2\,(e^{\mu\theta} - 1)$ $\qquad (\because T_1 = T_2 e^{\mu\theta})$

Slack side tension $T_2 = \dfrac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

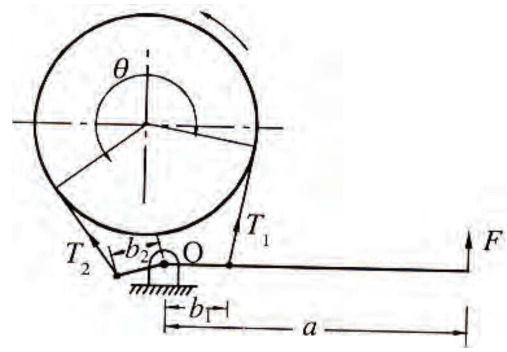
$$F = \frac{F_\theta}{a}\left[\frac{b_2\ e^{\mu\theta} - b_1}{e^{\mu\theta} - 1}\right]$$

For the brake to be self locking, the force F at the end of the lever must be equal to zero or negative.

$$\therefore = \frac{F_\theta}{a}\left[\frac{b_2\ e^{\mu\theta} - b_1}{e^{\mu\theta} - 1}\right] \le 0$$

The condition for self-locking is $b_2\ e^{\mu\theta} \le b_1$

Counter clockwise rotation of the drum:



Taking moments about the fulcrum O,

$$T_2 \times b_2 = T_1 \times b_1 + F \times a$$

$$T_2 \ b_2 \ - T_2 \ e^{\mu\theta} = Fa \qquad (\because T_1 = T_2 e^{\mu\theta})$$

Or $$F = \frac{T_2 \ (b_2 - b_1 \ e^{\mu\theta})}{a}$$

Braking force $F_\theta = T_1 - T_2 = T_2 \ e^{\mu\theta} - T_2 = T_2 \ (e^{\mu\theta} - 1) \qquad (\because T_1 = T_2 e^{\mu\theta})$

Slack side tension $T_2 = \dfrac{F_\theta}{e^{\mu\theta} - 1}$

Substitute the value of T_2 in equation (1), we get

$$F = \frac{F_\theta}{a}\left[\frac{b_2 - b_1 \ e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$$

For self-brake locking, the force F must be zero or negative.

i.e., $$\frac{F_\theta}{a}\left[\frac{b_2 - b_1 \, e^{\mu\theta}}{e^{\mu\theta} - 1}\right] \leq 0$$

The condition for self-locking is

$$b_2 \leq b_1 \ e^{\mu\theta}$$

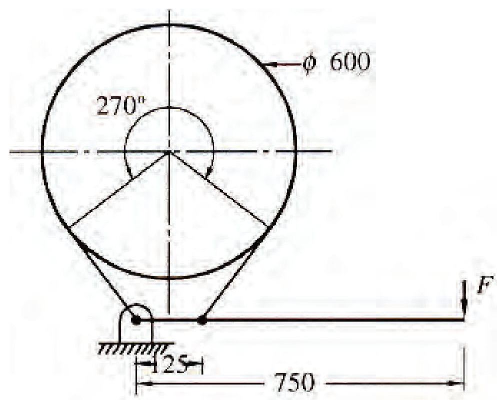
If $b_1 = b_2 = b$, the brake is called two way brake and is shown in figure. This type of brake can be used in either direction of rotation with same effort.

$\therefore$ Force at the end of lever $F = \dfrac{F_\theta \ b}{a}\left[\dfrac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1}\right]$

Example:
A simple band brake operates on a drum 0.6 m in diameter rotating at 200 rpm. The coefficient of friction is 0.25 and the angle of contact of the band is 270°. One end of the band is fastened to a fixed pin and the other end to 125 mm from the fixed pin. The brake arm is 750 mm long.

(i) What is the minimum pull necessary at the end of the brake arm to .stop the wheel if 35 kW is being absorbed? What is the direction of rotation for minimum pull?

(ii) Find the width of 2.4 mm thick steel band if the maximum tensile stress is not to exceed 55 N/mm^2.



Data: D = 0.6 m = 600 mm, n = 200 rpm, p = 0.25, 0= 270°, a = 750 mm, b = 125 mm, N = 35kW, h = 2.4 mm, $\sigma_d = 55\ N/mm^2$

Solution:

Frictional torque $M_1 = \dfrac{9550\,N}{n}$

$$= \frac{9550 \times 35}{200} = 1671.25\,N - m = 1671.25 \times 10^3\ \ N - mm$$

Braking for $F_\theta = \dfrac{M_1}{R} = \dfrac{2M_1}{D} = \dfrac{2 \times 1671.25 \times 10^3}{600} = 5570.83\,N$

Ratio of tensions $\dfrac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 270 \times \pi 180} = 3.248$

Clockwise rotation:

Force $F = \dfrac{F_\theta\ b}{a}\left[\dfrac{1}{e^{\mu\theta} - 1}\right]$

$$= \frac{5570.83 \times 125}{750}\left[\frac{1}{3.248 - 1}\right] = 413.02\ N$$

Counter clockwise rotation:

$$\text{Force} \quad F = \frac{F_\theta\ b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$$

$$= \frac{5570.83 \times 125}{750}\left[\frac{3248}{3.248 - 1}\right] = 1341.5\ N$$

Therefore the minimum pull $F = 413.02$ N

The direction of rotation is clockwise.

Braking force $F_\theta = T_1 - T_2 = T_1 - \frac{T_1}{e^{\mu\theta}} = T_1\,(1 - \frac{1}{e^{\mu\theta}})$

i.e., $5570.83 = T_1\left[1 - \frac{1}{3.248}\right]$

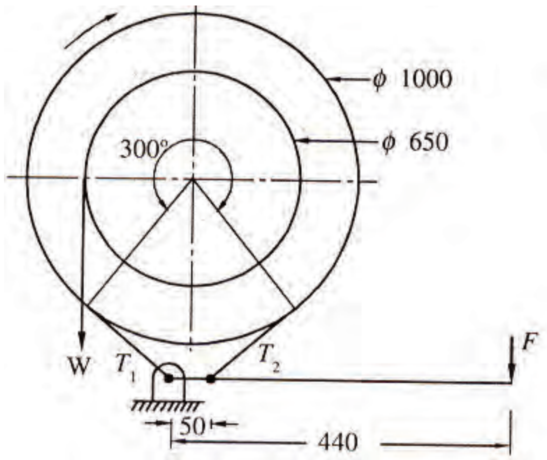
$\therefore$ Tight side tension $T_1 = 8048.96\ N$

Width of band $w = \frac{T_1}{h\ \sigma_d}$

$$= \frac{8048.96}{2.4 \times 55} = 60.98\ mm = 62\ mm$$

Problem:
In a simple band brake, the length of lever is 440 mm. The tight end of the band is attached to the fulcrum of the lever and the slack end to a pin 50 mm from the fulcrum. The diameter of the brake drum is 1 m and the arc of contact is 300°. The coefficient of friction between the band and the drum is 0.35. The brake drum is attached to a hoisting drum of diameter 0.65 m that sustains a load of 20 kN. Determine;

1. Force required at the end of lever to just support the load.
2. Required force when the direction of rotation is reversed.
3. Width of stee! band if the tensile stress is limited to 50 N/mnr.



Data: a = 440 mm, b = 50 mm, D= 1 m = 1000 mm, D_b = 0.65 m = 650 mm,
$\theta = 300°$, $\mu = 0.35$, W = 20kN = 20x 10^3 N, $\sigma_d = 50 N/mm^2$

Solution:

Torque on hoisting drum $M_1 = W\ R_b = \dfrac{WD_b}{2}$

$$= \frac{20 \times 10^3 \times 650}{2} = 6.5 \times 10^6\ N - mm$$

The hoisting drum and the brake drum are mounted on same shaft.

$\therefore$ Torque on brake drum M1 = 6.5 x 10^6 *N-mm*

Braking for $F_\theta = \dfrac{M_1}{R} = \dfrac{2M_1}{D}$

$$= \frac{2 \times 6.5 \times 10^6}{1000} = 13000\,N$$

Ratio of tensions $\dfrac{T_1}{T_2} = e^{\mu\theta} = e^{0.35 \times 300 \times \pi 180} = 6.25$

Clockwise rotation:

Force at the end of lever $F = \dfrac{F_\theta\ b}{a}\left[\dfrac{1}{e^{\mu\theta} - 1}\right]$

$$= \frac{13000 \times 50}{440}\left[\frac{1}{6.25 - 1}\right] = 281.38\ N$$

Counter clockwise rotation:

$$\text{Force} \quad F = \frac{F_\theta\, b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right]$$

$$= \frac{13000 \times 50}{440}\left[\frac{6.25}{6.25-1}\right] = 1758.66\ N$$

Thickness of band h = 0.005 D

= 0.005 x 1000 = 5 mm

$$\text{Braking force} \quad F_\theta = T_1 - T_2 = T_1 - \frac{T_1}{e^{\mu\theta}} = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)$$

$$\text{i.e.,} \quad 13000 = T_1\left[1 - \frac{1}{6.25}\right]$$

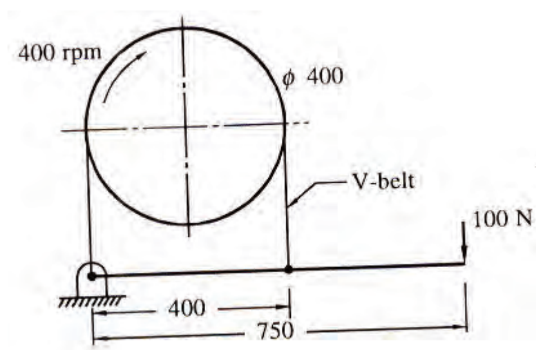
$\therefore$ Tight side tension T_1 = 15476.2 N

$$\text{Width of band } w = \frac{T_1}{h\,\sigma_d}$$

$$= \frac{15476.2}{5 \times 50} = 61.9\ mm = 62\ mm$$

Problem:

A band brake shown in figure uses a V-belt. The pitch diameter of the V-grooved pulley is 400 mm. The groove angle is 45° and the coefficient of friction is 0.3. Determine the power rating.



Data: D - 400 mm, 2a = 45°, α =22.5°, μ -0.3, F = 100N, b = 400 mm,
a = 750 mm, θ = 180° = π rad, n = 400 rpm

Solution:

Ratio of tensions for V-belt, $\frac{T_1}{T_2} = e^{\mu\theta / Sin\ \alpha}$

$$= e^{0.3 \times \pi / Sin\ 22.5} = 11.738$$

For clockwise rotation,

$$\text{Force} \quad F = \frac{F_\theta\ b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$$

$$\text{i.e.,} \qquad 100 = \frac{F_\theta \times 400}{750}\left[1 - \frac{1}{11.738 - 1}\right]$$

$\therefore$ Braking force $F_\theta = 2013.4\ N$

$$\text{Torque on the drum} \quad M_1 = F_\theta \quad R = \frac{F_\theta\ D}{2}$$

$$= \frac{2013.4 \times 400}{2} = 402680\ N-mm = 402.68\ N-m$$

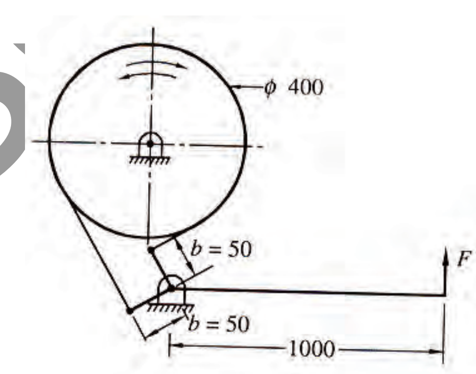
$$\text{Power rating} \qquad N = \frac{M_1\ n}{9550}$$

$$= \frac{402.68 \times 400}{9550} = 16.87\ kW$$

Problem

Figure shows a two way band brake. It is so designed that it can operate equally well in both clockwise and counter clockwise rotation of the brake drum. The diameter of the drum is 400 mm and the coefficient of friction between the band and the drum is 0.3. The angle of contact of band brake is 270° and the torque absorbed in the band brake is 400 N-m. Calculate;

1. Force F required at the end of the lever.
2. Width of the band if the allowable stress in the band is 70 MPa.



Data: D = 400 mm, μ= 0.3, θ = 270°, M_t = 400 N-m = 400 x 10^3 N-mm,
σ_d = 70 MPa = 70 N/mm2, b = b_l = b_2 = 50mm, a = 1000mm

Solution:

Braking force $F_\theta = T_1 - T_2 = \frac{2M_t}{D} = \frac{2 \times 400 \times 10^3}{400} = 2000\ N$

Ratio of tensions $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 270 \times \pi 180} = 4.1112$

Force at the end of lever $F = \frac{F_\theta\ b}{a}\left[\frac{e^{\mu\theta}+1}{e^{\mu\theta}-1}\right]$

$$= \frac{2000 \times 50}{1000}\left[\frac{4.1112+1}{4.112-1}\right] = 164.28\ N$$

Band thickness h = 0.005 D

= 0.005 x 400 = 2 mm

Braking force $F_\theta = T_1 - T_2 = T_1(1 - \frac{1}{e^{\mu\theta}})$ $\left[\because \frac{T}{T_{2\ 1}} = e^{\mu\theta}\right]$

i.e., $T_1\left[1 - \frac{1}{4.1112}\right] = 2000$

$\therefore$ Tight side tension T_1 = 2642.84 N

Width of band $w = \frac{T_1}{h\ \sigma_d}$

$$= \frac{2642.84}{2 \times 70} = 18.88\ mm = 20\ mm$$

DESIGN OF MACHINE ELEMENTS - II

CHAPTER VII

LUBRICATION AND BEARINGS

7.1 Why to study Friction, Wear & Lubrication?

Moving parts of every machine is subjected to friction and wear. Friction consumes and wastes energy. Wear causes changes in dimensions and eventual breakdown of the machine element and the entire machine. The loss of just a few milligrams of material in the right place, due to wear can cause a production machine or an automobile to be ready for replacement. If we imagine the amount of material rendered useless by way of wear, it is startling! Lots of materials ranging from Antimony to zinc, including titanium, vanadium, iron, carbon, copper, aluminum etc., would be lost. It is therefore essential to conserve the natural resources through reduction in wear. Lubrication plays a vital role in our great and complex civilization.

7.2 Bearings

A bearing is machine part, which support a moving element and confines its motion. The supporting member is usually designated as bearing and the supporting member may be journal. Since there is a relative motion between the bearing and the moving element, a certain amount of power must be absorbed in overcoming friction, and if the surface actually touches, there will be a rapid wear.

7.2.1 Classification:

Bearings are classified as follows:

Depending upon the nature of contact between the working surfaces:-

Sliding contact bearings and
Rolling contact bearings.

SLIDING BEARINGS:

Hydrodynamically lubricated bearings
Bearings with boundary lubrication
Bearings with Extreme boundary lubrication.
Bearings with Hydrostatic lubrication.

Rolling element bearings:

Ball bearings
Roller bearings
Needle roller bearings

Based on the nature of the load supported:

- Radial bearings - Journal bearings
- Thrust bearings
  - Plane thrust bearings
  - Thrust bearings with fixed shoes
  - Thrust bearings with Pivoted shoes
- Bearings for combined Axial and Radial loads.

Journal bearing:

It is one, which forms the sleeve around the shaft and supports a bearing at right angles to the axis of the bearing. The portion of the shaft resting on the sleeve is called the journal. Example of journal bearings are-

- Solid bearing
- Bushed bearing, and
- Pedestal bearing.

Solid bearing:

A cylindrical hole formed in a cast iron machine member to receive the shaft which makes a running fit is the simplest type of solid journal bearing. Its rectangular base plate has two holes drilled in it for bolting down the bearing in its position as shown in the figure7.1. An oil hole is provided at the top to lubricate the bearing. There is no means of adjustment for wear and the shaft must be introduced into the bearing endwise. It is therefore used for shafts, which carry light loads and rotate at moderate speeds.

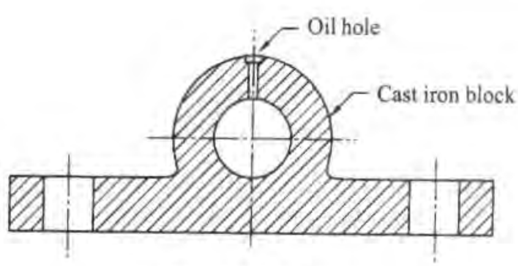


Fig. 7.1 Solid bearing

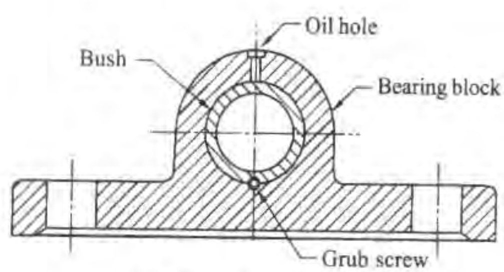


Fig. 7.2 Bushed bearing

Bushed bearing:

It consists of mainly two parts, the cast iron block and bush; the bush is made of soft material such as brasss, bronze or gunmetal. The bush is pressed inside the bore in the cast iron block and is prevented from rotating or sliding by means of grub- screw as shown if the figure 7.2. When the bush gets worn out it can be easily replaced. Elongated holes in the base are provided for lateral adjustment.

Pedestal bearing:

It is also called Plummer block. Figure 7.3 shows half sectional front view of the Plummer block. It consists of cast iron pedestal, phosphor bronze bushes or steps made in two halves and cast iron cap. A cap by means of two square headed bolts holds the halves of the steps together. The steps are provided with collars on either side in order to prevent its axial movement. The snug in the bottom step, which fits into the corresponding hole in the body, prevents the rotation of the steps along with the shaft. This type of bearing can be placed any where along the shaft length.

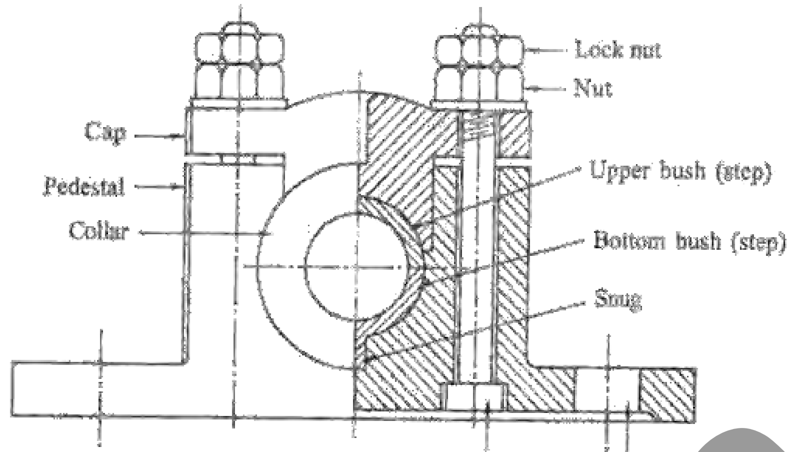


Fig. 7.3 Pedestal bearing

Thrust bearing:

It is used to guide or support the shaft, which is subjected to a load along the axis of the shaft. Since a thrust bearing operates without a clearance between the conjugate parts, an adequate supply of oil to the rubbing surfaces is extremely important. Bearings designed to carry heavy thrust loads may be broadly classified in to two groups-

· **Foot step bearing, and**
· **Collar bearing**

Footstep bearing:
Footstep bearings are used to support the lower end of the vertical shafts. A simple form of such bearing is shown in fig 7.4. It consists of cast iron block into which a gunmetal bush is fitted. The bush is prevented from rotating by the snug provided at its neck. The shaft rests on a concave hardened steel disc. This disc is prevented from rotating along with the shaft by means of pin provided at the bottom.

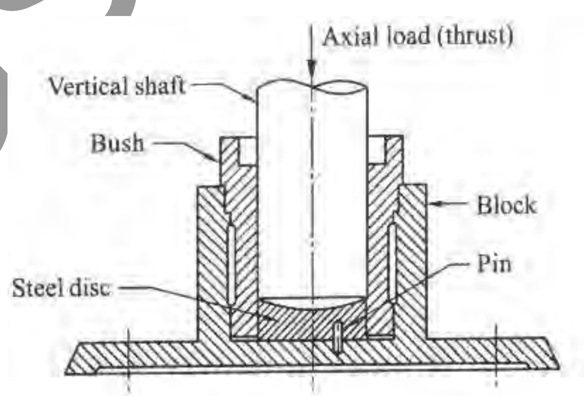


Fig. 7.4 Foot step bearing

Collar bearing:

The simple type of thrust bearing for horizontal shafts consists of one or more collars cut integral with the shaft as shown in fig.7.5. These collars engage with corresponding bearing surfaces in the thrust block. This type of bearing is used if the load would be too great for a step bearing, or if a thrust must be taken at some distance from the end of the shaft. Such bearings may be oiled by reservoirs at the top of the bearings.

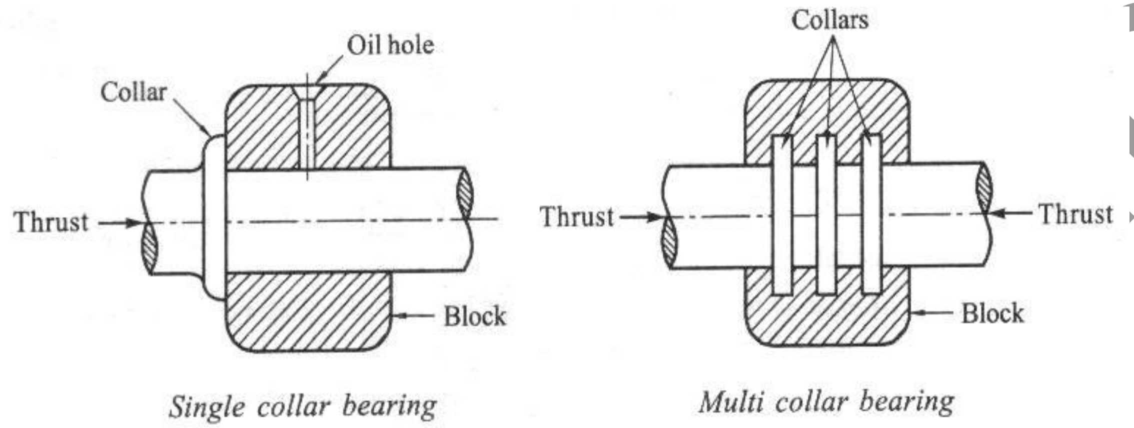


Fig.7.5 Collar bearings

Thrust bearings of fixed inclination pad and pivoted pad variety are shown in figure 7.6 a & b. These are used for carrying axial loads as shown in the diagram. These bearings operate on hydrodynamic principle.

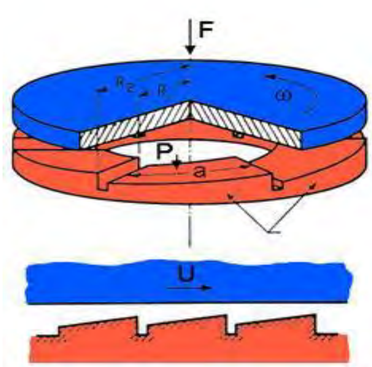


Fig.7.6a Fixed-incline-pads thrust bearing

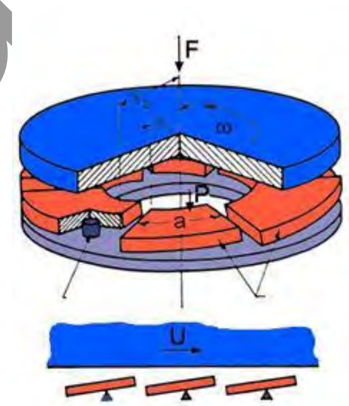


Fig.7.6b Pivoted-pads thrust bearing

Rolling contact bearings:

The bearings in which the rolling elements are included are referred to as rolling contact bearings. Since the rolling friction is very less compared to the sliding friction, such bearings are known as anti friction bearings.

Ball bearings:

It consists of an inner ring which is mounted on the shaft and an outer ring which is carried by the housing. The inner ring is grooved on the outer surface called inner race and the outer ring is grooved on its inner surface called outer race. In between the inner and outer race there are number of steel balls. A cage pressed steel completes the assembly and provides the means of equally spacing and holding the balls in place as shown in the figure 7.7. Radial ball bearings are used to carry mainly radial loads, but they can also carry axial loads.

Cylindrical roller bearings

The simplest form of a cylindrical roller bearing is shown in fig 7.8. It consists of an inner race, an outer race, and set of roller with a retainer. Due to the line contact between the roller and the raceways, the roller bearing can carry heavy radial loads.

Tapered roller bearings:

In tapered roller bearings shown in the fig. 7.9, the rollers and the races are all truncated cones having a common apex on the shaft centre to assure true rolling contact. The tapered roller bearing can carry heavy radial and axial loads. Such bearings are mounted in pairs so that the two bearings are opposing each other's thrust.

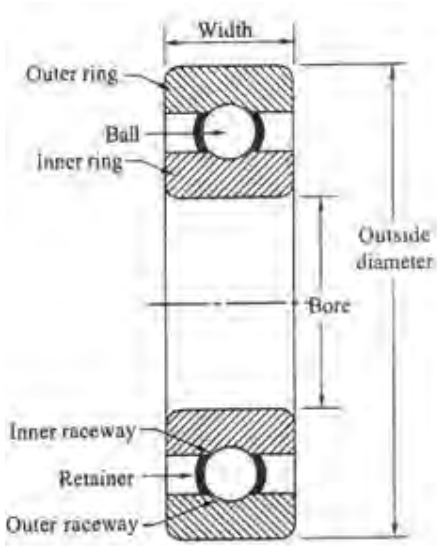


Fig. 7.7 Ball bearing

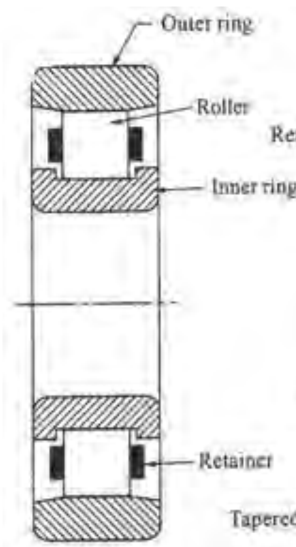


Fig 7.8 Roller bearing

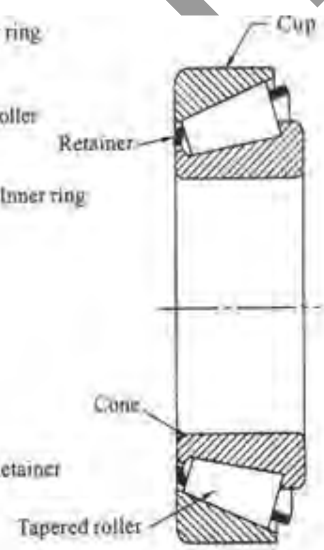


Fig. 7.9 Tapered Roller bearing

7.2.2 Advantages of sliding contact bearings:

They can be operated at high speeds.
They can carry heavy radial loads.
They have the ability to withstand shock and vibration loads.
Noiseless operation.

Disadvantages:

High friction losses during staring.
More length of the bearing.
Excessive consumption of the lubricant and high maintenance.

7.2.3 Advantages rolling contact bearings:

Low starting and low running friction.
It can carry both radial as well as thrust loads.
Momentary over loads can be carried without failure.
Shaft alignment is more accurate than in the sliding bearings.

Disadvantages:

More noisy at high speeds.
Low resistance to shock
loads. High initial cost.
Finite life due to eventual failure by fatigue

7.3 Solid Friction

- Resistance force for sliding
  - Static coefficient of friction
  - Kinetic coefficient of friction
- Causes
  - Surface roughness (asperities)
  - Adhesion (bonding between dissimilar materials)
- Factors influencing friction
  - Sliding friction depends on the normal force and frictional coefficient, independent of the sliding speed and contact area
- Effect of Friction
  - Frictional heat (burns out the bearings)
  - Wear (loss of material due to cutting action of opposing motion)
- Engineers control friction
  - Increase friction when needed (using rougher surfaces)
  - Reduce friction when not needed (lubrication)

The coefficients of friction for different material combinations under different conditions are given in table 7.1.

TABLE 7.1

COEFFICIENTS OF FRICTION

Material	μ
Perfectly clean metals in vacuum	Seizure $\mu > 5$
Clean metals in air	0.8-2
Clean metals in wet air	0.5-1.5
Steel on dry bearing metals (e.g. lead, bronze)	0.1-0.5
Steel on ceramics	0.1-0.5
Ceramics on ceramics (e.g. carbides on carbides)	0.05-0.5
Polymers on polymers	0.05-1.0
Metals and ceramics on polymers (PE, PTFE, PVC)	0.04-0.5
Boundary lubrication of metals	0.05-0.2
High-temperature lubricants (MoS_2, graphite)	0.05-0.2
Hydrodynamic lubrication	0.001-0.005

7.4 Lubrication:

Prevention of metal to metal contact by means of an intervening layer of fluid or fluid like material.

Types of sliding lubrication:

- Sliding with Fluid film lubrication.
- Sliding with Boundary lubrication.
- Sliding with Extreme boundary lubrication.
- Sliding with clean surfaces.

7.4.1 <u>Hydrodynamic / thick film lubrication / fluid film lubrication</u>

- Metal to Metal contact is prevented. This is shown in figure 7.10.
- Friction in the bearing is due to oil film friction only.
- Viscosity of the lubricant plays a vital role in the power loss, temperature rise & flow through of the lubricant through the bearing.
- The principle operation is the Hydrodynamic theory.
- This lubrication can exist under moderately loaded bearings running at sufficiently high speeds.

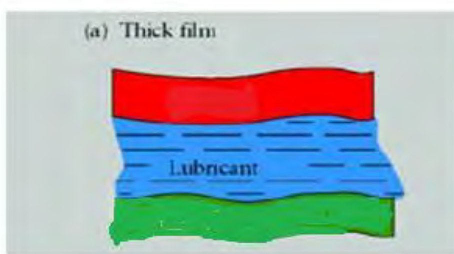


Fig.7.10 Thick Film Lubrication

<u>7.4.2 Boudary lubrication (thin film lubrication)</u>

During starting and stopping, when the velocity is too low, the oil film is not capable of supporting the load. There will be metal to metal contact at some spots as shown in figure 7.11. Boundary lubrication exists also in a bearing if the load becomes too high or if the viscosity of the lubricant is too low. Mechanical and chemical properties of the bearing surfaces and the lubricants play a vital role.

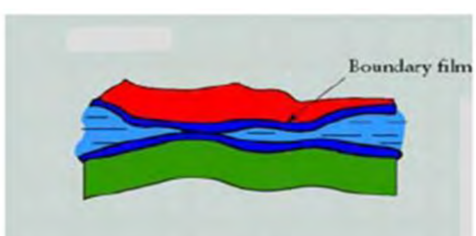


Fig.7.11 Boundary Lubrication

Oiliness of lubricant becomes an important property in boundary lubrication. Anti oxidants and Anti-corrosives are added to lubricants to improve their performance. Additives are added to improve the viscosity index of the lubricants.

Oiliness Agents

Increase the oil film's resistance to rupture, usually made from oils of animals or vegetables.

The molecules of these oiliness agents have strong affinity for petroleum oil and for metal surfaces that are not easily dislodged.

Oiliness and lubricity (another term for oiliness), not related to viscosity, manifest itself under boundary lubrication; reduce friction by preventing the oil film breakdown.

Anti-Wear Agents

Mild EP additives protect against wear under moderate loads for boundary lubrications Anti-wear agents react chemically with the metal to form a protective coating that reduces friction, also called as anti-scuff additives.

7.4.3 Extreme boundary lubrication

Under certain conditions of temperature and load, the boundary film breaks leading to direct metal to metal contact as shown in figure 7.12. Seizure of the metallic surfaces and destruction of one or both surfaces begins. Strong intermolecular forces at the point of contact results in tearing of metallic particles. "Plowing" of softer surfaces by surface irregularities of the harder surfaces. Bearing material properties become significant. Proper bearing materials should be selected.

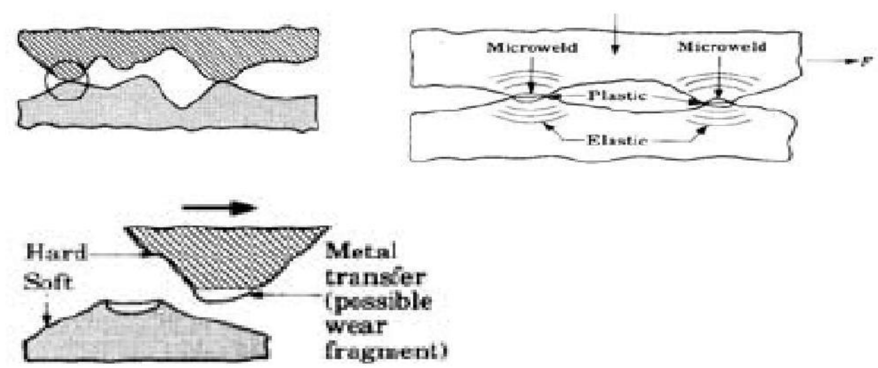


Fig.7.12 Extreme Boundary Lubrication

Extreme-Pressure Agents

Scoring and pitting of metal surfaces might occur as a result of this case, seizure is the primarily concern. Additives are derivatives of sulphur, phosphorous, or chlorine. These additives prevent the welding of mating surfaces under extreme loads and temperatures.

Stick-Slip Lubrication

A special case of boundary lubrication when a slow or reciprocating action exists. This action is destructive to the full fluid film. Additives are added to prevent this phenomenon causing more drag force when the part is in motion relative to static friction. This prevents jumping ahead phenomenon.

7.4.4 Solid film lubrication

When bearings must be operated at extreme temperatures, a solid film lubricant such as graphite or molybdenum di-sulphide must be used because the ordinary mineral oils are not satisfactory at elevated temperatures. Much research is currently being carried out in an effort to find composite bearing materials with low wear rates as well as small frictional coefficients.

7.4.5. Hydrostatic lubrication

Hydrostatic lubrication is obtained by introducing the lubricant, which is sometimes air or water, into the load-bearing area at a pressure high enough to separate the surfaces with a relatively thick film of lubricant. So, unlike hydrodynamic lubrication, this kind of lubrication does not require motion of one surface relative to another. Useful in designing bearings where the velocities are small or zero and where the frictional resistance is to be an absolute minimum.

7.4.6 Elasto Hydrodynamic lubrication

Elasto-hydrodynamic *lubrication* is the phenomenon that occurs when a lubricant is introduced between surfaces that are in rolling contact, such as mating gears or rolling bearings. The mathematical explanation requires the Hertzian theory of contact stress and fluid mechanics.

7.5 Newton's Law of Viscous Flow

In Fig. 7.13 let a plate A be moving with a velocity U on a film of lubricant of thickness h. Imagine the film to be composed of a series of horizontal layers and the force F causing these layers to deform or slide on one another just like a deck of cards. The layers in contact with the moving plate are assumed to have a velocity U; those in contact with the stationary surface are assumed to have a zero velocity. Intermediate layers have velocities that depend upon their distances y from the stationary surface.

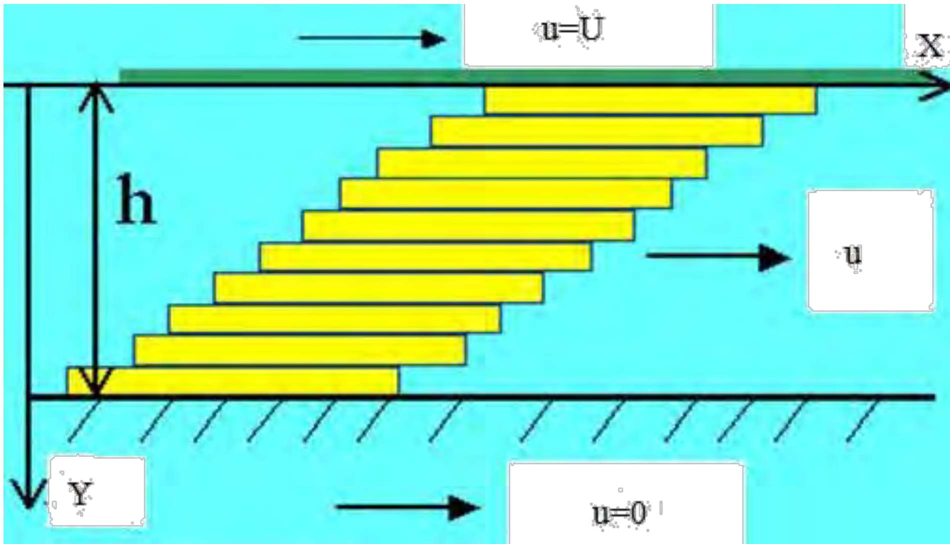


Fig.7.13 Viscous flow

where Z is the constant of proportionality and defines *absolute viscosity,* also called *dynamic viscosity*. The derivative du/dy is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient. The viscosity Z is thus a measure of the internal frictional resistance of the fluid.

For most lubricating fluids, the rate of shear is constant, and $du/dy = U/h$. Fluids exhibiting this characteristic are known as a Newtonian fluids.

Therefore $\tau = F/A = Z\,(U/h)$.

The absolute viscosity is measured by the pascal-second (Pa · s) in SI; this is the same as a Newton-second per square meter.

The poise is the cgs unit of dynamic or absolute viscosity, and its unit is the dyne second per square centimeter (dyn · s/cm2). It has been customary to use the centipoises (cP) in analysis, because its value is more convenient. The conversion from cgs units to SI units is as follows:

$$\mathbf{Z\,(Pa \cdot s) = (10)^{-3}\,\mathit{Z}\,(cP)}$$

Kinematic Viscosity is the ratio of the absolute Viscosity to the density of the lubricant.

$$\mathbf{Z_k = Z} / \rho$$

The ASTM standard method for determining viscosity uses an instrument called the Saybolt Universal Viscosimeter. The method consists of measuring the time in seconds for 60 mL of lubricant at a specified temperature to run through a tube 17.6 micron in diameter and 12.25 mm long. The result is called the *kinematic viscosity,* and in the past the unit of the square centimeter per second has been used. One square centimetre per second is defined as a **stoke.**

The kinematic viscosity based upon seconds Saybolt, also called *Saybolt Universal viscosity* (SUV) in seconds, is given by:

$$\mathbf{Zk = (0.22t - 180/t)}$$

where Z_k is in centistokes (cSt) and *t* is the number of seconds Saybolt.

7.6 Viscosity -Temperature relation

Viscous resistance of lubricating oil is due to intermolecular forces. As the temperature increases, the oil expands and the molecules move further apart decreasing the intermolecular forces. Therefore the viscosity of the lubricating oil decreases with temperature as shown in the figure.7.14. If speed increases, the oil's temperature increases and viscosity drops, thus making it better suited for the new condition. An oil with high viscosity creates higher temperature and this in turn reduces viscosity. This, however, generates an equilibrium condition that is not optimum. Thus, selection of the correct viscosity oil for the bearings is essential.

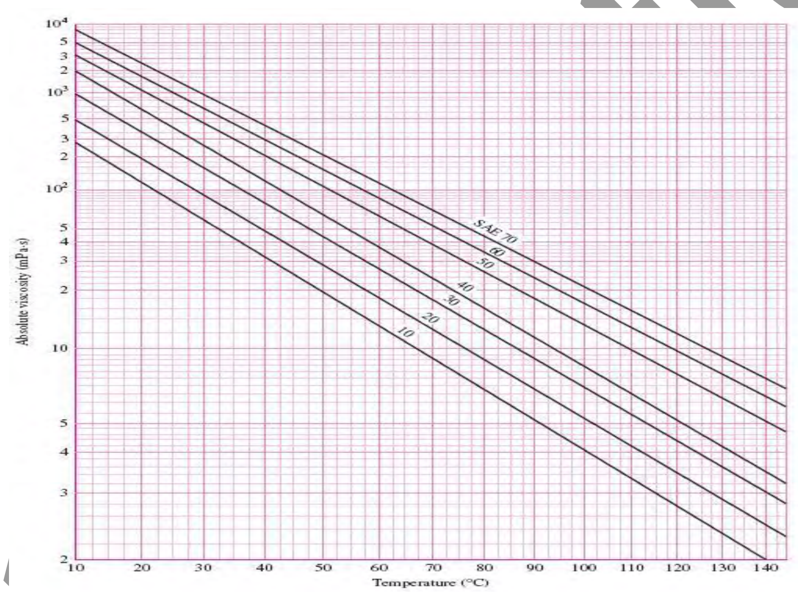


Fig.7.14 Viscosity temperature relationship

Viscosity index of a lubricating oil

Viscosity Index (V.I) is value representing the degree for which the oil viscosity changes with temperature. If this variation is small with temperature, the oil is said to have a high viscosity index. The oil is compared with two standard oils, one having a V.I. of 100 and the other Zero. A viscosity Index of 90 indicates that the oil with this value thins out less rapidly than an oil with V.I. of 50.

7.7 Types of lubricants

- Vegetable or Animal oils like Castor oil, Rapeseed oil, palm oil, Olive oil etc.
- Animal oils like lard oil, tallow oil, whale oil, etc.
- Mineral oils-petroleum based- Paraffinic and Naphthenic based oils

Properties of lubricants

Availability in wide range of viscosities. High Viscosity index. Should be Chemically stable with bearing material at all temperatures encountered. Oil should have sufficient Specific heat to carry away heat without abnormal rise in temperature. Reasonable cost.

Selection Guide for Lubricants

The viscosity of lubricating oil is decisively for the right thickness of the lubricating film (approx. 3-30μm) under consideration of the type of lubricant supply.

Low sliding speed ⇒ High Viscosity

High sliding speed ⇒ Low viscosity

High bearing clearance ⇒ High Viscosity

High load (Bearing pressures) ⇒ Higher Viscosity

7.8 Bearing materials

Relative **softness** (to absorb foreign particles), reasonable strength, **machinability** (to maintain tolerances), **lubricity**, **temperature and corrosion resistance**, and in some cases, **porosity** (to absorb lubricant) are some of the important properties for a bearing material.

A bearing element should be *less than one-third as hard* as the material running against it in order to provide **embedability** of abrasive particles.

A bearing material should have high compression strength to withstand high pressures without distortion and should have good fatigue strength to avoid failure due to pitting.

e.g. in Connecting rod bearings, Crank shaft bearings, etc. A bearing material should have conformability. Soft bearing material has *conformability.* Slight misalignments of bearings can be self-correcting if plastic flow occurs easily in the bearing metal. Clearly there is a compromise between load-bearing ability and conformability.

In bearings operating at high temperatures, possibility of oxidation of lubricating oils leading to formation of corrosive acids is there. The bearing material should be **corrosion resistant**. Bearing material should have easy **availability and low cost.** The bearing material should be soft to allow the dirt particles to get embedded in the bearing lining and avoid further trouble. This property is known as **Embeddability.**

Different Bearing Materials

- **Babbitt or White metal --** usually used as a lining of about 0.5mm thick bonded to bronze, steel or cast iron.
  - Lead based & Tin based Babbitts are available.
  - Excellent conformability and embeddability
  - Good corrosion resistance.
  - Poor fatigue strength

- **Copper Based alloys** - most common alloys are copper tin, copper lead, phosphor bronze: harder and stronger than white metal: can be used **<u>un-backed as a solid bearing</u>.**
- **Aluminum based alloys** - running properties not as good as copper based alloys but cheaper.

  - **Ptfe -** suitable in very light applications
  - **Sintered bronze** - Sintered bronze is a porous material which can be impregnated with oil, graphite or Ptfe. Not suitable for heavily loaded applications but useful where lubrication is inconvenient.
  - **Nylon** - similar to Ptfe but slightly harder: used only in very light applications.

Triple-layer composite bearing material consists of 3 bonded layers: steel backing, sintered porous tin bronze interlayer and anti-wear surface as shown in figure 7.15. High load capacities and low friction rates, and are oil free and anti-wear.

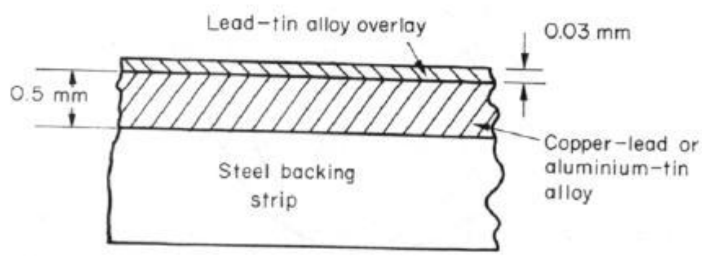


Fig.7.15 Tri-metal Bearing

If oil supply fails, frictional heating will rapidly increase the bearing temperature, normally lead to metal-to-metal contact and eventual seizure. Soft bearing material (low melting point) will be able to shear and may also melt locally. **Protects the journal** from severe surface damage, and helps to avoid component breakages (sudden locking of mating surfaces).

<u>7.9 Petroff's Equation for lightly Loaded Bearings</u>

The phenomenon of bearing friction was first explained by Petroff on the assumption that the shaft is concentric. This can happen when the radial load acting on the bearing is zero or very small, speed of the journal is very high and the viscosity of the lubricant is very high. Under these conditions, the eccentricity of the bearing (the offset between journal center and bearing center) is very small and the bearing could be treated as a concentric bearing as shown in figure 7.16.

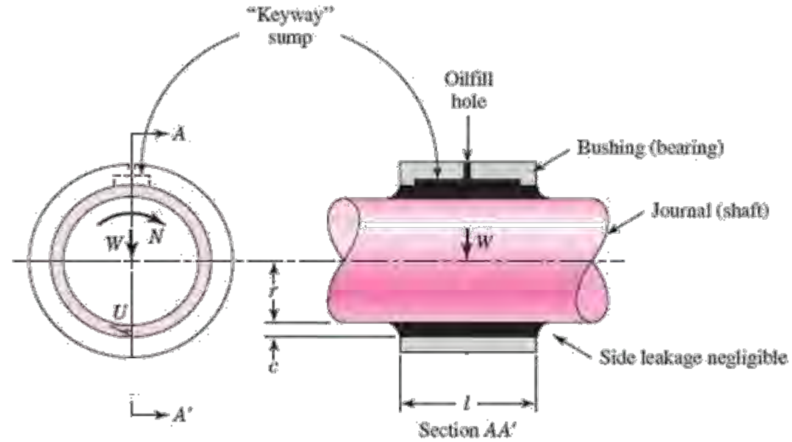


Fig.7.16 Concentric Bearing

Let us now consider a shaft rotating in a guide bearing. It is assumed that the bearing carries a very small load that the clearance space is completely filled with oil, and that leakage is negligible (Fig. 7.16). Let the radius of the shaft be *r*, and the length of the bearing by *l.* If the shaft rotates at N' rev/s, then its surface velocity is $U = 2\pi r N'$. Since the shearing stress in the lubricant is equal to the velocity gradient times the viscosity,

$$\tau = \mathbf{Z\ U/h = 2\pi r N' Z/c}$$

where the radial clearance *c* has been substituted for the distance **h.**

F= Frictional force= τ A= $\mathbf{(2\pi rN'Z/c)\ (2\pi rl) = (\ 4\pi 2r^{2} lZN'/c\ )}$

Frictional torque= Fr = $\mathbf{(\ 4\pi 2r^{3} lZN'/c\ )}$

The coefficient of friction in a bearing is the ratio of the frictional force
F to the Radial load W on the bearing.

f = F/W = $\mathbf{(\ 4\pi 2r^{3} lZN'/cW\ )}$

The unit bearing pressure in a bearing is given by p= W/2rL= Load/ Projected Area of the Bearing.
Or **W= 2prL**
Substituting this in equation for f and simplifying

$$\mathbf{f = 2\pi^{2}\ (ZN'/p)\ (r/c)}$$

This is the Petroff's equation for the coefficient of Friction in Lightly Loaded bearings.

Example on lightly loaded bearings

E1. A full journal bearing has the following specifications:

- Journal Diameter:46 mm
- Bearing length: 66 mm
- Radial clearance to radius ratio: 0.0015
- speed : 2800 r/min
- Radial load: 820 N.
- Viscosity of the lubricant at the operating temperature: 8.4

cP Considering the bearing as a lightly loaded bearing,

Determine (a) the friction torque (b) Coefficient of friction under given operating conditions and (c) power loss in the bearing.

Solution:

Since the bearing is assumed to be a lightly loaded bearing, Petroff's equation for the coefficient of friction can be used.

$$\mathbf{f = 2\pi^2 (ZN'/p) (r/c)}$$

N^I = 2800/60=46.66 r/sec.

Z = 8.4 cP = 8.4 x 10^{-3} Pa.sec

r = 46/2 =23 mm = 0.023 m

r/c= 1/ (c/r) = 1/0.0015= 666.66

P= w/2rL= 820/ 2X0.023X0.066= 270092 Pa.

Substituting all these values in the equation for f, **f = 0.019**

T=Frictional torque: Frictional force x Radius of the Journal

= (f W) r

= 0.019 x 820 x 0.023

= **0.358 N-m**

Power loss in the bearing = T x N^I / 1000 kW

= 0.358 x 46.66/ 1000

= **0.016 kW**

7.10 TOWER'S EXPERIMENT

The present theory of hydrodynamic lubrication originated in the laboratory of Beauchamp Tower in the early 1880s in England. Tower had been employed to study the friction in railroad journal bearings and learn the best methods of lubricating them. It was an accident or error, during the course of this investigation, that prompted Tower to look at the problem in more detail and that resulted in a discovery that eventually led to the development of the theory.

Fig 7.17 is a schematic drawing of the journal bearing that Tower investigated. It is a partial bearing, having a diameter of 4 in, a length of 6 in, and a bearing arc of 157◦, and having bath-type lubrication, as shown. The coefficients of friction obtained by Tower in his investigations on this bearing were quite low, which is now not surprising. After testing this bearing, Tower later drilled a 1/2 -in-diameter lubricator hole through the top. But when the apparatus was set in motion, oil flowed out of this hole. In an effort to prevent this, a cork stopper was used, but this popped out, and so it was necessary to drive a wooden plug into the hole. When the wooden plug was pushed out too, Tower, at this point, undoubtedly realized that he was on the verge of discovery. A pressure gauge connected to the hole indicated a pressure of more than twice the unit bearing load. Finally, he investigated the bearing film pressures in detail throughout the bearing width and length and reported a distribution similar to that of Fig. 7.18.

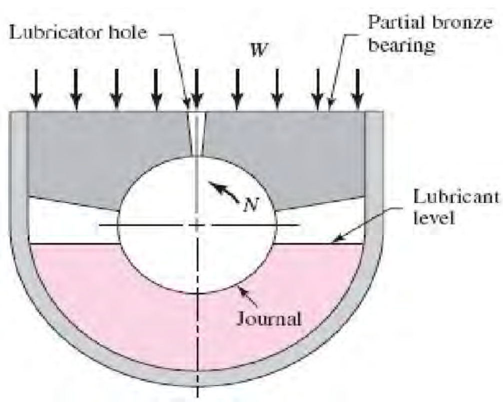


Fig.7.17 Tower's Experiment

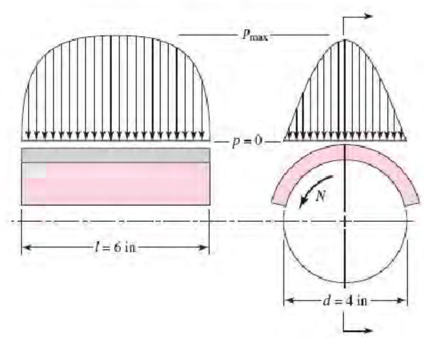


Fig.7.18 Pressure distribution in the oil film

7.11 HYDRODYNMIC JOURNAL BEARINGS

Concept

The film pressure is created by the moving surface itself pulling the lubricant into a wedge-shaped zone at a velocity sufficiently high to create the pressure necessary to separate the surfaces against the load on the bearing.

One type occurs when the rate of shear across the oil film is a constant value and the line representing the velocity distribution is a straight line. In the other type the velocity distribution is represented by a curved line, so that the rate of shear in different layers across the oil film is different. The first type takes place in the case of two parallel surfaces having a relative motion parallel to each other as shown in Fig.7.19.

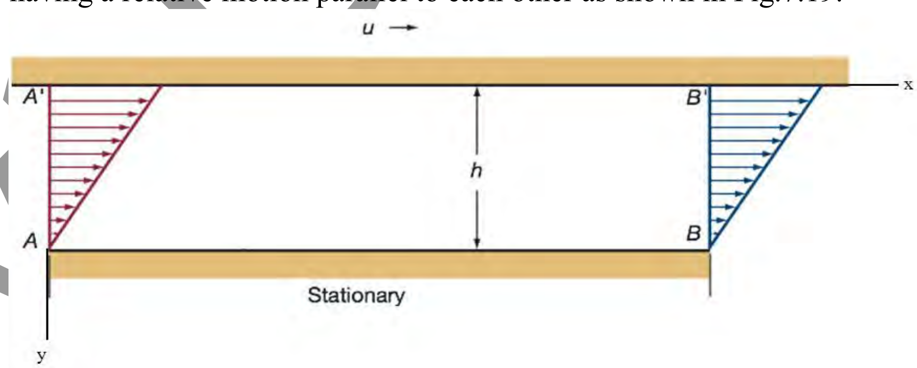


Fig. 7.19 Velocity profiles in a parallel-surface slider bearing.

There is no pressure development in this film. This film cannot support an external Load.

The second type of velocity distribution across the oil film occurs if pressure exists in the film. This pressure may be developed because of the change of volume between the surfaces so that a lubricant is squeezed out from between the surfaces and the viscous resistance of flow builds up the pressure in the film as shown in Fig 7.20 or the pressure may be developed by other means that do not depend upon the motion of the surfaces or it may develop due to the combination of factors. What is important to note here is the fact that pressure in the oil film is always present if the velocity distribution across the oil film is represented by a curved line

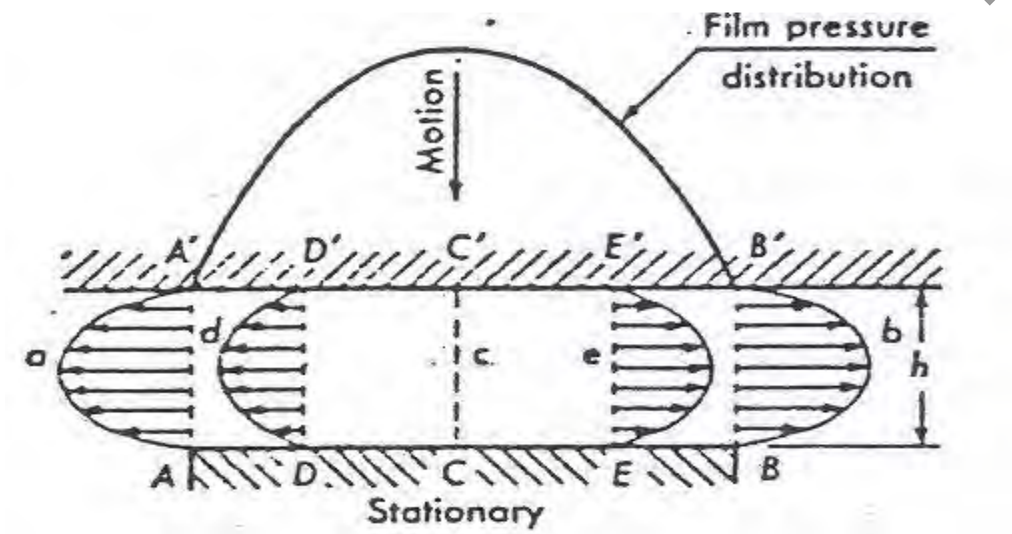


Fig.7.20 Flow between two parallel surfaces

Plate AB is stationary while A′ B′ is moving perpendicular to AB.

Note that the velocity distribution is ***Curvilinear*.** This is a ***pressure induced flow.*** This film can support an External load.

Hydrodynamic film formation

Consider now the case of two non parallel planes in which one is stationary while the other is in motion with a constant velocity in the direction shown in Fig 7.21. Now consider the flow of lubricant through the rectangular areas in section AA' and BB' having a width equal to unity in a direction perpendicular to the paper.

The volume of the lubricant that the surface A'B' tends to carry into the space between the surfaces AB and A'B' through section AA' during unit time is AC'A'. The volume of the lubricant that this surface tends to discharge from space through section BB' during the same period of time is BD'B'. Because the distance AA' is greater than BB' the volume AC'A' is greater than volume BC'B' by a volume AEC'. Assuming that the fluid is incompressible and that there is no flow in the direction perpendicular to the motion, the actual volume of oil carried into the space must be equal to the discharge from this space. Therefore the excess volume of oil is carried into these space is squeezed out through the section AA' and BB' producing a constatnt pressure – induced flow through these sections.

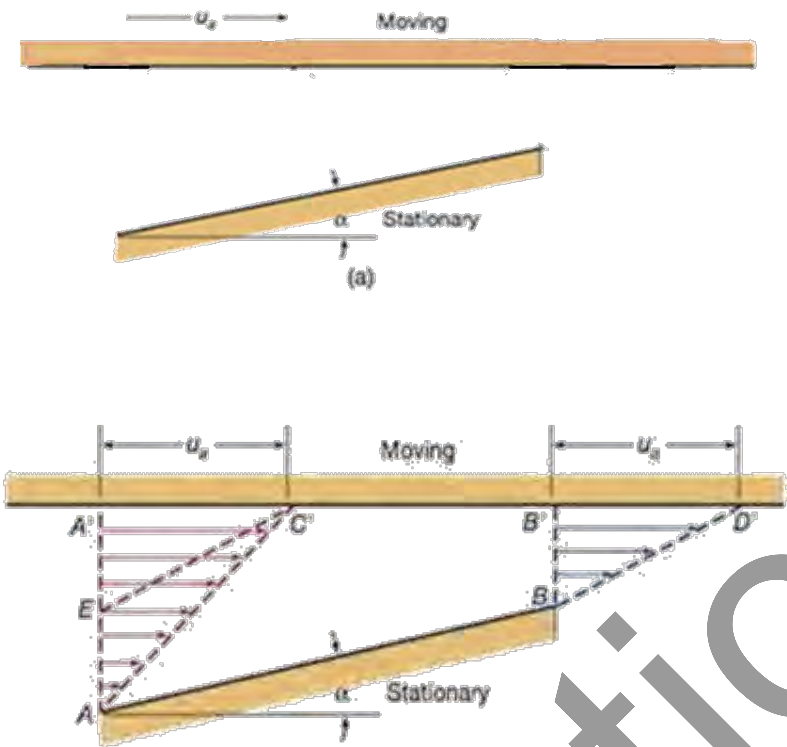


Fig. 7.21 Velocity distribution only due to moving plate

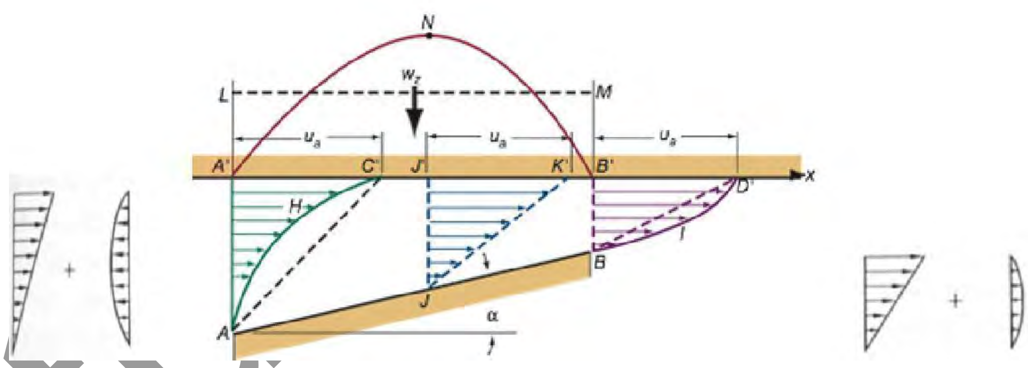


Fig.7.22 Resultant Velocity Distribution

The actual velocity distribution in section AA' and BB' is the result of the combined flow of lubricant due to viscous drag and due to pressure –induced flow. The resultant velocity distributions across these sections are as shown in Fig 7.22.

The curve A'NB' shows the general character of the pressure distribution in the oil film and the line LM shows the mean pressure in the oil film. Because of the pressure developed in the oil film the, plane A’B’ is able to support the vertical load W applied to this plane, preventing metal to metal contact between the surfaces AB and A’B’. This load is equal to the product of projected area of the surface AB and mean pressure in the oil film.

Conditions to form hydrodynamic lubrication

There must be a wedge-shaped space between two relative moving plates;

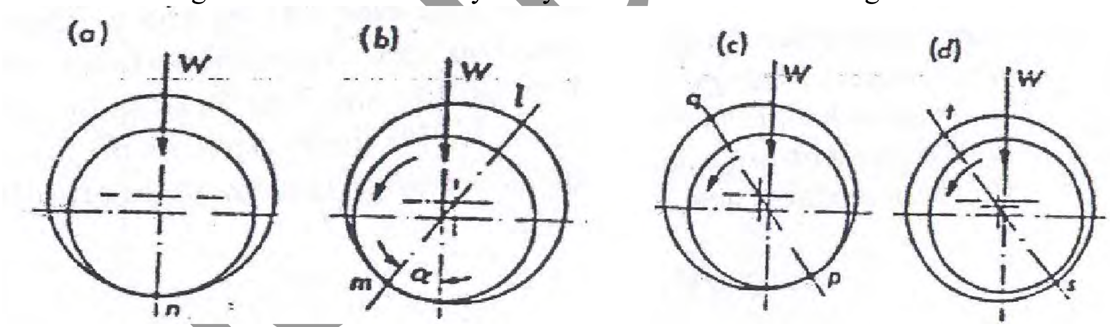
There must be a relative sliding velocity between two plates, and the lubricant must flow from big entrance to small exit in the direction of the moving plate;

The lubricant should have sufficient viscosity, and the supply of the lubricant is abundant.

Formation of oil film in a Journal bearing

Imagine a journal bearing with a downward load on the shaft that is initially at rest and then brought up to operating speed. At rest (or at slow shaft speeds), the journal will contact the lower face of the bearing as shown in the figure 7.23. This condition is known as boundary lubrication and considerable wear can occur. As shaft speed increases, oil dragged around by the shaft penetrates the gap between the shaft and the bearing so that the shaft begins to “float” on a film of oil. This is the transition region and is known as thin-film lubrication. The journal may occasionally contact the bearing particularly when shock radial load occur. Moderate wear may occur at these times. At high speed, the oil film thickness increases until there comes a point where the journal does not contact the bearing at all. This is known as thick film lubrication and no wear occurs because there is no contact between the journal and the bearing.

The various stages of formation of a hydrodynamic film is shown in figure7.23.



Journal at rest | Journal position during starting | Journal position after further increase in speed | Journal position under operating conditions

Fig.7.23 Formation of hydrodynamic oil film in a Journal bearing

Pressure distribution around an idealised journal bearing

A typical pressure distribution around the journal in a hydrodynamic bearing is as shown in the Fig. 7.24.

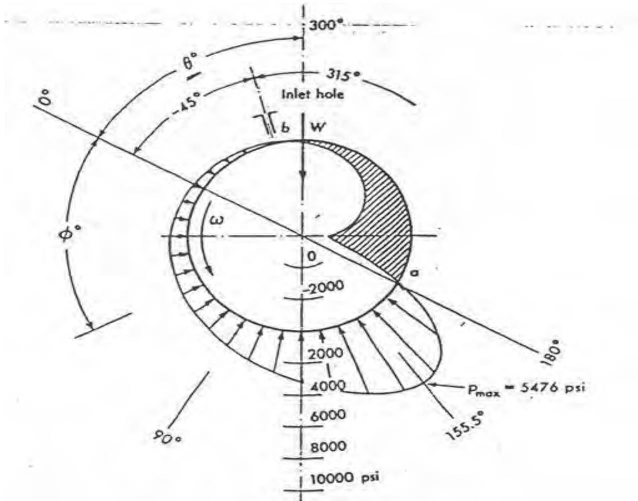


Fig.7.24 Bearing pressure distribution in a journal bearing

Design Considerations

In the first group are those whose values either are given or are under the control of the designer. These are:

- The viscosity Z
- The load per unit of projected bearing area, *p*
- The speed *N*□
- The bearing dimensions *r*, *c*, and *l*

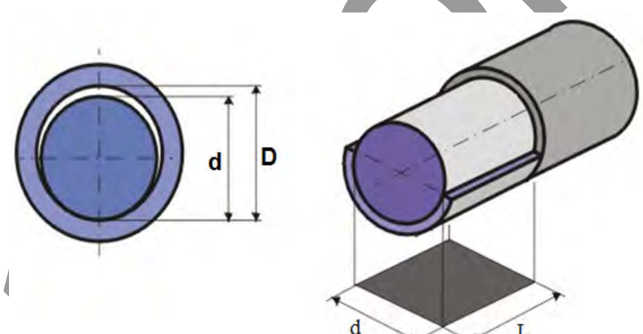
In the second group are the dependent variables. The designer cannot control these except indirectly by changing one or more of the first group. These are:

- The coefficient of friction *f*
- The temperature rise *T*
- The volume flow rate of oil *Q*
- The minimum film thickness h_0

7.12 Bearing Design Parameters:

- Bearing Pressure
- Bearing Modulus
- Sommerfeld Number
- Minimum Film Thickness variable
- Coefficient of friction variable
- Flow variable

Bearing Pressure



BEARING PRESSURE = Load / Projected area of the bearing

= W / L*d = W/2*r*L

Application	Unit Load psi	Unit Load MPa
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

If L/d is > 1, the bearing is said to be a long bearing.
If L/d is < 1, the bearing is said to be a short bearing.
If L/d is = 1, the bearing is said to be a Square bearing.

If the length of the bearing is very large compared to its diameter, L/d= ∞, such a bearing is said to be an idealized bearing with no side leakage.

The load carrying capacity of long bearing is better than that of the short bearings. From the point of view of reduced side leakage, long bearings are preferable. However, space requirements, manufacturing tolerances, Heat carrying capacity and shaft deflections are better met with short bearings.

For general purpose machinery L/d= 1 to 2.

Bearing Modulus (ZN^I/p)

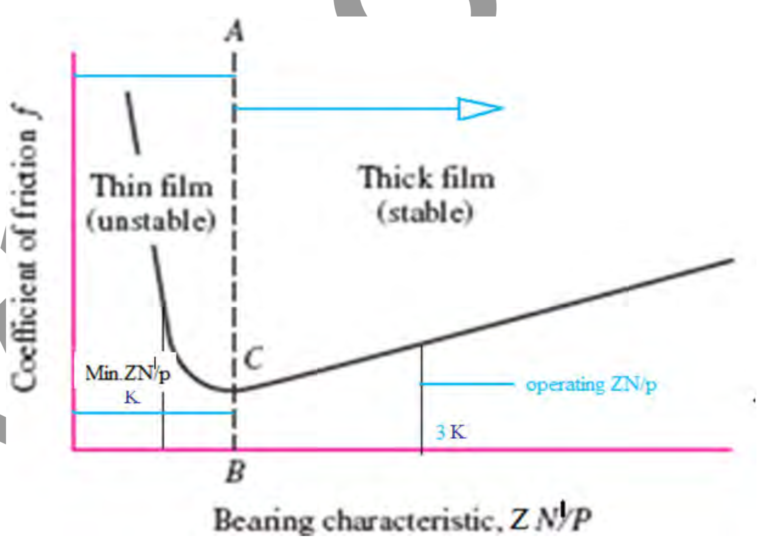


Fig.7.25 Variation of Coefficient of friction with Bearing modulus

Bearing Modulus is a dimensionless parameter on which the coefficient friction in a bearing depends (Fig.7.25). In the region to the left of Point C, operating conditions are severe and mixed lubrication occurs. Small change in speed or increase in load can reduce ZN'/p and a small education in ZN'/p can increase the coefficient of friction drastically. This increases heat which reduces the viscosity of the lubricant. This further reduces ZN'/p leading to further increase in friction.

This has a compounding effect on the bearing leading to destruction of Oil film and resulting in metal to metal contact. In order to prevent such conditions, the bearing should operate with a ZN'/p at least three times the minimum value of the bearing modulus (K).

Suppose we are operating to the right of the line BA and there is an increase in lubricating temperature. This results in lower viscosity and hence a smaller value of the ZN'/p. The coefficient of friction decreases, and consequently the lubricating temperature drops. Thus the region to the right of line BA defines “stable lubrication” because the variations are self correcting.

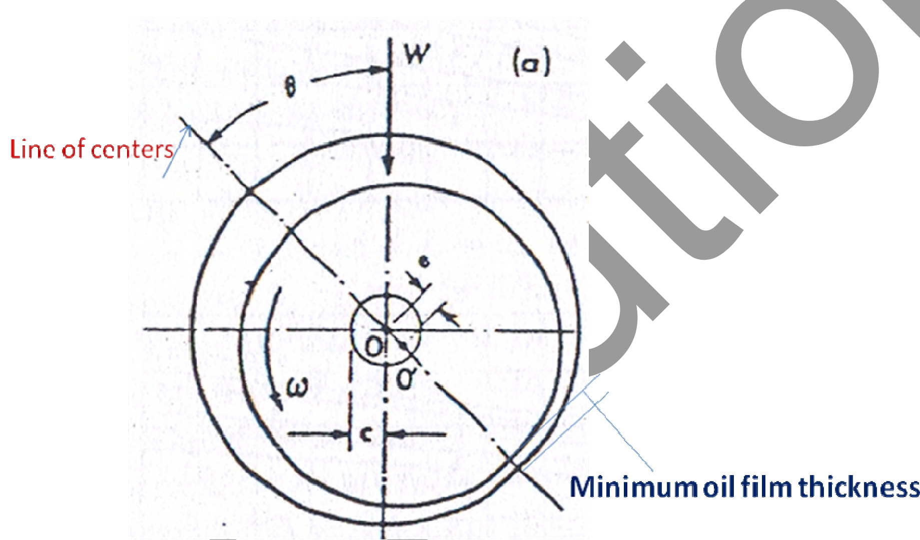


Fig. 7.26 Bearing Design Parameters

Attitude angle locates the position of minimum film thickness with respect to load line

O' = Journal or the shaft center
O = Bearing center
e = Eccentricity

$$0 \le e \le c$$

The radial clearance or half of the initial difference in diameters is represented by c which is in the order of 1/1000 of the journal diameter.
n = e/c, and is defined as **eccentricity ratio**
If n = 0, then there is no load, if n = 1, then the shaft touches the bearing surface under externally large loads.

Using the above figure 7.26, the following relationship can be obtained for 'h'

$h = c(1 + n\cos\theta)$

The maximum and minimum values of 'h' are:

$\mathbf{h_{max} = c + e = c(1+n)}$

$\mathbf{h_{min} = c - e = c(1-n)}$

The relationship between attitude, radial clearance and minimum oil film thickness is given by:

$\mathbf{n = 1 - (h_o/c)}$

Sommerfeld number

The bearing characteristic number, or the Sommerfeld number, is defined by

$\mathbf{S = (ZN'/p)\,(r/c)^2}$

The Sommerfeld number is very important in lubrication analysis because it contains many of the parameters that are specified by the designer.

The parameter **r/c** is called the radial clearance ratio.

The relation between Sommerfeld number and Attitude of the bearing is shown in the figure 7.27.

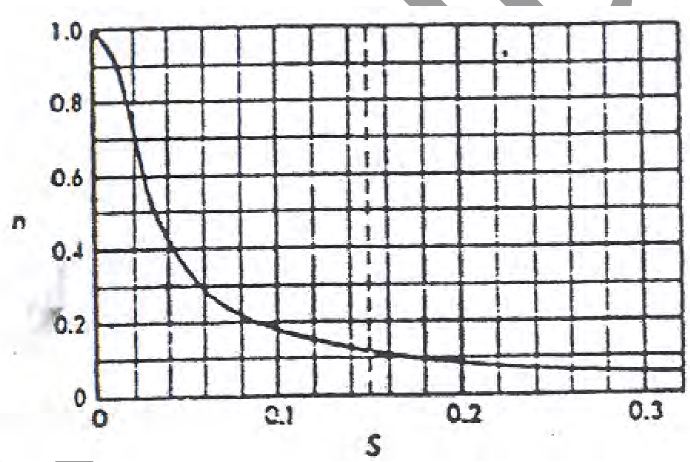


Fig.7.27 Attitude v/s Sommerfeld number

If S=0.15 or greater than 0.15, the bearing is a **Lightly loaded bearing**.
If S is lesser than 0.15, the bearing is a **Heavily loaded bearing.**

In **Raimondi and Boyd method,** the performance of a bearing is expressed in terms of dimensionless parameters The results of their work is available in the form of charts and tables.

From petroff's equation we know that,

f = 2π² (ZN′/p) (r/c) and therefore

(r/c) f = 2π² (ZN′/p) (r/c)² = 2 π² S

$$f = 2\pi^2 (ZN'/p)(r/c)$$

$$(r/c)\,f = 2\pi^2 (ZN'/p)(r/c)^2 = 2\pi^2 S$$

(r/c)f is a dimensionless variable called Coefficient of friction variable. Figure 7.28 shows the variation coefficient of friction variable with respect to Sommerfeld number.

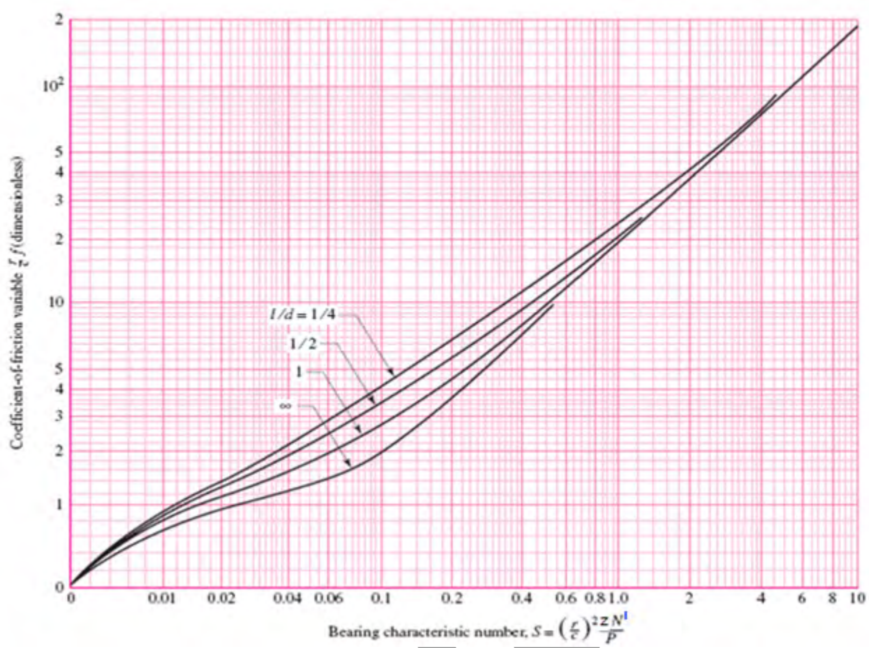


Fig.7.28 Friction Variable - Sommerfeld Number chart

h_o/c is a dimensionless variable and is known as Minimum Film Thickness variable. Figure 7.29 shows the variation Minimum film thickness variable with respect to Sommerfeld number.

Dimensionless **Flow Variable** is given by:

FV= Q / r c N' L

Where Q= Total Flow through the bearing

Figure 7.30 shows the variation flow variable with respect to Sommerfeld number.

Q_s = Side leakage
Q_s/Q = Ratio of Side leakage to Total flow through the Bearing (fig.7.31)

The Maximum pressure P_{max} developed can be determined from the ratio of P to P_{max}.(fig.7.32)

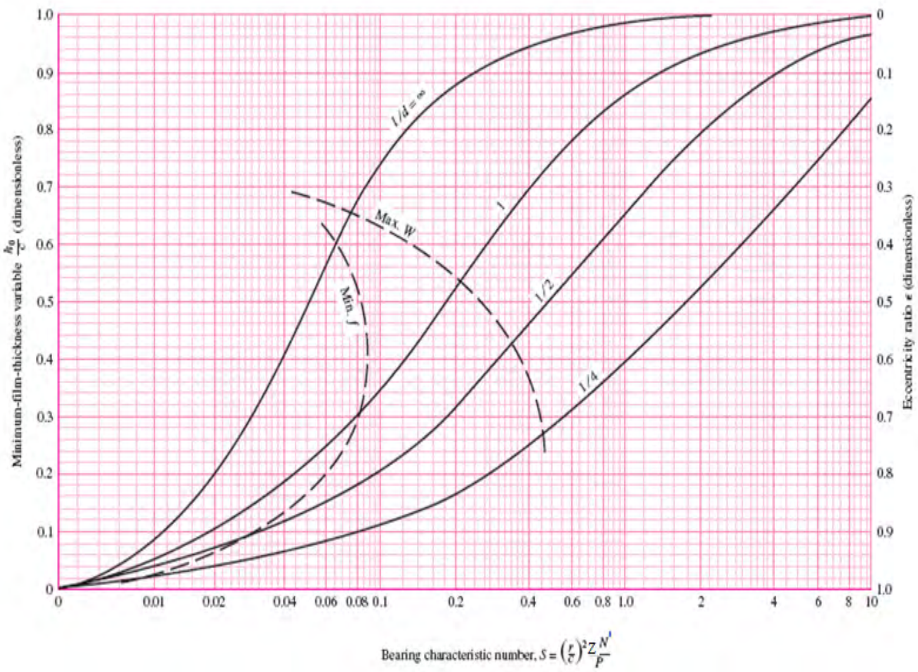


Fig.7.29 Minimum film thickness Variable - Sommerfeld Number chart

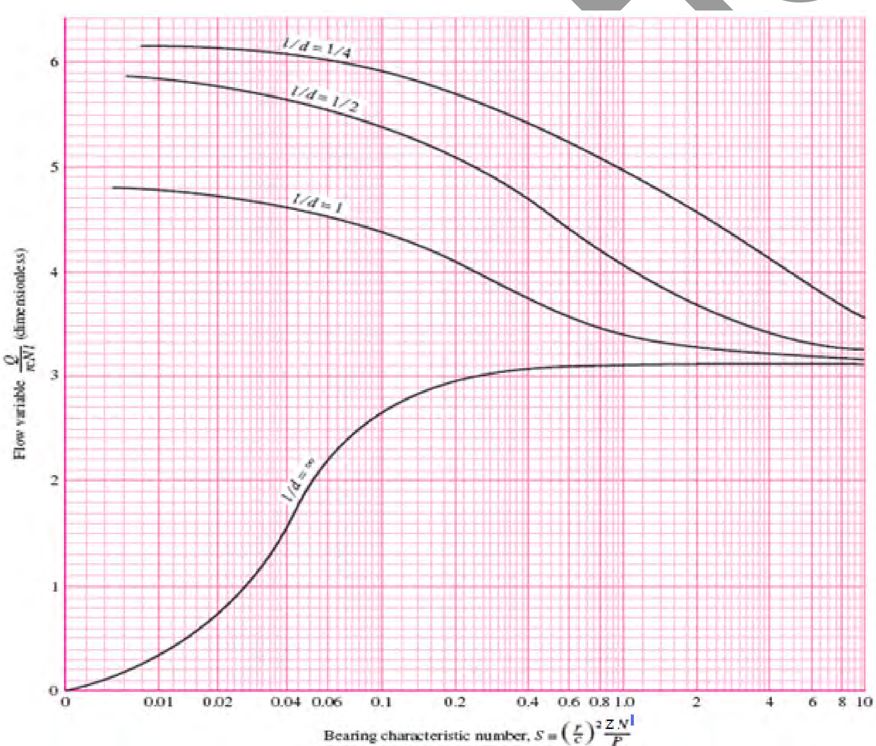


Fig.7.30 Flow Variable - Sommerfeld Number chart

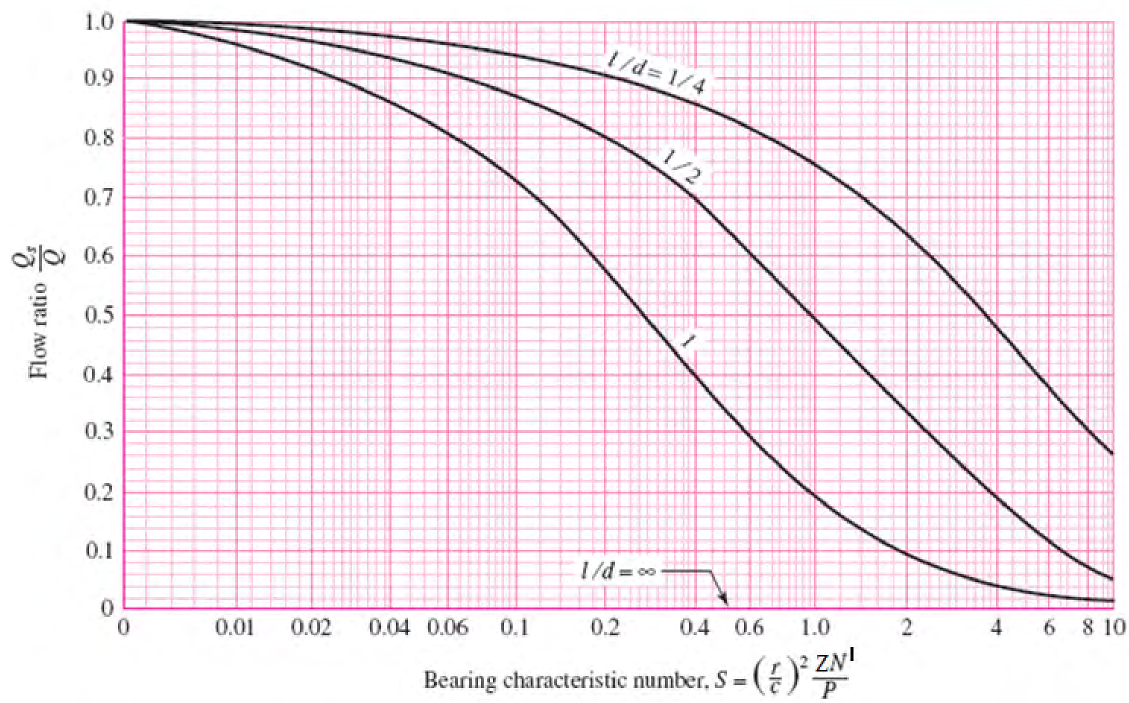


Fig.7.31 Ratio of Side Flow to Total Flow - Sommerfeld Number chart

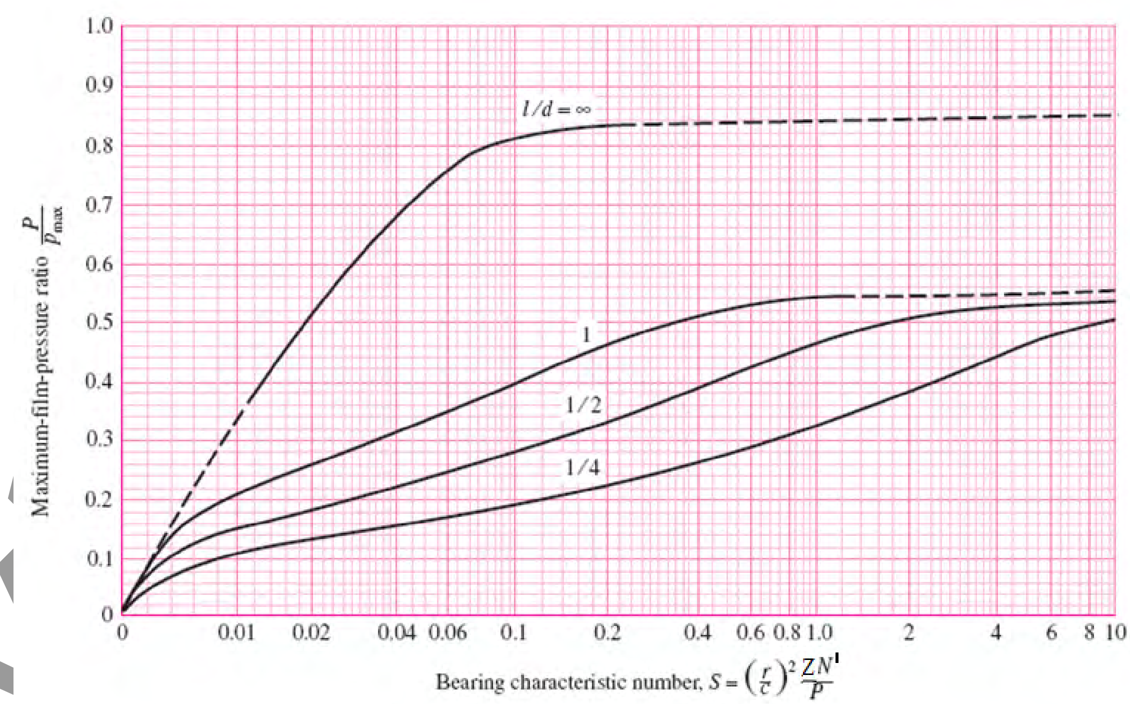
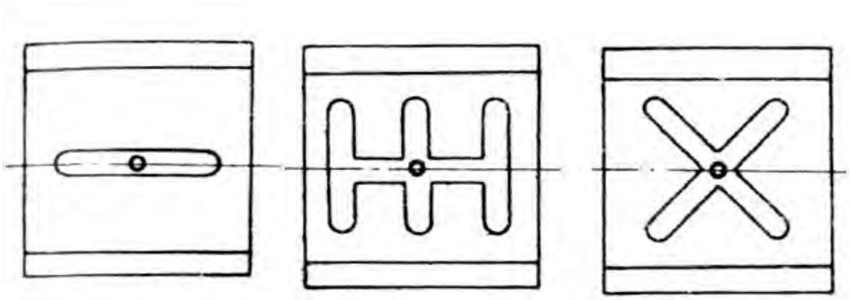


Fig.7.32 Maximum Film Pressure- Sommerfeld Number chart

Typical oil groove patterns



Some typical groove patterns are shown in the above figure. In general, the lubricant may be brought in from the end of the bushing, through the shaft, or through the bushing. The flow may be intermittent or continuous. The preferred practice is to bring the oil in at the center of the bushing so that it will flow out both ends, thus increasing the flow and cooling action.

7.13 Thermal aspects of bearing design

Heat is generated in the bearing due to the viscosity of the oil. The frictional heat is converted into heat, which increases the temperature of the lubricant. Some of the lubricant that enters the bearing emerges as a side flow, which carries away some of the heat. The balance of the lubricant flows through the load-bearing zone and carries away the balance of the heat generated. In determining the viscosity to be used we shall employ a temperature that is the average of the inlet and outlet temperatures, or

$$T_{av} = T_1 + T/2$$

where Ti is the inlet temperature and T is the temperature rise of the lubricant from inlet to outlet. The viscosity used in the analysis must correspond to $\mathbf{T_{av}}$.

Self contained bearings:

These bearings are called ***selfcontained*** bearings because the lubricant sump is within the bearing housing and the lubricant is cooled within the housing. These bearings are described as *pillow-block* or *pedestal* bearings. They find use on fans, blowers, pumps, and motors, for example. Integral to design considerations for these bearings is dissipating heat from the bearing housing to the surroundings at the same rate that enthalpy is being generated within the fluid film.

Heat dissipated based on the projected area of the bearing:

Heat dissipated from the bearing, J/S $\mathbf{H_D = K_2\, ld\, (t_B - t_A) = C''ld}$

Where $C'' = K_2 (t_B - t_A)$ a coefficient from fig15.16 or table 15.10 (From data hand book)

Another formula to determine the heat dissipated from the bearing

$$\mathbf{H_D = ld\, (T+18)^2 / K_3}$$

Where $K_3 = 0.2674 \times 10^6$ for bearings of heavy construction and well ventilated

$= 0.4743 \times 10^6$ for bearings of light construction in still air air

T= t_B - t_A
Where,
t_B = Bearing surface temperature
t_A = Ambient temperature

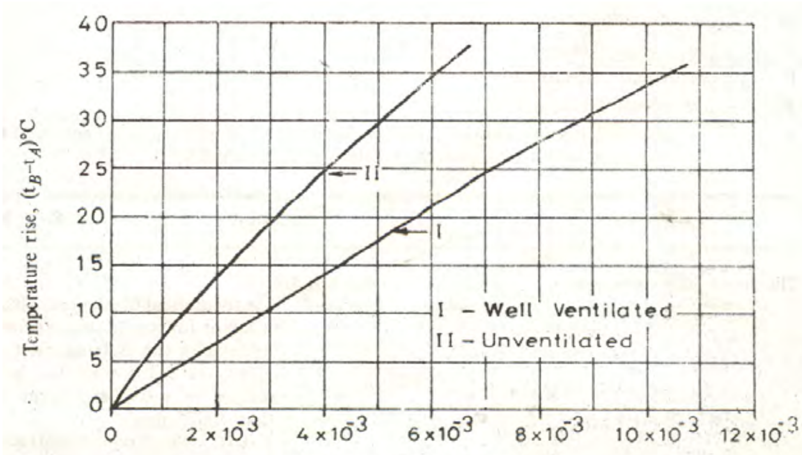


Fig. 15.16 Coefficient. C″

For good performance thye following factors should be considered.

- **Surface finish of the shaft (journal):** This should be a fine ground finish and preferably lapped.
- **Surface hardness of the shaft:** It is recommended that the shaft be made of steel containing at least 0.35-0.45% carbon. For heavy duty applications shaft should be hardened.
- **Grade of the lubricant:** In general, the higher the viscosity of the lubricant the longer the life. However the higher the viscosity the greater the friction, so high viscosity lubricants should only be used with high loads. In high load applications, bearing life may be extended by cutting a grease groove into the bearing so grease can be pumped in to the groove.
- **Heat dissipation:** Friction generates heat and causes rise in temperature of the bearing and lubricant. In turn, this causes a reduction in the viscosity of the lubricating oil and could result in higher wear. Therefore the housing should be designed with heat dissipation in mind. For example, a bearing mounted in a Bakelite housing will not dissipate heat as readily as one mounted in an aluminium housing.
- **Shock loads:** Because of their oil-cushioned operation, sliding bearings are capable of operating successfully under conditions of moderate radial shock loads. However excessive prolonged radial shock loads are likely to increase metal to metal contact and reduce bearing life. Large out of balance forces in rotating members will also reduce bearing life.
- **Clearance:** The bearings are usually a light press fit in the housing. A shouldered tool is usually used in arbour press. There should be a running clearance between the journal and the bush. A general rule of thumb is to use a clearance of 1/1000 of the diameter of the journal.
- **Length to diameter ratio(l/d ratio):** A good rule of thumb is that the ratio should lie in the range 0.5-1.5. If the ratio is too small, the bearing pressure will be too high and it will be difficult to retain lubricant and to prevent side leakage. If the ratio is too

high, the friction will be high and the assembly misalignment could cause metal to metal contact.

Examples on journal bearing design

Example EI:

Following data are given for a 360º hydrodynamic bearing:

Radial load=3.2 kN
Journal speed= 1490 r.p.m.
Journal diameter=50 mm
Bearing length=50mm
Radial clearance=0.05 mm
Viscosity of the lubricant= 25 cP

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate:

- Power lost in friction;
- The coefficient of friction;
- Minimum oil film thickness
- Flow requirement in 1/min; and
- Temperature rise.

Solution:

P= W/Ld = 3.2x1000/ (50x50) =1.28 MPa.= 1.28x10^6 Pa

Sommerfeld number = S= (ZN′/p) $(r/c)^2$

r/c =25/0.0.05 = 500
Z= 25 cP = 25x10^{-3} Pa.sec
= 1490/60= 24.833 r/sec. Substituting the above values, we get
S=0.121

For S= 0.121 & L/d=1,
Friction variable from the graph= (r/c) f= 3.22
Minimum film thickness variable= h_o /c =0.4
Flow variable= Q/rcN′L= 4.33

f = 3.22x0.05/25= 0.0064
Frictional torque= T= fWr = 0.0064x3200x 0.025
= 0.512 N-m
Power loss in the Bearing= 2π N′ T/ 1000 kW
= 0.080 kW

h_o = 0.4x 0.05= 0.02 mm

Q/r c N^l L= 4.33 from which we get,

Q= 6720.5 mm^3 / sec.

Determination of dimensionless variables is shown in the following figures.
Assume that all the heat generated due to friction is carried away by the lubricating oil.

Heat generated = 80 watt = $mC_p\ \Delta T$

where:

m= mass flow rate of lubricating oil= ρQ in kg/sec

Cp= Specific heat of the oil= 1760 J/kg ºC

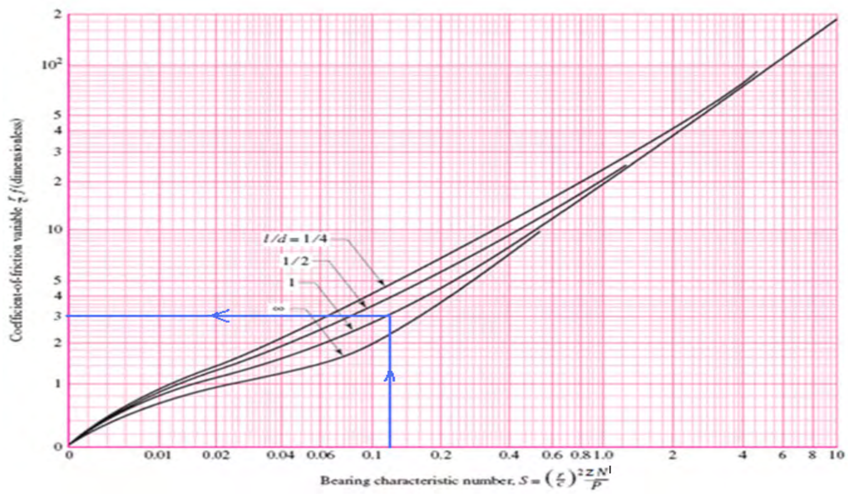
ΔT= temperature rise of the oil

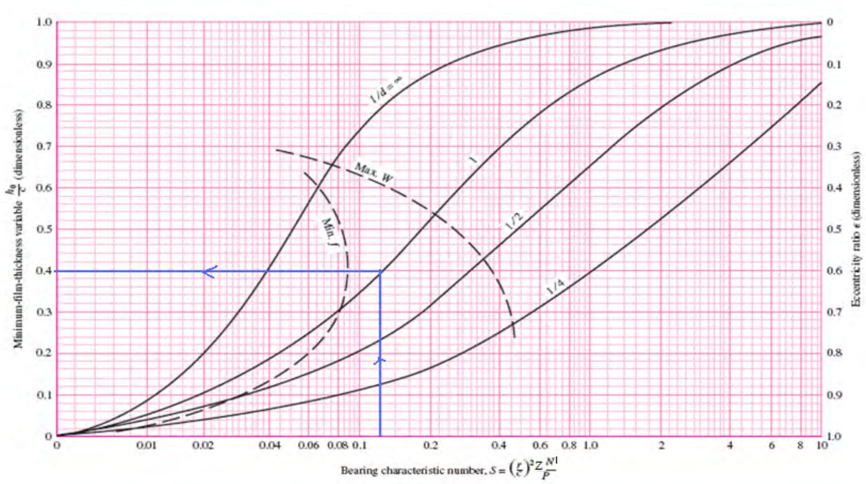
$\rho = 860\text{x}10^{-9}$ kg/mm^3

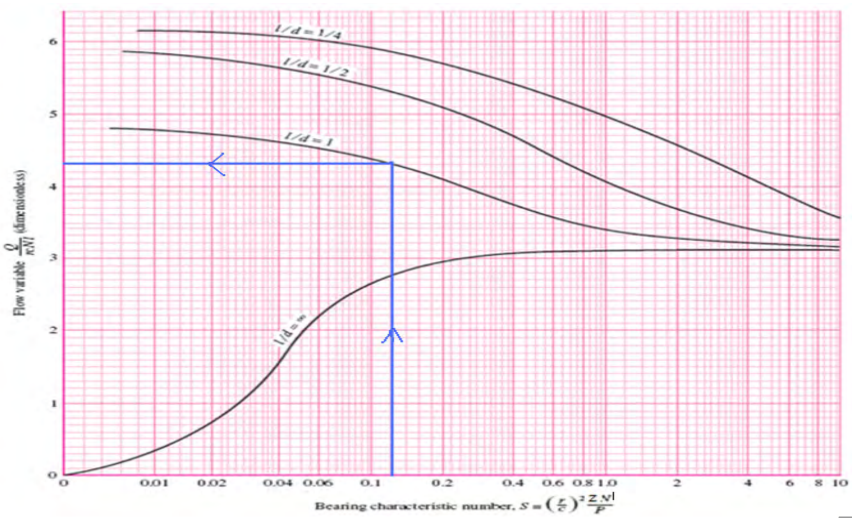
Substituting the above values,

ΔT= 7.9 ºC

The Average temperature of the oil= $T_i + \Delta T/2$ = 27+ ‹7.9/2›=30.85 ºC







Example E2:

A 50 mm diameter hardened and ground steel journal rotates at 1440 r/min in a lathe turned bronze bushing which is 50 mm long. For hydrodynamic lubrication, the minimum oil film thickness should be five times the sum of surface roughness of journal bearing. The data about machining methods are given below:

	Machining method	surface Roughness(c.l.a)
Shaft	grinding	1.6 micron
Bearing	turning/boring	0.8 micron

The class of fit is H8d8 and the viscosity of the lubricant is 18 cP. Determine the maximum radial load that the journal can carry and still operate under hydrodynamic conditions.

Solution:

Min. film thickness = h_o= 5 [0.8+1.6] = 12 micron = 0.012 mm
For H8 d8 fit, referring to table of tolerances,
Ø50 H8 = Min. hole limit = 50.000 mm
Max. hole limit = 50.039 mm
Mean hole diameter= 50.0195 mm
Ø 50 d8 = Max. shaft size = 50- 0.080= 49.920 mm
Min. shaft size = 50- 0.119= 49.881 mm
Mean shaft diameter= 49.9005 mm.
Assuming that the process tolerance is centered,
Diametral clearence= 50.0195- 49.9005= 0.119 mm
Radial clearence= 0.119/2= 0.0595 mm

h_o /c = 0.012/ 0.0595 = 0.2
L/d = 50/50= 1
From the graph, Sommerfeld number= 0.045

$S = (ZN'/p)\,(r/c)^2 = 0.045$

r/c= 25/0.0595= 420.19

$Z = 18$ cP= $18x10^{-3}$ Pa.sec
N′= 1440/60= 24 r/sec
From the above equation, Bearing pressure can be calculated.
p= $1.71x10^{6}$ Pa = 1.71 MPa.

The load that the bearing can carry:
W= pLd = 1.71x 50x 50= 4275 N

Example E3:

The following data are given for a full hydrodynamic journal bearing:
Radial load=25kN
Journal speed=900 r/min.
Unit bearing pressure= 2.5 MPa
(l/d) ratio= 1:1
Viscosity of the lubricant=20cP
Class of fit=H7e7
Calculate: 1.Dimensions of bearing
2. Minimum film thickness and
3. Requirement of oil flow

Solution:
N = 900/60= 15 r/sec
P=W/Ld
2.5= 25000/Ld=25000/d ^{2}
As L=d.
d= 100 mm & L=100 mm

For H7 e7 fit, referring to table of tolerances,
Ø100 H7 = Min. hole limit = 100.000 mm
Max. hole limit = 100.035 mm
Mean hole diameter= 100.0175 mm
Ø 100 e7 = Max. shaft size = 100- 0.072= 99.928 mm
Min. shaft size = 100- 0.107= 99.893 mm
Mean shaft diameter= 99.9105 mm
Assuming that the process tolerance is centered,
Diametral clearence= 100-0175- 99.9105= 0.107 mm
Radial clearence= 0.107/2= 0.0525mm

Assume r/c = 1000 for general bearing applications.
C= r/1000=50/1000 = 0.05 mm.
Z= 20 cP= $20x10^{-3}$ Pa.sec
N^{l} = 15 r/sec
P= 2.5 MPa= 2.5 $x10^{6}$ Pa
S= (ZN/p) $(r/c)^{2}$ =0.12

For L/d=1 & S=0.12, Minimum Film thickness variable= h_o /c = 0.4

h_o = 0.4x 0.05= 0.02 mm

Example E4:
A journal bearing has to support a load of 6000N at a speed of 450 r/min. The diameter of the journal is 100 mm and the length is 150mm.The temperature of the bearing surface is limited to 50 ºC and the ambient temperature is 32 ºC. Select a suitable oil to suit the above conditions.

Solution:

N^l = 450/60 =7.5 r/sec, W=6000 N, L=150mm, d=100 mm,
t_A = 32 ºC, t_B = 50 ºC.
Assume that all the heat generated is dissipated by the bearing.

Use the Mckee's Equation for the determination of coefficient of friction.

f=Coefficient of friction= K_a (ZN^l /p) (r/c) 10^{-10} +Δf

p= W/Ld= 6000/100x150 = 0.4 MPa.
K_a = 0.195x 10^6 for a full bearing
Δf = 0.002
r/c= 1000 assumed
U= $2\pi rN^l$ = 2x3.14x 50x7.5= 2335 mm/sec= 2.335 m/sec.
f = 0.195x 10^6 x (Z * 7.5 / 0.4) x 1000 x 10^{-10} +0.002
f = 0.365Z+0.002

Heat generated= f *W*U
Heat generated= (0.365Z+ 0.002)x6000x2.335

Heat dissipated from a bearing surface is given by:

H_D= ld $(T+18)^2$/ K_3

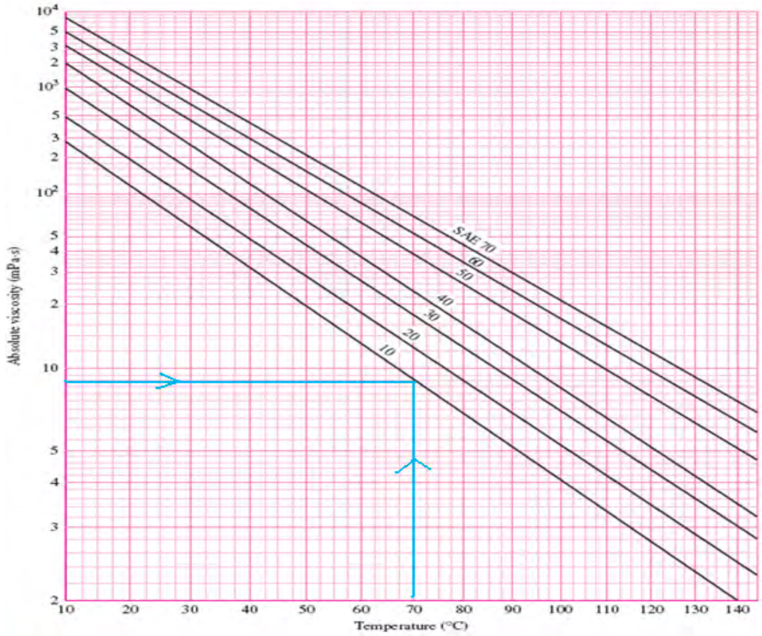
Where K_3= 0.2674x 10^6 for bearings of heavy construction and well ventilated
= 0.4743x10^6 for bearings of light construction in still air air

T= t_B - t_A = 50-32 =18ºC
H_D = 150x100($18+18)^2$ / 0.2674x10^6 =72.7 Watt

H_D = Hg for a self contained bearing.

72.7 = (0.365Z+ 0.002)x6000x2.335
Z= 0.0087 Pa.Sec.

Relation between oil temp, Amb. temp, & Bearing surface temperature is given by
$t_B - t_A$= ½ (t_O- t_A)
t_O = oil temperature= 68 ºC



Select SAE 10 Oil for this application

ME | VI

06ME61

Sixth Semester B.E. Degree Examination, May/June 2010

Design of Machine Elements - II

Time: 3 hrs. Max. Marks:100

Note: *1.Answer any FIVE full questions, selecting at least TWO questions from each part.*
2.Use of design data hand book is permitted.

PART – A

1 a. Derive expressions for extreme fibre stresses in a curved beam subjected to pure bending moment. **(08 Marks)**

b. Determine the combined stresses at the inner and outer fibers at the critical section of a crane hook which is required to lift loads upto 50 kN. The hook has trapezoidal C.S. with inner and outer sides of 90mm and 40mm respectively. Depth is 120mm. The center of curvature of the section is at a distance of 100mm from the inner side of the section and the load line passes through the centre of curvature. Also, determine the factor of safety according to max shear stress theory, if τ_{all} = 80 MPa. **(12 Marks)**

2 a. With reference to pressure vessels, what is autofrettage? Explain. **(04 Marks)**

b. A high pressure cylinder consists of an inner cylinder of ID and OD of 200mm and 300mm respectively. It is jacketed by an outer cylinder of OD 400mm. The difference between the OD of the inner cylinder and inner dia of the jacket before assembly is 0.25mm. $E = 2.07\times10^5$ MPa. Calculate the shrinkage pressure and stresses induced in cylinders due to shrinkage pressure. In service, the cylinder is further subjected to an internal pressure of 200 MPa. Plot the resultant stress distribution. **(16 Marks)**

3 a. Derive an expression for shearing stress induced in a helical spring subjected to a compressive load, P. **(07 Marks)**

b. Write a note on Wahl stress correction factor. **(03 Marks)**

c. A semi-elliptic multi-leaf spring is used for the suspension of the rear axle of a truck. It consists of two extra full length leaves and 10 graduated length leaves including the master leaf. The center to center distance between the spring eyes is 1.2m. The leaves are made of steel with σ_{yt} = 1500 MPa. $E = 2.07\times10^5$ MPa and FOS is 2.5. The spring is to be designed for a maximum force of 30 kN. The leaves are prestressed so as to equalize stresses in all leaves. Determine

i) C.S. of leaves ii) Initial nip iii) Initial pre-load required to close the gap
iv) Deflection of the spring. **(10 Marks)**

4 a. List the advantages and disadvantages of helical gears. **(03 Marks)**

b. It is required to transmit 15 kW power from a shaft running at 1200 rpm to a parallel shaft with speed reduction of 3. The centre distance of shafts is to be 300mm. The material used for pinion in steel (σ_d = 200 MPa) and for gear is CI (σ_d = 140 MPa). Service factor is 1.25 and tooth profile is 20° full depth involute. Design the spur gear and check the design for dynamic load and wear. **(17 Marks)**

PART – B

5 a. List the advantages and disadvantages of worm gear drive. **(03 Marks)**

b. A pair of straight tooth bevel gears at right angles is to transmit 5 kW at 1500 rpm of the pinion at a speed ratio of 3. Diameter of the pinion is 75mm. The tooth form is 14½° involute. Pinion is made of steel (σ_d = 160 MPa) and gear of CI (σ_d = 80 MPa). Design the gear pair and check the design for dynamic load and wear. **(17 Marks)**

6 a. Derive power transmitting capacity of a single plate clutch for
i) Uniform pressure condition and ii) Uniform wear condition. **(10 Marks)**

b. A single block brake with a torque capacity of 250 N.m is shown Fig.Q6(b). The brake drum rotates at 100 rpm and the coefficient of friction is 0.35. Calculate:
i) The actuating force and the hinge-pin reaction. ii) the rate of heat generated during the braking action and iii) The dimensions of the block, if the intensity of pressure between the block and brake drum is 1 MPa. The length of the block is twice its width. **(10 Marks)**

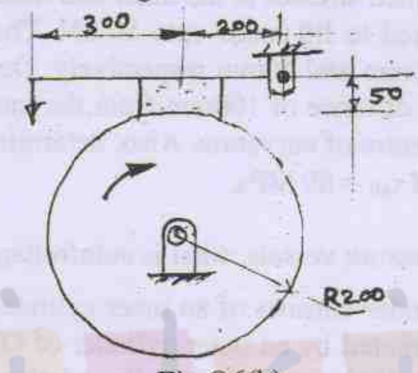


Fig.Q6(b)

7 a. Derive the Petnoff's equation coefficient of friction, in a sliding contact bearing. **(06 Marks)**

b. Following data refers to a 360° hydrodynamic bearing:
Radial load = 3.2 kN Journal speed = 1500 rpm
Journal diameter = bearing length = 50 mm Radial clearance = 0.05 mm
Viscosity of lubricant = 25 cP.
Assume that the total heat generated in the bearing is carried away by total oil flow in the bearing. Calculate:
i) Coefficient of friction ii) Power loss in friction iii) Minimum film thickness
iv) Flow requirement v) Temperature rise. **(14 Marks)**

8 a. Derive the expression for power rating of a V-belt drive. **(08 Marks)**

b. The following data is given for a V-belt drive connecting a 20 kW motor to a compressor:

	Motor pulley	Compressor pulley
Pitch dia (mm)	300	900
Speed (rpm)	1440	480
Coefficient of friction	0.2	0.2

The center distance between pulleys is 1 m. C.S. of belt is trapezoidal with parallel sides being 12mm and 22mm respectively and depth is 14mm. The density of the composite belt is 0.97 gm/cc and the allowable tension per belt is 850 N. Determine he number of belts required for this application. **(12 Marks)**

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6th SEM (MECH)

USN

06ME61

Sixth Semester B.E. Degree Examination, December 2010

Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Use of design data handbook is permitted.
3. Missing data, if any, may suitably be assumed with justification.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Derive an expression for normal stresses due to bending at the extreme fibres on the cross section of a curved machine member. **(08 Marks)**

b. Determine the value of 't' in the cross section of a curved machine member shown in Fig.1(b), so that the normal stresses due to bending at extreme fibres are numerically equal. Also determine the normal stresses so induced at extreme fibres due to a bending moment of 10 kN-m. **(12 Marks)**

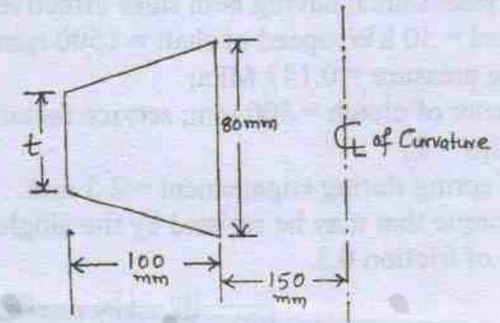


Fig.1(b).

2 a. A cast steel cylinder of 350 mm inside diameter is to contain liquid at a pressure of 13.5 N/mm^2. It is closed at both ends by flat cover plates which are made of alloy steel and are attached by bolts.

i) Determine the wall thickness of the cylinder, if the maximum hoop stress in the material is limited to 55 MPa.

ii) Calculate the minimum thickness necessary of the cover plates if the working stress is not to exceed 65 MPa. **(08 Marks)**

b. A shrink fit assembly, formed by shrinking one cylinder over another, is subjected to an external pressure of 60 N/mm^2. Before the fluid is admitted, the internal and external diameters of the assembly are 120 mm and 200 mm respectively and the diameter at the junction is 160 mm. If after shrinking on, the contact pressure at the junction is 8 N/mm^2, determine using Lame's equations, the stresses at inner, mating and outer surfaces of the assembly after the fluid has been admitted. **(12 Marks)**

3 a. A semi elliptical laminated spring has effective length of 1 m. The spring has to sustain a load of 75 kN. The spring has 3 full length leaves and 16 graduated leaves. If the leaves are prestressed such that the stress induced in all the leaves is same and are limited to 400 MPa, when maximum load is applied. The width of the leaves is 9 times the thickness. Assume E = 200 GPa. Determine :

i) The width and thickness of the leaves.

ii) The initial space that has to be provided between full length leaves and the graduated leaves before the band is applied.

iii) Load on the clip to close the initial gap. **(10 Marks)**

b. A load of 2 kN is dropped axially on a close coiled helical spring from a height of 250 mm. The spring has 20 effective turns and it is made of 25 mm diameter wire. The spring index is 8. Find the maximum shear stress induced in the spring and the amount of compression produced. Take G = 82.7 GPa. **(10 Marks)**

4 a. State the advantages of gear drives when compared to chain or belt drives. **(04 Marks)**

b. Design a helical gear pair to transmit a power of 15 kW from a shaft rotating at 1000 rpm to another shaft to be run at 360 rpm. Assume involute profile with a pressure angle of 20°. The material for pinion is forged steel SAE 1030 whose $\sigma_0 = 172.375$ MPa and the material for the gear is cast steel 0.20% C untreated with $\sigma_o = 137.34$ MPa. The gears operate under a condition of medium shocks for a period of 10 hrs per day. Check for dynamic load, if load factor C = 580 N/mm and also for wear load. **(16 Marks)**

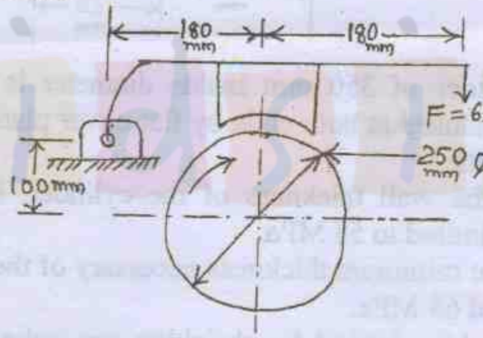
PART – B

5 Design a pair of bevel gears to connect two shafts at 60°. The power transmitted is 25 kW at 900 rpm of pinion. The reduction ratio desired is 5:1. The teeth are 20° full depth involute and pinion has 24 teeth. Check the design for dynamic and wear considerations. **(20 Marks)**

6 a. Design a single plate clutch having both sides effective form the following data :
Power transmitted = 30 kW; speed of shaft = 1500 rpm;
Allowable lining pressure = 0.147 MPa;
Maximum diameter of clutch = 300 mm; service factor = 1.5;
Number of springs = 9;
Compression of spring during engagement = 2.5 mm. **(12 Marks)**

b. Determine the torque that may be resisted by the single block brake shown in Fig.6(b) below for a coefficient of friction 0.3. **(08 Marks)**

Fig.6(b).



7 a. Explain mechanism of hydrodynamic journal bearing. **(04 Marks)**

b. A full journal bearing 50 mm in diameter and 50 mm long operates at 1000 rpm and carries a load 5 kN. The radial clearance is 0.025 mm. The bearing is lubricated with SAE 30 oil and the operating temperature of oil is 80°C. Assume the attitude angle as 60°, determine :
i) Bearing pressure ; ii) Sommerfeld number; iii) Attitude ; iv) Minimum film thickness; v) Heat generated ; vi) Heat dissipated if the ambient temperature is 20°C and vii) Amount of artificial cooling if necessary. Use McKnee's and Pederson's equations. **(16 Marks)**

8 a. A 25 mm 6 x 37 steel wire rope is used in a mine of 80 m deep. The velocity of the cage is 2 m/sec, and the time required to accelerate the cage to the desired velocity is 10 secs. The diameter of the drum is 1.25 m. Determine the safe load that the hoist can handle by assuming a factor of safety as 8. Neglect the impact load on the rope. **(12 Marks)**

b. A leather belt 125 mm wide and 6 mm thick, transmits power from a pulley 750 mm diameter which runs at 500 rpm. The angle of the lap is 150° and the coefficient of friction between the belt and the pulley is 0.3. If the belt density is 1000 kg/m^3 and the stress in the belt is not to exceed 2.75 N/mm^2, find the power that can be transmitted by the belt. Also find the initial tension in the belt. **(08 Marks)**

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USN | | | | | | | | | | 06ME61

Sixth Semester B.E. Degree Examination, December 2011

Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Use of machine data hand book is permitted.

PART – A

1 a. Give the differences between a straight beam and curved beam. **(04 Marks)**

b. The cross section of a steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of the section is 64 mm. The centre of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable strength is limited to 60 MPa. **(16 Marks)**

2 a. Design a helical compression spring to sustain an axial load that fluctuates between 1.5 kN and 2 kN with an associated deflection of 15 mm during the fluctuation of load. **(10 Marks)**

b. An automotive leaf spring is to be designed to consist of 10 graduated leaves and 2 full length leaves. The spring is to support a central load of 5 kN over a span of 1100 mm with the actual band with of 100 mm. The width and thickness of leaves limiting the maximum equalized stress induced in the leaves to 350 MPa. Also determine the initial gap to be provided between the full length and graduated leaves before the assembly. **(10 Marks)**

3 a. Derive Lame's equation for thick cylinder. **(10 Marks)**

b. A circular plate made of steel and of diameter 200 mm with thickness 10 mm is subjected to a load inducing a pressure of 4 MPa. Taking $E = 201$ kN /mm^2, Poisson's ratio = 0.3, determine :

i) The maximum stress, its location and maximum deflection when the edges of the plate are supported

ii) The maximum stress, its location and maximum deflection when the edge of the plate is fixed. **(10 Marks)**

4 a. Derive the Lewis equation for the beam strength of a gear tooth. Also list the assumptions. **(04 Marks)**

b. Design a pair of spur gears to transmit 20 kW of power at a pinion speed of 1000 rpm. The required velocity ratio is 3.5 : 1. 20° stub involutes tooth profile to be used. The static design stress for the pinion is 100 MPa and for the gear is 70 MPa. The pinion has 16 teeth. Determine the module, face width, and pitch circle diameters of the gears based on a service factor is 1.25. **(16 Marks)**

PART – B

5 a. Explain with a sketch, the formulative number of teeth based on bevel gears. **(04 Marks)**

b. A pump is driven by a 30 kW motor through a pair of right angled bevel gear. The speed of the motor is 1200 rpm. The pinion on the motor has a pitch circle diameter of 150 mm and carries 30 teeth and the gear on the pump shaft carries 40 teeth. The pinion made of C_{45} steel untreated where as the gear is made of 0.2 % carbon steel untreated. The teeth are generated to have 20° full depth involute. Check whether the gear pair is safe from the stand point of bending strength. **(16 Marks)**

6 a. A multiple clutch as steel on bronze is to transmit 8 kW at 1440 rpm. The inner diameter of the contact is 80 mm and the outer dia of contact is 140 mm. The clutch operates in oil with expected co–efficient of friction of 0.1, the average allowable pressure is 0.35 MPa. Assume uniform wear theory and determine the following :

i) No. of steel and bronze plates

ii) Axial force required

iii) Actual maximum pressure. **(10 Marks)**

b. A friction cone clutch has to transmit a torque of 200 N/m at 1440 rpm. The large diameter of the cone is 350 mm, the cone pitch angle is 6.25°. The face width is 65 mm. The co–efficient of friction is 0.2. Determine :

i) The axial force required to transmit the torque

ii) The average normal pressure on the contact surface with the maximum torque is transmitted. **(10 Marks)**

7 a. Discuss the mechanism of fluid film lubrication. **(04 Marks)**

b. Design a journal bearing to withstand a load of 5886 N. speed of the journal is 1000 rpm. The journal is made of hardened steel and bearing is made of babbit. Operating temperature is 70°C and ambient temperature is 30°C. Check the design for thermal equilibrium and also determine the power loss at the bearing. The lubricant used is of grade SAE 40 $\ell/d = 1.5$. **(16 Marks)**

8 a. Select a V belt drive to transmit a power of 6 kW from a shaft rotating at 1500 rpm to a parallel shaft to be run at 375 rpm. The distance between the shaft centres is 500 mm. The pitch dia of the smaller grooved pulley car be taken to be 150 mm. The factor of application is to be taken as 1.2. **(10 Marks)**

b. Select a standard v–belt to transmit 30 kW from an AC induction motor rotating at 1500 rpm to a centrifugal pump rotating at 750 rpm. The drive operates continuously for 8 hr /day. Calculate the number of belts. **(10 Marks)**

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USN | | | | | | | | | |

06ME61

Sixth Semester B.E. Degree Examination, June 2012
Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO from each part A and B.***
2. Use of data hand book permitted.
3. Missing data, if any, may be suitably assumed.

PART – A

1 a. Compare the stresses due to a bending moment applied on a straight beam and a curved beam. **(05 Marks)**

b. The parallel sides of a trapezoidal cross section of a crane hook of capacity 50 kN are 100 mm and 60 mm. the depth of the section being 120 mm. The radius of curvature of the inner fibre is 150 mm as shown is the Fig.Q1(b). Determine the stresses at the extreme fibres of the cross section of the crane hook. **(15 Marks)**

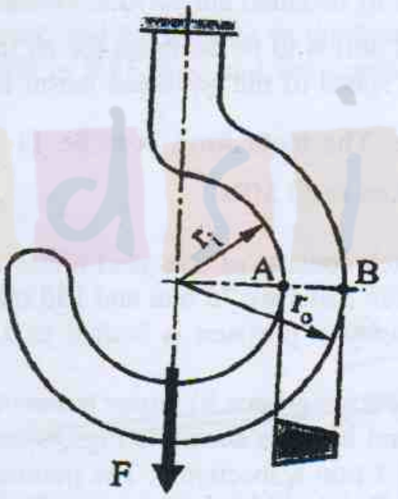

Fig.Q1(b)

2 a. In an air operated press, the piston rod of the operating cylinder must exert a force of 4000 N. The air pressure in the cylinder is 0.7 MPa. Calculate the bore of the cylinder, assuming that overall friction due to stuffing box and piston packing is equivalent to 8% of the maximum force exerted by the piston rod. Determine the thickness of the cylinder assuming that it is a seamless tubing with an allowable stress of 21 MPa. **(06 Marks)**

b. A steel hub 440 mm out side diameter, 250 mm inside diameter and 300 mm length has an interference fit with a shaft of 250 mm diameter. The torque to be transmitted is 30×10^4 N-m. The permissible stress for the material of the shaft and hub is 120 MPa. The coefficient of friction is 0.18. Determine:

i) The contact pressure
ii) Interference required
iii) The tangential stress at the inner and outer surface of the hub.
iv) Force required to assemble
v) Radial stress at the outer and inner diameter of the hub. **(14 Marks)**

3 a. Derive an expression for the stress induced is a helical spring, with usual notations. **(06 Marks)**

b. A carriage weighing 25000 N is moving on track with a linear velocity of 3.6 km/hour. If is brought to rest by two helical compression springs is the from of a bumper by under going a compression of 180 mm. The springs may be assumed to have a spring index of 6 and a permissible shear strength of 450 MPa. Design the spring and determine the diameter of the wire, mean coil diameter and the length of the spring. Assume the modulus of rigidity of the spring material as 81.4 GPa. **(14 Marks)**

4 a. Sketch and explain the different forms of involute gear tooth. **(05 Marks)**

b. A cast steel pinion with an allowable stress of 103 MPa rotating at 900 r/min is to drive a cast iron gear at 1440 r/min. The teeth are 20° stub involute and the maximum power to be transmitted is 25 kW. The allowable stress for cast iron gear is 56 MPa. Determine the module, number of teeth on the gears and face-width from the stand point of strength, dynamic load and wear. **(15 Marks)**

PART – B

5 a. Explain the advantages of worm drive. Write a note on materials used for worm and worm wheel. **(05 Marks)**

b. A speed reduced unit is to be designed for an input power of 0.75 kW with a transmission ratio of 27. The speed of the hardened worm is 1750 r/min. The worm wheel is made of phosphor bronze. The tooth form is to be $14\frac{1}{2}^\circ$ involute. The allowable stress for the wheel may be taken as 80 MPa. **(15 Marks)**

6 a. A multiplate clutch consists of five steel plates and four bronze plates. The inner and outer diameter of friction disks are 75 mm and 150 mm respectively. The coefficient of friction is 0.1 and the intensity of pressure is limited to 0.3 N/mm^2. Assuming uniform wear theory. Calculate:

i) The required operating force ii) Power transmitting capacity at 750 r/min. **(10 Marks)**

b. A differential band brake is shown is Fig.Q6(b). The width and thickness of the steel band are 100 mm and 3 mm respectively. The permissible tensile stress is the band is limited to 50 MPa. The coefficient of friction between the friction lining and the drum is 0.25. Calculate:

i) Tensions is the band ii) The actuating force iii) Torque capacity of the brake. **(10 Marks)**

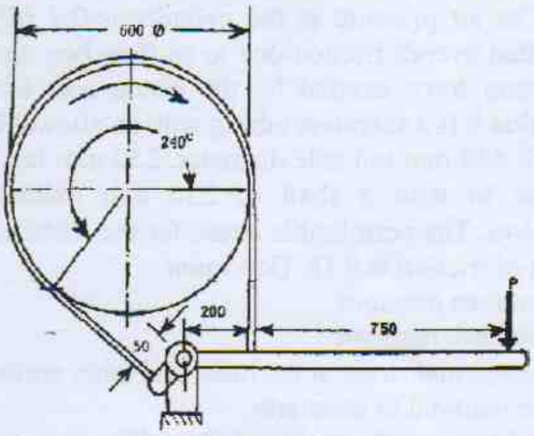

Fig.Q6(b)

7 a. Explain the properties a good bearing material should posses. List the different types of bearing materials. **(06 Marks)**

b. The following data are given for a full journal bearing:
Radial load: 25 kN
L/d ratio: 1:1
Unit bearing pressure: 2.5 MPa.
Viscosity of the lubricant: 20 Cp.
Class of fit: H7 e7.
Calculate:
   i) Dimensions of the bearing
   ii) Minimum oil film thickness.
   iii) Requirement of oil flow.
Assume that the process to clearance is centered. **(14 Marks)**

8 a. Explain the advantages and applications of chain drives. **(05 Marks)**

b. The layout of the leather belt drive transmitting 15 kW power is shown in Fig.Q8(b). The centre distance between the pulleys is twice the diameter of the big pulley. The belt should operate at a velocity of 20 m/sec and the stresses in the belt should not exceed 2.25 MPa. The density of the leather belt is 0.95 g/cc and the coefficient of friction is 0.35. The thickness of the belt is 5 mm.
Calculate:
   i) Diameter of the pulleys
   ii) The length and width belts.
   iii) Belt tensions.

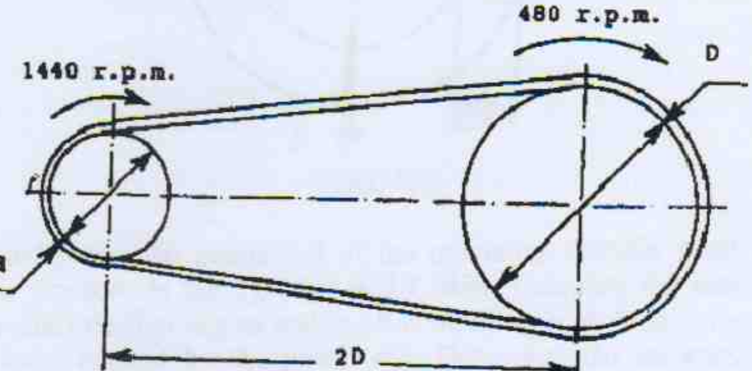

Fig.Q8(b)

(15 Marks)

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USN | | | | | | | | | | | **10ME62**

Sixth Semester B.E. Degree Examination, June/July 2015
Design Machine Elements – II

Time: 3 hrs.   Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Use of machine design data handbook is permitted.

PART – A

1 a. Determine the value of 't' in the cross section of a curved machine member shown in Fig. Q1(a), so that the normal stresses due to bending at extreme fibers are numerically equal. Also determine the normal stresses so induced at extreme fibers due to a bending moment of 10 KN – m. **(10 Marks)**

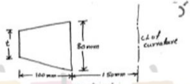

Fig. Q1(a)

b. A cast iron cylindrical pipe of outside diameter 300 mm and inside diameter 200 mm is subjected to an internal fluid pressure of 20 N/mm^2 and external fluid pressure of 5 N/mm^2. Determine the tangential and radial stresses at the inner, middle and outer surface. Sketch the tangential and radial stress distribution across its thickness. **(10 Marks)**

2 a. A nylon core flat belt 200 mm wide weighing 20 N/m, connecting a 300mm diameter pulley to a 900 mm diameter driven pulley at a shaft spacing of 6 m, transmits 55.2 kW at a belt speed of 25 m/sec i) calculate the belt length and the angles of wrap ii) compute the belt tensions based on a co-efficient of friction 0.38. **(10 Marks)**

b. Two shafts one metre apart are connected by a V – belt to transmit 90 kW at 1200 rpm of a driver pulley of 300 mm effective diameter. The driven pulley rotates at 400 rpm. The angle of groove is 40° and the co-efficient of friction between the belt and the pulley rim is 0.25. The area of the belt section is 400 mm^2 and the permissible stress is 2.1 MPa. Density of belt material is 1100 kg/m^3. Calculate the number of belts required and the length of the belt. **(10 Marks)**

3 a. A railway wagon weighting 50 kN and moving with a speed of 8 km/hr has to be stopped by four buffer springs in which the maximum compression allowed is 220 mm. Find the number of turns or coils in each spring of mean diameter 150mm. The diameter of spring wire is 25 mm. Take G = 84 GPa. Also find the shear stress. **(10 Marks)**

b. A multi leaf spring with camber is fitted to the chassis of an automobile over a span of 1.2 m to absorb shocks due to a maximum load of 20 kN. The spring material can sustain a maximum stress of 0.4 GPa. All the leaves of the spring were to receive the same stress. The spring is required at least 2 full length leaves out of 8 leaves. The leaves are assembled with bolts over a span of 150 mm width at the middle. Design the spring for a maximum deflection of 50 mm. **(10 Marks)**

4 Design a bronze spur gear 81.4 MN/m^2 and mild steel pinion 101 MN/m^2 to transmit 5 KW at 1800 rpm. The velocity ratio is 3.5 : 1. Pressure angle is 14½°. Not less than 15 teeth are to be used on either gear. Determine the module and face width. Also suggest suitable surface hardness for the weaker member based on dynamic and wear considerations. **(20 Marks)**

PART – B

5 a. A pair of mitre gears have pitch diameter 280 mm and face width of 36 mm and run at 250 rpm. The teeth are of $14\frac{1}{2}°$ involute and accurately cut and transmit 6 KW. Neglecting friction angle, find the following : i) outside diameter of gears ii) resultant tooth load tangent to pitch cone iii) radial load on the pinion iv) trust on the pinion. Assume low carbon cast steel 0.2 %C heat treated as the material for both the gears. **(12 Marks)**

b. The following data refer to a worm and worm gear drive that has to transmit 15 KW at 1750 rpm of the worm. Centre distance = 200 mm number of starts = 4, transmission ratio = 20 pitch circle diameter of worm = 80 mm, axial module = 8 mm tooth form = 20° FDI. The worm gear has an allowable bending stress of 55 MPa. The worm is made of hardened and ground steel. Determine : i) the number of teeth on the worm gear ii) the lead angle iii) face width of the worm gear based on the beam strength of the worm gear. **(08 Marks)**

6 a. In a multiple disc clutch the radial width of the friction material is to be 0.2 of maximum radius. The co-efficient of friction is 0.25. The clutch is to transmit 60 KW at 3000 rpm. Its maximum diameter is 250 mm and the axial force is limited to 600 N. Determine i) number of driving and driven discs ii) mean unit pressure on each contact surface. Assume uniform wear. **(10 Marks)**

b. A differential band brake shown in Fig. Q6(b) operates on a drum diameter of 500 mm. The drum rotates at 300 rpm in counter clockwise direction and absorbs 36 KW, $\mu = 0.25$ determine : i) force F required to operate the brake ii) width of band required for this brake if thickness is 5 mm and allowable tensile stress on band material is 72 N/mm^2 iii) design the lever if the maximum force is twice that of calculated force. Use C30 steel ($\sigma_u = 540$ MPa) and FOS = 4 based on ultimate stress. And also depth equal to thrice the width. **(10 Marks)**

Fig.6Q(b)

7 a. Derive Pertroff's equation for a lightly loaded bearing. **(10 Marks)**

b. A full journal bearing 50 mm in diameter and 50 mm long operates at 1000 rpm and carries a load 5 kN. The radial clearance is 0.025 mm. The bearing is lubricated with SAE 30 oil and the operating temperature of oil is 80°C. Assume the attitude angle as 60°. Determine : i) bearing pressure ii) sommerfield number iii) attitude iv) minimum film thickness v) heat generated vi) heat dissipated if the ambient temperature is 20°C vii) amount of artificial cooling if necessary. **(10 Marks)**

8 Design a suitable aluminium alloy piston with two compression rings and one oil ring for a petrol engine of following particulars :

Cylinder diameter	= 0.10 m
Peak gas pressure	= 3.2 MPa
Mean effective pressure	= 0.8 MPa
Average side thrust	= 2400 N
Skirt bearing pressure	= 0.22 MPa
Bending stress in piston crown	= 36 MPa
Crown temperature difference	= 70°C.
Heat dissipated through crown	= 157 kJ/m^{2}s = 157 KW/m^2
Allowable radial pressure	= 0.04 MPa
Bending piston on rings	= 90 MPa
Heat conductivity k	= 160 W/m/°C

Assume any further data required for the design. **(20 Marks)**

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USN | | | | | | | | | | **10ME62**

Sixth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of data handbook is permitted.
3. Missing data may be suitably assumed.

PART – A

1 a. A curved link mechanism made from a round steel bar is shown in Fig.Q1(a). The material for link is plain carbon steel 30C8 with an allowable yield strength of 400 MPa. Determine the factor of safety. **(10 Marks)**

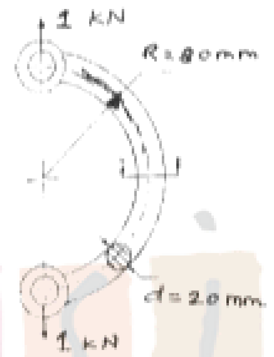

Fig.Q1(a)

b. A high pressure cylinder consists of a steel tube with inner and outer diameter of 20 mm and 40 mm respectively. It is jacketed by an outer steel tube with an outer diameter of 60 mm. The tubes are assembled by shrinking process in such a way that maximum principal stress induced in any tube is limited to 100 MPa. Calculate the shrinkage pressure and original dimensions of the tubes. Take the Young's modulus as 207 GPa. **(10 Marks)**

2 a. Write a note on construction of flat and 'V' belt. **(05 Marks)**

b. It is required to design a 'V' belt drive to connect a 7.5 kW, 1440 r/min induction motor to a fan, running at approximately 480 r/min for a service of 24 hr/day. Space is available for a centre distance of about 1 m. Determine the pitch length of the belt and number of belts required. **(15 Marks)**

3 a. Enumerate the applications of springs. Also derive an expression for the deflection of a close coiled helical spring. **(06 Marks)**

b. A spring is subjected to a load varying from 500 N and 1200 N. It is to be made of oil tempered cold drawn wire. Design factor based on Wahl's line is 1.25. The spring index is to be 6. The compression in the spring for the maximum load is 30 mm. Determine the wire diameter, mean coil diameter and free length of the spring. Take the yield stress in shear as 700 MPa and endurance stress in shear as 350 MPa for the material of the wire. **(14 Marks)**

4 a. Write a note on design of gears based on dynamic loading and wear. **(06 Marks)**

b. A cast steel 24 teeth spur pinion operating at 1150 r/min transmits 3 kW to a cast steel 56 teeth spur gear. The gears have the following specifications:

Module : 3 mm	Allowable stress : 100 MPa
Face width : 35 mm	Tooth form : 14½° full depth profile
Factor of dynamic loading, C= 350N/mm	Wear load factor, K = 0.28 MPa.

Determine the induced stress in the weaker gear. Also determine the dynamic load and wear load. Comment on the results. **(14 Marks)**

PART – B

5 a. Write a note on formative number of teeth in bevel gear. **(04 Marks)**

b. Hardened steel worm rotates at 1250 r/min and transmits power to a phosphor bronze gear with a transmission ratio of 15:1. The centre distance is to be 225 mm. Design the gear drive and give estimated power input ratings from the stand point of strength, endurance and heat dissipation. The teeth are of 14½° full depth involute. **(16 Marks)**

6 a. A cone clutch has a semi cone angle of 12°. It is to transmit 10 kW power at 750 r/min, the width of the face is one fourth of the mean diameter of friction lining. If the normal intensity of pressure between contacting surfaces is not to exceed 0.085 N/mm^2 and the coefficient of friction is 0.2, assuming uniform wear conditions, calculate the dimensions of the clutch. **(10 Marks)**

b. A band break arrangement is shown in Fig.Q6(b). It is used to generate a maximum braking torque of 200 N-m. Determine the actuating force 'P', if the coefficient of friction is 0.25. The angle of wrap of the band is 270°. Determine the maximum intensity of pressure, if the band width is 30 mm. **(10 Marks)**

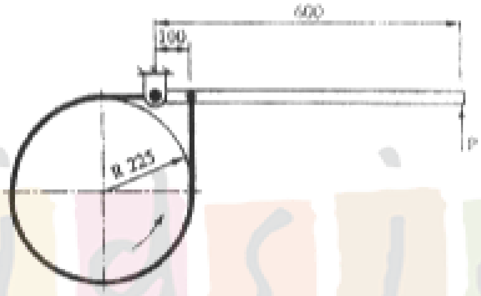

Fig.Q6(b)

7 a. Explain the following types of lubrication:
(i) Hydrodynamic lubrication (ii) Hydrostatic lubrication
(iii) Boundary lubrication (iv) Elasto hydro dynamic lubrication. **(08 Marks)**

b. The following data are given for a 360° hydro-dynamic bearing:

Bearing diameter : 50.02 mm Journal diameter : 49.93 mm
Bearing length : 50 mm Journal speed : 1440 r/min
Radial load = 8 kN Viscosity of lubricant : 12 cp.

The bearing is machined on a lathe from bronze casting, while the steel journal is hardened and ground. The surface roughness values for turning and grinding are 0.8 and 0.4 microns respectively. For thick film lubrication the minimum film thickness should be five times the sum of surface roughness values for the journal and the bearing. Calculate:
(i) The permissible minimum film thickness
(ii) The actual film thickness under the operating conditions
(iii) Power loss in friction.
(iv) Flow requirement. **(12 Marks)**

8 a. Explain the considerations given in the design of pistons for IC engines. **(05 Marks)**

b. Design a trunk piston for an IC engine. The piston is made of cast iron with an allowable stress of 38.5 MPa. The bore of the cylinder is 200 mm and the maximum explosion pressure is 0.4 MPa. The permissible bending stress of the material of the gudgeon pin is 100 MPa. The bearing pressure in the gudgeon pin bearing of the connecting rod is to be taken as 200 MPa. **(15 Marks)**

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USN

10ME62

Sixth Semester B.E. Degree Examination, Dec.2014/Jan.2015

Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of design data hand book permitted.
3. Missing data, if any may be suitably assumed.

PART – A

1 a. Determine the dimensions of I-section, shown in Fig. Q1 (a) in which maximum fibre stresses are numerically equal in pure bending, given $b_i + b_0 = 120$ mm **(10 Marks)**

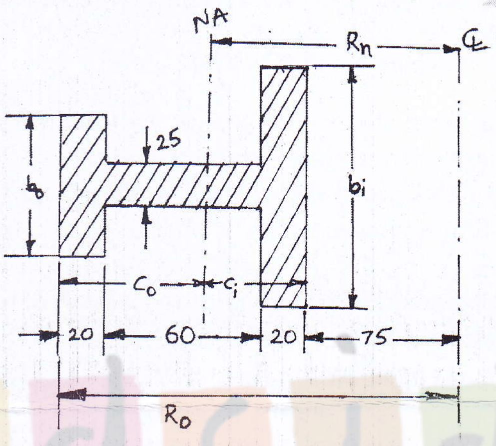

Fig. Q1 (a)

b. A solid shaft of 125 mm diameter is to be pressed into a steel flange which has an outside diameter of 150 mm and length of 100 mm. Take E = 210 GPa, ν = 0.3. Determine,
i) Pressure between hub and shaft.
ii) Proper size of bore, if maximum stress does not exceed 160 MPa.
iii) Force required to press the parts together. Assume μ = 0.1 for press fit.
iv) Torque capacity of press fit. **(10 Marks)**

2 a. Select a V-belt drive to transmit 18 kW at 1500 rpm to another pulley to run at 750 rpm. The dia of smaller pulley is 100 mm. The centre distance is 2 times the diameter of larger pulley. **(10 Marks)**

b. Select a number of wire ropes required to be used with a drum diameter of 2 meter. A load of 30 kN is to be lifted by 25 mm diameter 6×7 ropes form a height of 450 meter. A velocity of 15 m/sec is to be attained in 10 second. Assume the skip weight to be 30% of load capacity and factor of safety of 6. **(10 Marks)**

3 a. A free end of a torsional spring deflects through 60° when subjected to a torque of 6 N-m. The allowable stress in the spring material is 400 MPa and the spring index is 6. Determine the wire diameter and the number of effective turns. Take $E = 206.8\times10^3$ N/mm^2. **(08 Marks)**

b. A Belleville spring made from 5 mm thick steel sheet having outside diameter 150 mm and inside diameter 70 mm. The height of the spring is 10 mm. Using γ = 0.3, $E = 20\times10^4$ MPa. Find i) The deflectionn of spring. ii) The load the spring can carry. iii) Stress at the outer edge.
Limit the maximum stress at the inner edgee to 450 MPa. **(12 Marks)**

4 a. Design a pair of spur gear to transmit 40 kW at 4000 rpm of pinion to the gear 800 rpm. Select Chromium – Nickel steel for both gears. Assume pinion teeth as 20 and service factor as 1.5. Determine dynamic load, wear load and recommend BHN values. Assume $\alpha = 20^0$ FDI. **(12 Marks)**

b. A 24 teeth cast steel gear pinion ($\sigma_{0_1} = 51.7$ MPa) drives a high grade C.I. gear having ($\sigma_{0_2} = 31$ MPa) 50 teeth. The teeth are 20° full depth involute in the normal plane. The helix angle is 45°. Normal module is 3 mm. Find the safe power that can be transmitted by these gears at a pinion speed of 500 rpm. Assume face width is 10 times normal module and scant lubrication $C_W = 1.25$. **(08 Marks)**

PART – B

5 a. Design a pair of mitre bevel gears to transmit 9 kW at 1200 rpm. The pitch line velocity of gear is not exceed 15 m/sec. **(12 Marks)**

b. A hardened steel worm at 1250 rpm transmits power to a phosphor bronze gear with a transmission ratio of 20 : 1. The centre distance is 200 mm. Determine the input power capacity. Assume $\alpha = 14\frac{1}{2}^\circ$ FDI. **(08 Marks)**

6 a. Design a cone clutch to transmit 15 kW at 1200 rpm. Assume semi cone angle as 12.5°, co-efficient of friction for lining 0.4 and P = 0.2 MPa. **(08 Marks)**

b. A single band brake shown in Fig. Q6 (b) is to be designed to stop the rotation of a shaft transmitting a power of 45 kW at a rated speed of 500 rpm. Selecting suitable materials determine,

i) Dimensions of rectangular cross section of band.
ii) Dimensions of rectangular cross section of brake lever. (Assume $h_1 = 2\,b_1$).
iii) Diameter of fulcrum pin.

Assume $l_p = 1.5$ dp, bearing stress $\sigma_b = 10$ MPa. **(12 Marks)**

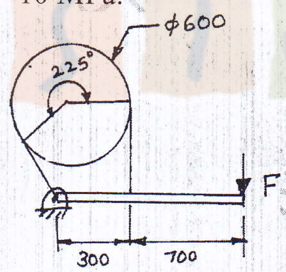

Fig. Q6 (b)

7 a. A full journal bearing of length 100 mm and the journal diameter of 80 mm supports a load of 2.5 kN at 600 rpm. What viscosity oil should be used to limit the bearing surface temperature to 60°C . The room temperature is 20°C and the clearance ratio is 0.001, use Mckee's equation. **(08 Marks)**

b. Determine the power loss for a Petroff bearing 100 mm in diameter and 150 mm long. The radial clearance is 0.05 mm. Speed of the journal is 1000 rpm. The lubricating oil is SAE10 and bearing operating temperature is 60°C. **(12 Marks)**

8 a. List and explain the functions of parts of internal combustion engine. **(04 Marks)**

b. Design a cast iron trunk type piston for four stroke internal combustion engine for the following data: Cylinder bore = 150 mm, Stroke = 200 mm, Indicated mean effective pressure = 6 bar, Fuel consumption = 4 kg/hr, Maximum gas pressure = 5 MPa,
Higher calorific value of fuel = 4100 KJ/kg, Speed of engine = 600 rpm,
Mechanical efficiency = 75% , Allowable stress for piston = 30 N/mm^2,
Allowable tensile stress for piston ring = 90 MPa,
Allowable bending stress in piston pin = 80 MPa. **(16 Marks)**

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USN | | | | | | | | | | | **10ME62**

Sixth Semester B.E. Degree Examination, Dec.2016/Jan.2017
Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of design data hand book is permitted.

PART – A

1 a. Crane hook of trapezoidal cross-section with an inner side of 120mm and outer side of 60mm. The depth of the section is 90mm. The centre of curvature is at a distance of 120mm from the inner edge of the section and the line of action of load is at a distance of 135mm from the inner edge. Determine the safe load that the hook can carry if it is made of steel having an allowable stress of 90 MPa. **(10 Marks)**

b. A 100mm inside and 150mm outside sleeve is press fitted on to a shaft of 100mm diameter? The modulus of elasticity of material is 210 GPa and Poisson ratio is 0.28. The contact pressure is not to exceed 60 MPa. Determine:
i) Tangential stress at inner and outer surface of the sleeve and outside diameter of the shaft.
ii) The radial stresses in the sleeve and shaft.
iii) The original diameters of the shaft and hub before press fit.
iv) The total interference. **(10 Marks)**

2 a. For a flat belt drive, the following data are given power transmitted = 9kW, speed of motor = 1500rpm, speed of driven pulley = 500 rpm, velocity of belt 16 m/sec, load factor = 1.2, density of leather = 9.8 kN/m^3. Small diameter to thickness of belt ratio = 36, factor of safety = 10, ultimate strength of belt material = 24 MPa, centre distance = 2.1m, coefficient of friction = 0.36. Design the belt. **(10 Marks)**

b. Select a r-belt drive to transmit 9 kW from a shaft rotating at 1200rpm to a parallel shaft to run at 300rpm. The diameter of smaller pulley is 120mm. The centre distance between shafts is 1.2m. **(10 Marks)**

3 a. Design a rectangular section helical spring to mount a buffer to sustain a load of 30kN. The deflection under load is 90mm. The spring is made of Z-nickel having a torsional ultimate stress of 830 MPa. The longer side of rectangle is twice the shorter side and the spring is wound with longer side of rectangle parallel to the axis. The spring index is 10. Take factor of safety = 2.5 and G = 75.51 GPa. **(12 Marks)**

b. A laminated spring having 6 graduated leaves is simply supported at ends at a distance of 0.9m. It is made of steel having allowable bending stress of 360 MPa. The width and thickness of leaves are 90mm and 6mm. Find the safe load that can be carried by this spring at the middle and the deflection under that load. Take $\in$ = 206 GPa. **(08 Marks)**

4 Design a pair of steel spur gears required to transmit 12kW at 2000 rpm of pinion. The velocity ratio received is 2.5:1. The allowable static stress for both may be taken as 138 MPa. Not less than 24 teeth are to be used on either gear. The teeth are 20° stub teeth. **(20 Marks)**

PART – B

5 Two shafts inclined at 60° are connected by a pair of bevel gears to transmit 9kW at 900rpm of 24 tooth cast steel pinion having allowable static stress of 138 MPa. The gear is made of high grade CI having allowable static stress of 103 MPa and is to run at 300rpm. The teeth are 14 ½° involute form. Design the gears completely. **(20 Marks)**

6 a. A multiplate clutch consists of 5 steel and 4 bronze plates. The inner and outer diameters of friction discs are 75mm and 150mm respectively. The coefficient of friction is 0.1 and allowable pressure is to be limited to 0.3 MPa. Assuming uniform pressure. Calculate:
i) The required axial force.
ii) Power that can be transmitted at 750 rpm. **(10 Marks)**

b. A 360mm radius brake drum contacts a single shoe as shown in Fig.Q.6(b) and sustains a power of 23.5 kW at 1000 rpm. Determine:
i) The normal force F_n on the shoe.
ii) The tangential force.
iii) The operating force for clockwise rotation.
iv) The value of distance 'C' for the brake to be self locking and
v) The rate of heat generated. **(10 Marks)**

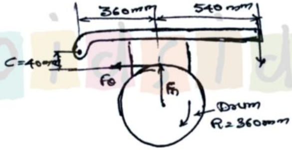

Fig.Q.6(b)

7 a. Derive the Petroff's equation for coefficient of friction. **(08 Marks)**

b. A full journal bearing 90mm diameter and 150mm long has a radial load of 2MPa per unit projected area. Shaft speed is 500rpm. The bearing is operating with SAE 20 oil at 50°C. The specific gravity of oil at the operating temperature is 0.985. Calculate the following:
i) Minimum film thickness
ii) Heat lost due to friction
iii) Whether artificial cooling is necessary. **(12 Marks)**

8 Design a cast iron piston for a single acting four stroke engine for the following data:
Cylinder bore: 100mm, stroke = 125mm, maximum gas pressure = 5 N/mm^2, indicated mean effective pressure = 0.75 N/mm^2, mechanical efficiency = 80%, fuel consumption = 0.15 kg, per brake power per hour, higher calorific value of fuel = 42 × 10^3 kJ/kg, speed = 2000 rpm. **(20 Marks)**

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10ME62

Sixth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of data handbook is permitted.
3. Missing data may be suitably assumed.

PART – A

1 a. A curved link mechanism made from a round steel bar is shown in Fig.Q1(a). The material for link is plain carbon steel 30C8 with an allowable yield strength of 400 MPa. Determine the factor of safety. **(10 Marks)**

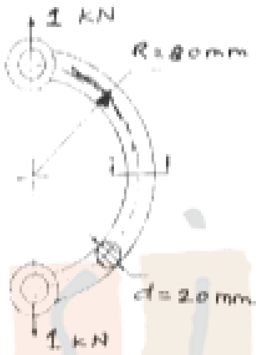

Fig.Q1(a)

b. A high pressure cylinder consists of a steel tube with inner and outer diameter of 20 mm and 40 mm respectively. It is jacketed by an outer steel tube with an outer diameter of 60 mm. The tubes are assembled by shrinking process in such a way that maximum principal stress induced in any tube is limited to 100 MPa. Calculate the shrinkage pressure and original dimensions of the tubes. Take the Young's modulus as 207 GPa. **(10 Marks)**

2 a. Write a note on construction of flat and 'V' belt. **(05 Marks)**

b. It is required to design a 'V' belt drive to connect a 7.5 kW, 1440 r/min induction motor to a fan, running at approximately 480 r/min for a service of 24 hr/day. Space is available for a centre distance of about 1 m. Determine the pitch length of the belt and number of belts required. **(15 Marks)**

3 a. Enumerate the applications of springs. Also derive an expression for the deflection of a close coiled helical spring. **(06 Marks)**

b. A spring is subjected to a load varying from 500 N and 1200 N. It is to be made of oil tempered cold drawn wire. Design factor based on Wahl's line is 1.25. The spring index is to be 6. The compression in the spring for the maximum load is 30 mm. Determine the wire diameter, mean coil diameter and free length of the spring. Take the yield stress in shear as 700 MPa and endurance stress in shear as 350 MPa for the material of the wire. **(14 Marks)**

4 a. Write a note on design of gears based on dynamic loading and wear. **(06 Marks)**

b. A cast steel 24 teeth spur pinion operating at 1150 r/min transmits 3 kW to a cast steel 56 teeth spur gear. The gears have the following specifications:

Module : 3 mm	Allowable stress : 100 MPa
Face width : 35 mm	Tooth form : $14\frac{1}{2}°$ full depth profile
Factor of dynamic loading, C= 350N/mm	Wear load factor, K = 0.28 MPa.

Determine the induced stress in the weaker gear. Also determine the dynamic load and wear load. Comment on the results. **(14 Marks)**

PART – B

5 a. Write a note on formative number of teeth in bevel gear. **(04 Marks)**

b. Hardened steel worm rotates at 1250 r/min and transmits power to a phosphor bronze gear with a transmission ratio of 15:1. The centre distance is to be 225 mm. Design the gear drive and give estimated power input ratings from the stand point of strength, endurance and heat dissipation. The teeth are of 14½° full depth involute. **(16 Marks)**

6 a. A cone clutch has a semi cone angle of 12°. It is to transmit 10 kW power at 750 r/min, the width of the face is one fourth of the mean diameter of friction lining. If the normal intensity of pressure between contacting surfaces is not to exceed 0.085 N/mm^2 and the coefficient of friction is 0.2, assuming uniform wear conditions, calculate the dimensions of the clutch. **(10 Marks)**

b. A band break arrangement is shown in Fig.Q6(b). It is used to generate a maximum braking torque of 200 N-m. Determine the actuating force 'P', if the coefficient of friction is 0.25. The angle of wrap of the band is 270°. Determine the maximum intensity of pressure, if the band width is 30 mm. **(10 Marks)**

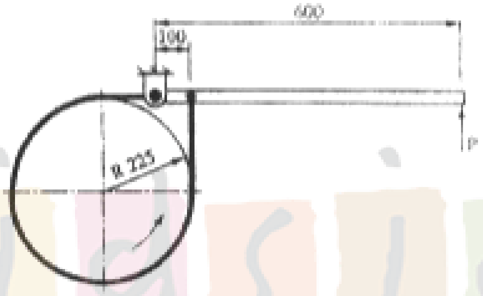

Fig.Q6(b)

7 a. Explain the following types of lubrication:

(i) Hydrodynamic lubrication (ii) Hydrostatic lubrication

(iii) Boundary lubrication (iv) Elasto hydro dynamic lubrication. **(08 Marks)**

b. The following data are given for a 360° hydro-dynamic bearing:

Bearing diameter : 50.02 mm Journal diameter : 49.93 mm

Bearing length : 50 mm Journal speed : 1440 r/min

Radial load = 8 kN Viscosity of lubricant : 12 cp.

The bearing is machined on a lathe from bronze casting, while the steel journal is hardened and ground. The surface roughness values for turning and grinding are 0.8 and 0.4 microns respectively. For thick film lubrication the minimum film thickness should be five times the sum of surface roughness values for the journal and the bearing. Calculate:

(i) The permissible minimum film thickness

(ii) The actual film thickness under the operating conditions

(iii) Power loss in friction.

(iv) Flow requirement. **(12 Marks)**

8 a. Explain the considerations given in the design of pistons for IC engines. **(05 Marks)**

b. Design a trunk piston for an IC engine. The piston is made of cast iron with an allowable stress of 38.5 MPa. The bore of the cylinder is 200 mm and the maximum explosion pressure is 0.4 MPa. The permissible bending stress of the material of the gudgeon pin is 100 MPa. The bearing pressure in the gudgeon pin bearing of the connecting rod is to be taken as 200 MPa. **(15 Marks)**

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USN | | | | | | | | | | 10ME62

Sixth Semester B.E. Degree Examination, June / July 2014
Design of Machine Elements – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. ***Design data handbook is permitted.***

PART – A

1 a. Determine the maximum tensile stress and maximum shear stress of the component shown in Fig. Q1(a) and indicate the location. **(10 Marks)**

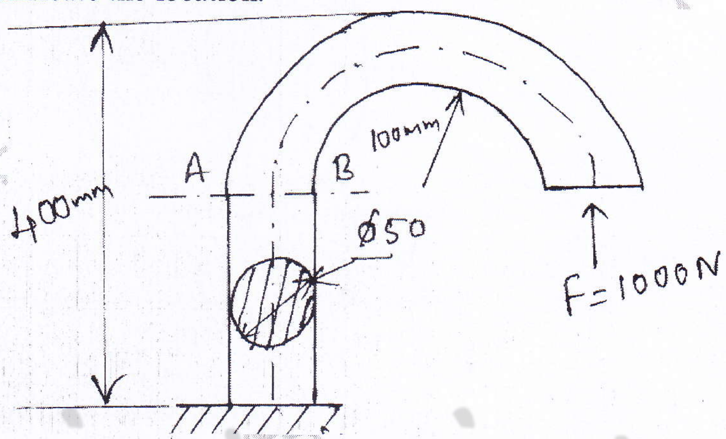

Fig. Q1 (a)

b. A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm^2. Calculate the tangential and radial stresses at the inner, middle and outer surface. **(10 Marks)**

2 a. A compressor is driven by a motor of 2.5 kW, running at 1200 rpm to a 400 rpm compressor. Select a suitable V-belt. **(10 Marks)**

b. Explain Hoisting tackle mechanism to raise and lowering load for a rope. **(10 Marks)**

3 a. Derive an expression for the stres induced in a helical spring with usual notations. **(10 Marks)**

b. Design a leaf spring for the following specification for a truck total load = 120 KN, number of springs = 4, material for the spring is chrome-vanadium steel permissible stress in 0.55 GPa. Span of spring = 1100 mm, width of central band = 100 mm and allowable deflection = 80 mm, number of full length leaves are 2 and graduated leaves 6. **(10 Marks)**

4 Design a pair of spur gear to transmit a power of 18 kW from a shaft running at 1000 rpm to a parallel shaft to be run at 250 rpm maintaining a distance of 160 mm between the shaft centres. Suggest suitable surface hardness for the gear pair. **(20 Marks)**

PART – B

5 A pair of bevel gear wheels with 20° pressure angle consists of 20 teeth pinion meshing with 30 teeth gear. The modulus is 4 mm while is 20 mm. The surface hardness of both pinion and gear is 400 BHN. The pinion rotates at 500 rpm and receives power from an electric motor. The starting torque of the motor is 150 percent of the rated torque. Determine the safe power that can be transmitted considering the dynamic load wear strength and endurance strength. The allowable bending stress may be taken as 240 MPa. **(20 Marks)**

6 a. A plate clutch with a maximum diameter of 600 mm has maximum lining pressure of 0.35 MPa. The power to be transmitted at 400 rpm is 135 kW and μ= 0.3. Find inside diameter and spring force required to engage the clutch, if the spring with spring index 6 and material of spring the wire diameter if 6 springs are used. **(10 Marks)**

b. The torque absorbed in the band break shown in Fig. Q6 (b) is 400×10^3 Nmm. Design the band and lever taking μ = 0.27 and diameter of drum as 400 mm. The allowable stress in band may be taken as 70 N/mm^2. **(10 Marks)**

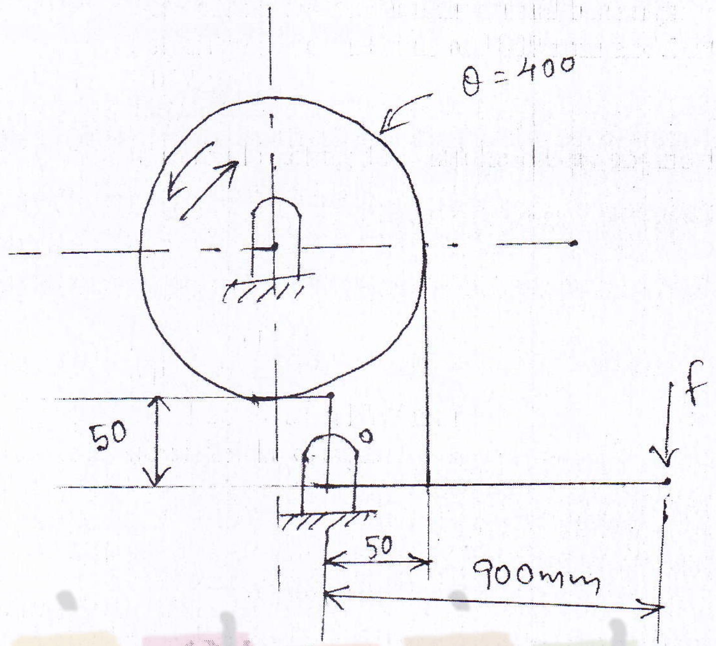

Fig. Q6 (b)

7 a. Derive Petroff's equation for co-efficient of friction in journal bearings. **(08 Marks)**

b. Design the main bearing of a steam turbine that runs at 1800 rpm and 70°C . The load on the bearing is estimated to be 2500 N. **(12 Marks)**

8 Design a cast iron piston for a single acting four stroke diesel engine from the following data:

Cylinder bore = 100 mm
Length of stroke = 125 mm
Speed = 2000 rpm
Brake mean effective pressure = 0.5 MPa,
Maximum gas pressure = 5 MPa,
Fuel consumption = 0.25 kg/ Brake Power in kW/hour
Assume any further data required for the design. **(20 Marks)**

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