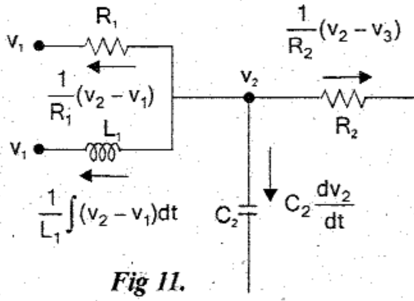


Fig 11.

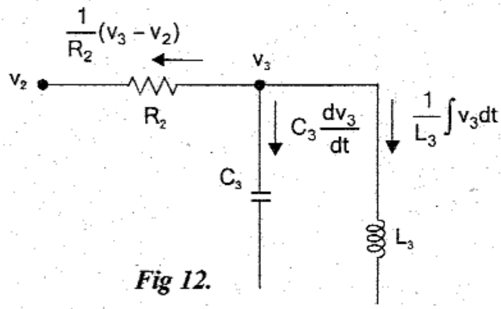


Fig 12.

The electrical analogous elements for the elements of mechanical rotational system are given below.

$T \rightarrow i(t)$	$\omega_1 \rightarrow v_1$	$J_1 \rightarrow C_1$	$B_1 \rightarrow 1/R_1$	$K_1 \rightarrow 1/L_1$
	$\omega_2 \rightarrow v_2$	$J_2 \rightarrow C_2$	$B_2 \rightarrow 1/R_2$	$K_3 \rightarrow 1/L_3$
	$\omega_3 \rightarrow v_3$	$J_3 \rightarrow C_3$		

The node basis equations using Kirchoff's current law for the circuit shown in fig 9 are given below (Refer fig 10, 1 and 12).

$$C_1\frac{dv_1}{dt} + \frac{1}{R_1}(v_1 - v_2) + \frac{1}{L_1}\int(v_1 - v_2)dt = i(t) \quad .....(10)$$

$$C_2\frac{dv_2}{dt} + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{R_2}(v_2 - v_3) + \frac{1}{L_1}\int(v_2 - v_1)\,dt = 0 \quad .....(11)$$

$$C_3\frac{dv_3}{dt} + \frac{1}{R_2}(v_3 - v_2) + \frac{1}{L_3}\int v_3\,dt = 0 \quad .....(12)$$

It is observed that the node basis equations (10), (11) and (12) are similar to the differential equations (4), (5) and (6 governing the mechanical system.

1.11 BLOCK DIAGRAMS

A control system may consist of a number of components. In control engineering to show the functions performed by each component, we commonly use a diagram called the block diagram. A ***block diagram*** of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. The elements of a block diagram are ***block***, ***branch point*** and ***summing point.***

BLOCK

In a block diagram all system variables are linked to each other through functional blocks. The ***functional block*** or simply ***block*** is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Figure 1.25 shows the block diagram of functional block.

The arrowhead pointing towards the block indicates the input, and the arrowhead leading away from the block represents the output. Such arrows are referred to as signals. The output signal from the block is given by the product of input signal and transfer function in the block.

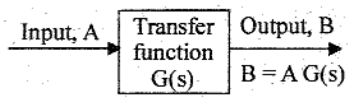


Fig 1.25 : *Functional block.*

SUMMING POINT

Summing points are used to add two or more signals in the system. Referring to figure 1.26, a circle with a cross is the symbol that indicates a summing operation.

The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

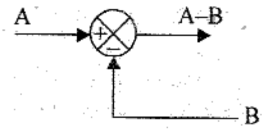


Fig 1.26 : Summing point.

BRANCH POINT

A ***branch point*** is a point from which the signal from a block goes concurrently to other blocks or summing points.

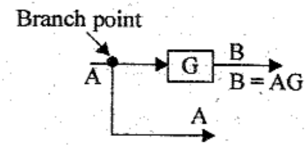


Fig 1.27 : Branch point.

CONSTRUCTING BLOCK DIAGRAM FOR CONTROL SYSTEMS

A control system can be represented diagramatically by block diagram. The differential equations governing the system are used to construct the block diagram. By taking Laplace transform the differential equations are converted to algebraic equations. The equations will have variables and constants. From the working knowledge of the system the input and output variables are identified and the block diagram for each equation can be drawn. Each equation gives one section of block diagram. The output of one section will be input for another section. The various sections are interconnected to obtain the overall block diagram of the system.

EXAMPLE 1.14

Construct the block diagram of armature controlled dc motor.

SOLUTION

The differential equations governing the armature controlled dc motor are (refer section 1.7),

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \qquad .....(1)$$

$$T = K_t i_a \qquad .....(2)$$

$$T = J\,\frac{d\omega}{dt} + B\omega \qquad .....(3)$$

$$e_b = K_b\,\omega \qquad .....(4)$$

$$\omega = \frac{d\theta}{dt} \qquad .....(5)$$

On taking Laplace transform of equation (1) we get,

$$V_a(s) = I_a(s)\,R_a + L_a\,s\,I_a(s) + E_b(s) \qquad .....(6)$$

In equation (6), $V_a(s)$ and $E_b(s)$ are inputs and $I_a(s)$ is the output. Hence the equation (6) is rearranged and the block diagram for this equation is shown in fig 1.

$$V_a(s) - E_b(s) = I_a(s)\,[R_a + s\,L_a]$$

$$\therefore I_a(s) = \frac{1}{R_a + s\,L_a}\,[V_a(s) - E_b(s)]$$

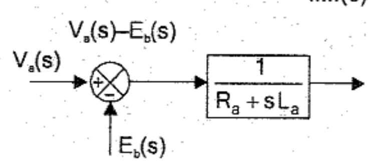


Fig 1.

On taking Laplace transform of equation (2) we get,

$$T(s) = K_t\,I_a(s) \qquad .....(7)$$

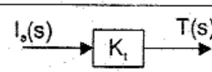


Fig 2.

In equation (7), $I_a(s)$ is the input and $T(s)$ is the output. The block diagram for this equation is shown in fig 2.

On taking Laplace transform of equation (3) we get,

$$T(s) = Js\,\omega(s) + B\,\omega(s) \quad .....(8)$$

In equation (8), T(s) is the input and $\omega(s)$ is the output. Hence the equation (8) is rearranged and the block diagram f this equation is shown in fig (3).

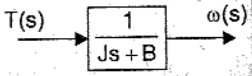


Fig 3.

$$T(s) = (Js + B)\,\omega(s)$$

$$\therefore \omega(s) = \frac{1}{Js + B}\,T(s)$$

On taking Laplace transform of equation (4) we get,

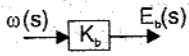


Fig 4.

$$E_b(s) = K_b\,\omega(s) \quad .....(9)$$

In equation (9), $\omega(s)$ is the input and $E_b(s)$ is the output. The block diagram for this equation is shown in fig 4.

On taking Laplace transform of equation (5) we get,

$$\omega(s) = s\,\theta(s) \quad .....(10)$$

In equation (10), $\omega(s)$ is the input and $\theta(s)$ is the output. Hence equation (10) is rearranged and the block diagram for th equation is shown in fig 5.

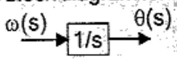


Fig 5.

$$\theta(s) = \frac{1}{s}\,\omega(s)$$

The overall block diagram of armature controlled dc motor is obtained by connecting the various sections shown in fig 1 fig 5. The overall block diagram is shown in fig 6.

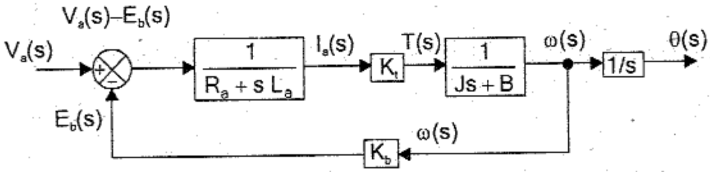


Fig 6 : Block diagram of armature controlled dc motor.

EXAMPLE 1.15

Construct the block diagram of field controlled dc motor.

SOLUTION

The differential equations governing the field controlled dc motor are (refer section 1.8),

$$v_f = R_f\,i_f + L_f\,\frac{di_f}{dt} \quad ....$$

$$T = K_{tf}\,i_f \quad ....$$

$$T = J\,\frac{d^2\theta}{dt^2} + B\,\frac{d\theta}{dt} \quad ....$$

On taking Laplace transform of equation (1) we get,

$$V_f(s) = R_f\,I_f(s) + L_f\,s\,I_f(s) \quad ....$$

In equation (4), $V_f(s)$ is the input and $I_f(s)$ is the output. Hence the equation (4) is rearranged and the block diagram for is equation is shown in fig 1.

$$V_f(s) = I_f(s)\,[R_f + sL_f]$$

$$\therefore I_f(s) = \frac{1}{R_f + s\,L_f}\,V_f(s)$$

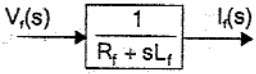


Fig 1.

On taking Laplace transform of equation (2) we get,

$$T(s) = K_{tf}\,I_f(s) \qquad .....(5)$$

In equation (5), $I_f(s)$ is the input and T(s) is the output. The block diagram for this equation is shown in fig 2.

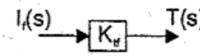


Fig 2.

On taking Laplace transform of equation (3) we get,

$$T(s) = J\,s^2\theta(s) + B\,s\,\theta(s) \qquad .....(6)$$

In equation (6), T(s) is input and θ(s) is the output. Hence equation (6) is rearranged and the block diagram for this equation is shown in fig 3.

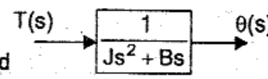


Fig 3.

$$T(s) = (J\,s^2 + Bs)\,\theta(s)$$

$$\therefore \theta(s) = \frac{1}{Js^2 + Bs}\,T(s)$$

The overall block diagram of field controlled dc motor is obtained by connecting the various section shown in fig 1 to 3. The overall block diagram is shown in fig 4.

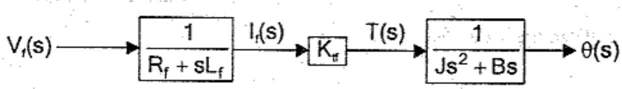


Fig 4 : Block diagram of field controlled dc motor.

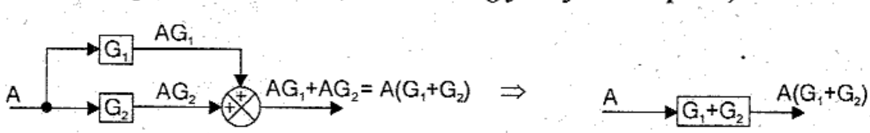
BLOCK DIAGRAM REDUCTION

The block diagram can be reduced to find the overall transfer function of the system. The following rules can be used for block diagram reduction. The rules are framed such that any modification made on the diagram does not alter the input-output relation.

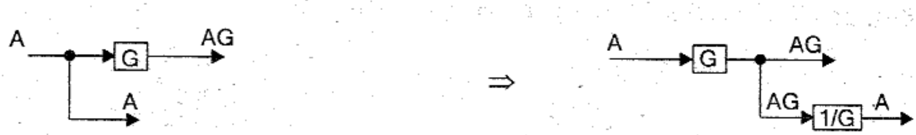
RULES OF BLOCK DIAGRAM ALGEBRA

Rule-1 : ***Combining the blocks in cascade***

Rule-2 : ***Combining Parallel blocks (or combining feed forward paths)***

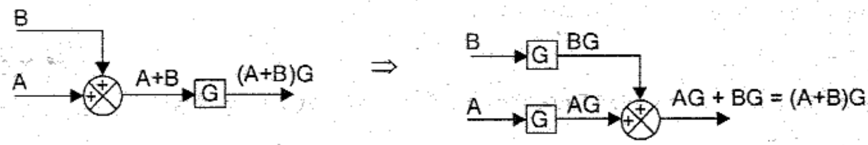


Rule-3 : ***Moving the branch point ahead of the block***

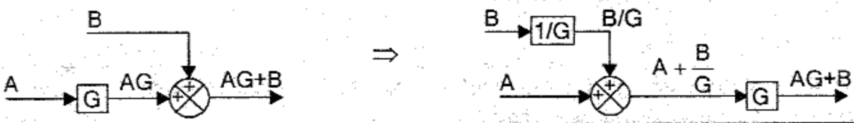


Rule-4 : ***Moving the branch point before the block***

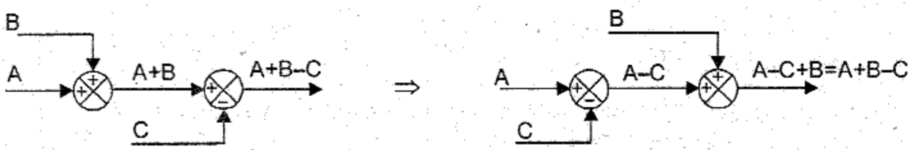
Rule-5 : ***Moving the summing point ahead of the block***



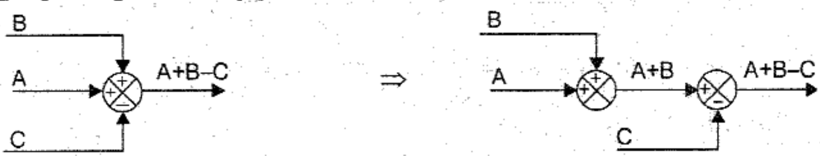
Rule-6 : ***Moving the summing point before the block***



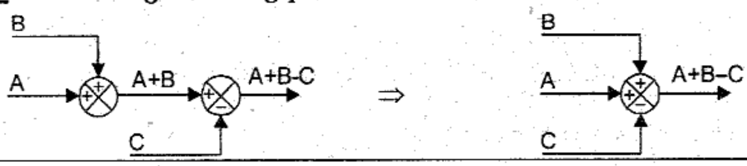
Rule-7 : ***Interchanging summing point***



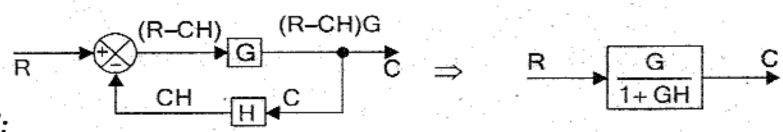
Rule-8 : ***Splitting summing points***



Rule-9 : ***Combining summing points***



Rule-10 : ***Elimination of (negative) feedback loop***

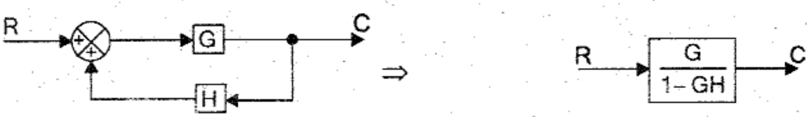


Proof:

$$C = (R - CH)G \quad \Rightarrow \quad C = RG - CHG \quad \Rightarrow \quad C + CHG = RG$$

$$\therefore C(1 + HG) = RG \quad \Rightarrow \quad \frac{C}{R} = \frac{G}{1 + GH}$$

Rule-11 : ***Elimination of (positive) feedback loop***



EXAMPLE 1.16

Reduce the block diagram shown in fig 1 and find C/R.

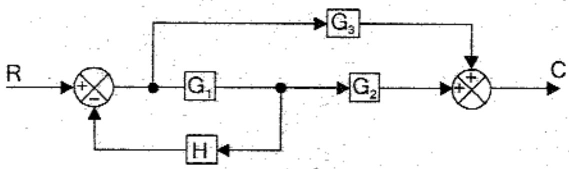
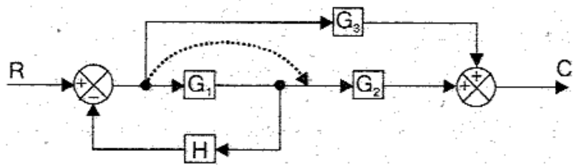


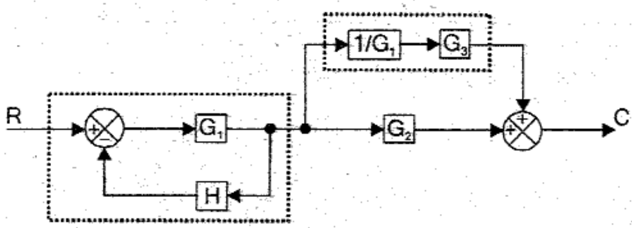
Fig 1.

SOLUTION

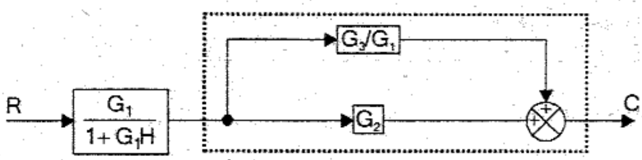
Step 1: Move the branch point after the block.



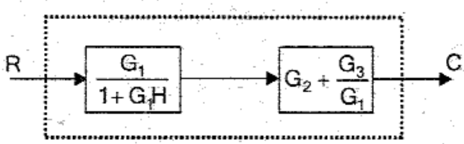
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining blocks in cascade



$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

RESULT

The overall transfer function of the system, $\frac{C}{R}=\frac{G_1G_2+G_3}{1+G_1H}$

EXAMPLE 1.17

Using block diagram reduction technique find closed loop transfer function of the system whose block diagram is sho in fig 1.

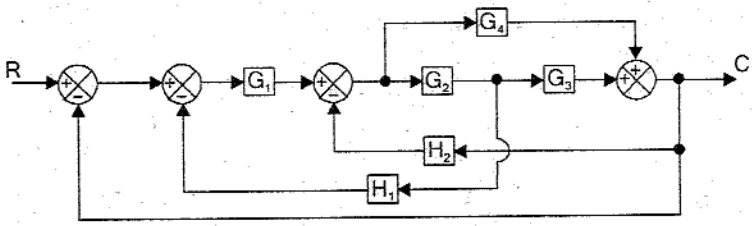
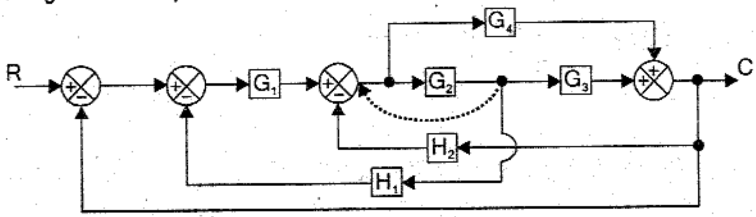


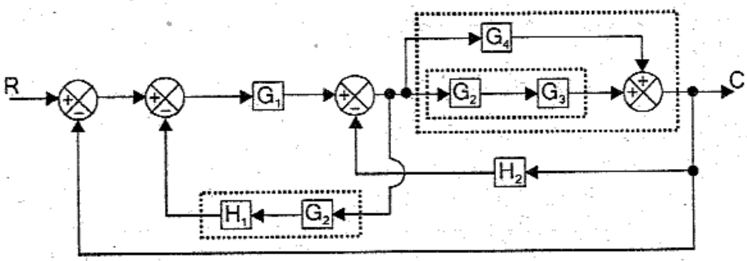
Fig 1.

SOLUTION

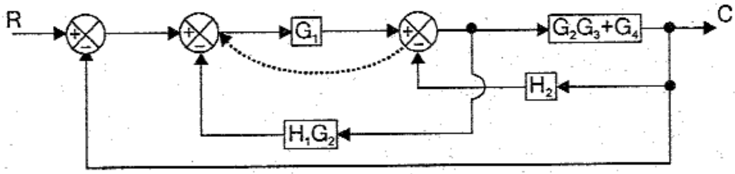
Step 1: Moving the branch point before the block



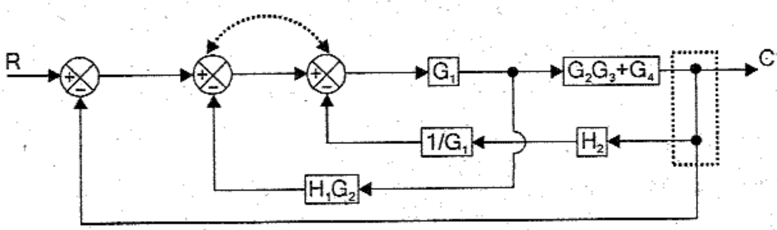
Step 2: Combining the blocks in cascade and eliminating parallel blocks



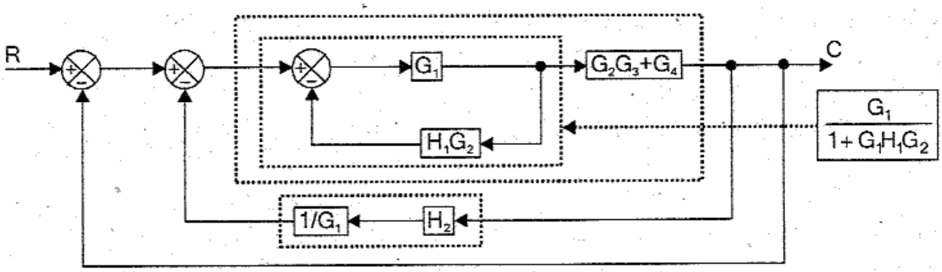
Step 3: Moving summing point before the block.



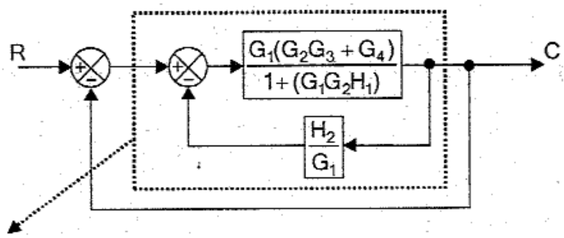
Step 4: Interchanging summing points and modifying branch points.



Step 5: Eliminating the feedback path and combining blocks in cascade

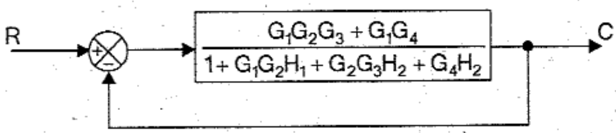


Step 6: Eliminating the feedback path



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\dfrac{H_2}{G_1}} \Rightarrow \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}} \Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminating the feedback path



$$\frac{C}{R}=\frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}} = \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

RESULT

The overall transfer function is given by,

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

EXAMPLE 1.18

Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig 1.

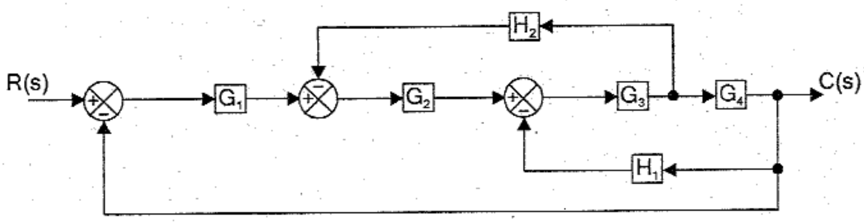
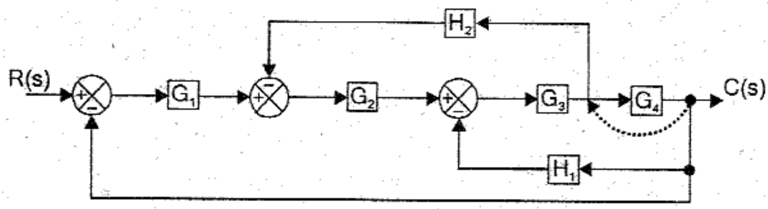


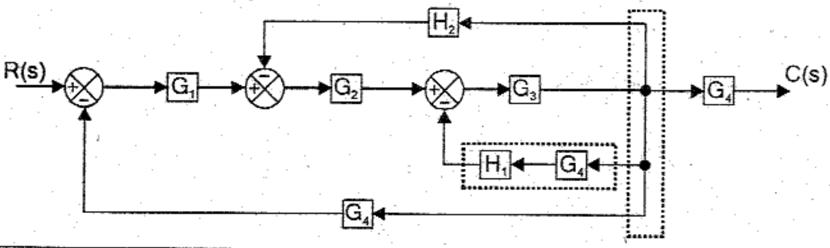
Fig 1.

SOLUTION

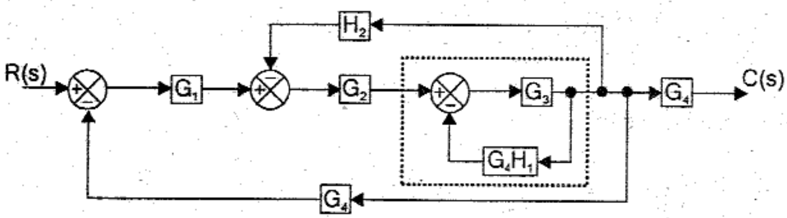
Step 1: Moving the branch point before the block



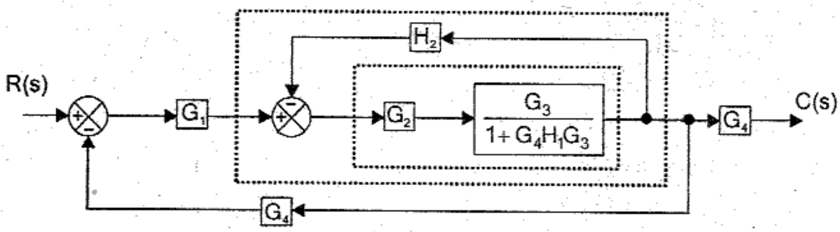
Step 2 : Combining the blocks in cascade and rearranging the branch points



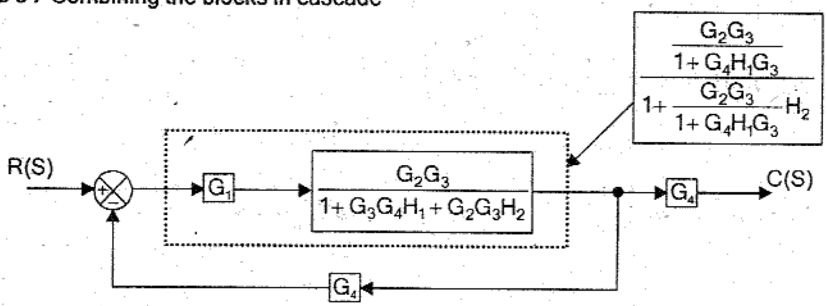
Step 3 : Eliminating the feedback path



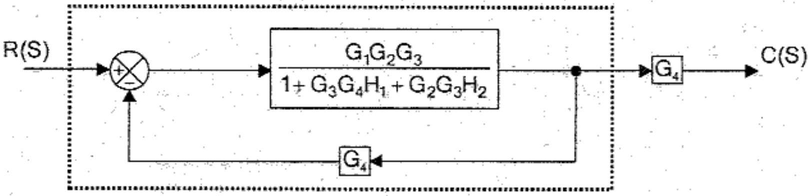
Step 4 : Combining the blocks in cascade and eliminating feedback path



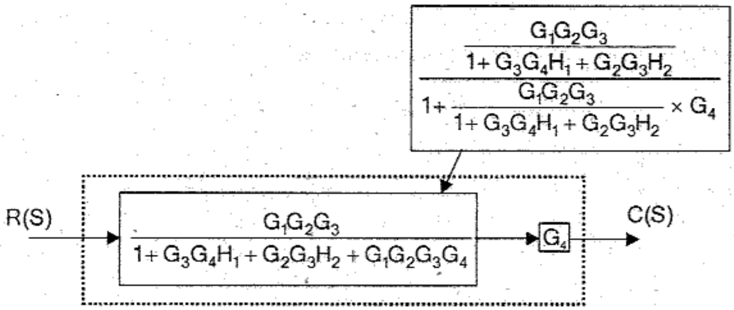
Step 5 : Combining the blocks in cascade



Step 6 : Eliminating the feedback path



Step 7 : Combining the blocks in cascade



$$\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

RESULT

The overall transfer function of the system is given by,

$$\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

EXAMPLE 1.19

For the system represented by the block diagram shown in fig 1. Evaluate the closed loop transfer function when the input R is (i) at station-I (ii) at station-II.

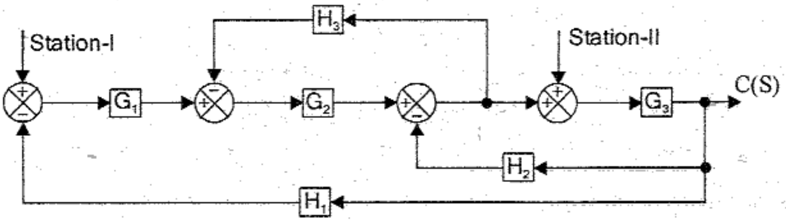


Fig 1.

SOLUTION

(i) Consider the input R is at station-I and so the input at station-II is made zero. Let the output be C_1. Since there is no input at station-II that summing point can be removed and resulting block diagram is shown in fig 2.

Step 1 : Shift the take off point of feedback H_3 beyond G_3 and rearrange the branch points

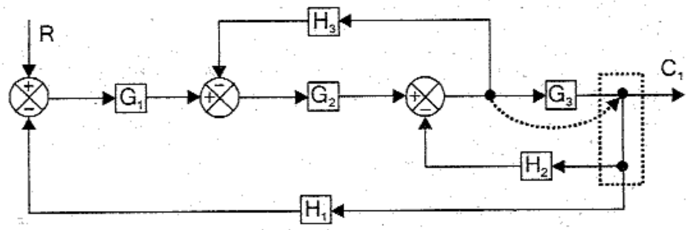
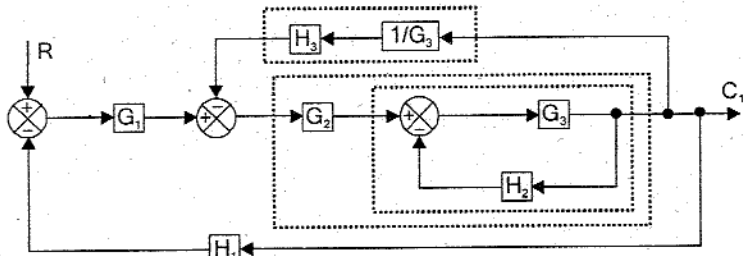
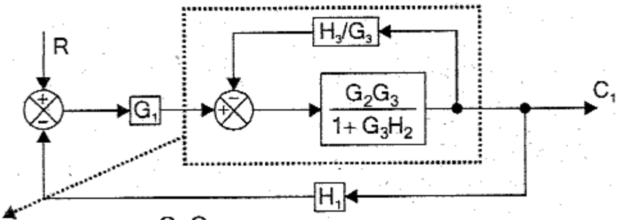


Fig 2.

Step 2 : Eliminating the feedback H_2 and combining blocks in cascade

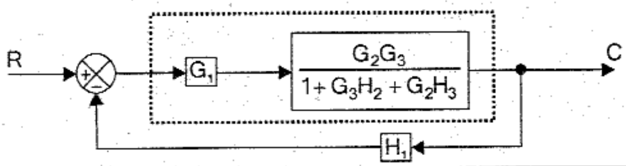


Step 3: Eliminating the feedback path

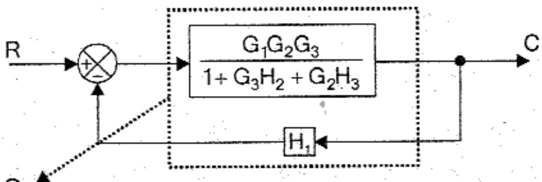


$$\frac{\dfrac{G_2G_3}{1+G_3H_2}}{1+\dfrac{G_2G_3}{1+G_3H_2}\times\dfrac{H_3}{G_3}} = \frac{\dfrac{G_2G_3}{1+G_3H_2}}{1+\dfrac{G_2H_3}{1+G_3H_2}} = \frac{G_2G_3}{1+G_3H_2+G_2H_3}$$

Step 4: Combining the blocks in cascade



Step 5: Eliminating feedback path H_1



$$\frac{\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}}{1+\dfrac{G_1G_2G_3}{1+G_3H_2+G_2H_3}\times H_1} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

$$\therefore \quad \frac{C_1(s)}{R(s)} = \frac{G_1G_2G_3}{1 + G_3H_2 + G_2H_3 + G_1G_2G_3H_1}$$

(ii) Consider the input R at station-II, the input at station-I is made zero. Let output be C_2. Since there is no input in station-I that corresponding summing point can be removed and a negative sign can be attached to the feedback path gain H_1. The resulting block diagram is shown in fig 3.

Step 1: Combining the blocks in cascade, shifting the summing point of H_2 before G_2 and rearranging the branch points.

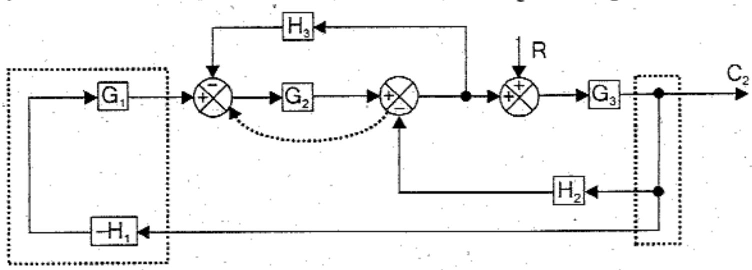
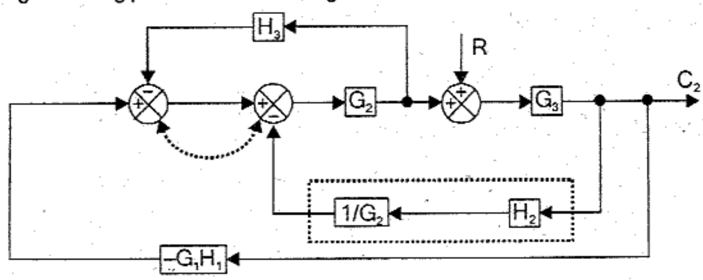
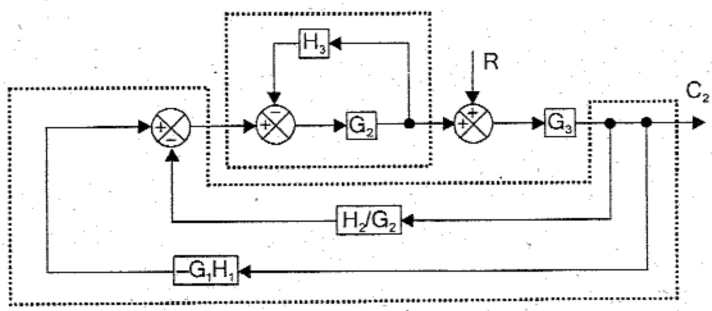


Fig 3.

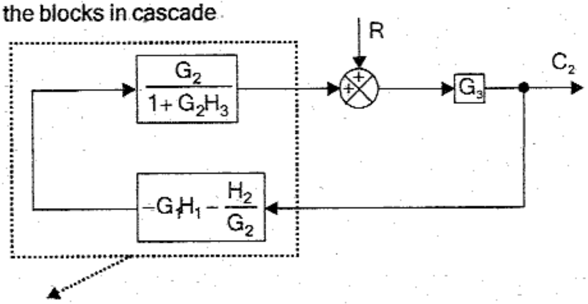
Step 2: Interchanging summing points and combining the blocks in cascade.



Step 3: Combining parallel blocks and eliminating feedback path

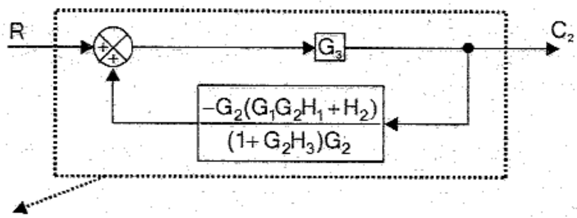


Step 4: Combining the blocks in cascade



$$\left(\frac{G_2}{1+G_2H_3}\right) \times \left(-G_1H_1 - \frac{H_2}{G_2}\right) = \left(\frac{G_2}{1+G_2H_3}\right) \times \left(\frac{-G_1H_1G_2 - H_2}{G_2}\right) = \frac{-G_2(G_1G_2H_1 + H_2)}{(1+G_2H_3)G_2}$$

Step 5: Eliminating the feedback path



$$\frac{G_3}{1-\left(\frac{-(G_1G_2H_1+H_2)}{1+G_2H_3}\right)G_3} = \frac{G_3}{\frac{1+G_2H_3+G_3(G_1G_2H_1+H_2)}{1+G_2H_3}} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

$$\therefore \; \frac{C_2}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

RESULT

The transfer function of the system with input at station-I is,

$$\frac{C_1}{R} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

The transfer function of the system with input at station-II is,

$$\frac{C_2}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

EXAMPLE 1.20

For the system represented by the block diagram shown in the fig 1, determine C_1/R_1 and C_2/R_1.

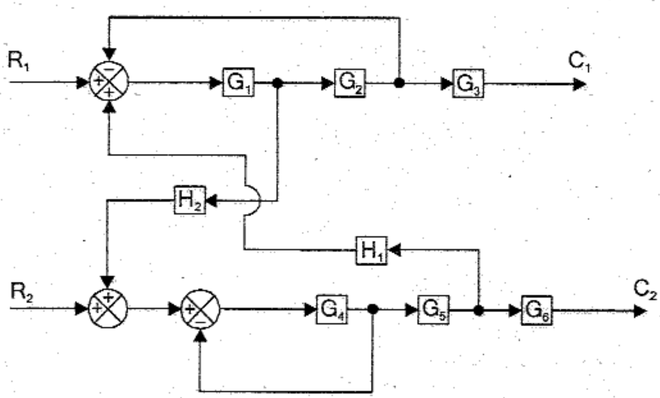


Fig 1

SOLUTION

Case (i) To find $\frac{C_1}{R_1}$

In this case set $R_2 = 0$ and consider only one output C_1. Hence we can remove the summing point which adds R_2 and need not consider G_6, since G_6 is on the open path. The resulting block diagram is shown in fig 2.

Step 1: Eliminating the feedback path

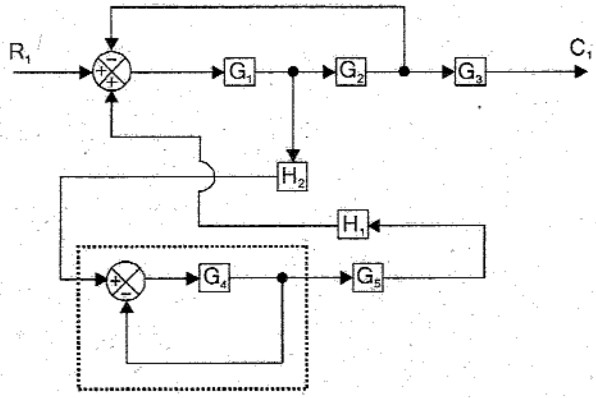
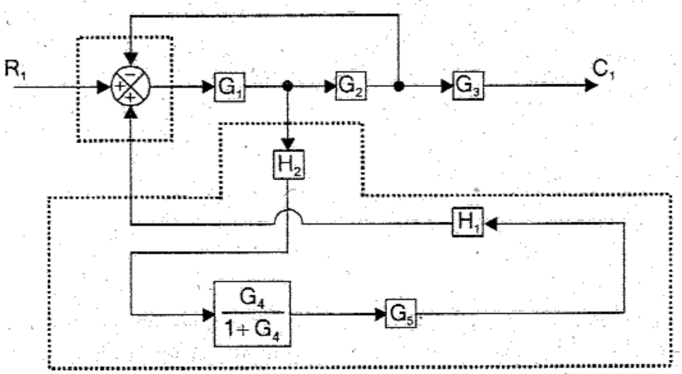
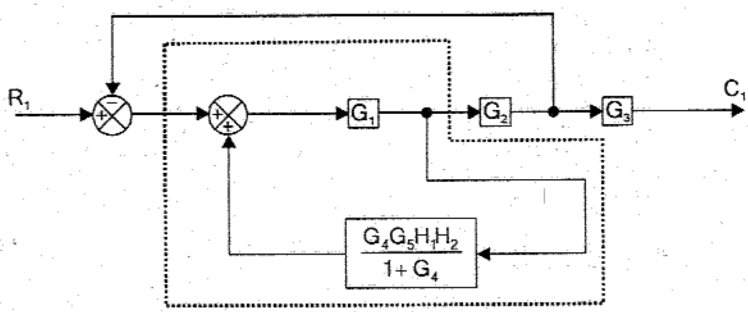


Fig 2.

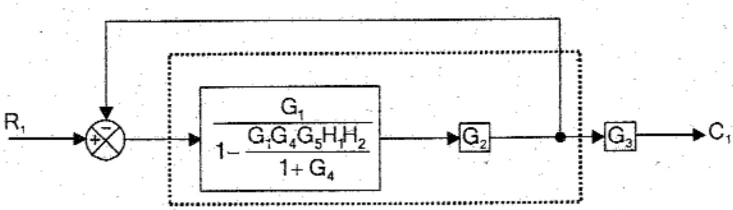
Step 2: Combining the blocks in cascade and splitting the summing point



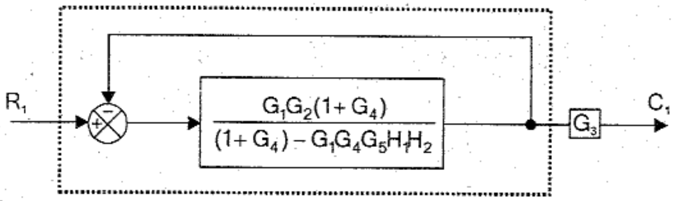
Step 3: Eliminating the feedback path

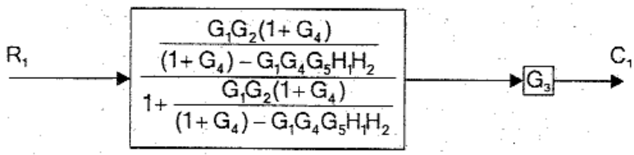


Step 4: Combining the blocks in cascade

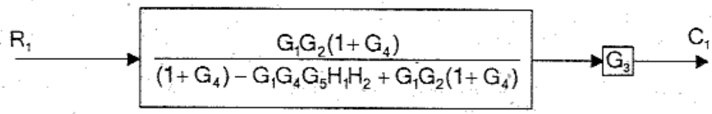


Step 5: Eliminating the feedback path





Step 6: Combining the blocks in cascade



$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)\,(1+G_4) - G_1G_4G_5H_1H_2}$$

Case 2 : To find $\frac{C_2}{R_1}$

In this case set $R_2 = 0$ and consider only one output C_2. Hence we can remove the summing point which adds R_2 and need not consider G_3, since G_3 is on the open path. The resulting block diagram is shown in fig 3.

Step 1: Eliminate the feedback path.

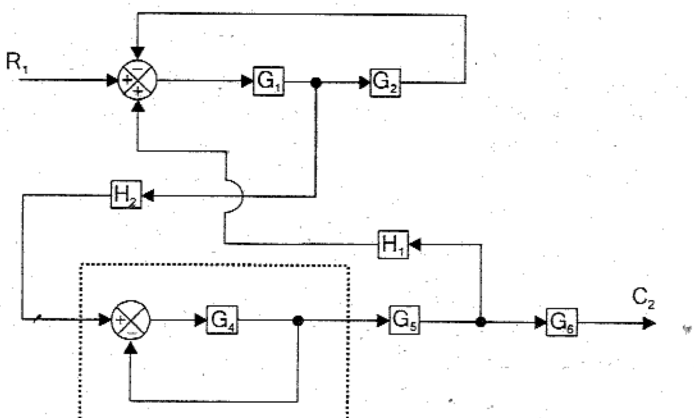
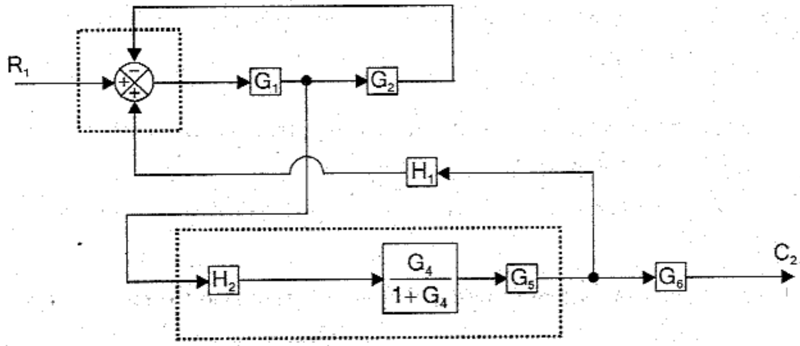
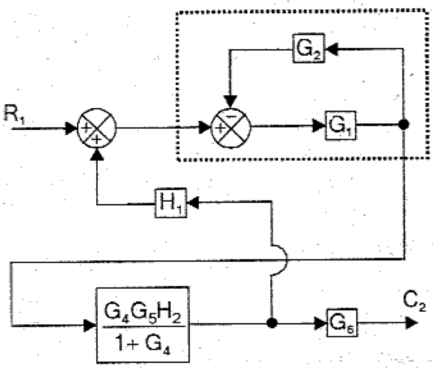


Fig 3.

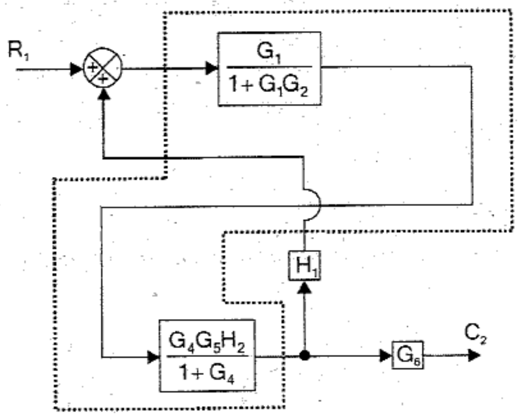
Step 2: Combining blocks in cascade and splitting the summing point



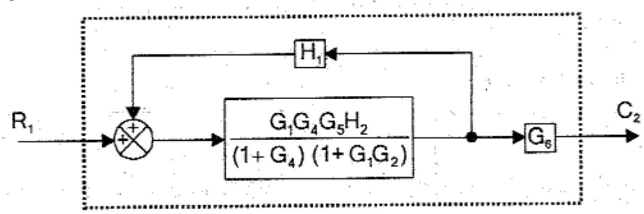
Step 3: Eliminating the feedback path

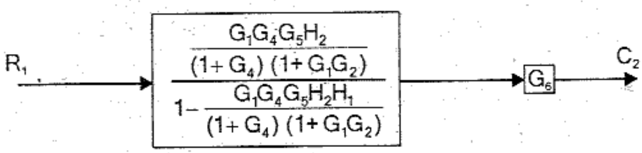


Step 4 : Combining the blocks in cascade

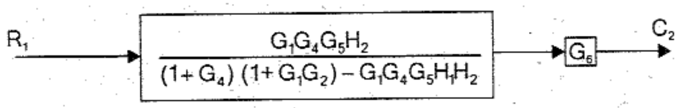


Step 5: Eliminating the feedback path





Step 6: Combining the blocks in cascade



$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)\,(1+G_1G_2) - G_1G_4G_5H_1H_2}$$

RESULT

The transfer function of the system when the input and output are R_1 and C_1 is given by,

$$\frac{C_1}{R_1} = \frac{G_1G_2G_3\,(1+G_4)}{(1+G_1G_2)\,(1+G_4) - G_1G_4G_5H_1H_2}$$

The transfer function of the system when the input and output are R_1 and C_2 is given by,

$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6\,H_2}{(1+G_4)\,(1+G_1G_2) - G_1G_4G_5H_1H_2}$$

EXAMPLE 1.21

Obtain the closed loop transfer function C(s)/R(s) of the system whose block diagram is shown in fig 1.

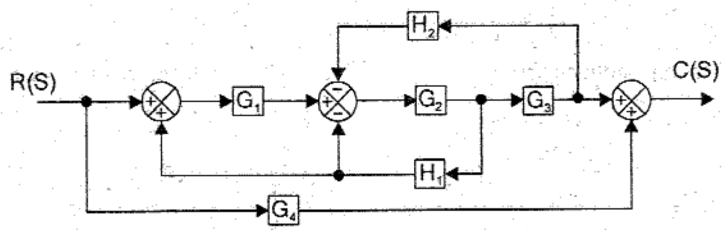
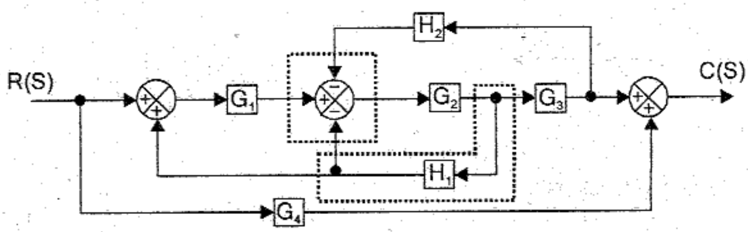


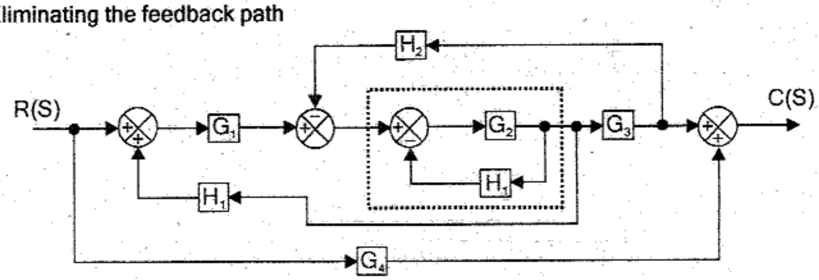
Fig 1.

SOLUTION

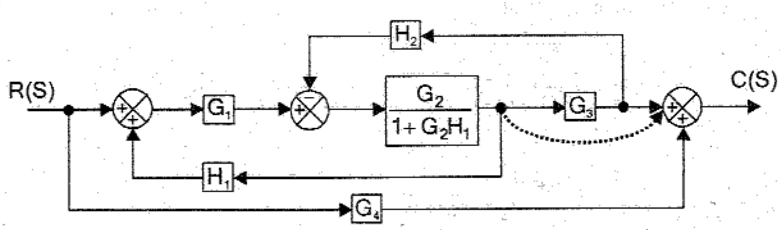
Step 1 : Splitting the summing point and rearranging the branch points



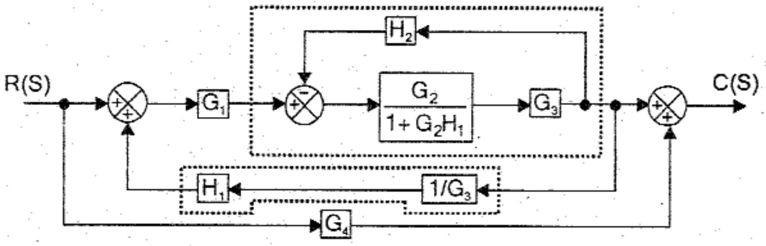
Step 2 : Eliminating the feedback path



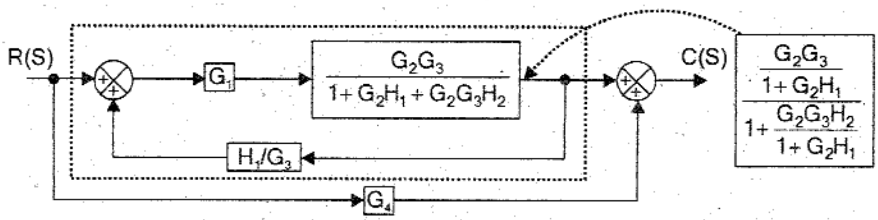
Step 3 : Shifting the branch point after the block.



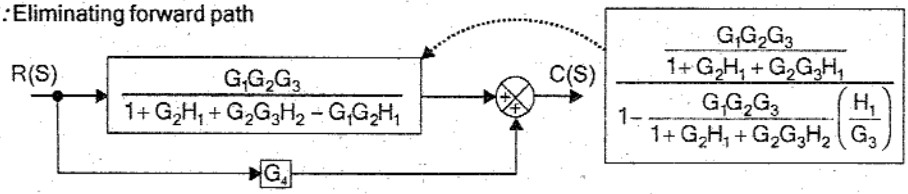
Step 4 : Combining the blocks in cascade and eliminating feedback path



Step 5 : Combining the blocks in cascade and eliminating feedback path



Step 6 : Eliminating forward path



$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 + G_2H_1 + G_2G_3H_2 - G_1G_2H_1} + G_4$$

RESULT

The transfer function of the system is $\dfrac{C(s)}{R(s)} = \dfrac{G_1G_2G_3}{1 + G_2H_1 + G_2G_3H_2 - G_1G_2H_1} + G_4$

EXAMPLE 1.22

The block diagram of a closed loop system is shown in fig 1. Using the block diagram reduction technique determine the closed loop transfer function C(s)/R(s).

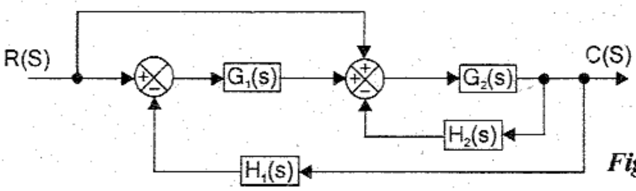
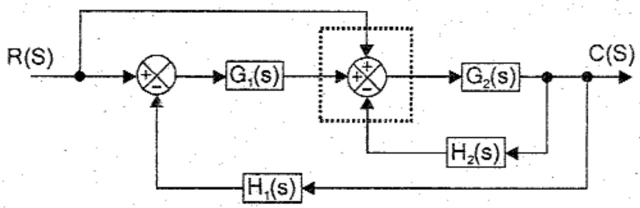


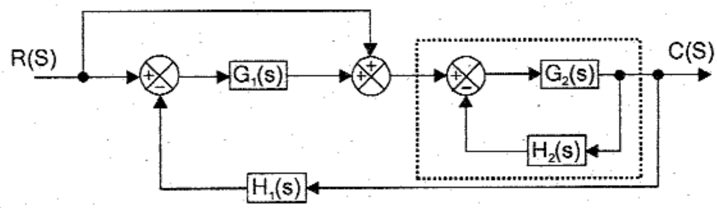
Fig 1.

SOLUTION

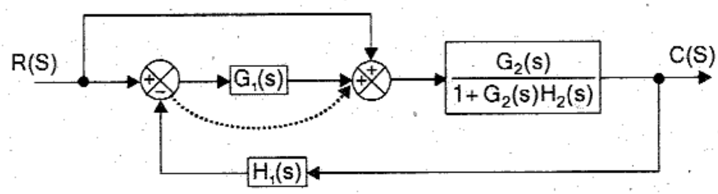
Step 1 : Splitting the summing point.



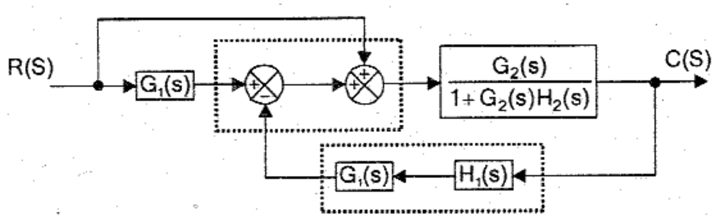
Step 2 : Eliminating the feedback path.



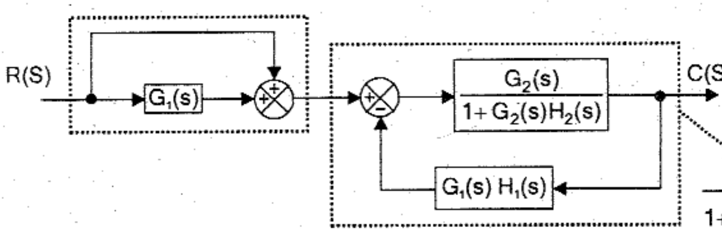
Step 3 : Moving the summing point after the block.



Step 4 : Interchanging the summing points and combining the blocks in cascade

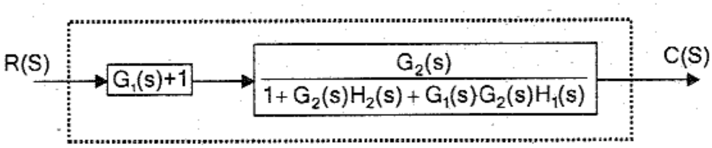


Step 5 : Eliminating the feedback path and feed forward path



$$\frac{\dfrac{G_2(s)}{1+G_2(s)H_2(s)}}{1+\dfrac{G_2(s)}{1+G_2(s)H_2(s)}G_1(s)H_1(s)}$$

Step 6 : Combining the blocks in cascade



$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\left[G_1(s)+1\right]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

RESULT

The transfer function of the system is,

$$\frac{C(s)}{R(s)} = \frac{G_2(s)\left[G_1(s)+1\right]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

EXAMPLE 1.23

Using block diagram reduction technique find the transfer function C(s)/R(s) for the system shown in fig 1.

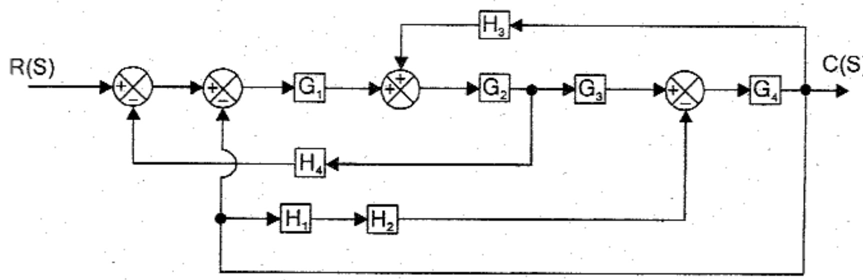
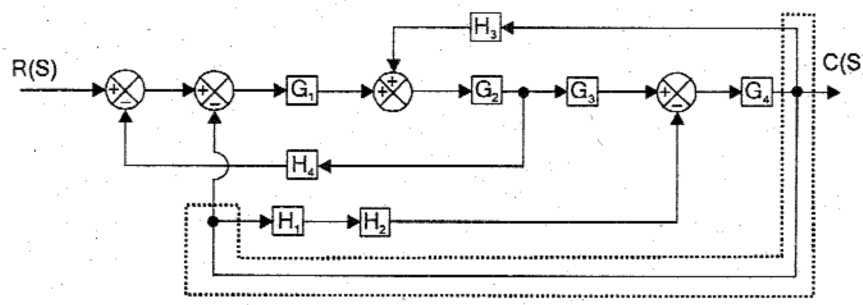


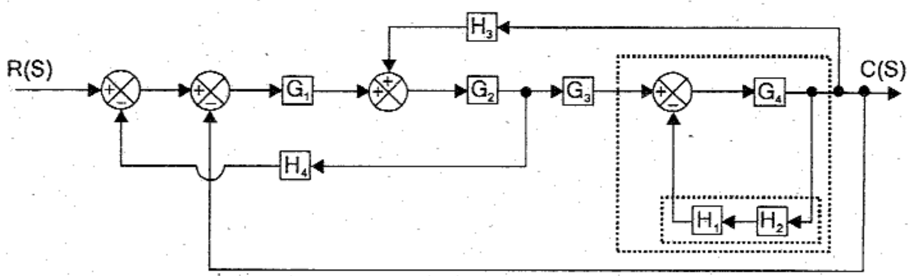
Fig 1.

SOLUTION

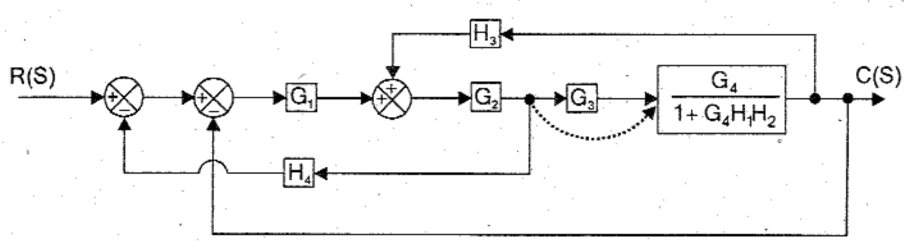
Step 1 : Rearranging the branch points



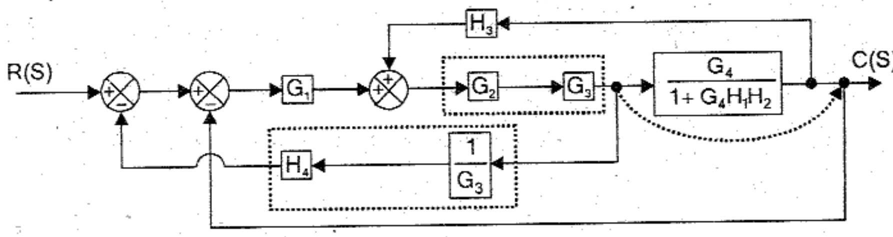
Step 2 : Combining the blocks in cascade and eliminating the feedback path.



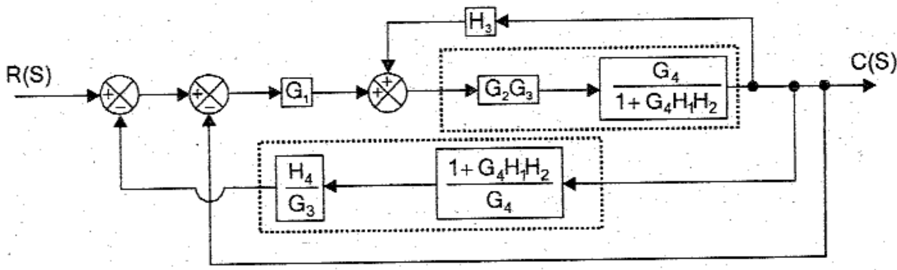
Step 3 : Moving the branch point after the block.



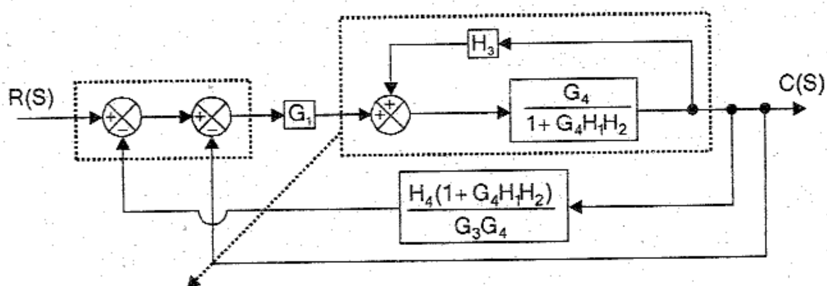
Step 4 : Moving the branch point and combining the blocks in cascade.



Step 5 : Combining the blocks in cascade

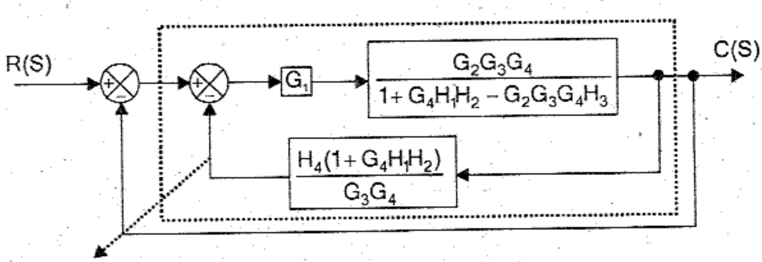


Step 6 : Eliminating feedback path and interchanging the summing points.



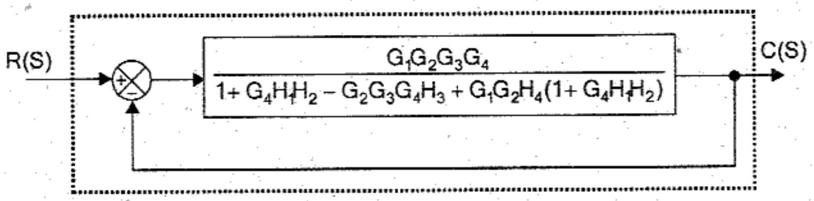
$$\frac{\dfrac{G_2G_3G_4}{1+G_4H_1H_2}}{1-\dfrac{G_2G_3G_4H_3}{1+G_4H_1H_2}} = \frac{G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}$$

Step 7 : Combining the blocks in cascade and eliminating the feedback path



$$\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}}{1+\left(\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}\right)\left(\dfrac{H_4(1+G_4H_1H_2)}{G_3G_4}\right)} = \frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}$$

Step 8 : Eliminating the unity feedback path.



$$\therefore \frac{C(s)}{R(s)}=\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}{1+\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}}$$

$$=\frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)+G_1G_2G_3G_4}$$

$$=\frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

RESULT

The transfer function of the system is,

$$\frac{C(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

1.12 BLOCK DIAGRAM REDUCTION USING MATLAB

TRANSFER FUNCTION OF A SYSTEM

Let, G(s) be the transfer function of a system. When the transfer function is a rational function of "s", then using MATLAB the transfer function can be obtained from the coefficients of the numerator and denominator polynomials as shown below. Let, the general form of G(s) be as shown below.

$$G(s)=\frac{b_0s^M+b_1s^{M-1}+b_2s^{M-2}+.........+b_{M-1}s+b_M}{a_0s^N+a_1s^{N-1}+a_2s^{N-2}+.........+a_{N-1}s+a_N}$$

First, the coefficients of the numerator and denominator polynomials are declared as two arrays as shown below.

```
num_cof  =  [b0  b1  b2  ........  bM];
den_cof  =  [a0  a1  a2  ........  aN];
```

Next, the transfer can be obtained using the following commands of MATLAB.

```
G  =  tf('s');
G  =  ([num_cof],  [den_cof])
```

TRANSFER FUNCTION OF CASCADE / PARALLEL / FEEDBACK SYSTEM

Consider two systems with transfer functions $G_1(s)$ and $G_2(s)$. Let the two transfer functions be rational function of "s" as shown below.

$$G_1(s)=\frac{b_0s^M+b_1s^{M-1}+b_2s^{M-2}+.........+b_{M-1}s+b_M}{a_0s^N+a_1s^{N-1}+a_2s^{N-2}+.........+a_{N-1}s+a_N}$$

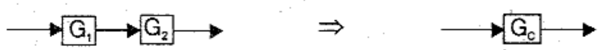
$$G_2(s) = \frac{d_0 s^M + d_1 s^{M-1} + d_2 s^{M-2} + \ldots\ldots\ldots + d_{M-1}s + d_M}{c_0 s^N + c_1 s^{N-1} + c_2 s^{N-2} + \ldots\ldots\ldots + c_{N-1}s + c_N}$$

When the two systems are connected as cascade / parallel / feedback system, then the over transfer function of cascaded system / parallel system / feedback system can be obtained using MATLA

In order to obtain the overall transfer function, first the coefficients of the numerator and denomina polynomials of $G_1(s)$ and $G_2(s)$ are declared as arrays as shown below.

```
num_cof1  =  [b0  b1  b2  ........  bM];
den_cof1  =  [a0  a1  a2  ........  aN];
num_cof2  =  [d0  d1  d2  ........  dM];
den_cof2  =  [c0  c1  c2  ........  cN];
```

When the two systems are connected in cascade as shown below, then the overall transfer functio $G_C(s)$ of the cascaded system can be obtained using the following commands of MATLAB.

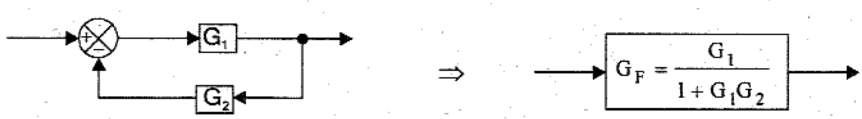



```
GC  =  tf('s');
[num_cofC,  den_cofC]  =  series(num_cof1,  den_cof1,  num_cof2,  den_cof2);
GC  =  ([num_cofC],  [den_cofC])
```

When the two systems are connected in parallel as shown below, then the overall transfer functio $G_P(s)$ of parallel system can be obtained using the following commands of MATLAB.


```
GP  =  tf('s');
[num_cofP,  den_cofP]=parallel(num_cof1,  den_cof1,  num_cof2,  den_cof2);
GP  =  ([num_cofP],  [den_cofP])
```

When the two systems are connected in feedback as shown below, then the overall trans function $G_F(s)$ of feedback system can be obtained using the following commands of MATLAB.




```
GF  =  tf('s');
[num_cofF,  den_cofF]  =  feedback(num_cof1,  den_cof1,  num_cof2,  den_cof2)
GF  =  ([num_cofF],  [den_cofF])
```

PROGRAM 1.1

Consider the transfer functions of the two systems given below.

$G_1(s)=8/(s^2+2s+9)$ and $G_2(s)=4/(s+6)$

Write a MATLAB program to find the overall transfer function if the two systems are connected as cascade system, parallel system and feedback system.

```
clc
clear all
G1=tf('s'); G2=tf('s'); GC=tf('s');GP=tf('s');GF=tf('s');
num_cof1=[0 0 8];
den_cof1=[1 2 9];
disp('System1');
G1=tf([num_cof1], [den_cof1])
num_cof2=[0 4];
den_cof2=[1 6];
disp('System2');
G2=tf([num_cof2], [den_cof2])
[num_cofC,den_cofC]=series(num_cof1,den_cof1,num_cof2,den_cof2);
disp('Cascade system');
GC=tf([num_cofC], [den_cofC])
[num_cofP,den_cofP]=parallel(num_cof1,den_cof1,num_cof2,den_cof2);
disp('Parallel system');
GP=tf([num_cofP], [den_cofP])
[num_cofF,den_cofF]=feedback(num_cof1,den_cof1,num_cof2,den_cof2);
disp('Feedback system');
GF=tf([num_cofF], [den_cofF])
```

OUTPUT

```
System1
Transfer function:
           8
     -------------
     s^2 + 2 s + 9

System2
Transfer function:
       4
     -----
     s + 6

Cascade system
Transfer function:
                  32
     ----------------------
     s^3 + 8 s^2 + 21 s + 54

Parallel system
Transfer function:
        4 s^2 + 16 s + 84
     ----------------------
     s^3 + 8 s^2 + 21 s + 54

Feedback system
Transfer function:
             8 s + 48
     ----------------------
     s^3 + 8 s^2 + 21 s + 86
```

1.13 SIGNAL FLOW GRAPH

The signal flow graph is used to represent the control system graphically and it was developed by **S.J. Mason.**

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. The signal flow graph of the system can be constructed using these equations.

It should be noted that the signal flow graph approach and the block diagram approach yield the same information. The advantage in signal flow graph method is that, using Mason's gain formula the overall gain of the system can be computed easily. This method is simpler than the tedious block diagram reduction techniques.

The signal flow graph depicts the flow of signals from one point of a system to another and gives the relationships among the signals. A signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable and each branch connected between two nodes acts as a signal multiplier. Each branch has a gain or transmittance. When the signal pass through a branch, it gets multiplied by the gain of the branch.

In a signal flow graph, the signal flows in only one direction. The direction of signal flow is indicated by an arrow placed on the branch and the gain (multiplication factor) is indicated along the branch.

EXPLANATION OF TERMS USED IN SIGNAL FLOW GRAPH

Node	:	A node is a point representing a variable or signal.
Branch	:	A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of a branch is the transmittance.
Transmittance	:	The gain acquired by the signal when it travels from one node to another is called transmittance. The transmittance can be real or complex.
Input node (Source)	:	It is a node that has only outgoing branches.
Output node (Sink)	:	It is a node that has only incoming branches.
Mixed node	:	It is a node that has both incoming and outgoing branches.
Path	:	A path is a traversal of connected branches in the direction of the branch arrows. The path should not cross a node more than once.
Open path	:	A open path starts at a node and ends at another node.
Closed path	:	Closed path starts and ends at same node.
Forward path	:	It is a path from an input node to an output node that does not cross any node more than once.
Forward path **gain**	:	It is the product of the branch transmittances (gains) of a forward path.
Individual loop	:	It is a closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once.
Loop gain	:	It is the product of the branch transmittances (gains) of a loop.
Non-touching Loops	:	If the loops does not have a common node then they are said to be non-touching loops.

PROPERTIES OF SIGNAL FLOW GRAPH

The basic properties of signal flow graph are the following :

(i) The algebraic equations which are used to construct signal flow graph must be in the form of cause and effect relationship.

(ii) Signal flow graph is applicable to linear systems only.

(iii) A node in the signal flow graph represents the variable or signal.

(iv) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.

(v) A mixed node which has both incoming and outgoing signals can be treated as an output node by adding an outgoing branch of unity transmittance.

(vi) A branch indicates functional dependence of one signal on the other.

(vii) The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.

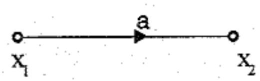
(viii) The signal flow graph of system is not unique. By rearranging the system equations different types of signal flow graphs can be drawn for a given system.

SIGNAL FLOW GRAPH ALGEBRA

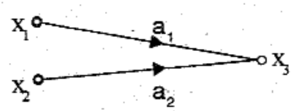
Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules. The guideline in developing the rules for signal flow graph algebra is that the signal at a node is given by sum of all incoming signals.

Rule 1 **:** Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.

Example:



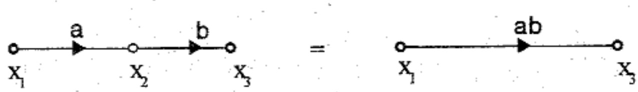
$x_2 = ax_1$



$x_3 = a_1x_1 + a_2x_2$

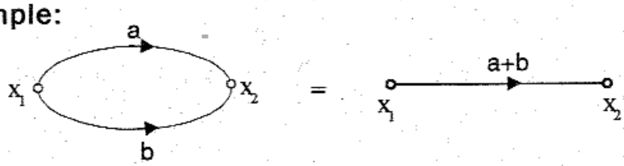
Rule 2 **:** Cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.

Example:



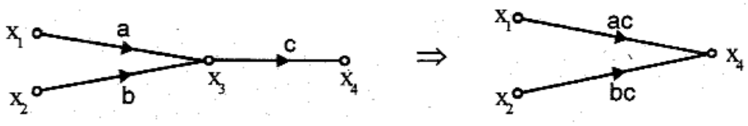
Rule 3 **:** Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittances.

Example:



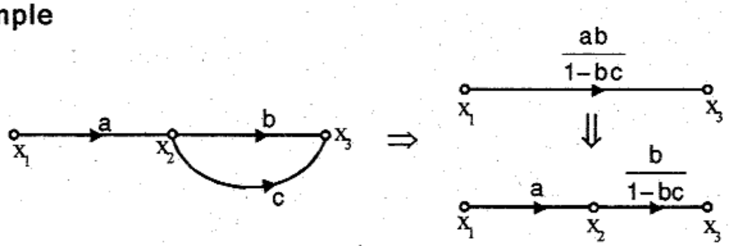
Rule 4 **:** A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node.

Example



Rule 5 **:** A loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input. This ratio gives the gain of resultant branch.

Example



***Proof*:**

$x_2 = ax_1 + cx_3$; $x_3 = bx_2$

Put, $x_2 = ax_1 + cx_3$ in the equation for x_3.

$\therefore x_3 = b\,(ax_1 + cx_3) \Rightarrow x_3 = abx_1 + bcx_3 \Rightarrow x_3 - bc\,x_3 = ab\,x_1 \Rightarrow x_3(1-bc) = ab\,x_1$

$$\therefore \frac{x_3}{x_1} = \frac{ab}{1-bc}$$

SIGNAL FLOW GRAPH REDUCTION

The signal flow graph of a system can be reduced either by using the rules of a signal flow graph algebra or by using Mason's gain formula.

For signal flow graph reduction using the rules of signal flow graph, write equations at every node and then rearrange these equations to get the ratio of output and input (transfer function).

The signal flow graph reduction by above method will be time consuming and tedious. **S.J.Mason** has developed a simple procedure to determine the transfer function of the system represented as a signal flow graph. He has developed a formula called by his name **Mason's gain formula** which can be directly used to find the transfer function of the system.

MASON'S GAIN FORMULA

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system.

Let, R(s) = Input to the system

C(s) = Output of the system

Now, Transfer function of the system, $T(s) = \frac{C(s)}{R(s)}$ .....(1.34)

Mason's gain formula states the overall gain of the system [transfer function] as follows,

Overall gain, $T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ .....(1.35) where, $T = T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

$$\Delta = 1 - (\text{Sum of individual loop gains}) + \begin{pmatrix}\text{Sum of gain products of all possible}\\ \text{combinations of two non - touching loops}\end{pmatrix} - \begin{pmatrix}\text{Sum of gain products of all possible}\\ \text{combinations of three non - touching loops}\end{pmatrix} + \ldots\ldots$$

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

CONSTRUCTING SIGNAL FLOW GRAPH FOR CONTROL SYSTEMS

A control system can be represented diagrammatically by signal flow graph. The differential equations governing the system are used to construct the signal flow graph.The following procedure can be used to construct the signal flow graph of a system.

1. Take Laplace transform of the differential equations governing the system in order to convert them to algebraic equations in s-domain.
2. The constants and variables of the s-domain equations are identified.
3. From the working knowledge of the system, the variables are identified as input, output and intermediate variables.
4. For each variable a node is assigned in signal flow graph and constants are assigned as the gain or transmittance of the branches connecting the nodes.
5. For each equation a signal flow graph is drawn and then they are interconnected to give overall signal flow graph of the system.

PROCEDURE FOR CONVERTING BLOCK DIAGRAM TO SIGNAL FLOW GRAPH

The signal flow graph and block diagram of a system provides the same information but there is no standard procedure for reducing the block diagram to find the transfer function of the system. Also the block diagram reduction technique will be tedious and it is difficult to choose the rule to be applied for simplification. Hence it will be easier if the block diagram is converted to signal flow graph and **Mason's gain formula** is applied to find the transfer function. The following procedure can be used to convert block diagram to signal flow graph.

1. Assume nodes at input, output, at every summing point, at every branch point and in between cascaded blocks.
2. Draw the nodes separately as small circles and number the circles in the order 1, 2, 3, 4, ... etc.
3. From the block diagram find the gain between each node in the main forward path and connect all the corresponding circles by straight line and mark the gain between the nodes.
4. Draw the feed forward paths between various nodes and mark the gain of feed forward path along with sign.
5. Draw the feedback paths between various nodes and mark the gain of feedback paths along with sign.

EXAMPLE 1.24

Construct a signal flow graph for armature controlled dc motor.

SOLUTION

The differential equations governing the armature controlled dc motor are (refer section 1.7).

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_b\,; \qquad T = K_t i_a\,; \qquad T = J\frac{d\omega}{dt} + B\omega\,; \qquad e_b = K_b\,\omega\,; \qquad \omega = d\theta/dt$$

On taking Laplace transform of above equations we get,

$$V_a(s) = I_a(s)\,R_a + L_a\,s\,I_a(s) + E_b(s) \qquad .....(1)$$

$$T(s) = K_t I_a(s) \qquad .....(2)$$

$$T(s) = J\,s\,\omega(s) + B\,\omega(s) \qquad .....(3)$$

$$E_b(s) = K_b\,\omega(s) \qquad .....(4)$$

$$\omega(s) = s\,\theta(s) \qquad .....(5)$$

The input and output variables of armature controlled dc motor are armature voltage $V_a(s)$ and angular displacement $\theta(s)$ respectively. The variables $I_a(s)$, $T(s)$, $E_b(s)$ and $\omega(s)$ are intermediate variables.

The equations (1) to (5) are rearranged & individual signal flow graph are shown in fig 1 to fig 5.

$$V_a(s) - E_b(s) = I_a(s)\,[R_a + s\,L_a]$$

$$\therefore I_a(s) = \frac{1}{R_a + s\,L_a}\,[V_a(s) - E_b(s)]$$

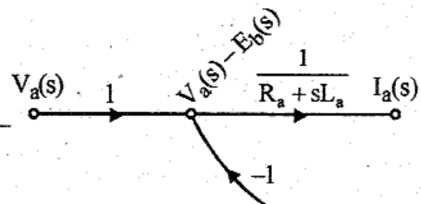


Fig 1

$$T(s) = K_t I_a(s)$$

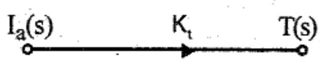


Fig 1

$$T(s) = \omega(s)\,[Js + B]$$

$$\therefore \omega(s) = \frac{1}{Js + B}\,T(s)$$

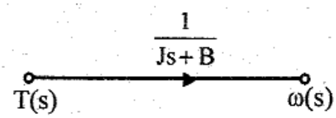


Fig 3

$$E_b(s) = K_b\,\omega(s)$$

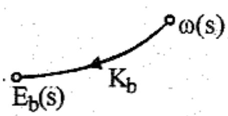


Fig 4

$$\omega(s) = s\theta(s)$$

$$\therefore \theta(s) = \frac{1}{s}\omega(s)$$

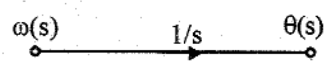


Fig 5

The overall signal flow graph of armature controlled dc motor is obtained by interconnecting the individual signal flow graphs shown in fig 1 to fig 5. The overall signal flow graph is shown in fig 6.

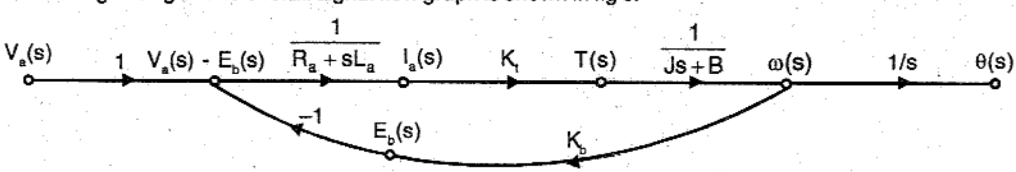


Fig 6 : Signal flow graph of armature controlled dc motor.

EXAMPLE 1.25

Find the overall transfer function of the system whose signal flow graph is shown in fig 1.

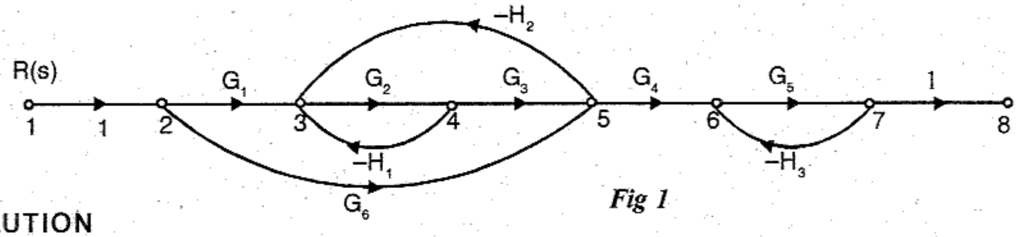


Fig 1

SOLUTION

Forward Path Gains

There are two forward paths. $\therefore$ K = 2

Let forward path gains be P_1 and P_2.

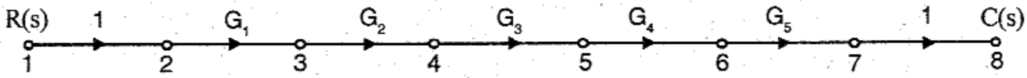


***Fig 2 :** Forward path-1.*

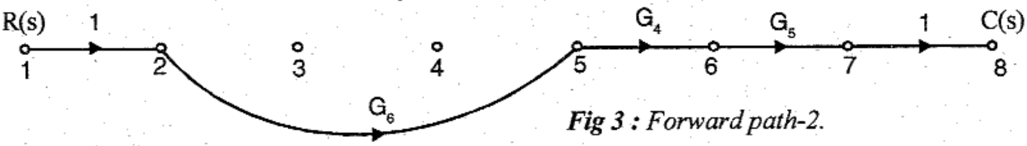


***Fig 3 :** Forward path-2.*

Gain of forward path-1, $P_1 = G_1\ G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_4 G_5 G_6$

Individual Loop Gain

There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.

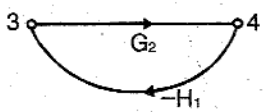


***Fig 4 :** Loop-1.*

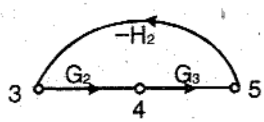


***Fig 5 :** Loop-2.*

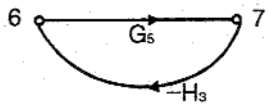


***Fig 6 :** Loop-3.*

Loop gain of individual loop-1, $P_{11} = -G_2 H_1$

Loop gain of individual loop-2, $P_{21} = -G_2 G_3 H_2$

Loop gain of individual loop-3, $P_{31} = -G_5 H_3$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be P_{12} and P_{22}.

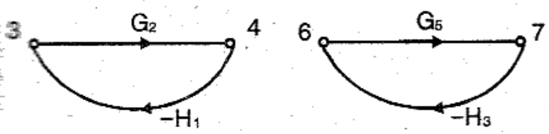


***Fig 7 :** First combination of 2 non-touching loops.*

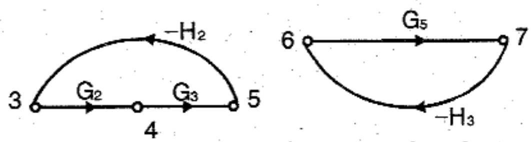


***Fig 8 :** Second combination of 2 non-touching loops.*

$$\left.\begin{array}{l}\text{Gain product of first combination}\\ \text{of two non touching loops}\end{array}\right\} P_{12} = P_{11}P_{31} = (-G_2H_1)\,(-G_5H_3) \;=\; G_2G_5H_1H_3$$

$$\left.\begin{array}{l}\text{Gain product of second combination}\\ \text{of two non touching loops}\end{array}\right\} P_{22} = P_{21}P_{31} = (-G_2G_3H_2)\,(-G_5H_3) \;=\; G_2G_3G_5H_2H_3$$

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in fig 9.

$$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$$

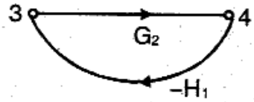


Fig 9

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \qquad \text{(Number of forward paths is 2 and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6\,(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

EXAMPLE 1.26

Find the overall gain of the system whose signal flow graph is shown in fig 1.

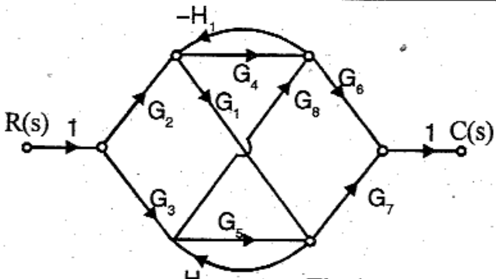


Fig 1

SOLUTION

Let us number the nodes as shown in fig 2.

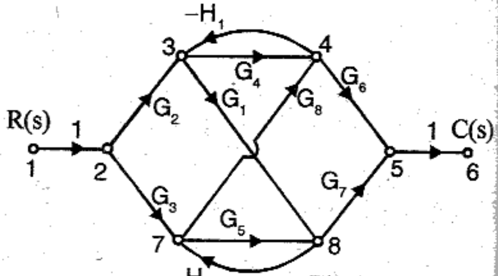


Fig 2

I. Forward Path Gains

There are six forward paths. $\therefore K = 6$

Let the forward path gains be P_1, P_2, P_3, P_4, P_5 and P_6.

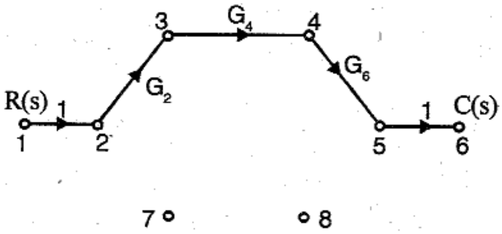


Fig 3 : Forward path-1.

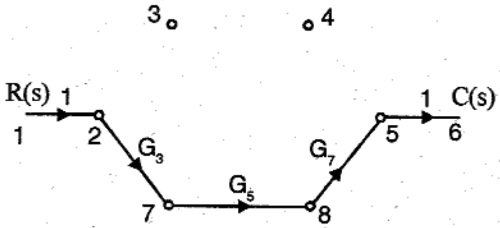


Fig 4 : Forward path-2.

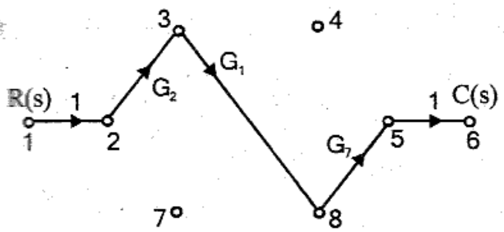


Fig 5 : Forward path-3

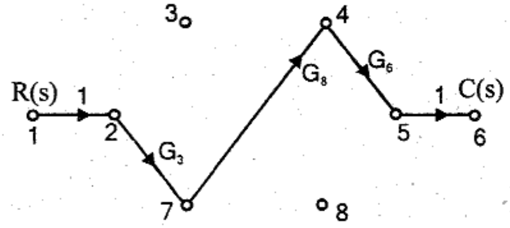


Fig 6 : Forward path-4

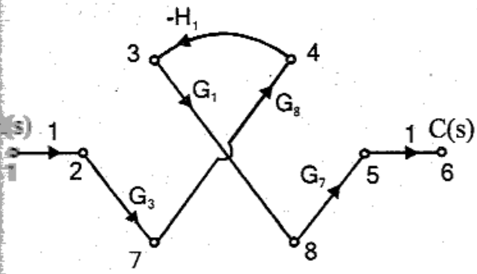


Fig 7 : Forward path-5

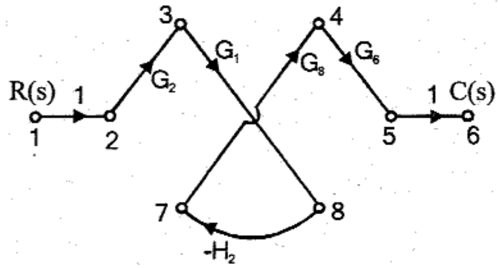


Fig 8 : Forward path-6

Gain of forward path-1, $P_1 = G_2 G_4 G_6$

Gain of forward path-2, $P_2 = G_3 G_5 G_7$

Gain of forward path-3, $P_3 = G_1 G_2 G_7$

Gain of forward path-4, $P_4 = G_3 G_8 G_6$

Gain of forward path-5, $P_5 = -G_1G_3G_7G_8H_1$

Gain of forward path-6, $P_6 = -G_1G_2G_6G_8H_2$

Individual Loop Gain

There are three individual loops.

Let individual loop gains be P_{11}, P_{21} and P_{31}.

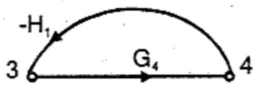


Fig 9 : Loop-1

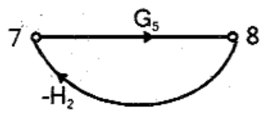


Fig 10 : Loop-2

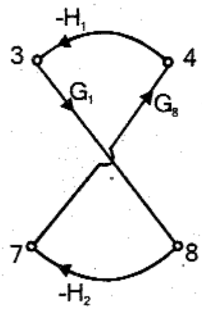


Fig 11 : Loop-3

Loop gain of individual loop-1, $P_{11} = -G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_5H_2$

Loop gain of individual loop-3, $P_{31} = G_1G_8H_1H_2$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let gain product of two non-touching loops be P_{12}.

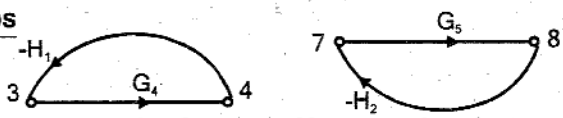


Fig 12 : Combination of 2 non-touching loops

$$\left.\begin{array}{l}\text{Gain product of first combination}\\ \text{of two non-touching loops}\end{array}\right\} P_{12} = P_{11}P_{21} = (-G_4H_1)\,(-G_5H_2) = G_4G_5H_1H_2$$

Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12} = 1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$= 1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2$$

The part of the graph non-touching forward path - 1 is shown in fig 13.

$$\therefore \Delta_1 = 1 - (-G_5H_2) = 1 + G_5H_2$$

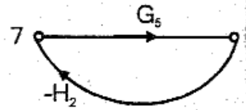


Fig 13

The part of the graph non-touching forward path -2 is shown in fig 14.

$$\therefore \Delta_2 = 1 - (-G_4H_1) = 1 + G_4H_1$$

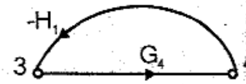


Fig 14

There is no part of the graph which is non-touching with forward paths 3, 4, 5 and 6.

$$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\left(\sum_K P_K \Delta_K\right) \qquad \text{(Number of forward paths is six and so K = 6)}$$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6)$$

$$= \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_1G_2G_7 + G_3G_6G_8 - G_1G_3G_7G_8H_1 - G_1G_2G_6G_8H_2}{1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}$$

EXAMPLE 1.27

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

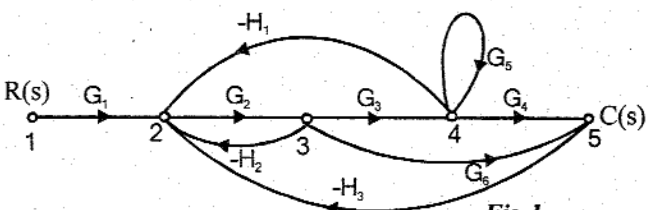


Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore K = 2$. Let the forward path gains be P_1 and P_2.

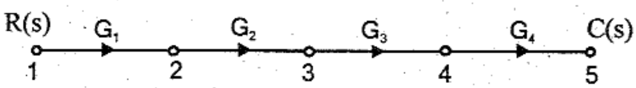


Fig 2 : Forward path-1

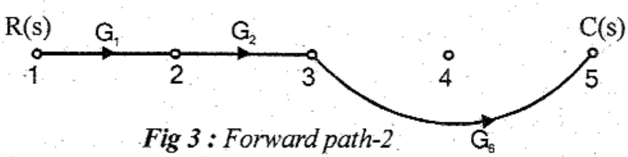


Fig 3 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

Individual Loop Gain

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

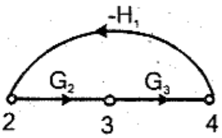


Fig 4 : loop-1

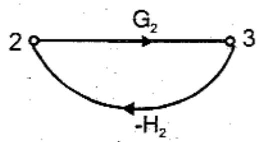


Fig 5 : loop-2

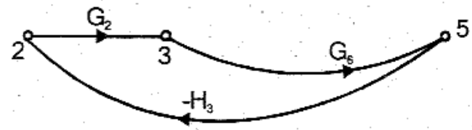


Fig 6 : loop-3

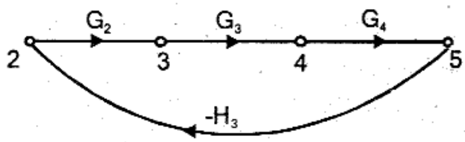


Fig 7 : loop-4

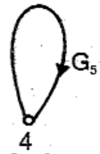


Fig 8 : loop-5

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.

Let the gain products of two non-touching loops be P_{12} and P_{22}.

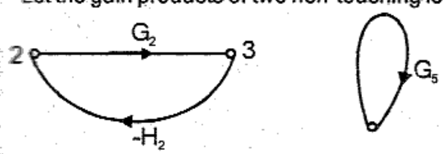


Fig 9 : First combination of two non-touching loops

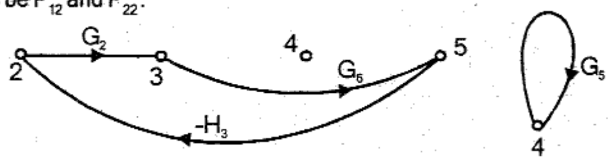


Fig 10 : Second combination of two non-touching loops

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = G_2G_5H_2$

Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})$$
$$= 1-(-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3)$$
$$+(-G_2H_2G_5 - G_2G_5G_6H_3)$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

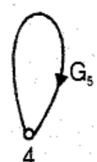


Fig 11

$\therefore \Delta_2 = 1 - G_5$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K \text{ (Number of forward path is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right] = \frac{1}{\Delta}\left[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1-G_5)\right]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

EXAMPLE 1.28

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

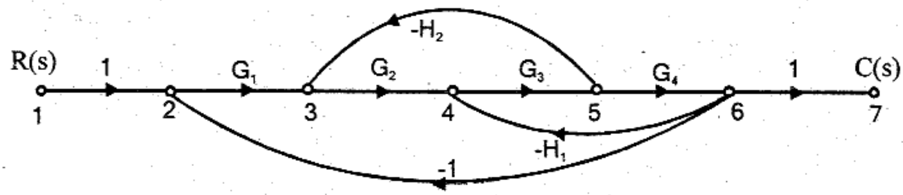


Fig 1

SOLUTION

I. Forward Path Gains

There is only one forward path. $\therefore K = 1$.

Let the forward path gain be P_1.

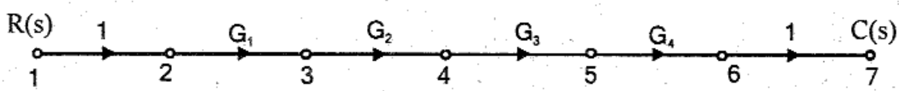


Fig 1 : Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

II. Individual Loop Gain

There are three individual loops. Let the loop gains be P_{11}, P_{21}, P_{31}.

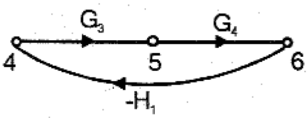


Fig 3 : loop-1

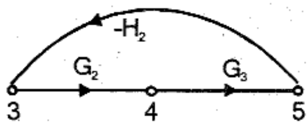


Fig 4 : loop-2

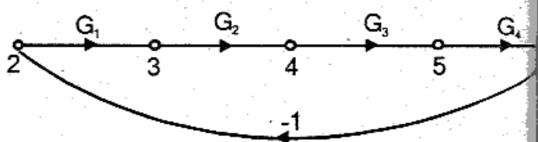


Fig 5 : loop-3

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$
$$= 1 - (-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)$$
$$= 1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4$$

Since no part of the graph is non-touching with forward path-1, $\Delta_1 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta} P_1\Delta_1 \text{ (Number of forward path is 1 and so K = 1)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

EXAMPLE 1.29

The signal flow graph for a feedback control system is shown in fig 1. Determine the closed loop transfer function C(s)/R(s).

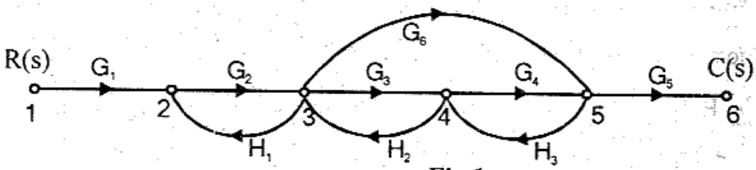


Fig 1

SOLUTION

Forward Path Gains

There are two forward paths. $\therefore$ K = 2.

Let forward path gains be P_1 and P_2.

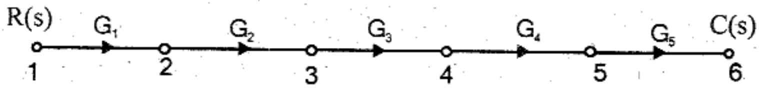


***Fig 2 :** Forward path-1*

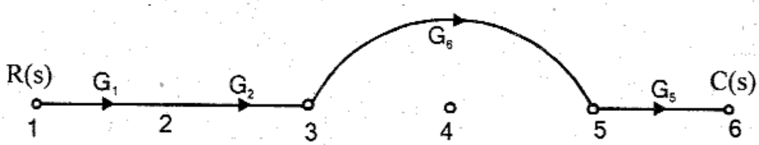


***Fig 3 :** Forward path-2*

Gain of forward path-1, $P_1 = G_1G_2G_3G_4G_5$

Gain of forward path-2, $P_2 = G_1G_2G_6G_5$

Individual Loop Gain

There are four individual loops. Let individual loop gains be P_{11}, P_{21}, P_{31} and P_{41}.

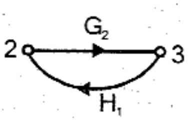


***Fig 4 :** loop-1*

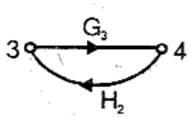


***Fig 5 :** loop-2*

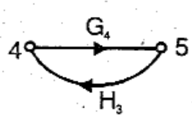


***Fig 6 :** loop-3*

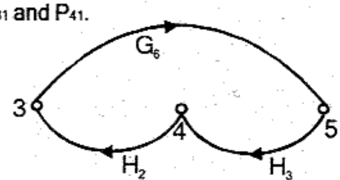


***Fig 7 :** loop-4*

Loop gain of individual loop-1, $P_{11} = G_2H_1$

Loop gain of individual loop-2, $P_{21} = G_3H_2$

Loop gain of individual loop-3, $P_{31} = G_4H_3$

Loop gain of individual loop-4, $P_{41} = G_6H_2H_3$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let the gain products of two non-touching loops be P_{12}.

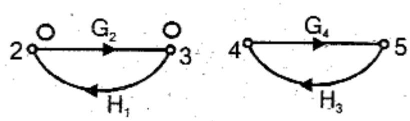


***Fig 8 :** First combination of two non touching loops*

Gain product of first combination of two non - touching loops $\Big\}$ $P_{12} = (G_2H_1)\,(G_4H_3)$

$$= G_2G_4H_1H_3$$

IV. Calculation of Δ and Δ_K

$$\begin{aligned}\Delta &= 1-(P_{11}+P_{21}+P_{31}+P_{41})+P_{12}\\ &= 1-(G_2H_1+G_3H_2+G_4H_3+G_6H_2H_3)+G_2G_4H_1H_3\\ &= 1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3\end{aligned}$$

Since there is no part of graph which is non-touching with forward path-1 and 2, $\Delta_1 = \Delta_2 = 1$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is two and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3}$$

EXAMPLE 1.30

Convert the given block diagram to signal flow graph and determine C(s)/R(s).

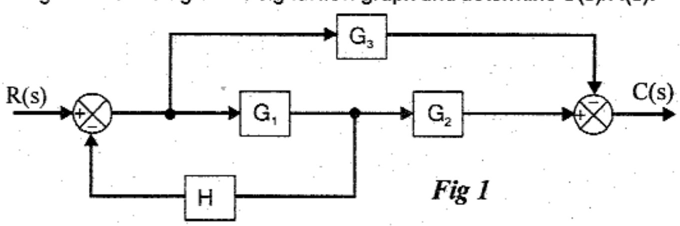


Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

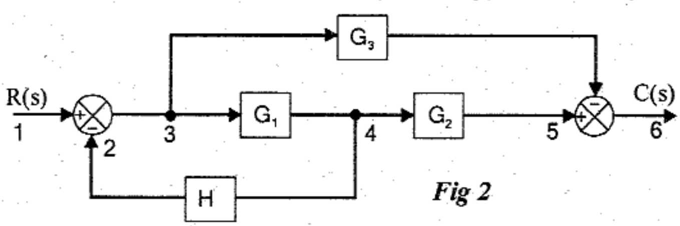


Fig 2

The signal flow graph of the above system is shown in fig 3.

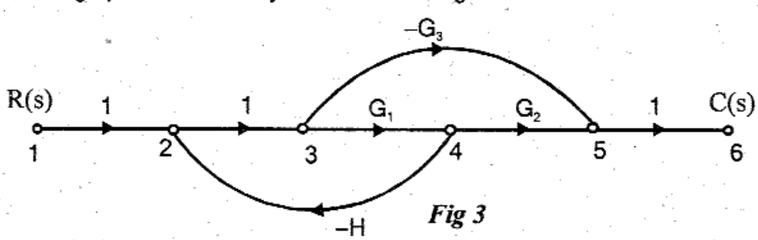


Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore$ K=2

Let the forward path gains be P_1 and P_2.

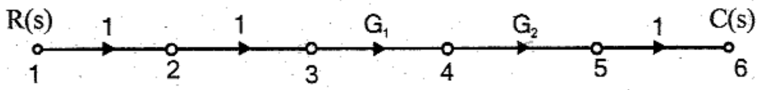


Fig 4 : Forward path-1

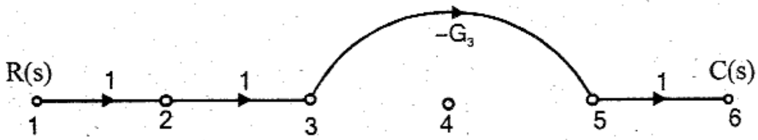


Fig 5 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2$

Gain of forward path-2, $P_2 = -G_3$

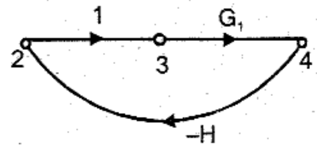


Fig 3 : loop-1

II. Individual Loop Gain

There is only one individual loop. Let the individual loop gain be P_{11}.

Loop gain of individual loop-I, $P_{11} = -G_1H$.

III. Gain Products of Two Non-touching Loops

There are no combinations of non-touching Loops.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}] = 1 + G_1H$$

Since there are no part of the graph which is non-touching with forward path-1 and 2,

$$\Delta_1 = \Delta_2 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{G_1G_2 - G_3}{1+G_1H}$$

EXAMPLE 1.31

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

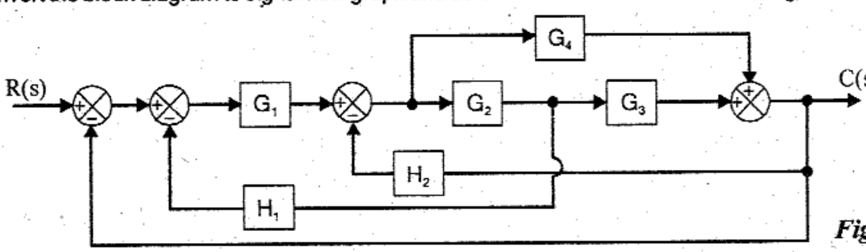


Fig 1

SOLUTION

The nodes are assigned at input, ouput, at every summing point & branch point as shown in fig 2.

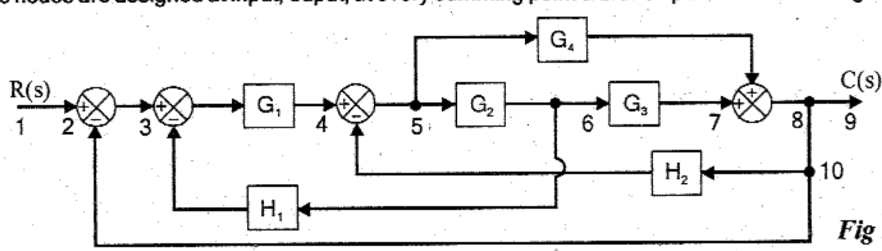


Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

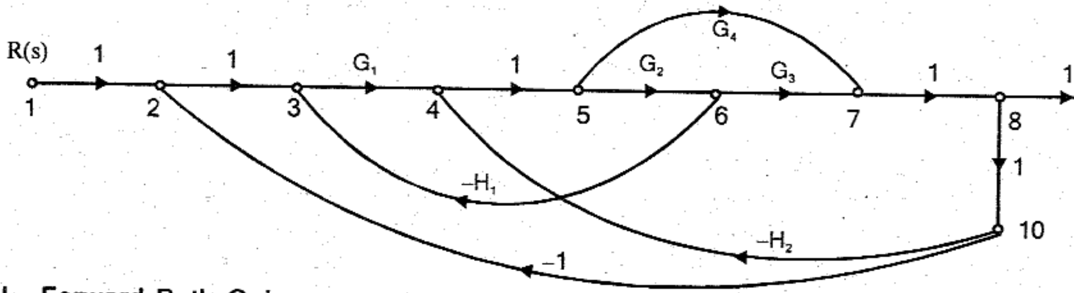


Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore$ K=2.

Let the gain of the forward paths be P_1 and P_2.

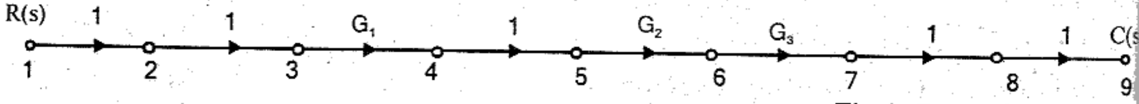


Fig 4 : Forward path-1

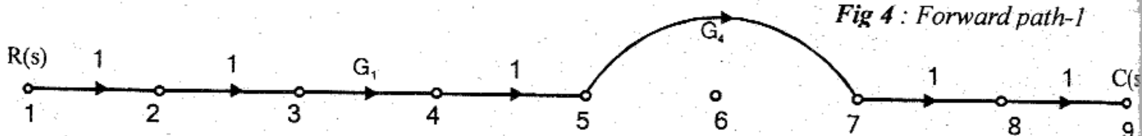


Fig 5 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_1G_4$

II. Individual Loop Gain

There are five individual loops. Let the individual loop gain be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

Loop gain of individual loop-1, $P_{11} = -G_1G_2G_3$

Loop gain of individual loop-2, $P_{21} = -G_2G_1H_1$

Loop gain of individual loop-3, $P_{31} = -G_2G_3H_2$

Loop gain of individual loop-4, $P_{41} = -G_1G_4$

Loop gain of individual loop-5, $P_{51} = -G_4H_2$

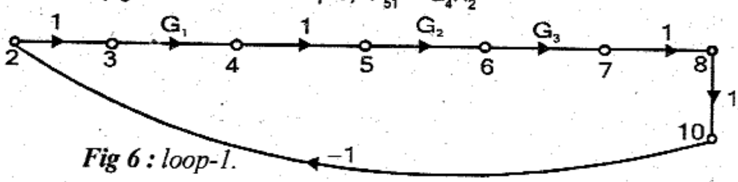


Fig 6 : loop-1.

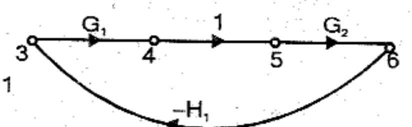


Fig 7 : loop-2.

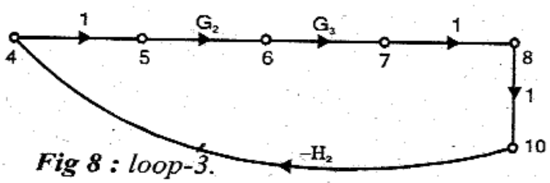


Fig 8 : loop-3.

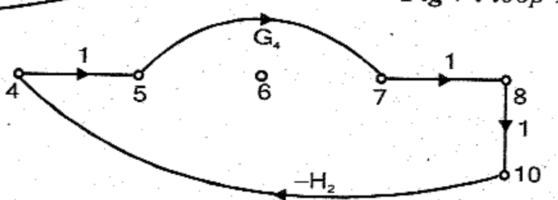


Fig 10 : loop-5.

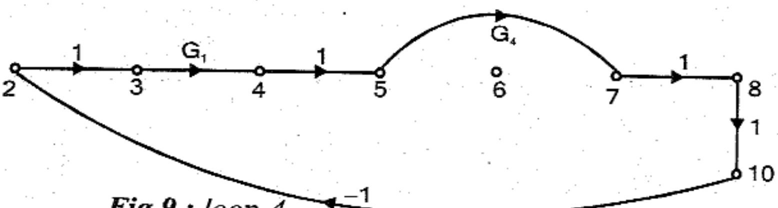


Fig 9 : loop-4.

Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.,.

Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}+P_{21}+P_{31}+P_{41}+P_{51}] = 1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_1G_4+G_4H_2$$

Since no part of graph is non touching with forward paths-I and 2, $\Delta_1 = \Delta_2 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_1G_4+G_4H_2}$$

EXAMPLE 1.32

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

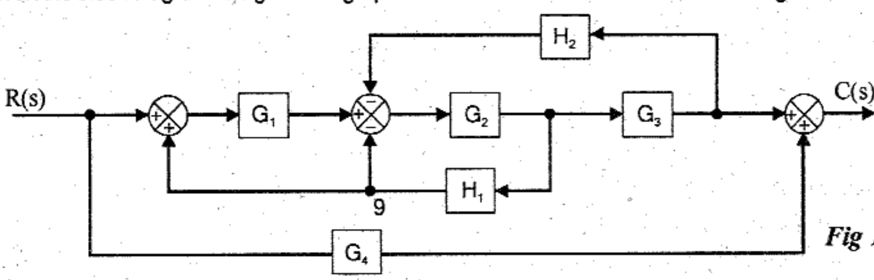


Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

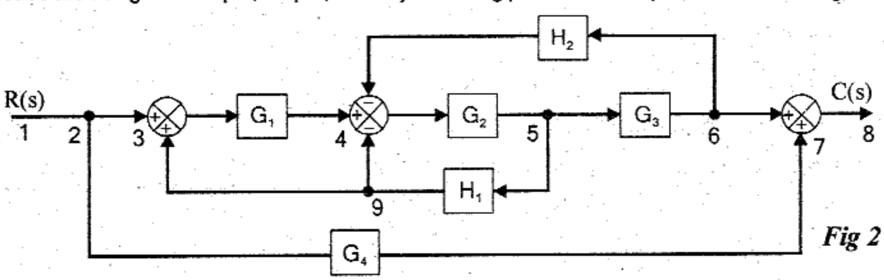


Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

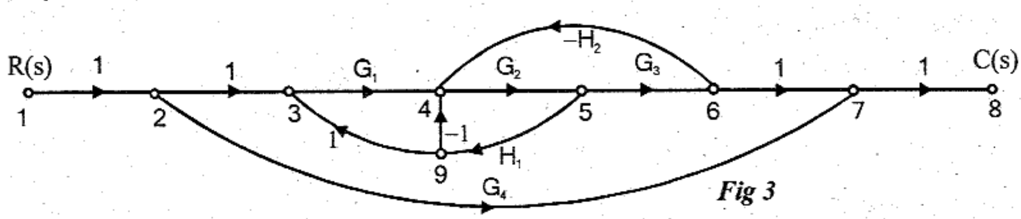


Fig 3

Forward Path Gains

There are two forward path, $\therefore$ K=2.

Let the forward path gains be P_1 and P_2.

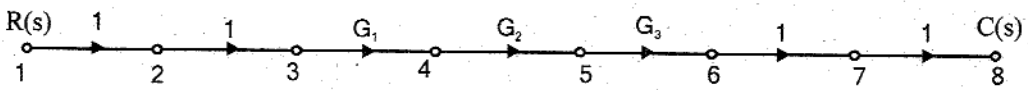


Fig 4 : Forward path-1.

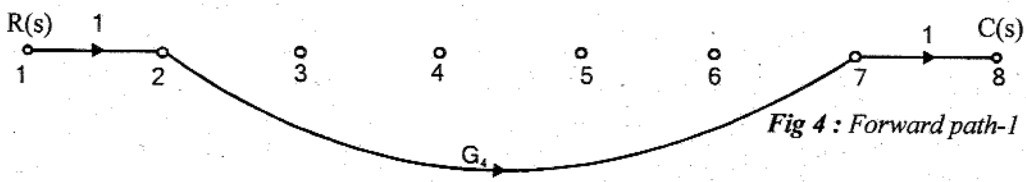


Fig 4 : Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_4$

II. Individual Loop Gain

There are three individual loops with gains P_{11}, P_{21} and P_{31}.

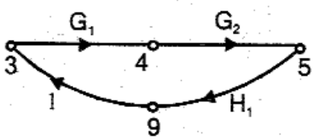


Fig 6 : loop-1.

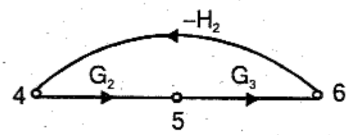


Fig 7 : loop-2.

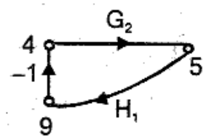


Fig 8 : loop-3.

Gain of individual loop-1, $P_{11} = G_1G_2H_1$

Gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Gain of individual loop-3, $P_{31} = -G_2H_1$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two-non touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_K

$$\Delta = 1-[P_{11}+P_{21}+P_{31}] = 1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

Since no part of graph touches forward path-1, $\Delta_1 = 1$.

The part of graph non touching forward path-2 is shown in fig 9.

$$\therefore \Delta_2 = 1-[G_1G_2H_1 - G_2G_3H_2 - G_2H_1]$$

$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

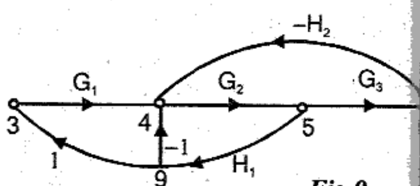


Fig 9

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] \text{ (Number of forward paths is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4(1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1)]$$

$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4 - G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1]$$

$$= \frac{G_1G_2G_3 + G_4 - G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1}{1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1}$$

EXAMPLE 1.33

Draw a signal flow graph and evaluate the closed loop transfer function of a system whose block diagram is shown in fig 1.

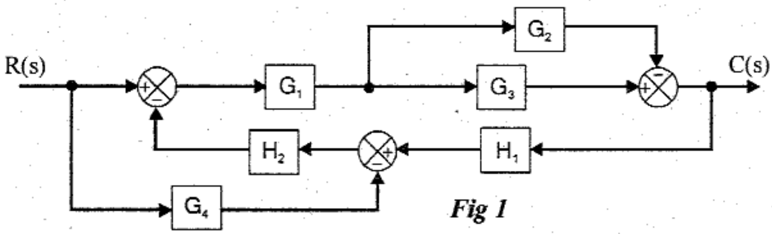


Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

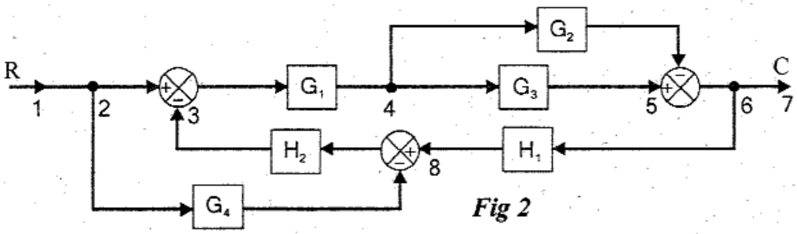


Fig 2

The signal flow graph for the block diagram of fig 2, is shown in fig 3.

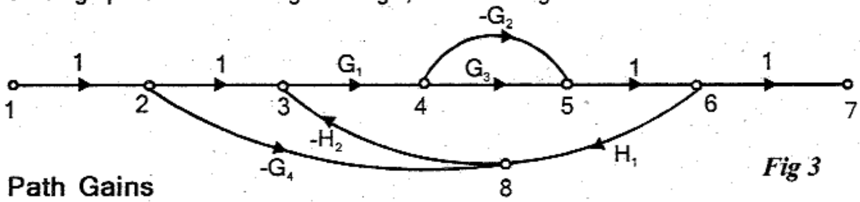


Fig 3

Forward Path Gains

There are four forward paths, $\therefore K = 4$

Let the forward path gains be P_1, P_2, P_3 and P_4.

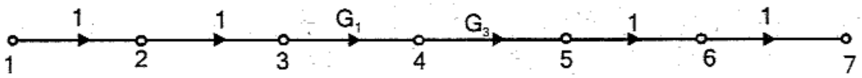


Fig 4 : Forward path-1.

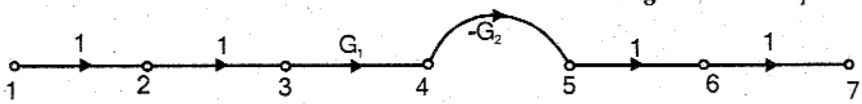


Fig 5 : Forward path-2.

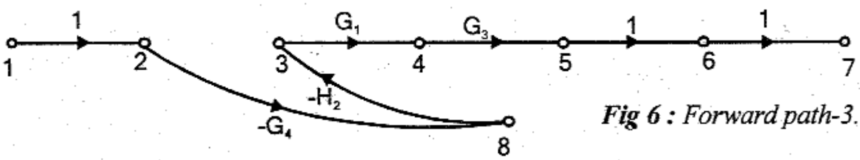


Fig 6 : Forward path-3.

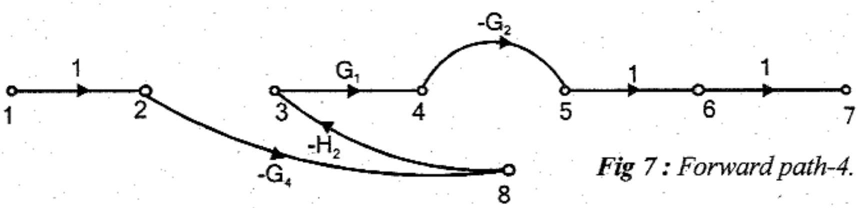


Fig 7 : Forward path-4.

Gain of forward path-1, $P_1 = G_1G_3$
Gain of forward path-2, $P_2 = -G_1G_2$
Gain of forward path-3, $P_3 = G_1G_3G_4H_2$
Gain of forward path-4, $P_4 = -G_1G_2G_4H_2$

II. Individual Loop Gain

There are two individual loops, let individual loop gains be P_{11} and P_{21}.

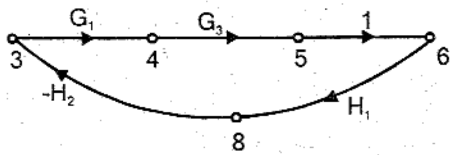


Fig 7 : loop-1

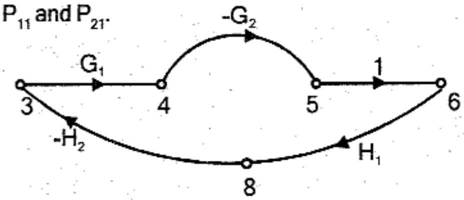


Fig 7 : loop-2

Loop gain of individual loop-1, $P_{11} = -G_1G_3H_1H_2$
Loop gain of individual loop-2, $P_{21} = G_1G_2H_1H_2$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [\text{sum of individual loop gain}] = 1 - (P_{11} + P_{21})$$
$$= 1 - [-G_1G_3H_1H_2 + G_1G_2H_1H_2] = 1 + G_1G_3H_1H_2 - G_1G_2H_1H_2$$

Since no part of graph is non touching with the forward paths, $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$.

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{P_1 + P_2 + P_3 + P_4}{\Delta} \text{ (Number of forward paths is 4 and so K = 4)}$$

$$= \frac{G_1G_3 - G_1G_2 + G_1G_3G_4H_2 - G_1G_2G_4H_2}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}$$

$$= \frac{G_1(G_3 - G_2) + G_1G_4H_2(G_3 - G_2)}{1 + G_1H_1H_2(G_3 - G_2)} = \frac{G_1(G_3 - G_2)(1 + G_4H_2)}{1 + G_1H_1H_2(G_3 - G_2)}$$

1.14 SHORT QUESTIONS AND ANSWERS

Q1.1 ***What is system?***

When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a system.

Q1.2 ***What is control system?***

A system consists of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity, then the system is called control system. The output quantity is called controlled variable or response and input quantity is called command signal or excitation.

Q1.3 ***What are the two major type of control systems?***

The two major type of control systems are open loop and closed loop systems.

Q1.4 ***Define open loop system.***

The control system in which the output quantity has no effect upon the input quantity are called open loop control system. This means that the output is not fedback to the input for correction.

Q1.5 ***Define closed loop system.***

The control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called closed loop control systems.

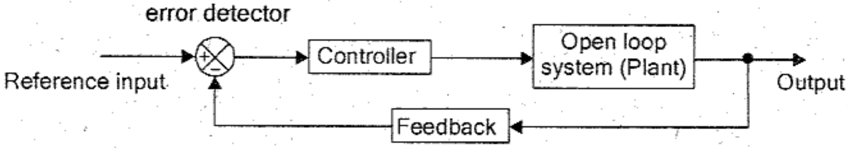
Q1.6 ***What is feedback? What type of feedback is employed in control system?***

The feedback is a control action in which the output is sampled and a proportional signal is given to input for automatic correction of any changes in desired output.

Negative feedback is employed in control system.

Q1.7 ***What are the components of feedback control system?***

The components of feedback control system are plant, feedback path elements, error detector and controller.



Q1.8 ***Why negative feedback is invariably preferred in a closed loop system?***

The negative feedback results in better stability in steady state and rejects any disturbance signals. It also has low sensitivity to parameter variations. Hence negative feedback is preferred in closed loop systems.

Q1.9 ***What are the characteristics of negative feedback?***

The characteristics of negative feedback are as follows :

(i) accuracy in tracking steady state value.

(ii) rejection of disturbance signals.

(iii) low sensitivity to parameter variations.

(iv) reduction in gain at the expense of better stability.

Q1.10. ***What is the effect of positive feedback on stability?***

The positive feedback increases the error signal and drives the output to instability. But sometimes the positive feedback is used in minor loops in control systems to amplify certain internal signals or parameters.

Q1.11. ***Distinguish between open loop and closed loop system.***

Open loop	Closed loop
1. Inaccurate & unreliable.	1. Accurate & reliable.
2. Simple and economical.	2. Complex and costly.
3. Changes in output due to external disturbances are not corrected automatically.	3. Changes in output due to external disturbances are corrected automatically.
4. They are generally stable.	4. Great efforts are needed to design a stable system.

Q1.12 ***What is servomechanism?***

The servomechanism is a feedback control system in which the output is mechanical position (or time derivatives of position e.g. velocity and acceleration).

Q1.13 ***State the principle of homogenity (or) State the principle of superposition.***

The principle of superposition and homogenity states that if the system has responses $c_1(t)$ and $c_2(t)$ for the inputs $r_1(t)$ and $r_2(t)$ respectively then the system response to the linear combination of these input $a_1r_1(t) + a_2r_2(t)$ is given by linear combination of the individual outputs $a_1c_1(t) + a_2c_2(t)$, where a_1 and a_2 are constants.

Q1.14 ***Define linear system.***

A system is said to be linear, if it obeys the principle of superposition and homogenity, which states that the response of a system to a weighed sum of signals is equal to the corresponding weighed sum of the responses of the system to each of the individual input signals. The concept of linear system is diagrammatically shown below.

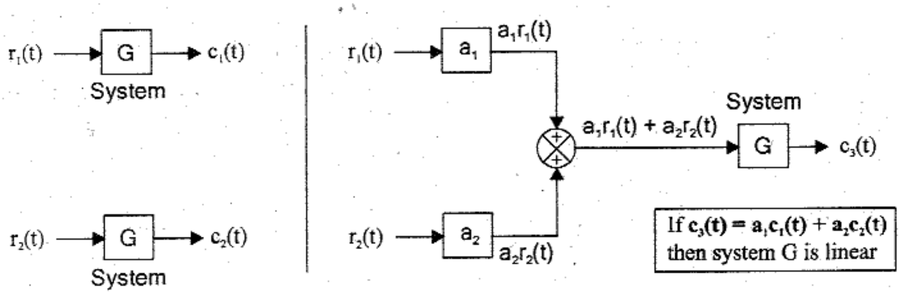


Fig Q1.14 : Principle of linearity and superposition.

Q1.15 ***What is time invariant system?***

A system is said to be time invariant if its input-output characteristics do not change with time. A linear time invariant system can be represented by constant coefficient differential equations. (In linear time varying systems the coefficients of the differential equation governing the system are function of time).

Q1.16 ***Define transfer function.***

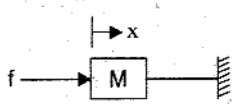
The transfer function of a system is defined as the ratio of Laplace transform of output to Laplace transform of input with zero initial conditions. (It is also defined as the Laplace transform of the impulse response of system with zero initial conditions).

Q1.17 ***What are the basic elements used for modelling mechanical translational system?***

The model of mechanical translational system can be obtained by using three basic elements mass, spring and dashpot.

Q1.18 ***Write the force balance equation of ideal mass element.***

Let a force f be applied to an ideal mass M. The mass will offer an opposing force, f_m which is proportional to acceleration.

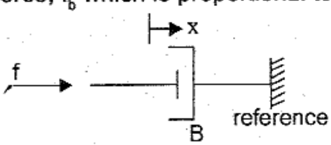


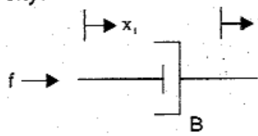
$$\therefore\ f = f_m = M\frac{d^2x}{dt^2}$$

Q1.19 ***Write the force balance equation of ideal dashpot.***

Let a force f be applied to an ideal dashpot, with viscous frictional coefficient B. The dashpot will offer an opposing force, f_b which is proportional to velocity.

$$f = f_b = B\frac{dx}{dt}$$

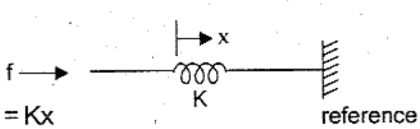




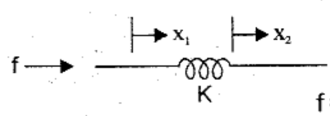
$$f = f_b = B\frac{d}{dt}(x_1 - x_2)$$

Q1.20 ***Write the force balance equation of ideal spring.***

Let a force f be applied to an ideal spring with spring constant K. The spring will offer an opposing force, f_k which is proportional to displacement.



$$f = f_k = Kx$$



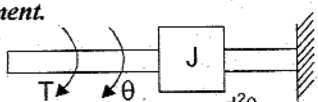
$$f = f_k = K(x_1 - x_2)$$

Q1.21 What are the basic elements used for modelling mechanical rotational system?

The model of mechanical rotational system can be obtained using three basic elements mass with moment of inertia, J, dash-pot with rotational frictional coefficient, B and torsional spring with stiffness, K.

Q1.22 Write the torque balance equation of an ideal rotational mass element.

Let a torque T be applied to an ideal mass with moment of inertia, J. The mass will offer an opposing torque T_j which is proportional to angular acceleration.

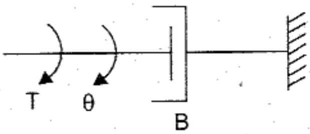


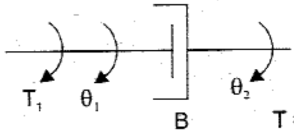
$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Q1.23 Write the torque balance equation of an ideal rotational dash-pot.

Let a torque T be applied to a rotational dash-pot with frictional coefficient B. The dashpot will offer an opposing torque which is proportional to angular velocity.

$$T = T_b = B\frac{d\theta}{dt}$$



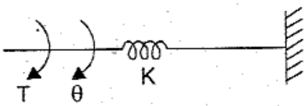


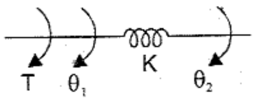
$$T = T_b = B\frac{d}{dt}(\theta_1 - \theta_2)$$

Q1.24 Write the torque balance equation of ideal rotational spring.

Let a torque T be applied to an ideal rotational spring with spring constant K. The spring will offer an opposing torque T_k which is proportional to angular displacement.

$$T_k = K\theta$$





$$T = T_k = K(\theta_1 - \theta_2)$$

Q1.25 Name the two types of electrical analogous for mechanical system.

The two types of analogies for the mechanical system are force-voltage and force-current analogy.

Q1.26 Write the analogous electrical elements in force-voltage analogy for the elements of mechanical translational system.

Force, f	→ Voltage, e	Frictional coefficient, B	→ Resistance, R
Velocity, v	→ Current, i	Stiffness, K	→ Inverse of capacitance, 1/C
Displacement, x	→ Charge, q	Newton's second law, $\Sigma f = 0$	→ Kirchoff 's voltage law, $\Sigma v = 0$
Mass, M	→ Inductance, L		

Q1.27 Write the analogous electrical elements in force-current analogy for the elements of mechanical translational system.

Force, f	→ Current, i	Frictional coefficient, B	→ Conductance, G = 1/ R
Velocity, v	→ Voltage, v	Stiffness, K	→ Inverse of Inductance, 1/L
Displacement, x	→ Flux, ϕ	Newton's second law, $\Sigma f = 0$	→ Kirchoff's current law, $\Sigma i = 0$
Mass, M	→ Capacitance, C		

Q1.28 Write the analogous electrical elements in torque-voltage analogy for the elements of mechanical rotational system.

Torque, T	→ Voltage, e	Stiffness of spring, K	→ Inverse of capacitance, 1/C
Angular velocity, ω	→ Current, i	Frictional coefficient, B	→ Resistance, R
Moment of inertia, J	→ Inductance, L	Newton's second law, $\Sigma T = 0$	→ Kirchoff 's voltage law, $\Sigma v = 0$.
Angular displacement, θ	→ Charge, q		

Q1.29 ***Write the analogous electrical elements in torque-current analogy for the elements of mechanica rotational system.***

Torque, T	→ Current, i	Frictional coefficient , B	→ Conductance, G = 1/R
Angular velocity, ω	→ Voltage, v	Stiffness of spring, K	→ Inverse of inductance, 1/L
Angular displacement, θ	→ Flux, ϕ	Newton's second law, $\Sigma T = 0$	→ Kirchoff 's current law, $\Sigma i =$
Moment of inertia, J	→ Capacitance, C		

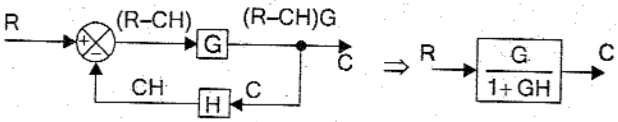
Q1.30 ***What is block diagram? What are the basic components of block diagram?***

A block diagram of a system is a pictorial representation of the functions performed by each componen of the system and shows the flow of signals. The basic elements of block diagram are block, branch point and summing point.

Q1.31 ***What is the basis for framing the rules of block diagram reduction technique?***

The rules for block diagram reduction technique are framed such that any modification made on the diagram does not alter the input output relation.

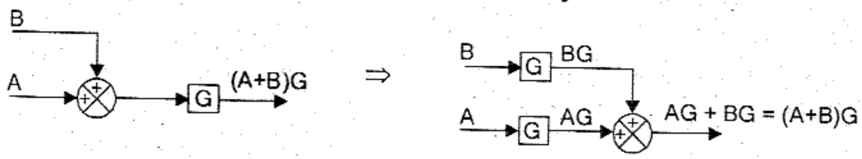
Q1.32 ***Write the rule for eliminating negative feedback loop.***



Proof

$$C = (R - CH)\,G$$
$$C = RG - CHG$$
$$C + CHG = RG$$
$$C(1 + HG) = RG$$
$$\frac{C}{R} = \frac{G}{1+GH}$$

Q1.33 ***Write the rule for moving the summing point ahead of a block.***



Q1.34 ***What is a signal flow graph?***

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. The signal flow graph of the system can be constructed using these equations.

Q1.35 ***What is transmittance?***

The transmittance is the gain acquired by the signal when it travels from one node to another node in signal flow graph.

Q1.36 ***What is sink and source?***

Source is the input node in the signal flow graph and it has only outgoing branches. Sink is a output node in the signal flow graph and it has only incoming branches.

Q1.37 ***Define non-touching loop.***

The loops are said to be non-touching if they do not have common nodes.

Q1.38 ***What are the basic properties of signal flow graph?***

The basic properties of signal flow graph are,

(i) Signal flow graph is applicable to linear systems.

(ii) It consists of nodes and branches. A node is a point representing a variable or signal. A branch indicates functional dependence of one signal on the other.

(iii) A node adds the signals of all incoming branches and transmits this sum to all outgoing branches.

(iv) Signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.

(v) The algebraic equations must be in the form of cause and effect relationship.

Q1.39 *Write the Mason's gain formula.*

Mason's gain formula states that the overall gain of the system [transfer function] as follows,

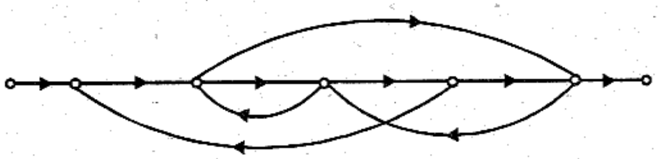
Overall gain, $T = \frac{1}{\Delta}\sum_K P_K \Delta_K$

$T = T(s) =$ Transfer function of the system
$K =$ Number of forward paths in the signal flow graph
$P_K =$ Forward path gain of K^{th} forward path

$$\Delta = 1 - \begin{bmatrix}\text{sum of individual} \\ \text{loop gains}\end{bmatrix} + \begin{bmatrix}\text{sum of gain products of all possible} \\ \text{combinations of two non-touching loops}\end{bmatrix}$$
$$- \begin{bmatrix}\text{sum of gain products of all possible} \\ \text{combinations of three non-touching loops}\end{bmatrix} + \ldots..$$

$\Delta_K = \Delta$ for that part of the graph which is not touching K^{th} forward path

Q1.40 *For the given signal flow graph, identify the number of forward path and number of individual loop.*



Number of forward paths = 2
Number of individual loops = 4

1.15 EXERCISES

E1.1 For the mechanical system shown in fig E1.1derive the transfer function. Also draw the force-voltage and force-current analogous circuits.

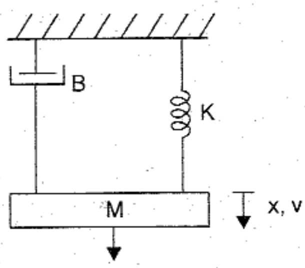


Fig E1.1

E1.2 For the mechanical system shown in fig E1.2 draw the force-voltage and force-current analogous circuits.

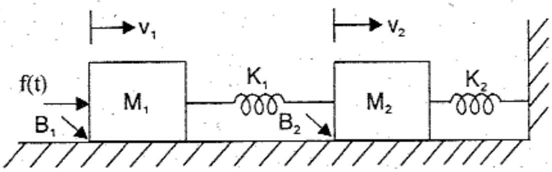


Fig E1.2

E1.3 Write the differential equations governing the mechanical system shown in fig E1.3(a) & (b). Also draw the force-voltage and force-current analogous circuit.

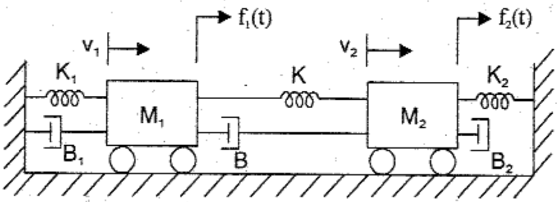


Fig E1.3(a)

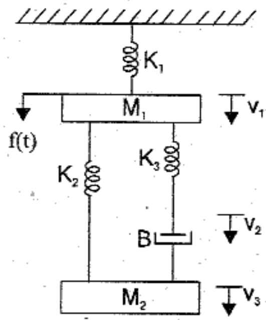


Fig E1.3(b)

E1.4 Consider the mechanical translational system shown in fig E1.4, Draw(a) force-voltage and (b) force-current analogous circuits.

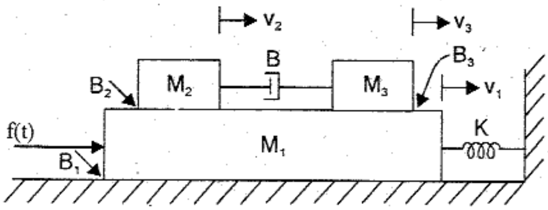


Fig E1.4

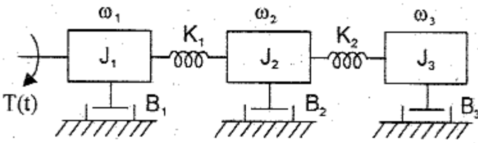


Fig E1.5

E1.5 Write the differential equations governing the rotational mechanical system shown in fig E1.5. Also draw the torque-voltage and torque-current analogous circuits.

E1.6 In an electrical circuit the elements resistance, capacitance and inductance are connected in parallel across the voltage source E as shown in fig E1.6, Draw(a) Translation mechnical analogous system (b) Rotational mechanical analogous system.

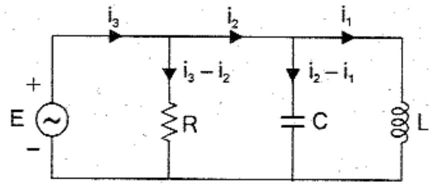


Fig E1.6

E1.7 Consider the block diagram shown in fig E.1.7(a), (b) (c) & (d). Using the block diagram reduction technique, find C/R.

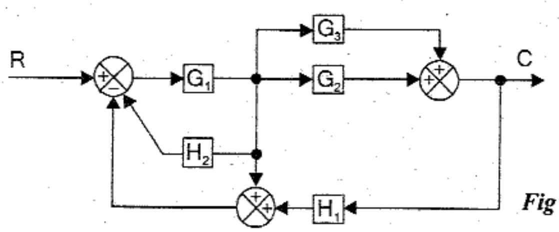


Fig E1.7(a)

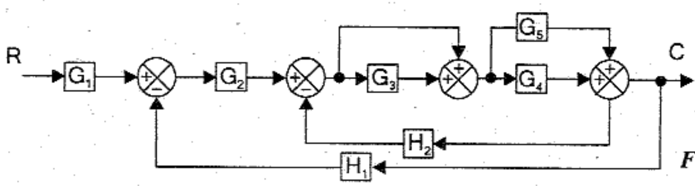


Fig E1.7(b)

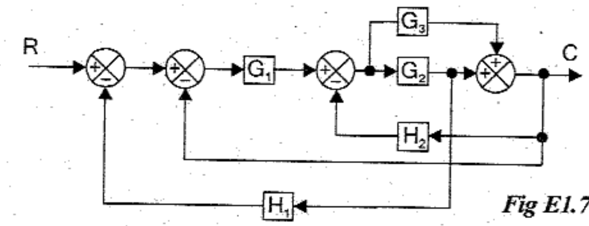


Fig E1.7(c)

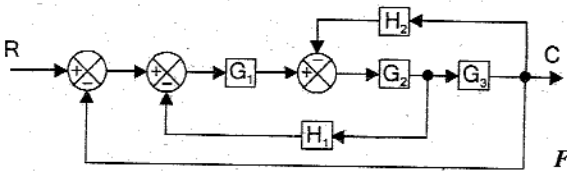


Fig E1.7(d)

E 8 Convert the block diagram shown in fig E1.8 to signal flow graph and find the transfer function of the system.

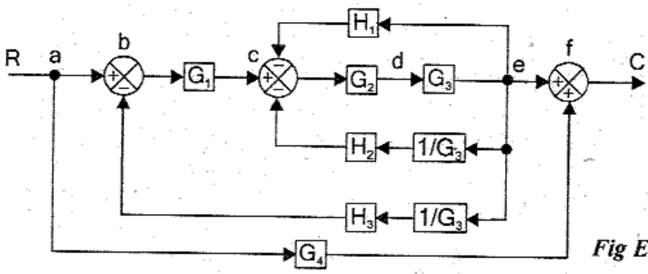


Fig E1.8

E 9 Consider the system shown in fig E1.9(a), (b), (c) & (d). obtain the transfer function using Mason's gain formula.

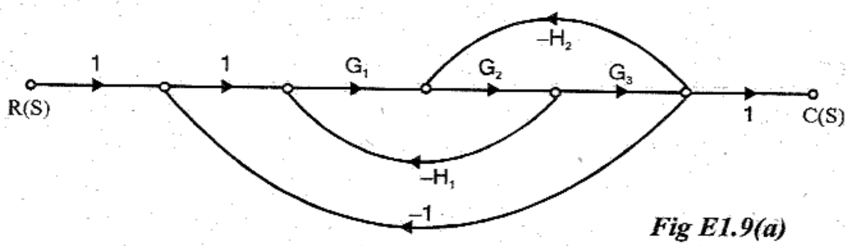


Fig E1.9(a)

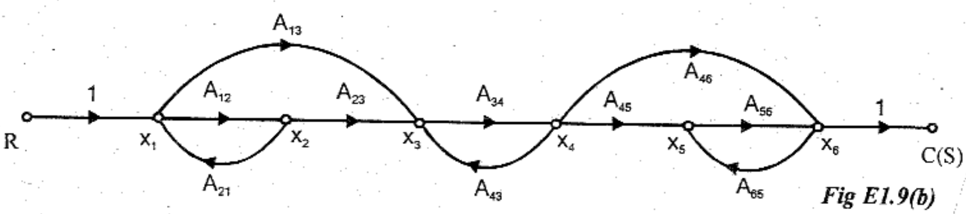


Fig E1.9(b)

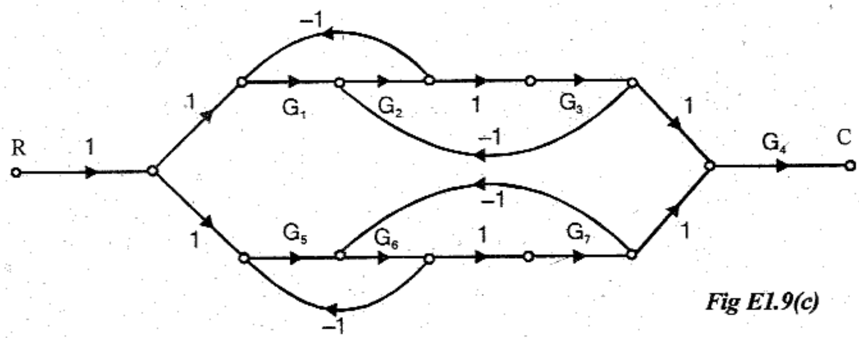


Fig E1.9(c)

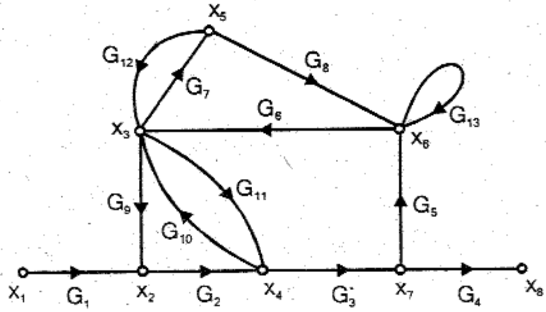


Fig E1.9(d)

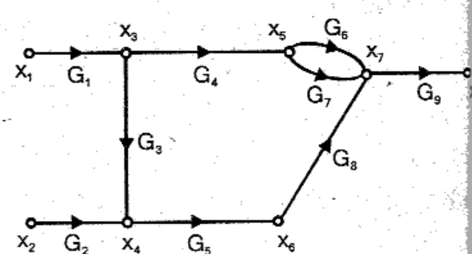


Fig E1.10

E1.10 Consider the signal flow graph shown in fig E.1.10 obtain $\frac{X_8}{X_1}$ and $\frac{X_8}{X_2}$

E1.11 Find the transfer functions of the networks shown in fig E1.11(a), (b), (c) & (d).

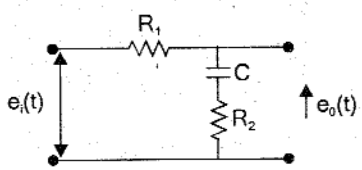


Fig E1.11(a)

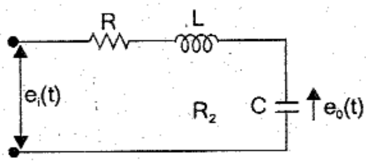


Fig E1.11(b)

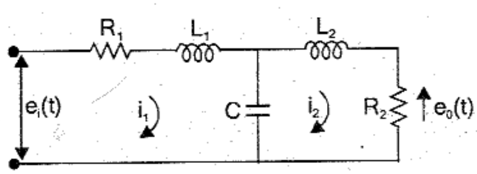


Fig E1.11(c)

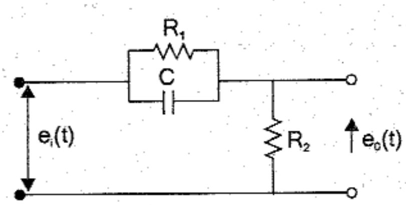


Fig E1.11(d)

E1.12 Find the transfer function of the circuit shown in fig E1.12.

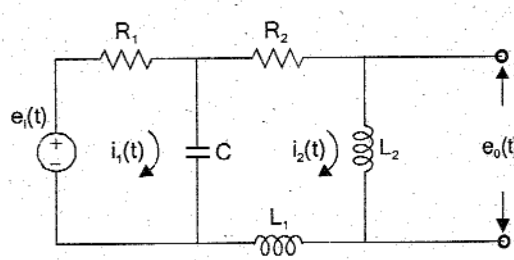
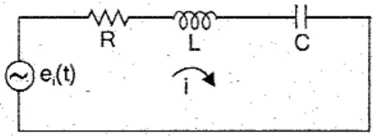


Fig E1.12

ANSWER FOR EXERCISE PROBLEMS

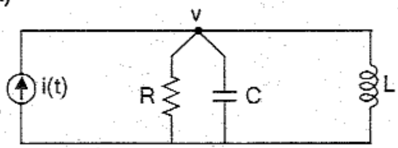
The transfer function is $\dfrac{X(s)}{F(s)} = \dfrac{1}{(Ms^2 + Bs + K)}$



$f(t) \rightarrow e(t)$ $\quad M \rightarrow L \quad K \rightarrow 1/C$

$v \rightarrow i$ $\quad B \rightarrow R$

Force-voltage analogous circuit

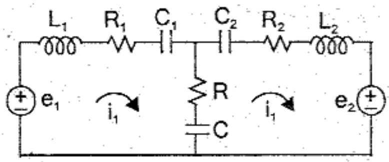


$f(t) \rightarrow i(t)$ $\quad M \rightarrow C \quad K \rightarrow 1/L$

$v \rightarrow v$ $\quad B \rightarrow 1/R$

Force-current analogous circuit

2



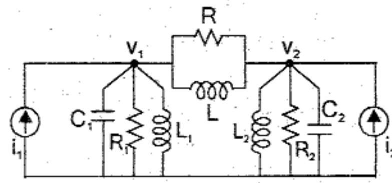
$f_1 \rightarrow e_1$ $\quad M_1 \rightarrow L_1 \quad B \rightarrow R$

$f_2 \rightarrow e_2$ $\quad M_2 \rightarrow L_2 \quad K_1 \rightarrow 1/C_1$

$v_1 \rightarrow i_1$ $\quad B_1 \rightarrow R_1 \quad K_2 \rightarrow 1/C_2$

$v_2 \rightarrow i_2$ $\quad B_2 \rightarrow R_2 \quad K \rightarrow 1/C$

Force-voltage analogous circuit



$f_1 \rightarrow i_1$ $\quad M_1 \rightarrow C_1 \quad B \rightarrow 1/R$

$f_2 \rightarrow i_2$ $\quad M_2 \rightarrow C_2 \quad K_1 \rightarrow 1/L_1$

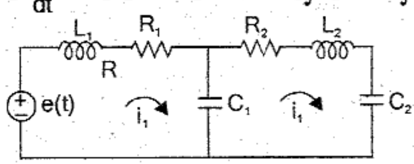
$v_1 \rightarrow v_1$ $\quad B_1 \rightarrow 1/R_1 \quad K_2 \rightarrow 1/L_2$

$v_2 \rightarrow v_2$ $\quad B_2 \rightarrow 1/R_2 \quad K \rightarrow 1/L$

Force-current analogous circuit

3 a) $M_1 \dfrac{dv_1}{dt} + B_1 v_1 + B(v_1 - v_2) + K_1 \int v_1 dt + K \int (v_1 - v_2) dt = f_1(t)$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + B(v_2 - v_1) + K_2 \int v_2 dt + K \int (v_2 - v_1) dt = f_2(t)$$

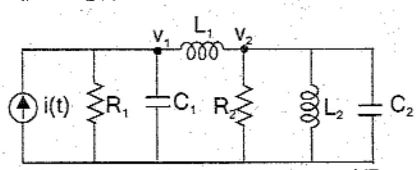


$f(t) \rightarrow e(t)$ $\quad M_1 \rightarrow L_1 \quad B_2 \rightarrow R_2$

$v_1 \rightarrow i_1$ $\quad M_2 \rightarrow L_2 \quad K_1 \rightarrow 1/C_1$

$v_2 \rightarrow i_2$ $\quad B_1 \rightarrow R_1 \quad K_2 \rightarrow 1/C_2$

Force-voltage analogous circuit



$f(t) \rightarrow i(t)$ $\quad M_1 \rightarrow C_1 \quad B_2 \rightarrow 1/R_2$

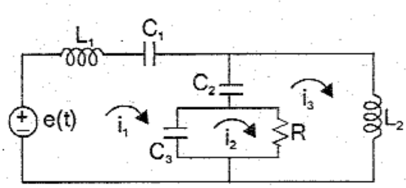
$v_1 \rightarrow v_1$ $\quad M_2 \rightarrow C_2 \quad K_1 \rightarrow 1/L_1$

$v_2 \rightarrow v_2$ $\quad B_1 \rightarrow 1/R_1 \quad K_2 \rightarrow 1/L_2$

Force-current analogous circuit

3 b) $M_1 \dfrac{dv_1}{dt} + K_1 \int v_1 dt + K_2 \int (v_1 - v_3) dt + K_3 \int (v_1 - v_2) dt = f(t)$

$K_3 \int (v_2 - v_1) dt + B(v_2 - v_3) = 0;$ $\qquad M_2 \dfrac{dv_3}{dt} + B(v_3 - v_2) + K_2 \int (v_3 - v_1) dt = 0$



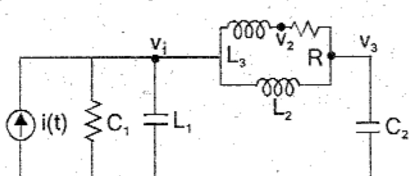
$f(t) \rightarrow e(t)$ $\quad M_1 \rightarrow L_1 \quad K_1 \rightarrow 1/C_1$

$v_1 \rightarrow i_1$ $\quad M_2 \rightarrow L_2 \quad K_2 \rightarrow 1/C_2$

$v_2 \rightarrow i_2$ $\quad B \rightarrow R \quad K_3 \rightarrow 1/C_3$

$v_3 \rightarrow i_3$

Force-voltage analogous circuit



$f(t) \rightarrow i(t)$ $\quad M_1 \rightarrow C_1 \quad K_1 \rightarrow 1/L_1$

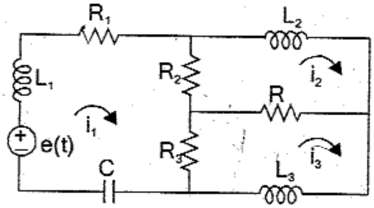
$v_1 \rightarrow v_1$ $\quad M_2 \rightarrow C_2 \quad K_2 \rightarrow 1/L_2$

$v_2 \rightarrow v_2$ $\quad B \rightarrow 1/R \quad K_3 \rightarrow 1/L_3$

$v_3 \rightarrow v_3$

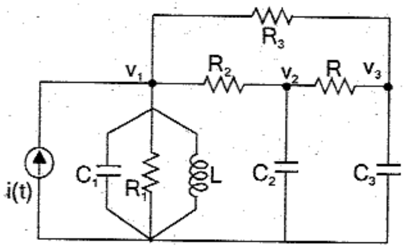
Force-current analogous circuit

E1.4



$f(t) \to e(t)$ $M_1 \to L_1$ $B_1 \to R_1$
$v_1 \to i_1$ $M_2 \to L_2$ $B_2 \to R_2$
$v_2 \to i_2$ $M_3 \to L_3$ $B_3 \to R_3$
$v_3 \to i_3$ $B \to R$ $K \to 1/C$

Force voltage-analogous circuit

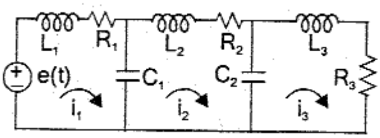


$f(t) \to i(t)$ $M_1 \to C_1$ $B_1 \to 1/R_1$
$v_1 \to v_1$ $M_2 \to C_2$ $B_2 \to 1/R_2$
$v_2 \to v_2$ $M_3 \to C_3$ $B_3 \to 1/R_3$
$v_3 \to v_3$ $B \to 1/R$ $K \to 1/L$

Force-current analogous circuit

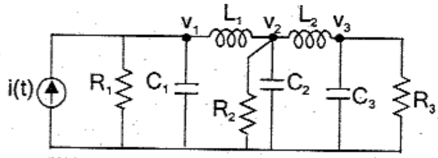
E1.5 $J_1\dfrac{d\omega_1}{dt} + B_1\omega_1 + K_1\int(\omega_1 - \omega_2)dt = T(t);$ $J_2\dfrac{d\omega_2}{dt} + B_2\omega_2 + K_1\int(\omega_2 - \omega_1)dt + K_2\int(\omega_2 - \omega_3)dt = 0$

$$J_3\frac{d\omega_3}{dt} + B_3\omega_3 + K_2\int(\omega_3 - \omega_2)dt = 0$$



$T(t) \to e(t)$ $J_1 \to L_1$ $B_1 \to R_1$ $K_1 \to 1/C_1$
$\omega_1 \to i_1$ $J_2 \to L_2$ $B_2 \to R_2$ $K_2 \to 1/C_2$
$\omega_2 \to i_2$ $J_3 \to L_3$ $B_3 \to R_3$ $\omega_3 \to i$

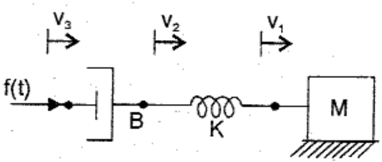
Torque-voltage analogous circuit



$T(t) \to i(t)$ $J_1 \to C_1$ $B_1 \to 1/R_1$ $K_1 \to 1/L_1$
$\omega_1 \to v_1$ $J_2 \to C_2$ $B_2 \to 1/R_2$ $K_2 \to 1/L_2$
$\omega_2 \to v_2$ $J_3 \to C_3$ $B_3 \to 1/R_3$ $\omega_3 \to v_3$

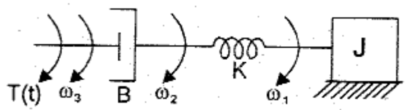
Torque-current analogous circuit

E1.6



$e(t) \to f(t)$ $i_1 \to v_1$ $i_3 \to v_3$ $R \to B$
$i_2 \to v_2$ $L \to M$ $1/C \to K$

Analogous mechanical translational system



$e(t) \to T(t)$ $i_1 \to \omega_1$ $i_3 \to \omega_3$ $R \to B$
$i_2 \to \omega_2$ $L \to J$ $1/C \to K$

Analogous mechanical rotational system

E1.7

(a) $$\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_1H_2 + G_1 + G_1G_2H_1 + G_1G_3H_1}$$

(b) $$\frac{C}{R} = \frac{G_1G_2(1+G_3)(G_4+G_5)}{1 + (1+G_3)(G_4+G_5)H_2 + (1+G_3)(G_4+G_5)G_2H_1}$$

(c) $$\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_1G_2 + G_1G_3 + G_2H_2 + G_3H_2}$$

(d) $$\frac{C}{R} = \frac{G_1G_2G_3}{1 + G_2G_3H_2 + G_1G_2H_1 + G_1G_2G_3}$$