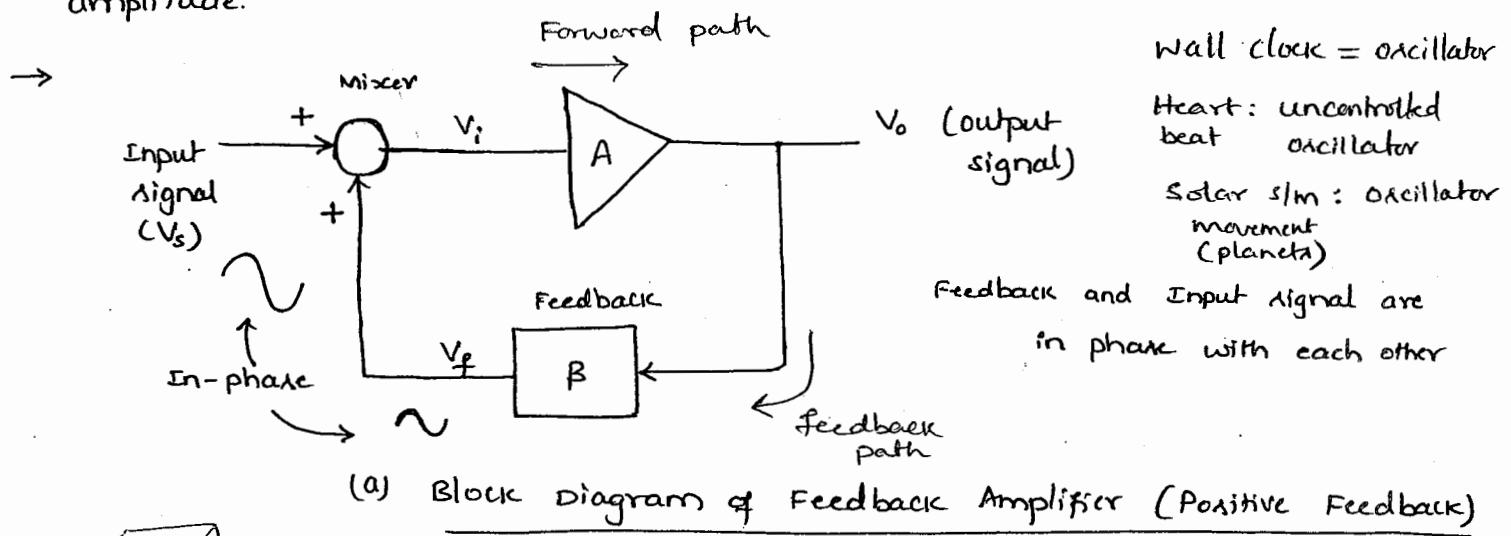


Oscillator Operation

- Feedback
 - Positive feedback : Input signal and part of output signal are in phase
 - Negative feedback : Input signal and part of the output signal are out of phase
- Positive feedback results in oscillations. The circuit that generate oscillations of desired amplitude and frequency are called oscillators.
- An oscillator is an electronic circuit which uses a positive feedback and generates the output which oscillates with constant frequency and amplitude.



DC voltage → Oscillator → oscillation

$$A = \text{open loop gain} = \frac{V_o}{V_i} \quad \text{--- (1)}$$

$$A_f = \text{closed loop gain} = \frac{V_o}{V_s} \quad \text{--- (2)}$$

From Fig. (a), $V_i = V_s + V_f = \beta V_o$

$$V_i = V_s + \beta V_o$$

$$\text{or } V_s = V_i - \beta V_o \quad \text{--- (3)}$$

Substituting (3) in (2) we get

$$A_f = \frac{V_o}{V_i - \beta V_o}$$

Dividing Numerator and Denominator by V_i

$$A_f = \frac{\frac{V_o}{V_i}}{\frac{V_i}{V_i} - \frac{\beta V_o}{V_i}} = \frac{A}{1 - \beta A}$$

$$A_f = \frac{A}{1 - BA}$$

→ If $A = \text{constant}$, $\beta \uparrow$, $AB \uparrow$, $\cancel{A_f} \downarrow$ $A_f \uparrow$

→ At $\beta = \frac{1}{A}$, $(1 + BA) = 0$, $A_f \rightarrow \infty$

⇒ The circuit produces output without an input

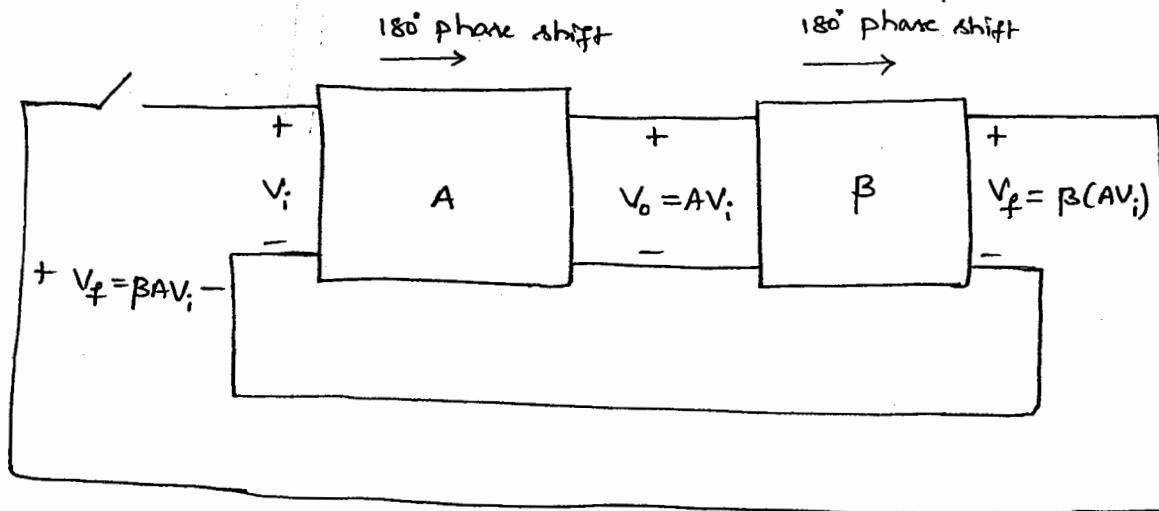
$$\text{since } A_f = \frac{V_o}{V_s} = \infty$$

The circuit gets driven by the feedback signal and produces oscillation. The circuit stops amplifying and starts oscillation

Imagine old pendulum clock with key & oscillations

- The positive feedback increases the closed loop gain, A_f and at $\beta = \frac{1}{A}$, the circuit works as an oscillator.
- It must be noted that β is always a fraction and $\beta < 1$ and circuit adjusts itself such that $AB = 1$ and works as an oscillator.

Barkhausen criterion for oscillation



(a) Feedback circuit used as an oscillator

To understand how the circuit works as an oscillator, consider the feedback circuit shown in Fig. (a)

→ When the switch at the amplifier input is open, no oscillation occurs.

→ Consider that we have a fictitious voltage at the amplifier input V_i

- This results in an output voltage, $V_o = AV_i$ after the amplifier stage.

- The feedback voltage, $V_f = -BV_o$

where negative sign indicates 180° phase shift between V_o and V_f .

$$V_f = -B(AV_i)$$

$$V_f = -BAV_i$$

For an oscillator, $V_i = 0$ and $V_f = V_i$

Therefore,
$$\boxed{-BA = 1} \quad \text{--- ①}$$

The condition $-BA = 1$ is called Barkhausen condition

Eqn. ① can be written as

$$AB = -1 + j0$$

$$|AB| = 1$$

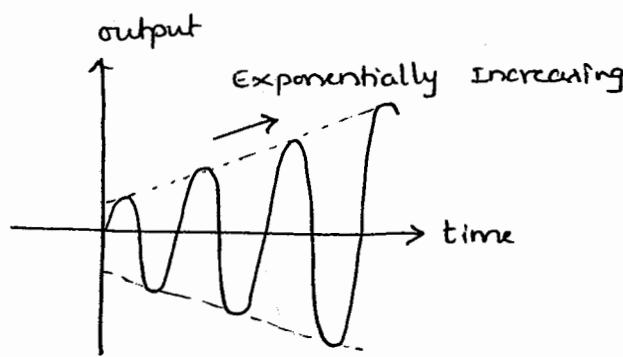
and to have phase same as V_i i.e. positive feedback total phase shift around the loop must be 360°

The Barkhausen criterion states that

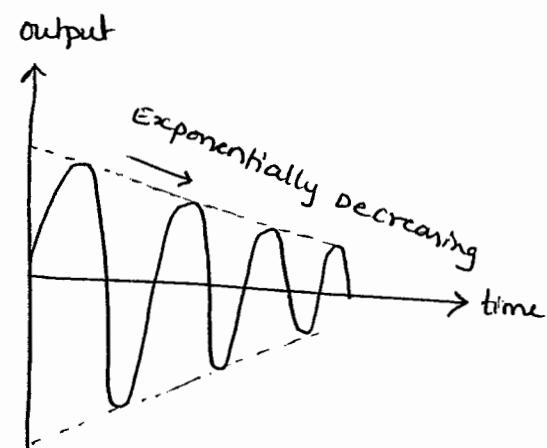
1. The total phase shift around a loop, as the signal proceeds from input through amplifier, feedback network back to input again, completing a loop, is precisely 0° or 360° an integral multiple of 360° radians.

2. The magnitude of the product of the open-loop gain of the amplifier (A) and the feedback factor (β) is unity i.e $|AB| = 1$

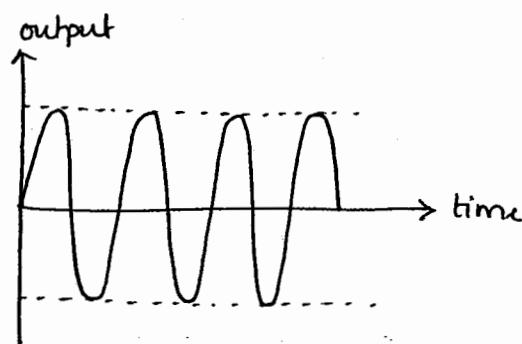
Effect of $|AB|$ on oscillations



(a) $|AB| > 1$



(b) $|AB| < 1$



(c) $|AB| = 1$

→ Total phase shift around a loop : 0° or 360°

$$|AB| > 1$$

output oscillations : Increasing in amplitude

\Rightarrow System is unstable

→ Total phase shift around a loop : 0° or 360°

$$|AB| < 1$$

output oscillations : oscillations are damped with decreasing amplitude

\Rightarrow System is unstable.

→ total phase shift around loop : 0° or 360°

$$|AB| = 1$$

Barkhausen condition are satisfied

The circuit works as an oscillator producing sustained oscillations

- In reality, no input signal is needed to start the oscillator going, only the condition $\beta A = 1$ must be satisfied for self sustained oscillations to result.
- In practice, $\beta A > 1$ the system starts oscillating by amplifying noise voltage.
- If $\beta A \approx 1$, nearly sinusoidal waveform is produced

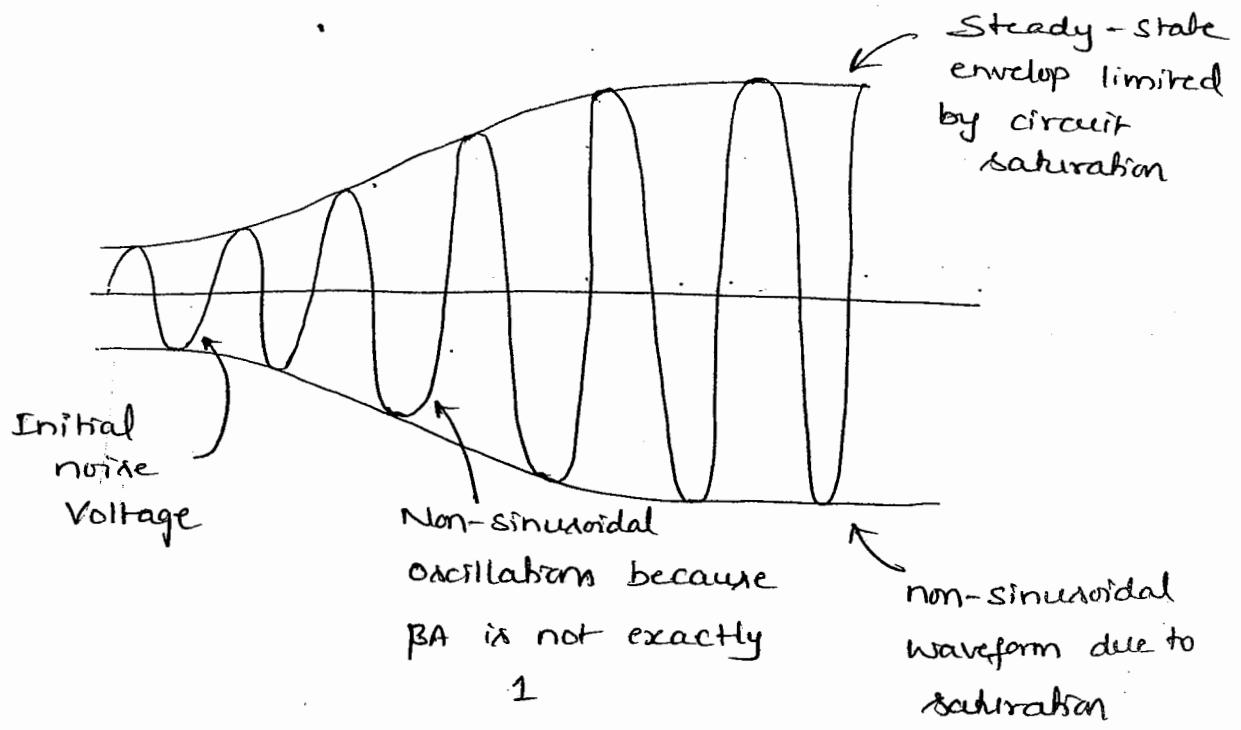
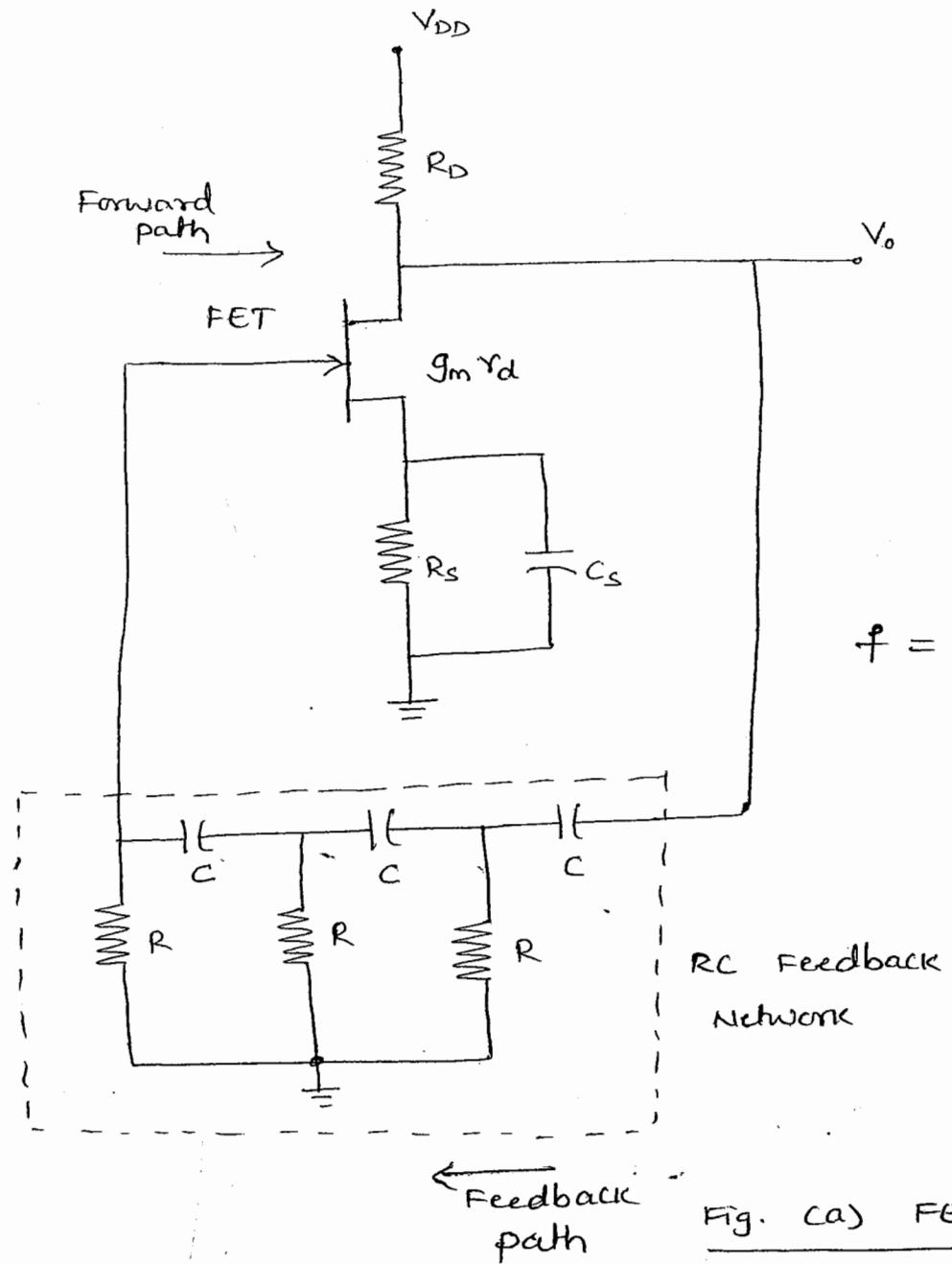


Fig.(a) Buildup of Steady-state oscillations

- Fig. (a) shows how the noise signal results in a buildup of a steady-state oscillation condition.

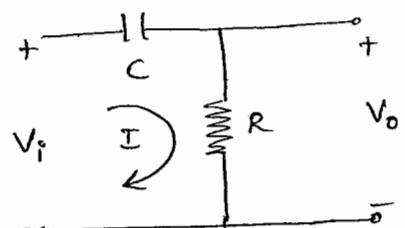
FET phase shift oscillator



$$f = \frac{1}{2\pi\sqrt{6} RC}$$

Fig. (a) FET phase-shift oscillator

$$\text{W.K.T} \quad X_C = \frac{1}{2\pi f C} \Omega$$



Total impedance of RC network Z

$$Z = R - j X_C = R - j \left(\frac{1}{2\pi f C} \right)$$

$$|Z| = \sqrt{R^2 + X_C^2} \quad \phi = \tan^{-1} \left(\frac{X_C}{R} \right)$$

By using proper values of R and C , the angle ϕ is adjusted to 60°

→ If one RC network produces phase shift, $\phi = 60^\circ$ then to produce a phase shift of 180° three RC networks are connected in cascade. (4)

→ A practical version of FET phase shift oscillator is shown in Fig. (a)

→ The amplifier stage is self-biased with a capacitor bypassed source resistor R_s and drain bias resistor, R_D

→ The FET device parameters of interest are g_m and r_d . From FET amplifier theory, the amplifier gain magnitude is given by

$$|A| = g_m R_L$$

Where $R_L = \frac{R_D r_d}{R_D + r_d}$

→ Input impedance of FET amplifier stage is infinite. As long as the operating frequency is low enough, the capacitive impedance may be neglected

→ The feedback network is again three stage RC network having gain

$$|\beta| = \frac{1}{29}$$

$$|A| > 29$$

The frequency of oscillator is given by, $f = \frac{1}{2\pi RC\sqrt{6}}$

(P) It is desired to design a phase shift oscillator having $g_m = 5000 \mu S$, $r_d = 40k\Omega$ and a feedback circuit value of $R = 10k\Omega$. Select the value of C for oscillator operation at 1kHz and R_D for $A > 29$ to ensure oscillator action.

Solution:

$$f = \frac{1}{2\pi RC\sqrt{6}} \Rightarrow C = \frac{1}{2\pi R f \sqrt{6}}$$

$$C = \frac{1}{6.28 \times 10 \times 10^3 \times 1 \times 10^3 \times 2.45} = 6.5 \text{ nF}$$

$$|A| = g_m R_L$$

$$R_L = \frac{|A|}{g_m} = \frac{40}{5000 \times 10^{-6}} = 8 \text{ k}\Omega$$

$$R_L = \frac{R_D r_d}{R_D + r_d} \Rightarrow 8 \text{ k}\Omega = \frac{R_D \times r_d}{R_D + r_d}$$

$$8 \text{ k}\Omega = \frac{40 \text{ k} \times R_D}{R_D + 40 \text{ k}\Omega} \Rightarrow \boxed{R_D = 10 \text{ k}\Omega}$$

Advantages

1. circuit is simple to design
2. can produce output over audio-freq. range
3. produces sinusoidal output waveform.
4. Fixed freq. oscillator

Disadvantages

1. To vary, f the values of R and C must be varied of all three sections simultaneously
 \Rightarrow practically difficult
 \Rightarrow frequency cannot be varied.
2. Frequency stability is poor since various components changes with temperature.

G Wein Bridge Oscillator

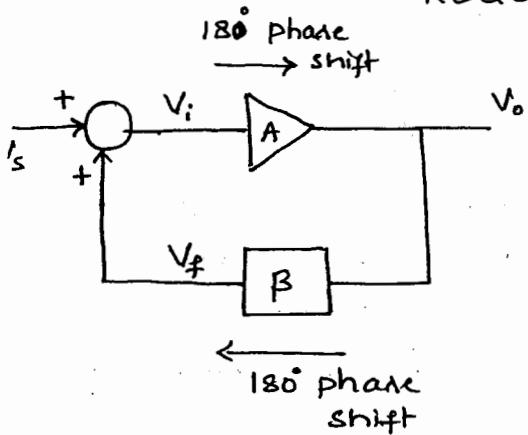
Generally in an oscillator,

Amplifier stage introduces : 180° phase shift

Feedback network introduces : 180° phase shift

Total phase shift around the loop : 0° or 360°

"REQUIRED CONDITION FOR OSCILLATOR"



However,

Wein-bridge oscillator uses a non-inverting amplifier

Therefore, Amplifier stage : 0° phase shift

Feedback network : 0° phase shift

Total phase shift around the loop : 0°

G Basic version of wein-bridge oscillator

→ Note the basic bridge connection

→ Amplifier output is applied at the bridge input at points a and c

→ The bridge circuit output at points b and d is the input to the amplifier stage (op-amp)

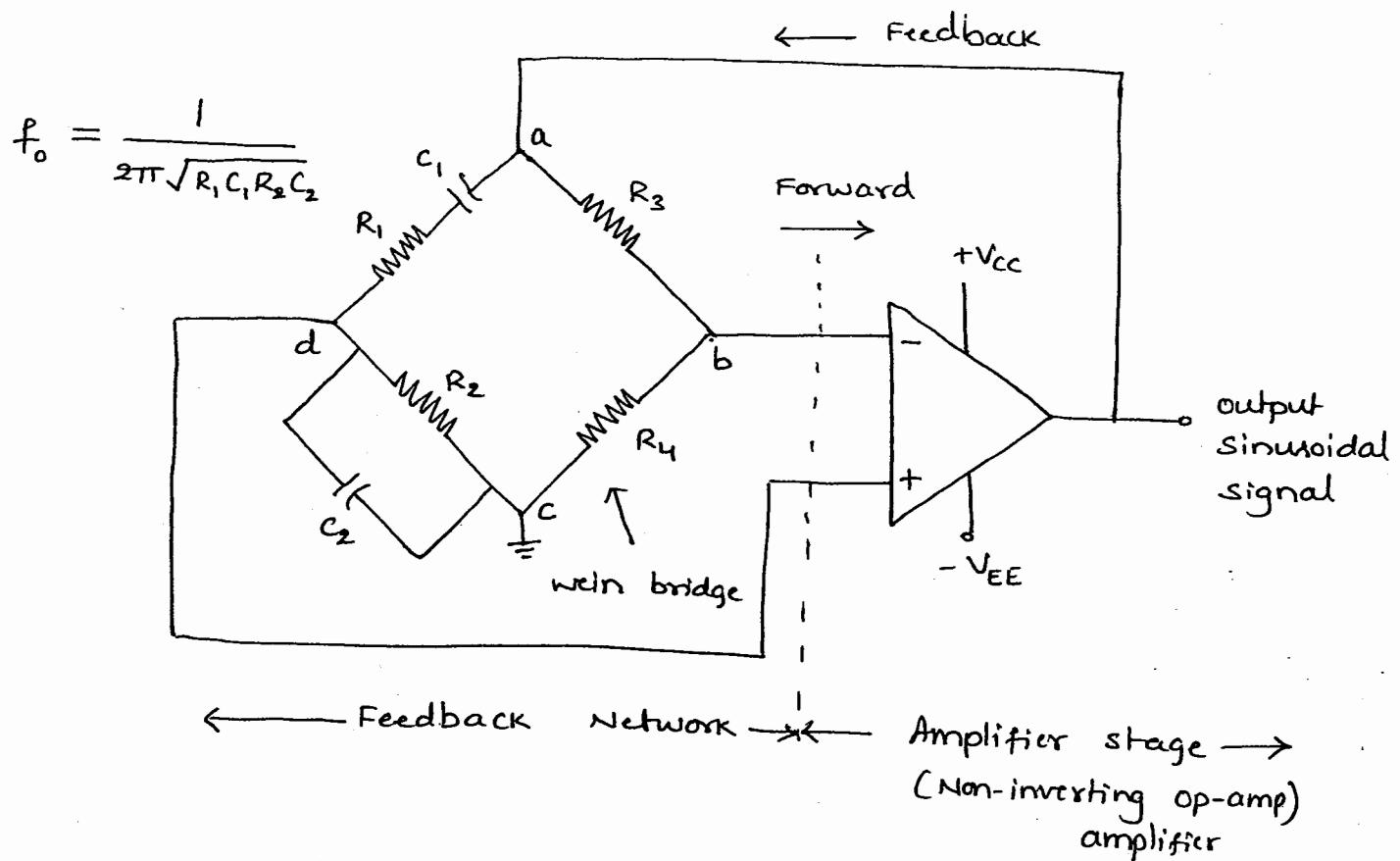


Fig. (a) Wein bridge oscillator circuit using an op-amp amplifier.

→ The amplifier supplies its own input through the wein bridge as the feedback.

→ The two arms of the bridge, namely

R_1 and C_1 : series

R_2 and C_2 : parallel

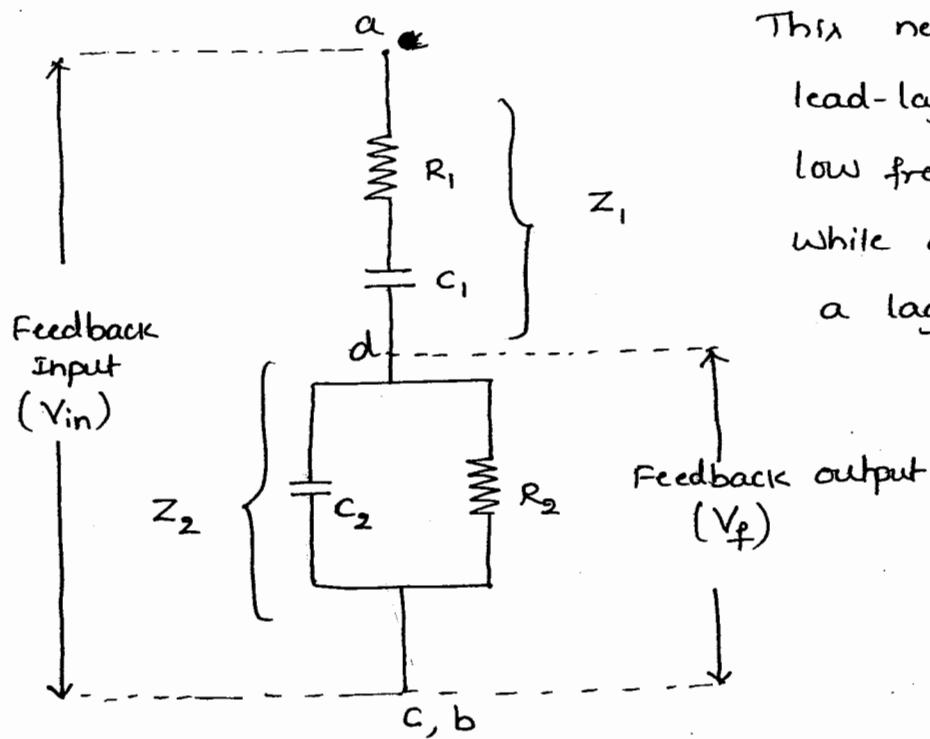
are called frequency sensitive arm

⇒ The components of these two arm decides the frequency of the oscillator.

Gain of the Feedback network

Feedback input : points a and c
(V_{in})

Feedback output : points b and d
(V_f)



This network is an example of lead-lag network, because at low freq., it acts like a lead while at high freq., it acts like a lag network.

(b) Feedback Network of Wein bridge oscillator

g Derivation for Frequency of oscillation

From Fig. (b), the impedance $Z_1 = R_1 + \frac{1}{j\omega C_1}$ (series)

$$\text{or } Z_1 = \frac{1 + j\omega R_1 C_1}{j\omega C_1} \quad \textcircled{1}$$

Similarly, the impedance Z_2 is given by

$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} \quad (\text{parallel})$$

$$Z_2 = \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$Z_2 = \frac{R_2}{1 + j\omega R_2 C_2} \quad \textcircled{2}$$

Replacing $j\omega = s$, equation $\textcircled{1}$ and $\textcircled{2}$ may be rewritten as

$$Z_1 = \frac{1 + SR_1 C_1}{SC_1} \quad \textcircled{3}$$

and

$$Z_2 = \frac{R_2}{1 + SR_2 C_2} \quad \textcircled{4}$$

From Fig. (b), $I = \frac{V_{in}}{Z_1 + Z_2}$

 $V_f = I \cdot Z_2$

$V_f = \frac{V_{in}}{Z_1 + Z_2} Z_2$

$B = \frac{V_f}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$

Substituting the values of Z_1 and Z_2 from eqn. (3) and (4)

$$B = \frac{\left[\frac{R_2}{1 + SR_2 C_2} \right]}{\left[\frac{1 + SR_1 C_1}{SC_1} \right] + \left[\frac{R_2}{1 + SR_2 C_2} \right]}$$

$$= \frac{\left[\frac{R_2}{1 + SR_2 C_2} \right]}{(1 + SR_1 C_1)(1 + R_2 SC_2) + SC_1 R_2}$$

$$B = \frac{SC_1 R_2}{(1 + SR_1 C_1)(1 + SR_2 C_2) + SC_1 R_2}$$

$$= \frac{SC_1 R_2}{1 + SR_1 C_1 + SR_2 C_2 + S^2 R_1 C_1 R_2 C_2 + SC_1 R_2}$$

$$B = \frac{SC_1 R_2}{1 + S(R_1 C_1 + R_2 C_2 + C_1 R_2) + S^2 R_1 C_1 R_2 C_2}$$

Replacing $S = j\omega$ and $S^2 = -\omega^2$, since $j^2 = -1$

$$B = \frac{j\omega C_1 R_2}{(1 - \omega^2 R_1 R_2 C_1 C_2) + j\omega (R_1 C_1 + R_2 C_2 + C_1 R_2)}$$

Rationalising the expression

$$B = \frac{j\omega C_1 R_2 [(1 - \omega^2 R_1 R_2 C_1 C_2) - j\omega (R_1 C_1 + R_2 C_2 + C_1 R_2)]}{(1 - \omega^2 R_1 R_2 C_1 C_2)^2 + \omega^2 (R_1 C_1 + R_2 C_2 + C_1 R_2)^2}$$

$$\beta = \frac{\omega^2 C_1 R_2 (R_1 C_1 + R_2 C_2 + C_1 R_2) + j\omega C_1 R_2 (1 - \omega^2 R_1 R_2 C_1 C_2)}{(1 - \omega^2 R_1 R_2 C_1 C_2)^2 + \omega^2 (R_1 C_1 + R_2 C_2 + C_1 R_2)^2}$$

we know that the required phase shift from the ^(S) feedback network is zero. Therefore, the imaginary part of eqn. ⑤ must be zero.

$$\frac{\omega C_1 R_2 (1 - \omega^2 R_1 R_2 C_1 C_2)}{(1 - \omega^2 R_1 R_2 C_1 C_2)^2 + \omega^2 (R_1 C_1 + R_2 C_2 + C_1 R_2)^2} = 0$$

$$\Rightarrow (1 - \omega^2 R_1 R_2 C_1 C_2) = 0$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

or

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

— ⑥

$$f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$

since $\omega = 2\pi f$

— ⑦

This is the frequency of oscillation of an Wein-bridge oscillator. From the above equation, it is clear that R_1 , R_2 , C_1 and C_2 decides the frequency of oscillation.

Alternatively,

The two arms

R_1 and C_1 : Series

R_2 and C_2 : parallel

} Decides the freq. of oscillation.

Case (i) : If $R_1 = R_2 = R$, and

The frequency of oscillation, f is given by

$$f = \frac{1}{2\pi\sqrt{R^2C^2}} = \frac{1}{2\pi RC}$$

$$\boxed{f = \frac{1}{2\pi RC}}$$

(8)

$$\left| \begin{array}{l} R_1 = R_2 = R \\ C_1 = C_2 = C \end{array} \right.$$

The gain of the feedback network for this condition is given by

$$\beta = \frac{\omega^2 RC (3RC) + j\omega RC (1 - \omega^2 R^2 C^2)}{(1 - \omega^2 R^2 C^2) + \omega^2 (3RC)^2} \quad (9)$$

$$\text{From Eqn. (8)} \quad f = \frac{1}{2\pi RC} \quad \text{or} \quad \omega = \frac{1}{RC}$$

Substituting in eqn. (9)

$$\beta = \frac{3}{0 + \frac{1}{R^2 C^2} (3RC)^2} = \frac{3}{9}$$

$$\Rightarrow \boxed{\beta = \frac{1}{3}}$$

→ The positive sign of the β indicates that the phase shift by the feedback network is 0°

→ To satisfy the Barkhausen criterion for the sustained oscillations, we can write

$$|AB| \geq 1$$

$$|A| \geq \frac{1}{|\beta|}$$

$$|A| \geq \frac{1}{(\frac{1}{3})} \Rightarrow \boxed{|A| \geq 3}$$

This is the required gain of the amplifier stage without any phase shift.

case (ii) if $R_1 \neq R_2$, and
 $C_1 \neq C_2$ then

$$f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$

Substituting in eqn. (5) we get

$$\beta = \frac{C_1 R_2}{(R_1 C_1 + R_2 C_2 + C_1 R_2)}$$

$$|AB| \geq 1$$

$$|A| \geq \frac{R_1 C_1 + R_2 C_2 + C_1 R_2}{C_1 R_2}$$

- (P) Calculate the resonant freq. of the Wein-bridge oscillator for $R_1 = 51k\Omega = R_2$, $C_1 = C_2 = 0.001 \mu F$

Solution:

$$f = \frac{1}{2\pi R C}$$

$$f = \frac{1}{2\pi (51 \times 10^3) (0.001 \mu F)}$$

$$f = 3120.7 \text{ Hz.}$$

- (P) Design the RC elements of a Wein-bridge oscillator for operation at frequency $f_0 = 10 \text{ kHz}$

Solution: $f_0 = 10 \text{ kHz}$

$$f = f_0 = \frac{1}{2\pi R C}$$

choose $R = 100 \text{ k}\Omega$, calculate the required value of C

$$C = \frac{1}{2\pi f R} = \frac{1}{2\pi \times 10 \text{ kHz} \times 100 \text{ k}\Omega}$$

$$C = 159 \text{ pF}$$

§ Crystal Oscillators

→ Piezoelectric effect :

The influence of mechanical pressure, the voltage gets generated across the opposite faces of the crystal.

→ Crystal : Naturally available or synthetically manufactured

→ Mechanical force \longrightarrow A. C. Voltage
(Electrical signal) 

A. C. Voltage \longrightarrow Mechanical distortion

→ Crystal : Greater stability and produces constant freq. of oscillations.

→ A crystal oscillator is basically a tuned circuit oscillator using a piezoelectric crystal as its resonant tank circuit

→ Crystal oscillator

crystal = quartz, originally cut to operate at a specific frequency.

⊕ Greater freq. stability

⊕ used in watches, communication transmitters and receivers.

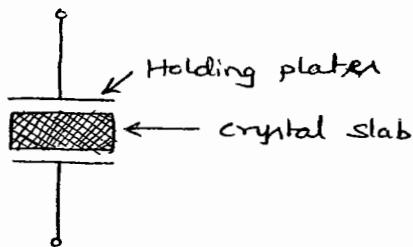
Crystal substance: ① Quartz : ~~inexpensive~~ available in nature
 → used in RF oscillators & filters
 (Crystal oscillator)
 ② Rochelle Salt : Greatest piezoelectric activity (Ex. Microphones)
 ③ Tourmaline : Expensive ^{loud speaker, Headsets.}

§ Construction Details

→ Nature shape of quartz crystal : Hexagonal Prism

But for practical use : cut into Rectangular slab.

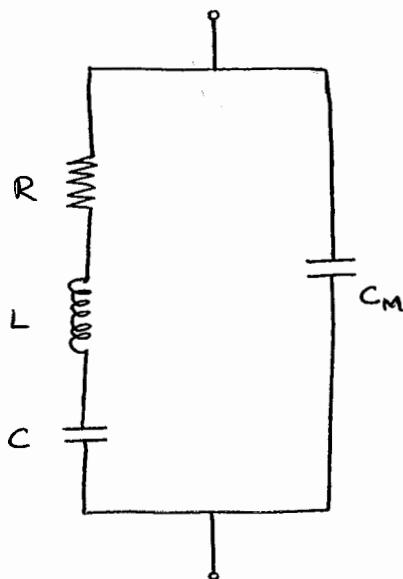
→ Mount slab between two metal plates



→ Fig. (a) shows the symbolic representation of a practical crystal.

Fig. (a) Symbolic Representation of a crystal

A. C. Equivalent circuit



→ When crystal is not vibrating, it is equivalent to a capacitance due to mechanical mounting of a crystal.

$$\frac{1}{C_m}$$

→ When crystal is vibrating

R → Internal frictional losses

L → Mass of the crystal represented as inertia.

C → In vibrating condition, the stiffness.

$$C \parallel C_m$$

→ RLC forms a resonating circuit. The expression for resonating frequency :

$$f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{Q^2}{1+Q^2}}$$

where Q = Quality factor of a crystal

$$Q = \frac{\omega L}{R}$$

$$Q \approx 20,000$$

upto 10^6 can be achieved using

$$\Rightarrow \sqrt{\frac{Q^2}{R}}$$

Module # 4 : Feedback and Oscillator Circuits

Refer chapter #14 from the text titled "Electronic Devices and circuit theory" authored by Robert L. Boylestad and Louis Nashelsky.

Topics: - Feedback concepts

- ① Driving a vehicle: Feedback s/m
- ② Rainfall cycle

- Feedback connection types
- Practical feedback circuits
- oscillator operation
- FET phase shift oscillator
- Wein bridge oscillator
- Tuned oscillator circuit
- crystal oscillator
- UJT construction
- UJT oscillator

9 Feedback concepts

- Depending on the relative polarity of the signal being fed back into a circuit, one may have negative or positive feedback.
- Negative feedback results in decreased voltage gain, for which a number of circuit features are improved.
- Positive feedback drives a circuit into oscillation as in various types of oscillator circuits.
- A typical feedback connection is shown in Fig. (a)

V_s : Input signal applied to a mixer

V_f : Feedback signal

β : Feedback network

V_i : Input voltage to the amplifier

V_o : Amplifier output

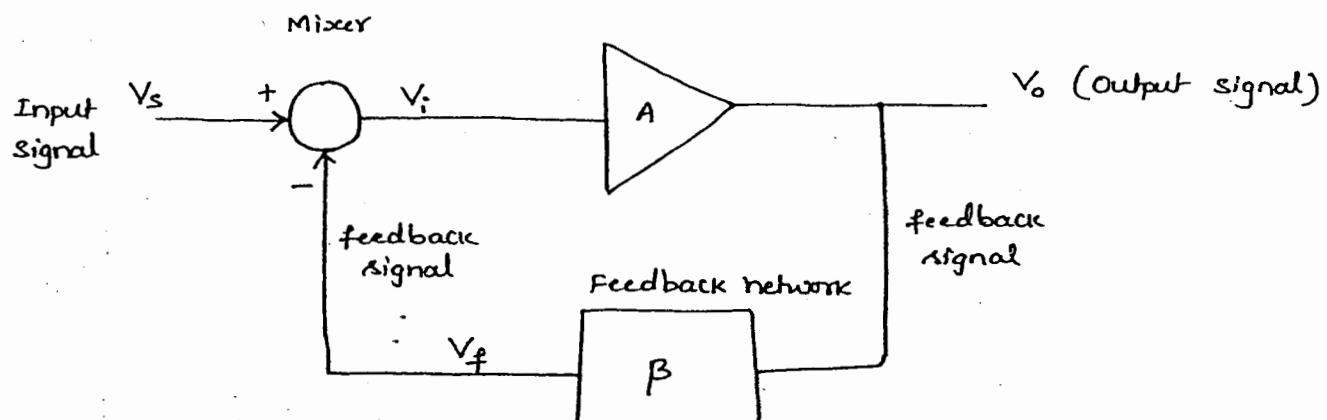
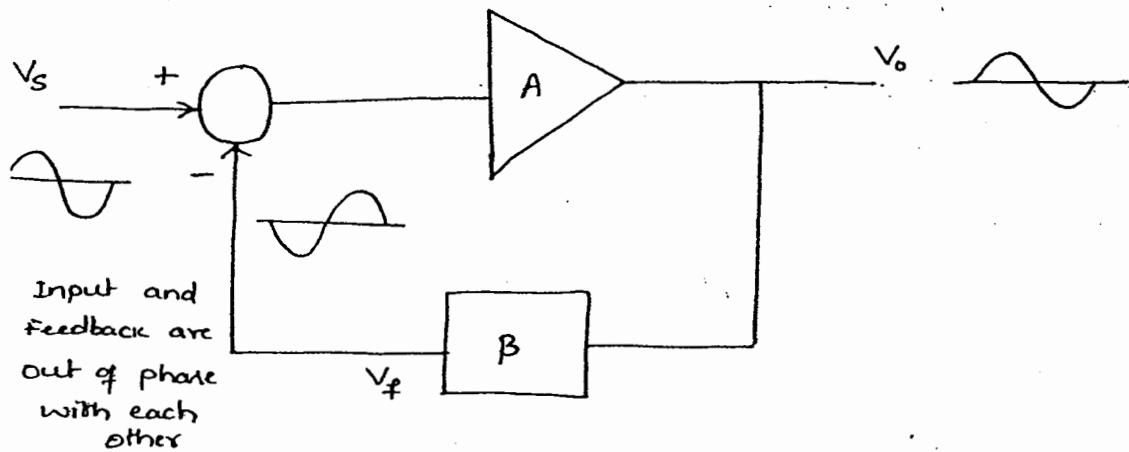


Fig. (a) simple Block Diagram of Feedback Amplifier

- The input signal, V_s is applied to a mixer network where it is combined with a feedback signal, V_f . The difference of these signals, $V_i = V_s - V_f$ is then fed as input to the amplifier. A portion of the amplifier output, V_o is connected to the feedback network, β , which provides a reduced portion of the output as feedback signal to the input mixer network.
- Negative feedback : If the feedback signal is of opposite polarity ~~the~~ to the input signal as shown in Fig. (b), negative feedback results.



(b) Concept of Negative Feedback

→ Although negative feedback results in reduced overall voltage gain, a number of improvements are obtained that are listed below:

1. Higher input impedance
2. Better stabilized voltage gain
3. Improved frequency response
4. Lower output impedance.
5. Reduced noise
6. More linear operation

5 Feedback connection Types

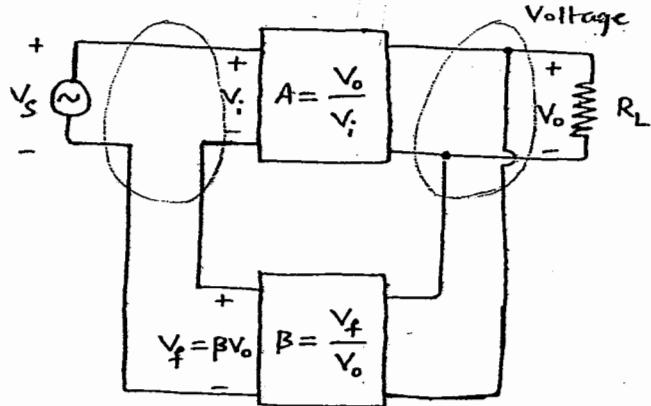
There are four basic ways of connecting the feedback signal.

Both voltage and current can be fed back to the input either in series or parallel.

I = Input, O = Output

1. Voltage - Series feedback : V-series (I-O)
2. Voltage - Shunt feedback : V-shunt (I-O)
3. Current - Series feedback : I-series (I-O)
4. Current - Shunt feedback : I-shunt (I-O)

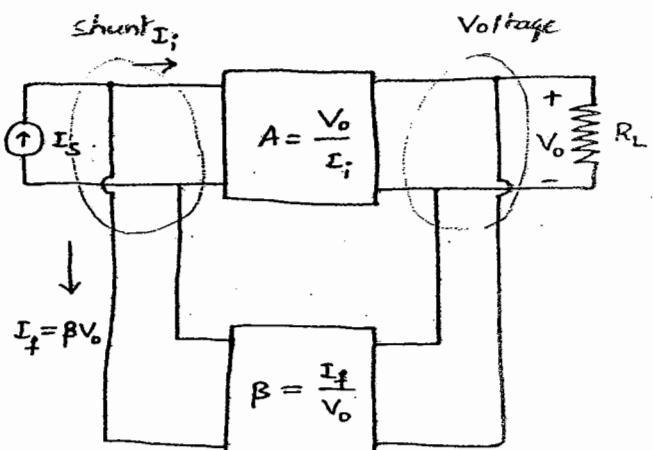
Series



(a) Voltage-Series Feedback

Gain
with
Feedback

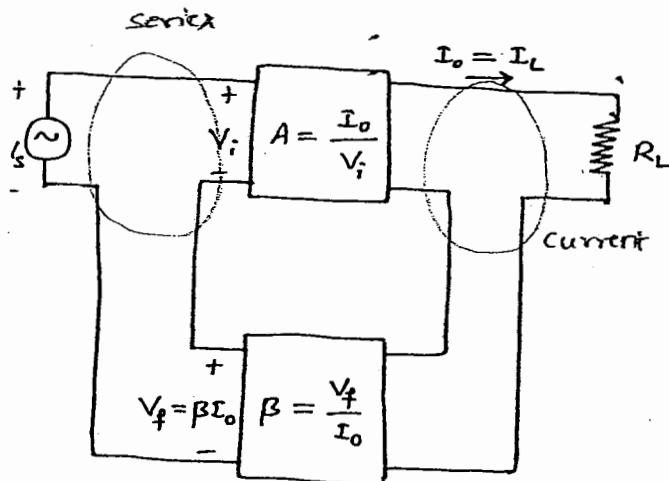
$$A_f = \frac{V_o}{V_s}$$



(b) Voltage-Shunt Feedback

Gain
with
Feedback

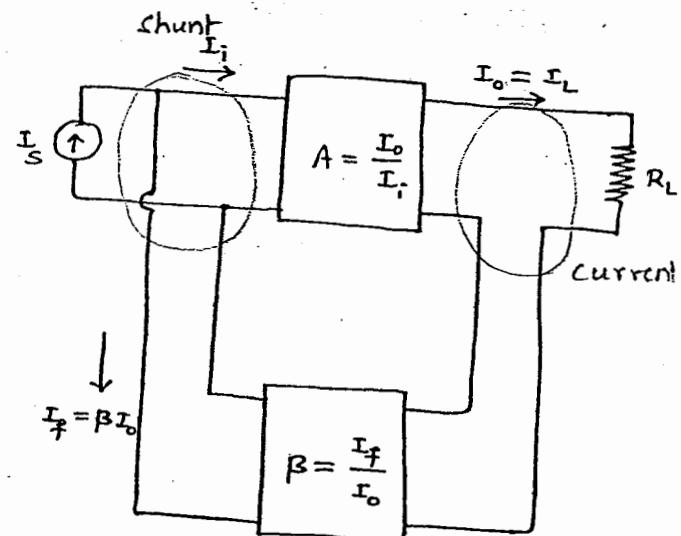
$$A_f = \frac{V_o}{I_s}$$



(c) current-series feedback

Gain with
Feedback

$$A_f = \frac{I_o}{V_s}$$



(d) current-shunt feedback

Gain with
Feedback

$$A_f = \frac{I_o}{I_s}$$

→ In the list above,

#1: Voltage refers to connecting output voltage as input to the feedback network.

#2: current refers to tapping off some output current through the feedback network.

#3: Series refers to connecting the feedback signal in series with the input signal voltage.

#4: Shunt refers to connecting the feedback signal in ~~series~~ shunt (parallel) with an input current source.

Type of Feedback

1. Series Feedback
2. Shunt Feedback
3. Voltage Feedback
4. Current Feedback

typically,

Effect on I/p or O/p Impedance

$$Z_{if} \uparrow$$

$$Z_{if} \downarrow$$

$$Z_{of} \downarrow$$

$$Z_{of} \uparrow$$

Desired for cascaded amplifier : $Z_{if} \uparrow$ and $Z_{of} \downarrow$
Both are provided by Voltage-series feedback

| Parameters | V-Series | V-shunt | I-Series | I-shunt |
|----------------------------------|-------------------|-------------------|-------------------|-------------------|
| Gain without feedback: (A) | $\frac{V_o}{V_i}$ | $\frac{V_o}{I_i}$ | $\frac{I_o}{V_i}$ | $\frac{I_o}{I_i}$ |
| Feedback (β) : | $\frac{V_f}{V_o}$ | $\frac{I_f}{V_o}$ | $\frac{V_f}{I_o}$ | $\frac{I_f}{I_o}$ |
| Gain with feedback: (A_f) | $\frac{V_o}{V_s}$ | $\frac{V_o}{I_s}$ | $\frac{I_o}{V_s}$ | $\frac{I_o}{I_s}$ |

Gain with Feedback

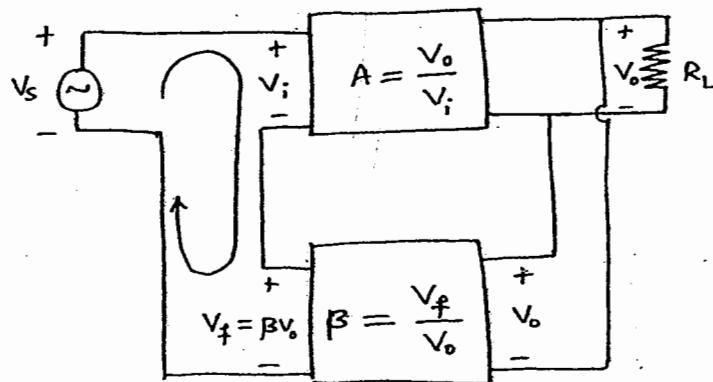
The gain of each of the feedback circuit connection is derived as follows:

Let A = Gain without feedback of the amplifier stage

β = Gain of the feedback network

A_f = Gain of the amplifier stage with feedback

i. Voltage-series Feedback : The voltage-series feedback connection is shown in Fig. (a) with a part of the output voltage fed back in series with the input signal. This results in the reduction of overall gain.



(a) Voltage-series Feedback
connection

If there is no feedback ($V_f = 0$), the voltage gain of the amplifier stage is

$$A = \frac{V_o}{V_s} = \frac{V_o}{V_i} \quad \text{--- (1)}$$

If a feedback signal, V_f , is connected in series with the input, then

Applying KVL for the input loop shown

$$-V_s + V_i + V_f = 0$$

or

$$V_i = V_s - V_f \quad \text{--- (2)}$$

$$\text{From (1)} \quad V_o = A V_i$$

Eqn. (2)

$$V_o = A (V_s - V_f)$$

$$V_o = A V_s - A V_f \quad \text{--- } \beta V_o$$

$$V_o = A V_s - A (\beta V_o)$$

$$V_o + A \beta V_o = A V_s$$

$$V_o [1 + A \beta] = A V_s$$

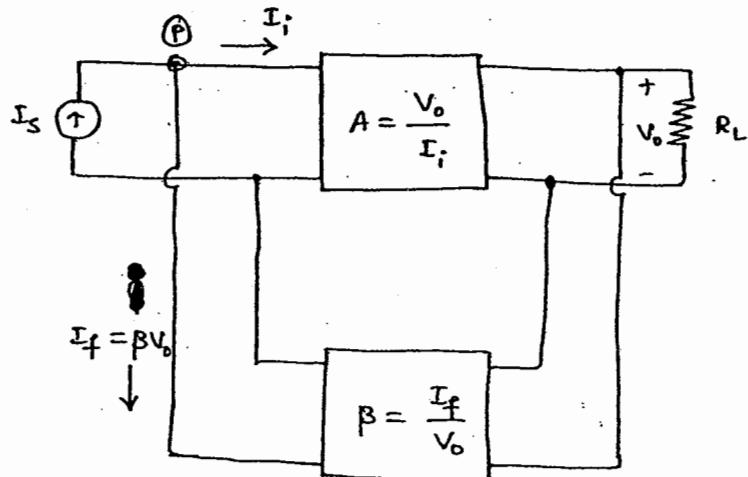
Therefore, the overall gain with feedback is

$$A_f = \frac{V_o}{V_s} = \frac{A}{1 + \beta A} \quad \text{--- (3)}$$

From Eqn. (3), it is evident that the gain with feedback is the amplifier gain reduced by the factor, $(1 + \beta A)$

5 Voltage-Shunt Feedback : The gain with feedback for the

network shown in Fig. (b) is,



$$A_f = \frac{V_o}{I_s} \quad \text{--- (1)}$$

Applying KCL at node P

$$I_s = I_i + I_f \quad \text{--- (2)}$$

$$A = \frac{V_o}{I_i} \Rightarrow V_o = A I_i \quad \text{--- (3)}$$

Substitute (2) and (3) in (1)

Fig. (b) Voltage - Shunt Feedback connection

(7)

$$A_f = -\frac{A I_i}{I_i + I_f} = \frac{\beta V_o}{V_o}$$

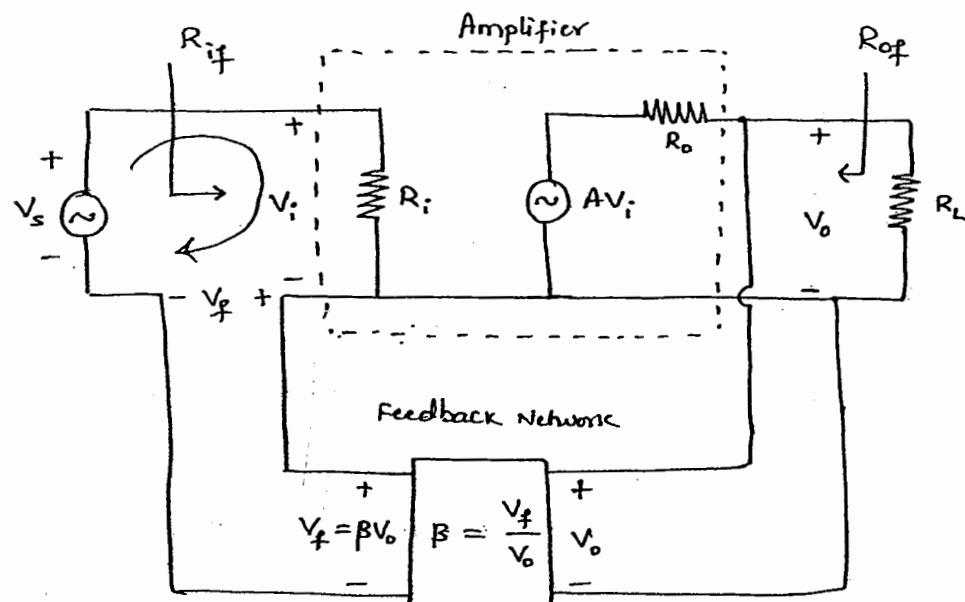
$$A_f = \frac{A I_i}{I_i + \beta V_o} = \frac{A I_i}{A I_i}$$

$$A_f = \frac{A I_i}{I_i + \beta A I_i} = \frac{A I_i}{I_i (1 + \beta A)}$$

$$A_f = \frac{A}{1 + \beta A} \quad \text{--- (4)}$$

Eqn. (4) shows that the gain with feedback is the amplifier gain reduced by the factor, $(1 + \beta A)$

Impedance with Feedback : Input Impedance



$$A = \frac{V_o}{V_i}; \quad A_f = \frac{V_o}{V_s}$$

$$\beta = \frac{V_f}{V_o}; \quad A_f = \frac{A}{1 + \beta A}$$

(a) Voltage-series Feedback connection

→ A more detailed voltage-series feedback connection is shown in fig.

→ The input impedance can be determined as follows :

$$\text{Input current, } I_i = \frac{V_i}{Z_i} = \frac{\text{Input voltage to amplifier}}{\text{Amplifier input impedance}}$$

Applying KVL for the input loop

$$V_t - V_s + V_i = 0$$

$$V_i = V_s - V_f \quad \text{--- (2)}$$

Substituting (2) in (1), we get

$$I_i = \frac{V_s - V_f}{Z_i}$$

$$\text{But } V_f = \beta V_o$$

$$\text{Therefore, } I_i = \frac{V_s - \beta V_o}{Z_i}, \quad \text{but } A = \frac{V_o}{V_i} \Rightarrow V_o = AV_i$$

$$I_i = \frac{V_s - \beta AV_i}{Z_i}$$

$$I_i Z_i = V_s - \beta AV_i$$

$$V_s = I_i Z_i + \beta AV_i$$

$$V_s = I_i Z_i + \beta A I_i Z_i$$

$$V_s = I_i [Z_i + (\beta A) Z_i]$$

$$\frac{V_s}{I_i} = Z_{if} = Z_i (1 + \beta A)$$

Input Impedance
with feedback

$$Z_{if} = Z_i (1 + \beta A)$$

Input Impedance
without feedback

--- (3)

From eqn. (3), it is clear that the input impedance with voltage series feedback is seen to be the value of the ~~feedback~~ input impedance without feedback multiplied by the factor, $(1 + \beta A)$. The eqn. (3) applies to both voltage-series and current-series feedback configurations.

§ Voltage shunt Feedback : Input Impedance

(5)

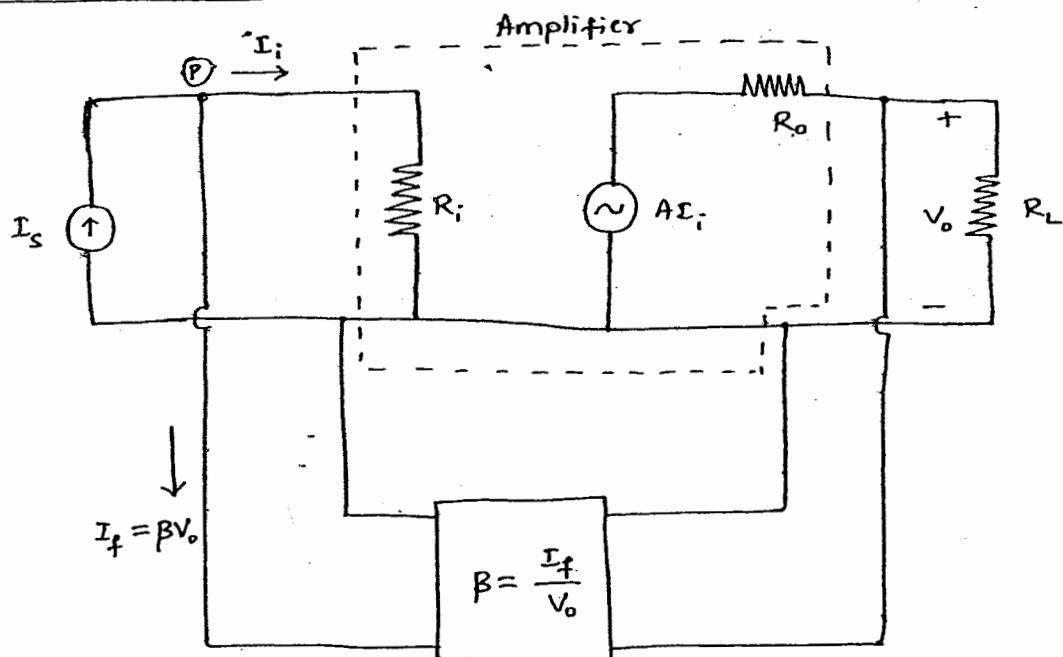


Fig. (b) Voltage-shunt Feedback connection

→ A more detailed voltage-shunt feedback connection is shown in Fig. (b)

→ The input impedance can be determined as follows :

From Definition, Input impedance with feedback = ~~Z_{if}~~ Z_{if}

$$Z_{if} = \frac{V_i}{I_s} \quad \text{--- (1)}$$

Applying KCL to the node, P

$$I_s = I_i + I_f \quad \text{--- (2)}$$

Substituting (2) in (1), we get

$$Z_{if} = \frac{V_i}{I_i + I_f} = \frac{V_i}{\beta V_o}$$

$$Z_{if} = \frac{V_i}{I_i + \beta V_o}$$

Dividing ~~multiplying~~ Numerator and Denominator by I_i

$$Z_{if} = \frac{\frac{V_i}{I_i}}{\frac{I_i + \beta V_o}{I_i}} = \frac{\frac{V_i}{I_i}}{\frac{I_i}{I_i} + \frac{\beta V_o}{I_i}} = \frac{Z_i}{1 + \beta A}$$

$$Z_{if} = \frac{Z_i}{1 + \beta A_f} \quad (3)$$

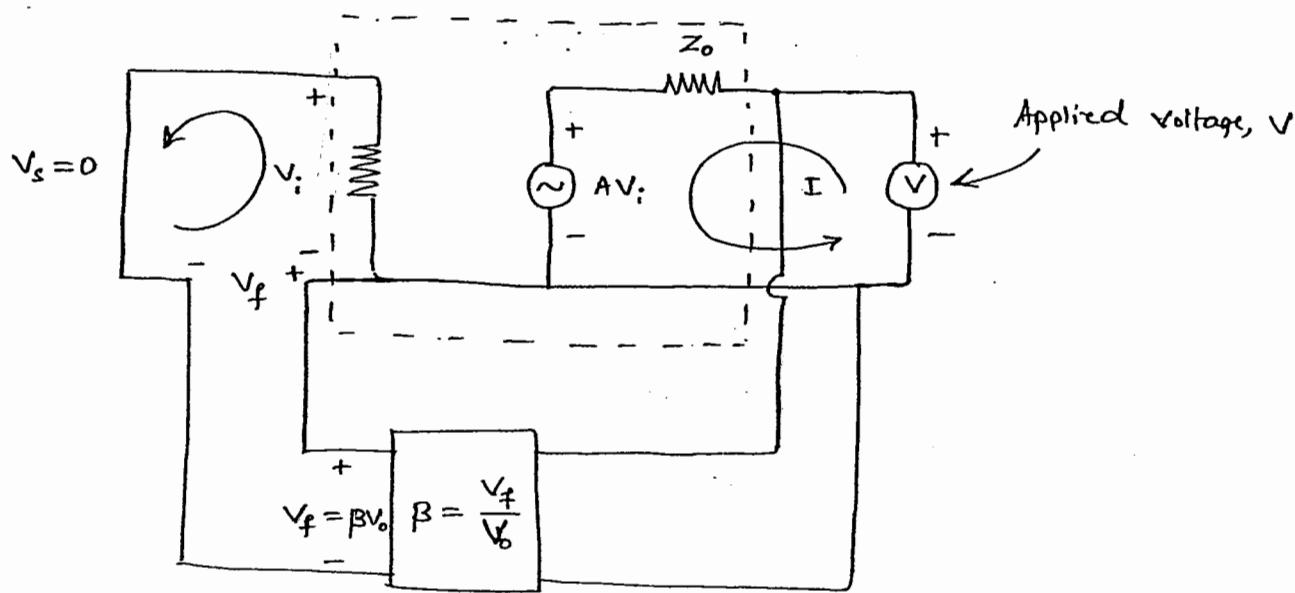
The input impedance with feedback is reduced by the factor, $(1 + \beta A_f)$. The equation (3) applies to both voltage-shunt and current-shunt feedback connection.

§ Output Impedance with Feedback

- The output impedance depends on whether voltage or current feedback is used.
- For a voltage feedback, the output impedance is decreased and for the current feedback, the output impedance is increased.

§ Voltage-Series Feedback

- The Fig. (a) shows the detailed connection of voltage-series feedback.
- The output impedance is determined by applying a voltage, V , resulting in a current, I with V_s shorted out (i.e. $V_s = 0$)



(a) Voltage-series Feedback with $V_s = 0$

and applying a voltage, V in the output circuit

Applying KVL for the loop shown

$$-V + IZ_0 + AV_i = 0$$

$$V = IZ_0 + AV_i \quad \text{--- (1)}$$

Applying KVL to the input loop results in

$$-V_f - V_i = 0, \quad \text{since } V_s = 0$$

$$\text{or } V_i = -V_f \quad \text{--- (2)}$$

Substituting (2) in (1) gives

$$V = IZ_0 + A(-V_f)$$

$$V = IZ_0 - AV_f \quad \text{--- (3)}$$

$$V = IZ_0 - A(\beta V)$$

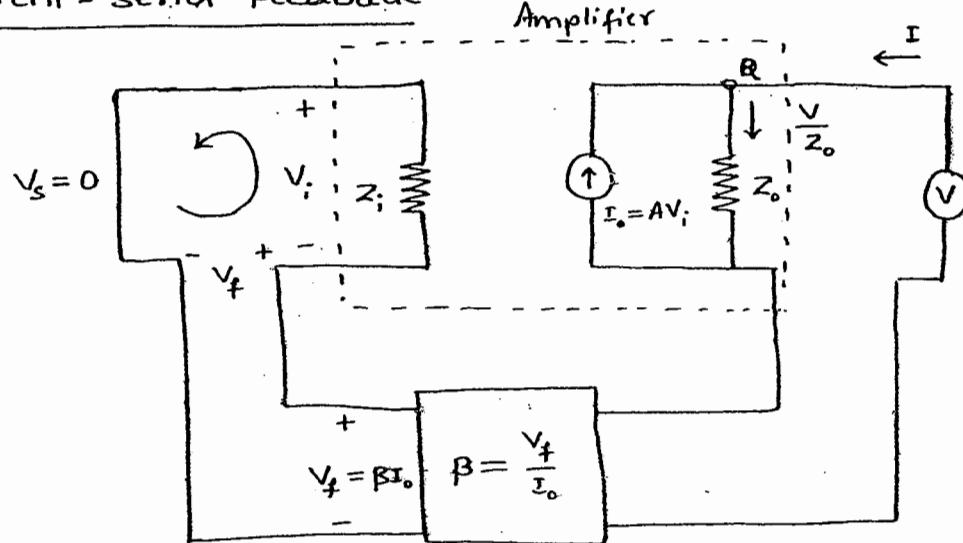
$$\text{Rewriting eqn. } V + \beta A V = IZ_0$$

$$V(1 + \beta A) = IZ_0$$

$$\boxed{\frac{V}{I} = Z_{of} = \frac{Z_0}{1 + \beta A}} \quad \text{--- (3)}$$

The output impedance of a voltage-series feedback is reduced from that without feedback by the factor $(1 + \beta A)$

Current-Series Feedback



- The output impedance for the current-series feedback can be determined by applying a signal v to the output with the input source, v_s shorted out (i.e. $v_s = 0$).
- The applied voltage, v results in a current, I . The output impedance is obtained by disconnecting the load resistance R_L and looking into the output terminals. The ratio of v to I gives the output impedance.
- Fig. (b) shows the more detailed connection of current-series feedback. The output impedance is determined as follows:

Apply KCL at the output node, Q

$$I + I_o = \frac{v}{Z_o}$$

\parallel
AV;

$$I = \frac{v}{Z_o} - AV_i \quad \text{--- (1)}$$

The input voltage is determined as follows:

Applying KVL to the input loop

$$-V_f - V_i + 0 = 0$$

\parallel
 V_s

$$V_i = -V_f$$

$$\text{but } V_f = \beta I_o = -\beta I \quad (\text{because } I_o = -I)$$

$$\text{Therefore, } V_i = -(-\beta I) = \beta I \quad \text{--- (2)}$$

Substituting the value of V_i from (2) in eqn. (1)

$$I = \frac{v}{Z_o} - A\beta I$$

$$\frac{v}{Z_o} = I(1 + \beta A)$$

$$Z_{of} = \boxed{\frac{v}{I} = Z_o(1 + \beta A)}$$

→ A summary of the effect of feedback on input and output impedance is provided in the table below:

Table : Effect of Feedback connection on I_{IP} and O/P Impedance

| | Voltage - series | current - series | voltage - shunt | current - shunt |
|------------|---|---|---|---|
| Z_{if} : | $Z_i (1 + \beta A)$ (Increased) ↑ | $Z_i (1 + \beta A)$ (Increased) ↑ | $\frac{Z_i}{(1 + \beta A)}$ (Decreased) ↓ | $\frac{Z_i}{(1 + \beta A)}$ (Decreased) ↓ |
| Z_{of} : | $\frac{Z_o}{(1 + \beta A)}$ (Decreased) ↓ | $Z_o (1 + \beta A)$ (Increased) ↑ | $\frac{Z_o}{(1 + \beta A)}$ (Decreased) ↓ | $Z_o (1 + \beta A)$ (Increased) ↑ |

- (P) Determine the voltage gain, input, and output impedance with feedback for voltage-series feedback having $A = -100$, $R_i = 10\text{k}\Omega$, and $R_o = 20\text{k}\Omega$ for feedback of (a) $\beta = -0.1$ and (b) $\beta = -0.5$

Solution: (a) The mathematical expression for A_f for voltage series feedback is given by

$$A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.1)(-100)} = \frac{-100}{11} = -9.09$$

$$\boxed{A_f = -9.09}$$

$$Z_{if} = Z_i (1 + \beta A) = 10\text{k}\Omega (11) = 110\text{k}\Omega$$

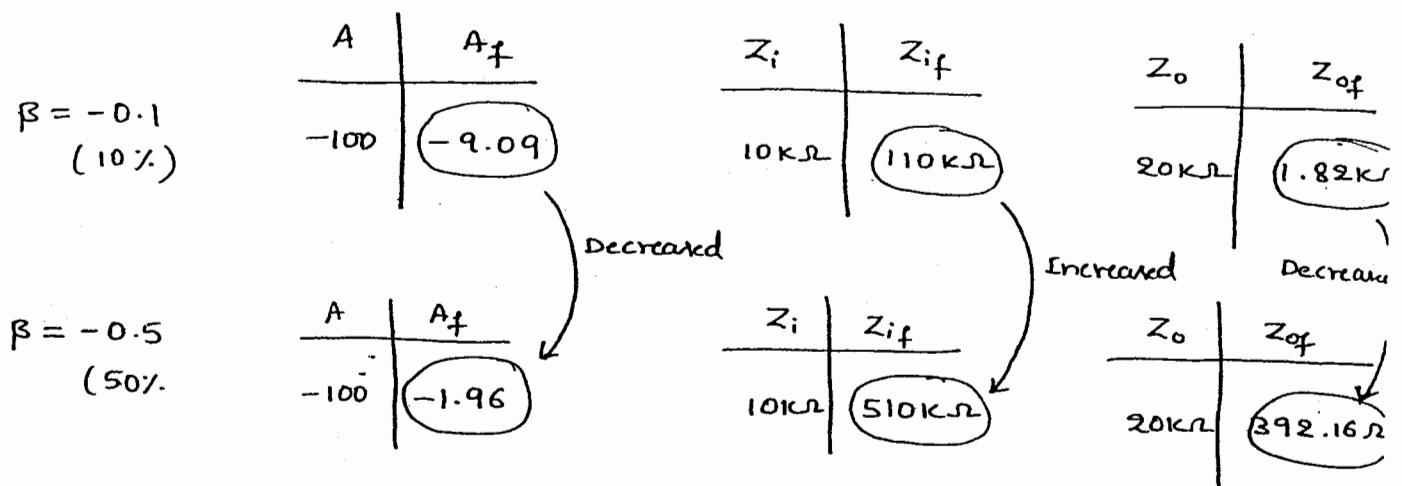
$$Z_{of} = \frac{Z_o}{1 + \beta A} = \frac{20\text{k}\Omega}{11} = 1.82\text{k}\Omega$$

$$(b) A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.5)(-100)} = \frac{-100}{51} = -1.96$$

$$Z_{if} = Z_i (1 + \beta A) = 10\text{k}\Omega (51) = 510\text{k}\Omega$$

$$Z_{of} = \frac{Z_o}{1 + \beta A} = \frac{20\text{k}\Omega}{51} = 392.16 \Omega$$

→ The example demonstrates the trade-off of gain for improved input and output impedance



- Reduced gain by a factor of 11 (100 to 9.09) is complemented by a reduced output resistance and increased input resistance by the same factor of 11.
- Reducing the gain by a factor of 51 provides a gain of only 2 but with input resistance increased by the factor of 51 (to over 500k Ω) and output resistance reduced from 20k Ω to under 400 Ω .
- Feedback offers the designer the choice of trading away some of the available amplifier gain for other improved circuit features.

§ Reduction in Frequency Distortion

15

- For a negative feedback amplifier $\beta A \gg 1$

$$\text{The gain with feedback, } A_f = \frac{A}{1 + \beta A} \Rightarrow A_f \approx \frac{1}{\beta}$$

- From the above approximation, it follows that if the $\beta A \gg 1$ network is purely resistive, the gain with feedback is not dependent on frequency, even though the basic amplifier gain is a frequency dependent.
- In a negative voltage feedback amplifier, the freq. distortion arising because of varying amplifier gain with freq. is considerably reduced.

§ Reduced in noise and non-linear distortion

The factor $(1 + \beta A)$ reduces both input noise and resulting non-linear distortion for considerable improvement. Thus, noise and non-linear distortion also reduced by the same factor as the gain.

§ Effect of negative feedback on Gain and Bandwidth

- overall gain with negative feedback is

$$A_f = \frac{A}{1 + \beta A} \approx \frac{A}{\beta A} = \frac{1}{\beta} \quad \text{for } \beta A \gg 1$$

as long as $\beta A \gg 1$, the overall gain is approximately $\frac{1}{\beta}$

- For a practical amplifier, the open-loop gain (A) drops off at high freq. due to the active device and circuit capacitances
 - The gain also drop off at low frequencies for capacitively coupled amplifier stages.
 - Once the open-loop gain, A drops low enough and the factor βA is no longer much larger than 1. Therefore

Bandwidth

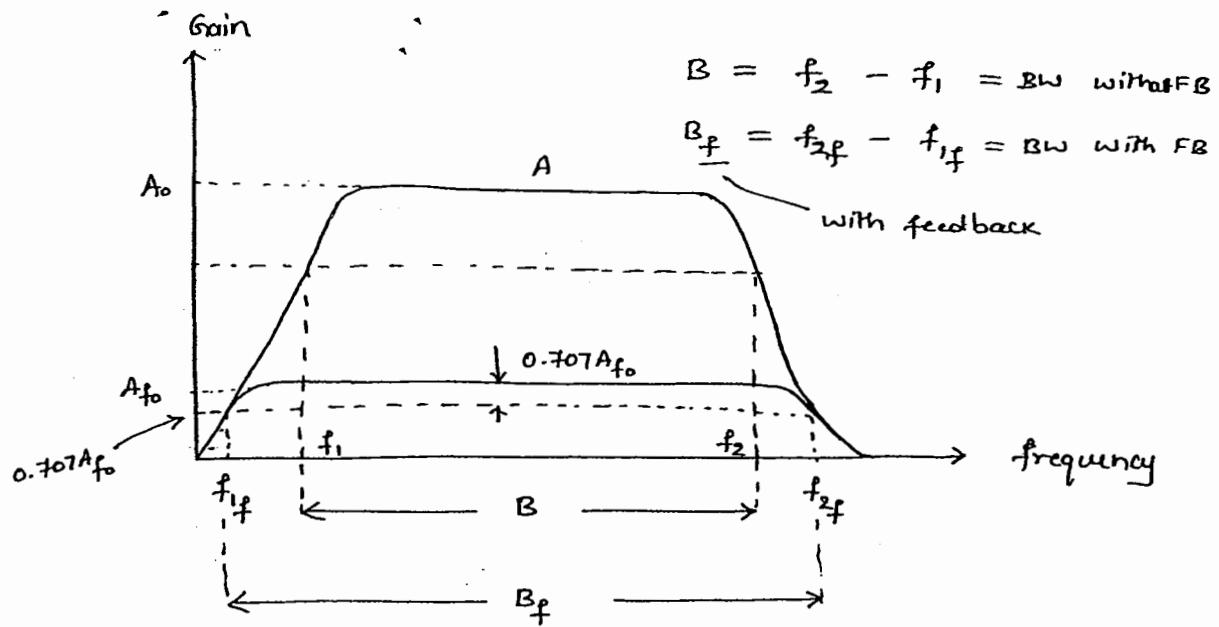


Fig. (a) Effect of Negative Feedback on Gain and Bandwidth

The bandwidth of an amplifier is given by

$$B = B_{BW} = \text{upper cutoff frequency} - \text{lower cutoff frequency}$$

It can be shown that : $f_{1f} = \frac{f_1}{(1 + \beta A)} = \text{lower cutoff freq. with feedback}$

$$f_{2f} = f_2 (1 + \beta A) = \text{upper cutoff freq. with feedback}$$

Bandwidth without feedback : $B = f_2 - f_1$

Bandwidth with feedback : $B_f = f_{2f} - f_{1f}$

$$B_f = f_2 (1 + \beta A) - \frac{f_1}{(1 + \beta A)}$$

$\Rightarrow B_f > B$, Bandwidth with feedback is larger than the bandwidth without feedback.

Gain Stability with Feedback

The transfer gain or simply gain of an amplifier is not constant as it depends on the factors such as operating point, temperature, etc. This lack of stability in amplifiers can be

reduced by introducing negative feedback.

$$\text{We know that : } A_f = \frac{A}{1 + BA}$$

Differentiating both sides w.r.t. A we get,

$$\frac{dA_f}{dA} = \frac{(1 + BA) \cdot 1 - BA}{(1 + BA)^2} = \frac{1}{(1 + BA)^2}$$

$$dA_f = \frac{dA}{(1 + BA)^2}$$

Dividing both sides by A_f we get,

$$\frac{dA_f}{A_f} = \frac{dA}{(1 + BA)^2} \times \frac{1}{A_f} = \frac{A}{1 + BA}$$

$$\frac{dA_f}{A_f} = \frac{dA}{(1 + BA)^2} \times \frac{(1 + BA)}{A}$$

$$\boxed{\left| \frac{dA_f}{A_f} \right| = \frac{1}{|1 + BA|} \left| \frac{dA}{A} \right|} \quad \text{--- (1)}$$

where $\left| \frac{dA_f}{A_f} \right|$ = Fractional change in amplification with feedback

$\left| \frac{dA}{A} \right|$ = Fractional change in amplification without feedback

→ From eqn. (1) it is clear that change in the gain with feedback is less than the change in gain without feedback by factor $(1 + BA)$

→ The Eqn. (1) may be approximated as follows:

For $BA \gg 1$

$$\left| \frac{dA_f}{A_f} \right| = \left| \frac{1}{BA} \right| \left| \frac{dA}{A} \right|$$

This shows that magnitude of the relative change in gain with feedback $\left| \frac{dA_f}{A_f} \right|$ is reduced by the factor $|BA|$ compared to that with out feedback,

$$\left| \frac{dA}{A} \right|$$

Sensitivity of transfer gain = $\frac{\text{Fractional change in amplification with } F/B}{\text{Fractional change in amplification w/o F/B}}$

$$\text{Therefore, Sensitivity} = \frac{\left| \frac{dA_f}{A_f} \right|}{\left| \frac{dA}{A} \right|} = \frac{1}{|1 + \beta A|} \approx \frac{1}{|\beta A|}$$

The reciprocal of sensitivity is called the denensitivity, D

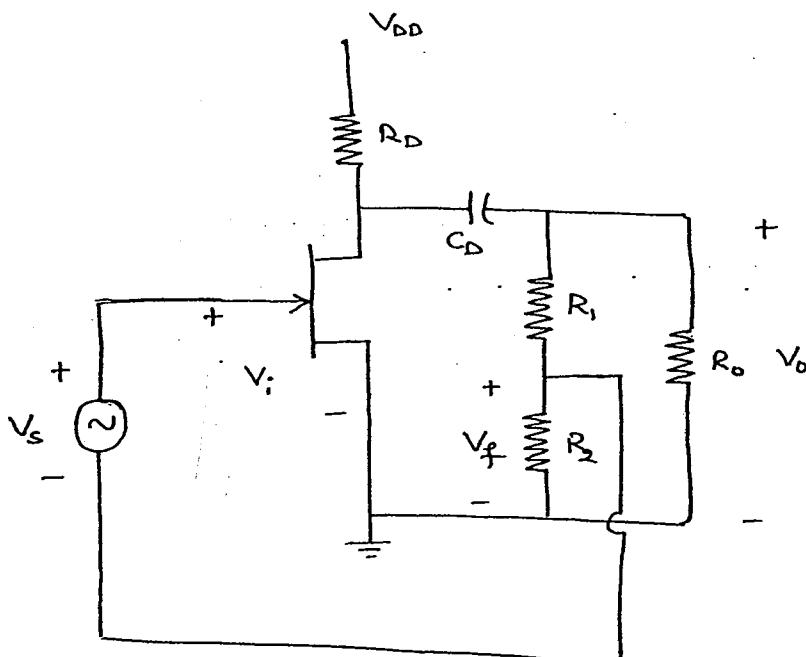
$$\text{Therefore, } D = (1 + \beta A)$$

The stability of the amplifier increases with increase in D

9 Practical Feedback circuits

Practical feedback circuits will provide a means of demonstrating the effect of feedback has on various connection types.

Voltage - series Feedback



- A part of V_o is obtained using R_1 & R_2
- R_1 and R_2 forms feedback network
- V_f is in series with V_i

(a) Voltage series feedback of an FET Amplifier Stage

- An FET amplifier stage with voltage series feedback is shown in Fig. (a)
- From the Fig. (a), it is clear that, a part of the output voltage (V_o) is obtained using a feedback network of

resistors R_1 and R_2 .

- The feedback voltage is connected in series with the signal source, V_s their difference being the input signal, V_i .
- Without feedback the amplifier gain is

$$A = \frac{V_o}{V_i} = -g_m R_L \quad \text{--- (1)}$$

where $R_L = R_D \parallel R_o \parallel (R_1 + R_2)$

- The feedback voltage, V_f is given by

$$V_f = -V_o \frac{R_2}{R_1 + R_2}$$

$$\frac{V_f}{V_o} = \frac{-R_2}{R_1 + R_2} = \beta$$

Therefore, $\beta = \frac{V_f}{V_o} = \frac{-R_2}{R_1 + R_2}$

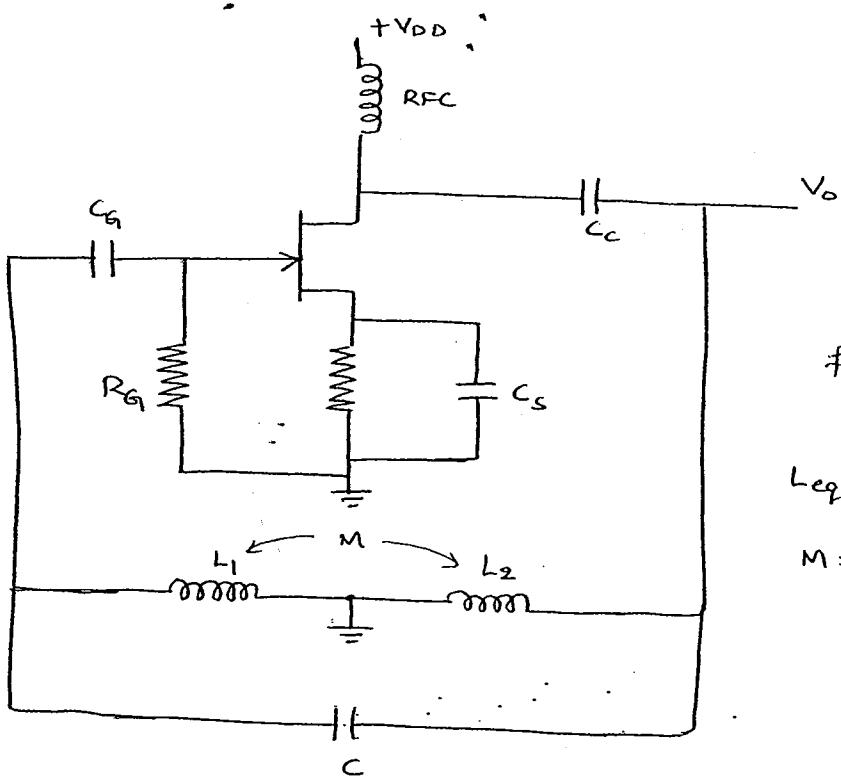
- The expression for gain is given by

$$A_f = \frac{A}{1 + A\beta} = \frac{-g_m R_L}{1 + (-g_m R_L) \left(\frac{-R_2}{R_1 + R_2} \right)}$$

$$A_f = \frac{-g_m R_L}{1 + \left(\frac{R_2 R_L}{R_1 + R_2} \right) g_m}$$

For $BA \gg 1$, we have $A_f \approx \frac{1}{\beta} = - \left(\frac{R_1 + R_2}{R_2} \right)$

FET Hartley oscillator



$$f_0 = \frac{1}{2\pi\sqrt{L_{eq}C}}$$

$$L_{eq} = L_1 + L_2 + 2M$$

M = Mutual coupling

(a) FET Hartley oscillator

→ The active device FET is used in the amplifier stage
 ⇒ FET hartley oscillator

$$x_1 = j\omega L_1, \quad x_2 = j\omega L_2, \quad \text{and} \quad x_3 = \frac{1}{j\omega C}$$

$$\text{W.K.T: } x_1 + x_2 + x_3 = 0$$

Solving for ω

~~$$\frac{1}{2\pi\sqrt{L_{eq}C}}$$~~

$$f = \frac{1}{2\pi\sqrt{C L_{eq}}}$$

$$L_{eq} = L_1 + L_2 + 2M$$

→ The RFC is the radio freq. choke

for $f = \text{high}$, $x_{RFC} = \infty$, open circuit

for $f = \text{low}$, $x_{RFC} = 0$, short circuit
 (DC)

⇒ Achieved isolation b/w

→ FET amplifier : 180° phase shift

LC feedback network : 180° phase shift

⇒ necessary conditions for oscillation.

- (P) In a transistorized Hartley oscillator the two inductances are 2 mH and 20 μH, while the frequency is to be changed from 950 kHz to 2050 kHz. calculate the range over which the capacitor is to be varied.

Solution

The freq. is given by

$$f = \frac{1}{2\pi \sqrt{C L_{eq}}}$$

$$L_{eq} = L_1 + L_2 = 2 \times 10^{-3} + 20 \times 10^{-6} = 0.00202$$

$$f = f_{max} = 2050 \text{ kHz}$$

$$2050 \times 10^3 = \frac{1}{2\pi \sqrt{C \times 0.00202}} \Rightarrow C = 2.98 \text{ pF}$$

$$f = f_{min} = 950 \text{ kHz}$$

$$950 \times 10^3 = \frac{1}{2\pi \sqrt{C \times 0.00202}} \Rightarrow C = 13.89 \text{ pF}$$

⇒ C must be varied from 2.98 pF to 13.89 pF to get the required freq. variation.

- (P) In a hartley oscillator, $L_1 = 15 \text{ mH}$ and $C = 50 \text{ pF}$. calculate L_2 for a frequency of 168 kHz. The mutual inductance between L_1 and L_2 is 5 μH. Also find the required gain of the transistor to be used for the oscillations.

Solution For Hartley oscillator $f = \frac{1}{2\pi \sqrt{C(L_1 + M + L_2)}}$

$$L_{eq} = L_1 + L_2 + 2M$$

$$168 \times 10^{-3} = \frac{1}{2\pi \sqrt{L_{eq} \times 50 \times 10^{-12}}}$$

$$\Rightarrow L_{eq} = 17.95 \text{ mH}$$

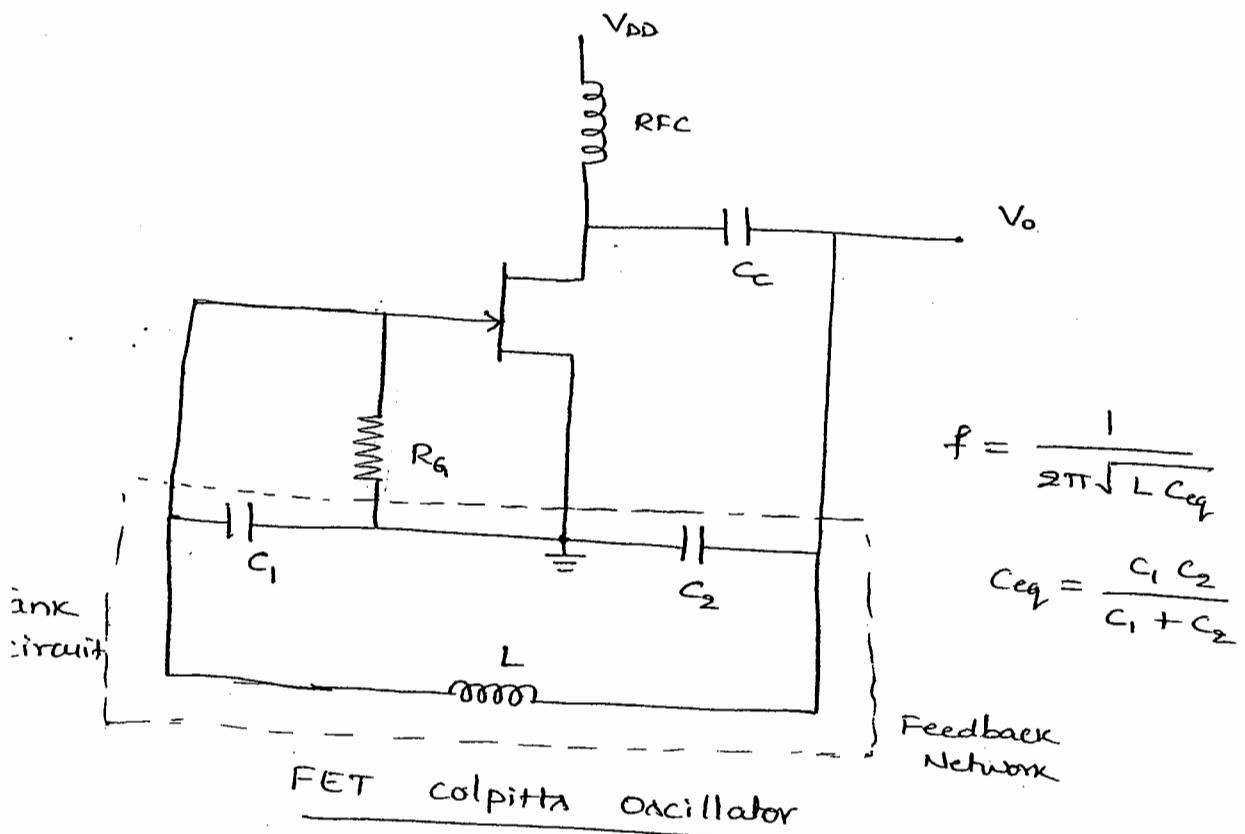
$$17.95 \times 10^{-3} = L_2 + 15 \times 10^{-3} + 5 \times 10^{-6}$$

$$\Rightarrow L_2 = 2.945 \text{ mH}$$

Now gain = $h_{fe} = \frac{L_1 + M}{L_2 + M} = \frac{15 \times 10^{-3} + 5 \times 10^{-6}}{2.945 \times 10^{-3} + 5 \times 10^{-6}}$

$$h_{fe} = 5.08$$

Colpitts Oscillator



→ The basic circuit is similar to that of Hartley oscillator, except the tank circuit

→ The amplifier stage : 180° phase shift

(14)

→ The tank circuit : 180° phase shift

⇒ necessary condition for oscillation

$$\text{Therefore, } f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

$$\text{where } C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$h_{fe} = \frac{C_2}{C_1} \Rightarrow \text{gain of amplifier stage}$$

required for oscillation

- (P) In a Colpitt's oscillator, $C_1 = 1 \text{ nF}$ and $C_2 = 1000 \text{ nF}$. Find the value of L for the freq. of 100 kHz.

$$\text{Soln: } C_1 = 1 \text{ nF}$$

$$C_2 = 1000 \text{ nF}$$

$$f = 100 \text{ kHz}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{1 \times 10^{-9} \times 1000 \times 10^{-9}}{1001 \times 10^{-9}} = 9.99 \times 10^{-11}$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

$$100 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 9.99 \times 10^{-11}}}$$

$$\Rightarrow L = 2.5355 \text{ mH}$$

- (P) In a Colpitt's oscillator, $C_1 = C_2 = C$ and $L = 100 \mu\text{H}$. The freq. of oscillation is 500 kHz. Determine the value of C .

$$\text{Soln: } f = \frac{1}{2\pi\sqrt{LC_{eq}}} \Rightarrow C_{eq} = 1.0132 \times 10^{-9} \text{ F}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \quad \text{since} \quad C_1 = C_2 = C$$

$$C_{eq} = \frac{C \cdot C}{C + C} = \frac{C}{2}$$

$$\Rightarrow C = 2.026 \text{ nF}$$

$$C = \frac{1}{6.28 \times 10 \times 10^3 \times 1 \times 10^3 \times 2.45} = 6.5 \text{ nF}$$

$$|A| = g_m R_L$$

$$R_L = \frac{|A|}{g_m} = \frac{40}{5000 \times 10^{-6}} = 8 \text{ k}\Omega$$

$$R_L = \frac{R_D r_d}{R_D + r_d} \Rightarrow 8 \text{ k}\Omega = \frac{R_D \times r_d}{R_D + r_d}$$

$$8 \text{ k}\Omega = \frac{40 \text{ k} \times R_D}{R_D + 40 \text{ k}\Omega} \Rightarrow R_D = 10 \text{ k}\Omega$$

Advantages

1. circuit is simple to design.
2. can produce output over audio-freq. range
3. produces sinusoidal output waveform.
4. Fixed freq. oscillator

Disadvantages

1. To vary, f the values of R and C must be varied of all three sections simultaneously
 - ⇒ practically difficult
 - ⇒ frequency cannot be varied.
2. Frequency stability is poor since various components changes with temperature.

Module # 4 Feedback and Oscillator Circuits

(1)

Refer chapter #14 from the text titled "Electronic Devices and Circuit Theory" authored by Robert L. Boylestad and Louis Nashelsky.

Topics: - Feedback concepts

- ① Driving a vehicle: Feedback s/m
- ② Rainfall cycle

- Feedback connection types
- Practical feedback circuits
- Oscillator operation
- FET phase shift oscillator
- Wein bridge oscillator
- Tuned oscillator circuit
- Crystal oscillator
- UJT construction
- UJT oscillator

§ Feedback concepts

- Depending on the relative polarity of the signal being fed back into a circuit, one may have negative or positive feedback.
- Negative feedback results in decreased voltage gain, for which a number of circuit features are improved.
- Positive feedback drives a circuit into oscillation as in various types of oscillator circuits.
- A typical feedback connection is shown in Fig. (a)

V_s : Input signal applied to a mixer

V_f : Feedback signal

V_i : Input voltage to the amplifier

V_o : Amplifier output

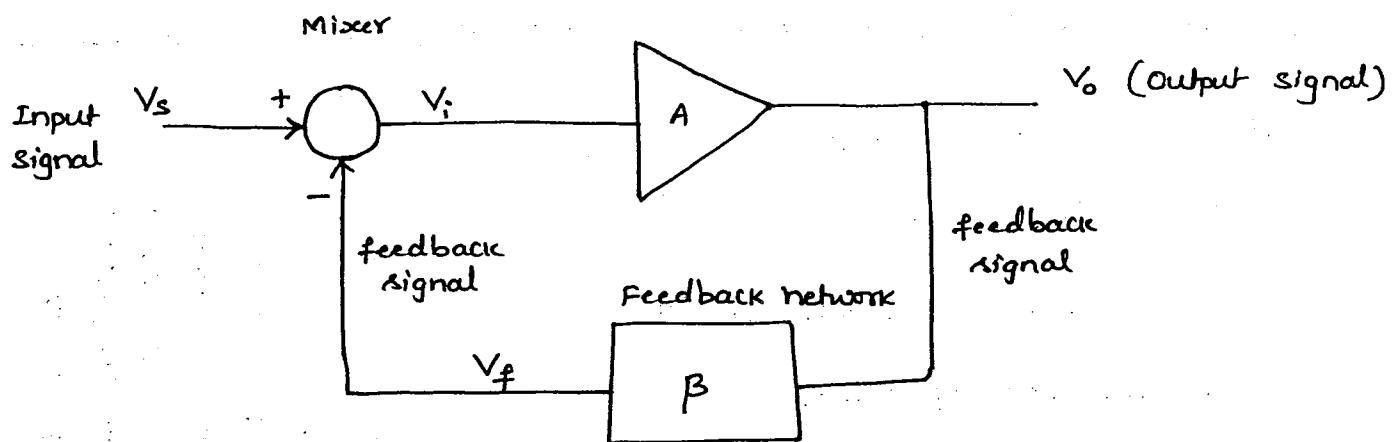
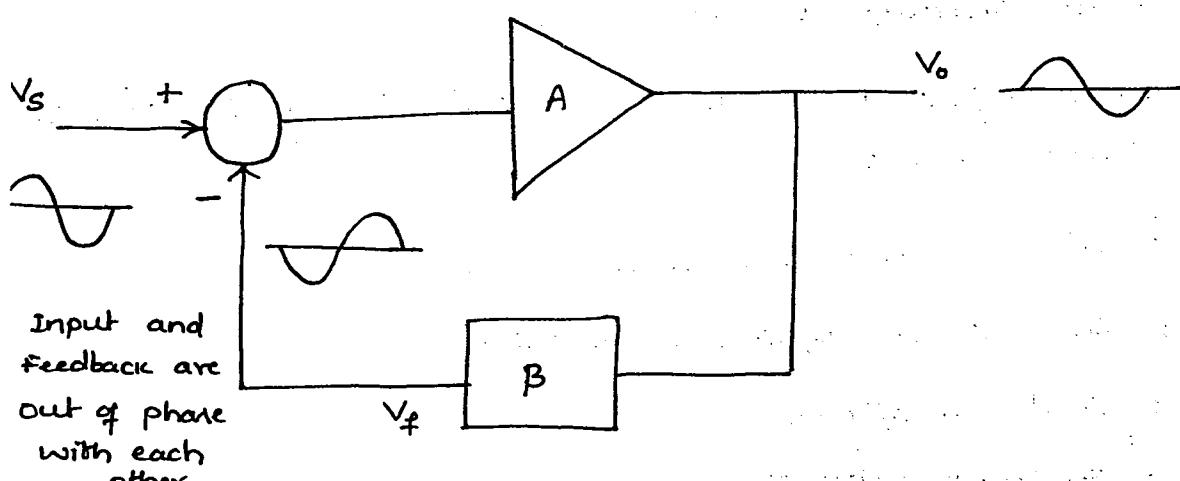


Fig. (a) Simple Block Diagram of Feedback Amplifier

→ The input signal, V_s is applied to a mixer network where it is combined with a feedback signal, V_f . The difference of these signals, $V_i = V_s - V_f$ is then fed as input to the amplifier. A portion of the amplifier output, V_o is connected to the feedback network, B , which provides a reduced portion of the output as feedback signal to the input mixer network.

→ Negative feedback : If the feedback signal is of opposite polarity ~~the~~ to the input signal as shown in Fig. (b), negative feedback results.



→ Although negative feedback results in reduced overall voltage gain, a number of improvements are obtained that are listed below:

1. Higher input impedance
2. Better stabilized voltage gain
3. Improved frequency response
4. Lower output impedance.
5. Reduced noise
6. More linear operation

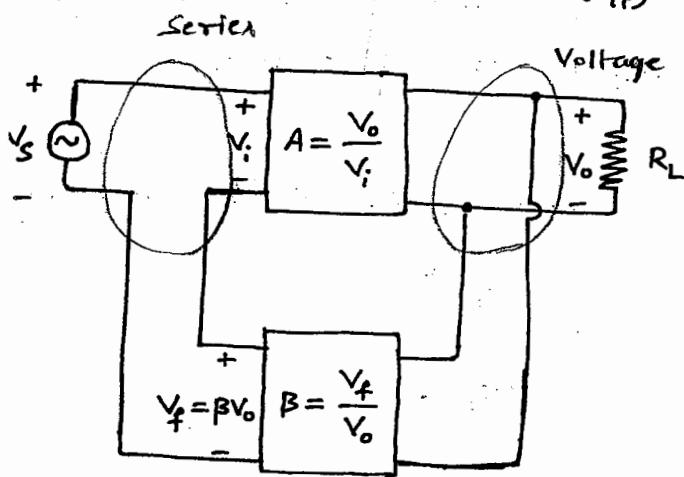
5 Feedback connection Types

There are four basic ways of connecting the feedback signal.

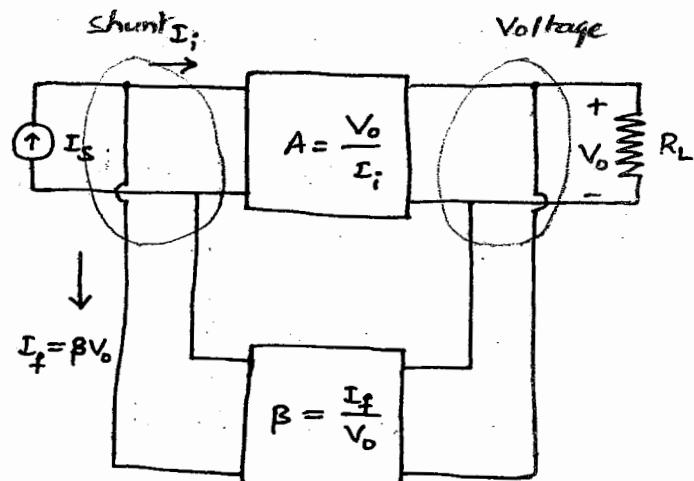
Both voltage and current can be fed back to the input either in series or parallel.

I = Input, O = Output

1. Voltage - Series feedback : V-series (I-O)
2. Voltage - Shunt feedback : V-shunt (I-O)
3. Current - Series feedback : I-series (I-O)
4. Current - Shunt feedback : I-shunt (I-O)



(a) Voltage-Series Feedback



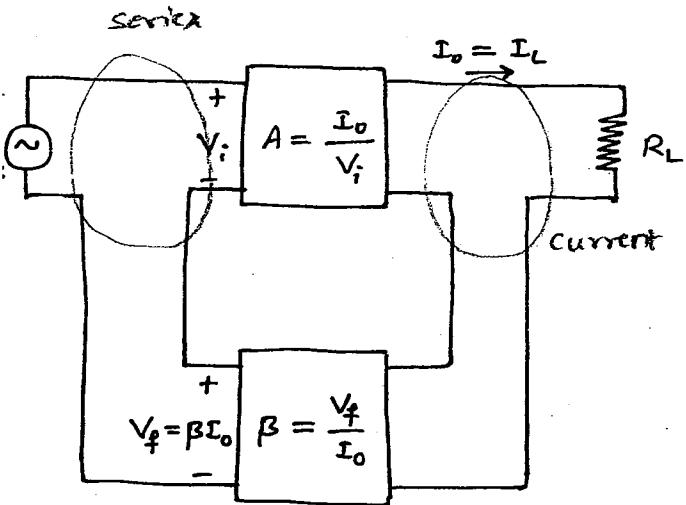
(b) Voltage-Shunt Feedback

Gain with
Feedback

$$A_f = \frac{V_o}{V_s}$$

Gain with
Feedback

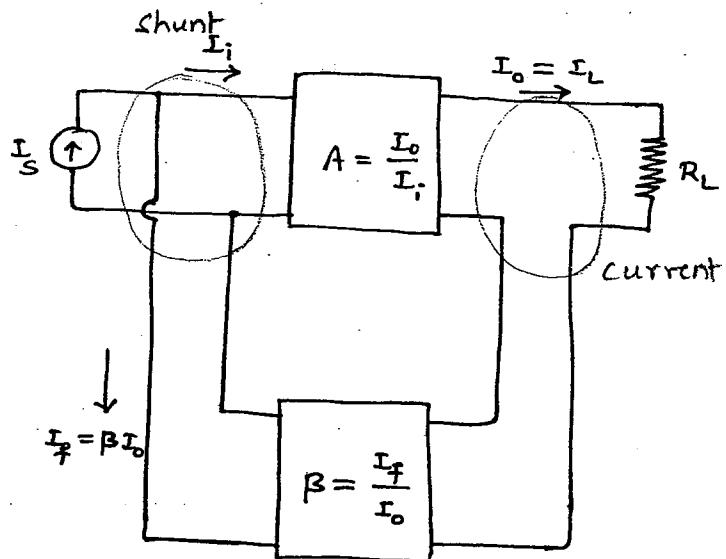
$$A_f = \frac{V_o}{I_s}$$



(c) current-series Feedback

Gain with
Feedback

$$A_f = \frac{I_o}{V_s}$$



(d) current-shunt Feedback

Gain with
Feedback

$$A_f = \frac{I_o}{I_s}$$

→ In the list above,

#1: Voltage refers to connecting output voltage as input to the feedback network.

#2: current refers to tapping off some output current through the feedback network.

#3: Series refers to connecting the feedback signal in series with the input signal voltage.

#4: Shunt refers to connecting the feedback signal in ~~series~~ shunt (parallel) with an input current source.

Type of Feedback

1. Series Feedback
2. Shunt Feedback
3. Voltage Feedback
4. Current Feedback

Effect on I/f or O/f Impedance

$$Z_{if} \uparrow$$

$$Z_{if} \downarrow$$

$$Z_{of} \downarrow$$

$$Z_{of} \uparrow$$

Typically,

Desired for cascaded amplifier : $Z_{if} \uparrow$ and $Z_{of} \downarrow$

Parameters

V-Series

V-shunt

I-series

I-shunt

Gain without feedback: $\frac{V_o}{V_i}$

(A)

$$\frac{V_o}{I_i}$$

$$\frac{I_o}{V_i}$$

$$\frac{I_o}{I_i}$$

Feedback (β) : $\frac{V_f}{V_o}$

$$\frac{I_f}{V_o}$$

$$\frac{V_f}{I_o}$$

$$\frac{I_f}{I_o}$$

Gain with feedback: $\frac{V_o}{V_s}$

(A_f)

$$\frac{V_o}{I_s}$$

$$\frac{I_o}{V_s}$$

$$\frac{I_o}{I_s}$$

Gain with Feedback

The gain of each of the feedback circuit connection is derived as follows :

Let A = Gain without feedback of the amplifier stage

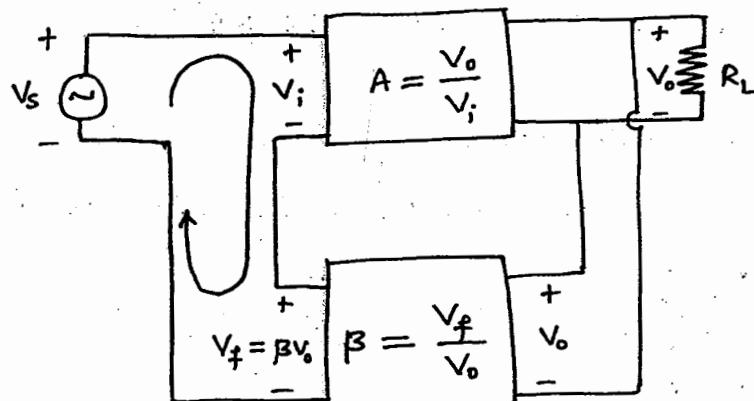
β = Gain of the feedback network

A_f = Gain of the amplifier stage with feedback

1. Voltage-series Feedback : The voltage-series feedback connection

shown in Fig. (a) with a part of the output voltage fed back in series with the input signal. This results in the reduction of overall gain.

If there is no feedback ($V_f = 0$), the voltage gain of the amplifier stage is



(a) Voltage-series Feedback connection

$$A = \frac{V_o}{V_s} = \frac{V_o}{V_i} \quad \text{--- (1)}$$

If a feedback signal, V_f , is connected in series with

Applying KVL for the input loop shown

$$-V_s + V_i + V_f = 0$$

or $V_i = V_s - V_f \quad \text{--- (2)}$

From (1) $V_o = A V_i$

Eqn. (2)

$$V_o = A (V_s - V_f)$$

$$V_o = A V_s - A V_f = \beta V_o$$

$$V_o = A V_s - A (\beta V_o)$$

$$V_o + A \beta V_o = A V_s$$

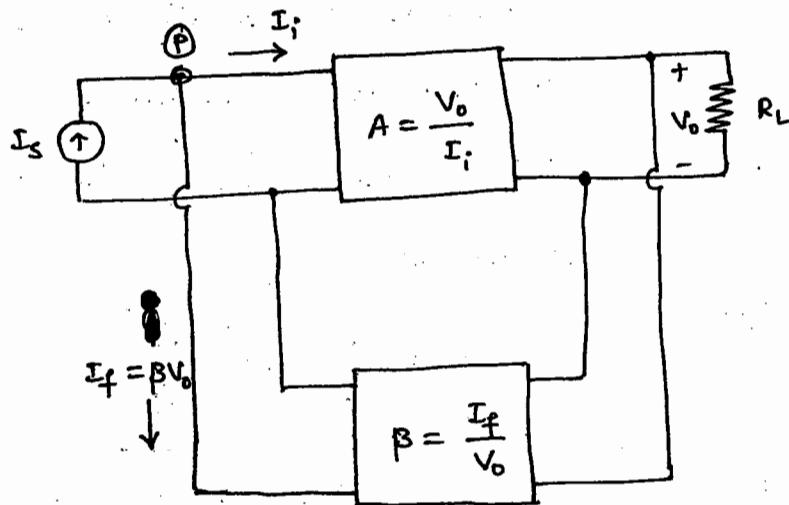
$$V_o [1 + A \beta] = A V_s$$

Therefore, the overall gain with feedback is

$$A_f = \frac{V_o}{V_s} = \frac{A}{1 + \beta A} \quad \text{--- (3)}$$

From Eqn. (3), it is evident that the gain with feedback is the amplifier gain reduced by the factor, $(1 + \beta A)$

5 Voltage-Shunt Feedback : The gain with feedback for the network shown in Fig. (b) is



$$A_f = \frac{V_o}{I_s} \quad \text{--- (1)}$$

Applying KCL at node P

$$I_s = I_i + I_f \quad \text{--- (2)}$$

$$A = \frac{V_o}{I_i} \Rightarrow V_o = A I_i \quad \text{--- (3)}$$

Substitute (2) and (3) in (1)

$$A_f = \frac{A I_i}{I_i + I_f} = \frac{A I_i}{I_i + \beta V_o}$$

$$A_f = \frac{A I_i}{I_i + \beta V_o} = \frac{A I_i}{A I_i + \beta V_o}$$

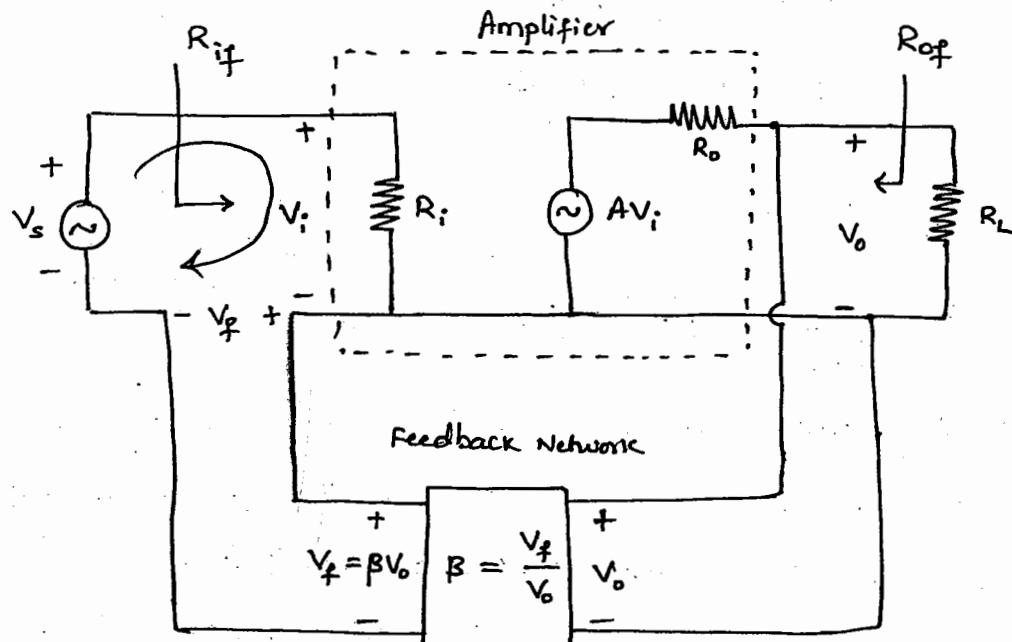
$$A_f = \frac{A I_i}{I_i + \beta A I_i} = \frac{A I_i}{I_i (1 + \beta A)}$$

$$A_f = \frac{A}{1 + \beta A}$$

(4)

Eqn. (4) shows that the gain with feedback in the amplifier gain reduced by the factor, $(1 + \beta A)$

Impedance with Feedback : Input impedance



$$A = \frac{V_o}{V_i} ; A_f = \frac{V_o}{V_s}$$

$$\beta = \frac{V_f}{V_o} \quad A_f = \frac{A}{1 + \beta A}$$

(a) Voltage-Series Feedback connection

→ A more detailed voltage-series feedback connection is shown in Fig. 1a

→ The input impedance can be determined as follows:

$$\text{Input current, } I_i = \frac{V_i}{Z_i} = \frac{\text{Input Voltage to amplifier}}{\text{Amplifier Input impedance}}$$

Applying KVL for the input loop

$$V_i = V_s - V_f \quad \text{--- (2)}$$

Substituting (2) in (1), we get

$$I_i = \frac{V_s - V_f}{Z_i}$$

$$\text{But } V_f = \beta V_o$$

$$\text{Therefore, } I_i = \frac{V_s - \beta V_o}{Z_i}, \quad \text{but } A = \frac{V_o}{V_i} \Rightarrow V_o = AV_i$$

$$I_i = \frac{V_s - \beta A V_i}{Z_i}$$

$$I_i Z_i = V_s - \beta A V_i$$

$$V_s = I_i Z_i + \beta A V_i$$

$$V_s = I_i Z_i + \beta A I_i Z_i$$

$$V_s = I_i [Z_i + (\beta A) Z_i]$$

$$\frac{V_s}{I_i} = Z_{if} = Z_i (1 + \beta A)$$

Input Impedance
with feedback

$$Z_{if} = Z_i (1 + \beta A)$$

Input Impedance
without feedback

--- (3)

From eqn. (3), it is clear that the input impedance with voltage series feedback is seen to be the value of the ~~feedback~~ input impedance without feedback multiplied by the factor, $(1 + \beta A)$. The eqn. (3) applies to both voltage-series and current-series feedback configurations.

9 Voltage shunt Feedback : Input Impedance

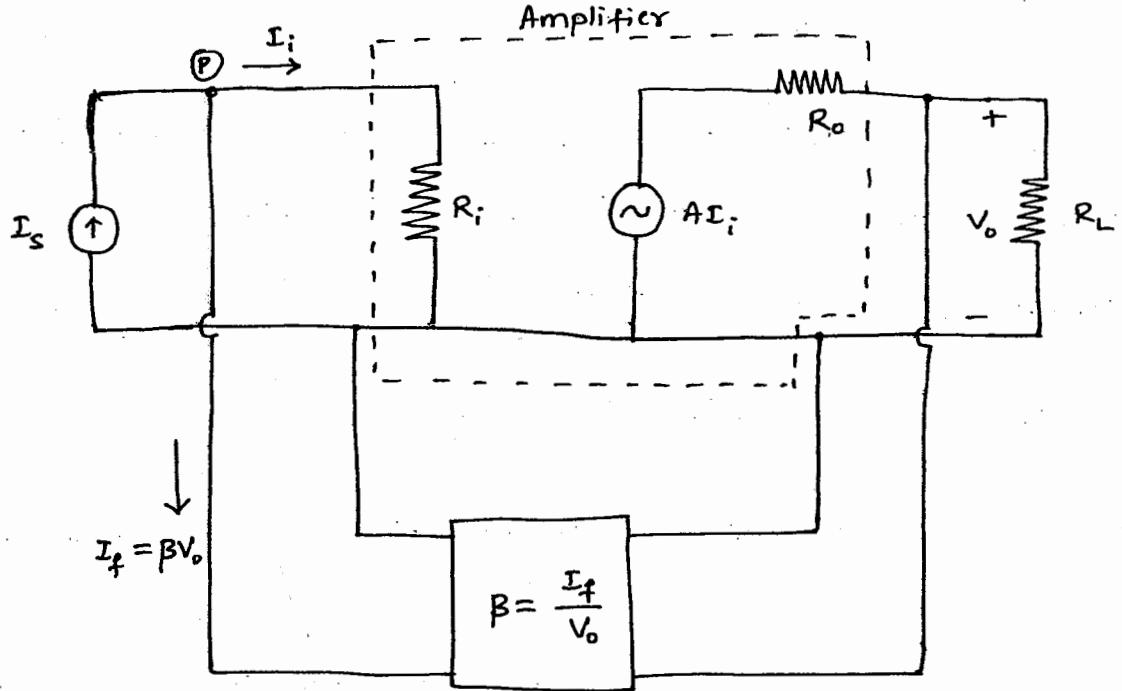


Fig. (b) Voltage-shunt feedback connection

→ A more detailed voltage-shunt feedback connection is shown in Fig. (b)

→ The input impedance can be determined as follows :

From definition, Input impedance with feedback = ~~Z_{if}~~ Z_{if}

$$Z_{if} = \frac{V_i}{I_s} \quad \text{--- (1)}$$

Applying KCL to the node, P

$$I_s = I_i + I_f \quad \text{--- (2)}$$

Substituting (2) in (1), we get

$$Z_{if} = \frac{V_i}{I_i + I_f} = \frac{V_i}{\beta V_o}$$

$$Z_{if} = \frac{V_i}{I_i + \beta V_o}$$

Dividing ~~multiplying~~ Numerator and Denominator by I_i

$$Z_{if} = \frac{\frac{V_i}{I_i}}{\frac{I_i + \beta V_o}{I_i}} = \frac{\frac{V_i}{I_i}}{1 + \frac{\beta V_o}{I_i}} = \frac{Z_i}{1 + \beta A}$$

$$Z_{if} = \frac{Z_i}{1 + BA} \quad \text{--- (3)}$$

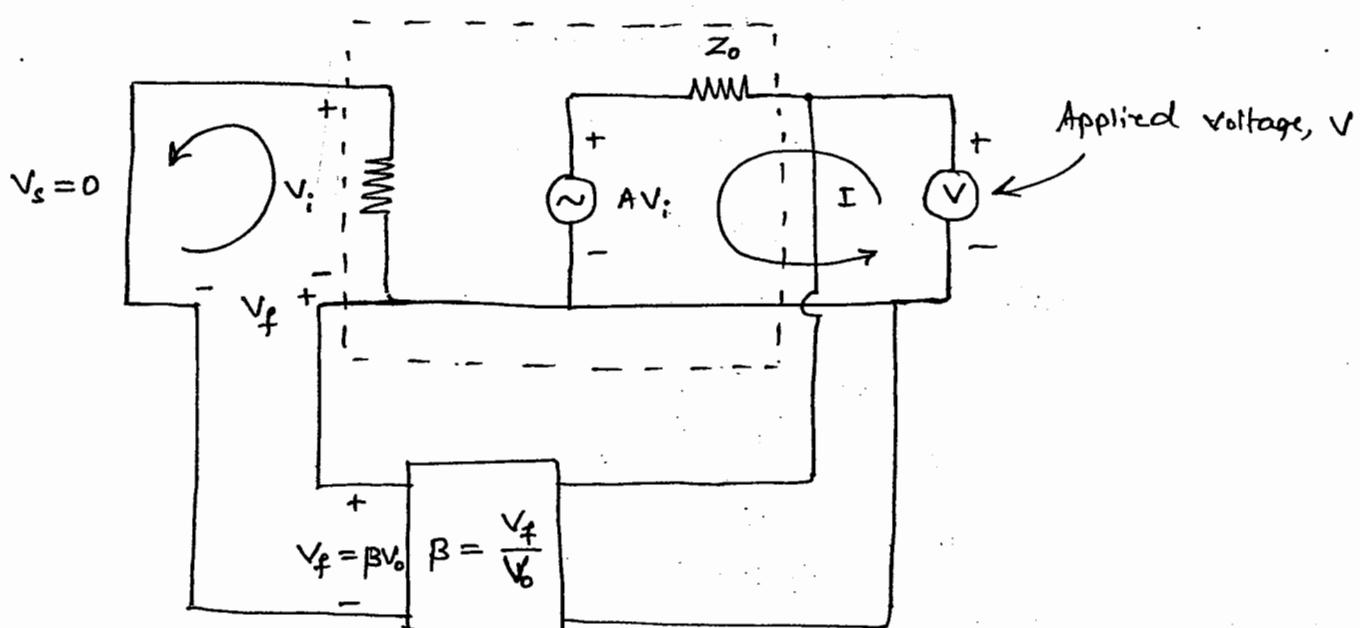
The input impedance with feedback is reduced by the factor, $(1 + BA)$. The equation (3) applies to both voltage-shunt and current-shunt feedback connection.

9 Output Impedance with Feedback

- The output impedance depends on whether voltage or current feedback is used.
- For a voltage feedback, the output impedance is decreased and for the current feedback, the output impedance is increased.

9 Voltage-Series Feedback

- The Fig. (a) shows the detailed connection of voltage-series feedback
- The output impedance is determined by applying a voltage, V , resulting in a current, I with V_s shorted out (i.e. $V_s = 0$)



(a) Voltage-series Feedback with $V_s = 0$

and applying a voltage, V in the output

Applying KVL for the loop shown

$$-V + IZ_0 + AV_i = 0$$

$$V = IZ_0 + AV_i \quad \text{--- (1)}$$

Applying KVL to the input loop results in

$$-V_f - V_i = 0, \quad \text{since } V_s = 0$$

$$\text{or } V_i = -V_f \quad \text{--- (2)}$$

Substituting (2) in (1) gives

$$V = IZ_0 + A(-V_f)$$

$$V = IZ_0 - AV_f \quad \text{--- (3)}$$

$$V = IZ_0 - A(\beta V)$$

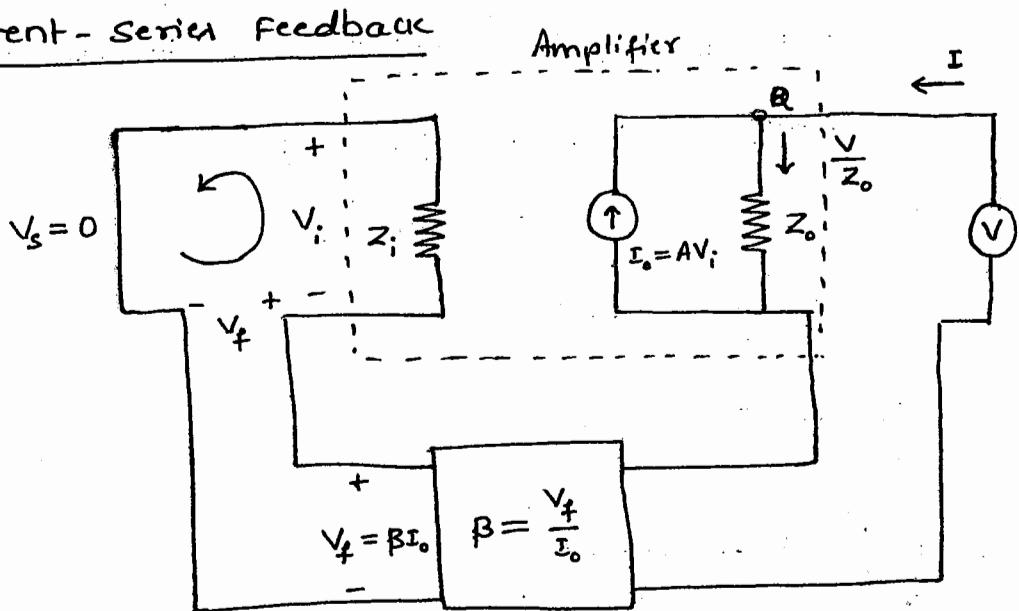
$$\text{Rewriting eqn. } V + \beta AV = IZ_0$$

$$V(1 + \beta A) = IZ_0$$

$$\boxed{\frac{V}{I} = Z_{of} = \frac{Z_0}{1 + \beta A}} \quad \text{--- (3)}$$

The output impedance of a voltage-series feedback is reduced from that without feedback by the factor $(1 + \beta A)$

Current-Series Feedback



- The output impedance for the current-series feedback can be determined by applying a signal v to the output with the input source, v_s shorted out (i.e. $v_s = 0$).
- The applied voltage, v results in a current, I . The output impedance is obtained by disconnecting the load resistance R_L and looking into the output terminals. The ratio of v to I gives the output impedance.
- Fig. (b) shows the more detailed connection of current-series feedback. The output impedance is determined as follows:

Apply KCL at the output node, Q

$$I + I_o = \frac{v}{Z_o}$$

||

$$AV_i$$

$$I = \frac{v}{Z_o} - AV_i \quad \text{--- (1)}$$

The input voltage is determined as follows:

Applying KVL to the input loop

$$-V_f - V_i + v_s = 0$$

||

V_s

$$V_i = -V_f$$

$$\text{but } V_f = \beta I_o = -\beta I \quad (\text{Because } I_o = -I)$$

$$\text{Therefore, } V_i = -(-\beta I) = \beta I \quad \text{--- (2)}$$

Substituting the value of V_i from (2) in eqn. (1)

$$I = \frac{v}{Z_o} - A\beta I$$

$$\frac{v}{Z_o} = I(1 + \beta A)$$

$$Z_{of} = \boxed{\frac{v}{I} = Z_o(1 + \beta A)}$$

→ A summary of the effect of feedback on input and output impedance is provided in the table below:

Table : Effect of Feedback connection on I/p and O/p Impedance

| | Voltage - series | current - series | voltage - shunt | current - shunt |
|------------|---|---|---|---|
| Z_{if} : | $Z_i (1 + \beta A)$ (Increased) ↑ | $Z_i (1 + \beta A)$ (Increased) ↑ | $\frac{Z_i}{(1 + \beta A)}$ (Decreased) ↓ | $\frac{Z_i}{(1 + \beta A)}$ (Decreased) ↓ |
| Z_{of} : | $\frac{Z_o}{(1 + \beta A)}$ (Decreased) ↓ | $Z_o (1 + \beta A)$ (Increased) ↑ | $\frac{Z_o}{(1 + \beta A)}$ (Decreased) ↓ | $Z_o (1 + \beta A)$ (Increased) ↑ |

- (P) Determine the voltage gain, input, and output impedance with feedback for Voltage-series feedback having $A = -100$, $R_i = 10\text{k}\Omega$, and $R_o = 20\text{k}\Omega$ for feedback of (a) $\beta = -0.1$ and
(b) $\beta = -0.5$

Solution: (a) The mathematical expression for A_f for voltage series feedback is given by

$$A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.1)(-100)} = \frac{-100}{11} = -9.09$$

$$\boxed{A_f = -9.09}$$

$$Z_{if} = Z_i (1 + \beta A) = 10\text{k}\Omega (11) = 110\text{k}\Omega$$

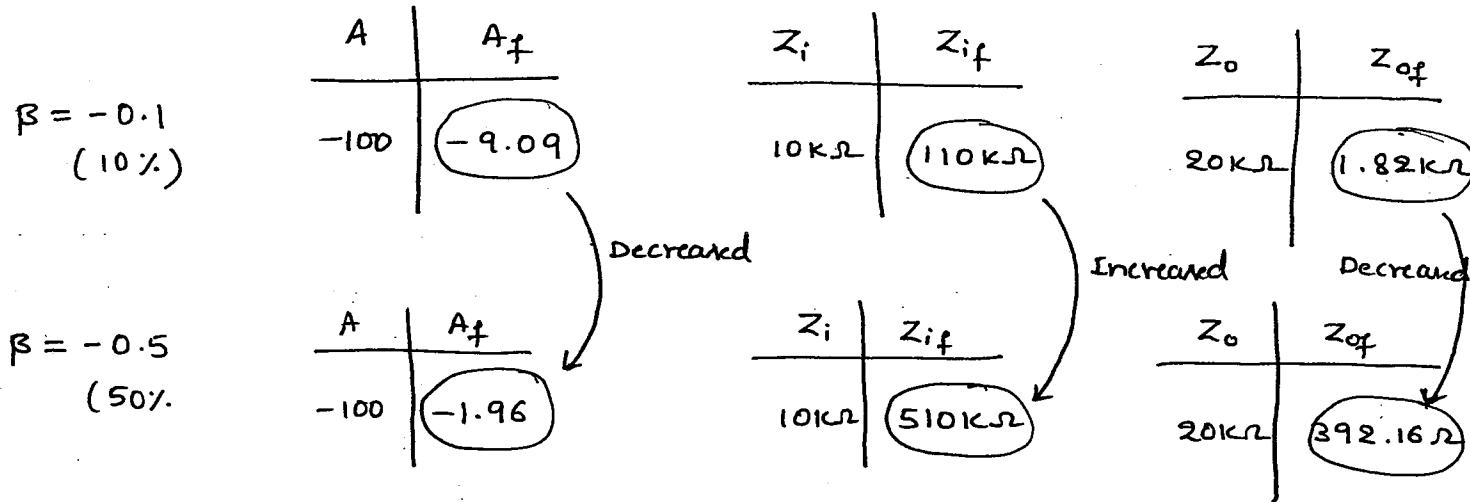
$$Z_{of} = \frac{Z_o}{1 + \beta A} = \frac{20\text{k}\Omega}{11} = 1.82\text{k}\Omega$$

$$(b) A_f = \frac{A}{1 + \beta A} = \frac{-100}{1 + (-0.5)(-100)} = \frac{-100}{51} = -1.96$$

$$Z_{if} = Z_i (1 + \beta A) = 10\text{k}\Omega (51) = 510\text{k}\Omega$$

$$\Rightarrow Z_o = \frac{20\text{k}\Omega}{51} = 392.16\Omega$$

→ The example demonstrates the trade-off of gain for improved input and output impedance



- Reduced gain by a factor of 11 (100 to 9.09) is complemented by a reduced output resistance and increased input resistance by the same factor of 11.
- Reducing the gain by a factor of 51 provides a gain of only 2 but with input resistance increased by the factor of 51 (to over 500k Ω) and output resistance reduced from 20k Ω to under 400 Ω .
- Feedback offers the designer the choice of trading away some of the available amplifier gain for other improved circuit features.

§ Reduction in Frequency Distortion

→ for a negative feedback amplifier $BA \gg 1$

$$\text{The gain with feedback, } A_f = \frac{A}{1+BA} \Rightarrow A_f \approx \frac{1}{B}$$

- From the above approximation, it follows that if the feedback network is purely resistive, the gain with feedback is not dependent on frequency, even though the basic amplifier gain is a frequency dependent.
- In a negative voltage feedback amplifier, the freq. distortion arising because of varying amplifier gain with freq. is considerably reduced.

§ Reduced in noise and non-linear distortion

The factor $(1+BA)$ reduces both input noise and resulting non-linear distortion for considerable improvement. Thus, noise and non-linear distortion also reduced by the same factor as the gain.

§ Effect of negative feedback on Gain and Bandwidth

→ overall gain with negative feedback is

$$A_f = \frac{A}{1+BA} \approx \frac{A}{BA} = \frac{1}{\beta} \quad \text{for } BA \gg 1$$

as long as $BA \gg 1$, the overall gain is approximately $\frac{1}{\beta}$

- For a practical amplifier, the open-loop gain (A) drops off at high freq. due to the active device and circuit capacitances
- The gain also drop off at low frequencies for capacitively coupled amplifier stages.
 - Once the open-loop gain, A drops low enough and the factor BA is no longer much larger than 1. Therefore

Bandwidth

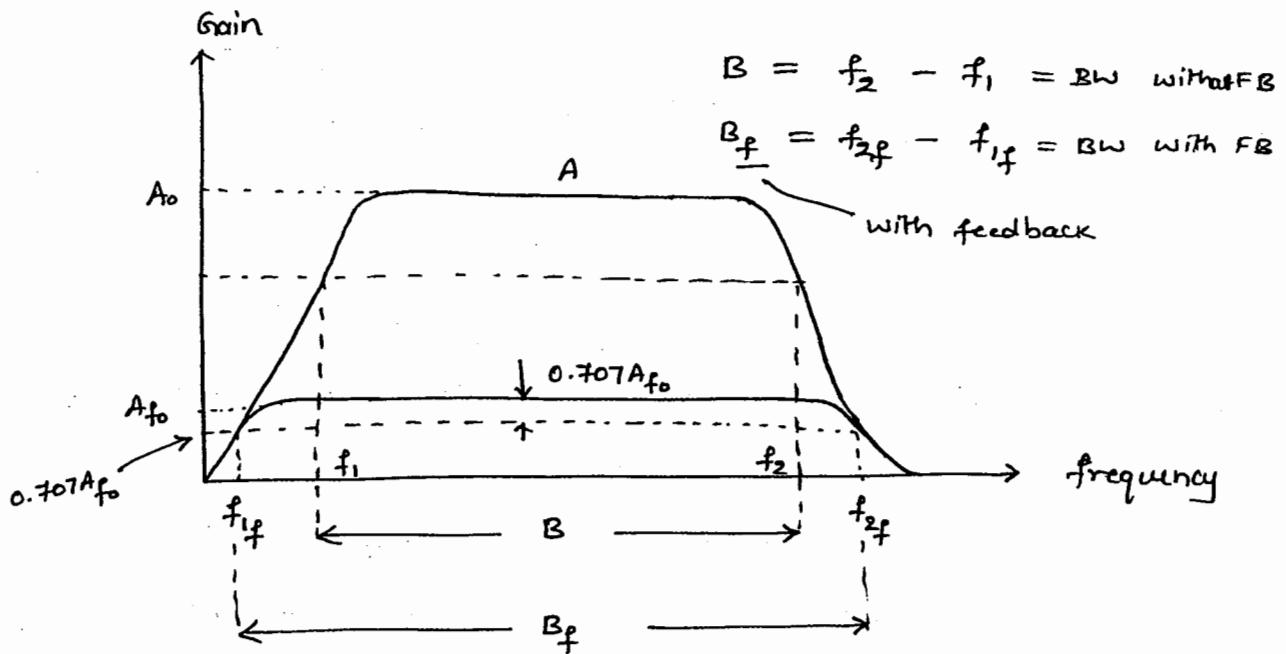


Fig. (a) Effect of Negative Feedback on Gain and Bandwidth

The bandwidth of an amplifier is given by

$$B = BW = \text{upper cutoff frequency} - \text{lower cutoff frequency}$$

It can be shown that : $f_{1f} = \frac{f_1}{(1 + \beta A)} = \text{lower cutoff freq. with feedback}$

$$f_{2f} = f_2 (1 + \beta A) = \text{upper cutoff freq. with feedback}$$

Bandwidth without feedback : $B = f_2 - f_1$

Bandwidth with feedback : $B_f = f_{2f} - f_{1f}$

$$B_f = f_2 (1 + \beta A) - \frac{f_1}{(1 + \beta A)}$$

$\Rightarrow B_f > B$, Bandwidth with feedback is larger than the bandwidth without feedback.

9 Gain Stability with Feedback

The transfer gain or simply gain of an amplifier is not constant as it depends on the factors such as operating point, temperature, etc. This lack of stability in amplifiers can be

reduced by introducing negative feedback. (9)

We know that : $A_f = \frac{A}{1 + \beta A}$

Differentiating both sides w.r.t A we get,

$$\frac{dA_f}{dA} = \frac{(1 + \beta A) \cdot 1 - \beta A}{(1 + \beta A)^2} = \frac{1}{(1 + \beta A)^2}$$

$$dA_f = \frac{dA}{(1 + \beta A)^2}$$

Dividing both sides by A_f we get,

$$\frac{dA_f}{A_f} = \frac{dA}{(1 + \beta A)^2} \times \frac{1}{A_f} = \frac{A}{1 + \beta A}$$

$$\frac{dA_f}{A_f} = \frac{dA}{(1 + \beta A)^2} \times \frac{(1 + \beta A)}{A}$$

$$\boxed{\left| \frac{dA_f}{A_f} \right| = \frac{1}{|1 + \beta A|} \left| \frac{dA}{A} \right|} \quad \text{--- (1)}$$

where $\left| \frac{dA_f}{A_f} \right|$ = Fractional change in amplification with feedback

$\left| \frac{dA}{A} \right|$ = Fractional change in amplification without feedback

→ From eqn. (1) it is clear that change in the gain with feedback is less than the change in gain without feedback by factor $(1 + \beta A)$

→ The Eqn. (1) may be approximated as follows:

For $\beta A \gg 1$

$$\left| \frac{dA_f}{A_f} \right| = \left| \frac{1}{\beta A} \right| \left| \frac{dA}{A} \right|$$

This shows that magnitude of the relative change in gain with feedback $\left| \frac{dA_f}{A_f} \right|$ is reduced by the factor $|\beta A|$ compared to

the change in gain without feedback $\left| \frac{dA}{A} \right|$

BJT and FET Frequency Response Characteristics:

- Logarithms and Decibels:

- Logarithms taken to the base 10 are referred to as *common logarithms*, while logarithms taken to the base e are referred to as *natural logarithms*. In summary:

$$\text{Common logarithm: } x = \log_{10} a$$

$$\text{Natural logarithm: } y = \log_e a$$

The two are related by

$$\log_e a = 2.3 \log_{10} a$$

- Some relationships hold true for logarithms to any base

$$\log_{10} 1 = 0$$

$$\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b$$

$$\log_{10} \frac{1}{b} = -\log_{10} b$$

$$\log_{10} ab = \log_{10} a + \log_{10} b$$

- The background surrounding the term *decibel* (dB) has its origin in the established fact that power and audio levels are related on a logarithmic basis.
- That is, an increase in power level, say 4 to 16 W, does not result in an audio level increase by a factor of $16/4 = 4$. It will increase by a factor of 2 as derived from the power of 4 in the following manner: $(4)^2 = 16$.
- The term *bel* was derived from the surname of Alexander Graham Bell. For standardization, the bel (B) was defined by the following equation to relate power levels P_1 and P_2 :

$$G = \log_{10} \frac{P_2}{P_1} \quad \text{bel}$$

P. L

$4 \rightarrow 16$

A. L

$4 \times$

$4^2 = 16 \checkmark$

- It was found, however, that the bel was too large a unit of measurement for practical purposes, so the decibel (dB) was defined such that 10 decibels=1 bel. Therefore,

$$G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1} \quad \text{dB}$$

bel → decibels

- There exists a second equation for decibels that is applied frequently. It can be best described through the system with R_i , as an input resistance.

$$G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10} \left(\frac{V_2}{V_1} \right)^2$$

↓
 power
 voltage

and

$$G_{\text{dB}} = 20 \log_{10} \frac{V_2}{V_1} \quad \text{dB}$$

$$A_{v_T} = |A_{v_1}| \cdot |A_{v_2}| \cdots |A_{v_n}|$$

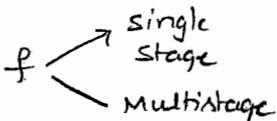
$$G_V = 20 \log_{10} |A_{v_T}| =$$

$$20 \log_{10} |A_{v_1}| + 20 \log_{10} |A_{v_2}| + \cdots + 20 \log_{10} |A_{v_n}|$$

One of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages. In words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage.

$$G_{dB_T} = G_{dBi_1} + G_{dBi_2} + \cdots + G_{dBi_n} \quad \text{dB}$$

- General Frequency Considerations:



$$f_{low} : C_C, C_E/C_S$$

No SC
 $X_C = \text{High}$

- freq. Dependent param
- stray capacitance of active device C_E

LIMITS HIGH FREQUENCY

- The frequency of the applied signal can have a pronounced effect on the response of a single-stage or multistage network. The analysis thus far has been for the midfrequency spectrum.
- At low frequencies, we shall find that the coupling and bypass capacitors can no longer be replaced by the short-circuit approximation because of the increase in reactance of these elements.
- The frequency-dependent parameters of the small-signal equivalent circuits and the stray capacitive elements associated with the active device and the network will limit the high-frequency response of the system.
- An increase in the number of stages of a cascaded system will also limit both the high- and low-frequency responses.
- For any system, there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the midband value.
- To fix the frequency boundaries of relatively high gain, 0.707 $A_{v_{mid}}$ was chosen to be the gain at the cutoff levels. The corresponding frequencies f_1 and f_2 are generally called the *corner, cutoff, bandbreak, or half-power frequencies*. The multiplier 0.707 was chosen because at this level the output power is half the midband power output, that is, at midfrequencies.

- The bandwidth (or passband) of each system is determined by f_1 and f_2 , that is,

$$\text{bandwidth (BW)} = f_2 - f_1$$

- For applications of a communications nature (audio, video), a decibel plot of the voltage gain versus frequency is more useful.
- Before obtaining the logarithmic plot, however, the curve is generally normalized as shown in Fig. 9.6. In this figure, the gain at each frequency is divided by the midband value. Obviously, the midband value is then 1 as indicated. At the half-power frequencies, the resulting level is $0.707 = 1/\sqrt{2}$.

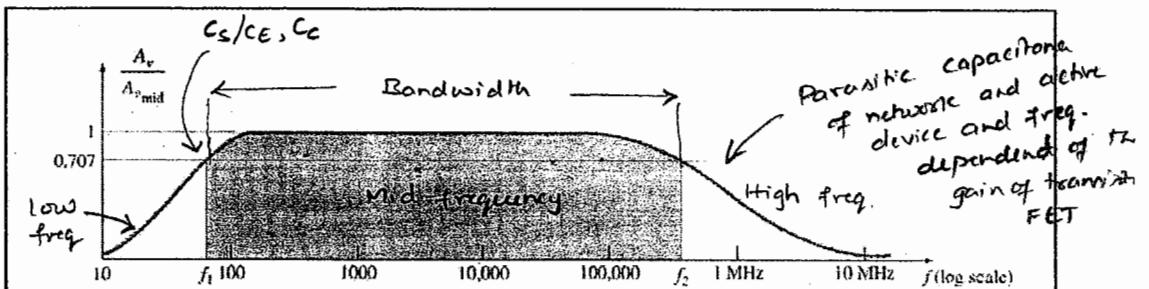


Fig. 9.6 Normalized gain versus frequency plot.

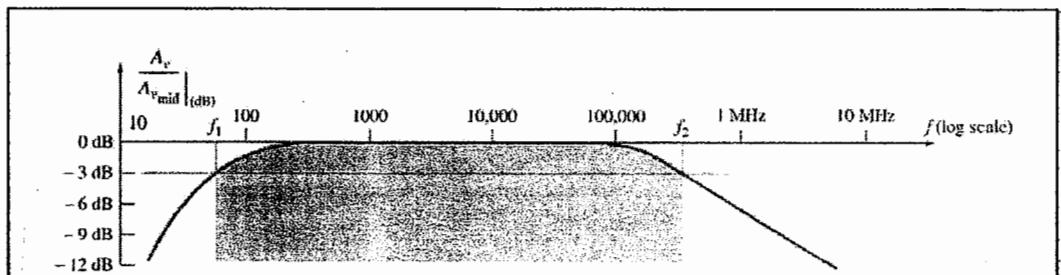
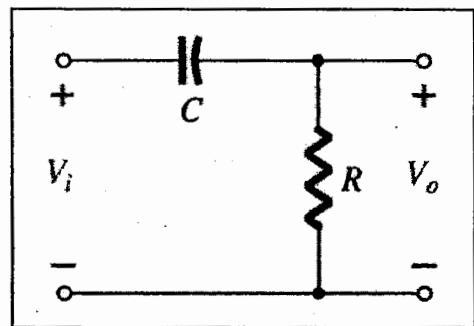


Fig. 9.7 Decibel plot of the normalized gain versus frequency plot of Fig. 9.6.

- Low Frequency Analysis:

- In the low-frequency region of the single-stage amplifier, it is the R-C combinations formed by the network capacitors C_C , C_E , and C_B and the network resistive parameters that determine the cutoff frequencies.
- The analysis, therefore, will begin with the series R-C combination of the given Fig. and the development of a procedure that will result in a plot of the frequency response with a minimum of time and effort.



- 1) At very high frequencies,

$$X_C = \frac{1}{2\pi f C} \approx 0 \Omega$$

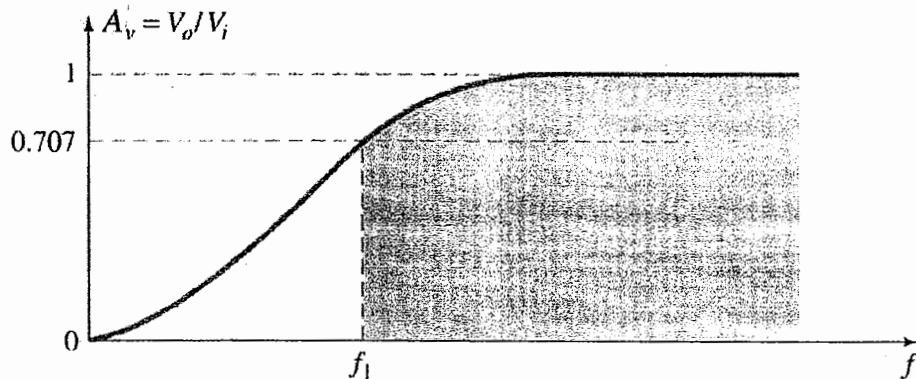
A short-circuit equivalent can be substituted for the capacitor. The result is that $V_o = V_i$ at high frequencies.

- 2) At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(0)C} = \infty \Omega$$

An open-circuit approximation can be applied, with the result that $V_o = 0$ V.

- 3) Between the two extremes, the ratio $A_v = V_o/V_i$ will vary as shown below. As the frequency increases, the capacitive reactance decreases and more of the input voltage appears across the output terminals.



- The output and input voltages are related by the voltage-divider rule in the following manner:

$$\mathbf{V}_o = \frac{\mathbf{RV}_i}{\mathbf{R} + \mathbf{X}_c}$$

- The magnitude of V_o determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_c^2}}$$

- For the special case where $X_c = R$,

$$\begin{aligned} V_o &= \frac{RV_i}{\sqrt{R^2 + X_c^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}} V_i \\ \Rightarrow |A_v| &= \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707 = -3dB \end{aligned}$$

- The frequency of which $X_c = R$ (the output will be 70.7% of the input) is determined as:

$$\begin{aligned} X_c &= \frac{1}{2\pi f_1 C} = R \Rightarrow f_1 = \frac{1}{2\pi CR} \\ A_v &= \frac{R}{R - jX_c} = \frac{1}{1 - j\left(\frac{X_c}{R}\right)} = \frac{1}{1 - j\left(\frac{1}{\omega CR}\right)} = \frac{1}{1 - j\left(\frac{1}{2\pi f_1 C R}\right)} \\ \Rightarrow A_v &= \frac{1}{1 - j\left(\frac{f_1}{f}\right)} = \frac{1}{\underbrace{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}_{\text{magnitude of } A_v} \angle \tan^{-1}\left(\frac{f_1}{f}\right)} \xrightarrow{\text{phase by which } V_o \text{ leads } V_i} = -10 \log \left[1 + \left(\frac{f_1}{f} \right)^2 \right] dB \end{aligned}$$

- For frequencies where $f \ll f_1$ or $(f_1/f)^2 \gg 1$, the equation above can be approximated as

$$A_v = -10 \log \left[\left(\frac{f_1}{f} \right)^2 \right] dB = -20 \log \left(\frac{f_1}{f} \right) dB$$

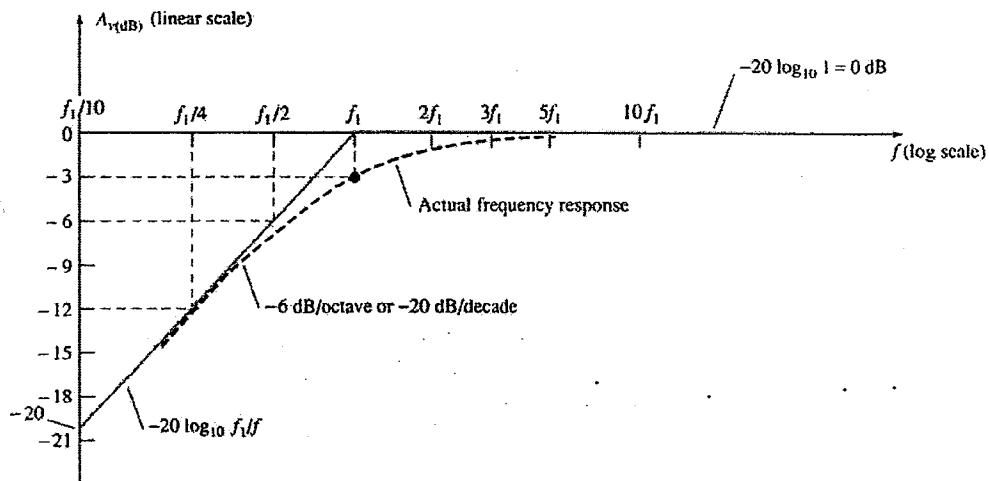
- Ignoring the previous condition for a moment, a plot on a frequency log scale will yield a result of a very useful nature for future decibel plots.

At $f = f_1 \Rightarrow -20 \log(1) = 0 \text{ dB}$

At $f = \frac{f_1}{2} \Rightarrow -20 \log(2) \approx -6 \text{ dB}$

At $f = \frac{f_1}{4} \Rightarrow -20 \log(4) \approx -12 \text{ dB}$

At $f = \frac{f_1}{10} \Rightarrow -20 \log(10) = -20 \text{ dB}$

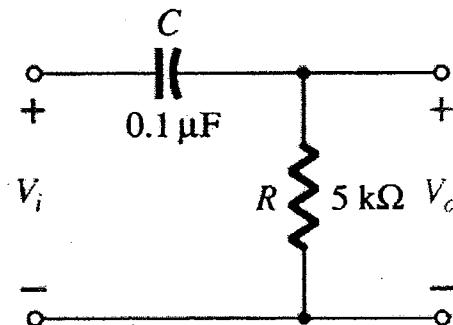


-A change in frequency by a factor of 2, equivalent to 1 octave, results in a 6-dB change in the ratio as noted by the change in gain from $f_1/2$ to f_1 .

-For a 10:1 change in frequency, equivalent to 1 decade, there is a 20-dB change in the ratio as demonstrated between the frequencies of

Ex. For the given network:

- Determine the break frequency.
- Sketch the asymptotes and locate the -3-dB point.
- Sketch the frequency response curve.



Solution:

$$(a) f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi(5 \times 10^3 \Omega)(0.1 \times 10^{-6} F)} \approx 318.5 \text{ Hz}$$

(b) See Figure below

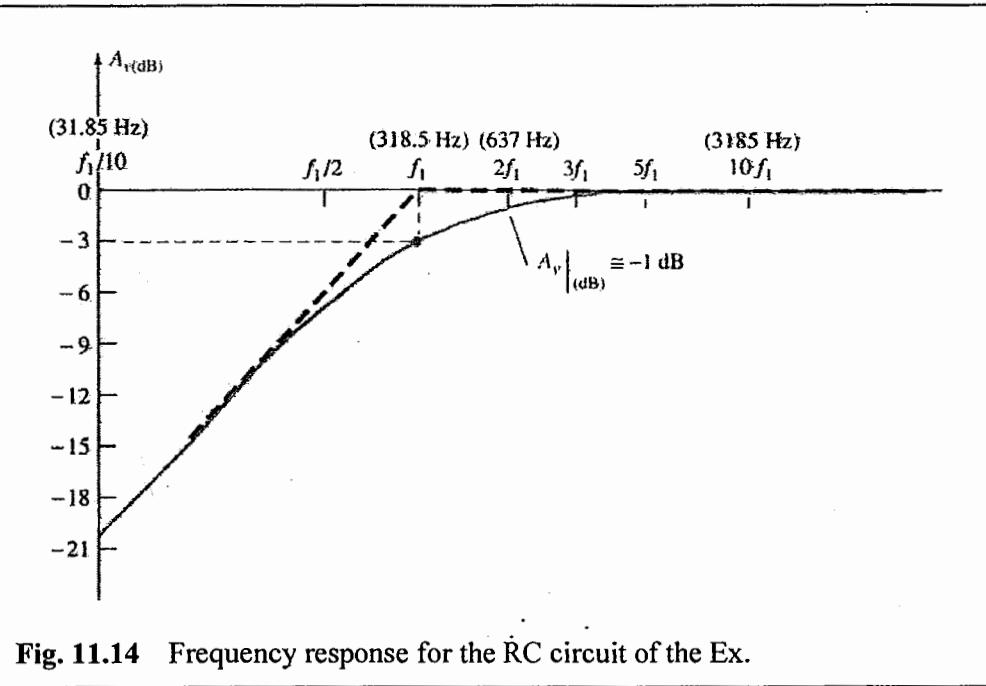


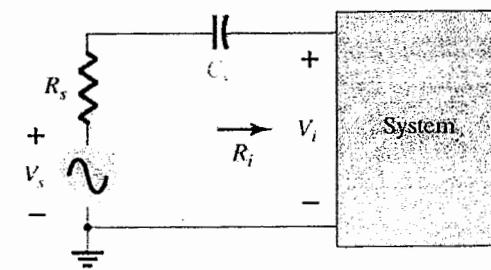
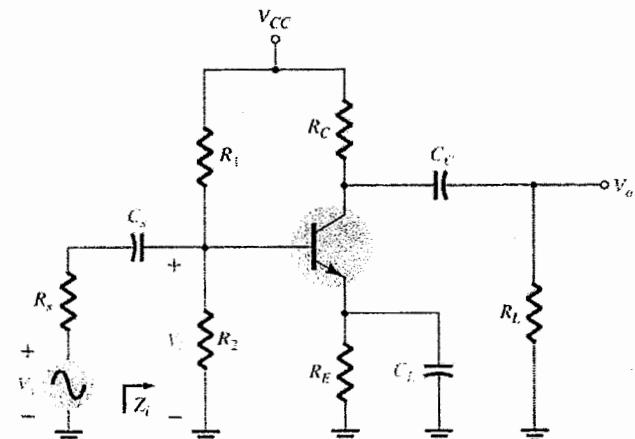
Fig. 11.14 Frequency response for the RC circuit of the Ex.

- Low Frequency Analysis-BJT Amplifiers:

- The analysis of this section will employ the loaded voltage-divider BJT bias configuration, but the results can be applied to any BJT configuration.
- It will simply be necessary to find the appropriate equivalent resistance for the $R-C$ combination (for the capacitors C_s , C_C , and C_E which will determine the low-frequency response).

1) The effect of C_s :

- Since C_s is normally connected between the applied source and the active device, the total resistance is now $R_s + R_i$, and the cutoff frequency will be modified to be as:



$$f_{ls} = \frac{1}{2\pi(R_s + R_i)C_s}$$

- At mid or high frequencies, the reactance of the capacitor will be sufficiently small to permit a short-circuit approximation for the element. The voltage V_i will then be related to V_s by

$$V_i |_{mid} = V_s \frac{R_i}{R_i + R_s}$$

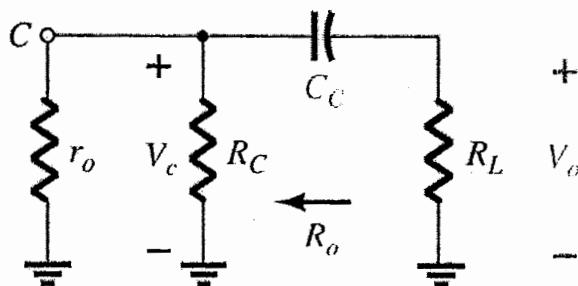
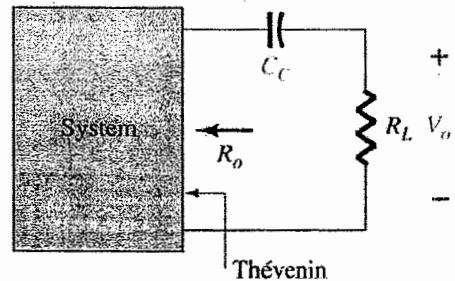
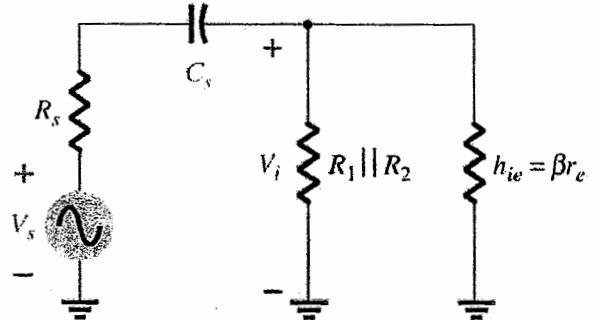
- The voltage V_i applied to the input of the active device can be calculated using the voltage-divider rule:

$$V_i = V_s \frac{R_i}{R_s + R_i - jX_{Cs}}$$

2) The effect of C_C :

- Since C_C the coupling capacitor is normally connected between the output of the active device and the applied load, the total resistance is now $R_o + R_L$, and the cutoff frequency will be modified to be as:

$$f_{LC} = \frac{1}{2\pi(R_o + R_L)C_C}$$



3) The effect of C_E :

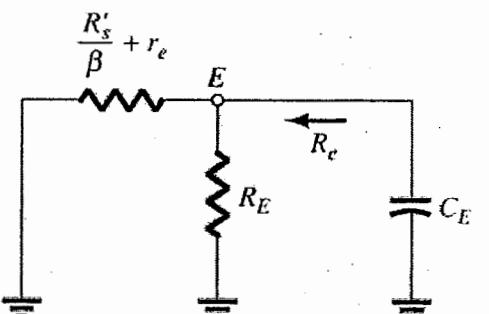
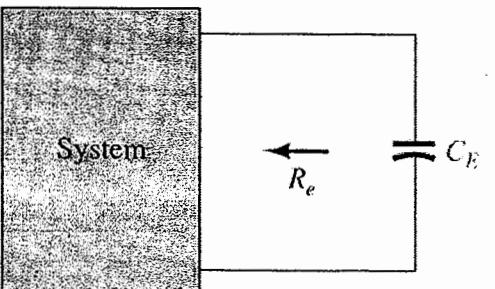
- To determine f_{LE} , the network "seen" by C_E must be determined as shown in the Fig. below. Once the level of R_e is established, the cutoff frequency due to C_E can be determined using the following equation:

$$f_{LE} = \frac{1}{2\pi(R_e)C_E}$$

where R_e could be calculated as:

$$R_e = R_E // \left(\frac{R_s // R_1 // R_2}{\beta} + r_e \right)$$

$$\text{where } R_s // R_1 // R_2 = R'_s$$



Ex. (a) Determine the lower cutoff frequency for the voltage-divider BJT bias configuration network using the following parameters:

$C_s = 10\mu F$, $C_E = 20\mu F$, $C_C = 1\mu F$, $R_s = 1k\Omega$, $R_1 = 40k\Omega$, $R_2 = 10k\Omega$, $R_E = 2k\Omega$, $R_C = 4k\Omega$, $R_L = 2.2 k\Omega$, $\beta = 100$, $r_o = \infty$, $V_{CC} = 20V$.

(b) Sketch the frequency response using a Bode plot.

Solution:

(a) Determining r_e for dc conditions:

$$\beta R_E = (100)(2 k\Omega) = 200 k\Omega \gg 10R_2 = 100 k\Omega$$

The result is:

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10 k\Omega (20 V)}{10 k\Omega + 40 k\Omega} = \frac{200 V}{50} = 4 V$$

with $I_E = \frac{V_E}{R_E} = \frac{4 V - 0.7 V}{2 k\Omega} = \frac{3.3 V}{2 k\Omega} = 1.65 \text{ mA}$

so that

$$r_e = \frac{26 \text{ mV}}{1.65 \text{ mA}} \cong 15.76 \Omega$$

and

$$\beta r_e = 100(15.76 \Omega) = 1576 \Omega = 1.576 \text{ k}\Omega$$

$$\text{Midband Gain } A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4 \text{ k}\Omega) \| (2.2 \text{ k}\Omega)}{15.76 \text{ }\Omega} \cong -90$$

$$\begin{aligned} \text{The input impedance } Z_i &= R_i = R_1 \| R_2 \| \beta r_e \\ &= 40 \text{ k}\Omega \| 10 \text{ k}\Omega \| 1.576 \text{ k}\Omega \\ &\cong 1.32 \text{ k}\Omega \end{aligned}$$

and

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

$$\text{or } \frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32 \text{ k}\Omega}{1.32 \text{ k}\Omega + 1 \text{ k}\Omega} = 0.569$$

$$\begin{aligned} \text{so that } A_{v_s} &= \frac{V_o}{V_s} = \frac{V_o}{V_i} \frac{V_i}{V_s} = (-90)(0.569) \\ &= -51.21 \end{aligned}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40 \text{ k}\Omega \| 10 \text{ k}\Omega \| 1.576 \text{ k}\Omega \cong 1.32 \text{ k}\Omega$$

$$\begin{aligned} f_{Ls} &= \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1 \text{ k}\Omega + 1.32 \text{ k}\Omega)(10 \mu\text{F})} \\ f_{Ls} &\cong 6.86 \text{ Hz} \end{aligned}$$

C_C

$$\begin{aligned} f_{Lc} &= \frac{1}{2\pi(R_C + R_L)C_C} \\ &= \frac{1}{(6.28)(4 \text{ k}\Omega + 2.2 \text{ k}\Omega)(1 \mu\text{F})} \\ &\cong 25.68 \text{ Hz} \end{aligned}$$

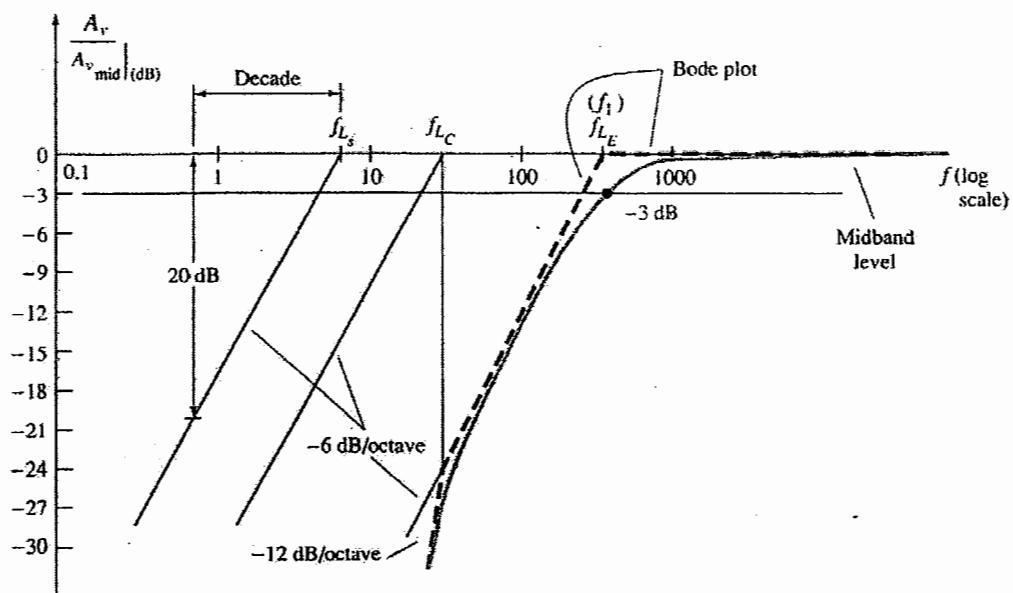
C_E

$$R'_s = R_s \| R_1 \| R_2 = 1 \text{ k}\Omega \| 40 \text{ k}\Omega \| 10 \text{ k}\Omega \cong 0.889 \text{ k}\Omega$$

$$\begin{aligned} R_e &= R_E \left| \left(\frac{R'_s}{\beta} + r_e \right) \right| = 2 \text{ k}\Omega \left| \left(\frac{0.889 \text{ k}\Omega}{100} + 15.76 \text{ }\Omega \right) \right| \\ &= 2 \text{ k}\Omega \| (8.89 \text{ }\Omega + 15.76 \text{ }\Omega) = 2 \text{ k}\Omega \| 24.65 \text{ }\Omega \cong 24.35 \text{ }\Omega \end{aligned}$$

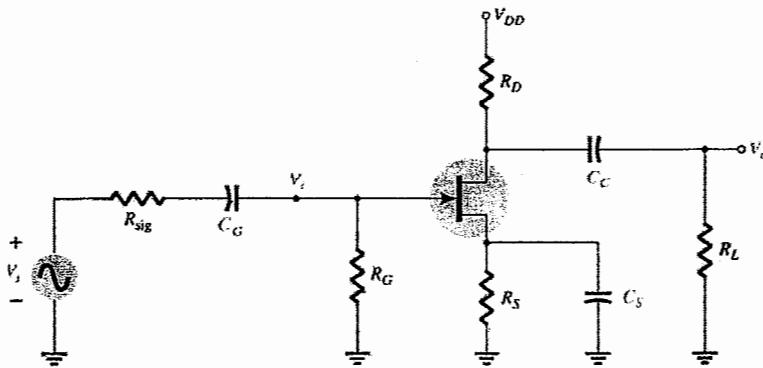
$$f_{L_E} = \frac{1}{2\pi R_E C_E} = \frac{1}{(6.28)(24.35 \text{ }\Omega)(20 \mu\text{F})} = \frac{10^6}{3058.36} \cong 327 \text{ Hz}$$

(b)



- Low Frequency Analysis-FET Amplifiers:

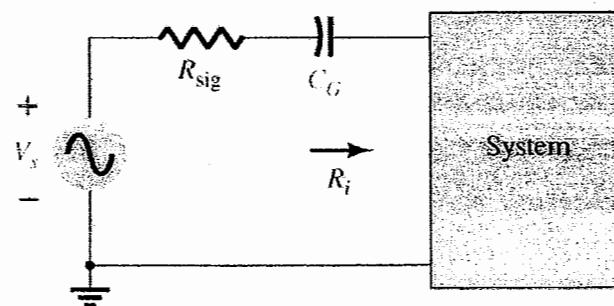
- The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier. There are again three capacitors of primary concern as appearing in the given network: C_G , C_C , and C_S .



D) The effect of C_G :

The cutoff frequency determined by C_G will then be

$$f_{LG} = \frac{1}{2\pi(R_G + R_{sig})C_G}$$

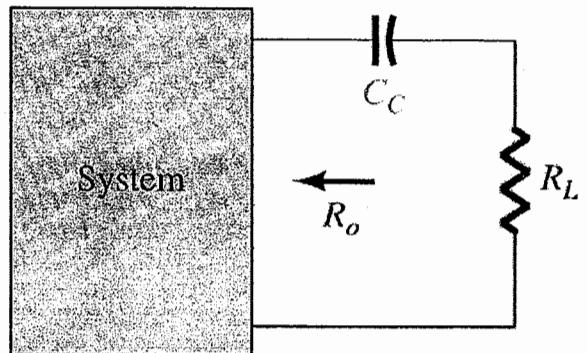


2) The effect of C_C :

The cutoff frequency determined by C_C will then be

$$f_{LC} = \frac{1}{2\pi(R_L + R_o)C_C}$$

where $R_o = R_D // r_d$



3) The effect of C_S :

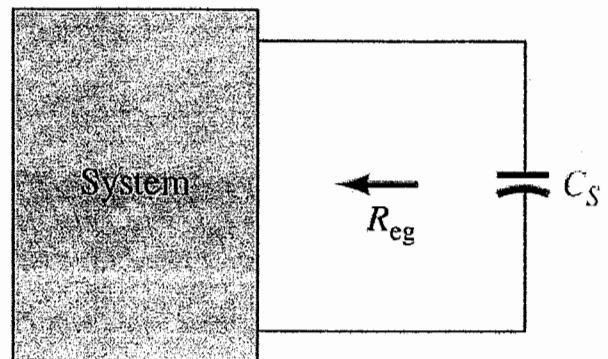
The cutoff frequency determined by C_S will then be

$$f_{LS} = \frac{1}{2\pi(R_{eg})C_S}$$

where $R_{eg} = \frac{R_s}{1 + \left[\frac{R_s(1 + g_m r_d)}{(r_d + R_G // R_L)} \right]}$

which for $r_d \approx \infty$ becomes

$$R_{eg} = R_s // \left(\frac{1}{g_m} \right)$$



Ex. (a) Determine the lower cutoff frequency for the CS FET bias configuration network using the following parameters:

$C_G = 0.01\mu F$, $C_C = 0.5\mu F$, $C_S = 2\mu F$, $R_{sig} = 10k\Omega$, $R_G = 1M\Omega$, $R_D = 4.7k\Omega$, $R_s = 1k\Omega$, $R_L = 2.2 k\Omega$, $I_{DSS} = 8mA$, $V_P = -4V$, $r_d = \infty$, $V_{DD} = 20V$.

(b) Sketch the frequency response using a Bode plot.

Solution:

(a) DC Analysis: Plotting the transfer curve of $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ and superimposing the curve defined by $V_{GS} = -I_D R_S$ will result in an intersection at $V_{GS_Q} = -2V$ and $I_{D_Q} = 2 mA$. In addition,

$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8 \text{ mA})}{4 \text{ V}} = 4 \text{ mS}$$

$$g_m = g_{m0} \left(1 - \frac{V_{GS_Q}}{V_P} \right) = 4 \text{ mS} \left(1 - \frac{-2 \text{ V}}{-4 \text{ V}} \right) = 2 \text{ mS}$$

$$C_G \quad f_{L_G} = \frac{1}{2\pi (10 \text{ k}\Omega + 1 \text{ M}\Omega)(0.01 \mu\text{F})} \cong 15.8 \text{ Hz}$$

$$C_C \quad f_{L_C} = \frac{1}{2\pi (4.7 \text{ k}\Omega + 2.2 \text{ k}\Omega)(0.5 \mu\text{F})} \cong 46.13 \text{ Hz}$$

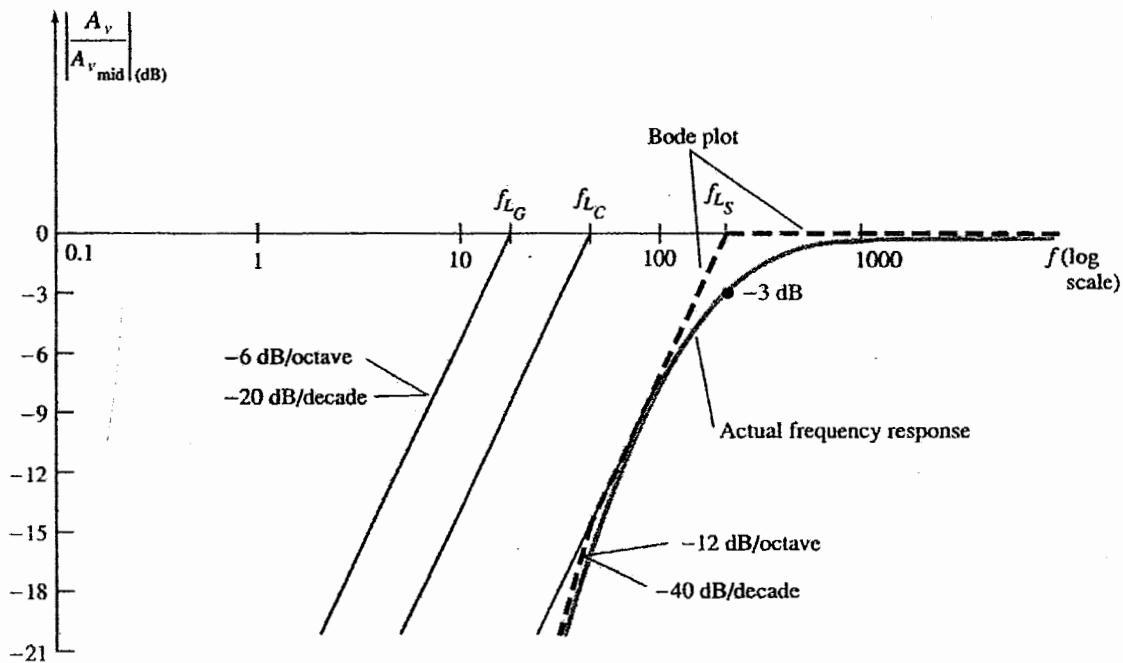
$$C_S \quad R_{\text{eq}} = R_S \parallel \frac{1}{g_m} = 1 \text{ k}\Omega \parallel \frac{1}{2 \text{ mS}} = 1 \text{ k}\Omega \parallel 0.5 \text{ k}\Omega = 333.33 \text{ }\Omega$$

$$f_{L_S} = \frac{1}{2\pi (333.33 \text{ }\Omega)(2 \mu\text{F})} = 238.73 \text{ Hz}$$

Since f_{L_S} is the largest of the three cutoff frequencies, it defines the low cutoff frequency for the network.

(b) The midband gain of the system is determined by

$$\begin{aligned} A_{v_{\text{mid}}} &= \frac{V_o}{V_i} = -g_m(R_D \parallel R_L) = -(2 \text{ mS})(4.7 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega) \\ &= -(2 \text{ mS})(1.499 \text{ k}\Omega) \\ &\cong -3 \end{aligned}$$



- ***Miller Effect capacitance:***

In the high-frequency region, the capacitive elements of importance are the interelectrode (between terminals) capacitances internal to the active device and the wiring capacitance between leads of the network. The large capacitors of the network that controlled the low-frequency response have all been replaced by their short-circuit equivalent due to their very low reactance levels.

- Miller input capacitance

$$C_{Mi} = (1 - A_v) C_f$$

where C_f is the feedback capacitance.

- Miller output capacitance

$$C_{Mo} = \left(1 - \frac{1}{A_v}\right) C_f \underset{|A_v \gg 1}{\approx} C_f$$

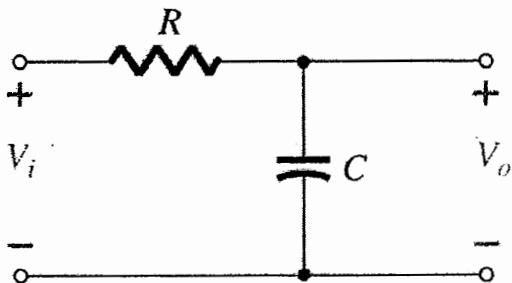
where C_f is the feedback capacitance.

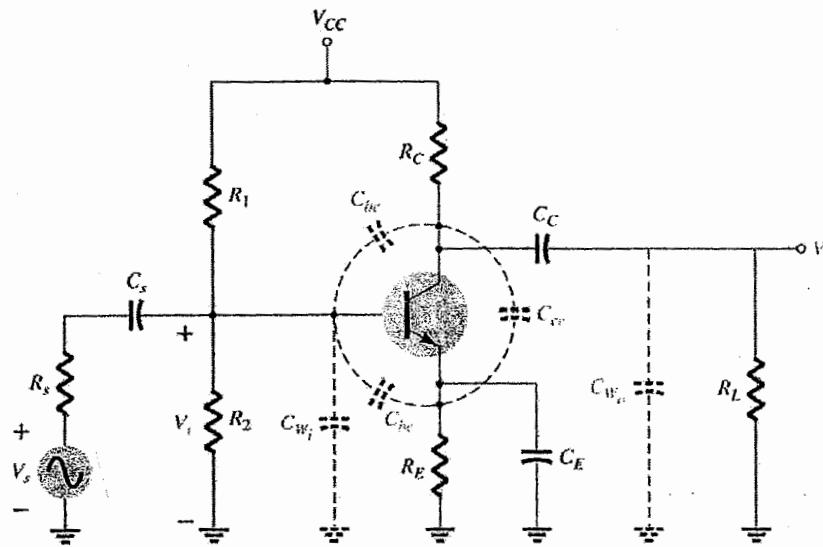
- ***High Frequency Analysis-BJT Amplifiers:***

In the high-frequency region, the RC network of concern has the configuration appearing in given Fig. At increasing frequencies, the reactance X_C will decrease in magnitude, resulting in a shorting effect across the output and a decrease in gain.

The derivation leading to the corner frequency for this RC configuration follows along similar lines to that encountered for the low-frequency region. The most significant difference is in the general form of A_v appearing below:

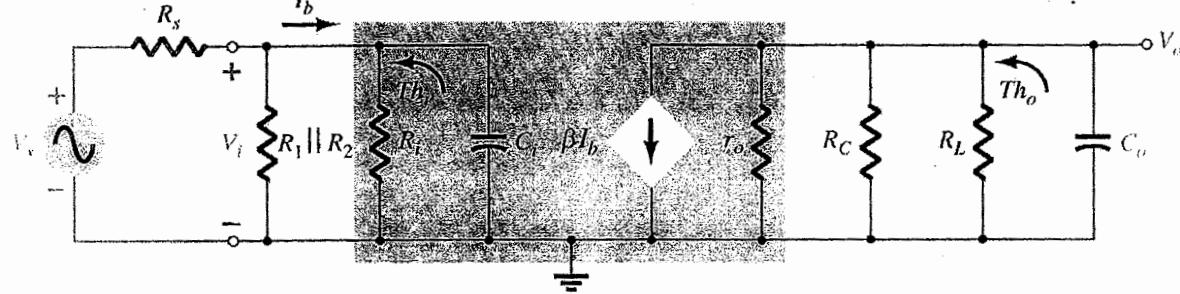
$$A_v = \frac{1}{1 + j \left(\frac{f}{f_2} \right)}$$





$$C_i = C_{W_i} + C_{be} + C_{M_i}$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$



- In the above Fig., the various parasitic capacitances (C_{be} , C_{bc} , C_{ce}) of the transistor have been included with the wiring capacitances (C_{W_i} , C_{W_o}) introduced during construction.
- In the high-frequency equivalent model for the network, note the absence of the capacitors C_s , C_C , and C_E , which are all assumed to be in the short-circuit state at these frequencies.
- The capacitance C_i includes the input wiring capacitance C_{W_i} , the transition capacitance C_{be} , and the Miller capacitance C_{M_i} .
- The capacitance C_o includes the output wiring capacitance C_{W_o} , the parasitic capacitance C_{ce} , and the output Miller capacitance C_{M_o} .
- In general, the capacitance C_{be} is the largest of the parasitic capacitances, with C_{ce} the smallest. In fact, most specification sheets simply provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of a particular type of transistor in a specific area of application.

1) For the input network, C_i :

For the input network, the -3-dB frequency is defined by

$$f_{Hi} = \frac{1}{2\pi R_{Th_1} C_i}$$

where

$$R_{Th_1} = R_s \| R_1 \| R_2 \| R_i$$

$$C_i = C_{Wi} + C_{be} + C_{Mi} = C_{Wi} + C_{be} + (1 - A_v) C_{be}$$

2) For the output network, C_o :

$$f_{Ho} = \frac{1}{2\pi R_{Th_2} C_o}$$

$$R_{Th_2} = R_C \| R_L \| r_o$$

$$C_o = C_{Wo} + C_{ce} + C_{Mo}$$

Ex. For the given network, with the following parameters:

$C_s = 10\mu F$, $C_E = 20\mu F$, $C_C = 1\mu F$, $R_s = 1k\Omega$, $R_1 = 40k\Omega$, $R_2 = 10k\Omega$, $R_E = 2k\Omega$, $R_C = 4k\Omega$, $R_L = 2.2 k\Omega$, $\beta = 100$, $r_o = \infty$, $V_{CC} = 20V$.

with the addition of

$C_{be} = 36pF$, $C_{bc} = 4pF$, $C_{ce} = 1pF$, $C_{Wi} = 6pF$, $C_{Wo} = 8pF$

(a) Determine f_{Hi} and f_{Ho} .

(b) Sketch the total frequency response for the low- and high-frequency regions.

Solution:

(a) from previous example

$$R_i = 1.32 k\Omega, \quad A_{v_{out}}(\text{amplifier}) = -90$$

and

$$R_{Th_1} = R_s \| R_1 \| R_2 \| R_i = 1 k\Omega \| 40 k\Omega \| 10 k\Omega \| 1.32 k\Omega \\ \cong 0.531 k\Omega$$

with

$$\begin{aligned}
 C_i &= C_{W_i} + C_{be} + (1 - A_v)C_{be} \\
 &= 6 \text{ pF} + 36 \text{ pF} + [1 - (-90)]4 \text{ pF} \\
 &= 406 \text{ pF}
 \end{aligned}$$

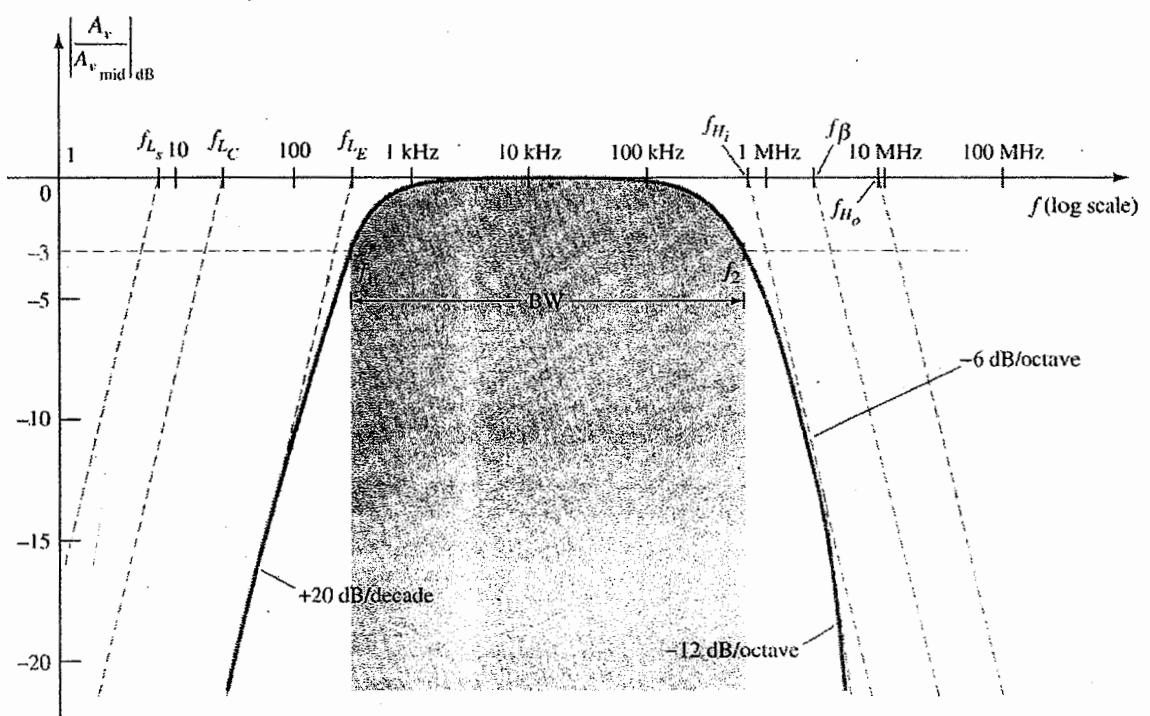
$$\begin{aligned}
 f_{H_i} &= \frac{1}{2\pi R_{Th_1} C_i} = \frac{1}{2\pi(0.531 \text{ k}\Omega)(406 \text{ pF})} \\
 &= 738.24 \text{ kHz}
 \end{aligned}$$

$$R_{Th_2} = R_C \| R_L = 4 \text{ k}\Omega \| 2.2 \text{ k}\Omega = 1.419 \text{ k}\Omega$$

$$\begin{aligned}
 C_o &= C_{W_o} + C_{ce} + C_{M_o} = 8 \text{ pF} + 1 \text{ pF} + \left(1 - \frac{1}{-90}\right)4 \text{ pF} \\
 &= 13.04 \text{ pF}
 \end{aligned}$$

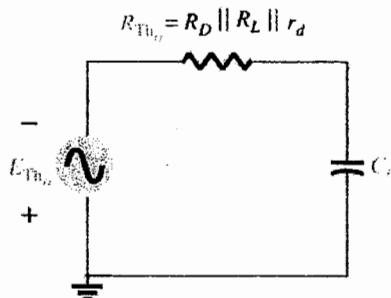
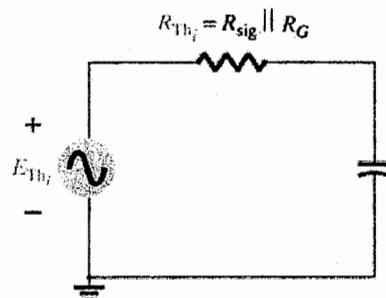
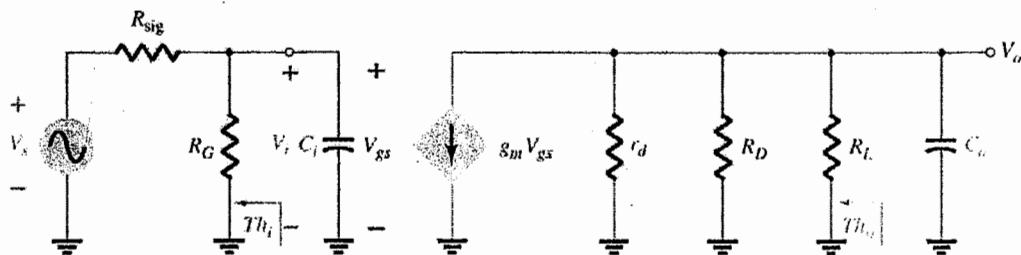
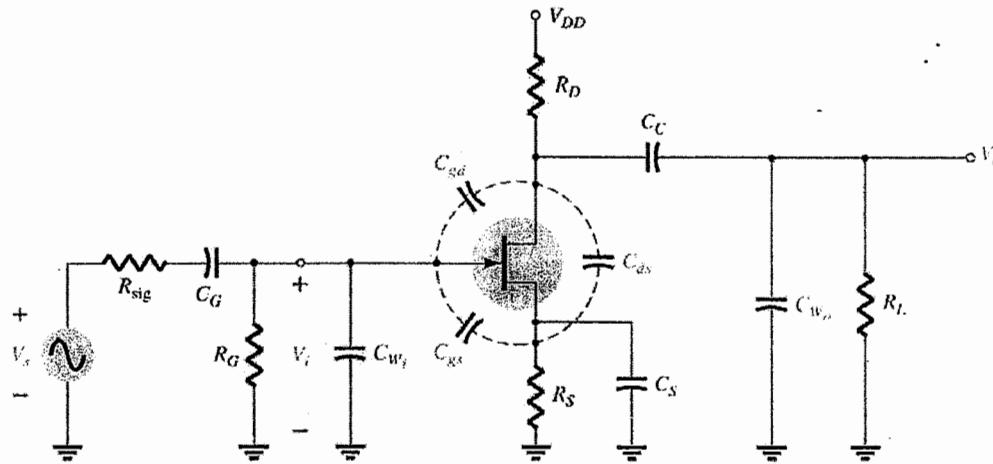
$$\begin{aligned}
 f_{H_o} &= \frac{1}{2\pi R_{Th_2} C_o} = \frac{1}{2\pi(1.419 \text{ k}\Omega)(13.04 \text{ pF})} \\
 &= 8.6 \text{ MHz}
 \end{aligned}$$

(b)



- **High Frequency Analysis-FET Amplifiers:**

- The analysis of the high-frequency response of the FET amplifier will proceed in a very similar manner to that encountered for the BJT amplifier. There are interelectrode and wiring capacitances that will determine the high-frequency characteristics of the amplifier.
- The capacitors C_{gs} and C_{gd} typically vary from 1 to 10 pF, while the capacitance C_{ds} is usually quite a bit smaller, ranging from 0.1 to 1 pF.
- At high frequencies, C_i will approach a short-circuit equivalent and V_{gs} will drop in value and reduce the overall gain. At frequencies where C_o approaches its short circuit equivalent, the parallel output voltage V_o will drop in magnitude.



$$f_{H_i} = \frac{1}{2\pi R_{Th_1} C_i}$$

and

$$R_{Th_1} = R_{sig} \| R_G$$

with

$$C_i = C_{W_i} + C_{gs} + C_{M_i}$$

and

$$C_{M_i} = (1 - A_v) C_{gd}$$

and for the output circuit,

$$f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o}$$

with

$$R_{Th_2} = R_D \| R_L \| r_d$$

and

$$C_o = C_{W_o} + C_{ds} + C_{M_o}$$

and

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_{gd}$$

Ex. (a) Determine the high cutoff frequencies for the CS FET bias configuration network using the following parameters:

$C_G = 0.01\mu F$, $C_C = 0.5\mu F$, $C_S = 2\mu F$, $R_{sig} = 10k\Omega$, $R_G = 1M\Omega$, $R_D = 4.7k\Omega$, $R_S = 1k\Omega$, $R_L = 2.2 k\Omega$, $I_{DSS} = 8mA$, $V_p = -4V$, $r_d = \infty$, $V_{DD} = 20V$.
 $C_{gd} = 2pF$, $C_{gs} = 4pF$, $C_{ds} = 0.5pF$, $C_{W_i} = 5pF$, $C_{W_o} = 6pF$

Solution:

(a) From the previous example

$$R_{Th_1} = R_{sig} \| R_G = 10 k\Omega \| 1 M\Omega = 9.9 k\Omega$$

$$A_v = -3.$$

$$\begin{aligned}
 C_i &= C_{W_i} + C_{gs} + (1 - A_v)C_{gd} \\
 &= 5 \text{ pF} + 4 \text{ pF} + (1 + 3)2 \text{ pF} \\
 &= 9 \text{ pF} + 8 \text{ pF} \\
 &= 17 \text{ pF}
 \end{aligned}$$

$$\begin{aligned}
 f_{H_i} &= \frac{1}{2\pi R_{Th_1} C_i} \\
 &= \frac{1}{2\pi(9.9 \text{ k}\Omega)(17 \text{ pF})} = 945.67 \text{ kHz}
 \end{aligned}$$

$$\begin{aligned}
 R_{Th_2} &= R_D \parallel R_L \\
 &= 4.7 \text{ k}\Omega \parallel 2.2 \text{ k}\Omega \\
 &\approx 1.5 \text{ k}\Omega
 \end{aligned}$$

$$\begin{aligned}
 C_o &= C_{W_o} + C_{ds} + C_{M_o} = 6 \text{ pF} + 0.5 \text{ pF} + \left(1 - \frac{1}{-3}\right)2 \text{ pF} = 9.17 \text{ pF} \\
 f_{H_o} &= \frac{1}{2\pi(1.5 \text{ k}\Omega)(9.17 \text{ pF})} = 11.57 \text{ MHz}
 \end{aligned}$$

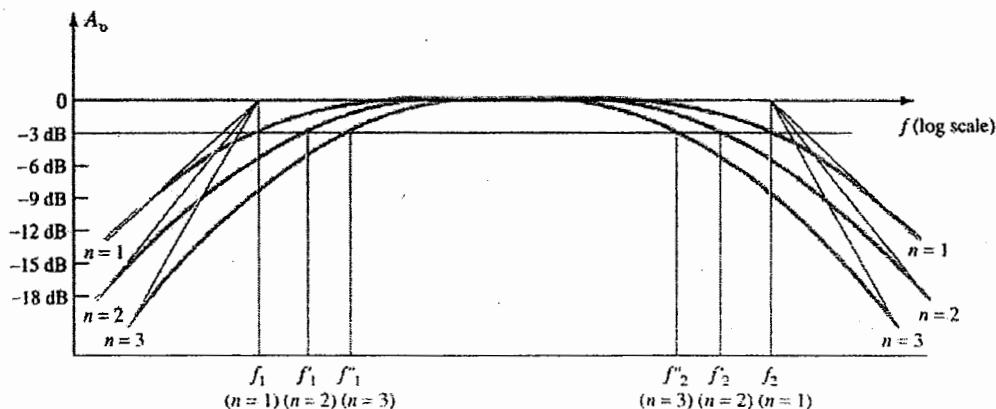
Multistage Frequency Effect:

- For Low Frequency region

$$f'_1 = \frac{f_1}{\sqrt{2^{\left(\frac{1}{n}\right)} - 1}}$$

- For High Frequency region

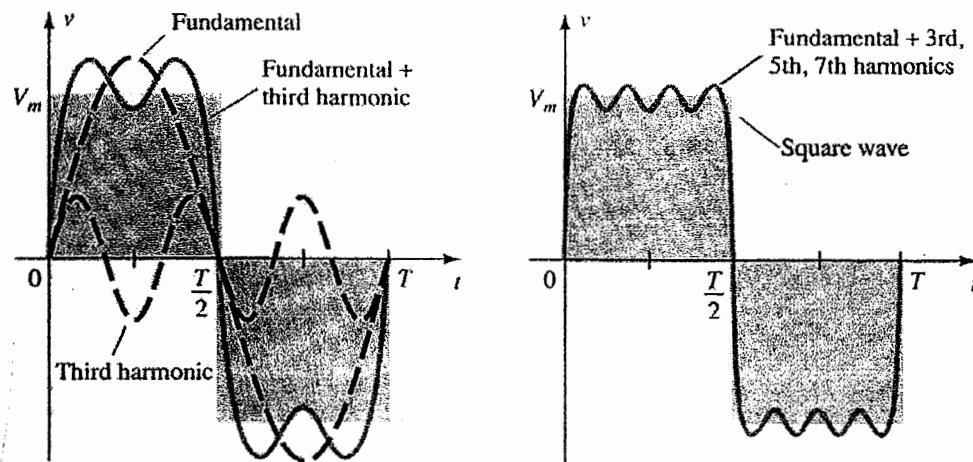
$$f'_2 = \left(\sqrt{2^{\left(\frac{1}{n}\right)} - 1} \right) f_2$$



- Square wave Testing:

- Experimentally, the sense for the frequency response can be determined by applying a square wave signal to the amplifier and noting the output response.
- The reason for choosing a square-wave signal for the testing process is best described by examining the Fourier series expansion of a square wave composed of a series of sinusoidal components of different magnitudes and frequencies. The summation of the terms of the series will result in the original waveform. In other words, even though a waveform may not be sinusoidal, it can be reproduced by a series of sinusoidal terms of different frequencies and magnitudes.

$$v = \frac{4}{\pi} V_m \left(\sin 2\pi f_s t + \frac{1}{3} \sin 2\pi(3f_s)t + \frac{1}{5} \sin 2\pi(5f_s)t + \frac{1}{7} \sin 2\pi(7f_s)t + \frac{1}{9} \sin 2\pi(9f_s)t + \dots + \frac{1}{n} \sin 2\pi(nf_s)t \right)$$

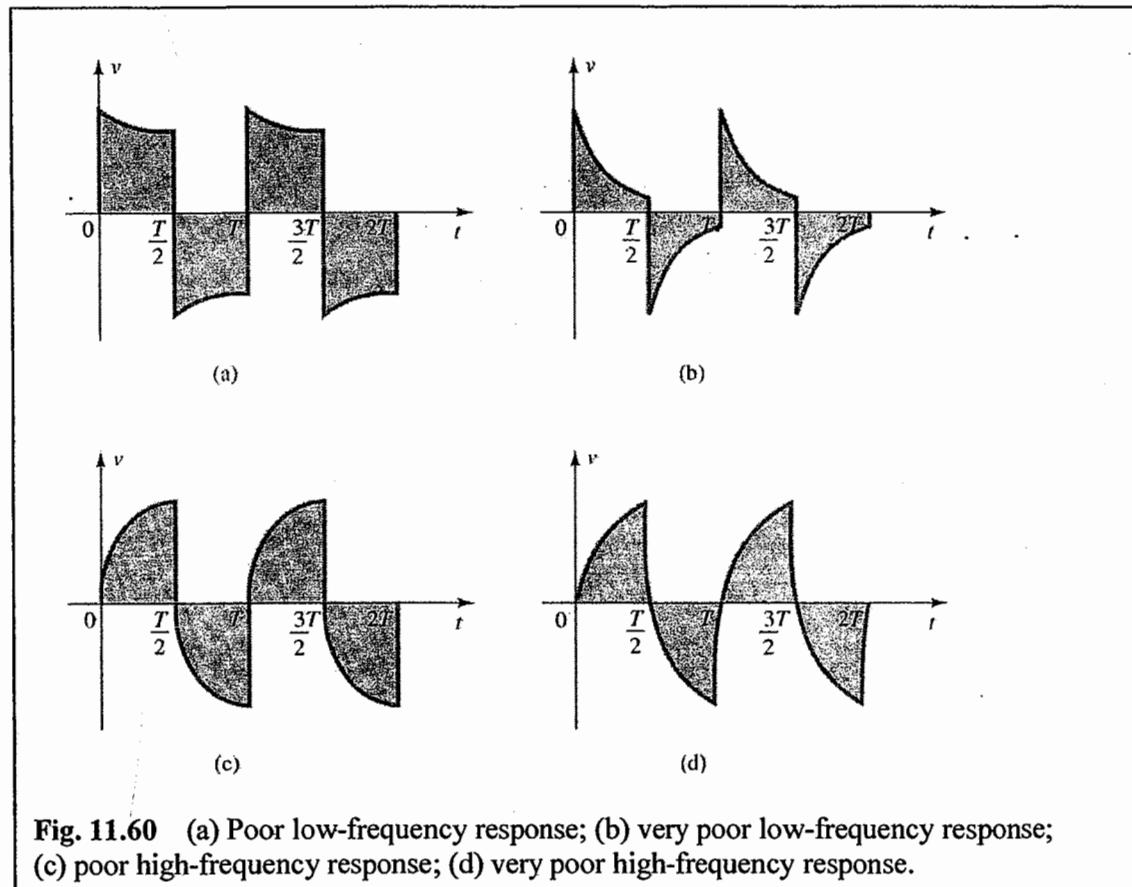


- Since the ninth harmonic has a magnitude greater than 10% of the fundamental term, the fundamental term through the ninth harmonic are the major contributors to the Fourier series expansion of the square-wave function.

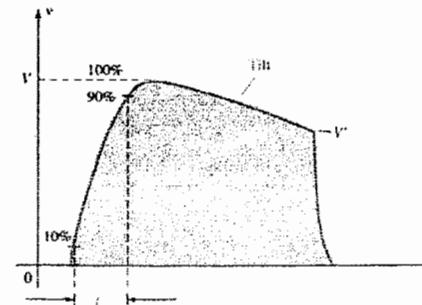
Ex. For a specific application (Audio amplifier with 20kHz Bandwidth), what is the maximum frequency could be amplified?

$$\frac{20\text{kHz}}{9} = 2.2\text{kHz}$$

- If the response of an amplifier to an applied square wave is an undistorted replica of the input, the frequency response (or BW) of the amplifier is obviously sufficient for the applied frequency.
- If the response is as shown in Fig. 11.60a and b, the low frequencies are not being amplified properly and the low cutoff frequency has to be investigated.
- If the waveform has the appearance of Fig. 11.60c, the high-frequency components are not receiving sufficient amplification and the high cutoff frequency (or BW) has to be reviewed.



- The actual high cutoff frequency (or BW) can be determined from the output waveform by carefully measuring the rise time defined between 10% and 90% of the peak value, as shown in the Fig. below.
- Substituting into the following equation



will provide the upper cutoff frequency, and since $\text{BW} = f_{Hi} - f_{Lo} \approx f_{Hi}$, the equation also provides an indication of the BW of the amplifier.

$$\text{BW} \cong f_{Hi} = \frac{0.35}{t_r}$$

- The low cutoff frequency can be determined from the output response by carefully measuring the tilt and substituting into one of the following equations:

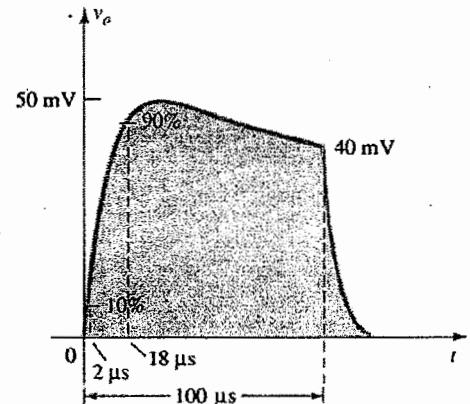
$$\begin{aligned}\% \text{tilt} &= P\% = \frac{V - V'}{V} \times 100\% \\ &= P = \frac{V - V'}{V} \quad (\text{decimal form})\end{aligned}$$

- The low cutoff frequency is then determined from

$$f_{Lo} = \frac{P}{\pi} f_s$$

Ex. The application of a 1-mV, 5-kHz square wave to an amplifier resulted in the output waveform of the given Fig.

- Write the Fourier series expansion for the square wave through the ninth harmonic.
- Determine the bandwidth of the amplifier.



Solution:

$$\begin{aligned}(a) \quad v_i &= \frac{4 \text{ mV}}{\pi} \left(\sin 2\pi (5 \times 10^3)t + \frac{1}{3} \sin 2\pi(15 \times 10^3)t + \frac{1}{5} \sin 2\pi(25 \times 10^3)t \right. \\ &\quad \left. + \frac{1}{7} \sin 2\pi(35 \times 10^3)t + \frac{1}{9} \sin 2\pi(45 \times 10^3)t \right)\end{aligned}$$

$$(b) \quad t_r = 18 \mu s - 2 \mu s = 16 \mu s$$

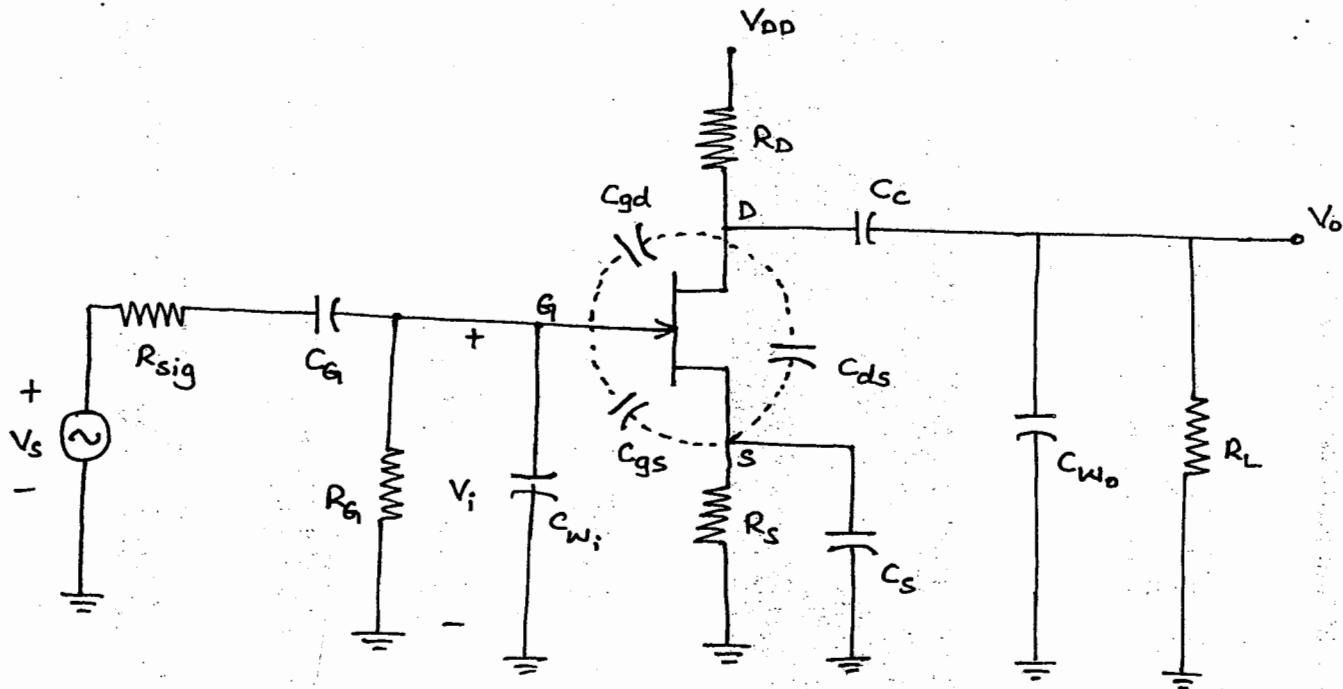
$$\text{BW} = \frac{0.35}{t_r} = \frac{0.35}{16 \mu s} = 21,875 \text{ Hz} \cong 4.4f_s$$

$$(c) \quad P = \frac{V - V'}{V} = \frac{50 \text{ mV} - 40 \text{ mV}}{50 \text{ mV}} = 0.2$$

$$f_{Lo} = \frac{P}{\pi} f_s = \left(\frac{0.2}{\pi} \right) (5 \text{ kHz}) = 318.31 \text{ Hz}$$

High-Frequency Response - FET Amplifier

- The analysis of the high freq. response of the FET amplifier will proceed in a similar manner to that encountered for the BJT amplifier.
- There are electrode and wiring capacitances that will determine the high frequency characteristics of the amplifier.
- The capacitors C_{gs} and C_{gd} typically vary from 1PF to 10 PF, while the capacitance C_{ds} is usually quite a bit smaller, ranging from 0.1 PF to 1 PF.

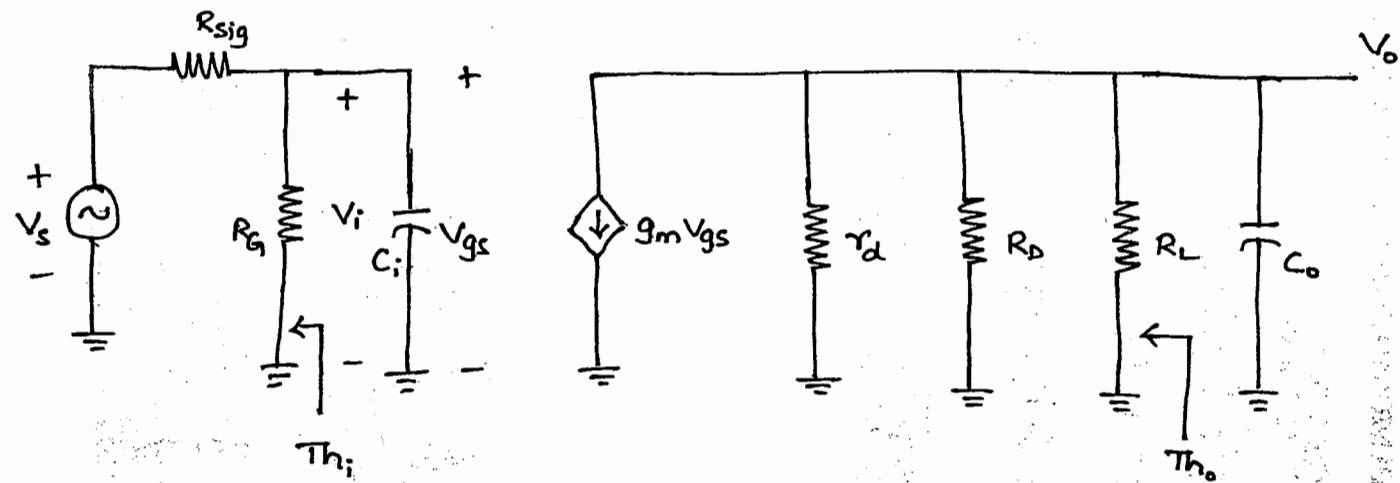


(a) Capacitive Elements that affect the High-frequency Response of a JFET amplifier

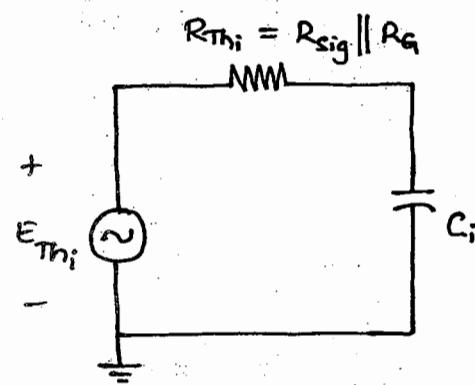
- The network of Fig. (a) is an inverting amplifier
- A Miller effect capacitance will appear in the high freq. ac equivalent network shown in Fig. (b)
- At high frequencies, C_i will approach a short-circuit equivalent

and V_{GS} will drop in value and reduce the overall gain.

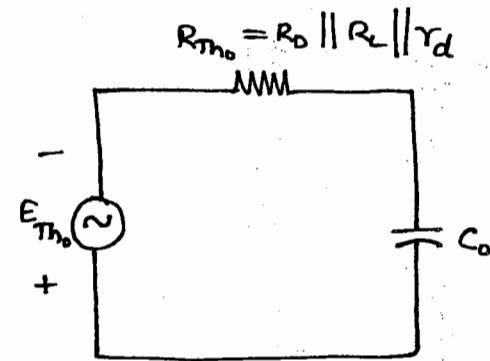
→ At frequencies where C_o approaches its short circuit equivalent, the parallel output voltage V_o will drop in magnitude.



(b) High Frequency ac equivalent circuit for Fig. (a)



(c) Input circuit



(d) Output circuit

Thevenin Equivalent circuit

→ The cutoff freq. defined by the input and output circuits can be obtained by first finding the Thevenin equivalent circuits shown in Fig. (c) and (d).

(i) For the input circuit,

$$f_{T,i} = \frac{1}{2\pi R_{Th_i} C_i}$$

and $R_{Th_i} = R_{sig} \parallel R_G$

$$C_i = C_{w,i} + C_{gs} + C_{m,i}$$

and $C_{M_i} = (1 - A_v) C_{gd}$

(ii) For the output circuit,

$$f_{H_0} = \frac{1}{2\pi R_{Th_0} C_o}$$

$$R_{Th_0} = R_D \parallel R_L \parallel r_d$$

$$C_o = C_{w_0} + C_{ds} + C_{M_0}$$

$$C_{M_0} = \left(1 - \frac{1}{A_v}\right) C_{gd}$$

Example 1: Determine the high-cut off frequencies for the network shown in Fig. (a) using the following parameters.

$$C_G = 0.01 \mu F, \quad C_C = 0.5 \mu F, \quad C_S = 2 \mu F, \quad g_m = 2 \text{ ms}$$

$$R_{sig} = 10 k\Omega, \quad R_g = 1 M\Omega, \quad R_D = 4.7 k\Omega, \quad R_S = 1 k\Omega,$$

$$R_L = 2.2 k\Omega, \quad I_{DSS} = 8 \text{ mA}, \quad V_p = -4 \text{ V}, \quad r_d = 80 \Omega$$

$V_{DD} = 20 \text{ V}$ with the addition of

$$C_{gd} = 2 \text{ pF}, \quad C_{gs} = 4 \text{ pF}, \quad C_{ds} = 0.5 \text{ pF}, \quad C_{w_i} = 5 \text{ pF}, \quad C_{w_0} = 6 \text{ pF}$$

Solution:

$$R_{Th_i} = R_{sig} \parallel R_g = 10 k\Omega \parallel 1 M\Omega = 9.9 k\Omega$$

$$A_v = -g_m (R_D \parallel R_L) = -2 \text{ ms} (4.7 k\Omega \parallel 2.2 k\Omega)$$

$$A_v = -3$$

$$C_i = C_{w_i} + C_{gs} + (1 - A_v) C_{gd}$$

$$= 5 \text{ pF} + 4 \text{ pF} + (1 + 3) 2 \text{ pF}$$

$$C_i = 17 \text{ pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi \times 9.9 k\Omega \times 17 \times 10^{-12} \text{ F}} = 945.67 \text{ kHz}$$

$$R_{Th_0} = R_D \parallel R_L$$

$$R_{Th_0} = 4.7k\Omega \parallel 2.2k\Omega \cong 1.5k\Omega$$

$$C_0 = C_{W_0} + C_{ds} + C_{M_0} = 6 \text{ pF} + 0.5 \text{ pF} + \left(1 - \frac{1}{3}\right) 2 \text{ pF}$$

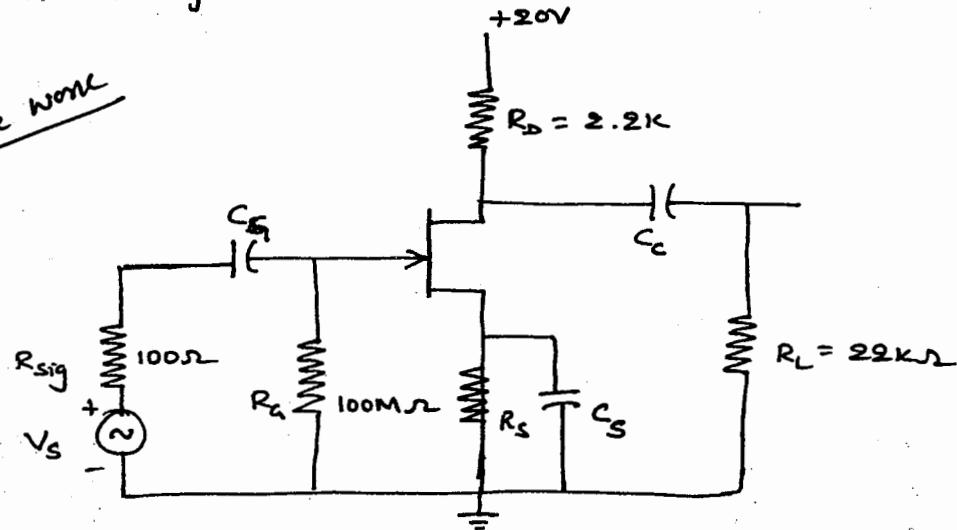
$$C_0 = 9.17 \text{ pF}$$

$$f_{Ho} = \frac{1}{2\pi (1.5k\Omega) (9.17 \text{ pF})} = 11.57 \text{ MHz}$$

The results shows that the input capacitance with its Miller effect capacitance will determine the upper cutoff frequency.

(P) Determine the high freq. response of the amplifier circuit shown in Fig. below:

Home work



$$V_{GS} = -8V$$

$$I_{DSS} = 80 \text{ nA}$$

$$g_m = 6 \text{ ms}$$

$$\tau_d = \infty$$

$$C_{GD} = 2 \text{ pF}$$

$$C_{GS} = 4 \text{ pF}$$

$$C_{W1} = 0$$

$$C_{W0} = 0$$

$$C_{DS} = 0$$

Solution: $f_{H1} = 53 \text{ MHz}$
 $f_{H0} = 36.74 \text{ MHz}$

(P) An amplifier consists of three identical stages in cascade, the bandwidth of overall amplifier extends from 20 Hz to 20 kHz. calculate the bandwidth of individual stage.

Solution:

$$(i) f_1' = \frac{f_1}{\sqrt{2^{1/n} - 1}}$$

$$f_1 = f_1' (\sqrt{2^{1/n} - 1}) = 20 \sqrt{2^{1/3} - 1} = 10.196 \text{ Hz}$$

→ lower 3-dB freq. of single stage.

$$(ii) f_2' = f_2 (\sqrt{2^{1/n} - 1})$$

$$f_2 = \frac{f_2'}{\sqrt{2^{1/n} - 1}} = \frac{20 \times 10^3}{\sqrt{2^{1/3} - 1}} = 39.23 \text{ kHz}$$

→ upper 3-dB freq. of single stage amplifier

$$(iii) \text{ Bandwidth} = f_2 - f_1 = 39.23 \times 10^3 - 10.196 \text{ Hz}$$

$$\boxed{\text{BW} = 39.218 \text{ kHz}}$$

(P) calculate the overall lower 3-dB and upper 3-dB frequencies for a three stage amplifier having an individual $f_1 = 40 \text{ Hz}$ and $f_2 = 2 \text{ MHz}$.

Solution: overall lower 3-dB freq.: f_1'

$$f_1' = \frac{f_1}{\sqrt{2^{1/n} - 1}} = \frac{40}{\sqrt{2^{1/3} - 1}} = 78.458 \text{ Hz}$$

overall higher 3-dB freq.: f_2'

$$f_2' = f_2 (\sqrt{2^{1/n} - 1}) = 2 \times 10^6 (\sqrt{2^{1/3} - 1})$$

$$f_2' = 1.0196 \text{ MHz}$$

- (P) A four stage amplifier has a lower 3-dB freq. for an individual stage of $f_1 = 40 \text{ Hz}$ and individual upper 3-dB frequency of $f_2 = 2.5 \text{ MHz}$. calculate the overall lower 3-dB and upper 3-dB freq. of this full amplifier.

Solution: $f_1' = \frac{f_1}{\sqrt{2^{1/n} - 1}} = \frac{40}{\sqrt{2^{1/4} - 1}} = 91.95 \text{ Hz}$

$$f_2' = f_2 \sqrt{2^{1/n} - 1} = 2.5 \times 10^6 \times \sqrt{2^{1/4} - 1} = 1.087 \text{ MHz}$$

- (P) A two stage cascaded amplifier system is built with a stage voltage gains 25 and 40. Both stages have the same BW of 220 kHz. with identical lower cutoff freq. of 500 Hz. Find the overall gain band-width product.

Solution: The overall voltage gain = $A_v = A_{v1} \times A_{v2} = 25 \times 40$

$$A_v = 1000$$

For each stage: $f_1 = 500 \text{ Hz}$, $f_2 = ?$, $\text{BW} = 220 \text{ kHz}$

$$\text{BW} = f_2 - f_1 \Rightarrow 220 \text{ kHz} = f_2 - 500 \text{ Hz}$$

$$f_2 = 219.5 \text{ kHz}$$

overall lower 3-dB freq. = $f_1' = \frac{f_1}{\sqrt{2^{1/n} - 1}} = \frac{500}{\sqrt{2^{1/2} - 1}}$

$$f_1' = 776.88 \text{ Hz}$$

overall upper 3-dB freq. = $f_2' = f_2 (\sqrt{2^{1/n} - 1})$

$$f_2' = 219.5 \sqrt{2^{1/2} - 1} = 141.59 \text{ kHz}$$

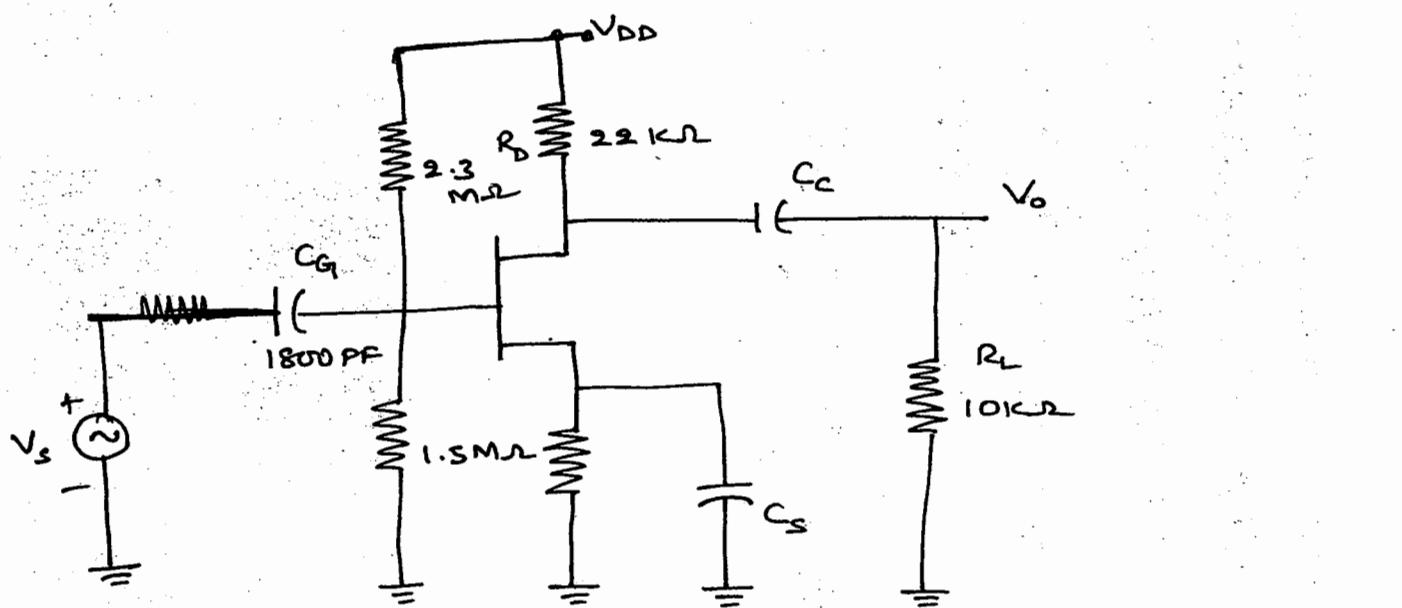
overall BW = $f_2' - f_1' = 141.59 \text{ kHz} - 776.88 \text{ Hz}$
 $= 140.815 \text{ kHz}$

overall gain bandwidth product = $\pi f_L C_{in}$

$$= 1000 \times 140.815 \text{ kHz}$$

$$= 140.815 \times 10^6$$

- (P) For the circuit shown in Fig. find cutoff frequencies due to C_g , C_s , and C_C and due to interelectrode capacitance, C_{gs} and C_{gd} . Given: $g_m = 0.49 \text{ mA/V}$, $C_{gd} = 9.38 \text{ pF}$, $C_{gs} = 1.8 \text{ pF}$, $r_d = \infty \Omega$, $C_{ds} = C_{ws} = C_{w_0} = 0$



Solution: $f_{L_G} = 97.378 \text{ Hz}$ $A_v = -g_m (R_D || R_L)$

$$f_{L_C} = 0.33 \text{ Hz}$$
 $A_v = -3.369$

$$f_{L_S} = 1.629 \text{ Hz}$$

$$C_{in} = C_{gd} (1 - A_v)$$

$$C_{in} = 40.98 \text{ pF}$$

$$C_{out} = 12.16 \text{ pF}$$

$$f_{out} = 1.904 \text{ MHz}$$

Module 3 BJT and JFET Frequency Response

①

Introduction

→ So far,

Exam Marks : 32 (with choice)

< i.e two questions, out of which student has to answer one >

The analysis has been limited to the particular frequency.

→ The frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at higher frequencies are now investigated.

i.e Larger capacitive elements : Low freq. effects
(C/L)

Smaller capacitive elements : High freq. effects.
(BJT / JFET)

active
device

→ The logarithmic scale will be defined and used throughout the analysis, since the analysis will extend through a wide freq. range

→ The industry uses a decibel scale on its freq. plots. Therefore the concept of the decibel is introduced.

→ The freq. response analysis of both BJT and FETs are covered in the same chapter.

3 LOGARITHMS

- In this chapter, there is a need to become comfortable with the logarithmic function.
- The plotting of a variable between wide limits, comparing levels and dealing with unwieldy numbers are impractical.
- The positive features of the logarithm function are:
 - comparing levels without dealing with unwieldy numbers
 - Identifying levels of particular importance in the design.
 - Ease of analysis and review procedure.
- To understand the relationship between the variables of a logarithm function, consider the following mathematical equations:

$$\begin{array}{|c|} \hline a = b^x \\ \hline x = \log_b a \\ \hline \end{array} \quad \text{--- } ①$$

Variables a , b , and x are same in each eqn.

$a = b^x$: a is determined by taking the base b to the x power

$x = \log_b a$: The x will result if the log of a is taken to the base b

Example: If $b = 10$ and $x = 2$, then

$$a = b^x = (10)^2 = 100$$

$$x = \log_b a = \log_{10} 100 = 2$$

To find the power of a number : 10,000, then

$$10,000 = 10^x$$

The level of x is determined using logarithms

$$\text{i.e. } x = \log_{10} 10,000 = 4$$
$$\Rightarrow x = 4$$

Note: For electrical/electronic industry/scientific research, the base in the logarithm equation is chosen as either 10 or the number $e = 2.71828\dots$.

Logarithm taken to the base 10 : referred as "Common Logarithm"

Logarithm taken to the base e : referred as "Natural Logarithm"

$$\boxed{\text{Common logarithm} : x = \log_{10} a} \quad \text{--- (2)}$$

$$\boxed{\text{Natural logarithm} : y = \log_e a} \quad \text{--- (3)}$$

The common and natural logarithm are related by

$$\boxed{\log_e a = 2.3 \log_{10} a} \quad \text{--- (4)}$$

In Scientific calculators use the following key :

Common logarithm :

Natural logarithm :

Example ①

Using the calculator, determine the logarithm of the following numbers to the base indicated:

(a) $\log_{10} 10^6 = 6$

(b) $\log_e e^3 = 3$

(c) $\log_{10} 10^{-2} = -2$

The logarithm of a number taken to a power is simply the power of the number, if the number matches the base of the logarithm.

→ The logarithm of a number does not increase in the same linear fashion as the number

Example: $8000 = 125 \times 64$

The number 8000 is 125 times larger than 64

But, $\log 8000 = 3.903$ }
 $\log 64 = 1.806$ }
 $\log 8000 = 2.16 \times \log 64$

$\Rightarrow 3.903 = 2.16 \times 1.806$ "Non-linear relationship"

$$\log_{10} 10^0 = 0$$

$$\log_{10} 10^1 = 1$$

$$\log_{10} 10^2 = 2$$

$$\log_{10} 10^3 = 3$$

$$\log_{10} 10,000 = 4$$

$$\log_{10} 100,000 = 5$$

$$\log_{10} 1,000,000 = 6$$

$$\log_{10} 10,000,000 = 7$$

$$\log_{10} 100,000,000 = 8$$

This shows that logarithm of a number increases only as the exponent of a number

Note: The antilogarithm of a number is obtained by the func

$$10^x \text{ or } e^x$$

↳ Properties of Logarithms : A review

This chapter employs the common logarithm. Therefore, we review few properties of common logarithms. The same relationship hold true for logarithms to any base.

$$\rightarrow \log_{10} 1 = 0, \text{ because } 10^0 = 1$$

$$\rightarrow \log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b$$

for special case, $a = 1$ becomes

$$\log_{10} \frac{1}{b} = \underline{\log_{10} 1} - \log_{10} b = \underline{\underline{0}} - \log_{10} b$$

\Rightarrow For any b greater than 1, the logarithm of a number less than 1 is always negative

$$\rightarrow \log_{10} ab = \log_{10} a + \log_{10} b$$

Example: Determine the logarithm of the following numbers:

$$(a) \log_{10} 0.5 \quad (b) \log_{10} \frac{4000}{250} \quad (c) \log_{10} (0.6 \times 30)$$

Home work

\rightarrow The use of log scales can significantly expand the range of variation of a particular variable on graph.

\rightarrow Most available graph paper:

1. Semilog (one half): one of the two scales is a log
2. Double log (log-log): Both scales are log scales

\rightarrow Semilog Scale: (use semilog graph)

- Vertical scale: Linear scale with equal divisions

- observe the spacing between the lines of a log plot on the graph.

- $\log_{10} 2 \approx 0.3$ }
 $\log_{10} 1 = 0$ }
The distance from 1 to
is 30% of the span

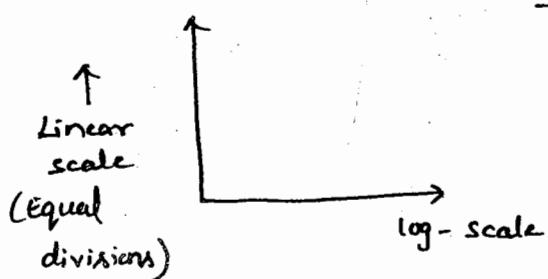
- $\log_{10} 3 = 0.4771$ }
 ~~$\log_{10} 2 = 0.3010$~~ }
 $\log_{10} 1 = 0$ }
The distance from 1 to
is 48% of the span.
 \Rightarrow close to $\frac{1}{2}$ of distance

- $\log_{10} 4 = 0.6021 (\approx 60\%)$

\Rightarrow The distance from 1 to 4 is
60% of the span.

- $\log_{10} 5 \approx 0.7$

\Rightarrow 70% of the distance



- Between any two digits, the same compression of the lines appears as we progress from left to right
- The resulting numerical value and the Spacing is important to note, because the plot will typically have the tic marks, due to lack of Space
- ~~Due to lack of space~~
- The longer bars for this figure have the numerical values of 0.3, 3 and 30 associate with them. The next shorter bars have values of 0.5, 5, and 50 and the shortest bars 0.7, 7, and 70

5 DECIBELS

- The concept of decibels is increasingly important in the remaining sections.
- The term decibel has its origin in the fact that power and audio levels are related in a logarithmic basis.
- An increase in the power level ~~by~~ from 4W to 16W does not result in an audio level increase by a factor of $16/4 = 4$ but by a factor of 2, as derived from the power of 4 in the following manner. $4^2 = 16$

- For a change of 4W to 64W, the audio level increases by a factor of 3, since $4^3 = 64$.

In logarithmic form, the relationship can be written as

$$\log_4 64 = 3$$

- The term bel was derived from the surname of Alexander Graham Bell. For standardization, the bel (B) was defined by the following equation to relate power levels P_1 and P_2 :

$$G = \log_{10} \left(\frac{P_2}{P_1} \right) \text{ bel}$$

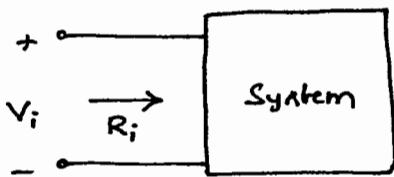
- It was found, however, that the bel was too large a unit of measurement for practical purposes, so the decibel was defined such that 10 decibels = 1 bel. Therefore,

$$G_{dB} = 10 \log_{10} \left(\frac{P_2}{P_1} \right) dB$$

- There exist a second equation for decibels that is applied frequently. It can be best described through the system with R_i , the input resistance of the system.

$$G_{dB} = 10 \log_{10} \left(\frac{P_2}{P_1} \right) = 10 \log \frac{V_2^2 / R_i}{V_1^2 / R_i} = 10 \log_{10} \left(\frac{V_2}{V_1} \right)^2$$

$$G_{dB} = 20 \log \left(\frac{V_2}{V_1} \right) dB$$



→ one of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages. In other words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage.

$$|Av_T| = |Av_1| \cdot |Av_2| \cdot |Av_3| \cdots \cdots |Av_n|$$

Applying the proper logarithmic relationship results in

$$G_V = 20 \log_{10} |Av_T| = 20 \log_{10} |Av_1| + 20 \log_{10} |Av_2| + 20 \log_{10} |Av_3| + \cdots + 20 \log_{10} |Av_n|$$

In words, the equation is

$$G_{dB_T} = G_{dB_1} + G_{dB_2} + G_{dB_3} + \cdots + G_{dB_n}$$

→ The associations between dB levels and voltage gains are as follows:

$$\text{Voltage gain of } 2 = \text{dB level : } +6 \text{ dB}$$

$$10 = \text{dB level : } +20 \text{ dB}$$

$$\frac{\text{Voltage Gain}}{} = \frac{V_o}{V_i} = Av$$

0.5

0.707

1

2

10

40

100

1000

$$\frac{\text{dB Level}}{} =$$

-6 dB

-3 dB

0

6

20

32

40

60

Comparing Av to dB

$$\text{where } Av = \frac{V_o}{V_i}$$

G LOW-FREQUENCY RESPONSE - FET AMPLIFIER

- The analysis of the FET amplifier in the low frequency region is similar to that of BJT amplifier.
- There are again three capacitors of primary concern:
 - (i) C_G
 - (ii) C_C , and
 - (iii) C_S

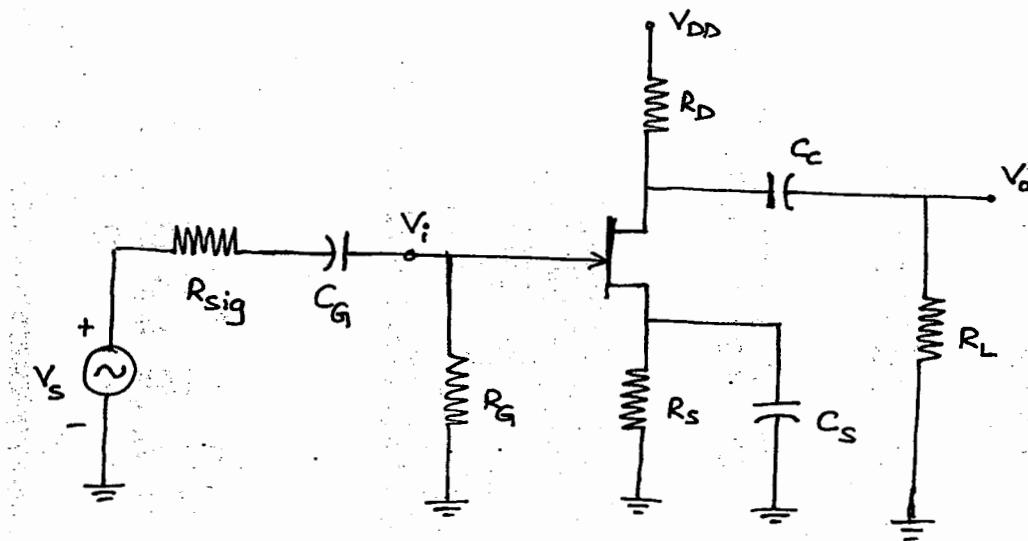


Fig. (a) Capacitive elements that affect the low frequency response of a JFET Amplifier

- The procedures and conclusions applied to the JFET amplifier shown in Fig. (a) can be applied to other FET configurations.

④ The Effect of C_G

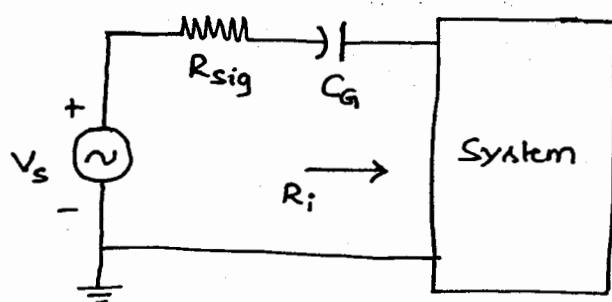


Fig (b) Determining the effect of C_G on the low frequency response

→ Note the coupling capacitor connected between the AC voltage source and the active device. The AC equivalent network is shown in Fig. (b) to determine the effect of C_G on the low frequency response.

→ The cutoff freq. determined by C_G is given by

$$f_{LG} = \frac{1}{2\pi C_G R_i}$$

→ For the network of Fig. (b), $R_i = R_G$

Typically, $R_G \gg R_{sig}$

⇒ The lower cutoff frequency is primarily determined by R_G and C_G

Also, R_G = large, permits relatively low level of C_G while maintaining a low cutoff freq. level for f_{LC}

④ The Effect of C_C

→ Note the C_C connected between the active device and the load of the network.

→ The cutoff freq. determined by Coupling capacitor, C_C is given by

$$f_{LC} = \frac{1}{2\pi (R_o + R_L) C_C}$$

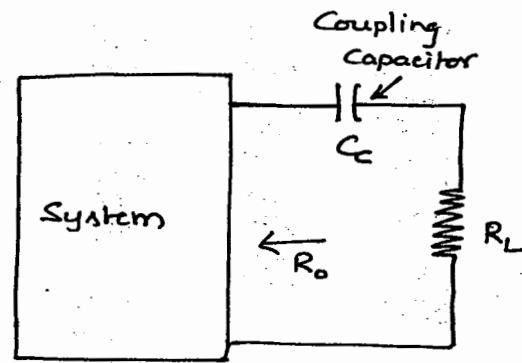
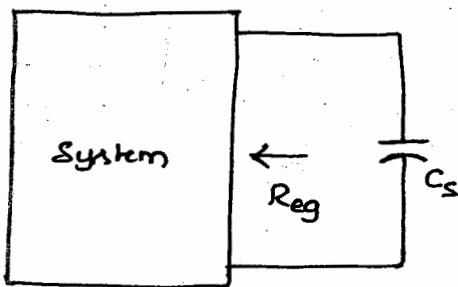


Fig. (c) Determining the effect of C_C on the low freq. Response

$$\text{where } R_o = R_D \parallel \gamma_d$$

④ The Effect of C_S

The cutoff frequency determined by C_S is given by



$$f_{LS} = \frac{1}{2\pi (R_{reg}) C_S}$$

$$\text{where } R_{reg} = \frac{R_S}{1 + \left[\frac{R_S(1 + g_m \gamma_d)}{\gamma_d + R_G \parallel R_L} \right]}$$

For $\gamma_d \approx \infty$

$$R_{reg} = R_S \parallel \frac{1}{g_m}$$

Fig. (d) Determining the effect of C_S on the low frequency Response

Miller Effect Capacitance

→ In the high freq. region the capacitive elements that are important are the

(i) Interelectrode capacitance (Between-terminals)

↳ Internal to the active device

(ii) Wiring capacitance

↳ Capacitance between leads of the network.

→ The large capacitors of the network that controlled the low freq. response are replaced by the short circuit equivalent due to their very low reactance level

$$X_C = \frac{1}{2\pi f C}$$

$C = \text{large}$

$f = \text{low} \rightarrow$

$X_C = \text{low}$

$\langle C_S, C_C, \text{and } C_E : SC \rangle$

→ For inverting amplifiers: (Op-amp based/CE Amplifier)

→ phase shift = 180° b/w input & output

$A_V = \text{Voltage gain} = \text{Negative}$

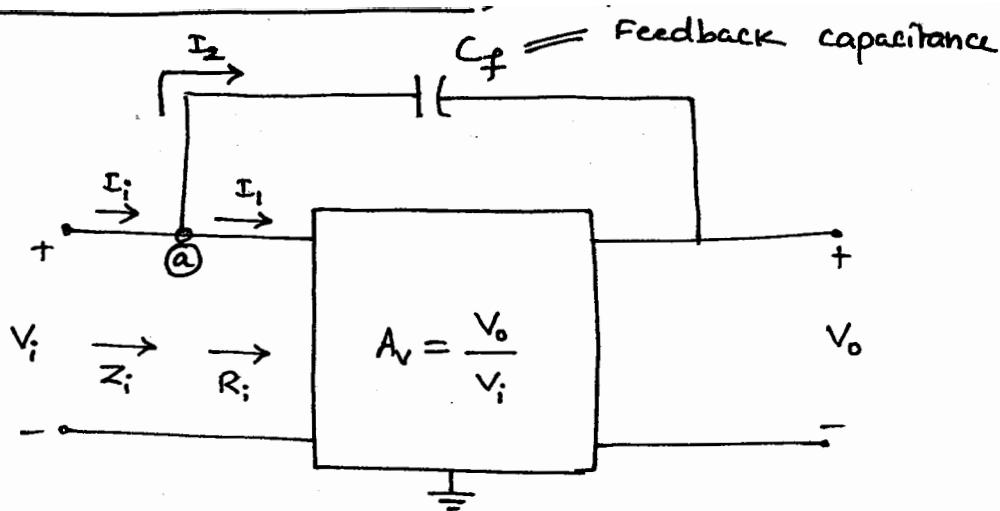
The input and output capacitance of an inverting amplifier is increased by interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier

$$\text{i.e. } C_{io}^{\uparrow} = f (C_{\text{interelectrode}}^{\uparrow}, A_V^{\uparrow})$$

Input-output
capacitance

Interelectrode
capacitance

Voltage
gain



(a) Network Employed in the derivation of an Equation for the Miller Input Capacitance

Applying KCL given at node a

$$I_i = I_1 + I_2 \quad \text{--- (1)}$$

$$\text{using ohm's law, } I_i = \frac{V_i}{Z_i}, \quad I_1 = \frac{V_i}{R_i} \quad \text{--- (2)}$$

$$\text{using nodal analysis, } I_2 = \frac{V_i - V_o}{X_{C_f}} = A_v V_i$$

$$I_2 = \frac{V_i - A_v V_i}{X_{C_f}}$$

$$I_2 = \frac{V_i (1 - A_v)}{X_{C_f}} \quad \text{--- (3)}$$

Substituting (2) and (3) in (1)

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i (1 - A_v)}{X_{C_f}}$$

cancel V_i on both LHS and RHS

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{(1 - A_v)}{X_{C_f}}$$

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{\frac{1}{X_{C_f}}}{\frac{1}{(1 - A_v)}} \quad \text{--- (4)}$$

X_{CM} = Miller capacitive reactance

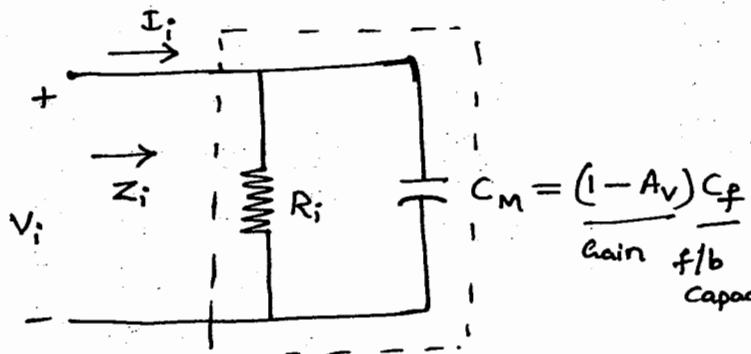
$$\text{But } \frac{x_{cf}}{(1-A_v)} = \frac{1}{\omega(1-A_v)C_f} = x_{cm} \quad \text{--- (5)}$$

$\xrightarrow{\text{cancel}}$

Substituting (5) in (4)

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{x_{cm}} \quad \text{--- (6)}$$

→ Establishes the equivalent network
of Fig. (b)



→ Eqn. (6) is the equivalent
+ input impedance of
the amplifier.

(b) Demonstrating the effect
of Miller Effect
Capacitance

→ Eqn. (6) includes R_i ,
f/b capacitor magnified
by the gain of the
amplifier

→ In general, the miller effect input capacitance is defined by

$$C_{mi} = (1 - A_v) C_f$$

This shows us that

"For any inverting amplifier, the input capacitance will be increased by a miller effect capacitance sensitive to the gain of the amplifier and the interelectrode capacitance between the input and output terminals of the active device"

→ At high frequencies,

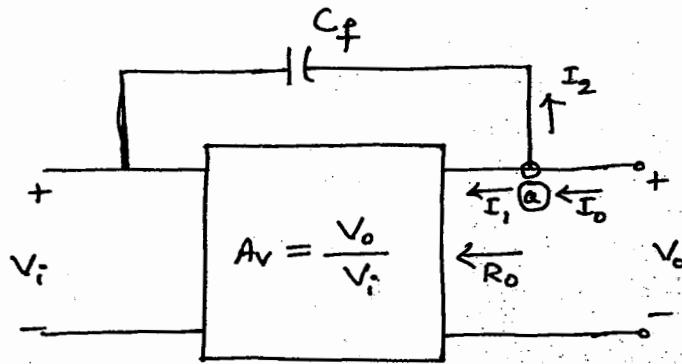
$$A_v = f(C_{mi})$$

$A_{v_{max}} = A_{v_{mid}}$, results in a highest level of C_{mi}

→ A positive value of A_V would result in a negative capacitance.

↳ Miller Output Capacitance, C_{MO}

→ The Miller effect will also increase the level of output capacitance, which must also be determined for high freq. cutoff.



(c) Network employed in the derivation of an equation for the Miller output capacitance

→ The parameters of importance to determine the output miller effect are shown in Fig. (c)

→ Applying KCL at node (a) of Fig. (c)

$$I_o = I_1 + I_2 \quad \text{--- (1)}$$

$$I_1 = \frac{V_o}{R_o} \quad \text{and} \quad I_2 \cong \frac{V_o - V_i}{x_{cf}} \quad \text{--- (2)}$$

The resistance R_o is large, therefore the first term of the equation (1) may be ignored, compared to the second term.

$$\text{i.e. } I_o \cong \frac{V_o - V_i}{x_{cf}} \quad \text{--- (3)}$$

$$I_o = \frac{V_o - \frac{V_o}{A_V}}{x_{cf}}$$

$$I_o = \frac{V_o \left(1 - \frac{1}{A_V}\right)}{x_{cf}}$$

$$\begin{aligned} A_V &= \frac{V_o}{V_i} \\ V_i &= \frac{V_o}{A_V} \end{aligned}$$

$$\frac{I_o}{V_o} = -\frac{1 - \frac{1}{A_v}}{x_{C_f}}$$

$$\frac{V_o}{I_o} = \frac{x_{C_f}}{1 - \frac{1}{A_v}} = \frac{1}{w C_f (1 - \frac{1}{A_v})}$$

$$\boxed{\frac{V_o}{I_o} = \frac{1}{w C_{M_o}}}$$

Therefore, the Miller output capacitance, C_{M_o} is given by

$$\boxed{C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_f}$$

If $A_v \gg 1$, then the above equation becomes

$$\boxed{C_{M_o} \approx C_f}$$

$|A_v| \gg 1$

PROBLEMS

1. Determine the lower cutoff freq. for the network shown in Fig. (a) using the following parameters:

$$C_G = 0.01 \mu F, \quad C_C = 0.5 \mu F, \quad C_S = 2 \mu F, \quad R_{sig} = 10 k\Omega$$

$$R_G = 1 M\Omega, \quad R_D = 4.7 k\Omega, \quad R_S = 1 k\Omega, \quad R_L = 2.2 k\Omega$$

$$V_P = -4V, \quad r_d = \infty, \quad V_{DD} = 20V, \quad V_{GSQ} = -2V, \quad I_{DQ} = 2mA$$

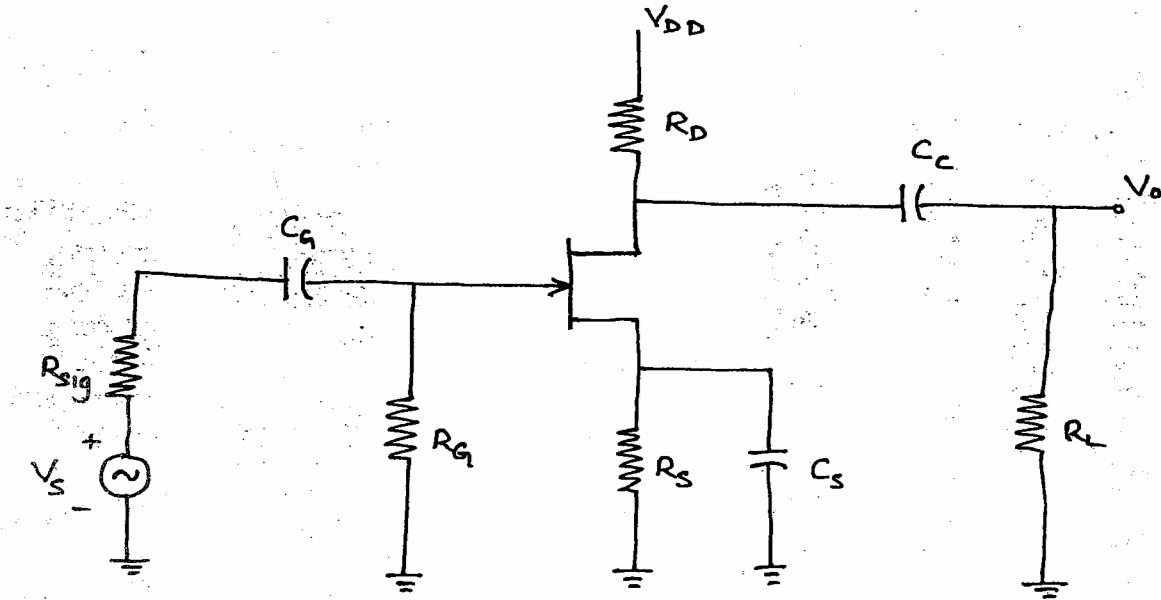


Fig. (a)

Solution: (a) $I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$

$$\text{or } I_{DSS} = \frac{I_D}{\left(1 - \frac{V_{GS}}{V_P}\right)^2} = \frac{2 \times 10^{-3}}{\left(1 - \frac{-2V}{-4V}\right)^2} = \frac{2 \times 10^{-3}}{0.25}$$

$$I_{DSS} = 8 \text{ mA}$$

(b) $g_{m_0} = \frac{2I_{DSS}}{|V_P|} = \frac{2 \times 8 \times 10^{-3}}{4V} = 4 \text{ ms}$

~~$$g_{m_0} = 4 \text{ ms}$$~~

$$g_{m_0} = 4 \text{ ms}$$

$$g_m = g_{m_0} \left(1 - \frac{V_{GSQ}}{V_P}\right) = 4 \text{ ms} \left(1 - \frac{-2V}{-4V}\right) = 2 \text{ ms}$$

$$g_m = 2 \text{ ms}$$

$$C_G : f_{L_G} = \frac{1}{2\pi (R_{sig} + R_i) C_s} = \frac{1}{2\pi (10k\Omega + 1M\Omega) (0.01\mu F)}$$

$$f_{L_G} \cong 15.8 \text{ Hz}$$

$$C_C : f_{L_C} = \frac{1}{2\pi (R_o + R_L) C_C} = \frac{1}{2\pi (4.7k\Omega + 2.2k\Omega) (0.5\mu F)}$$

$$f_{L_C} \cong 46.13 \text{ Hz}$$

$$C_S : f_{L_S} = \frac{1}{2\pi R_{eq} C_S} ; R_{eq} = R_s \parallel \frac{1}{g_m} = 1k\Omega \parallel 0.5k\Omega = 333.3$$

$$f_{L_S} = \frac{1}{2\pi (333.33 \Omega) (2\mu F)} = 238.73 \text{ Hz}$$

$$f_{L_S} \cong 238.73 \text{ Hz}$$

The f_{L_S} is largest of the three cutoff frequencies, it defines the low-cut frequency for the ~~circuit~~ network.

(c) The midband gain of the system is determined by

$$\begin{aligned} A_{V_{mid}} &= \frac{V_o}{V_i} = -g_m (R_o \parallel R_L) \\ &= -(2ms) (4.7k\Omega \parallel 2.2k\Omega) \\ &= -(2ms) (1.499k\Omega) \end{aligned}$$

$$A_{V_{mid}} \cong -3$$

2. Determine the lower cutoff frequency for the BJT amplifier for the following parameters:

$$C_s = 10\mu F, C_E = 20\mu F, C_C = 1\mu F$$

$$R_s = 1k\Omega, R_i = 40k\Omega, R_2 = 10k\Omega, R_E = 2k\Omega, R_C = 4k\Omega$$

$$R_L = 2.2k\Omega, \beta = 100, r_o = \infty \Omega, V_{CC} = 20V.$$

(a) To determine r_e for dc conditions, first apply test condition

$$\left. \begin{array}{l} \beta R_E = 100 \times 2k\Omega = 200k\Omega \\ 10R_2 = 10 \times 10k\Omega = 100k\Omega \end{array} \right\} \quad \beta R_E \gg 10R_2$$

Condition : $\beta R_E \gg 10R_2$, The condition is satisfied.

The dc base voltage is determined by

$$V_B = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{(10k\Omega)(20V)}{10k\Omega + 40k\Omega} = \frac{200V}{50} = 4V$$

$$V_B = 4V \quad 0.7V \text{ (for silicon BJT)}$$

$$I_E = \frac{V_E}{R_E} = \frac{V_B - V_{BE}}{R_E} = \frac{4V - 0.7V}{2k\Omega} = \frac{3.3V}{2k\Omega}$$

$$I_E = 1.65 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.65 \text{ mA}} = 15.76 \Omega$$

$$r_e = 15.76 \Omega$$

$$\beta r_e = 100 \times 15.76 \Omega = 1.576k\Omega$$

$$A_V = \frac{V_o}{V_i} = -\frac{R_C || R_L}{r_e} = -\frac{(4k\Omega) || (2.2k\Omega)}{15.76 \Omega} = -9$$

$$A_V = -90$$

$$Z_i = R_i = R_1 || R_2 || \beta r_e$$

$$= 40k\Omega || 10k\Omega || 1.576k\Omega$$

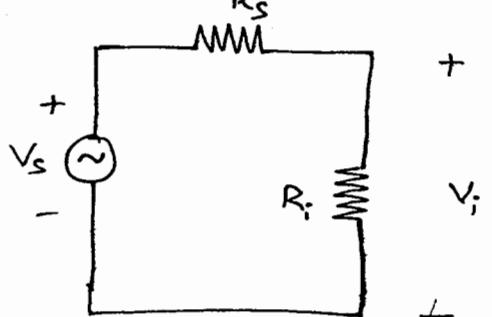
$$Z_i = 1.32k\Omega$$

Fig. Det. the effect of R_s on the gain A_{VS}

$$V_i = \frac{R_s V_s}{R_s + R_i}$$

$$\frac{V_i}{V_s} = \frac{R_i}{R_s + R_i} = \frac{1.32k\Omega}{1.32k\Omega + 1k\Omega}$$

$$\frac{V_i}{V_s} = 0.569$$



$$\text{So that } A_{V_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$$

$$A_{V_s} = (-90)(0.569) = -51.21$$

$$A_{V_s} = -51.21$$

$$\underline{\underline{C_s}} : f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1k\Omega + 1.32k\Omega)(10\mu F)}$$

$$f_{L_s} = 6.86 \text{ Hz}$$

$$\underline{\underline{C_C}} : f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C} = \frac{1}{(6.28)(4k\Omega + 2.2k\Omega)(1\mu F)}$$

$$f_{L_C} = 25.68 \text{ Hz}$$

$$\underline{\underline{C_E}} : R'_s = R_s \parallel R_1 \parallel R_2 = 1k\Omega \parallel 40k\Omega \parallel 10k\Omega \cong 0.889\Omega$$

$$R_e = R_E \parallel \left(\frac{R'_s}{\beta} + r_e \right) = 2k\Omega \parallel \left(\frac{0.889k\Omega}{100} + 15.76\Omega \right)$$

$$R_e = 2k\Omega \parallel (8.89\Omega + 15.76\Omega)$$

$$R_e = 2k\Omega \parallel 24.65\Omega$$

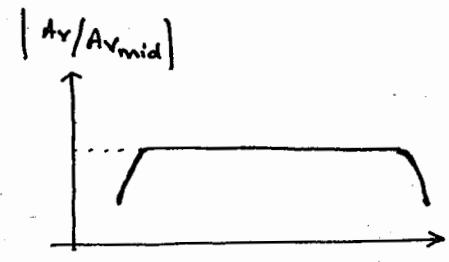
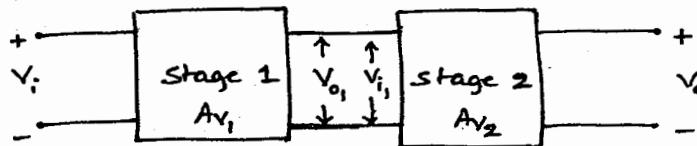
$$R_e \cong 24.35\Omega$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\Omega)(20\mu F)} = 327 \text{ Hz}$$

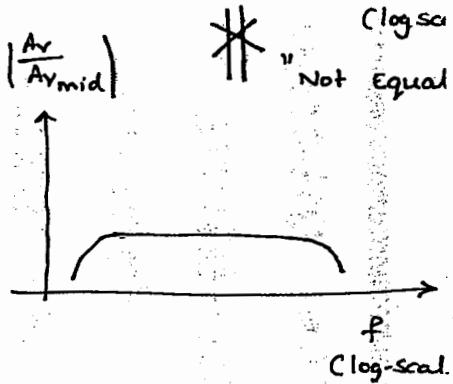
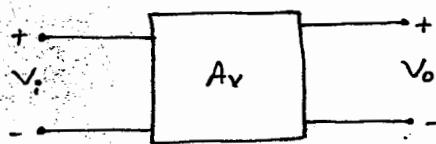
$$f_{L_E} = 327 \text{ Hz}$$

9 Multistage Frequency Effects

- There will be a significant change in the overall frequency response for the second transistor stage connected directly to the output of first stage.



Two Stage Amplifier

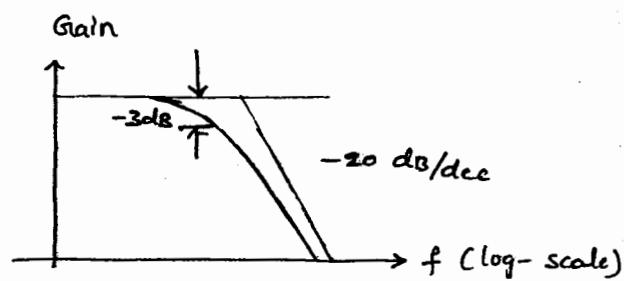
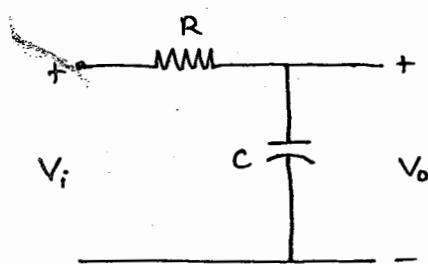


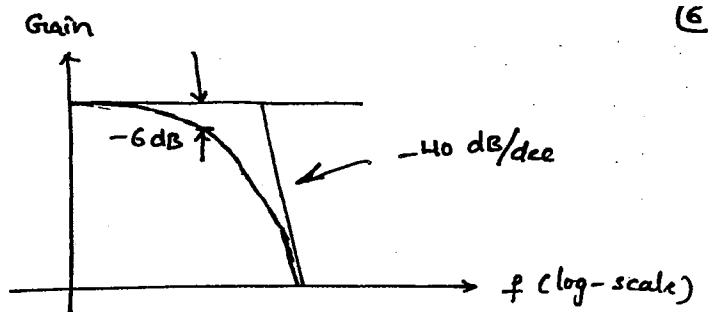
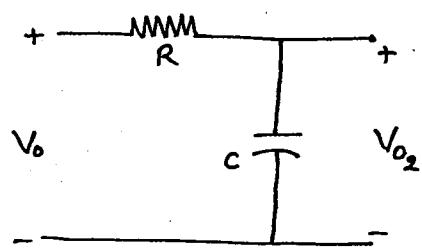
single stage Amplifier

- In the high freq. region, the output capacitance, C_o , must include the following :

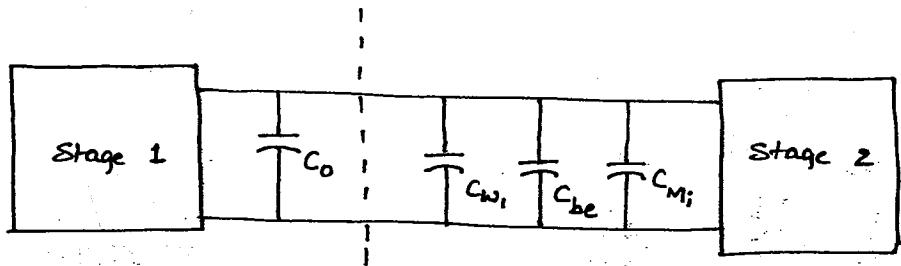
- (i) The wiring capacitance, C_{w1}
 - (ii) The parasitic capacitance, C_{be}
 - (iii) The Miller capacitance, C_M ;
- The C_o that includes for the following stage

- There will be additional low freq. cutoff levels due to the second stage, which will further reduce the overall gain of the system in this region.





→ Output of first stage affects the input of second stage.



→ For each additional stage, the upper cutoff freq. will be determined primarily by the stage having the lowest cutoff frequency. The lower cutoff freq. is primarily determined by the stage having the highest low cutoff freq.

⇒ Therefore, the poorly designed stage can affect an otherwise well designed cascaded system.

→ The effect of increasing the number of identical stages can be clearly demonstrated using the fig. (a)

- In each case, the upper and lower cutoff frequencies of each of the cascaded stages are identical.
- For a single stage, the cutoff frequencies are f_1 and f_2 .
- For two identical stages in cascade, the drop-off rate in the high and low frequency regions has increased to -12 dB/octave or -40 dB/decade .
- At f_1 and f_2 , the decibel drop is now -6 dB rather than the defined band freq. gain level of -3 dB .
- The -3 dB point has shifted to f'_1 and f'_2 as indicated in Fig. (a) with the resulting drop in the bandwidth.
- A -18 dB/octave or -60 dB/decade slope will result for a three stage system of identical stages with the indicated reduction in bandwidth (f''_1 and f''_2)

i.e $\underline{f_1}$: Single stage
 $\underline{\underline{\text{lower cutoff freq.}}}$

$\underline{f_2}$: Single stage upper
 $\underline{\underline{\text{cutoff freq.}}}$

$\underline{f'_1}$: Two stage lower
 $\underline{\underline{\text{cutoff freq.}}}$

$\underline{f'_2}$: Two stage upper
 $\underline{\underline{\text{cutoff freq.}}}$

$\underline{f''_1}$: Three stage lower
 $\underline{\underline{\text{cutoff freq.}}}$

$\underline{f''_2}$: Three stage upper
 $\underline{\underline{\text{cutoff freq.}}}$

f_1 and f_2 : Decibel drop = -3 dB : Slope = -20 dB/dec
 -6 dB/oct

f'_1 and f'_2 : $\frac{-11}{\text{---}}$ = -6 dB : Slope = -40 dB/dec
 -12 dB/oct

f''_1 and f''_2 : $\frac{-11}{\text{---}}$ = -9 dB : Slope = -60 dB/dec
 -18 dB/oct

- Assuming identical stages, we can determine an equation for each band freq. as a function of the number of stages, n

- For the low frequency region,

$$A_{V, \text{low, (overall)}} = A_{V_1, \text{low}} \cdot A_{V_2, \text{low}} \cdot A_{V_3, \text{low}} \cdots A_{V_n, \text{low}}$$

The overall voltage gain at lower freq. is given by $A_{V, \text{low, (overall)}}$

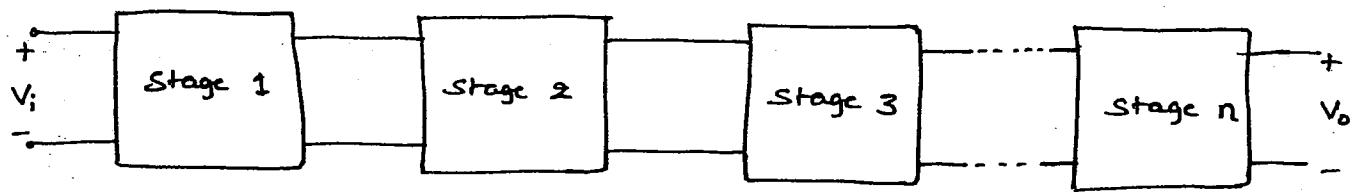


Fig. (b) : Cascade connection of n -identical stages
of Amplifier

Since all the stages are identical, then

$$A_{V_{\text{low}}} = A_{V_2 \text{ low}} = A_{V_3 \text{ low}} = \dots = A_{V_n \text{ low}}$$

Therefore, $A_{V_{\text{low, overall}}} = \cancel{A_{V_{\text{low}}}} (A_{V_{\text{low}}})^n$

The low freq. gain $A_{V_{\text{low}}}$ for one stage is given by

$$|A_{V_{\text{low}}}| = \frac{|A_{V_{\text{mid}}}|}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$$

$$\frac{|A_{V_{\text{low}}}|}{|A_{V_{\text{mid}}}|} = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$$

Therefore, for n -stages in cascade connection, we have

$$\left[\frac{|A_{V_{\text{low}}}|}{|A_{V_{\text{mid}}}|} \right]^n = \left[\frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \right]^n \quad \text{--- (1)}$$

Now setting the magnitude of the result equal to $\frac{1}{\sqrt{2}} = -3 \text{ dB}$
Eqn. (1) becomes for n -stages

$$\left[\frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \right]^n = \frac{1}{\sqrt{2}} = \left[\frac{|A_{V_{\text{low}}}|}{|A_{V_{\text{mid}}}|} \right]^n$$

$f = f_1$

Squaring on both sides

$$\sqrt{2} = \left[\sqrt{1 + \left(\frac{f_1}{f}\right)^2} \right]^n \quad \text{squaring on both sides}$$

$$2 = \left[1 + \left(\frac{f_1}{f'_1} \right)^2 \right]^n$$

Taking n^{th} root on both sides

$$2^{1/n} = \left[1 + \left(\frac{f_1}{f'_1} \right)^2 \right]^{n/n}$$

$$2^{\frac{1}{n}} = 1 + \left(\frac{f_1}{f'_1} \right)^2$$

$$\left(\frac{f_1}{f'_1} \right)^2 = 2^{\frac{1}{n}} - 1$$

$$\frac{f_1}{f'_1} = \sqrt{(2^{\frac{1}{n}} - 1)}$$

$$f'_1 = \frac{f_1}{\sqrt{(2^{\frac{1}{n}} - 1)}}$$

where f'_1 = lower 3-dB freq. of identical cascaded stages

f_1 = lower 3-dB freq. of single stage.

n = no. of stages.

g Overall Higher cutoff frequency of multi-stage Amplifier

→ For n -stages in cascade connection, we have

$$\left\{ \frac{|A_{V_{\text{high}}}|}{|A_{V_{\text{mid}}}|} \right\}^n = \left[\frac{1}{\sqrt{1 + \left(\frac{f}{f_2'} \right)^2}} \right]^n \quad \text{--- } ①$$

→ Let f_2' be the upper cutoff freq. for n -stage amplifier in cascade. Therefore, at $f_2' = f$, we have magnitude $= \frac{1}{\sqrt{2}}$
 $= -3$ dB.

$$\left\{ \frac{|A_{V_{\text{high}}}|}{|A_{V_{\text{mid}}}|} \right\}_{f=f_2'}^n = \left[\frac{1}{\sqrt{1 + \left(\frac{f_2'}{f_2} \right)^2}} \right]^n = \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \left[1 + \left(\frac{f_2'}{f_2} \right)^2 \right]^n$$

Squaring on both sides, we get

$$2 = \left[1 + \left(\frac{f_2'}{f_2} \right)^2 \right]^n$$

Taking n^{th} root on both sides, we get

$$2^{1/n} = \left[1 + \left(\frac{f_2'}{f_2} \right)^2 \right]^{n/n}$$

$$2^{1/n} = 1 + \left(\frac{f_2'}{f_2} \right)^2$$

$$2^{1/n} - 1 = \left(\frac{f_2'}{f_2} \right)^2$$

$$\frac{f_2'}{f_2} = \sqrt{(2^{1/n} - 1)}$$

$$f_2' = f_2 (\sqrt{(2^{1/n} - 1)})$$

→ Note the presence of $\sqrt{2^{1/n} - 1}$ in both f_1' and f_2' equation. The magnitude of this factor for various value of n are listed below

| n | $\sqrt{2^{1/n} - 1}$ |
|-----|----------------------|
| 2 | 0.64 |
| 3 | 0.51 |
| 4 | 0.43 |
| 5 | 0.39 |

→ For $n=2$, consider the upper cutoff freq. $f_2' = 0.64 f_2$ or 64% of the value obtained for a single stage

→ For $n=2$, the lower cutoff freq. $f_1' = \left(\frac{1}{0.64} \right) f_1 = 1.56 f_1$

- For $n=3$, $f_2 = 0.5f_1$ or approximately one half of the value of single stage.
- For $n=3$, $f'_1 = \left(\frac{1}{0.5}\right)f_1 = 1.96 f_1$ = twice the single stage value.
- A decrease in BW is not always associated with an increase in the no of stages, if the mid-band gain can remain fixed and independent of the no of stages.

i.e single stage amplifier

$$\text{Gain} = 100$$

$$\text{BW} = 10 \text{ KHz} = 10 \times 10^3$$

$$\text{GBW} = 100 \times 10 \text{ KHz} = 10^6$$

Two stage Amplifier

$$\text{Gain} = 10 \quad (10 \times 10 = 100) \quad \text{Same}$$

$$\text{Bandwidth} = 100 \text{ KHz}$$

$$\text{GBW} = 10 \times 10 \times 10^3 = 10^6$$

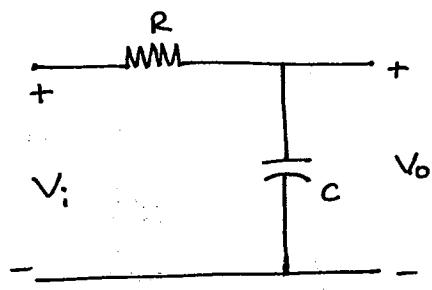
For the two stage amplifier, the same gain can be obtained by having two stages with a gain of 10 because $10 \times 10 = 100$

High Frequency Response - BJT Amplifier

The two factors that define the -3 dB cutoff point are

1. The network capacitance (parasitic and introduced)
2. The Frequency dependence of h_{fe} (β)

Network Parameters



→ In the high freq. region, the RC n/w configuration appears as shown in Fig. (a)

→ At ↑ freq. $X_C \downarrow$ in magnitude resulting in a shorting effect across the output

⇒ Gain ↓

→ The derivation leading to the

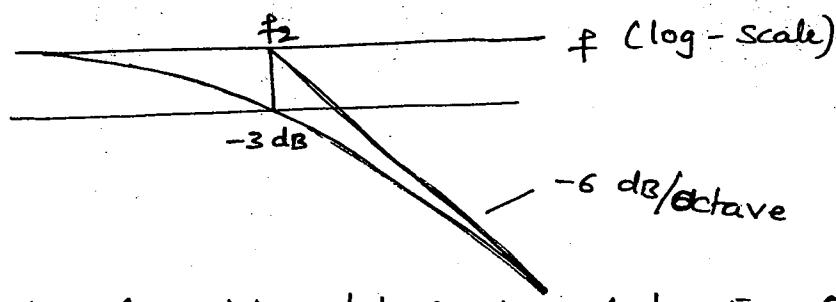
Fig. (a) RC combination

that will determine a

high - cutoff frequency

corner frequency for this RC configuration follows similar lines to that encountered for the low freq. region.

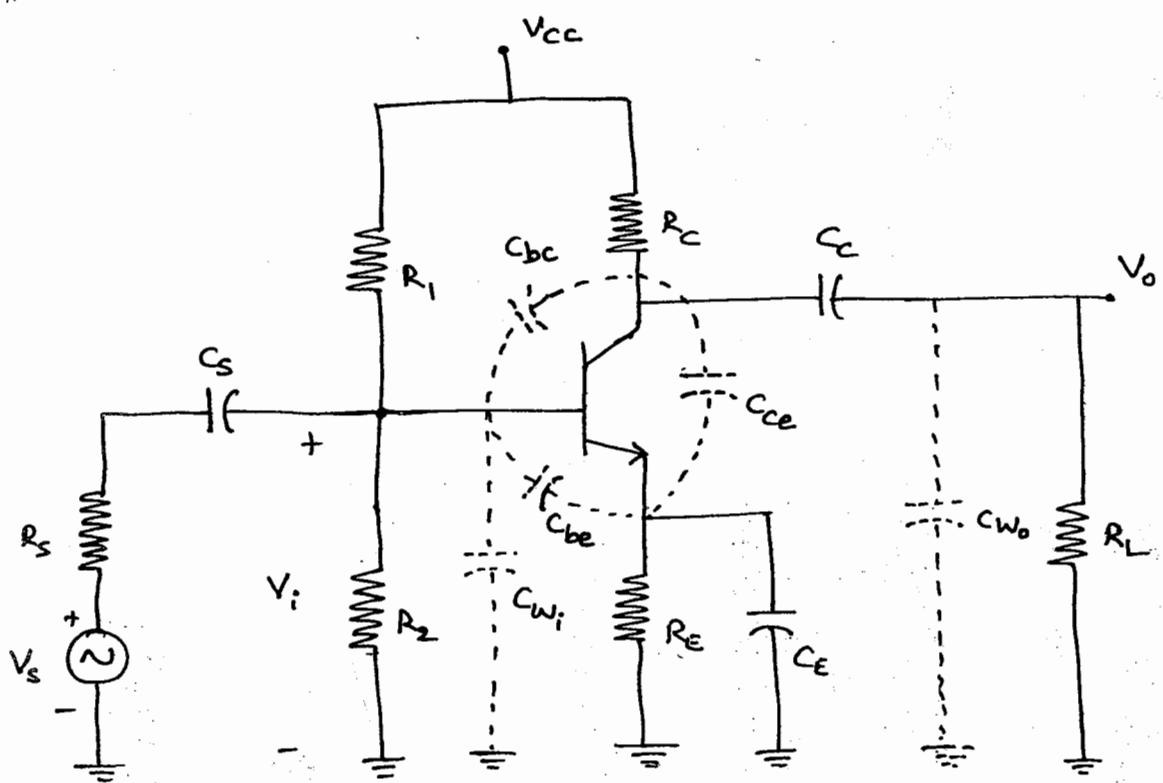
$$A_v = \frac{1}{1 + j\left(\frac{f}{f_2}\right)} \quad \text{--- (1)}$$



(b) Asymptotic plot as defined by Eqn. (1)

→ The magnitude plot drops off at 6 dB/octave with ↑ frequency.

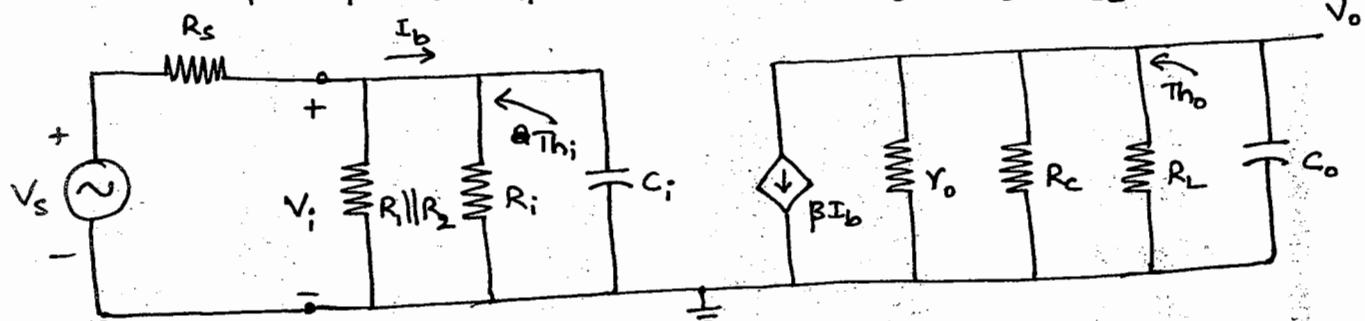
→ Note that f_2 is in the denominator of the freq. ratio



(c) The Network with the capacitors that affect the high Frequency Response

$$C_i = C_{wi} + C_{be} + C_{mi}$$

$$C_o = C_{wb} + C_{ce} + C_{mo}$$

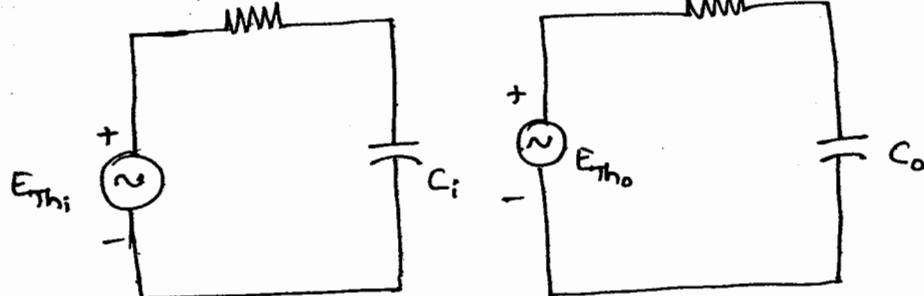


(d) High Frequency ac equivalent model for the Network of

Fig. (c)

$$R_{Thi} = R_s \parallel R_1 \parallel R_2 \parallel R_i$$

$$R_{Tho} = R_C \parallel R_L \parallel r_o$$



Thevenin's Equivalent
of input network
of Fig. (d)

Thevenin's Equivalent
of output network
of Fig. (d)

- The various parasitic capacitances (C_{be} , C_{bc} , C_{ce}) of the transistor have been included with the wiring capacitance (C_{w_i} , C_{w_o}) introduced during construction.
- In the high frequency equivalent model for the network, note the absence of the capacitors C_s , C_c , and C_E which are all assumed to be in short-circuit state at these frequencies.
- The capacitance, C_i includes the input wiring capacitance C_{w_i} , the transition capacitance C_{be} , and the Miller capacitance C_{M_i} .
- The capacitance C_o includes the output wiring capacitance C_{w_o} , the parasitic capacitance C_{ce} and the output Miller capacitance C_{M_o} .
- In general, the capacitance, C_{be} is the largest of the parasitic capacitances with C_{ce} the smallest. In fact, most specification sheets simply provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of a particular type of transistor in a specific area of application.

1. For the Input Network, C_i

The 3-dB freq. for the input network is defined by

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

where $R_{Th_i} = x_s \parallel r_1 \parallel R_2 \parallel r_2$

$$C_i = C_{W_i} + C_{be} + C_{M_i}$$

$$C_i = C_{W_i} + C_{be} + (1 - A_v) C_{bc}$$

2. For the output Network, C_o

$$f_{H_0} = \frac{1}{2\pi R_{Th_o} C_o}$$

$$R_{Th_o} = R_c \parallel R_L \parallel r_o$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

$$C_o = C_{W_o} + C_{ce} + \left(1 - \frac{1}{A_v}\right) C_{bc}$$

$$1 \gg \frac{1}{A_v}$$

Therefore, $C_o = C_{W_o} + C_{ce} + C_{bc}$

→ At very high freq., the capacitance reactance of C_o will decrease and consequently reduce the total impedance of the output parallel branches.

→ The frequencies f_{H_i} and f_{H_o} will define a -6 dB/octave asymptote

→ If the parasitic capacitors were the only elements to determine the higher cutoff frequency, the lowest frequency would be the determining factor.

→ The decrease in β_f^{\downarrow} or β_f^{\uparrow} with freq. must also be considered as to whether its break freq. is lower than f_{H_i} or f_{H_o} .