Module -2

Microactuation: Principal means of Microactuation, MEMS with Microactuators, Microaccelrometer, Microfluidic. (Text 1: 2.3, 2.4, 2.5, 2.6)

Engineering Science for Microsystem Design and Fabrication: Ions and Ionization, The Diffusion Process, Plasma Physics, Electrochemistry, Quantum Physics. (Text 1: 3.3, 3.6, 3.7, 3.8, 3.9)

Scaling Laws: Scaling in Geometry, Scaling in Rigid body Dynamics, Scaling in Electrostatic force, Electricity, Fluid mechanics, Heat Transfer.(Text 1: 6.2, 6.3, 6.4, 6.6, 6.7, 6.8)

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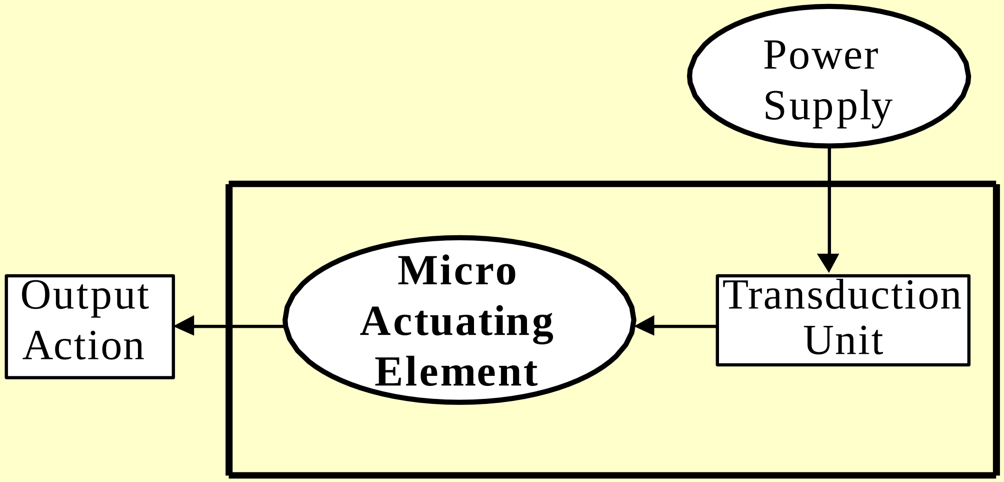
Working Principles for Microactuators



Power supply: Electrical current or voltage

Transduction unit: To covert the appropriate form of power supply into the desired form of actions of the actuating element

Actuating element: A material or component that moves with power supply

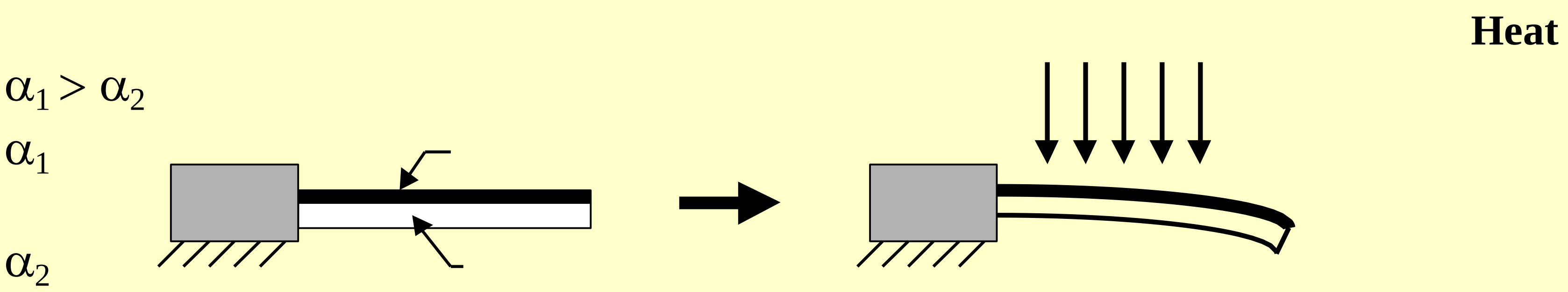
Output action: Usually in a prescribed motion

Actuation Using Thermal Forces

- Solids deform when they are subjected to a temperature change (ΔT)

- A solid rod with a length L will extend its length by $\Delta L = \alpha \Delta T$, in which α = coefficient of thermal expansion (CTE) – a material property.

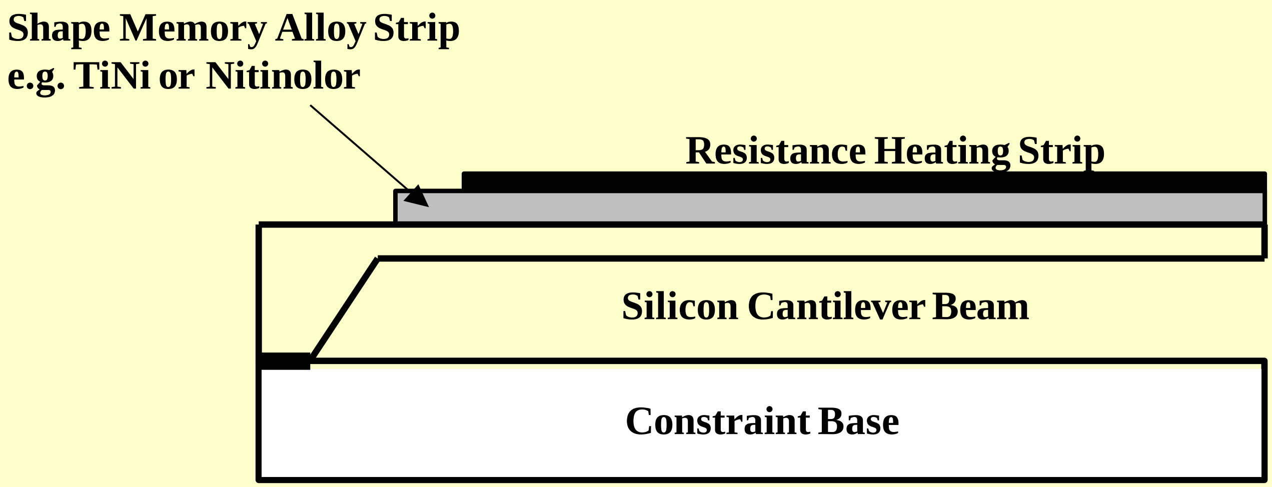
- When two materials with distinct CTE bond together and is subjected to a temperature change, the compound material will change its geometry as illustrated below with a compound beam:



- These compound beams are commonly used as microswitches and relays in MEMS products.

Actuation Using Shape Memory Alloys (SMA)

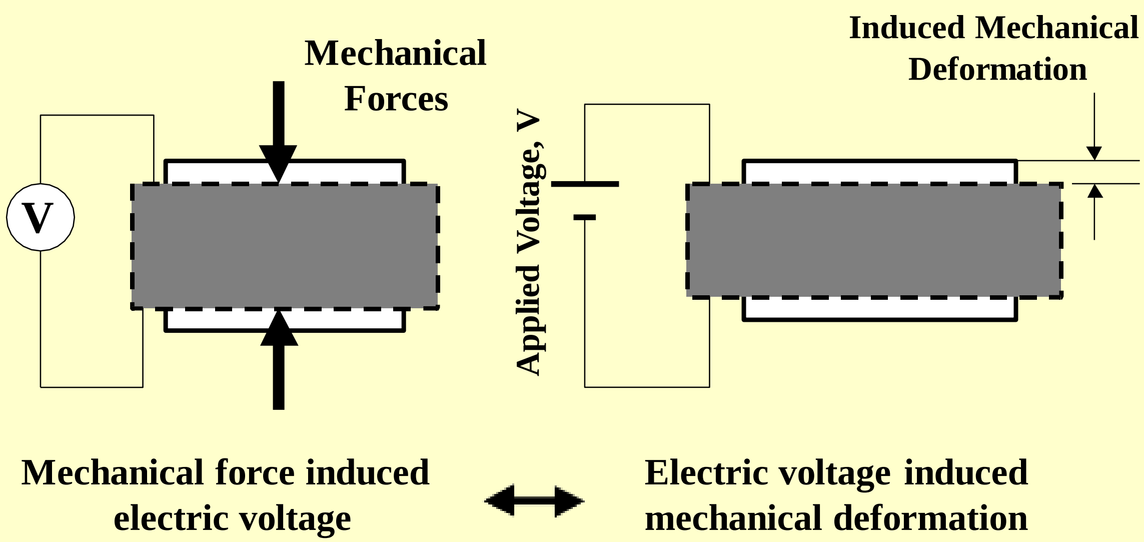
- SMA are the materials that have a "memory" of their original geometry (shape) at a typically elevated temperature of production.
- These alloys are deformed into different geometry at typically room temperature.
- The deformed SMA structures will return to their original shapes when they are heated to the elevated temperature at their productions.
- Ti-Ni is a common SMA.
- A microswitch actuated with SMA:



Actuation Using Piezoelectric Crystals

- A certain crystals, e.g., quartz exhibit an interesting behavior when subjected to a mechanical deformation or an electric voltage.

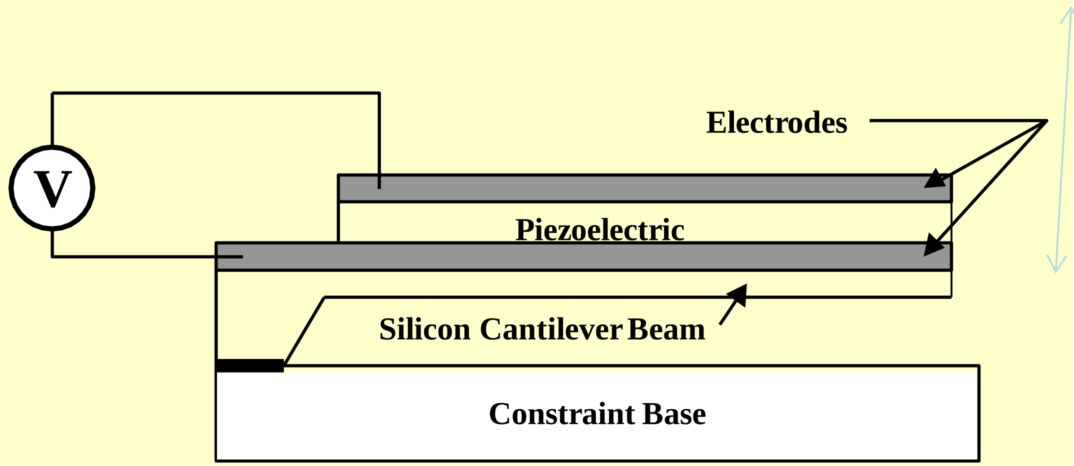
- This behavior may be illustrated as follows:



- This peculiar behavior makes piezoelectric crystals an ideal candidate for microactuation as illustrated in the following case:

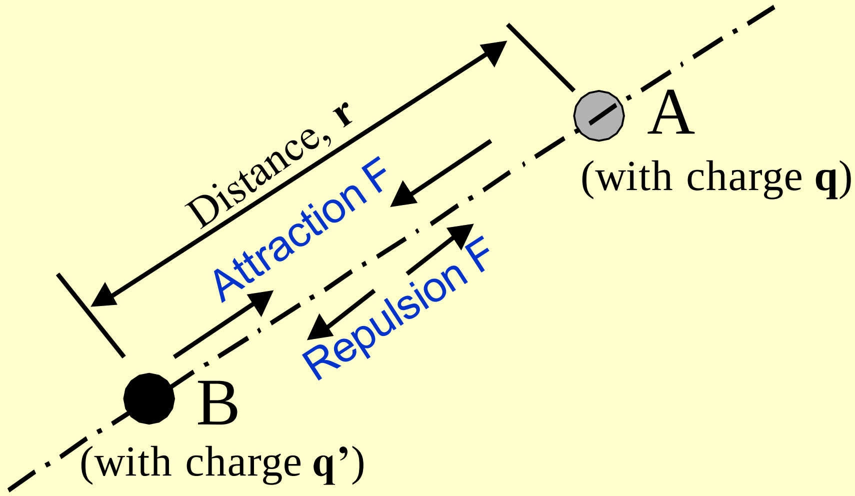
Actuation Using Piezoelectric Crystals-Cont'd

A micro relay or microelectrical switch



Actuation Using Electrostatic Forces

- **Electrostatic Force between Two Particles – The Coulomb's Law:**

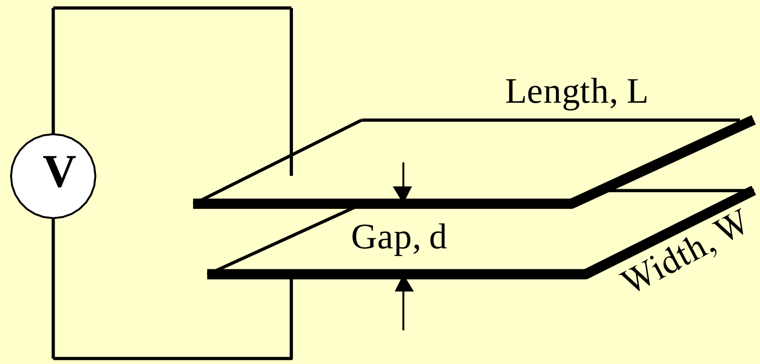


The attraction or repulsive force: $$F = \frac{1}{4\pi\varepsilon}\frac{qq'}{r^2}$$

where ε = permittivity of the medium between the two particles
= 8.85 x 10^{-12} $C^2/N\text{-}m^2$ or 8.85 pF/m in vacuum (= ε_o) r
= Distance between the particles (m)

Actuation Using Electrostatic Forces-Cont'd

● **Electrostatic Force <u>Normal</u> to Two Electrically Charged <u>Plates</u>:**



● The induced capacitance, C is: $$C = \varepsilon_r \, \varepsilon_o \frac{A}{d} = \varepsilon_r \, \varepsilon_o \frac{WL}{d}$$

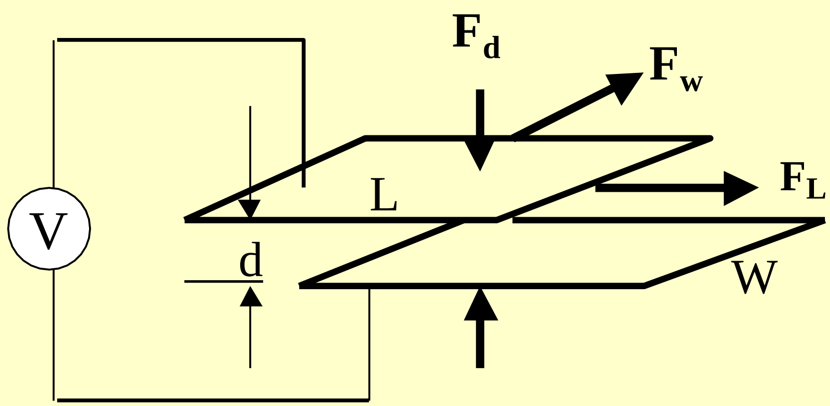
● The induced normal force, F_d is:

$$F_d = -\frac{1}{2} \frac{\varepsilon_r \, \varepsilon_o \, WL}{d^2} V^2$$

in which ε_r = relative permittivity of the dielectric material between the two plates (see Table 2.2 for values of ε_r for common dielectric materials).

Actuation Using Electrostatic Forces-Cont'd

- Electrostatic Force Parallel to Two Misaligned Electrically Charged Plates:



- Force in the "Width" direction:

$$F_w = -\frac{1}{2}\frac{\varepsilon_r\,\varepsilon_o\,L}{d}V^2$$

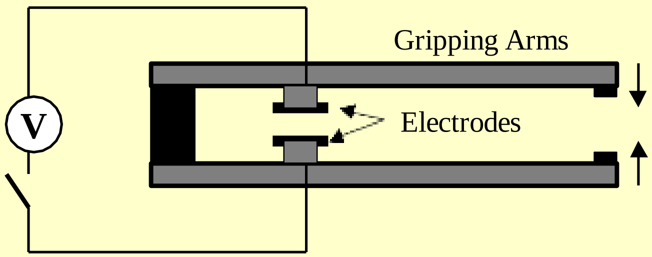
- Force in the "Length" direction:

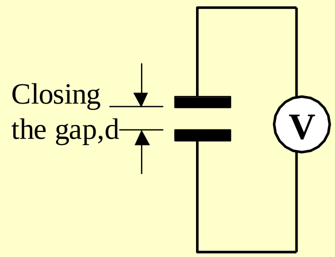
$$F_L = -\frac{1}{2}\frac{\varepsilon_r\,\varepsilon_o\,W}{d}V^2$$

Applications of Microactuations

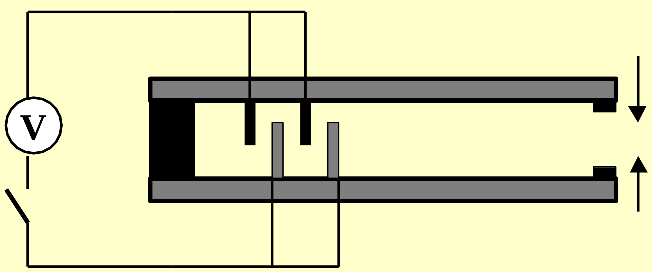
Microgrippers An essential component in microrobots in assembly microassemblies and surgery

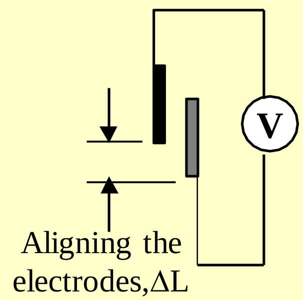
Two gripping methods:





The normal plate electrodes
- Not practical b/c requiring more space.

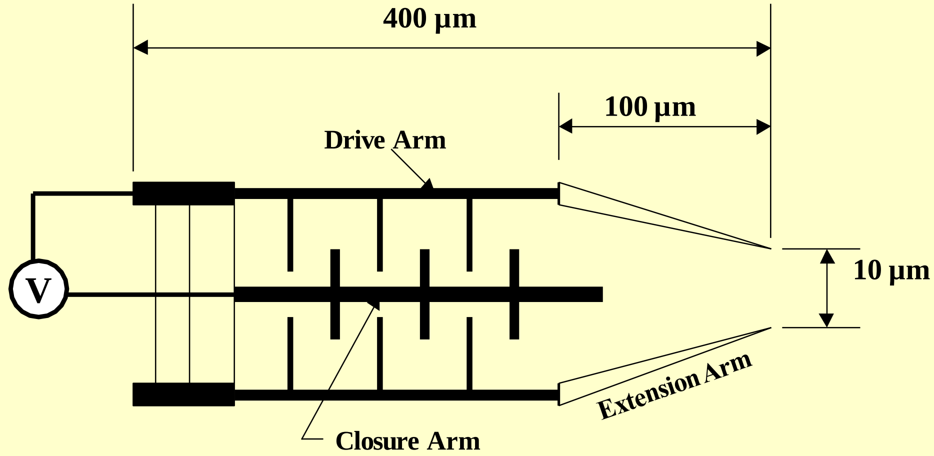




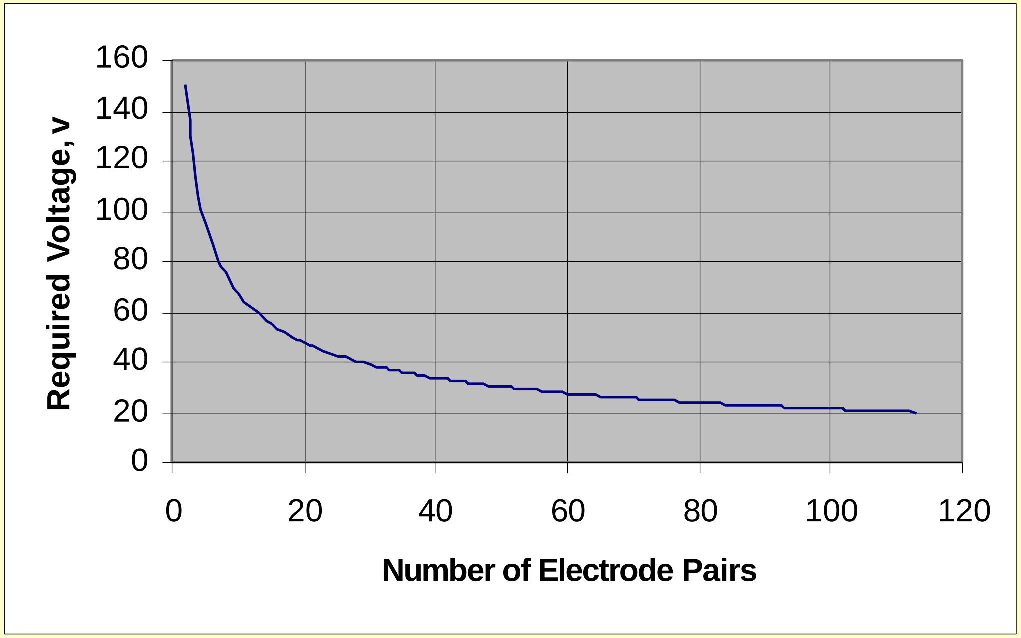
The sliding plate electrodes
- Popular method. Can have many sets to make “Comb drive” actuators

A Typical Microgripper with "Comb drive" Actuators:

Arrangement of electrodes:



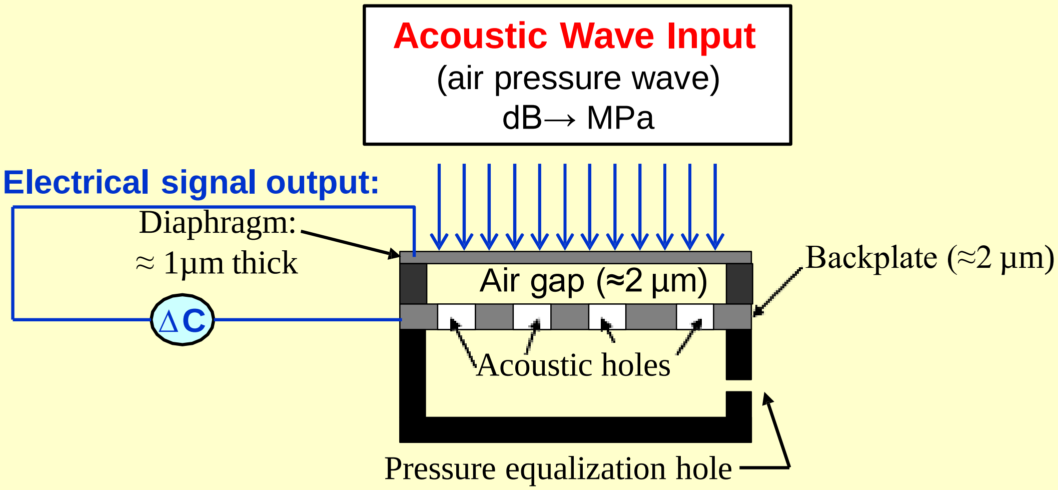
Drastic reduction in required actuation voltage with increase of number of pairs of electrodes:



Applications of Microactuations

Miniature Microphones

A niche market in mobile telecommunications and intelligent hearing aides



dB = unit of noise level:

$$dB = 20 \log_{10} \left(\frac{P}{P_o} \right)$$

where P = Air pressure (Pa)
P_o = Reference air pressure at threshold sound level

Most microphones are designed for 20-80 dB in the frequency range of 150-1000 Hz

A major challenge in MEMS microphone design and manufacture is the packaging and integration of MEMS and CMOS integrated circuits for signal conditioning and processing

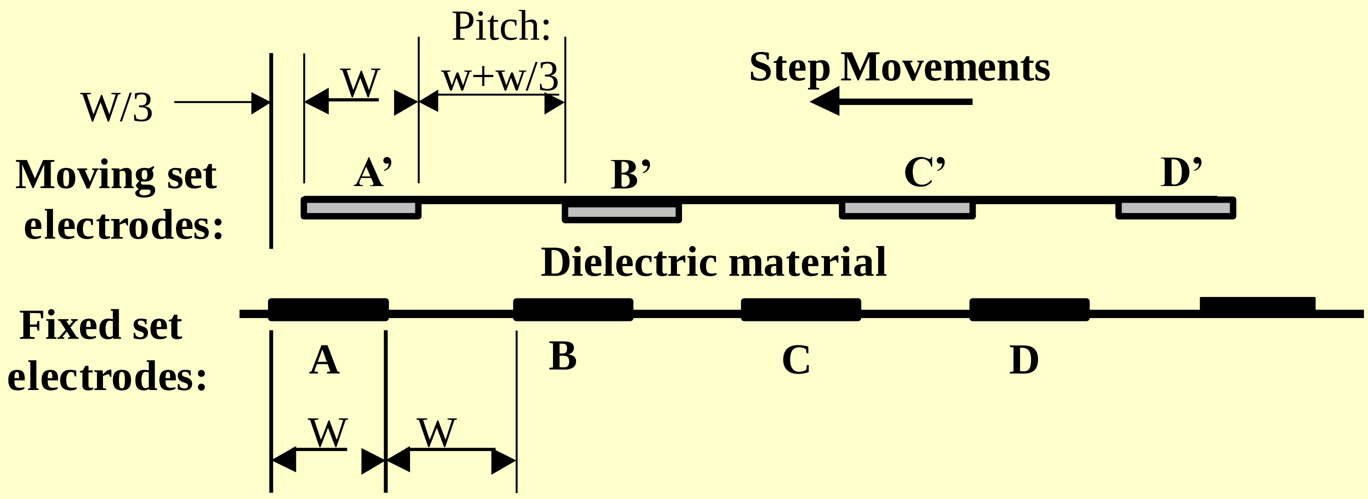
Applications of Microactuations

Micromotors

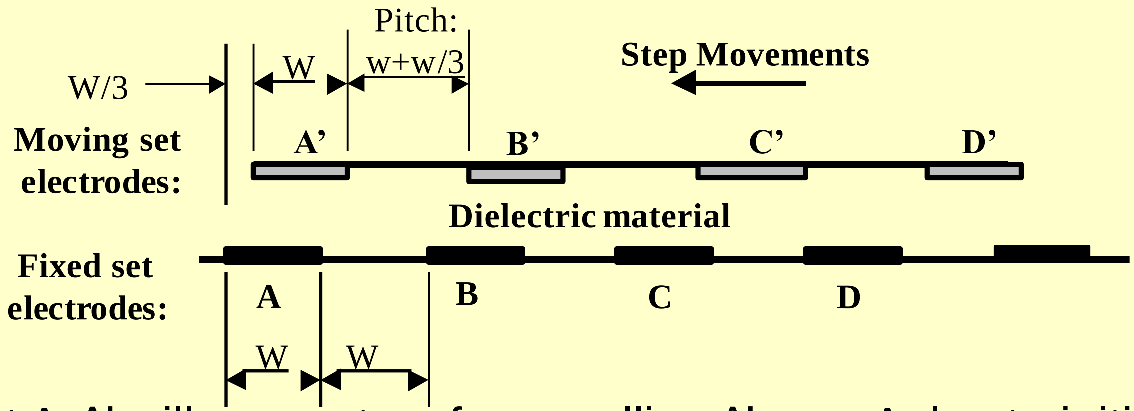
Unlike traditional motors, the *driving forces* for micro motors is primarily the parallel electrostatic forces between pairs of misaligned electrically charged plates (electrodes), as will be demonstrated in the following two cases:

Linear stepping motors:

- Two sets of electrodes in the form of plates separated by dielectric material (e.g. quartz film).
- One electrode set is fixed and the other may slide over with little friction.
- The two sets have slightly different pitch between electrodes



Applications of Micro Actuations-Cont'd

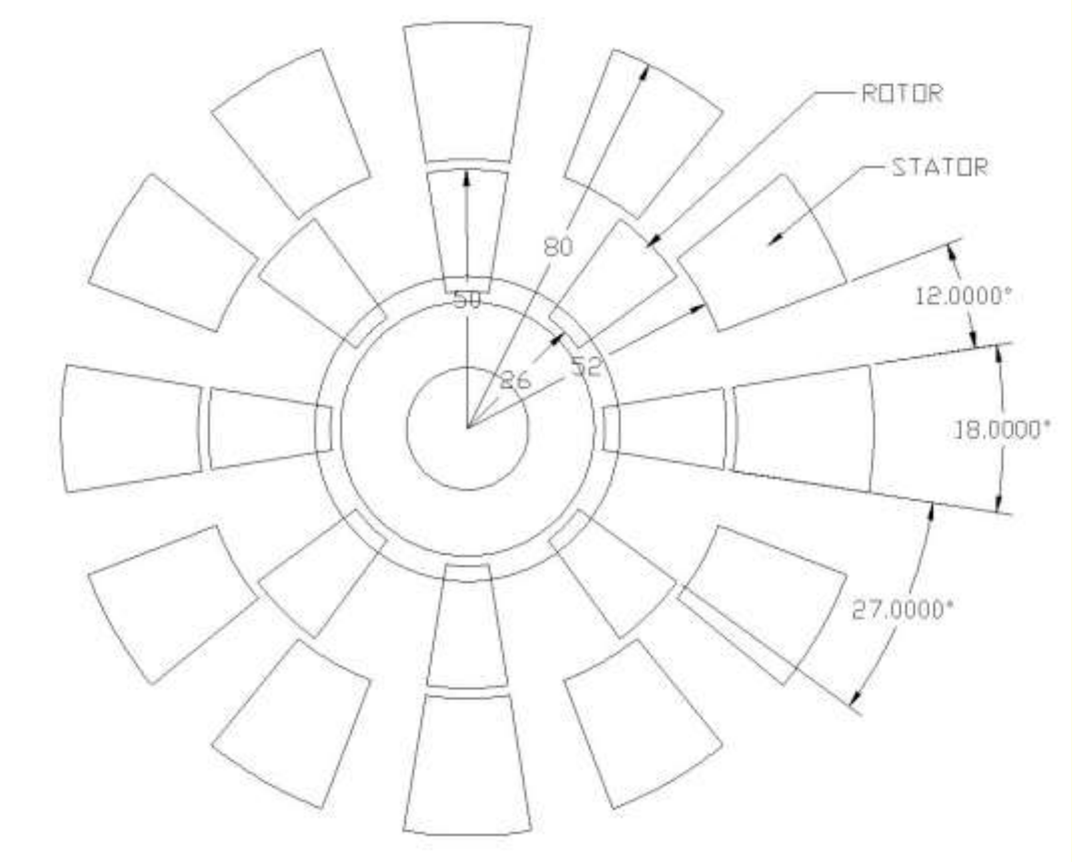


- Energize the set A-A' will generate a force pulling A' over A due to initial misalignment.
- Once A and A' are aligned, the pair B and B' become misaligned.
- Energize the misaligned B-B' will generate electrostatic force pulling B' over B.
- It is now with C' and C being misaligned.
- Energize C' and C will produce another step movement of the moving set over the stationary set.
- Repeat the same procedure will cause continuous movements of the moving sets
- The step size of the motion = w/3, or the size of preset mismatch of the pitch between the two electrode sets.

Applications of Micro Actuations-Cont'd

Rotary stepping motors:

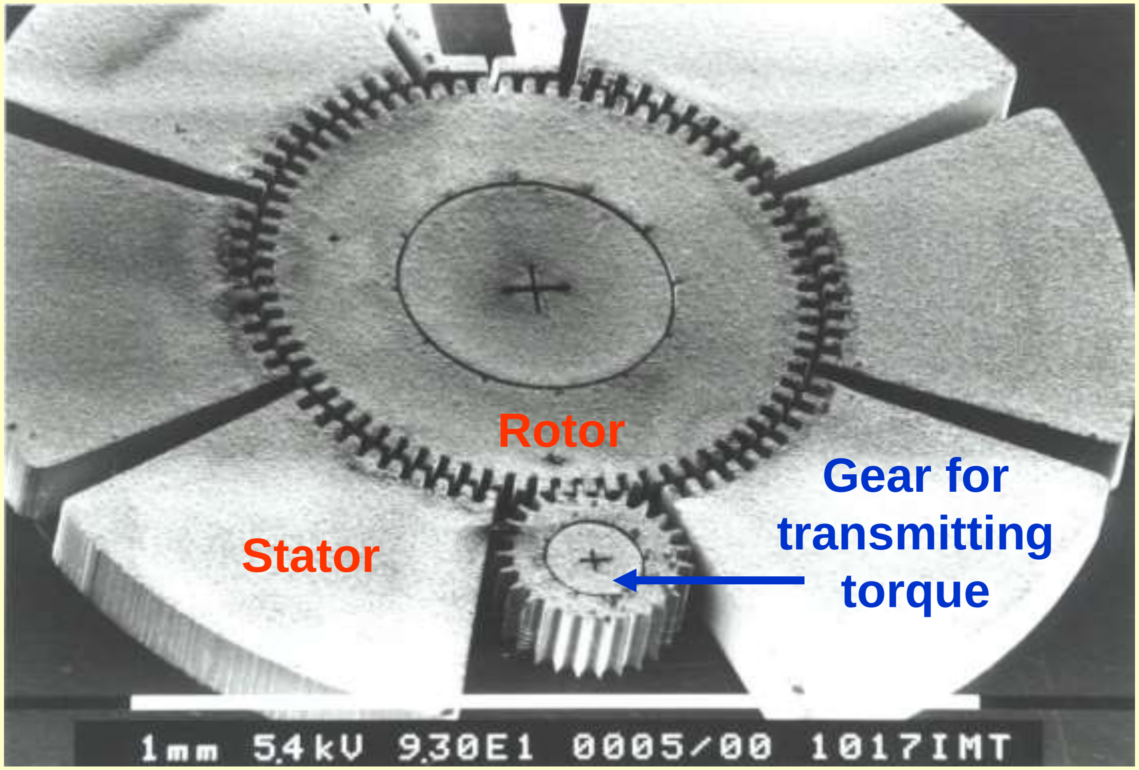
- Involve two sets of electrodes- one set for the rotor and the other for the stator.
- Dielectric material between rotor and stator is air.
- There is preset mismatch of pitches of the electrodes in the two sets.



Applications of Microactuations-Cont’d

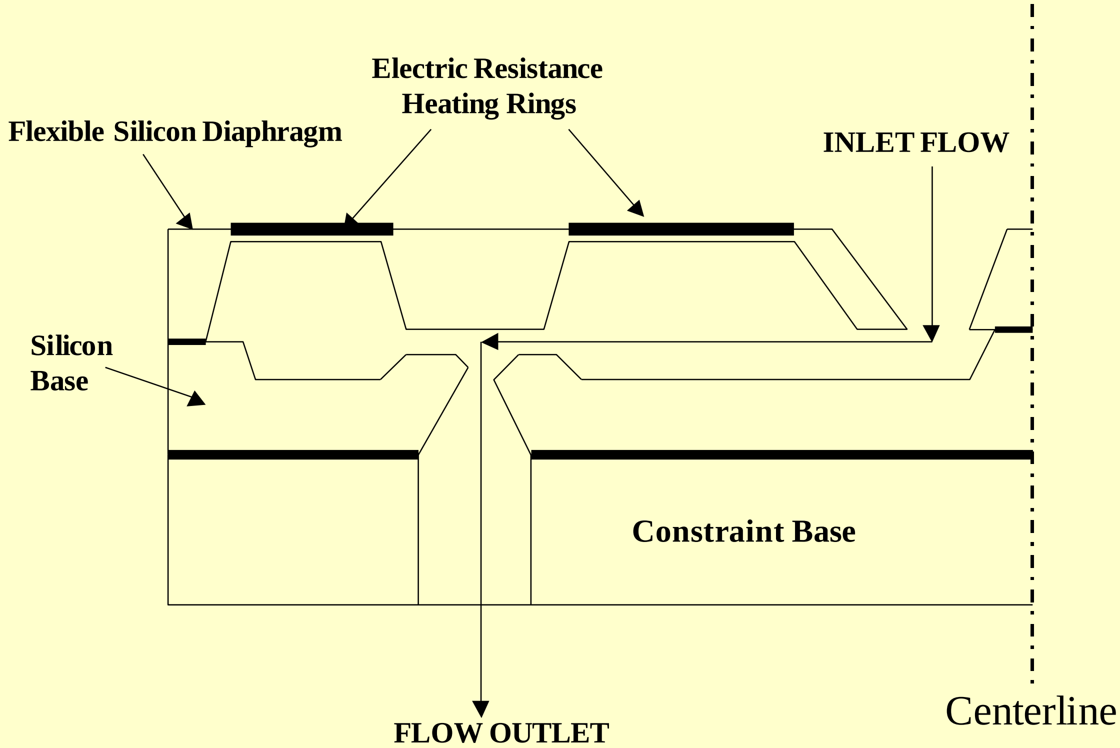
- Working principle of this rotary motor is similar to that in linear motors.

A micro motor produced by Karlsruhe Nuclear Research Center, Germany:



Microvalves

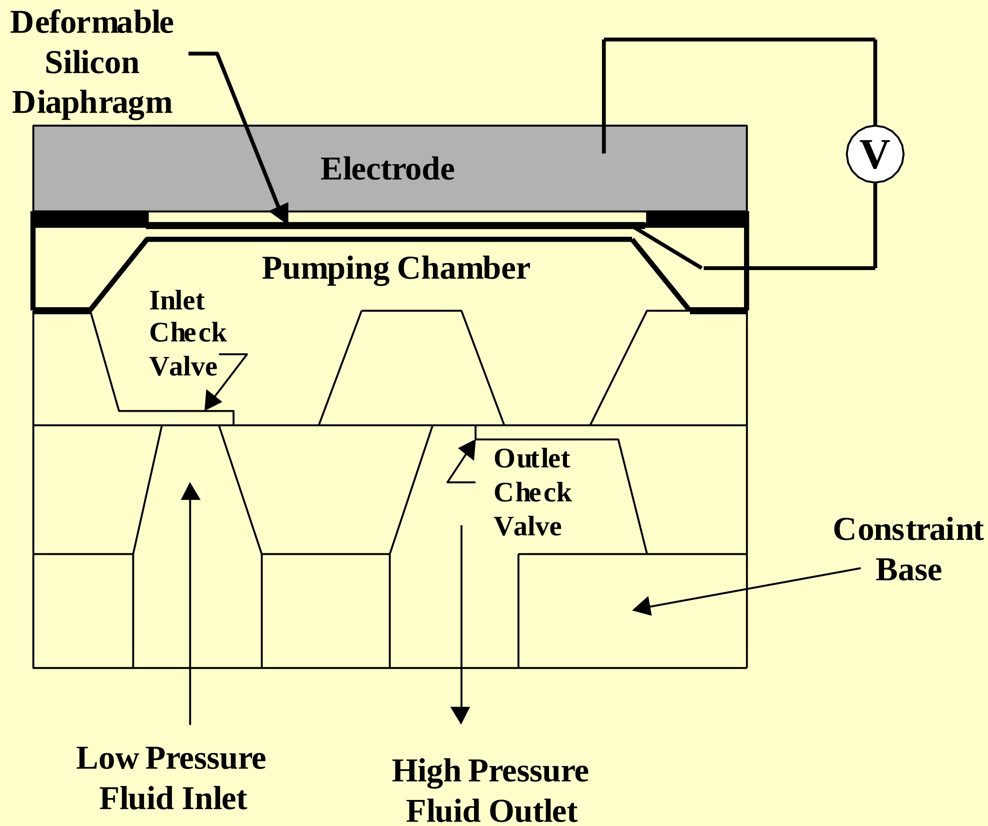
- A special microvalve designed by Jerman in 1990.
- Circular in geometry, with diaphragm of 2.5 mm in diameter x 10 µm thick.
- The valve is actuated by thermal force generated by heating rings.
- Heating ring is made of aluminum films 5 µm thick.
- The valve has a capacity of 300 cm^3/min at a fluid pressure of 100 psig.
- Power consumption is 1.5 W.



Micropumps

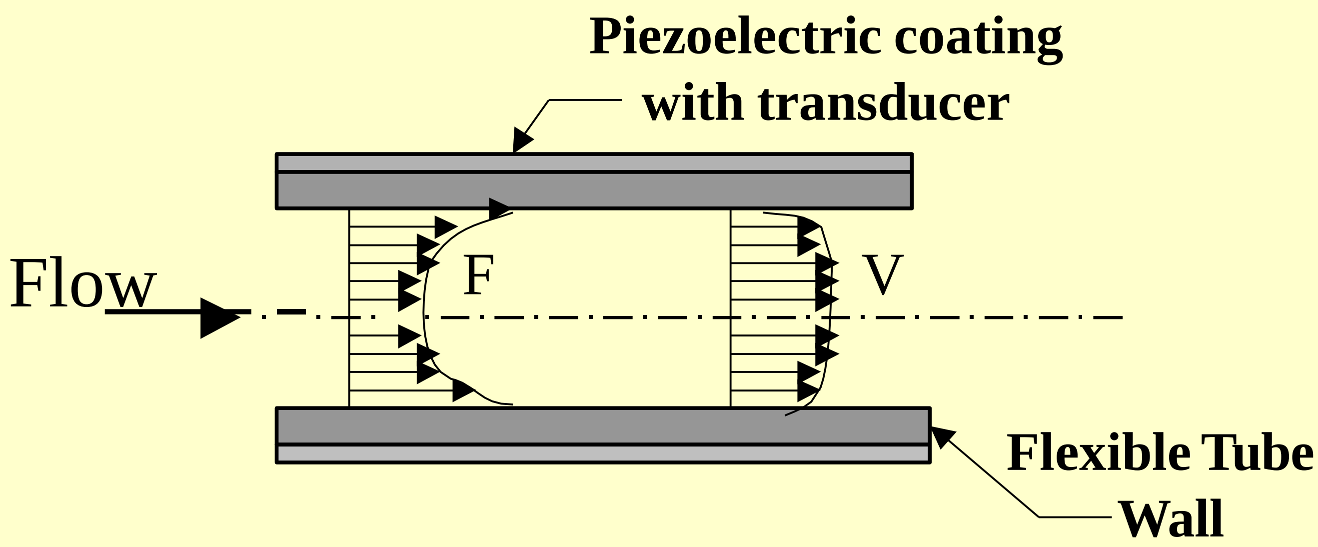
Electrostatically actuated micropump:

- An electrostatic actuated pump in 1992.
- The pump is of square geometry with 4 mm x 4mm x 25 µm thick.
- The gap between the diaphragm and the electrode is 4 µm.
- Pumping rate is 70 µL/min at 25 Hz.



Piezoelectrically actuated pump:

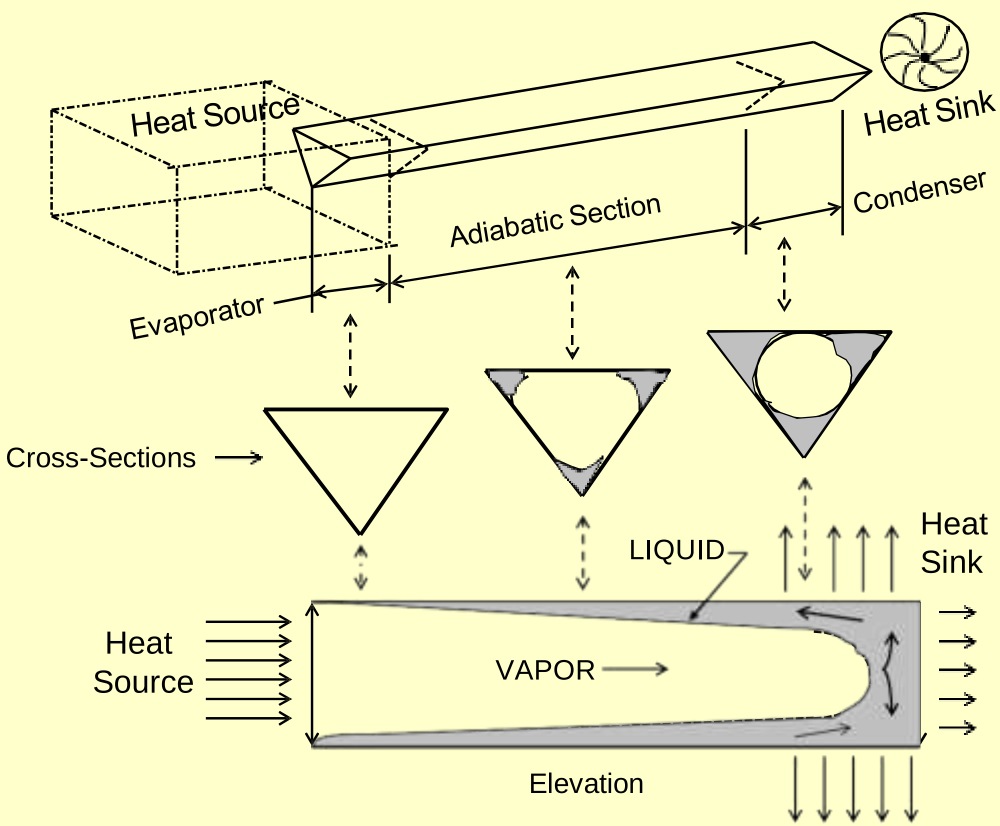
- An effective way to pump fluid through capillary tubes.
- Tube wall is flexible.
- Outside tube wall is coated with piezoelectric crystal film, e.g. ZnO with aluminum interdigital transducers (IDTs).
- Radio-frequency voltage is applied to the IDTs, resulting in mechanical squeezing in section of the tube (similar to the squeezing of toothpaste)
- Smooth flow with "uniform" velocity profile across the tube cross section.



Micro Heat Pipes

Heat pipes = Closed systems that transport heat from heat source @ higher temperature to heat sink @ lower temperature. They are often referred to as "Heat pumps."

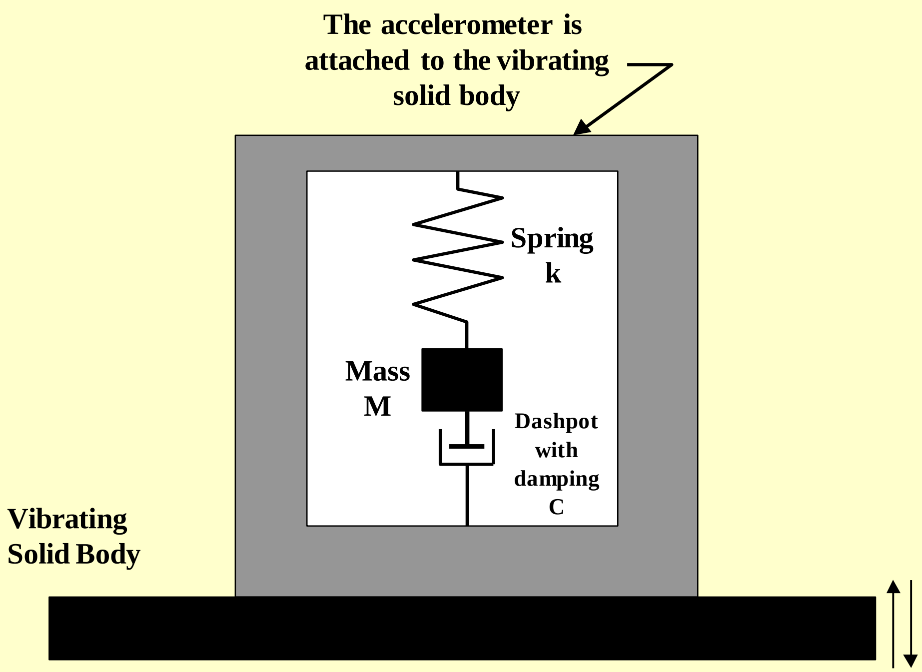
Micro heat pipes provide promising solution to effective heat dissipation in micro and molecular electronics circuits as will be presented in Chapter 12.



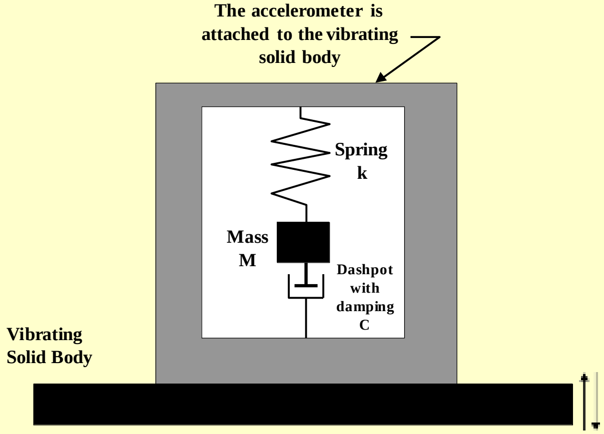
- A pipe with triangular or trapezoidal x-section ($d_p \approx 100$ µm) is in contacts with heat source, e.g., IC and a heat sink, e.g., ambient cool air with cooling air by a fan.
- The pipe contains liquid, e.g., Ethanol
- Liquid vaporizes near the heat source
- The vapor flows towards heat sink due to temperature difference
- The vapor condenses in the motion due to drop in temperature
- Vapor turns into liquid near the heat sink
- The condensed liquid moves in the sharp corners towards the heat sink due to the capillary effect
- The liquid vaporizes upon arriving at the heat sink
- The heat transport cycle repeats itself as long as temperature differences between the heat source and sink maintain.

Microaccelerometers

- Accelerometers are used to measure dynamic forces associated with moving objects.
- These forces are related to the velocity and acceleration of the moving objects.
- Traditionally an accelerometer is used to measure such forces.
- A typical accelerometer consists of a "proof mass" supported by a spring and a "dashpot" for damping of the vibrating proof mass:



Microaccelerometers-Cont'd



- The instantaneous displacement of the mass y(t) induced by the attached moving solid body is measured and recorded with respect to time, t.

- The associated velocity, V(t) and the acceleration α(t) may be obtained by the following derivatives:

$$V(t) = \frac{dy(t)}{dt} \quad and \quad \alpha(t) = \frac{dy(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

- The associated dynamic force of induced by the moving solid is thus obtained by using the Newton's law, i.e. F(t) = M α(t), in which M = the mass of the moving solid.

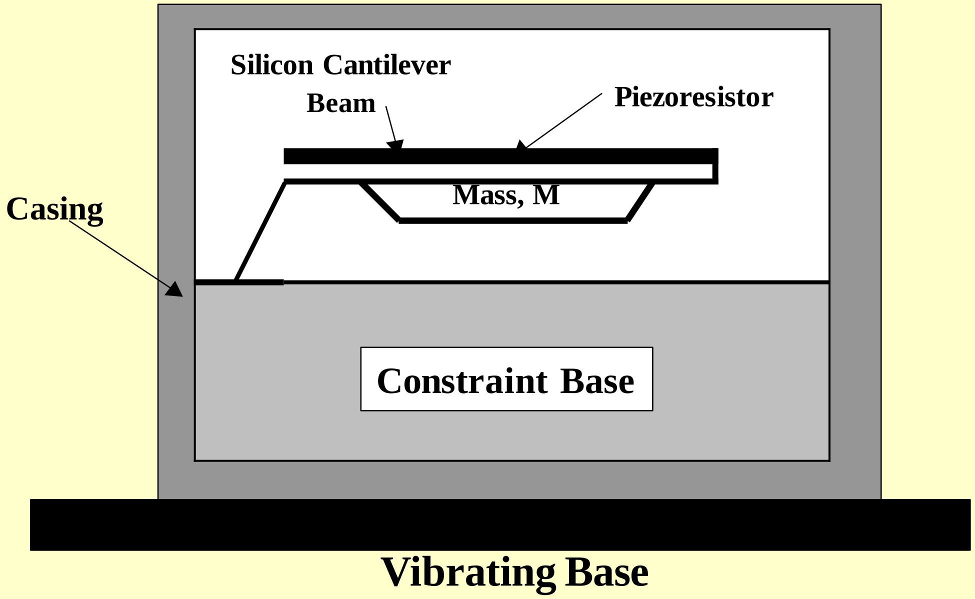
♦ In miniaturizing the accelerometers to the micro-scale, there is no room for the coil spring and the dashpot for damping on the vibrating mass.

♦ Alternative substitutes for the coil spring, dashpot, and even the proof mass need to be found.

Microaccelerometers-Cont'd

- There are two types micro accelerometers available.

(1) The cantilever beam accelerometer:



In this design: Cantilever beam = coil spring;
Surrounding viscous fluid = dashpot for damping of the proof mass

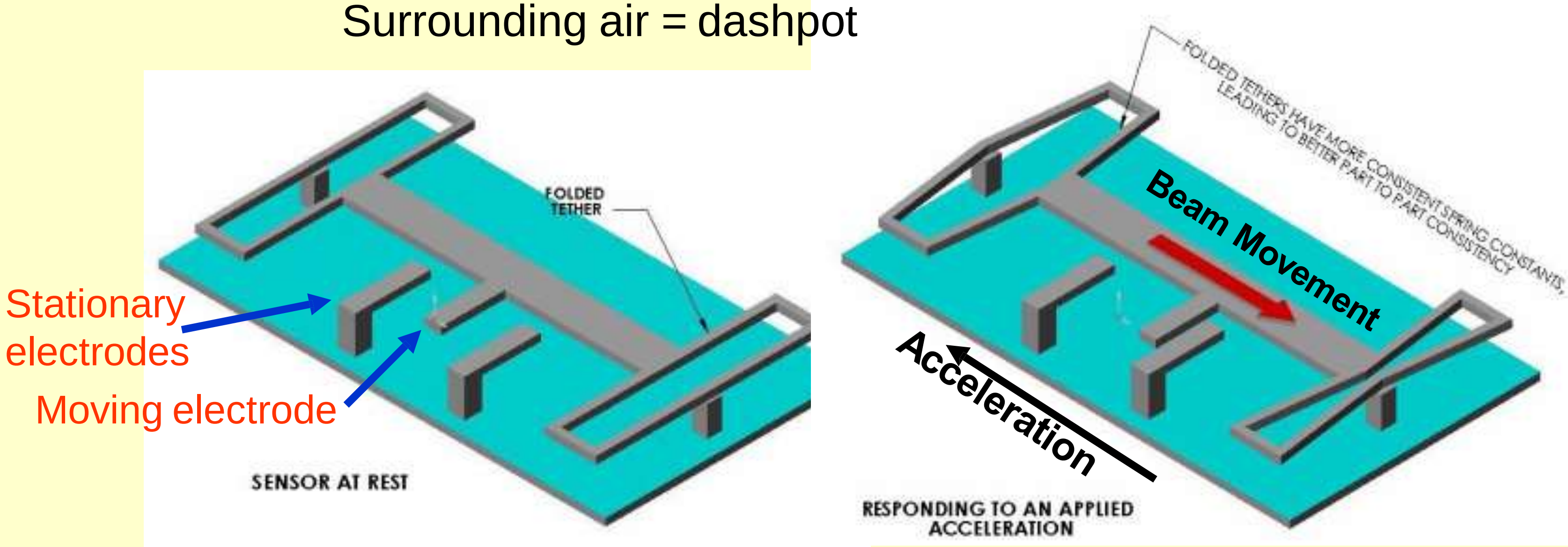
The movement of the proof mass is carried out by the attached piezoresistor.

Microaccelerometers-Cont'd

(2) Balanced force micro accelerometer:

- This is the concept used in the "air-bag" deployment sensor in automobiles

- In this design: Plate beam = proof mass;
  Two end tethers = springs
  Surrounding air = dashpot



- The movement of the proof mass is carried out by measuring the change of capacitances between the pairs of electrodes.

PART 2 of MODULE 2

Ions and ionization

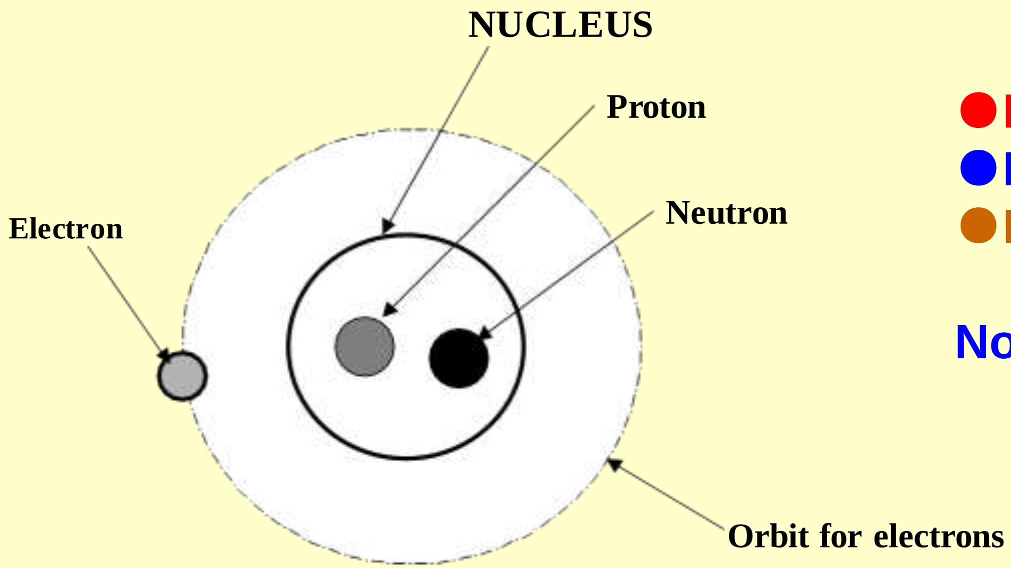
Molecular theory of matter and intermolecular forces

Doping of semiconductors

Diffusion process Plasma physics

Atomic Structure of Matter

Basic atomic structure



- **Protons** carry +ve charge
- **Electrons** carry –ve charge
- **Neutrons** carry no charge

No. of protons = No. of electrons

- **Nucleus contains protons and neutrons**

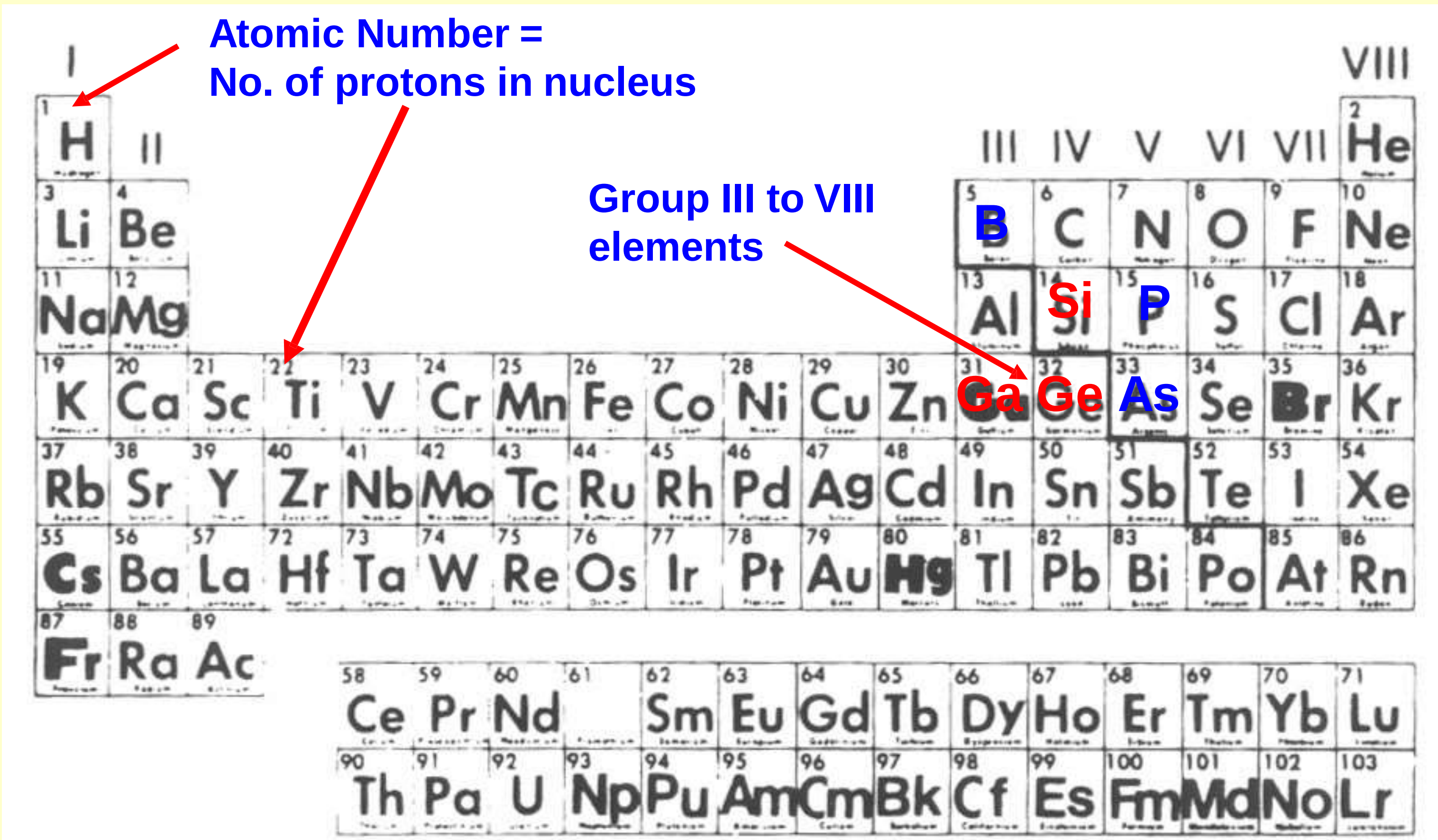
NOTE: There is no neutron in the nucleus of H_2 atoms.

- **The diameter of outer orbit:** 2 to 3x10^{-8} cm, or 0.2 to 0.3 nm.
- **Mass of protons:** 1.67x10^{-24} g
- **Mass of electrons:** 9.11x10^{-28} g

Atomic Structure of Matter-Cont'd

The periodic table of elements

Every thing on the Earth is made by 96 stable and 12 unstable elements.



Ions and Ionization

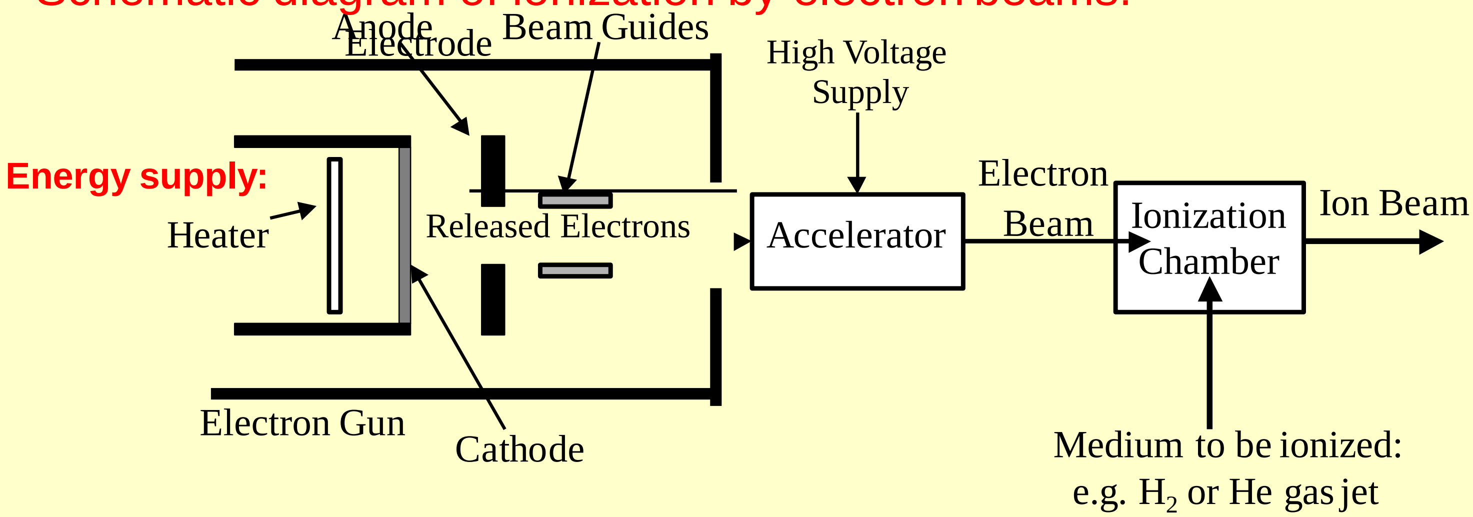
What is an ion?

An ion is an electrically charged atom or molecule.

+ve charged ions = atoms with more protons than electrons.

-ve charged ions = atoms with more electrons than protons. Ionization = The process of producing ions.

Schematic diagram of ionization by electron beams:

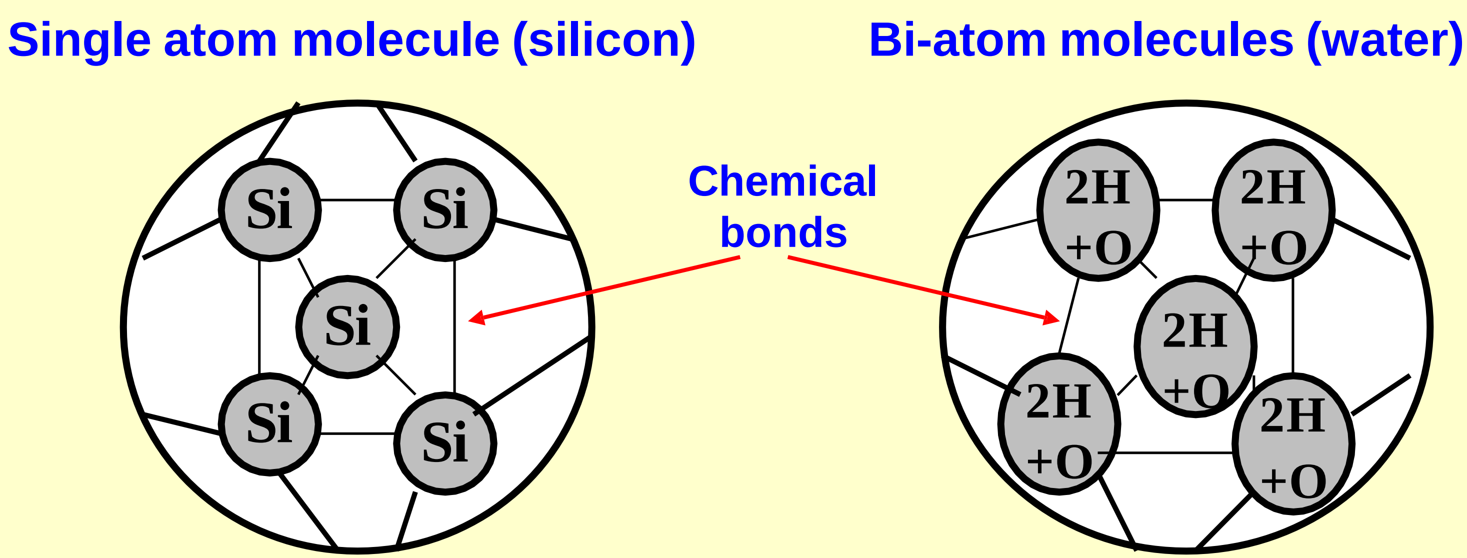


Molecular Theory of Matter

- All matters are made of large number of "particles" interconnected by deformable bonds.

- These "particles" are called molecules.

- By nature, some molecules are made by single atoms and some others involve multiple kinds of atoms.

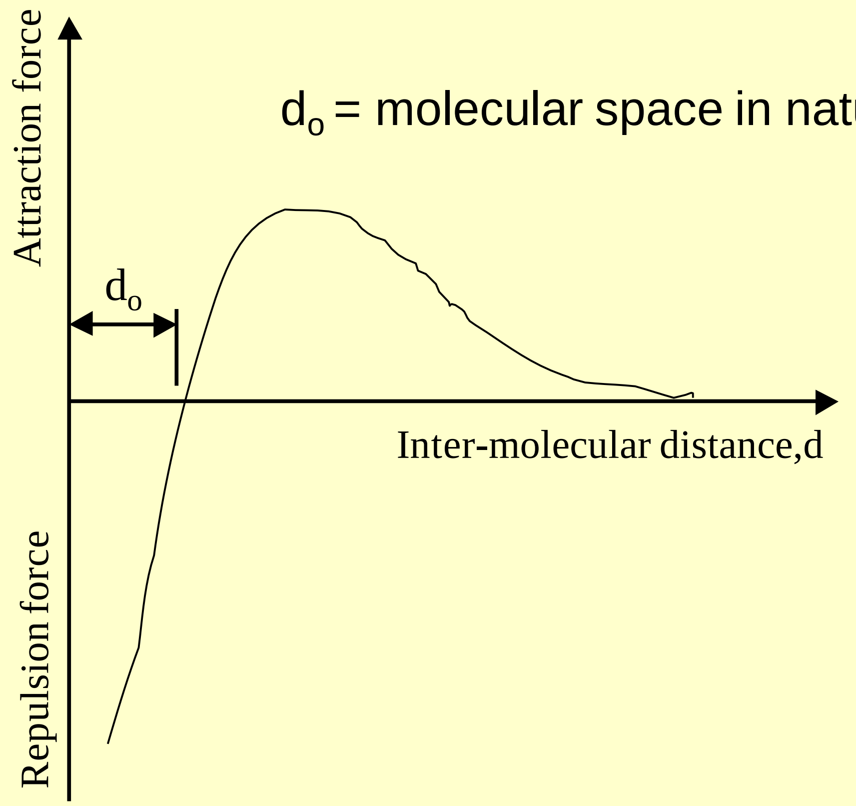


Inter-molecular Forces

- **The fact that molecular bonds are deformable indicates the existence of forces between molecules in a matter.**

- **These inter-molecular forces can be "attractions" or "repulsions"- determined by the distances between the molecules.**

- **Inter-molecular forces are often referred to as van der Waals forces.**



Doping of Semiconductors

- The process "**doping**" is a key process for producing transistors in microelectronics industry.
- Semiconducting materials are characterized by their electrical resistivity to be between electrically conductive and electrically insulators (or dielectric).
- They can be made electrically conductive by proper "**doping**" processes.
- Three classes of electrically conducting materials are:

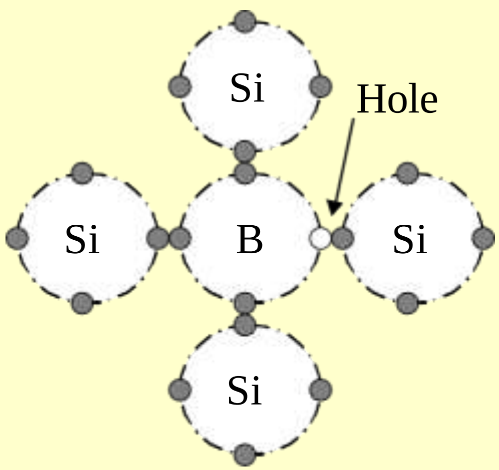
Materials	**Approximate Electrical Resistivity**, ρ (Ω-cm)	**Classifications**
Silver (Ag) Copper (Cu) Aluminum (Al) Platinum (Pt)	10^{-6} $10^{-5.8}$ $10^{-5.5}$ 10^{-5}	Conductors
Germanium (GE) **Silicon (Si)** **Gallium Arsenide (GaAs)** **Gallium Phosphide (GaP)**	$10^{1.5}$ $10^{4.5}$ $10^{8.0}$ $10^{6.5}$	**Semiconductors**
Oxide Glass Nickel (pure) Diamond Quartz (fused)	10^{9} $10^{10.5}$ 10^{13} 10^{14} 10^{18}	Insulators

Doping of Semiconductors-Cont'd

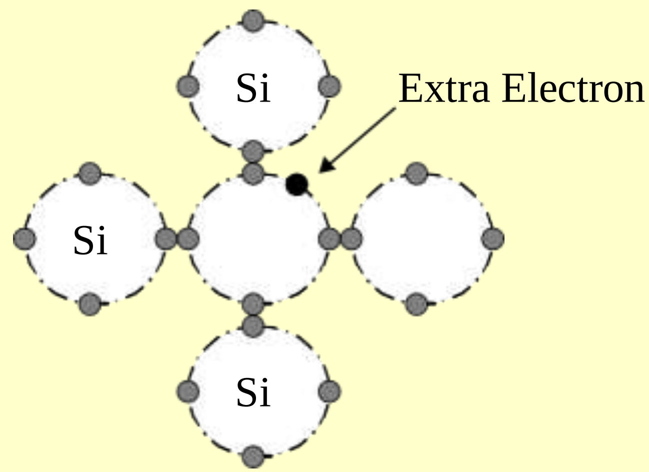
- Doping for common semiconductor, e.g. silicon (Si) involves <u>adding</u> atoms with different number of electrons to create unbalanced number of electrons in the base material (e.g. Si)

- The base material, after doping, with excessive electrons will carry –ve charge.
- The base material, after doping, with deficit in electron will carry +ve charge.

- Doping of silicon can be achieved by "ion implantation" or "diffusion" of Boron (B) atom for +ve charge or of Arsenide (As) or Phosphorous (P) for –ve charge.



P-type doping

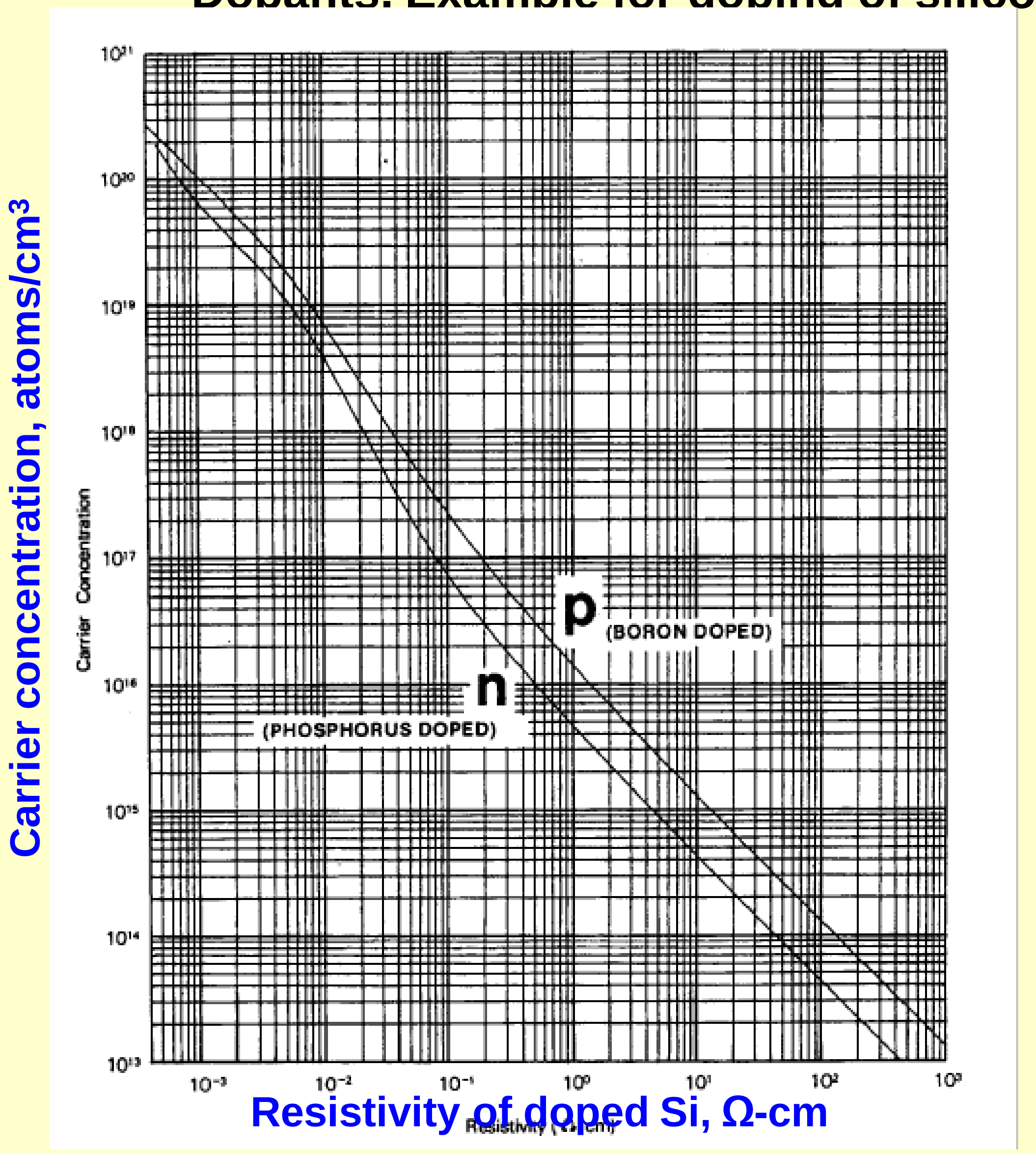


N-type doping

Doping of Semiconductors-Cont'd

Doping strength

It is determined by the concentration of atoms in the Dopants. Example for doping of silicon:



Diffusion Process

Diffusion process = **Introducing a controlled amount of foreign material into selected regions of another material.**

Diffusion processes may take place in:

Gas – gas (e.g. gas mixing and air pollution)
Liquid – liquid (e.g. spread of drop of ink in a pot of clear water)
Gas – solids (e.g. oxidation of metal)
Liquid – solids (e.g. corrosion of metal in water)

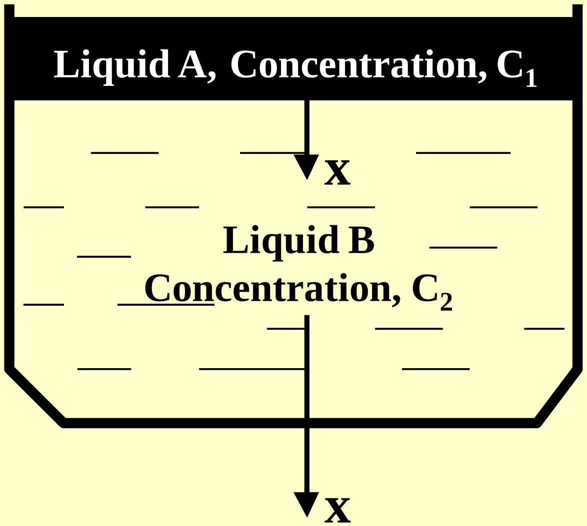
Three major applications of diffusion in microfabrication - a very important process:

- **Doping of semiconducting materials to produce p-n junctions and the production of piezoresistors.**
- **Oxidation of semiconducting materials.**
- **Chemical vapor deposition processes.**

Diffusion Process-Cont'd

Mathematical modeling by **Fick's law:**

A diffusion of liquid A into liquid B:



For the case $C_1 > C_2$:

$$C_a \propto \frac{C_{a,x_o} - C_{a,x}}{x_o - x} = -\frac{\Delta C}{\Delta x}$$

C_a = Concentration of A at a distance x away from the initial contacting surface/m²-s
Xo = position of the initial interface of A and B.
$C_{a,xo}$, C_{ax} = respective concentrations of A at x_o and x.

The above expression may be expressed in a different form of equation:

$$C_a = -D\frac{\Delta C}{\Delta x}$$

in which **D = diffusivity of A into B - a material constant for specific pair of materials in the process.**

The value of D usually increases with temperature → higher efficiency at elevated temperature

Diffusion Process-Cont'd

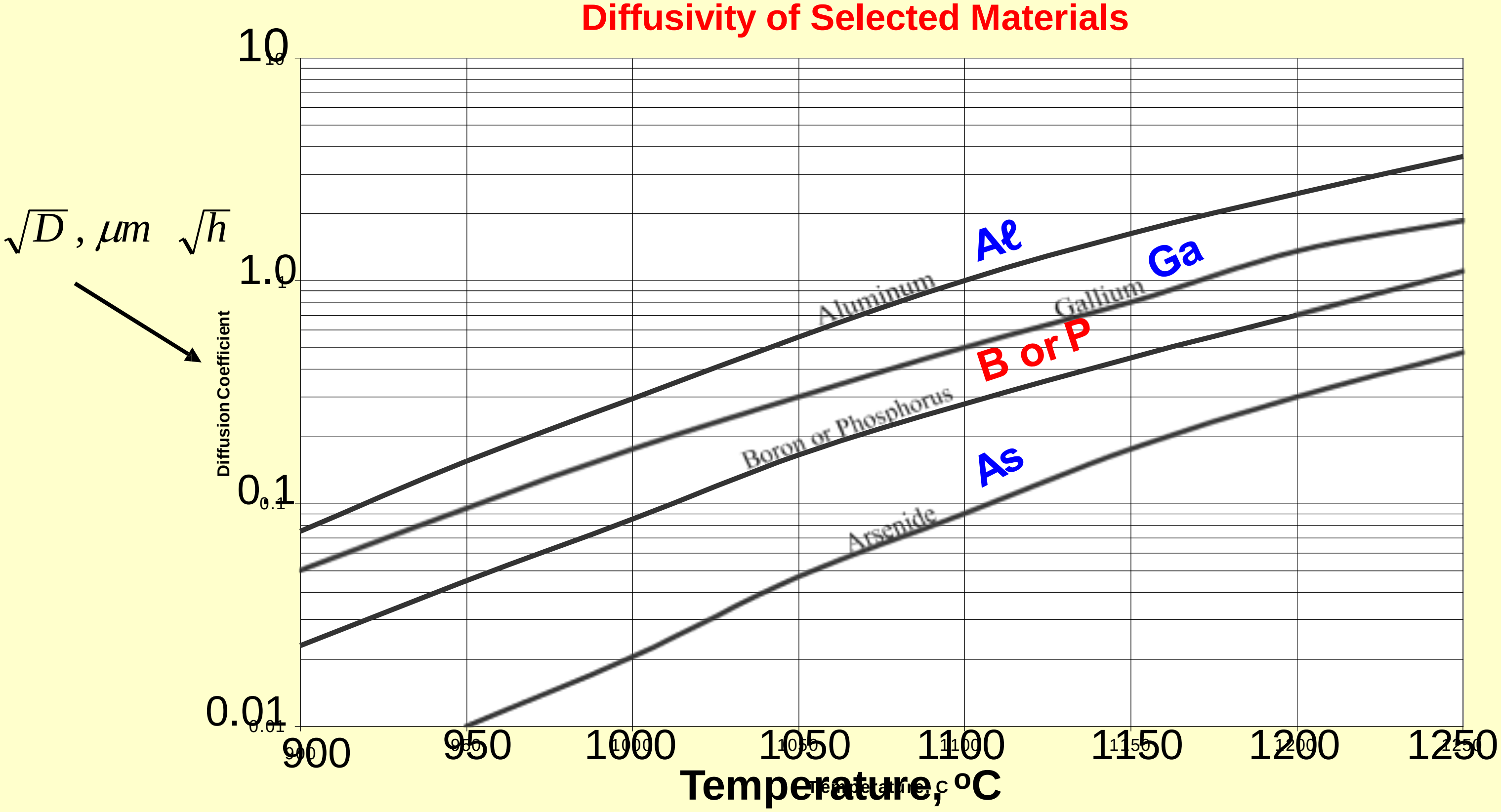
Solid-solid diffusion   e.g. in doping of silicon with B or As or P

Let J = the atoms (or molecules) of the foreign materials (B, As or P) to be diffused into base substrate material (e.g. Si) can be computed by:

$$J = -D \frac{\partial C}{\partial x} \qquad \text{atoms/m}^2\text{-s}$$

where D = diffusivity, or diffusion coefficient of the foreign material in the substrate material, m^2/s.

C = concentration of the foreign material in the substrate, atoms/m^3.



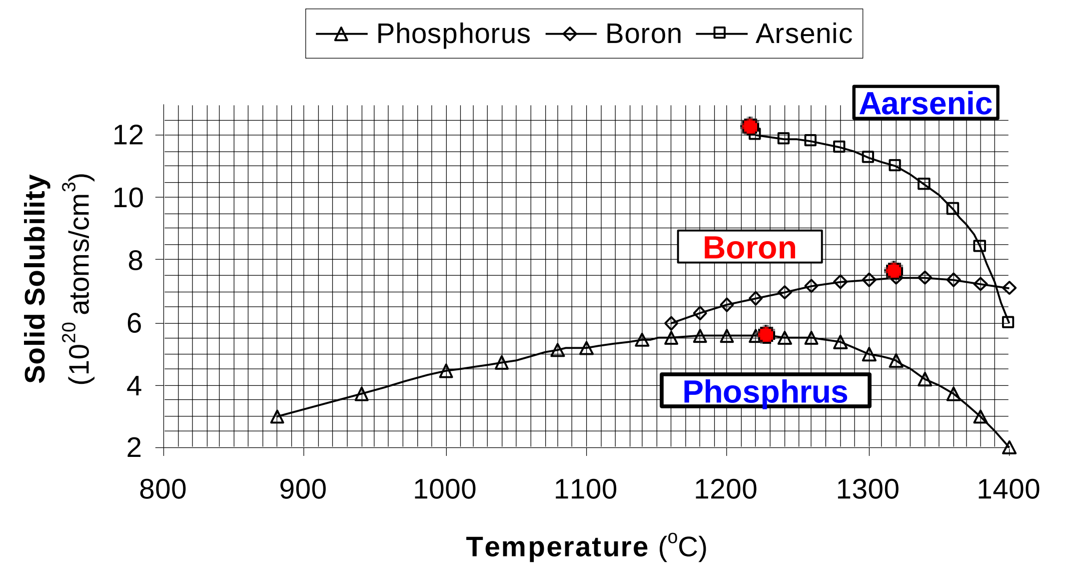
Diffusion Process-Cont'd

Solid Solubility

The theory of "Higher temperature→ Higher diffusion efficiency" does not always hold for solid-solid diffusion.

The "solubility" diagram below indicates, for example the temperatures at which maximum diffusion can take place are:

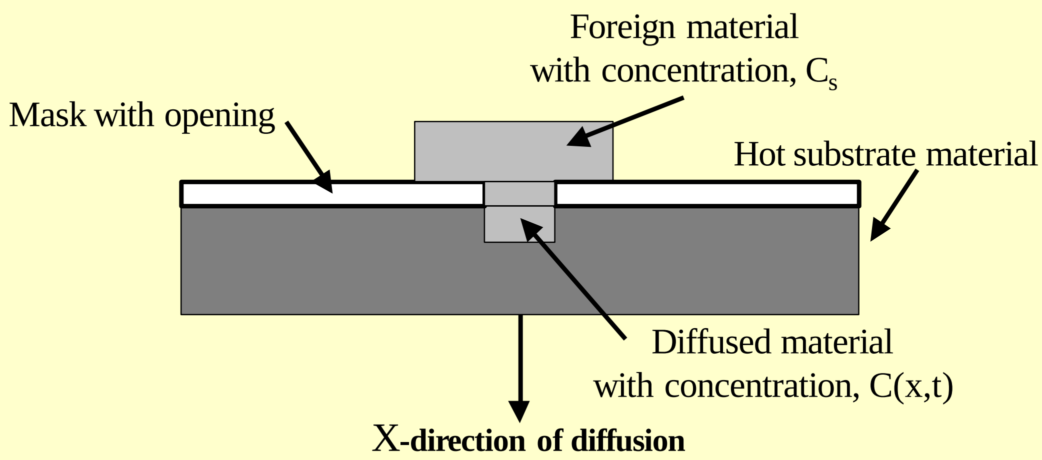
≈ 1220°C for As (-ve Si)
≈ 1350°C for B (+ve si)
≈ 1230°C for P (-ve Si) in doping silicon substrates



Diffusion Process-Cont'd

The Diffusion Equation

This equation is used to predict the concentration of the foreign material (e.g. B) in the substrate (e.g. Si) at given depth and time in a one-dimensional diffusion process.



$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}$$

with the following conditions:

$C(x,0) = 0$; $C(0,t) = C_s$; $C(\infty,t) = 0$

The solution of the partial differential equation satisfying the specific conditions is:

$$C(x,t) = C_s\, erfc\left(\frac{x}{2\sqrt{Dt}}\right)$$

where **erfc(X)** is the **complementary error function**, erfc(X) = 1-erf(X) in which **erf(X)** **Is the error function** with values available from mathematical handbooks.

Diffusion Process-Cont'd

The Error functions

X	erf(X)	X	erf(X)	X	erf(X)	X	erf(X)
0.0	0.0						
0.05	0.0564	0.55	0.5633	1.05	0.8624	1.55	0.9716
0.10	0.1125	0.60	0.6039	1.10	0.8802	1.60	0.9763
0.15	0.1680	0.65	0.6420	1.15	0.8961	1.65	0.9804
0.20	0.2227	0.70	0.6778	1.20	0.9103	1.70	0.9838
0.25	0.2763	0.75	0.7112	1.25	0.9229	1.75	0.9867
0.30	0.3286	0.80	0.7421	1.30	0.9340	1.80	0.9891
0.35	0.3794	0.85	0.7707	1.35	0.9438	1.85	0.9911
0.40	0.4284	0.90	0.7969	1.40	0.9523	1.90	0.9923
0.45	0.4755	0.95	0.8209	1.45	0.9597	1.95	0.9942
0.50	0.5205	1.00	0.8427	1.50	0.9661	2.00	0.9953

Example: erf(1.25) = 0.9229

Plasma Physics

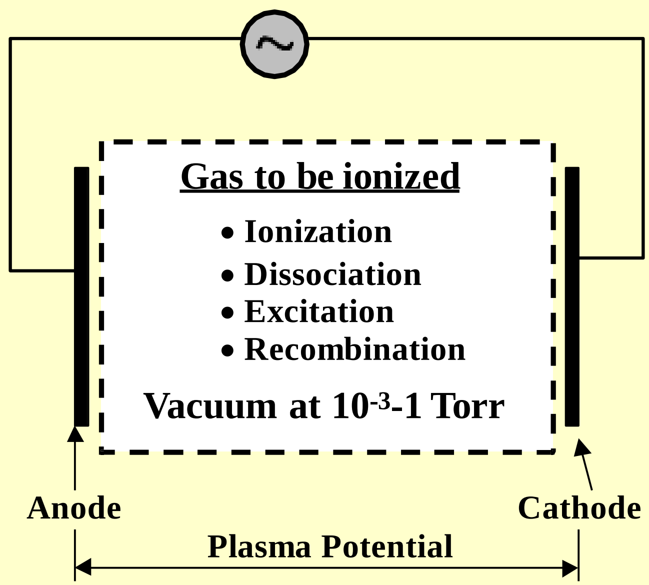
Plasma

- It is a **gas** containing **high energy ions** that carries electronic charges.

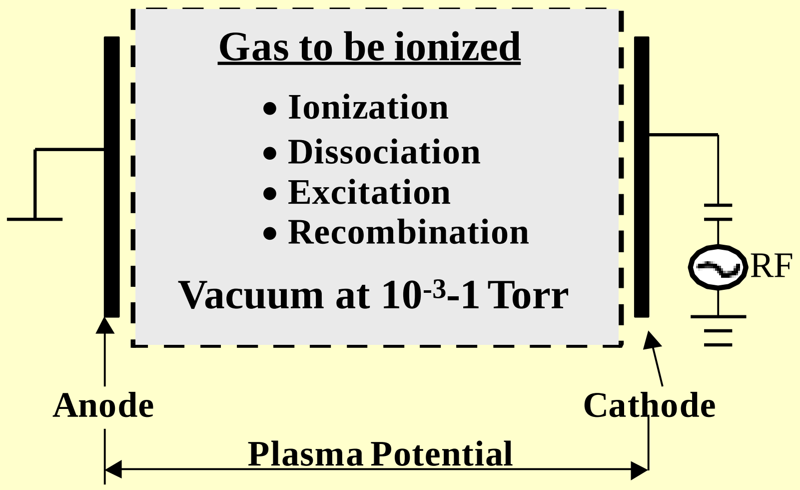
- Plasma is used to "knock out" substrate materials at desired localities - in a "**dry etching process**", or is used to carry chemicals in **chemical vapor deposition (CVD) process**.

Production of plasma

by high electric voltage:



by high energy RF:



PART 3 of MODULE 2

Content

- Scaling in Geometry
- Scaling in Rigid-Body Dynamics
- Scaling in Electrostatic Forces
- Scaling in Electricity
- Scaling in Fluid Mechanics
- Scaling in Heat Transfer

WHY SCALING LAWS?

Miniaturizing machines and physical systems is an ongoing effort in human civilization.

This effort has been intensified in recent years as market demands for:

Intelligent, Robust, Multi-functional and Low cost

consumer products has become more strong than ever.

The only solution to produce these consumer products is to package many components into the product –

making it necessary to miniaturize each individual components.

Miniaturization of physical systems is a lot more than just scaling down device components in sizes.

Some physical systems either cannot be scaled down favorably, or cannot be scaled down at all!

Scaling laws thus become the very first thing that any engineer would do in the design of MEMS and microsystems.

Types of Scaling Laws

1. **Scaling in Geometry:**

   Scaling of physical size of objects

2. **Scaling of Phenomenological Behavior**

   Scaling of both size and material characterizations

Scaling in Geometry

- **Volume (V)** and **surface (S)** are two physical parameters that are frequently involved in machine design.

- Volume leads to the **mass** and weight of device components.

- Volume relates to both **mechanical and thermal inertia**. Thermal inertia is a measure on how fast we can heat or cool a solid. It is an important parameter in the design of a thermally actuated device as described in Chapter 5.

- Surface is related to **pressure** and the **buoyant forces** in fluid mechanics. For instance, surface pumping by using piezoelectric means is a practical way for driving fluids flow in capillary conduits.

When the physical quantity is to be miniaturized, the design engineer must weigh the magnitudes of the possible consequences from the reduction on both the volume and surface of the particular device.

Scaling in Geometry

If we let ℓ = linear dimension of a solid, we will have:

The volume: $V \propto \ell^3$

The surface: $S \propto \ell^2$

$\longrightarrow$ **S/V = ℓ⁻¹** → $S/V = \ell^{-1}$

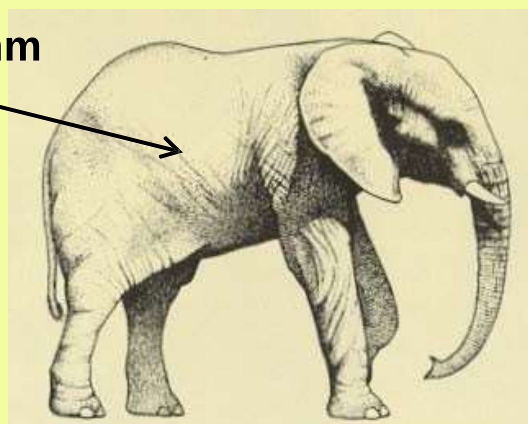
A 10 times reduction in length

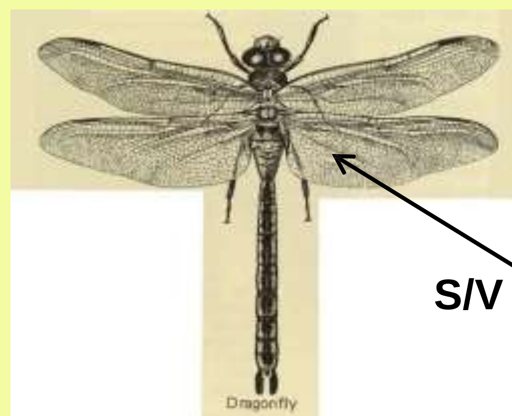
$\longrightarrow$ $10^3 = 1000$ time reduction in volume.

but $\longrightarrow$ $10^2 = 100$ time reduction in surface area.

Since volume, **V** relates to **mass** and surface area, **S** relates to **buoyancy force**:

S/V ≈ 10^{-4}/mm



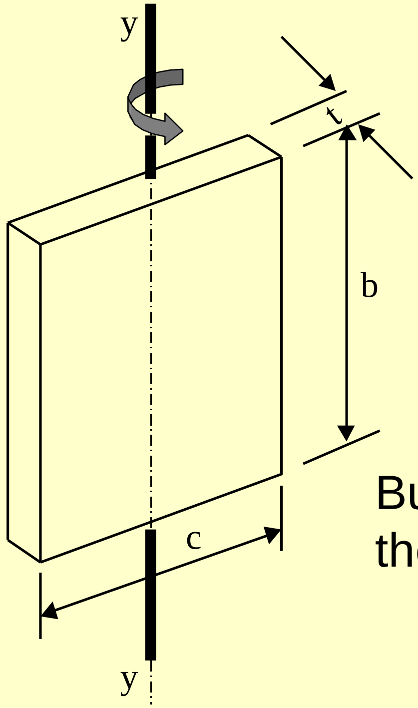


S/V ≈ 10^{-1}/mm

So, an elephant can never fly as easily as a dragonfly!!

Example 6.1: Example on scaling law in geometry of a MEMS device

What would happen to the required torque to turn a micro mirror with a 50% reduction in size?



Torque required to turn the mirror: $\tau \propto I_{yy}$

where $\mathbf{I_{yy}}$ = mass moment of inertia of the mirror about y-axis determined by the following expression:

$$I_{yy} = \frac{1}{12} M c^2$$

in which M = mass of the mirror and c = width of the mirror.

But the mass of the mirror, M = ρ(bct) with ρ = mass density of the mirror material (a fixed value). Thus, we have:

$$I_{yy} = \frac{1}{12} \rho b c^3 t$$

A 50% reduction in size would result in the following:

$$I'_{yy} = \frac{1}{12}\rho\left(\frac{1}{2}b\right)\left(\frac{1}{2}c\right)^3\left(\frac{1}{2}t\right) = \frac{1}{32}\left(\frac{1}{12}\rho b c^3 t\right) = \frac{1}{32} I_{yy}$$

Meaning a factor of 32 times reduction in required torque to rotate the mirror!!

Scaling in Rigid-Body Dynamics

- **Forces** are required to make parts to move such as in the case of micro actuators.

- **Power** is the source for the generation of forces.

- **An engineer needs to resolve the following issues when dealing with the design of a dynamics system such as an actuator :**
  - The required amount of a force to move a part,
  - How fast the desired movements can be achieved,
  - How readily a moving part can be stopped.

- **The resolution to the above issues is on the inertia of the actuating part.**

- **The inertia of solid is related to its mass and the acceleration that is required to initiate or stop the motion of a solid device component.**

- **In the case of miniaturizing these components, one needs to understand the effect of reduction in the size on the power (P), force (F) or pressure (p), and the time (t) required to deliver the motion.**

Scaling in Rigid Body Dynamics

Rigid body dynamics is applied in the design of micro actuations and micro sensors, e.g. micro accelerometers (inertia sensors).

It is important to know how size (scaling) affects the required forces (F), and thus power (P) in the performances of these devices.

The dynamic force (F) acting on a rigid body in motion with acceleration (a) (or deceleration) can be computed from Newton's 2nd law: F = M a

The acceleration (a) in the Newton's law can be expressed in the following way In scaling:

Let the displacement of the rigid body, $s \propto (\ell)$, in which ℓ = linear scale.

But velocity, v = s/t, and hence $v \propto (\ell)t^{-1}$, in which t is the required time.

From particle kinematics, we have: $$s = v_o t + \frac{1}{2} a t^2$$

where v_o = the initial velocity.
By letting $v_o = 0$, we may express: $$a = \frac{2s}{t^2}$$

Thus, the scaling of dynamic force, F is: $$F = Ma = \frac{2sM}{t^2} \propto (\ell)(\ell^3)\, t^{-2}$$

Trimmer Force Scaling Vector

William Trimmer in 1989 defined a **force scaling vector, F** as:

$$F = [\ell^F] = \begin{bmatrix} \ell^1 \\ \ell^2 \\ \ell^3 \\ \ell^4 \end{bmatrix}$$

He was able to relate this vector with other pertinent parameters in dynamics as:

Order	**Force scale, F**	**Acceleration, a**	**Time, t**	**Power density, P/V_o**
1	1	-2	1.5	-2.5
2	2	-1	1	-1
3	3	0	0.5	0.5
4	4	1	0	2

In which "order" means the index, α in the scaling of a quantity in linear dimension, i.e. $(\ell)^{\alpha}$. For example: Weight, **W** $\propto$ V (= $(\boldsymbol{\ell})^3$ for order 3); Pressure, **P** $\propto$ 1/A (= $(\boldsymbol{\ell})^{-2}$ for order 2). The +ve or –ve sign of the order indicates proportional or reverse proportional in scaling.

Scaling in Rigid-Body Dynamics – Cont'd

Trimmer force scaling vector-cont'd

- **Power density** (P/V_o):
  When scaling down a MEMS or a microsystem, one must make sure that the power used to drive the device or system is properly scaled down too.

- In the design practice, power density, rather than power, is used.

- **Power** is defined as **energy** produced or spent by the device per unit time, and energy is related to **work**, which is equals to the force required to move a mass by a distance. Mathematically, these relationships can be expressed as:

$$\frac{P}{V_o} = \frac{Fs}{tV_o}$$

in which, F = force, s = the displacement of the mass moved by the force, and t = time during which the energy is produced or consumed.

- The above expression is used to derive the "Force-scaling vector" as shown in the next slide.

Scaling in Rigid-Body Dynamics – Cont'd

Trimmer force scaling vector-cont'd

The power density:

$$\frac{P}{V_o} = \frac{[\ell^{F}][\ell^{1}]}{\{[\ell^{1}][\ell^{3}][\ell^{-F}]\}^{\frac{1}{2}}[\ell^{3}]} = [\ell^{1.5F}][\ell^{-4}] = [\ell^{F}]^{1.5}[\ell^{-4}]$$

$$= \begin{Bmatrix} \ell^{1} \\ \ell^{2} \\ \ell^{3} \\ \ell^{4} \end{Bmatrix}^{1.5} [\ell^{-4}] = \begin{Bmatrix} \ell^{-2.5} \\ \ell^{-1} \\ \ell^{0.5} \\ \ell^{2} \end{Bmatrix} \qquad (6.10)$$

Example 6.2

Estimate the associate changes in the acceleration (a) and the time (t) and the power supply (P) to actuate a MEMS component if its weight is reduced by a factor of 10.

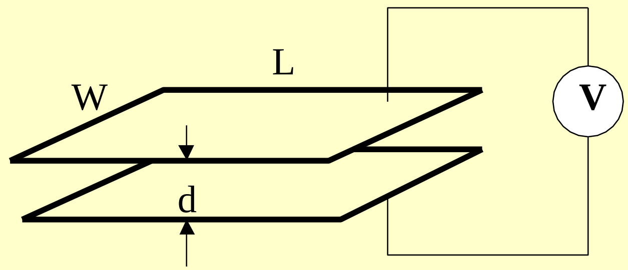
<u>Solution</u>:

Since **W** $\propto$ V (= $(\ell)^3$, so it involves <u>Order 3</u> scaling, from the table for scaling of dynamic forces, we get:

- There will be no reduction in the acceleration (ℓ^0).
- There will be ($\ell^{0.5}$) = $(10)^{0.5}$ = 3.16 reduction in the time to complete the motion.
- There will be ($\ell^{0.5}$) = 3.16 times reduction in power density (P/V_o).

The reduction of power consumption is 3.16 V_o. Since the volume of the component is reduced by a factor of 10, the power consumption after scaling down reduces by: P = 3.16/10 = 0.3 times.

Scaling in Electrostatic Forces



When two parallel electric conductive plates is charged by a voltage →
Electric potential field

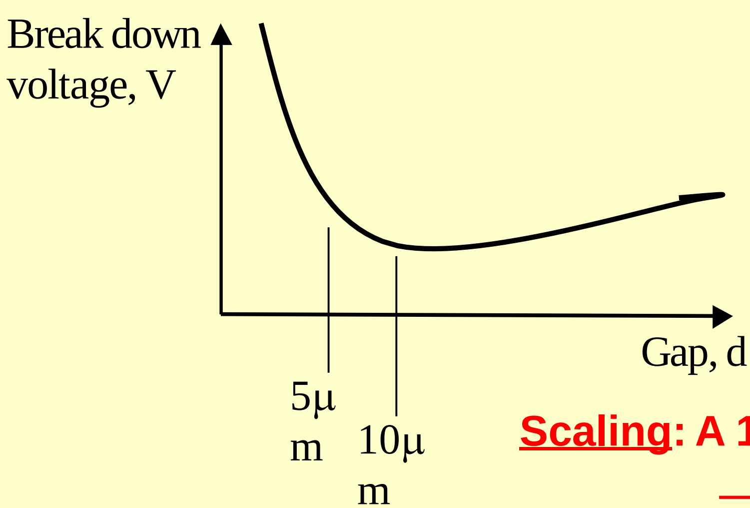
The corresponding potential energy is:

$$U = -\frac{1}{2}CV^2 = -\frac{\varepsilon_0 \varepsilon_r WL}{2d}V^2$$

Let ℓ = linear scale of the electrodes, we will have:

ε_o , $\varepsilon_r \propto \ell^0$ and W, L and d $\propto \ell^1$

The scaling of voltage, V can be approximated by the Paschen's effect illustrated as:



We will use a linear scaling for the voltage:
$V \propto \ell^1$

from which we get the scaling of the Potential energy, U to be:

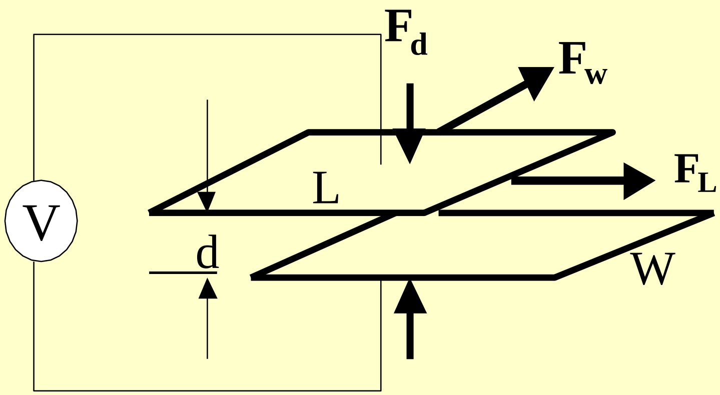
$$U \propto \frac{(\ell^0)(\ell^0)(\ell^1)(\ell^1)(\ell^1)^2}{\ell^1} = (\ell^3)$$

Scaling: A 10 times reduction of linear size of electrodes

→ 10^3 = 1000 times reduction in Potential energy!!

Scaling in Electrostatic Forces – Cont'd

Electrostatic forces in misaligned electrodes are obtained by:



$$F_d = -\frac{\partial U}{\partial d} = -\frac{1}{2}\frac{\varepsilon_0 \varepsilon_r WLV^2}{d^2} \propto l^2$$

$$F_w = -\frac{\partial U}{\partial W} = \frac{1}{2}\frac{\varepsilon_o \varepsilon_r LV^2}{d} \propto l^2$$

$$F_L = -\frac{\partial U}{\partial L} = -\frac{1}{2}\frac{\varepsilon_0 \varepsilon_r WV^2}{d} = \propto l^2$$

So, we may conclude that electrostatic forces:

F_d, F_W, and F_L $\propto l^2$

<u>Scaling</u>: A 10 times reduction in electrode linear dimensions

$\longrightarrow$ 10^2 = 100 times reduction in the magnitude of the electrostatic forces.

Scaling in Electricity

Electric resistance

$$R = \frac{\rho L}{A} \propto (\mathbf{l})^{-1}$$

in which ρ, L and A are respective electric resistivity of the material, the length and across-sectional area of the conductor

Resistive power loss

$$P = \frac{V^2}{R} \propto (\mathbf{l})^{1}$$

where V is the applied voltage.

Electric field energy

$$U = \frac{1}{2}\varepsilon E^2 \propto (\mathbf{l})^{-2}$$

where ε is the permeativity of dielectric , and E is the electric field strength $\propto (\mathbf{l})^{-1}$ **.**

Ratio of power loss to available power

$$\frac{P}{E_{av}} = \frac{(\mathbf{l})^{1}}{(\mathbf{l})^{3}} = (\mathbf{l})^{-2}$$

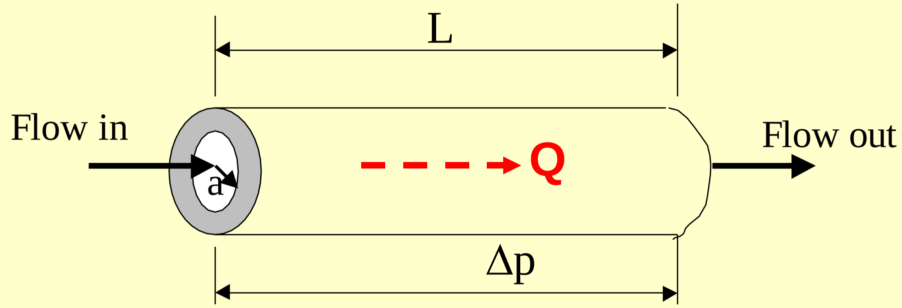
Scaling in Electricity

From "Nanosystems," K. Eric Drexler, John Wiley & Sons, Inc., New York, 1992
Chapter 2, 'Classical Magnitudes and Scaling Laws,' p. 34:

Electric Quantity	Index, α in ℓ^{α}
Current, i	2
Voltage, V	1
Resistance, R	-1
Capacitance, C	1
Inductance, L	1
Power, P	2

Scaling in Fluid Mechanics

Two important quantities in fluid mechanics in flows in capillary conduits:



A. Volumetric Flow, Q:

From Hagen-Poiseuille's equation in (5.17):

$$Q = \frac{\pi a^4 \Delta P}{8\mu L}$$ Leads to: $Q \propto a^4$

Meaning a reduction of 10 in conduit radius

→ 10^4 = 10000 times reduction in volumetric flow!

B. Pressure Drop, ΔP:

From the same Hagen-Poiseuille's equation, we can derive:

$$\Delta P = -\frac{8\mu V_{ave} L}{a^2}$$ Leads to: $\Delta P/L \propto a^{-3}$

<u>Scaling</u>: A reduction of 10 times in conduit radius

→ 10^3 = 1000 times increase in pressure drop per unit length!!

Scaling in Heat Conduction

Two concerns in heat flows in MEMS:

A. How conductive the solid becomes when it is scaling down?

This issue is related to thermal conductivity of solids.

Equation (5.51) indicates the thermal conductivity, k to be:

$$k = \frac{1}{3} C V \lambda \propto \left(\ell^{-3}\right)\left(\ell^{1}\right)\left(\ell^{3}\right) = \left(\ell^{1}\right)$$

B. How fast heat can be conducted in solids:

This issue is related to Fourier number defined as:

$$F_o = \frac{\alpha t}{L^2}$$

Leads to:

$$t = \frac{F_o}{\alpha} L^2 \propto \left(\ell^{2}\right)$$

Scaling: A 10 times reduction in size

$\longrightarrow$ 10^2 = 100 time reduction in time to heat the solid.