

Pneumatic

## UNIT - 1 - Review Of FLUID MECHANICS.

Continuity      Equation :-

Physical principle - Mass can neither be created nor be destroyed.

→ Let  $A_1$  be the cross sectional area of the stream-tube

@ pt 1.

→ Let  $v_1$  be flow velocity @ pt 1.

→ Consider all the fluid elements are in the plane  $A_1$ .

→ After lapse of time  $dt$ , these fluid element move through a certain distance  $v_1 dt$ .

→ The volume swept by the fluid is  $A_1 v_1 dt$ .

→ Mass of the gas  $dm$  in the volume is

$$dm = P_1 (A_1 v_1 dt) \rightarrow ①.$$

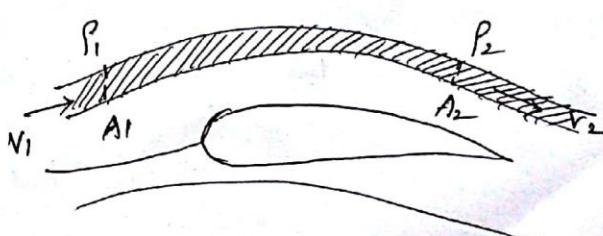
The mass of gas that has swept through area  $A_1$ , during  $dt$  is ①.

$$\text{Mass flow} = \frac{dm}{dt} = \dot{m} = P_1 A_1 v_1 \text{ Kg/s.}$$

$$\text{Mass flow through } A_2 = P_2 A_2 v_2$$

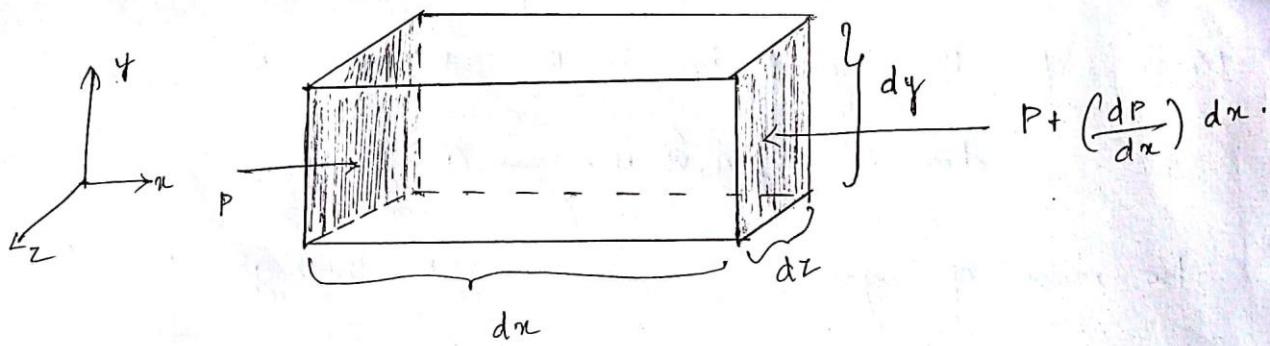
According to our principle

$$P_1 A_1 v_1 = P_2 A_2 v_2$$



### MOMENTUM EQUATION

- Physical Principle : Force = Mass  $\times$  Acceleration
- Consider a small fluid element moving along a stream
- has with velocity "v".
- At some instant, the element is located @ point "P"
- The force on this element is due to 3 phenomena:
1. Pressure acting in a normal direction on all the faces of the element.
  2. Frictional shear acting tangentially on all faces.
  3. Gravity acting on mass inside the element.



Consider left and right faces i.e. tr to x axis.

→ Pressure on left face is  $P$  and the area is  $dy \cdot dz$ .

Hence the force =  $P_A \times \text{Area}$

$$= P dy dz.$$

Pressure varies from  $p_1$  to  $p_2$ . The variation in pressure in  $x$  axis is given by  $\frac{dp}{dx}$ .

→ Change in pressure along the axis by a distance  $dx$  is given by  $\frac{dp}{dx} \cdot dx$ .

$P_H$  on right face is  $(P + \frac{dp}{dx} \cdot dx)$

$$\text{Force} = P \times \text{Area} \Rightarrow \left( P + \frac{dp}{dx} \cdot dx \right) dy dz \rightarrow ①.$$

Total force is given by:

$$F = P \cdot dy dz - \left[ P + \left( \frac{dp}{dx} \right) dx \right] dy dz$$

$$F = - \underbrace{\left( \frac{dp}{dx} \right) dx}_{=} dy dz \rightarrow ②$$

Mass of the fluid element is given by:

$$M = \underbrace{P dx dy dz}_{\text{Volume}} \rightarrow ③$$

Acceleration of fluid element

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$= \bar{V} \frac{d\bar{v}}{dx} \rightarrow ④$$

Sub 2, 3, 4 in basic eqn:  $F = ma$ .

$$-\frac{dp}{dx} dx dy dz = \bar{V} \frac{d\bar{v}}{dx} P dx dy dz$$

$$-\frac{dp}{dx} = \rho \bar{v} \frac{d\bar{v}}{dx}$$

$$-dp = \rho \bar{v} d\bar{v} \rightarrow (5)$$

(5) is the Euler's equation for steady inviscid flow.

In other words it is called Momentum eqn

$$\rightarrow \text{ so } dp + P dv = 0.$$

$$\rightarrow \int_1^2 dp = - \int_1^2 P dv.$$

$$P_1^2 = - [P v_{1/2}^2]_1^2 + C$$

$$P_2 - P_1 = -P \frac{[v_2^2 - v_1^2]}{2} + C$$

(or)

$$P_1 + \gamma_2 P v_1^2 = P_2 + \gamma_2 P v_2^2 \rightarrow (6)$$

Eqn (6) is called Bernoulli's eqn.

Energy Equation :-

Physical principle :- Energy can neither be created nor be destroyed

According to 1 law of Thermodynamics.

$$\delta q + \delta w = de \rightarrow (1)$$

According to thermodynamic equations.

$$\delta q = dh - v dp$$

If the flow is adiabatic. then  $\delta q = 0$ .

$$dh - \bar{v} dP = 0 \rightarrow (2) \quad \text{Chanc } \bar{v} \text{ is volume}$$

$$\text{For Euler's eqn} \quad dP = -Pvdv \rightarrow (3) \quad \text{sub (3) in (2)}$$

$$dh - \bar{v}(Pvdv) = 0 \quad \text{(here } v = v \text{ do it)}$$

$$dh + \bar{v} Pvdv = 0 \quad (here \quad P = \frac{1}{\bar{v}})$$

$$dh + \bar{v} x \frac{1}{\bar{v}} Pvdv$$

$$dh + Pv dv = 0 \rightarrow (4)$$

$$\text{On integrating (4)} \quad \int dh + \int v^2 dv = 0$$

$$(h_2 - h_1) + \left( \frac{v_2^2 - v_1^2}{2} \right) = c$$

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} \rightarrow (5)$$

$$(or) \quad h + \frac{v^2}{2} = \text{constant} \rightarrow (6).$$

here (5) & (6) is energy eqn for frictionless  
adiabatic flow

$$\text{if } h = c_P T$$

$$c_P T_1 + \frac{v_1^2}{2} = c_P T_2 + \frac{v_2^2}{2}$$

$$(iv) \quad \underline{\underline{c_P T + \frac{v^2}{2} = c}}$$

## Types of Flows :-

### Continuum      Versus Free      Molecular flow

→ The mean distance that the body travels between collision with neighbouring molecules is defined as Mean free Path  $\lambda$ .

→ If  $\lambda$  is order of magnitude smaller than the scale of the body measured by "d" (i.e)  $\lambda \ll d$ .

then molecules impact the surface so frequently that body can't differentiate individual molecular collision.

→ So surface feels the fluid as continuous medium and it is called as continuum flow.

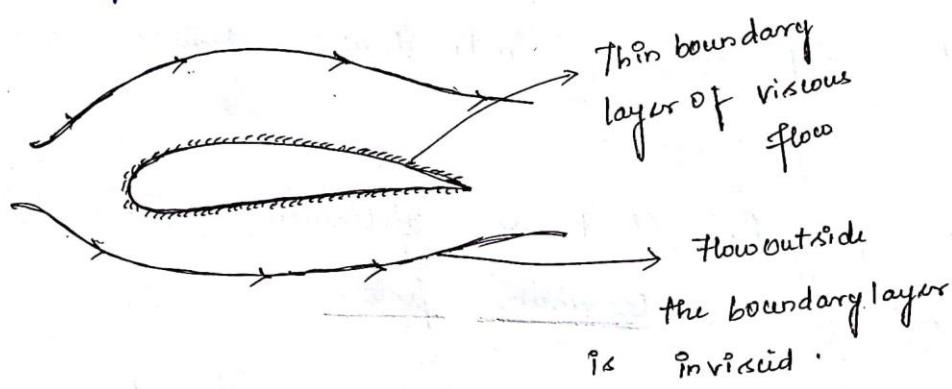
→ If  $\lambda$  is of small same order as "d" then the collision occurs infrequently & body surface can feel each molecular impact, such flow is free

### Molecular flow

## Inviscid Vs Viscid flow:

→ If the flow doesn't have any friction, thermal conduction (or) diffusion it is called as inviscid flow

- They don't exist in nature.
- Viscous flow are the one which involves friction, thermal conduction, diffusion.
- When Reynold's number goes to infinity we will consider the flow to be inviscid.
- In some case with high & finite Re number the flow is considered as inviscid, because the proportion are limited to this region adjacent to body surface called boundary layer.



### Compressible vs Incompressible flow?

→ Flow in which density is constant is called incompressible flow.

→ A flow where density  $\rho$  is variable is called compressible flow.

friction,

### Compressibility :-

- Consider a small element of fluid in volume  $V$ .
- The pressure exerted on sides of element by the fluid is "P".
- Assume the pressure is increased by  $dP$ .
- The volume of the element will be even more compressed by  $dV$ ; Here  $dV$  is  $(-v_c)$  because volume is decreased.
- Compressibility of the fluid  $\gamma$  is defined as

$$\gamma = -\frac{1}{V} \frac{dV}{dP}$$

- When gas is compressed temperature is increased depending on amount of heat transferred.
- If temperature is constant then Isothermal Compressibility is given by

$$\gamma_T = -\frac{1}{V} \left( \frac{dV}{dP} \right)_T$$

called

called

- In case of Isentropic compression (i.e) no heat and no other dissipative transport mechanism then Isentropic Compressibility is defined as

$$\gamma_s = -\frac{1}{V} \left( \frac{dV}{dP} \right)_s$$

→ consider fluid element of unit mass  
where  $v$  = specific volume & density is  $\rho = m/v = 1/v$

$$\text{so } v = 1/\rho.$$

$$\tau = -P \frac{d(\gamma g)}{dp}$$

$$= -P (-\gamma \rho^2) \frac{d\delta}{dp}$$

$$= \underline{\underline{(\gamma \rho) \frac{d\delta}{dp}}}.$$

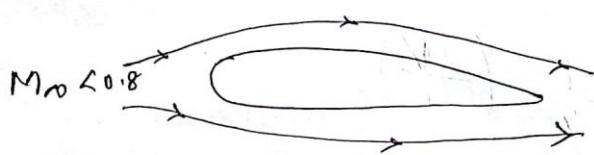
### Mach Number Regimes :-

- Subsonic flow.
- Transonic flow.
- Supersonic flow.
- Hypersonic flow.

### Subsonic flows :-

A flow field where the mach number is less than 1 @ every point. ( $M \leq 1$  everywhere).

- They are defined by smooth stream lines.
- Flow velocity is less than speed of sound so the disturbances in the flow propagate both up & downstream.



$$\frac{m}{v} = \frac{1}{v}$$

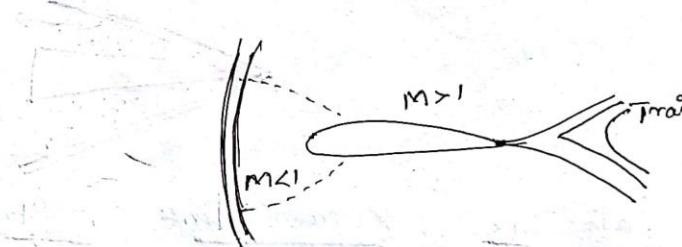
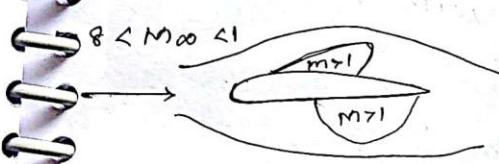
Transonic flows :-

(3)

→ Mixed region where  $M < 1$  and  $M > 1$

→ If  $M_\infty$  is subsonic but near the value of 1 then the flow will be supersonic ( $M > 1$ ) and supersonic pockets will be formed @ top and bottom surface of the airfoil which will be terminated by weak shockwaves after which flow will be subsonic again.

→ If  $M_\infty$  is increased more than 1 a bow shock is formed in front of the body and immediately behind shockwave the flow is subsonic.

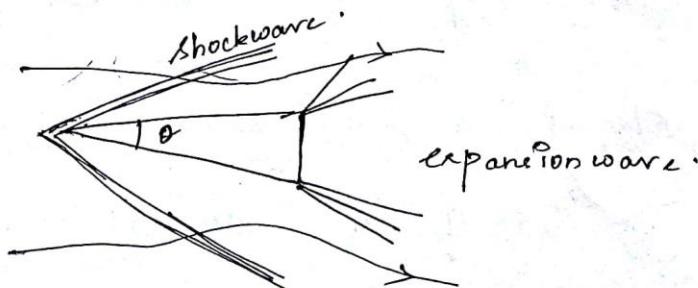


### Supersonic flows :-

→ Flow field is defined as supersonic if the mach number is greater than 1 @ every point.

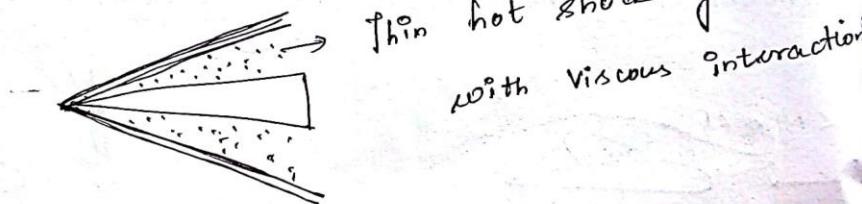
→ They are characterized by presence of shock waves across which flow properties changes.

$$M_\infty > 1.2$$



## Hypersonic flow :-

- It is defined as very high supersonic flow.
- The Mach number is considered to be  $M_\infty > 5$ .
- As shock wave  $\Gamma_{us}$  moves closer to the body surface the strength also increases which leads to  $\uparrow$  in temperature.
- The shock layer becomes very thin and starts to interact between the viscous boundary layer on the surface.



## Path line , Stream lines , Streak lines :

### Path line :-

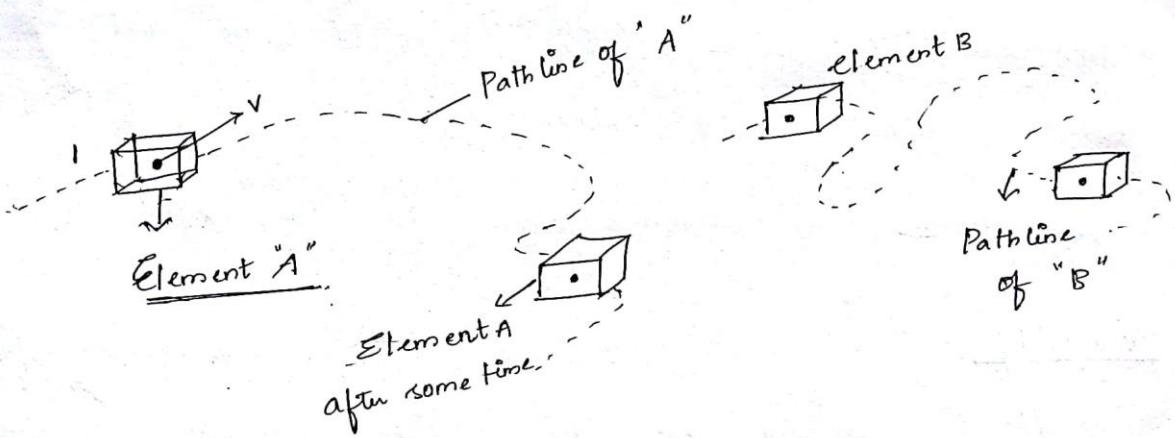
→ Consider an unsteady flow with velocity field given by  $\mathbf{v} = \mathbf{v}(x, y, z, t)$ .

→ Consider a fluid element moving through the flow field say element A

→ Element A passes through point 1

→ The path of 'A' as it moves downstream from point "1" is given by .

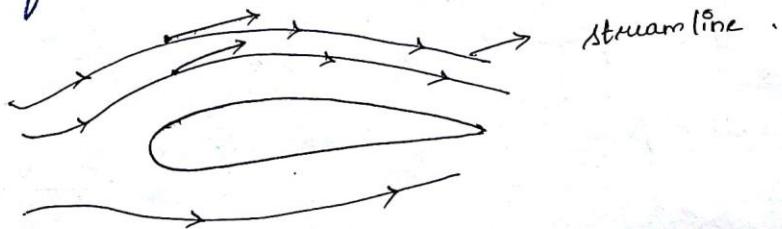
(4)



- Such a path is defined as pathline of element "A"
- Here both element A and B passes through point 1 but B @ some different time from element A.
- Due to unsteady flow, the velocity @ point 1 changes, Hence the path lines of 1 and 2 are different

### Stream line :-

- It is a curve whose tangent @ any point is the direction of velocity vector @ that point.
- Stream lines are drawn in such a way that their tangents @ those points will be tangential to the direction of velocity vectors.



- In case of steady flow there is no distribution between path line and stream line.
- They are the same curves in the space.



Vorticity:

- It is denoted as the twice of Angular Velocity.
- Denoted by  $\xi$ .

$$\boxed{\xi = \omega}$$

$$\text{so, } \xi = \left( \frac{du}{dy} - \frac{dv}{dz} \right) \hat{i} + \left( \frac{du}{dx} - \frac{dw}{dz} \right) \hat{j} + \left( \frac{dv}{dx} - \frac{dw}{dy} \right) \hat{k}$$

- $\xi = \nabla \times v$  → In a velocity field, the curl of the velocity is equal to vorticity.
- If  $\nabla \times v \neq 0$  At every point in the flow, the flow is called as rotational flow. It means the fluid elements have a finite angular velocity.

- If  $\nabla \times v = 0$  At every pt in the flow, the flow is irrotational.

$$\text{For a 2D flow } \xi = \xi_z k = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) k.$$

$$\text{For irrotational flow } \xi = 0; \text{ so } \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0.$$

6

→ For a steady flow inside a given stream tube, the mass flow inside inside a given tube is constant.

→ The imp property of the stream function is the derivative of  $\psi$  which gives the flow velocity.

Pg(7)

Velocity

Consider the same stream line  $s_1$  and assume the dist  $\Delta n$  is small means both streamlines are very close

→ The mass flow through stream tube / Unit depth is

$$\Delta \bar{\psi} = Pv \Delta n \quad [ \text{here } v \times \Delta n \rightarrow \text{vel} \times \text{dis} \\ \text{ie area covered} ]$$

$$\frac{\Delta \bar{\psi}}{\Delta n} = Pv. \quad (\text{by considering the limits} \\ \text{as } \Delta n \rightarrow 0)$$

$$Pv = \lim_{\Delta n \rightarrow 0} \frac{\Delta \psi}{\Delta n} = \frac{d \bar{\psi}}{dn}$$

Due to conservation of mass, the mass flow through  $\Delta n$  is equal to sum of mass flow through  $+ \Delta y$  and  $- \Delta x$ . [ $+ \Delta y$  coz Upward dir  
 $- \Delta x$  coz left dir].

$$\text{so } \Delta \bar{\psi} = Pv \Delta n = Pv \Delta y + Pv (-\Delta x)$$

$$d \bar{\psi} = Pudy - Pvdx$$

$$d \psi = \frac{\partial \bar{\psi}}{\partial x} dx + \frac{\partial \bar{\psi}}{\partial y} dy$$

## Stream Function :

→ Consider a 2d flow - steady flow. From the

eqn of stream line - diff eqn.

$$\frac{dy}{dx} = \infty \frac{v}{u}$$

If  $u, v$  are known function of  $x, y$  then eqn can be integrated into a algebraic form. (ie) obtained in term

$$f(x, y) = c$$

c - arbitrary constant with diff values for diff std

Denote the fn of  $x, y$  by  $\bar{\psi}$ .

$$\bar{\psi}(x, y) = c$$

function.

The fun  $\bar{\psi}(x, y)$  is called stream function.

consider stream line ab & cd. The stream function is

given by  $\bar{\psi} = c_1$  &  $\bar{\psi} = c_2$

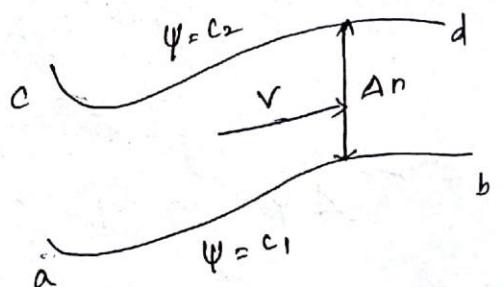
Now the mass flow

between the stream lines ab  
and cd per unit depth

is to

$$[\Delta \bar{\psi} = c_2 - c_1]$$

The mass flow is given per unit depth Lr to page



(6)

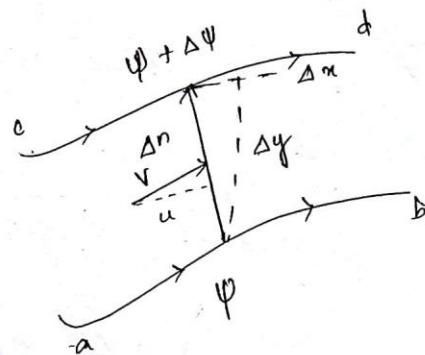
$$P_u = \frac{\partial \bar{\Psi}}{\partial y} ; \quad P_v = \frac{\partial \bar{\Psi}}{\partial x}.$$

If  $\bar{\Psi}(x, y)$  is known for given flow field then  
 @ any point in the flow.  $P_u$ , and  $P_v$  can be  
 obtained by diff  $\bar{\Psi}$  in the dir normal to  $u, v$ .

In terms of polar Co-ordinates

$$P_{v_n} = \gamma_n \frac{\partial \bar{\Psi}}{\partial \theta}$$

$$P_{v_\theta} = -\frac{\partial \bar{\Psi}}{\partial n}$$



Unit of  $\bar{\Psi}$  ~ In case of incompressible flow

$V = \frac{\partial (\Psi/\rho)}{\partial n}$ ; An ad stream function for  
 incompressible flow  $\Psi = \Psi/\rho$ .

so; 
$$\boxed{V = \frac{\partial \Psi}{\partial n}}$$

so,

$u = \frac{\partial \Psi}{\partial y}$
$v = -\frac{\partial \Psi}{\partial n}$

## Velocity Potential Eqn :-

Consider an irrotational flow

$$\xi = \nabla \times v = 0 \rightarrow \textcircled{1}$$

If  $\phi$  is a scalar function then  $\nabla \times (\nabla \phi) = 0 \rightarrow \textcircled{2}$

$$\text{here } v = \nabla \phi \rightarrow \textcircled{3}$$

Eqn \textcircled{3} states that for an irrotational flow there is a scalar function  $\phi$  such that velocity is given by the gradient of  $\phi$ .

$$\text{so } u_i + v_j + w_k = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k.$$

In cartesian co-ordinates.

$$u = \frac{\partial \phi}{\partial x}; v = \frac{\partial \phi}{\partial y}; w = \frac{\partial \phi}{\partial z}$$

In cylindrical co-ordinates

$$v_u = \frac{\partial \phi}{\partial r}; v_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}; v_z = \frac{\partial \phi}{\partial z}$$

In spherical co-ordinates

$$v_r = \frac{\partial \phi}{\partial r}; v_\theta = \frac{1}{r \sin \phi} \frac{\partial \phi}{\partial \theta}; v_\phi = \frac{1}{r \sin \phi} \frac{\partial \phi}{\partial \phi}$$

Angular  
 long.  
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## Angular Velocity:

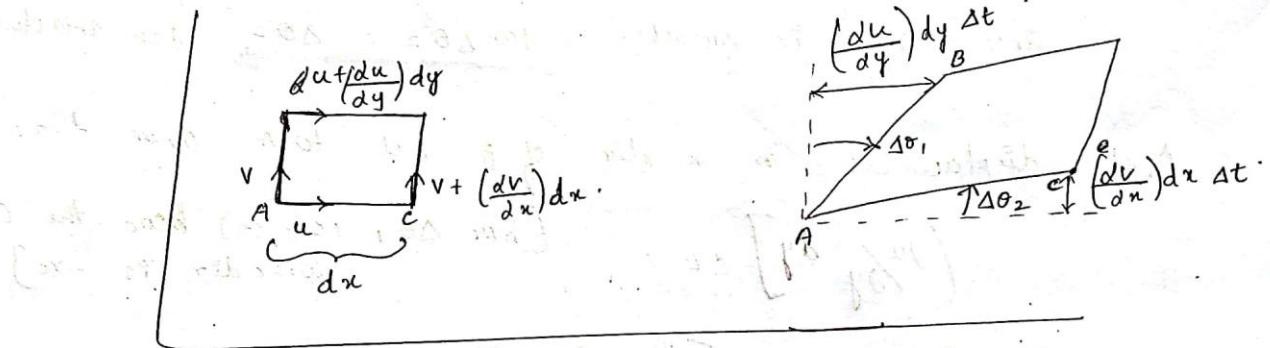
(7)

→ Consider a fluid flow in  $xy$  plane. Also consider a small element in flow.

→ At the time  $t$  the shape of fluid element is rectangular.

→ Assume that fluid element is moving upward & towards right.

→ Its position @ time  $t + \Delta t$  are -



Consider the line AC. It has rotated because during the time increment  $\Delta t$ , point C has moved differently from pt. A.

Consider Velocity @ pt A @ time  $t$  is  $v$ .

At pt C is @ a dist  $dx$  from pt A. So vertical component of velocity is  $v + (\frac{\partial v}{\partial x})dx$ .

$$\text{Distance} = \text{Velocity} \times \text{time}$$

Distance that pt A has moved in  $y$  dir =  $v\Delta t$

during time increment  $\Delta t$

Dist that pt C moved in  $y$  dir =  $\left[ v + \left( \frac{\partial v}{\partial x} \right) dx \right] \cdot \Delta t$   
during time incre  $\Delta t$

Net displacement in  $y$  dir of "c" rel to A =  $\left[ \frac{V - \frac{dv}{dx} dx}{\frac{du}{dx}} dt \right] = \left[ \left( \frac{dv}{dx} \right) dx \right] dt$

Continuity Eqn

By the figure the expression is given as

$$\tan \Delta\theta_2 = \frac{\int \left[ \left( \frac{dv}{du} \right) dx \right] dx}{dx} dt = \frac{\frac{dv}{du}}{dx} dt$$

here  $\Delta\theta_2$

Since  $\Delta\theta_2$  is small.  $\tan \Delta\theta_2 = \Delta\theta_2$ . coz anti-clockwise

Net displacement in  $x$  dir of B rel to A over time  $\rightarrow$  At the flow velo  
 $\left( \frac{du}{dy} dy \right) dt$  (here  $\Delta\theta_1$  is (-ve) becoz the clockwise dir is -ve)

$$\tan (\Delta\theta_1) = \frac{\int \left( \frac{du}{dy} dy \right) dy}{dy} dt = \frac{\frac{du}{dy}}{dy} dt$$

After moved

Since  $\Delta\theta_1$  is small  $\tan \theta_1 = \Delta\theta_1$

The Angular velocity @ AB and AC is defined as  $\frac{d\theta_1}{dt}$  by

$$so \quad \frac{d\theta_1}{dt} = \lim_{At \rightarrow 0} \frac{\Delta\theta_1}{At} = -\frac{du}{dy}$$

$$\frac{d\theta_2}{dt} = \lim_{At \rightarrow 0} \frac{\Delta\theta_2}{At} = \frac{dv}{dx}$$

let  $\omega_2$  be angular velocity  
therefore

$$\omega_2 = \frac{1}{2} \left[ \frac{d\theta_1}{dt} + \frac{d\theta_2}{dt} \right]$$

$$so, \quad \omega_2 = \frac{1}{2} \left[ \frac{\frac{du}{dy}}{dt} - \frac{dv}{dx} \right]$$

The resulting angular velo

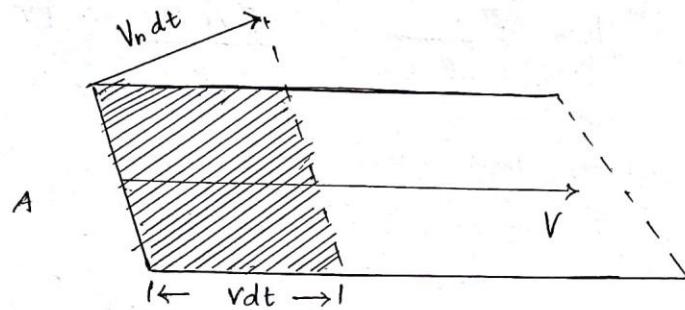
of fluid element is

$$\omega = \left( \omega_x i + \omega_y j + \omega_z k \right)$$

$$\omega = \frac{1}{2} \left[ \left( \frac{\frac{du}{dy} - \frac{dv}{dx}}{dt} \right) i + \left( \frac{\frac{du}{dz} - \frac{dw}{dx}}{dt} \right) j + \left( \frac{\frac{dw}{dy} - \frac{du}{dz}}{dt} \right) k \right]$$

Continuity Eqn in Control Volume Approach:-

(8)



→ Let the Area  $A$  be very small so that the flow velocity  $V$  is uniform across  $A$ ; consider the fluid elements with velocity "V" that pass thro "A".

→ After crossing  $A$  in time "dt" the distance moved will be  $v \cdot dt$

→ It had covered the shaded volume given by base Area  $\times$  height of cylinder  $V_n dt$ .

here  $V_n \rightarrow$  Vertical comp of velocity  $V$  to  $A$ .

$$\text{so, } \text{Volume} = (V_n dt) A$$

Mass inside the shaded volume

$$\text{Mass} = \rho (V_n dt) A \rightarrow ①$$

Mass flow through  $A$  is given by

$$\dot{m} = \frac{\rho (V_n dt) A}{dt} = \underline{\underline{\rho V_n A}} \rightarrow ②$$

Mass flux is defined as Mass flow per unit Area.

$$\text{so, } \text{Mass flux} = \frac{\dot{m}}{A} = \frac{Pv_n A}{A} = Pv_n. \rightarrow (3)$$

\* Basic principle states that mass can neither be created nor be destroyed.

→ so, Consider a flow field where all properties vary with time and space.

$$\text{so that } P = P(x, y, z, t).$$

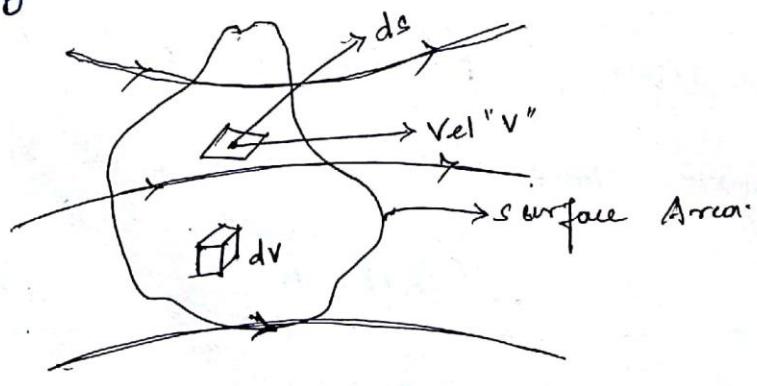
→ In this flow field consider fixed finite control volume. At a pt on the control surface, the flow velocity is  $v$ .

→ Elemental surface area ie  $ds$ .

→  $dv$  is an elemental volume inside control volume.

According to the principle,

Net mass flow out of control volume through surface 's' = Time rate of decrease of mass inside control volume "C" "v".



$\stackrel{10}{=}$

$\oint \nabla P \cdot d\ell$  can be written as  $\iiint_v \nabla(Pv) dv$ . (9)

Sub (8) into (7)

$$\text{or} \quad \iiint_v \frac{dP}{dt} dv + \iiint_v \nabla(Pv) dv = 0$$

$$\iiint_v \left[ \frac{dP}{dt} + \nabla(Pv) \right] dv = 0. \rightarrow (9)$$

The only way the integral to be zero for an arbitrary control volume is integral to be '0' @ all points within Control Volume.

$$\frac{dP}{dt} + \nabla(Pv) = 0.$$

Momentum Eqn - Control Volume Approach :-

Force - Time rate of change of momentum.

$$F = ma. \quad F = d/dt(mv).$$

Force  $F$  comes from two sources.

Body forces :- Gravity, electro magnetic force (Or) force acts at a distance on fluid inside v

Surface forces :- Pressure and shear stress.

Elemental mass flow across the area  $ds$  is

$$\rho v_n ds = \rho v ds.$$

→ Net mass flow out of entire Control surface  $S$  is

$$\oint_S \rho v ds = B. \rightarrow ⑤$$

The mass contained within elemental Volume  $ds$  is

$$\rho dv.$$

Total mass inside Control Volume is  $\iiint_V \rho dv$

Time rate of decrease of mass inside Volume is

$$-\frac{\partial}{\partial t} \iiint_V \rho dv. = c \rightarrow ⑥$$

Sub ⑤ and ⑥ in ④

$$\oint_S \rho v ds = -\frac{\partial}{\partial t} \iiint_V \rho dv.$$

$$\underline{\oint_S \rho ds + \frac{\partial}{\partial t} \iiint_V \rho dv = 0} \rightarrow \text{continuity equation}$$

in integral form.

Applying the divergence theorem

$$\text{i.e. } \oint_S A \cdot ds = \iiint_V (\nabla \cdot A) dv.$$

(10)

Let "f" represents the net body force per unit mass exerted on the fluid inside  $V$ .

The body force on elemental volume  $dv$  is

$$f \quad (\underline{P_f dv}) \rightarrow (1)$$

The total body force exerted on the fluid in the Control Volume is the summation of above over Volume  $V$ .

$$\underline{\text{Body force}} = \iiint_V P_f dv \rightarrow (2)$$

The elemental surface force due to pressure is  

$$[-P ds]$$
  
 (acting on elemental area  $ds$ )

Complete pressure force is given by

$$\underline{\text{Pressure force}} = - \iint_S P ds \rightarrow (3)$$

Total force experienced by fluid is  $(2) + (3)$ .

$$F = \iiint_V P_f dv - \iint_S P ds + F_{\text{viscous}} \rightarrow (4)$$

Time rate of change of momentum of fluid is through fixed control Volume.

Net flow of momentum out of control volume across surface  $S$

Time rate of change of momentum due to unsteady fluctuation of flow properties

\* The flow of momentum per second across  $S$

$$(P_{vds})_v \rightarrow ⑤$$

The net flow of momentum out of control volume through  $S$  is

$$g = \oint_S (P_{vds})_v \rightarrow ⑥$$

The momentum of the fluid in elemental volume

$$(P_{dr})_v \rightarrow ⑦$$

The momentum contained @ any instant inside control volume is

$$\iiint_v P_{dr} v.$$

Time rate of change due to unsteady flow is

$$H = \frac{d}{dt} \iiint_v P_{dr} v. \rightarrow ⑧$$

Combining ②, ③, ⑥ & ⑧ into basic Newton's eqn

$$\frac{d}{dt} (\rho v) = g + H \quad (11)$$

$$\int \int \int (Pv ds) \dot{v} + \frac{\partial}{\partial t} \int \int \int P v dv = - \int \int \int \frac{\partial P}{\partial s} ds + \int \int \int P f v dv + F_{vis} \quad \rightarrow (9)$$

The  $\int \int \int P ds$  can be written as  $\int \int \int v P dv$ .

so the above eqn in the  $x$  component can be

$$\int \int \int_v \frac{\partial (P u)}{\partial t} dv + \int \int_s (P v ds) u = - \int \int \int_v \frac{\partial P}{\partial u} dv + \int \int_v P f_x dv + (F_x)_{vis} \quad \rightarrow (10)$$

Applying divergence theorem.

$$\int \int_s (P v ds) u = \int \int_v (\rho u ds) \nabla v ds = \int \int_v \nabla (\rho u v) dv \quad \rightarrow (11)$$

sub (11) in (10)

$$\int \int_v \left[ \frac{\partial}{\partial t} (\rho u) + \nabla \cdot (\rho u v) + \cancel{\frac{\partial}{\partial x} P} - P f_x \right] dv - F_{vis} J dv = 0$$

As integrand is 0 @ all the points  $\rightarrow (12)$

$$\frac{\partial}{\partial t} (\rho u) + \nabla \cdot (\rho u v) = - \frac{\partial P}{\partial x} + P f_x + F_{x vis} = 0 \quad \rightarrow (13)$$

$$\frac{\partial (P v)}{\partial t} + \nabla \cdot (P v v) = - \frac{\partial P}{\partial y} + P f_y + F_{y vis} = 0 \quad \rightarrow (14)$$

$$\frac{\partial (P w)}{\partial t} + \nabla \cdot (P w v) = - \frac{\partial P}{\partial z} + P f_z + F_{z vis} = 0 \quad \rightarrow (15)$$

For steady ( $\frac{\partial}{\partial t} = 0$ ), inviscid ( $F_{vis} = 0$ ); flow with no body force ( $f = 0$ ) the eqn becomes

$$\left. \begin{aligned} \nabla \cdot (\rho u v) &= -\frac{\partial P}{\partial x} \\ \nabla \cdot (\rho v v) &= -\frac{\partial P}{\partial y} \\ \nabla \cdot (\rho w v) &= -\frac{\partial P}{\partial z} \end{aligned} \right] \rightarrow \textcircled{16}$$

So Momentum eqn for inviscid flow  $\textcircled{16}$  is called

as Euler's eqn

Momentum eqn for viscous flow (13, 14, 15) is

Called as Navier Stokes Eqn

Energy Equation :- (Control Volume Approach)

Energy can neither be created nor be destroyed. If we apply 1 law of thermodynamics flowing of heat we apply through fixed control volume.

but  $B_1 = \text{Rate of heat added to fluid inside}$

$B_2 = \text{Rate of work done on fluid inside}$

$B_3 = \text{Rate of change of energy of fluid as it flows through Control Volume}$

$\underline{T}$  law  $\Rightarrow [B_1 + B_2 = B_3]$

Let  $\dot{q}_v$  be volumetric rate of heat addition / unit mass with in elemental control volume (12)

$$\text{Rate of volumetric heating} = \iiint_V \dot{q}_v (P dV) \rightarrow 1$$

Total rate of change of heat addition is

$$B_1 = \iiint_V \dot{q}_v (P dV) + \dot{Q}_{vis} \quad \hookrightarrow 2$$

Pressure force on elemental area is  $-Pds$ . and the rate of work done on fluid passing through  $ds$  with velocity  $v$  is  $(-Pds)v$ .

$$\text{Rate of work done on fluid inside volume due to } PdS \text{ force} = - \iint_S (Pds)v \rightarrow 3$$

If  $F$  is the body force per unit mass, the rate of work done on volume due to body force  $(P_f dV)v$

$$\text{Work done} \rightarrow \text{Force} \times \text{dist/sec} = \text{Force} \times \text{Velocity}$$

$$\text{Rate of work done on fluid inside } v \text{ due to body force} = \iiint_V (P_f dV)v \rightarrow 4$$

so,

$$B_2 = 3 + 4$$

$$(If flow is viscous)$$

$$B_2 = - \iint_S (Pds)v + \iiint_V (P_f dr)v + W_{viscous}$$

Rate of change of total energy of the fluid  
it flows through control volume =  $B_3$ .

The elemental mass flow across  $ds$  is  $\rho v ds$ .  
so the elemental flow of total energy across  $ds$  is  
 $(\rho v ds) (e + \frac{v^2}{2})$

$$\text{Net rate of flow of total energy across C.S} = \iiint_s (\rho v ds) (e + \frac{v^2}{2}) \frac{dV}{dt} \quad \rightarrow (6)$$

→ Total energy contained in the elemental volume  $dV$   
is  $\rho (e + \frac{v^2}{2}) dV$  so

$$\iiint_v \rho (e + \frac{v^2}{2}) dV \rightarrow (7)$$

Time rate of change of total energy inside  $V$  is

$$B_3 = (6) + (8)$$

$$B_3 = \frac{\partial}{\partial t} \iiint_v \rho (e + \frac{v^2}{2}) dV + \iiint_s (\rho v ds) (e + \frac{v^2}{2}) \quad \rightarrow (9)$$

Energy equation in the integral form

(13)

$$\iiint_V \dot{q} \rho dv + Q_{visc} - \oint_S P \cdot v ds + \iiint_V P(f \cdot v) dv + \dot{W}_{viscous}$$

$$= \frac{d}{dt} \iiint_V P(e + \frac{v^2}{2}) dv + \oint_S P(e + \frac{v^2}{2}) v ds.$$

(10)

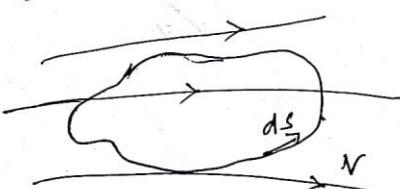
By applying divergence theorem, 2 integrands to "0"

$$\frac{d}{dt} \left[ P(e + \frac{v^2}{2}) \right] + \nabla \cdot \left[ P(e + \frac{v^2}{2}) v \right] = P\dot{q} - \nabla(Pv) + P(f \cdot v) + Q_{visc} + \dot{W}_{visc}$$

Assume ( $Q_{viscous}, W_{visc} = 0$ ), Adiabatic (no heat addition  $\dot{q} = 0$ ) without body forces  $f = 0$ , so the eqn reduces to .

$$\oint \nabla \cdot \left[ P(e + \frac{v^2}{2}) v \right] = -\nabla(Pv). \rightarrow (12).$$

Circulation :-



Consider a closed curve  $c$  in a flow field, let  $v$  and  $ds$  be the velocity and directed line

→ At a pt  $c$  the circulation is defined as

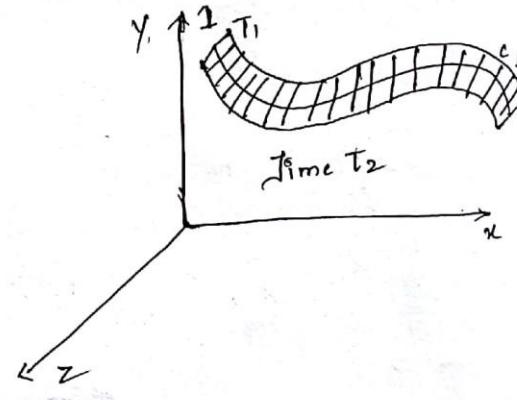
$$\Gamma = - \oint_c v \cdot ds.$$

The circulation is the negative of the line integral of velocity around a closed curve in a flow.

### Problems

#### Streak lines :

- Consider a fixed points in a flow such as point 'D'.
- Consider all fluid elements have passed through 'D'.
- Consider all fluid elements have passed through 'D' over a given time  $t_2 - t_1$ .
- These fluid elements are connected with each other like a string of elephant's trunk.
- If element A is first element and element B is second element, the line that connects element A and B is called as Streak line.



Soln

### Problems

①

Consider a convergent duct with an inlet area  $A_1 = 5 \text{ m}^2$ ; Air enters the duct with velocity  $v_1 = 10 \text{ m/s}$  and exit velocity of  $v_2 = 30 \text{ m/s}$ . What is the area of duct exit.

Soln

$$A_1 = 5 \text{ m}^2; v_1 = 10 \text{ m/s}; v_2 = 30 \text{ m/s}$$

By continuity eqn  $A_1 v_1 = A_2 v_2$

$$A_2 = \frac{A_1 v_1}{v_2}$$

$$A_2 = \frac{5 \times 10}{30}$$

$$\underline{\underline{A_2 = 1.67 \text{ m}^2}}$$

② Consider a convergent duct with inlet area

$$A_1 = 0.08 \text{ m}^2; \text{ exit area } A_2 = 0.771 \text{ m}^2 \quad P = 1.23 \text{ kg/m}^3$$

Velocity inlet  $v_1 = 110 \text{ m/s}$ ; exit velocity  $v_2 = 321 \text{ m/s}$

Calculate the density of air  $P_2$  @ exit

Soln.

$$P_1 A_1 v_1 = P_2 A_2 v_2$$

$$P_2 = P_1 \frac{A_1 v_1}{A_2 v_2} = \underline{\underline{0.83 \text{ kg/m}^3}}$$

(3). Consider an airfoil in a flow of air where  
 Pressure =  $2116 \text{ lb/ft}^2$ ;  $P = 0.002377 \text{ slug/ft}^2$ ,  
 $V = 100 \text{ mi/hr}$ . At a given point on airfoil the  
 Pressure is  $2070 \text{ lb/ft}^2$ , what is the velocity @ the

$$V_1 = 100 \text{ mi/hr}; \text{ WKT } 60 \text{ mi/hr} = 88 \text{ ft/sec}$$

$$\text{so } V_1 = 100 \left( \frac{88}{60} \right) = 146.7 \text{ ft/sec}$$

$$P_1 + \frac{Pr_1^2}{2} = P_A + \frac{Pr_A^2}{2}$$

$$V_A^2 = \left[ \frac{\rho (P_1 - P_A)}{P} + V_1^2 \right]$$

$$= \left[ \frac{\rho (2116 - 2070)}{0.002377} + (146.7)^2 \right]^{1/2}$$

$$= \underline{245.4 \text{ ft}}$$

(4) Same prob with diff values as follows

$$P_1 = 1.013 \text{ bar}; V = 160 \text{ kmph} \quad P = 1.23 \text{ kg/m}^3$$

$$P_A = 0.996 \text{ bar}$$

$$V_1 = 160 \left( \frac{1000}{3600} \right) = 44.4 \text{ m/s}$$

$$P_1 + \frac{Pr_1^2}{2} = P_A + \frac{Pr_A^2}{2}$$

$$V_A = \left[ \frac{\rho (P_1 - P_A)}{P} + V_1^2 \right]^{1/2} \Rightarrow \left[ \frac{\rho (1.013 - 0.996)}{1.23} + (44.4)^2 \right]^{1/2} V_A = 75.75 \text{ m/s.}$$

(5) Consider a velocity field given by  $u = y/(x^2+y^2)$   
 $v = -x/(x^2+y^2)$ . Calculate the eqn of streamline passing through pt  $(0, 5)$ .

Soln.  $\frac{dy}{dx} = \frac{v}{u} = -\frac{x}{y}$  so  $y dy = -x dx$ .

On integrating  $y^2 = -x^2 + C$ .

For stream line thro  $(0, 5)$

$$5^2 = 0 + C ; C = 25$$

so eqn is  $x^2 + y^2 = 25$ .

(6). For the velocity field given by  $u = y/(x^2+y^2)$  &  $v = -x/(x^2+y^2)$ . calculate the circulation around a circular path of radius 5m. Assume u & v are in m/s.

Since it is circular path, we use polar co-ordinates.

$$x = r \cos \theta ; y = r \sin \theta ; x^2 + y^2 = r^2$$

$$V_H = u \cos \theta + v \sin \theta \quad \& \quad V_\theta = -u \sin \theta + v \cos \theta ; \quad \text{so}$$

$$u = \frac{y}{x^2+y^2} = \frac{r \sin \theta}{r^2} = \frac{\sin \theta}{r}$$

$$v = -\frac{x}{x^2+y^2} = \frac{-r \cos \theta}{r^2} = -\frac{\cos \theta}{r}$$

$$V_H = \frac{\sin \theta}{r} \cos \theta + \left( -\frac{\cos \theta}{r} \right) \sin \theta = 0$$

$$V_\theta = -\frac{\sin \theta}{r} \sin \theta + \left( -\frac{\cos \theta}{r} \right) \cos \theta = -\gamma_n$$

$$\mathbf{V} \cdot d\mathbf{s} = V_R dr + r V_\theta d\theta = 0 + r (-\gamma_n) d\theta$$

$$= -d\theta$$

$$\Gamma = -\oint_c \mathbf{V} \cdot d\mathbf{s}$$

$$= -\int_0^{2\pi} -d\theta = \underline{\underline{2\pi m^2/s}}$$

## Fundamental Aerodynamic Variables :-

### MOD- 2 - AIRFOIL CHARACTERISTICS

- Pressure
- Density
- Temperature
- Velocity.

AERD  
AD.  
Shrinath

#### Pressure :-

- It is normal force per unit area exerted on a surface due to time rate of change of momentum of gas molecules impacting on the surface.
- Consider a point B in a volume of fluid.

$dA$  = elemental area at B.

$dF$  → Force on one side at pt B  $dA$ .

Pressure @ pt B is given by

$$P = \lim_{dA \rightarrow 0} \left( \frac{dF}{dA} \right)$$

#### Density :-

- It is mass per unit volume. It is a point property that can vary from point to point in the fluid.

- Consider a pt B in fluid.

$dv$  = elemental volume around B

$dm$  = Mass of fluid inside  $dv$

$$P = \lim_{dv \rightarrow 0} \left( \frac{dm}{dv} \right)$$

## Temperature :-

Temperature  $T$  of a gas is directly proportional to the kinetic energy, thus the temperature is given by Boltzmann Constant.

$$K.E = \frac{3}{2} kT ; k \rightarrow \text{Boltzmann Constant.}$$

High temp gas :- All molecules & atoms are randomly moving @ high speeds.

Low temp gas :- Random motion of molecules are slow.

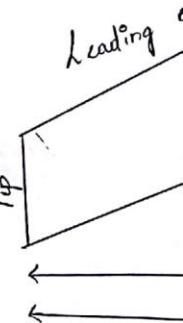
## Flow Velocity :-

→ Velocity of a gas @ any fixed point B in span is the velocity of an small fluid element as it sweeps through B.

→ Velocity is vector Quantity

→ Pressure, Density, Temp - scalar quantity.

→ Velocity is a pt Property which varies from point to pt in the flow.



Wing span

Chord

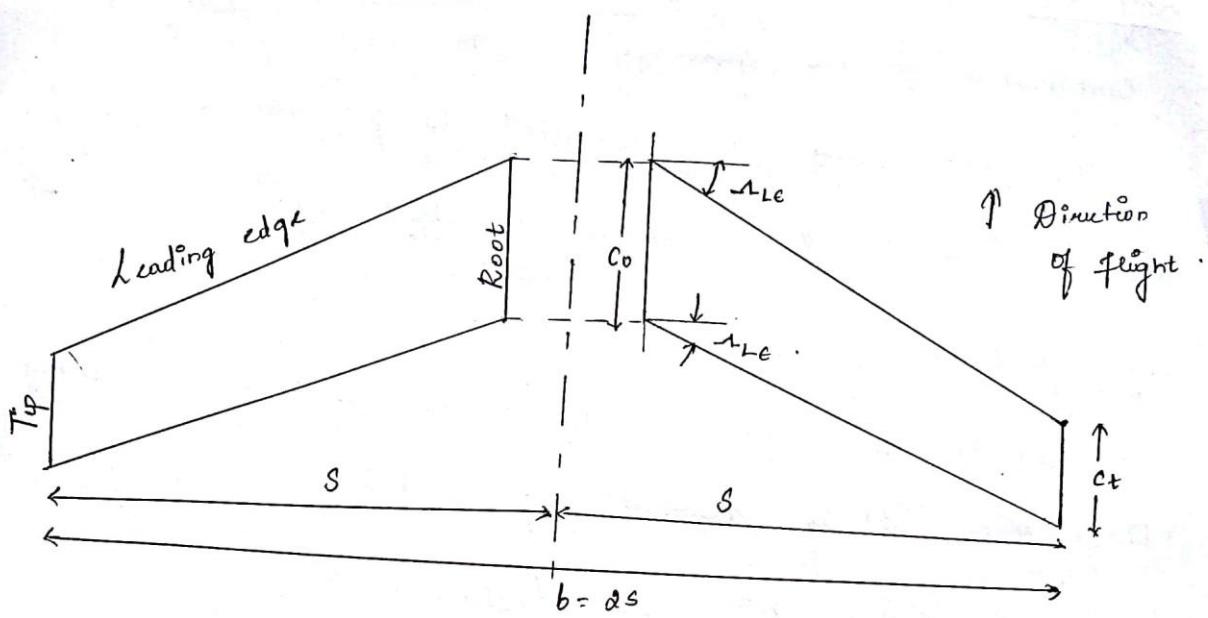
tip

Wing

→

and

## Wing Planform Geometry :-



The Planform of wing is the shape of the wing seen on plan view of Aircraft.

Wing span :- Distance between the extreme wing tips and denoted by "b".

Distance from each tip to Center line is "s".

Chords :- The two dimensions  $c_T$  and  $c_o$  are wing's tip and root chords.

→ Root chord distance is found by intersection of wing to the fuselage.

→ Ratio of  $c_T/c_o$  is called as Taper ratio and it is given by  $c_T/c_o < 1$ .

Wing Area: Plan Area of the wing including the continuation to the fuselage is the gross wing area's Tincidence

→ The plan area of extruded wing (i.e) excluding the continuation to fuselage is Net wing area "S"

### Aspect Ratio:

→ It is the measure of narrowness of the wing

Planform . It is denoted by A.R

$$A.R = \frac{b^2}{s} \Rightarrow \frac{(\text{Wing span})^2}{\text{Area}}$$

$$= \frac{b^2}{b \times c} = \boxed{\frac{b}{c}} \quad [\because \text{Area} = b \times c] \quad \underline{\text{Area}}$$

### Sweep back:

The sweep back angle of the wing is the angle between a line drawn along the space @ const fraction of chord from leading edge and a line fr to center line . Denoted by  $\phi$  (or)  $\alpha$

### Dihedral Angle:

→ If the wings are inclined upwards it is called as dihedral wing and inclined downwards is called as anhedral wing.

## Area & ~~Incidence, Twist, Washout and Washin.~~

→ The angle b/w the chord line of an aerofoil /wing and direction of flight / of undisturbed free stream is called geometric Angle of incidence "α".

→ If α of all sections are not same the wing is said to be twisted.

→ If incidence increases towards tip — Wash in wing

→ Decreases towards tailing tip — Wash out wing

## Aerodynamic Forces and Moments :-

Aerodynamic forces and moments on the body are due to two basic sources.

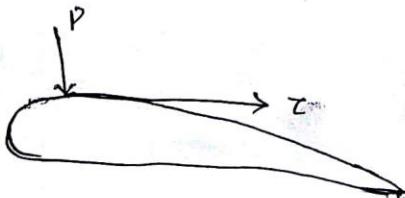
→ Pressure distributions over the body surfaces ( $P$ )

→ Shear stress distributions over body surface ( $\tau$ )

how  $P \rightarrow$  Acts normal to body.

$\tau \rightarrow$  Acts tangential to the body

The net effect of  $P$  &  $\tau$  integrated over complete body surface is resultant Aerodynamic force " $R$ "



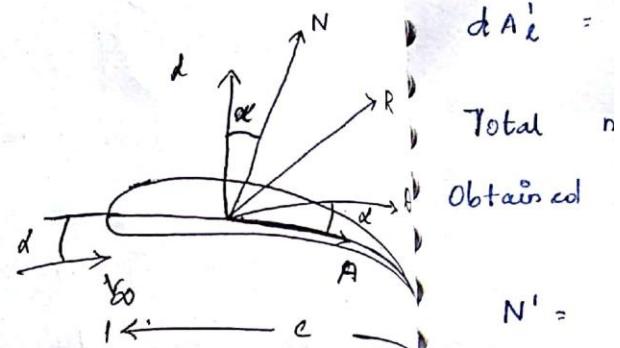
$V_\infty$  is the relative wind, defined as flow velocity far ahead of body, so called as "Free stream Velocity".

$\alpha$  = diff component of  $R$  Jr to  $V_\infty$   
||cl to  $V_\infty$ .

$D$  - Drag "

$N$  - Normal force " Jr to c  
||cl to c.

$A$  - Axial force "

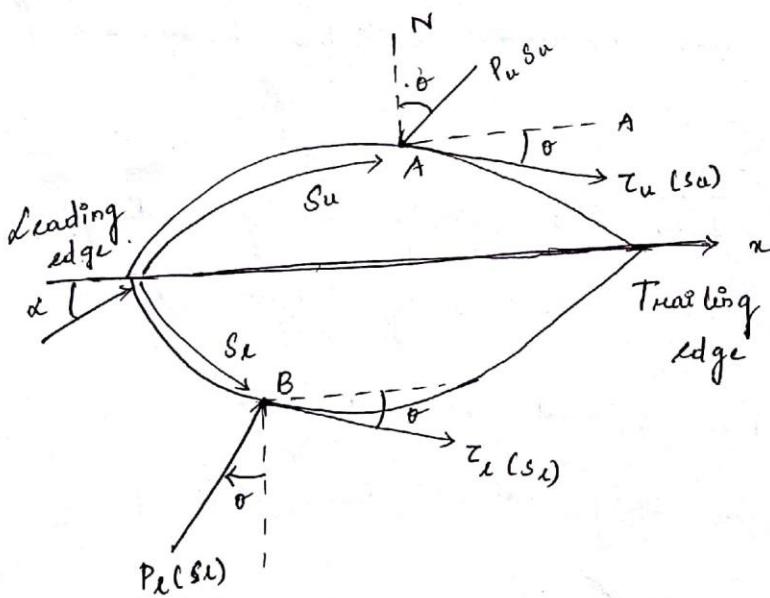


The Angle of attack is the angle between  $c$  and  $\alpha$ .

From the fig ②

$$L = N \cos \alpha - A \sin \alpha \rightarrow ①$$

$$D = N \sin \alpha + A \cos \alpha \rightarrow ②$$



Distance from the top along body surface to a pt A on upper surface is  $s_u$  and to the lower surface is  $s_x$ .

$$dN'_u = -p_u ds_u \cos \theta - \tau_u ds_u \sin \theta \rightarrow ③$$

$$dA'_u = -p_u ds_u \sin \theta + \tau_u ds_u \cos \theta \rightarrow ④$$

[ $ds_u \rightarrow$  elemental surface]

$$dN'_l = P_x ds_x \cos\theta - z_x ds_x \sin\theta \rightarrow (5)$$

$$dA'_l = P_x ds_x \sin\theta + z_x ds_x \cos\theta \rightarrow (6)$$

Total normal and Axial force per unit span are obtained by integrating (5) to (6).

$$N' = - \int_{LE}^{TE} (P_x \cos\theta + z_x \sin\theta) ds_x + \int_{LE}^{TE} (P_x \cos\theta - z_x \sin\theta) ds_x \rightarrow (7)$$

$$A' = \int_{LE}^{TE} (-P_x \sin\theta + z_x \cos\theta) ds_x + \int_{LE}^{TE} (P_x \sin\theta + z_x \cos\theta) ds_x$$

Moment per unit span about the leading edge due to  $P$  and  $z$  on the elemental Area  $ds$  over upper surface.

$$dM'_u = (P_x \cos\theta + z_x \sin\theta) x ds_u + (-P_x \sin\theta + z_x \cos\theta) y ds_u \rightarrow (8)$$

$$dM'_l = (-P_x \cos\theta + z_x \sin\theta) x ds_l + (P_x \sin\theta + z_x \cos\theta) y ds_l \rightarrow (9)$$

[here in eqn (7) take "-ve sign Commonly out and make it to null].

Moment about LE per unit span.

Let  $\rho_\infty$  and  $v_\infty$  be the density and velocity in the free stream far ahead of the body.

$$\rho_\infty = \frac{1}{2} \rho_\infty v_\infty^2$$

Let  $S$  be the surface area and  $l$  be the length.

Center of

$$\text{Lift Co-efficient } C_L = \frac{d}{q_{\infty} s}$$

$$\text{Drag Co-efficient } C_D = \frac{\theta}{q_{\infty} s}$$

$$\text{Normal force Co-eff } C_N = \frac{N}{q_{\infty} s}$$

$$\text{Axial force Co-eff } C_A = \frac{A}{q_{\infty} s}$$

$$\text{Moment Co-eff } C_M = \frac{M}{q_{\infty} s l}$$

$$\text{For a flat body } C_L = \frac{d'}{q_{\infty} c} ; \quad C_D = \frac{\theta'}{q_{\infty} c} ; \quad C_M = \frac{M}{q_{\infty} c^2 l}$$

Area  $S = C(c) = c$  for unit span.

$$\text{Pit. Co-eff} \Rightarrow C_p = \frac{P - P_{\infty}}{q_{\infty}} \quad \therefore P_{\infty} - \text{Free stream press.}$$

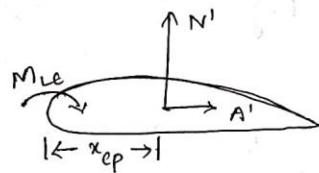
$$\text{Skin friction Co-eff } C_f = \frac{\tau}{q_{\infty}}$$

## Center of Pressure :-

If  $A'$  is placed on the chord line, then  $N'$  must be located @ a distance  $x_{cp}$  downstream of leading edge so that

$$M'_{LE} = -(x_{cp})N'$$

$$x_{cp} = \frac{-M'_{LE}}{N'}$$



→ Here  $x_{cp}$  is defined as the center of pressure. It is the location where the resultant of the distributed load acts on the body.

→ If moments were taken about center of pressure then the effect of loads will be zero.

→ If is the point on the body about which aerodynamic moments are zero.

→ Consider if  $\alpha$  is small so  $\sin\alpha=0$ ;  $\cos\alpha=1$

$$d = N\cos\alpha - A\sin\alpha \Rightarrow d = N'$$

$$\text{then } x_{cp} \approx \frac{-M'_{LE}}{d'}$$

As  $N'$  and  $d'$  decreases  $x_{cp}$  increases. As the force approaches to "0"  $x_{cp} \rightarrow \infty$ . So centre of pressure will be used in the field of Aerodynamics.

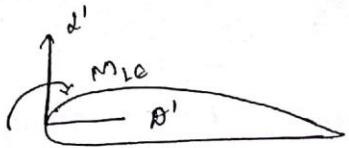


Fig ①

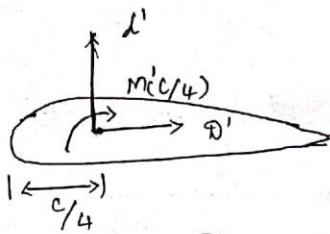


Fig ②

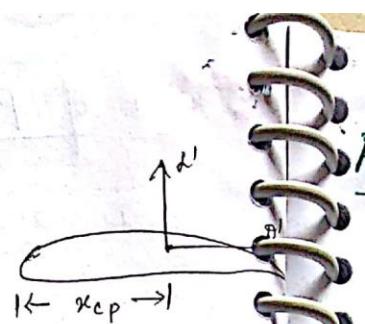


Fig ③.

Fig ① Moment about A.E as well as resultant forces placed @ A.E

Fig ② Finite value of moment @ ch. Quarter chord point and resultant force @ c/4

Fig ③ Resultant force @ center of pressure and moments are zero.

### Aerodynamic Center

- It is point of on the chord of the airfoil where moment is constant for all angle of attacks.
- Moments acting @ A.R center will be sum of the moments on the aerofoil + Moment due to lift.

### Pressure Co-efficient :-

→ Normally Pressure is a dimensioned, but for the use of Aerodynamics, we need non dimensionized Quantity

→ Non dimensionless Quantity of Pressure - Pressure Co-eff  $C_p$

$$C_p = \frac{P - P_\infty}{\rho V_\infty} \therefore \frac{1}{2} \rho V_\infty^2 = \frac{1}{2} P_\infty V_\infty^2$$

→ For incompressible flow  $C_p$  can be expressed in terms of Velocity Consider a body in a flow in a free stream of Pressure  $P_\infty$  and velocity  $V_\infty$ ; consider a point where Pressure & velocity  $P < v$ .

$$P_\infty + \frac{1}{2} \rho V_\infty^2 = P + \frac{1}{2} \rho v^2$$

$$(P - P_\infty) = \frac{1}{2} \rho (V_\infty^2 - v^2)$$

$$C_p = \frac{P - P_\infty}{\rho V_\infty} = \frac{\frac{1}{2} \rho (V_\infty^2 - v^2)}{\frac{1}{2} \rho V_\infty^2}$$

$$C_p = 1 - \left( \frac{v}{V_\infty} \right)^2 \Rightarrow (\text{Only for incompressible flow})$$

here the pressure co-eff @ a stagnation point where  $v=0$  will be 1 (so the max allowable  $C_p$  will be 1).

When  $\rho_p < P < P_\infty$  and  $v > V_\infty$   $C_p$  will be  $-ve$ .

$$P = P_{\infty} + \frac{1}{2} \rho_{\infty} C_P V^2$$

Here the pressure also depends on the  $C_P$  value.

### Problem :-

① Consider an Airflow with stream velocity  $V_{\infty} = 45.72$

The velocity @ a given pt is  $68.58 \text{ m/s}$ . calculate  $C_P$  @ this pt

$$C_P = 1 - \left( \frac{V}{V_{\infty}} \right)^2 = -1.25$$

② Consider an Aircraft when it is @ high angle of at

less than that when  $C_L$  becomes max. The peak negat

Pressure co-efficient which occurs @ a point on the

Airfoil is  $-5.3$

Typical Airfoil

a. Lift

b. Effect

c. Effect

d. Drag

e. Drag

f. Pitching

a. lift to

→ Consider

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→ Curv

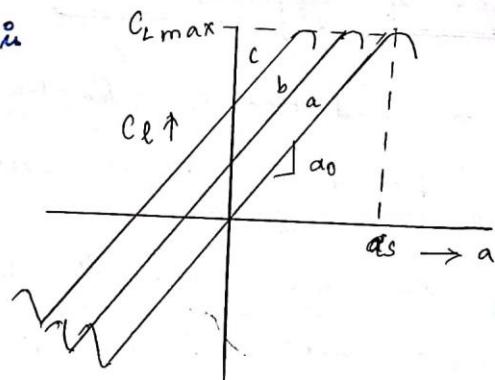
occurs @

Typical Airfoil Aerodynamic characteristics @ low speeds.

- Lift Co-efficient Versus Angle of incidence.
- Effect of Aspect ratio on  $C_L$ .
- Effect of Reynolds no. on  $C_L$ .
- Drag Co-efficient versus lift Co-efficient
- Drag Co-eff vs  $(\text{lift Co-eff})^2$
- Pitching moment Co-eff.

a. Lift Co-eff vs Incidence

→ Consider a curve A which is moderately thick (13%).



By the plot b/w  $C_L$  and  $\alpha$  it is seen that the curve after a certain point starts to fall, reaching the maximum value of  $C_L$  at an incidence

angle as called as stalling point

→ After which  $C_L$  decreases; The slope of the straight portion is called as lift slope  $\alpha_0$ .

→ Curves b and c are combined, so stalling point occurs @ less incidence angle.

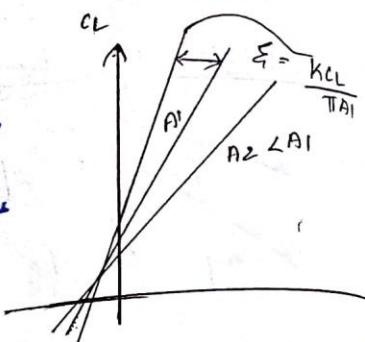
### b. Effect of Aspect Ratio on the $C_L$

Induced Angle of incidence is given by.

$$\Sigma = \frac{K_{CL}}{\pi A} ; A - \text{Aspect ratio.}$$

For highly swept wings of very low AR (less than 3), the lift curve slope becomes very small; mean  $C_{L\max} = 1.0$  occurring

(a) Stalling incidence of  $45^\circ$ .



### c. Effect of Reynolds Number on $C_L$ vs $\alpha$ Curve

→ Reduction of Re. No moves the transition point of the boundary layer rearwards on upper surface of wing.

→ At low value of Re. No the laminar boundary layer will move to adverse pressure gradient.

→ As laminar BL is much less able to overcome a adverse p<sub>m</sub> gradient, the flow will separate from surface @ lower angle of incidence.

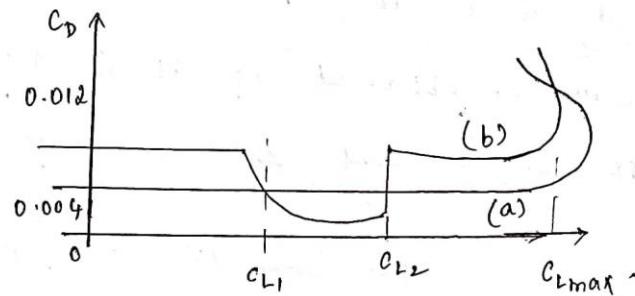
### d. Drag Co-ff. Vs lift Co-ff.

→ Curve (a) represents the typical Airfoil with CD Value constant over a region of CL increasing towards the end points of CL.

→ Curve B represents the variation found for low-drag Co-eff.

→ It is larger than for conventional type of airfoil but within a restricted range of lift Co-eff ( $C_{L1}$  to  $C_{L2}$ ) the profile Co-eff is less.

→ This range of  $C_L$  is favourable range for the section.



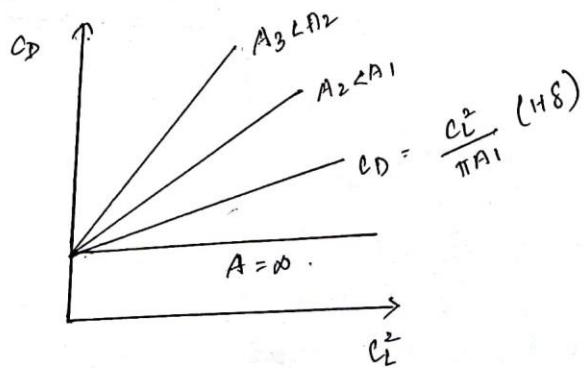
Figure

$C_L$  vs  $C_D$

### e. Drag Co-eff vs Lift Co-eff)

$C_D = \frac{C_L^2}{\pi A} (1 + \epsilon)$ . It states that a curve of  $C_D$  against  $C_L^2$  will be straight line of slope  $(1+\epsilon)\pi A$

→ For eg. if  $C_D$  is independent of  $C_L$ ; the  $C_D(C_L^2)$  curve for a family of wing of various A.R.



## F. Pitching Moment Co-eff

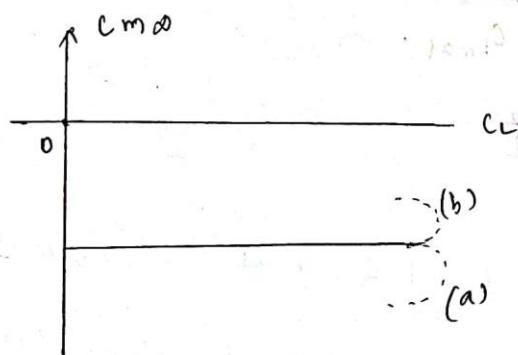
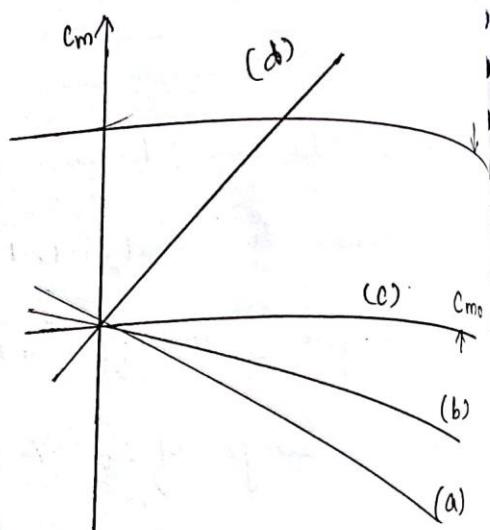
$$\frac{dC_m}{dC_L} = \text{constant}$$

Line (a) for which  $\frac{dC_m}{dC_L} = -\frac{1}{4}$

is measured about the leading edge.

Line (c) for which slope is "0" which measured about AD Center.

~~Line (B) will be obtained if it is taken a point between AD Center and leading edge.~~



For curve (A) the pitching

moment co-eff becomes negative near the stall, tending to decrease the

Unstall the wing. This is

Called as "stall break"

For (B) it is

Vice versa C\_m becomes less (-ve), so thereby initiating the stall. This is called "Unstable break"

①

MOD-3 - 2-D flows

Incompressible flows

Uniform flow :

- Consider a uniform flow with velocity  $V_\infty$  in  $x$  dir.
- Consider it as a incompressible flow so that  $\nabla \cdot \mathbf{v} = 0$
- It is also ir-rotational flow, so  $\nabla \times \mathbf{v} = 0$
- Velocity potential for uniform flow is  $\nabla \phi = \mathbf{v}$ .

$$\frac{\partial \phi}{\partial x} = u = V_\infty ; \quad \frac{\partial \phi}{\partial y} = v = 0 \rightarrow \textcircled{2}$$

$\hookrightarrow \textcircled{1}$

Integrating  $\textcircled{1}$  wrto  $x$  &  $\textcircled{2}$  wrto  $y$ .

$$\textcircled{3.a} \quad \phi = V_\infty x + f(y) ; \quad \phi = \text{const} + g(x) \rightarrow \textcircled{3.b}$$

$f(y)$  → function of  $y$  and  $g(x)$  → function of  $x$ .

By comparing  $\textcircled{3.a}$  and  $\textcircled{3.b}$ .

$$\phi = V_\infty x + \text{const} \rightarrow \textcircled{4}$$

Velocity Potential should be diff in order to obtain

Velocity so  $\mathbf{v} = \nabla \phi$  and const term vanishes

$$\text{so } \boxed{\phi = V_\infty x} \rightarrow \textcircled{5}$$

(1) Consider the stream function equation.

$$(ic) \frac{d\psi}{dy} = u = V_\infty \rightarrow \textcircled{7.a}$$

$$\frac{d\psi}{dx} = -v = 0 \rightarrow \textcircled{7.b}$$

On integrating both the equations

$$\boxed{\psi = V_\infty y} \rightarrow \textcircled{8}$$

Now convert both the term to polar co-ordin;

$$so, u = V_\infty \cos\theta ; v = V_\infty \sin\theta \rightarrow \textcircled{9}$$

[looking in  $\textcircled{6}$  and  $\textcircled{8}$  it is clear that the  $\psi$  any  
are the lines in stream lines where it is constant  
of  $y$  and  $\phi$  are the lines of constant  $x$ .]

$\rightarrow$  here  $\phi = \text{const}$  &  $\psi = \text{const}$  are mutually

Sub  $\textcircled{9}$  in  $\textcircled{6}$  and  $\textcircled{8}$

$$\phi = \underline{V_\infty V_\infty \cos\theta} ; \psi = \underline{V_\infty V_\infty \sin\theta}$$

Note that  $V_\infty$  is constant.

$$\overline{\overline{Q}} = \overline{V} \cdot \overline{A}$$

Since it is irrotational flow  $\nabla \times V = 0$

$$L = - \iint_S (\nabla \times V) \cdot dS$$

Circulation can be stated in other words as

$$L = \oint_C \phi dS = V_\infty \cdot a$$

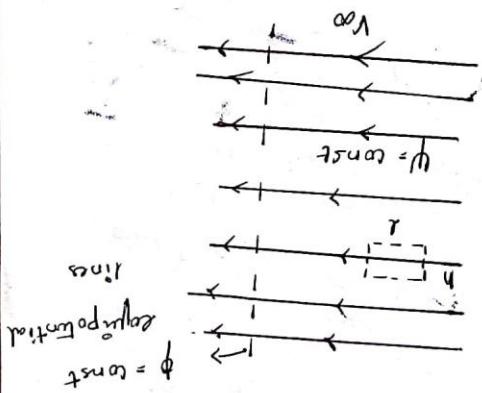
any closed curve is, here  $V_\infty$  is constant.

Other words the line integral of  $dS$  around

$$L = 0$$

$$(4) a + (7) a +$$

$$(7) a - V_\infty L = \oint_C V \cdot dS = 0$$



all the closed curves  $C$  in the

case be a rectangle with height  $a$  and width  $b$ .

$$L = - \oint_C V \cdot dS$$

(2)

on uniform flow.

Circulation in

constant

## Source flow..

→ Consider a 2d, incompressible flow, where all the stream lines are straight lines emanating from Central Point O.

→ Let the velocity of each stream line be vary with distance from point O.

→ Such type of flow is called Source flow.

→ Velocity Components in radial & tangential direction

$$V_R \text{ and } V_\theta$$

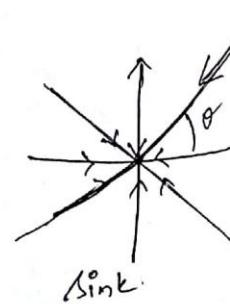
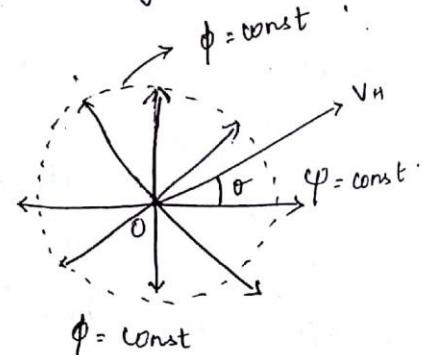
→ Here  $V_\theta = 0$ . ; It is a incompressible flow  $\rightarrow \nabla \cdot v = 0$

@ every point except the origin, where  $\nabla \cdot v$  becomes infinite; It is also irrotational @ every point.

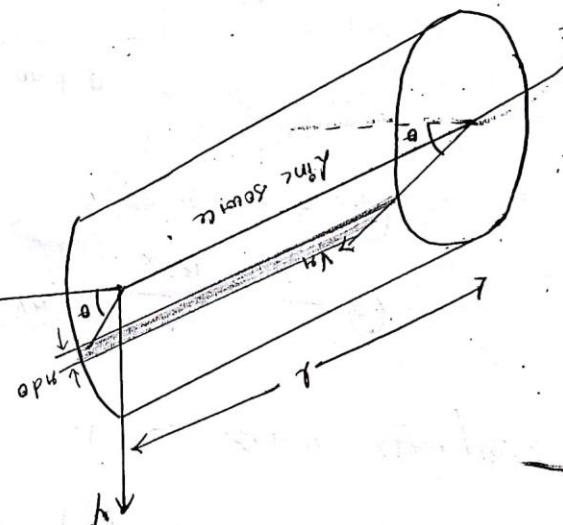
→ In a source flow the stream lines are directed away from the origin.

→ In case of sink flow, the stream lines are directed towards the origin.

→ Sink flow is a negative of source flow.



$\text{Flow rate } Q = \int_{2\pi}^0 \rho V_A (\alpha d\theta) L = \rho A V_A \int_0^{2\pi} (\alpha d\theta) L = \rho A V_A \alpha L$   
 Element mass flow across the surface area  $d\theta$  is  $\rho V_A ds$ .  
 Consider a depth of length  $L$  to page along Z-axis.  
 To find the surface of cylinder the mass flow across  
 the volume from source is proportional to the squared distance  
 from center to point  $(r)$  resulted to hours  $V_A = C/A$  where  
 Velocity is inversely proportional to the squared distance  $r$ .



$$Q = \rho V_A A$$

These bodies can be visualized.  
 Singularities over any arbitrary body, the flow field over  
 for which the strength is known value. So by placing these  
 by the source / sink. Thus the origin is the singular point  
 here the radial flow surrounding the origin is being induced  
 for which the source / sink. Thus the origin is the singular point  
 over any arbitrary body, the flow field over

③

NOTE:

Volume flow per second is given by  $\dot{V} = \frac{\dot{m}}{\rho}$

$$\dot{V} = 2\pi n l v_n \rightarrow (3)$$

Volume flow rate  $\mu m$  unit length given by  $\lambda$

$$\lambda = \frac{\dot{V}}{l} = \underline{\underline{2\pi n v_n}}$$

$$v_n = \frac{\lambda}{2\pi n} \rightarrow (4)$$

On comparing (1) and (4).

$$c = \lambda / 2\pi \quad \text{where } \lambda \rightarrow \text{source strength}$$

Velocity potential

$$\frac{d\phi}{dr} = v_n = \frac{\lambda}{2\pi r} \rightarrow (5)$$

$$\frac{1}{r} \frac{d\phi}{d\theta} = v_\theta = 0 \rightarrow (6)$$

Int. (5) < (6) wrt  $r$  and  $\theta$

$$\phi = \frac{\lambda}{2\pi} \ln r + f(\theta) \rightarrow (7)$$

$$\phi = \text{const} + f(\theta) \rightarrow (8)$$

$$\text{So } \phi = \underline{\underline{\frac{\lambda}{2\pi} \ln r}} \rightarrow (9)$$

Stream function :-

$$\frac{1}{r} \frac{d\psi}{d\theta} = v_n = \frac{\lambda}{2\pi r} \rightarrow (10)$$

$$-\frac{d\psi}{dr} = v_\theta = 0 \rightarrow (11)$$

Here  $\theta =$

is a equation

In  $\phi$  eq

Center @

Combination

$\rightarrow$  consider

strength  $\rightarrow$

$\rightarrow$  Superimpos

(max) on

$\rightarrow$  The sta

$\psi =$

The Velo

(4)

$$\Psi = \frac{1}{2\pi R} \theta \rightarrow \textcircled{12} \quad \text{By int } \textcircled{10} \text{ wrt } \theta$$

where

$$\Psi = \frac{-1}{2\pi} \theta = \text{constant}$$

Here  $\theta = \text{constant}$  value which in polar co-ordinates is a equation of straight line - from origin

In  $\phi$  eqn  $\theta$  is constant, so it is a circle with its center @ origin. so it is the equipotent line.

### Combination of Uniform flow with source and sink

→ Consider a polar Co-ordinate system, with a source of strength  $s$  @ origin.

→ Superimpose Uniform stream with velocity  $v_\infty$  (along x-axis) on the source flow.

→ The stream function is given by

$$\Psi = v_\infty R \sin \theta + \frac{1}{2\pi} \theta. [ \text{flow is incompressible and irrotational} ] \\ = \text{const} \rightarrow \textcircled{1}$$

The Velocity field is obtained by diff the eqn. ①.

$$V_R = \frac{\partial \Psi}{\partial R} = v_\infty \cos \theta + \frac{1}{2\pi R} \rightarrow \textcircled{2}$$

$$V_\theta = - \frac{\partial \Psi}{\partial \theta} = -v_\infty \sin \theta \rightarrow \textcircled{3}$$

Radial velocity from a source is  $\frac{A}{2\pi r}$

Free stream velocity in radial dir  $V_\infty \cos \theta$

The stagnation point is obtained by setting ② & ③ to zero

$$V_\infty \cos \theta + \frac{A}{2\pi r} = 0 \rightarrow ④$$

$$V_\infty \sin \theta = 0 \rightarrow ⑤.$$

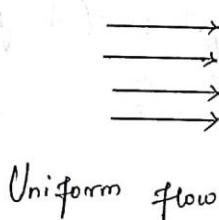
By equating we get  $\sin \theta = 0 \Rightarrow \theta = 0, \pi, \text{ all } \pi's$

Let us take  $\theta = \pi$ , on substituting to ④. We get

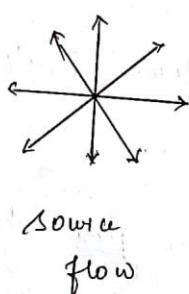
$$A = \frac{1}{2\pi V_\infty} \left[ V_\infty \cos(\pi) + \frac{A}{2\pi r} \right] \downarrow (-1)$$

$$A = \frac{-A}{2\pi V_\infty}$$

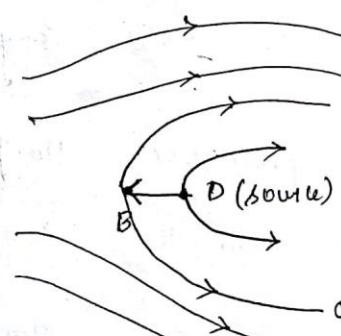
Note



Uniform flow



Source flow



Combined flow

Now from eqn of "A" the stagnation pt is located at a distance of  $\frac{1}{2\pi V_\infty}$  from source.

So DB ↑ when A is ↑ed

Stream  $V_\infty$  is ↑ed so that

be replaced

→ On the figure if  $A$  is increased and  $V_{\infty}$  = Same. (5)

the stagnation point will be thrown upstream

Vice versa if it is same and  $V_{00}$  is  $\neq$  0 then it will be thrown downstream.

$\hat{M} = \text{sub } \underline{x_1} \text{ and } \underline{x_0} \text{ in eqn of } \Psi$

$$\Psi = V_\infty \frac{1}{\alpha T V_\infty} \sin^0 \pi + \frac{1}{\alpha \pi} \pi = \text{const.}$$

$\psi = \frac{1}{d}$  → The stream line goes through  
stagnation point is given by  $\psi = \frac{1}{d}$ .

## NOTE

From the fig  $\rightarrow$  Any stream line of the combined flow could be replaced by solid surface of same shape.

→ Consider the line ABC, since it has stagnation

→ Consider point A-B stream line ABC is dividing stream line.

which separates free stream and fluid from source.

→ Entire region inside ABC could be replaced with a solid body of same shape and external flow will have no issues.

have no issues.  
 A line extends to  $(y = \lambda/2)$  forming semi infinite body

- Stream line extends to  $(y - x^2/2)^{1/2}$
- so that the flow over a semi infinite body should be replaced by the Uniform flow of  $v_1 v_\infty$  + source flow

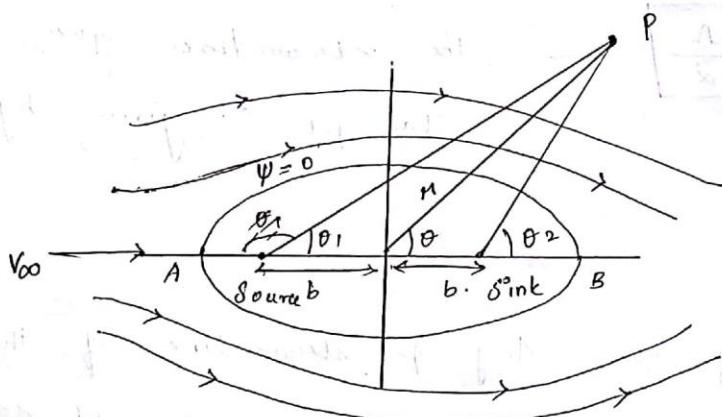
of strength  $-1$  @ D.

→ The resulting super position will represent the flow over the prescribed solid semi infinite body of shape ABC.

→ This is half body which stretches to infinity @ downstream to a line

→ If a sink of equal strength as source is added @ the the pt D, then the resulting body shaped will be closed.

→



In terms of polar co-ordinates.

$$\psi = V_\infty r \sin\theta + \frac{1}{2\pi} \theta_1 - \frac{1}{2\pi} \theta_2$$

Here,  $\theta_1$  &  $\theta_2$  are the strengths of source & sink respectively.

These four terms has opposite sign. If placed below zero.

Source

Sink

Source

Sink

(A)

## Doublet flow :-

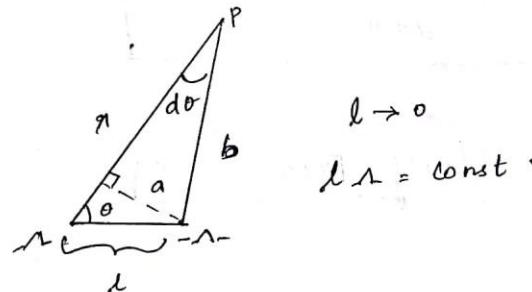
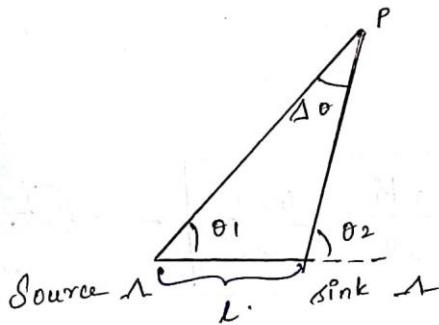
→ The special degenerate case of a source-sink pair leads to a singularity called doublet.

→ Consider a source of strength  $A$  and sink of strength  $-A$  separated by a distance " $l$ ". At any point  $P$  in the flow stream function is

$$\Psi = \frac{A}{2\pi} (\theta_1 - \theta_2) = -\frac{A}{2\pi} \Delta\theta \rightarrow ①$$

→ because  $(\theta_2 - \theta_1) = \Delta\theta$ . Let the distance  $l$  approach "0" where the magnitude of strength of both source & sink  $\rightarrow \infty$ .

→ So that the value of  $\Delta\theta$  should be constant. So that we obtain a special pattern called doublet.



$$\Psi = \lim_{l \rightarrow 0} \left( \frac{-A}{2\pi} \Delta\theta \right)$$

$K = l A = \text{const}$

→ ② The strength of the doublet is given by

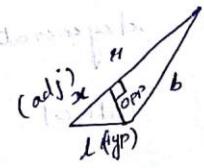
$$K = l A$$

In the limit  $\Delta\theta \rightarrow d\theta \rightarrow 0$ ,

Let  $a, b$  be the distance to point "P" from source and sink. Draw a line from sink to  $P$  & denote the length by  $c$

From figure  $a = l \sin \theta \Rightarrow \delta \sin \theta = a/l$

$$b = h - l \cos \theta$$



$$(b = h - a)$$

$$\cos \theta = a/l$$

$$(a = l \cos \theta)$$

$$d\theta = a/b$$

$$d\theta = \frac{1}{K} \frac{l \sin \theta}{h - l \cos \theta} \rightarrow (3)$$

$$\Psi = \lim_{l \rightarrow 0} - \frac{1}{d\pi} d\theta \quad [ \text{sub } (3) \text{ in } (1) ]$$

$K = \text{const}$

$$= \lim_{l \rightarrow 0} \left( \frac{-1}{d\pi} \frac{l \sin \theta}{(h - l \cos \theta)} \right) \quad [ \text{sub } K = \text{const} ]$$

$$= \lim_{l \rightarrow 0} - \frac{K \sin \theta}{d\pi(h - l \cos \theta)} \quad [ \text{sub } K = \text{const} ]$$

$$\Psi = - \frac{K \sin \theta}{d\pi h} \rightarrow (4)$$

Velocity Potential  $\phi$  /  $\psi$

$$\phi = \frac{+K}{d\pi} \frac{\cos \theta}{h} \rightarrow (5)$$

Stream lines of doublet are obtained from

$$\Psi = \frac{-K}{\sin \theta} - \frac{K \sin \theta}{d\pi h} = C \Rightarrow h = - \frac{K \sin \theta}{d\pi C}$$

From polar co-ordinates  $h = d \sin \theta$  is a circle with dia  
and center @  $d/2$ .

(7)

→ Stream function of combined flow is

$$\Psi = V_\infty R \sin \theta = \frac{K}{2\pi} \frac{\sin \theta}{R}$$

$$= V_\infty R \sin \theta \left[ 1 - \frac{K}{2\pi V_\infty R^2} \right] \rightarrow (1)$$

$$\text{Let } \frac{K}{2\pi V_\infty} = R^2$$

$$= V_\infty \sin \theta R \left[ 1 - \frac{R^2}{R^2} \right] \rightarrow (2).$$

The velocity field is obtained by diff (2)

$$(i) V_R = \frac{1}{R} \frac{\partial \Psi}{\partial \theta} = \frac{1}{R} \left[ V_\infty \cos \theta R \left( 1 - \frac{R^2}{R^2} \right) \right]$$

$$= \underline{\left( 1 - \frac{R^2}{R^2} \right) V_\infty \cos \theta}.$$

$$\begin{aligned} V_\theta &= - \frac{\partial \Psi}{\partial R} = - \left[ V_\infty R \sin \theta \frac{\partial R^2}{\partial R} + \left( 1 - \frac{R^2}{R^2} \right) V_\infty \sin \theta \right] \\ &= - \underline{\left( 1 + \frac{R^2}{R^2} \right) V_\infty \sin \theta}. \quad \left[ \text{Formula } d(uv) = udv + vdu \right] \end{aligned}$$

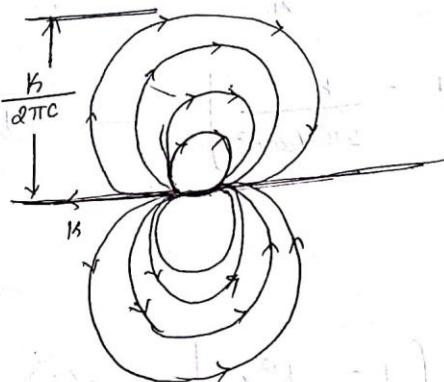
To locate the stagnation point.  $V_R = 0; V_\theta = 0$

$$\left( 1 - \frac{R^2}{R^2} \right) V_\infty \cos \theta = 0 \rightarrow (3)$$

$$\left( 1 + \frac{R^2}{R^2} \right) V_\infty \sin \theta = 0 \rightarrow (4)$$

→ Streamlines of a doublet are a family of circles with different radii corresponding to different values of  $p_0$ .

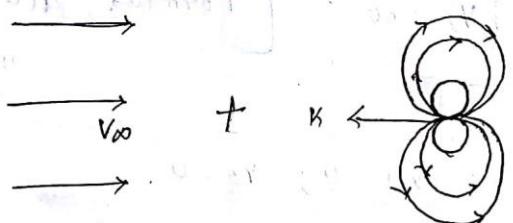
$$d = \frac{K}{2\pi c} ; \text{ Different circles correspond to diff values of } p_0$$



### Non Lifting flow over a Circular Cylinder

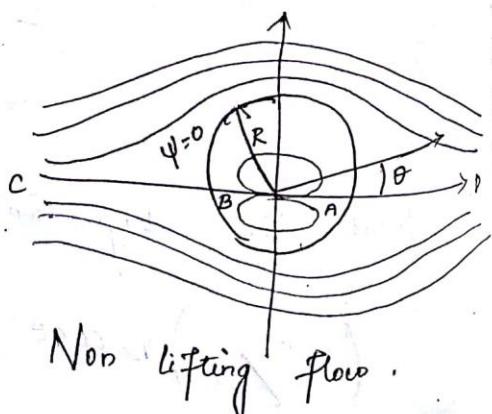
→ A combination of uniform flow and doublet flow produces flow over a circular cylinder.

→ Consider Uniform flow of velocity  $v_\infty$  and the doublet of strength  $K$ . The direction of doublet is upstream facing the Uniform flow.



Uniform flow

Doublet flow



Non lifting flow

(8)

→ So No pressure difference is created coz of which the net lift is "0" and the same way  $p_H$  @ front of cylinder is balanced by  $p_H$  @ back / rearward of cylinder which means "Zero drag".

→ In practical way Zero drag is impossible, where as in theory we have obtained zero drag, which is a paradox.

→ So the result of theoretical zero drag is called as

### D'Alembert's Paradox

→ But in practical we know that the drag created is due to the viscous effects. (ie) drag due to shear stresses.

→ The velocity distributions over the surface of the cylinder is obtained by substituting  $R = R$  in  $V_H$  &  $V_\theta$ .

$$\text{so } V_H = 0 \rightarrow (6) \quad (7)$$

$$V_\theta = -2V_\infty \sin\theta \rightarrow (8).$$

Note :- From the above eqn it is clear that the velocity is max @ 2 pts where  $\theta = \pi/2 \& 3\pi/2$  (ie) @ top and bottom of the cylinder.

By solving the eqns for stagnation point located @ A and B.

$(u, \theta) = (R, 0) \& (R, \pi)$ . They are given by  
 $\rightarrow$  The eqn of stream line through pt B is given by  
 sub  $R = A = R$  &  $\theta = \pi$  so that  $\underline{\psi = 0}$  and at pt A is  
 given by  $A = R$  &  $\theta = 0$ ; so  $\underline{\psi = 0}$

$$\underline{\psi = V_{\infty} R \sin \theta \left( 1 - \frac{R^2}{R^2} \right)} = 0 \rightarrow (5).$$

here  $R^2 = \frac{K}{2\pi V_{\infty}}$  where in polar co-ordinates  
 $r_1 = \text{const} = R$  is const of circle of radius R.

$\rightarrow$  Note that  $\psi = 0$  streamlines since it goes through the stag pt. it is dividing streamline.

$\rightarrow$  So flow inside the circle is given by doublet and the flow outside the circle is given by the Uniform flow.

$$R = \sqrt{\frac{K}{2\pi V_{\infty}}} \rightarrow (b)$$

### D'Alembert Paradox:

By looking into the figure,  $P_H$  dist is symmetrical about the axis, so that  $P_H$  dist over the top of the cylinder is balanced by the  $P_H$  dist @ bottom of the cylinder.

(9)

The pressure co-efficient is given by

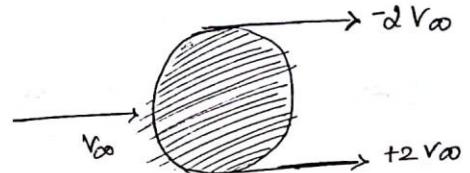
$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 \rightarrow ⑨$$

$$V_\theta = 2 V_\infty \sin\theta$$

$$\left(\frac{V_\theta}{V_\infty}\right) = 2 \sin\theta \rightarrow ⑩$$

Sub ⑩ in ⑨ to get

$$\underline{C_p = 1 - 4 \sin^2 \theta}$$



From the figure it is clear that no lift and no drag is produced over a circular cylinder.

Problem :-

Consider a non lifting flow over a circular cylinder. Calculate the location on surface of the cylinder where surface pressure equals the free stream pressure.

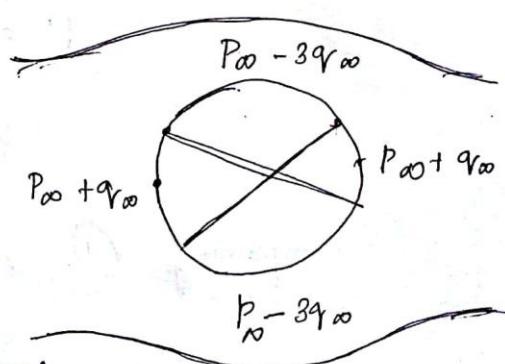
$$\text{When } P = P_\infty, C_p = 0.$$

$$C_p = 1 - 4 \sin^2 \theta = 0$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\theta = 150^\circ, 30^\circ, 210^\circ, 330^\circ$$



## Vortex Flow :-

→ Consider a flow where all the stream lines are concn. circles about a point. Let velocity along any given circular stream line be constant.

→ But let it vary from one stream line to other inversely with distance.

→ Such flow is called Vortex flow.

$$\text{From defn. } V_\theta = \frac{\text{Const}}{r}; V_r = 0.$$

Here  $\nabla \cdot v = 0$  @ every pt &  $\nabla \times v = 0$  @ every pt except origin

To evaluate the constant

$$V_\theta = \frac{\text{Const}}{r} = C/r. \rightarrow ①$$

Take the circulation around a given stream line.

of radius  $r_1$ .

$$\Gamma = \oint v \cdot ds = -V_\theta (2\pi r_1)$$

$$V_\theta = -\frac{\Gamma}{2\pi r_1}. \rightarrow ②$$

Comparing ① & ②.

$$\frac{C}{r_1} = \frac{\Gamma}{2\pi r_1} \Rightarrow \boxed{C = \frac{\Gamma}{d\pi}}$$

Now by relating circulation to vorticity.

→ Assume that the surface is in a flow field and the velocity @ pt P is  $v$ ; where pt P is in the surface. From Stokes theorem.

$$\Gamma = - \oint_C \mathbf{v} \cdot d\mathbf{s} = - \iint_S (\nabla \times \mathbf{v}) d\mathbf{s}. \rightarrow \textcircled{3}$$

here  $\Gamma = -2\pi C$   $\rightarrow \textcircled{4}$

$\underline{\underline{\text{Compare } \textcircled{3} \text{ & } \textcircled{4}}}$

$$2\pi C = \iint_S (\nabla \times \mathbf{v}) d\mathbf{s}.$$

Here both  $\nabla \times \mathbf{v}$  and  $d\mathbf{s}$  are in same dir so

$$\iint_S (\nabla \times \mathbf{v}) d\mathbf{s} = \iint_S |\nabla \times \mathbf{v}| d\mathbf{s}.$$

→ Surface integral is taken over the circular area inside the streamline along which the circulation  $\Gamma = -2\pi C$  is evaluated.

→ Here  $\Gamma$  is same for all circulation streamlines because it doesn't depends on "n".

→ Choose a circle as close to origin ie  $n \rightarrow 0$ ;  $\Gamma = \underline{\underline{-2\pi C}}$

→ The area inside the small circle is very small.

$$\text{so } \iint_S (\nabla \times v) ds \rightarrow |\nabla \times v| ds$$

$$\text{so } 2\pi C = (\nabla \times v) ds.$$

$$\nabla \times v = \frac{2\pi C}{ds}$$

when  $ds \rightarrow 0$  so that  $|\nabla \times v| \rightarrow \infty$ .

From here we can conclude that the vortex filaments by irrotational at every point except origin. (ie)  $\nabla \times v \rightarrow 0$ .

$$\text{here } \frac{\partial \phi}{\partial \theta} = V_R = 0$$

$$\frac{1}{\mu} \frac{\partial \phi}{\partial \theta} = V_\theta = \frac{\Gamma}{2\pi R}$$

On integrating

$$\boxed{\phi = \frac{\Gamma}{2\pi} \theta}$$

Stream func

$$\frac{1}{\mu} \frac{\partial \psi}{\partial \theta} = V_R = 0$$

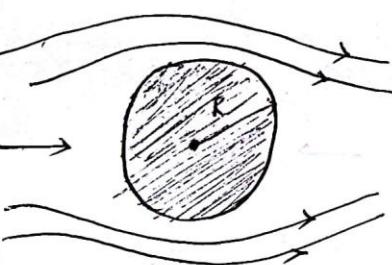
$$-\frac{\partial \psi}{\partial r} = V_\theta = -\frac{\Gamma}{2\pi R}$$

On Intg

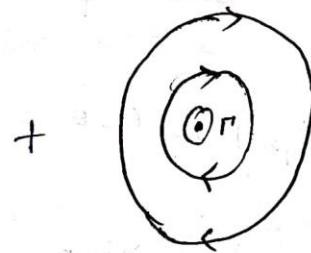
$$\boxed{\psi = \frac{\Gamma}{2\pi} \ln r}$$

## Lifting flow over a Cylinder -

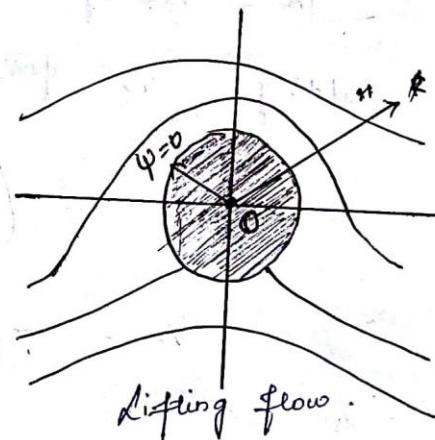
- We have concluded that there is no lift around a circular cylinder by combining Uniform flow and doublet.
- There are other flow patterns which may give rise to lift over a cylinder. One such flow pattern is by combination of Non lifting flow over a cylinder and Vortex of strength  $\Gamma$ .
- A cylinder spinning about its axis gives rise to the finite lift (i.e. RPM should be very high).
- Some time the non symmetric A.D forces acts over an asymmetric bodies.



Non lifting flow



Vortex flow



Lifting flow.

$$\text{For Non lifting flow} \Rightarrow \psi_1 = (V_\infty + \sin\theta) \left( 1 - \frac{R^2}{r^2} \right) \rightarrow ①$$

$$\text{Vortex flow of strength } \Gamma \Rightarrow \psi_2 = \frac{\Gamma}{2\pi} \ln r + \text{const} \rightarrow ②$$

Let  
Here a constant be  $\frac{-\Gamma}{2\pi} \ln R \rightarrow \textcircled{3}$

Combine \textcircled{2} and \textcircled{3}

$$\begin{aligned}\psi_2 &= \frac{\Gamma}{2\pi} \ln H - \frac{\Gamma}{2\pi} \ln R \\ &= \frac{\Gamma}{2\pi} (\ln H - \ln R) = \frac{\Gamma}{2\pi} \ln \left(\frac{H}{R}\right). \rightarrow \textcircled{4}\end{aligned}$$

$\Rightarrow$  since  
Quadrant

$$\text{So } \psi = \psi_1 + \psi_2.$$

$$\psi = V_{\infty} n \sin \theta \left(1 - \frac{R^2}{n^2}\right) + \frac{\Gamma}{2\pi} \ln \left(\frac{n}{R}\right) \rightarrow \textcircled{5}$$

There

Now

$\rightarrow$  In eqn if  $n=R$  then Value of  $\psi=0$  for all values

but  $\psi = \text{constant}$  is the eqn of the streamline.

$\rightarrow$  The velocity field is obtained by diff \textcircled{5} (or) by adding vel field of both flows.

$$\text{So } V_H = \left(1 - \frac{R^2}{n^2}\right) V_{\infty} \cos \theta \rightarrow \textcircled{6}$$

$$V_\theta = - \left(1 + \frac{R^2}{n^2}\right) V_{\infty} \sin \theta - \frac{\Gamma}{2\pi n}. \rightarrow \textcircled{7}$$

[NOTE :-

To locate stagnation points let  $V_H = V_\theta = 0$ .

so from  $V_H = 0$  we get  $\boxed{n = R}$ .

Sub in \textcircled{7}

$$V_\theta = 0 = - \left(1 + \frac{1}{R}\right) V_{\infty} \sin \theta = - \frac{\Gamma}{2\pi R}$$

(12)

$$-2V_\infty \sin\theta - \frac{\Gamma}{2\pi R} = 0$$

$$-2V_\infty \sin\theta = \frac{\Gamma}{2\pi R}$$

 $\theta$ 

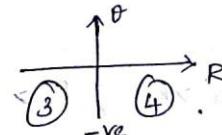
$$= \sin^{-1}$$

$$\frac{-\Gamma}{4\pi R V_\infty}$$

$$\boxed{\frac{-\Gamma}{4\pi R V_\infty}}$$

 $\theta$ 

$\Rightarrow$  since  $\theta$  is a (-ve) value, it must be in third / 4<sup>th</sup> Quadrant Only



→ There can be 2 stagnation pts

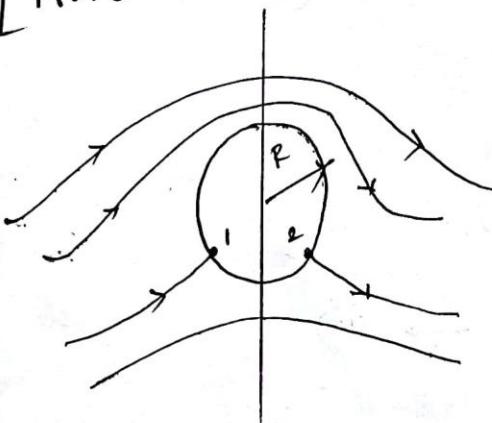
→ Now this result is valid only when  $\alpha \leq 1$  &  $\theta = 1$   
and not valid when  $\alpha \neq 1$ .  $b_{\infty} = \sin^{-1} (\alpha) (> 1) = \text{Unknown}$ .

→ for  $\alpha \leq 1$  the value of  $\alpha$  be 0.5.

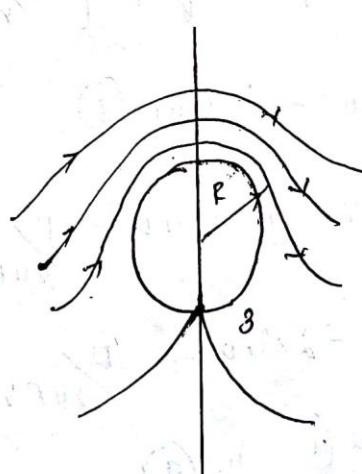
so that the  $\theta$  value will be  $\theta = 210^\circ / 330^\circ$   
because  $\sin(210/330) = -0.5$ .

and it should not exceed  $360^\circ$  coz circular cylinder.

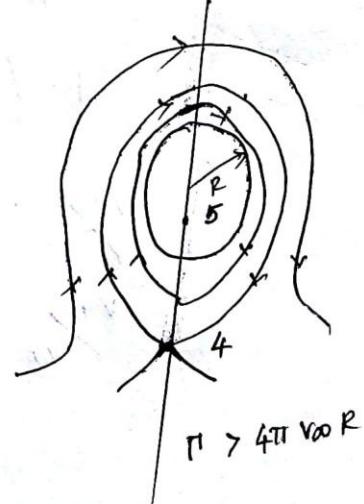
NOTE :



$$\Gamma < 4\pi V_\infty R$$



$$\Gamma = 4\pi V_\infty R$$



$$\Gamma > 4\pi V_\infty R$$

→ Now if the flow is  $x < 1$  then the  $\frac{r}{R} \times \Gamma < 4\pi V_\infty$   
 It means the free stream value is dominating (ie)  $V_\infty$   
 so that the stag pts will be under the cylinder's surface.

→ If  $x = 1$ ;  $\frac{r}{R} = 4\pi V_\infty R$  then the stag pts will be  
 Only one (ie) it will be just @ beneath of cylinder.

→ If  $x > 1$ ;  $\frac{r}{R} > 4\pi V_\infty R$  then the value of  $\frac{r}{R}$  (ie)  $V_\infty$  is dominating, so one stag point inside the cylinder.

One outside it

→ It means stag points depends on vortex strength  $\Gamma$  and free stream velocity  $V_\infty$ . So it is for incompressible flow over a cylinder, there are infinite number of possible corresponding to the infinite values of  $\Gamma$ .

$$V = V_\infty = -dV_\infty \sin\theta - \frac{\Gamma}{2\pi R} \quad (\text{sub } r=R \text{ in (7)})$$

$$\text{Pressure Co-efficient } C_p = k + \left( \frac{V}{V_\infty} \right)^2 \rightarrow (8)$$

$$= k - \frac{2\sin\theta}{V_\infty} \quad \text{From (8)} \rightarrow (9)$$

$$\frac{V_\theta}{V_\infty} = V_\infty \left( -d\sin\theta - \frac{\Gamma}{2\pi RV_\infty} \right)$$

$$\frac{V_\theta}{V_\infty} = -d\sin\theta - \frac{\Gamma}{2\pi RV_\infty} \quad \text{sub (10) in (9)} \rightarrow (10)$$

(13)

$$C_p = 1 - \left( -2 \sin \theta - \frac{\Gamma}{2\pi R V_\infty} \right)^2$$

$$C_p = 1 + \left[ 4 \sin^2 \theta + \frac{2\Gamma \sin \theta}{\pi R V_\infty} + \left( \frac{\Gamma}{2\pi R V_\infty} \right)^2 \right].$$

Normally A-D force coeff is obtained by intg the Pr coeff

over the surface, so

$$C_d = \frac{1}{c} \int_{L_e}^{T_e} (C_{p_u} - C_{p_e}) dy$$

$$= \frac{1}{c} \int_{L_e}^{T_e} C_{p_u} dy - \frac{1}{c} \int_{L_e}^{T_e} C_{p_e} dy.$$

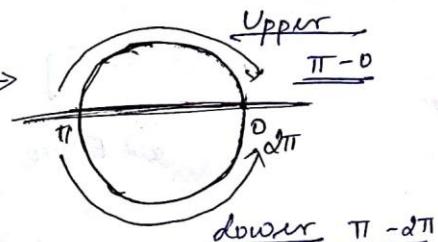
det  $y = R \sin \theta \quad ; \quad dy = R \cos \theta d\theta \quad ; \quad c = dR$

$$C_d = \frac{1}{2R} \int_{\pi}^{0} C_{p_u} R \cos \theta d\theta - \frac{1}{2} \int_{\pi}^{0} C_{p_e} \cos \theta d\theta$$

The limits here is given by

$$C_d = -\frac{1}{2} \int_0^{\pi} C_p \cos \theta d\theta - \frac{1}{2} \int_{\pi}^{2\pi} C_p \cos \theta d\theta$$

$$= -\frac{1}{2} \int_0^{2\pi} C_p \cos \theta d\theta$$



→ [B cos of symmetry  
( $C_{p_e} = C_{p_u} = C_p$ )]

Now sub the value of  $C_p$ .

$$= -\frac{1}{2} \int_0^{2\pi} \left[ 1 - \left( 4 \sin^2 \theta + \frac{2\Gamma \sin \theta}{\pi R V_\infty} + \left( \frac{\Gamma}{2\pi R V_\infty} \right)^2 \right) \right] \cos \theta d\theta$$

Converting eqn to 3 categories

$$= \int_0^{2\pi} \cos \theta \, d\theta = 0$$

$$= [\sin \theta]_0^{2\pi} = 0$$

$$\int_0^{2\pi} \sin^2 \theta \cos \theta \, d\theta = 0.$$

$$\text{Let } u = \sin \theta; \, du = \cos \theta \, d\theta$$

$$\int_0^{2\pi} u^2 \, du = 0.$$

$$= \left[ \frac{u^3}{3} \right]_0^{2\pi}$$

$$= \left[ \frac{\sin^3 \theta}{3} \right]_0^{2\pi}$$

$$= 0$$

$$\int_0^{2\pi} \sin \theta \cos \theta \, d\theta = 0$$

$$\int_0^{2\pi} \frac{\sin^2 \theta}{2} \, d\theta$$

$$\text{Let } u = \sin \theta \, du = \cos \theta \, d\theta$$

$$\int_0^{2\pi} u^2 \, du$$

$$\left[ \frac{u^3}{3} \right]_0^{2\pi}$$

$$\left[ \frac{\sin^3 \theta}{3} \right]_0^{2\pi}$$

$$= 0$$

$$\int_0^{2\pi} \sin \theta \, d\theta$$

$$[\cos \theta]_0^{2\pi}$$

$$1 - 1 = 0 \neq 0$$

So the value of  $C_d = 0$ .

Now

$$C_L = \frac{1}{c} \int_0^c C_{px} \, dx - \frac{1}{c} \int_0^c C_{pu} \, dx.$$

Converting it to polar co-ordinates.

$$x = R \cos \theta; \, dx = -R \sin \theta \, d\theta; \, c = 2R$$

$$C_L = -\frac{1}{2R} \int_{\pi}^{2\pi} C_{px} R \sin \theta \, d\theta + \frac{1}{2R} \int_0^{\pi} C_{pu} R \sin \theta \, d\theta$$

$$\Rightarrow C_{px} = C_{pu} = C_{px}.$$

$$C_L = -\frac{1}{2} \int_0^{2\pi} C_p \sin \theta \, d\theta$$

Now sub  $C_p$  and separate the eqn's to intg

From +

he

(14)

$$\begin{aligned}
 & \int_0^{2\pi} \sin \theta \, d\theta \\
 & [ \cos \theta ]_0^{2\pi} \\
 & 1 - 1 = 0 \\
 & \int_0^{2\pi} \sin^2 \theta \, d\theta \\
 & = \int_0^{2\pi} \frac{1 - \cos 2\theta}{2} \, d\theta \\
 & = \frac{1}{2} \int_0^{2\pi} (1 - \cos 2\theta) \, d\theta \\
 & = \frac{1}{2} \left[ \theta - \frac{1}{2} \int \cos 2\theta \, d\theta \right] \\
 & = \frac{1}{2} \left[ \theta - \frac{1}{2} \sin 2\theta \right] \\
 & = \text{Let } u = 2\theta ; \, du = 2d\theta \\
 & = \frac{1}{2} \int \frac{\cos u \, du}{2} \\
 & = \frac{1}{2} \left[ \frac{\sin u}{2} \right] \\
 & = \frac{1}{2} \left[ \frac{\sin 2\theta}{2} \right] \\
 & = \frac{1}{2} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} \\
 & = \frac{1}{2} \left[ 2\pi - 0 \right] \\
 & = \pi
 \end{aligned}$$

$$\begin{aligned}
 C_L &= \frac{1}{2} \int_0^{2\pi} CP \sin \theta \\
 &= \frac{1}{2} \cdot \frac{\Gamma}{\pi R V_\infty} \left[ -\frac{2 \Gamma \pi}{\pi R V_\infty} \right] \\
 &= \frac{\Gamma}{R V_\infty}
 \end{aligned}$$

From the defn  $I' = q_{\infty} s c_e$

$$= \frac{1}{2} P_\infty V_\infty^2 \delta c_e$$

here  $S \rightarrow \text{Area} \rightarrow 2R(1)$

$\left[ \begin{array}{l} 2R - \text{Radius} \\ 1 - \text{chord} \end{array} \right]$

$$= \frac{1}{2} P_\infty V_\infty^2 \frac{2R}{R V_\infty}$$

$$L' = \rho_0 V_\infty P'$$

→ With this result, lift per unit span is directly proportional to the circulation.

→ It is called as Kutta - Joukowski theorem.

### Magnus effect:

Let us spin the cylinder in the clockwise direction over the cylinder.

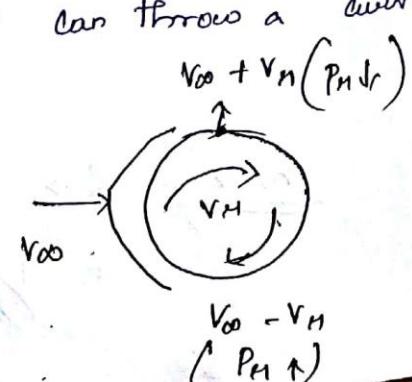
Now a finite lift can be measured between the fluid and surface of the cylinder as rotational motion.

→ Here the friction between the cylinder drags the fluid near the surface of some direction along with the non-spinning flow creates a velocity higher than usual velocity @ top of the cylinder and lesser vel @ bottom, which creates low pressure @ top and higher pressure @ bottom.

→ Because of which  $P_{\text{atm}}$  diff is created and lift is produced.

→ This is how a baseball pitcher can throw a curve ball and golfer can hit a hook.

→ This effect is called magnus effect.



## KUTTA JOUKOWSKI THEOREM & GENERATION OF LIFT

→ Consider an incompressible flow over an Aerofoil.

→ Let A be any curve in the flow enclosing the aerofoil.

→ The velocity field around the aerofoil will be such that the line integral of velocity around A will be finite.

→ lift according to the Kutta's theorem is given by

$$L' = \rho \alpha V_\infty \Gamma$$

→ The elementary flows are irrotational @ all the points except the vortex flow, which has infinite vorticity @ the origin.

→ Lifting flow over a cylinder is irrotational @ every point except at the origin.

→ It's only when we choose a curve that encloses the origin where  $\nabla \times V$  is infinite and yields a finite value of  $\Gamma$ .

→ The flow outside the aerofoil is irrotational, and circulation around any closed curve not enclosing the airfoil is 0.

→ From Kutta & Joukowski theorem the value of  $\Gamma$  must be evaluated around a closed curve enclosing the body.

→ The curve can be arbitrary but it must have the body inside it.

→ From the eqn  $d\Gamma = \rho v \alpha \Gamma$  we can't conclude circulation produces lift, rather lift is produced by pressure difference, where circulation is simply a defined quantity determined from same pressure.

→ But for a incompressible potential flow it is easier to determine the circulation around the body rather than calculating the surface pressure distribution.

Proposed method of finding circulation around a body  
based on circulation at trailing edge with circulation at leading edge  
and pressure for various bodies per branch iteration  
at each stage

## THE KUTTA CONDITION

→ Consider an Airfoil being experimeted and the flow pattern around the Airfoil is visualized as follows.

→ I Cond'n - The flow has just started over the Aerofoil and flow pattern just beginning to develop around it.

→ The flow tries to curl around the sharp trailing edge from bottom to top surface.

→ So the Velocity goes to max @ the corners.

II Cond'n - The real flow (ie) the I cond'n can't be adjusted, so Now the stagnation point will be moved towards the trailing edge.

→ This is the intermediate stage.

III Cond'n → Finally the steady flow is reached over the Aerofoil (ie) flow leaves the aerofoil in a very smooth way @ the trailing edge.

→ From this observation it is concluded that nature will select its own circulation so that flow leaves the trailing edge smoothly.

→ This condition is called as KUTTA'S CONDITION

→ On Application of Kutta Condition, two trailing edge

Kutta Cond

have been examined.

→ Consider the trailing edge with finite angle of  $\Gamma$  is

Denote the velocities @ top and bottom surface as  $v_1$ ,

→  $v_1$  is  $\parallel$  to top surface @ pt A

→ If

→  $v_2$  is  $\parallel$  to bottom surface @ pt A.

stagnation

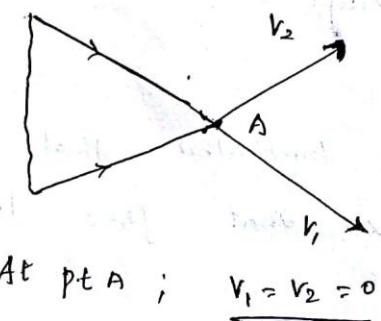
→ As per the finite trailing edge @ a pt "A" there are will be two different velocities  $v_1$  &  $v_2$  in two diff direction, which is not physically possible, so

the only possible solution is  $\underline{v_1 = v_2 = 0}$ .

→ For cusped type trailing edge the velocity @ bot top and bottom  $v_1$  &  $v_2$  are in the same direction and these  $v_1$  and  $v_2$  can be finite

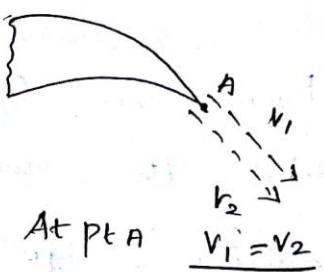
→ Pressure @ pt A should be a unique value

### Finite trailing edge



At pt A ;  $\underline{v_1 = v_2 = 0}$

### Cusped Trailing edge



At pt A  $\underline{v_1 = v_2}$

17

Kutta Condition are summarized as follows :-

→ For a given Al foil @ a given AOA, the value of  $\Gamma$  is such that the flow leaves T.E smoothly.

→ If trailing edge is finite then it is a stagnation point.

→ If T.E is cusped then the  $v_d$  @ top + bottom are equal in magnitude & direction as well as finite.

→ Imagine a Vortex sheet is placed on the Aerofoil then the strength of sheet is given by  $\gamma(s)$

→ For Trailing edge it is

$$\gamma(T_E) = v_1 - v_2$$

For Finite Angle Trailing edge :-

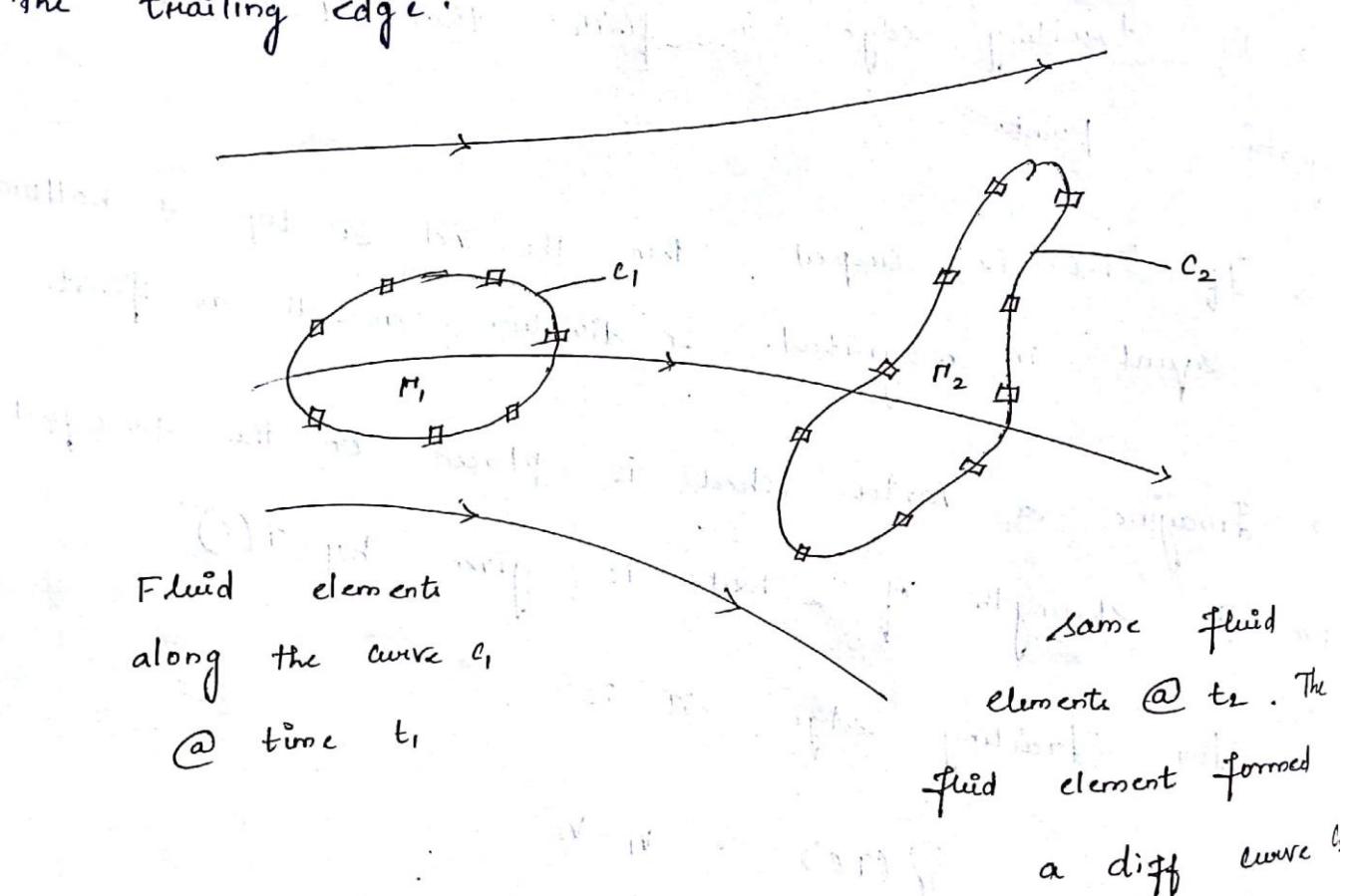
$$v_1 = v_2 = 0 ; \quad \gamma(T_E) = 0 // [v_1 - v_2].$$

For Cusped TE

$$v_1 = v_2 ; \quad \gamma(T_E) = v_1 = v_2 \\ = 0 //$$

## KELVIN'S CIRCULATION THEOREM

→ Kutta's Cond states that circulation around Aerofoil is just to ensure that the flow smoothly leaves the trailing edge.



→ Consider an arbitrary, inviscid, incompressible flow and Assume all body force  $\underline{f}_b = 0$

→ Choose a curve  $C_1$  and assume all fluid elements are on the curve so  $\Gamma_1 = - \oint_{C_1} \underline{V} \cdot d\underline{s}$  @ time  $t_1$

→ Choose After some time @  $t_2$  the fluid elements forms a diff curve  $C_2$  here the circulation is

$$\Gamma_2 = - \oint_{C_2} \underline{V} \cdot d\underline{s}$$

→ So Theorem states that circulation around a closed curve formed by a set of fluid elements remain the same as fluid elements move through out the flow.

So

$$\frac{D\Gamma}{Dt} = 0$$

### Starting Vortex

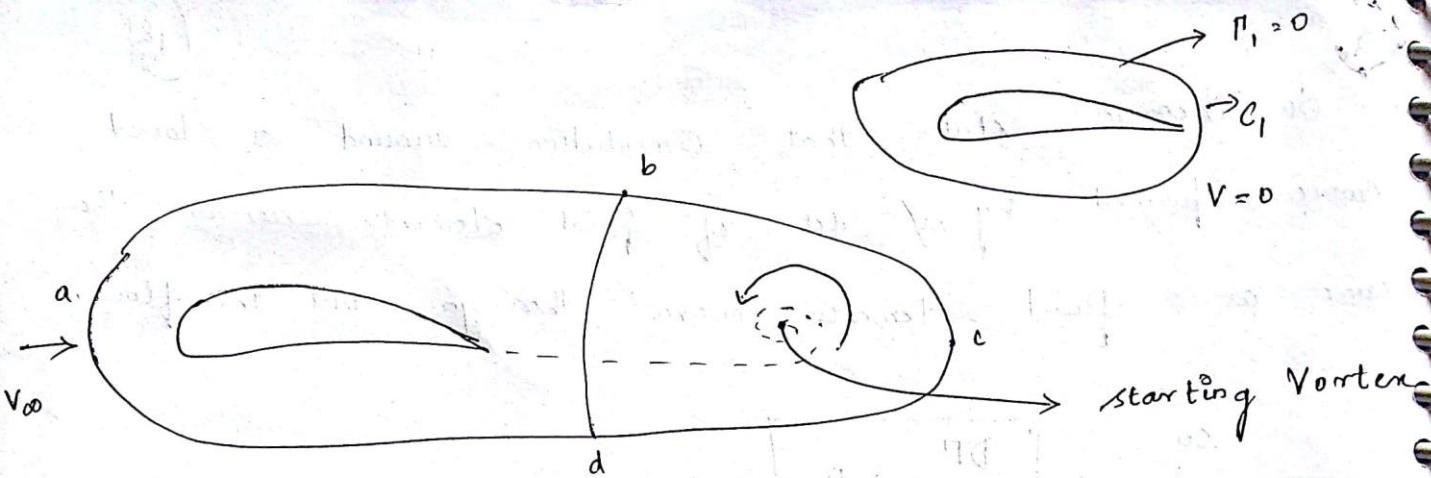
→ Consider an Airfoil in fluid @ rest; Circulation is 0 because  $\underline{v}=0$

→ Now start the flow in motion over the airfoil and the flow will start to curl around @ the trailing edge. so that  $v_\theta @ T.E$  is very high.

→ After sometime as flow becomes prominent this curled flow @ T.E will move downstream and becomes unstable (mean vorticity is unstable) and it tends to roll up to form a point vortex.

→ This vortex is called as "starting Vortex".

→ Once the flow becomes smooth and starts leaving T.E in smoother way Point Vortex is disappeared and higher velocity gradients disappear



→ Fluid elements initially makes up the wave  $\ell_1$  and it has moved downstream  $\ell_2$ . which is complete circuit abda.

→ We know that  $P_1 = 0$  around curve  $C_1$

→ Since same fluid element so  $P_2 = 0$  around  $C_2$

$$(i) \quad \Gamma_1 = \Gamma_2 = 0$$

→ Now split Airfoil in one wave and starting Vortex in other curve.

→ Curve abda enclosing Airfoil is  $C_4$  and circulation is  $\Gamma_4$ .

→ Curve bedb enclosing starting Vortex is  $C_3$  and  $\Gamma_3$

$$(ii) \quad \Gamma_3 + \Gamma_4 = \Gamma_2 \quad (\text{WRT } \Gamma_2 = 0)$$

$$\text{so } \underline{\Gamma_3 = -\Gamma_4}$$