UNIT-2

SINGLE SIDEBAND MODULATION

(SSB-MODULATION)

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★ INTRODUCTION - SINGLE SIDEBAND MODULATION-

→ Standard amplitude modulation and DSB-SC modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth

ie, $B.W = 2f_m$

→ In Both the cases (AM and DSB-SC) half of the transmission bandwidth is occupied by the upper sideband of the modulated wave, whereas the other half of the transmission bandwidth is occupied by the lower sideband of the modulated wave.

→ The upper and lower sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency 'f_c'

Thus, only only one sideband is necessary for transmission of information and if both the carrier and the other sideband are suppressed at the transmitter no information is lost.

∴ channel required the same BANDWIDTH as the message signal.

→ Thus, when only one sideband is transmitted the modulation is referred to as 'SINGLE SIDEBAND MODULATION'

ADVANTAGES OF SSB-

(1) SSB required is half the bandwidth required of AM and DSB-SC signals.

(2) Due to suppression of carrier and only one sideband → POWER is saved

(3) Reduced interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the signal will increase

(4) Fading does not occur in SSB transmission [Fading means that a signal alternately increases and decreases in strength as it is picked up by the receiver. It occurs because the carrier and sideband may reach the receiver shifted in time and phase w.r.t each other)

DISADVANTAGES OF SSB-

(1) The generation and reception of SSB signal is a complex process.

(2) The SSB modulation is expensive and highly complex to implement.

(3) Since carrier is absent, the SSB transmitter and receiver need to have an excellent frequency stability

APPLICATIONS OF SSB -

(1) SSB transmission is used in the applications where the power saving is required in mobile systems.

(2) SSB is also used in applications in which bandwidth requirements are low.

Ex: point to point communication, land, air, maritime mobile communications, TV, Telemetry, radio navigation, military communications.

★ FREQUENCY DOMAIN DESCRIPTION OF SSB MODULATED WAVES -

→ The frequency domain description of a SSB modulated wave depends on which sideband is transmitted.

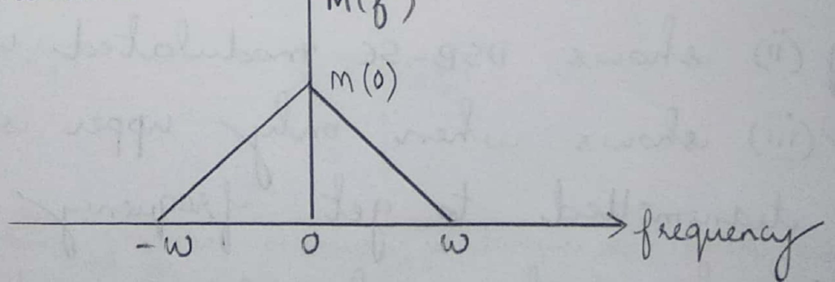
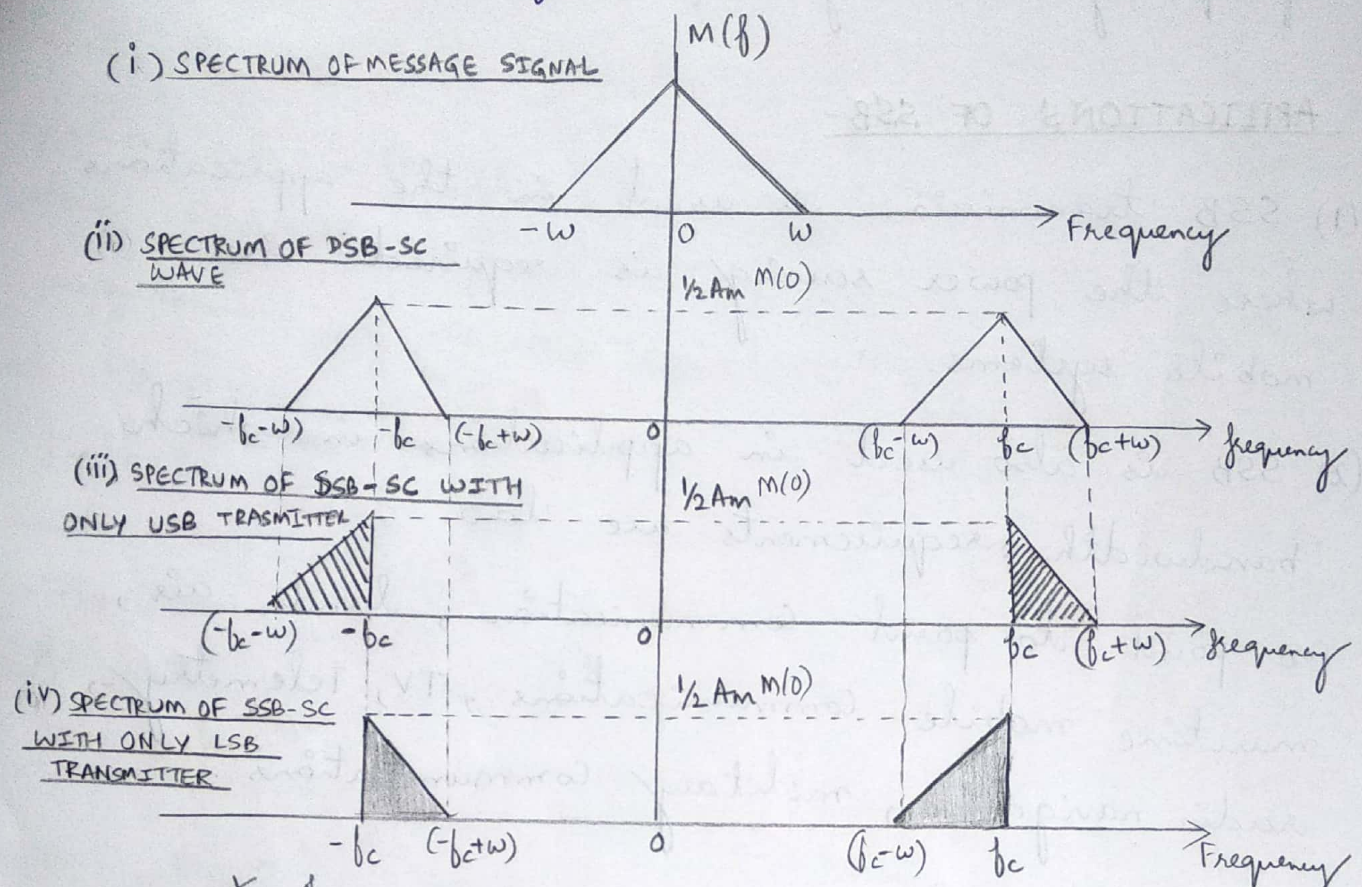
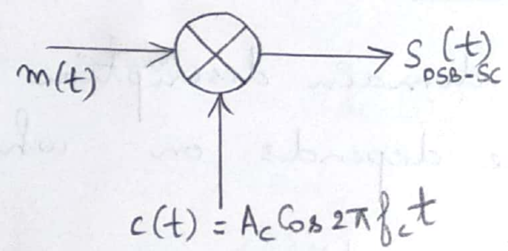


fig (i) SPECTRUM OF MESSAGE SIGNAL

→ fig (i) shows the spectrum of modulating signal M(f). The spectrum is limited to the band $-W \leq f \leq W$.



→ The spectrum of DSB-SC wave can be obtained by multiplying m(t) by the carrier wave $A_c \cos(2\pi f_c t)$



→ fig (ii) shows DSB-SC modulated wave

→ fig (iii) shows when only upper sideband is transmitted to get frequency spectrum
USB → represented in duplicate by frequencies above f_c and those below $-f_c$, only USB is transmitted

→ fig (iv) shows when only lower sideband is transmitted to get frequency spectrum
LSB → represented in duplicate by frequencies below f_c (+ve frequencies) and those above $-f_c$ (-ve frequencies), only LSB is transmitted

(Refer Text Book Simon Haykin Pg 287)

★ FREQUENCY DISCRIMINATION METHOD FOR GENERATING AN SSB MODULATED WAVE - (Filter method)

→ The most widely used method for generating SSB is the filter method

→ The filter method uses a Band pass filter having sufficient selectivity to pass one sideband and reject the other.

→ The transmitter stability and accuracy are determined by the carrier crystal oscillator, the balanced modulator and the side band filter circuits are also important for obtaining the output with desired sideband.

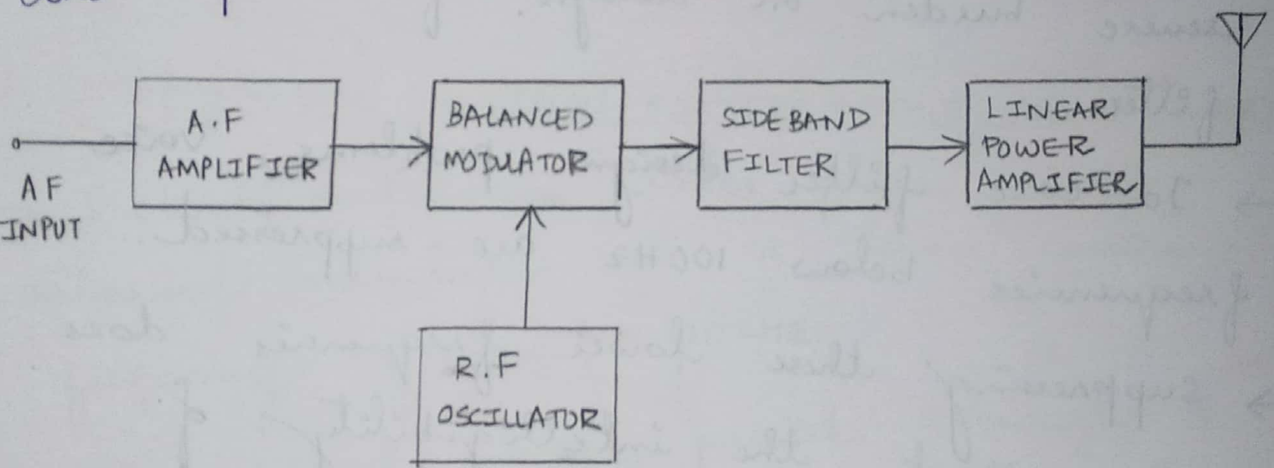


fig (i) BLOCK DIAGRAM OF SSB TRANSMITTER

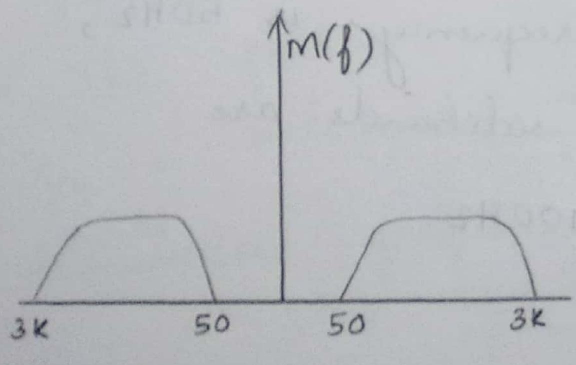


fig (a) SPECTRUM OF MESSAGE SIGNAL m(t)

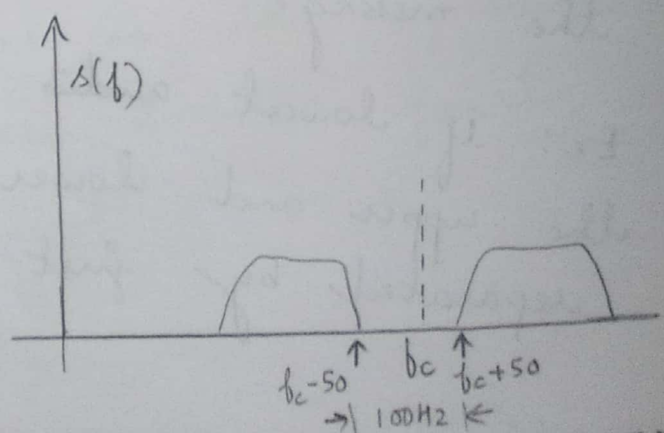


fig (b) SPECTRUM OF DSBSC WITH TWO SIDE BANDS

→ The block diagram of a basic SSB system is as shown in fig (i)

→ The carrier and message signal are applied to the balanced modulator

→ The DSBSC output of the balanced modulator is applied to the sideband filter

→ The filter is so designed so as to extract the desired sideband and reject the unwanted sideband.

→ The two sidebands may be separated only by a few Hertz, which has a severe burden on design of sideband filters.

→ To ease filter design problems, voice frequencies below 100 Hz are suppressed.

→ Suppressing these lower frequencies does not affect the intelligibility of the message

Ex: If lowest audio frequency is 50 Hz, the upper and lower sidebands are separated by just 100 Hz.

④

→ To ease the problem of filter design, modulation is carried out at a lower frequency and frequency is then increased by using a balanced mixer.

→ Such a transmitter is as shown in fig (ii)

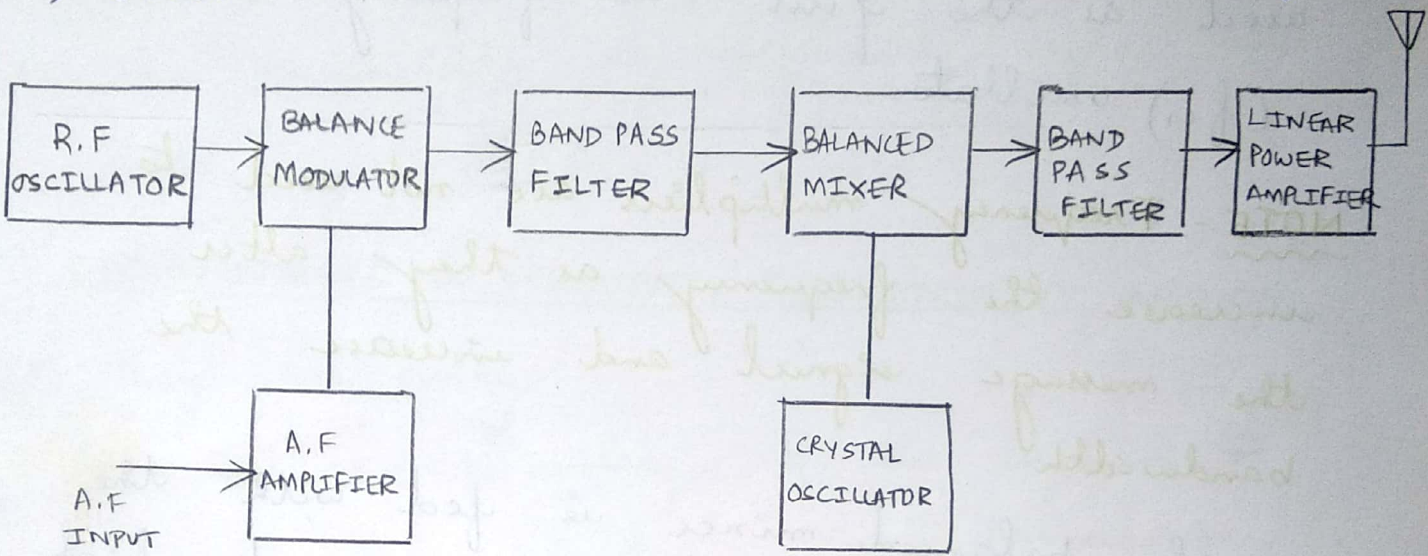


fig (ii) DUAL CONVERSION, FILTER METHOD SSB TRANSMITTER.

→ Initial modulation takes place in a balanced modulator at a lower carrier frequency of about 100 kHz.

→ This signal is then passed through a filter to eliminate one of the sidebands

→ The filtered signal is then up converted in a balanced mixer to the final transmitter frequency.

→ The signal is amplified in a linear power amplifier before being coupled to the antenna for radiation.

→ Since excellent stability can be obtained at 100 KHz, this frequency is commonly used as the first low frequency carrier (f_{c1}) oscillator.

NOTE - Frequency multipliers are not used to increase the frequency as they alter the message signal and increase the bandwidth.

→ The balanced mixer is fed with the oscillator operating at the required high carrier frequency (f_{c2})

→ At the output of the mixer the two sidebands are separated effectively by twice the first carrier frequency f_{c1}

→ This enables the removal of the unwanted sideband by the second band-pass filter easily.

(Filter method)

ADVANTAGES OF FREQUENCY DISCRIMINATOR METHOD -

(1) The filter method gives the adequate sideband suppression

(2) The sideband filter also helps to attenuate carrier if present in the output of balanced modulator

(3) The bandwidth is sufficiently flat and wide.

DISADVANTAGES OF FREQUENCY DISCRIMINATOR METHOD -

(1) They are bulky

(2) Due to the inability of the system to generate SSB at high radio frequencies, the frequency up conversion is necessary.

(3) Two expensive filters are to be used, one for each sideband.

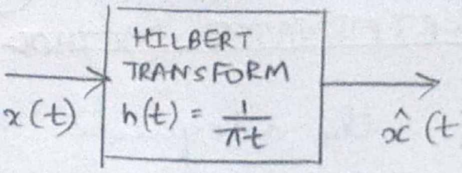
★ TIME DOMAIN DESCRIPTION - (of SSB wave)

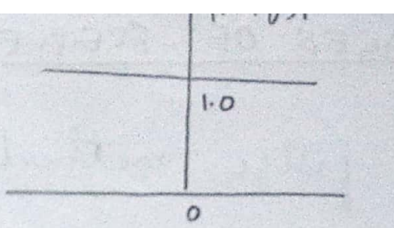
→ To describe SSB in time domain we need the concept of Hilbert transform and the pre-envelope of a signal.

NOTE -

(1) ★ HILBERT TRANSFORM -

→ when phase angles of all components of a given signal are shifted by $\pm 90°$, the resulting function of time is known as 'Hilbert transform'





AMPLITUDE RESPONSE

$$\hat{x}(t) = x(t) * h(t)$$

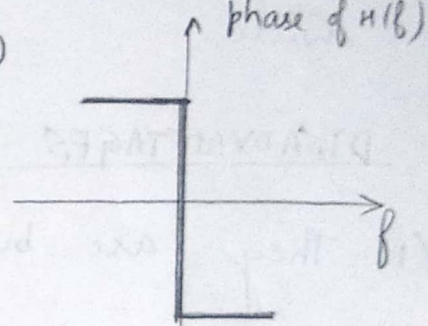
$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} \cdot d\tau} \quad \text{---- (1)}$$

where $\hat{x}(t)$ is the hilbert transform of $x(t)$

Taking fourier transform on both sides of eqn (1) we get,

$$\boxed{\hat{X}(f) = -j\, sgn(f) \cdot X(f)} \quad \text{---- (2)}$$



PHASE RESPONSE OF H(f)

(2)★ PRE-ENVELOPE.

→ The preenvelope of the signal $x(t)$ is defined as the complex valued function given as,

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \quad \text{------ (1)}$$

preenvelope for -ve frequency

$$x_-(t) = x(t) - j\hat{x}(t)$$

where,

$x(t)$ is real part of the pre envelope

$j\hat{x}(t)$ is imaginary part of preenvelope

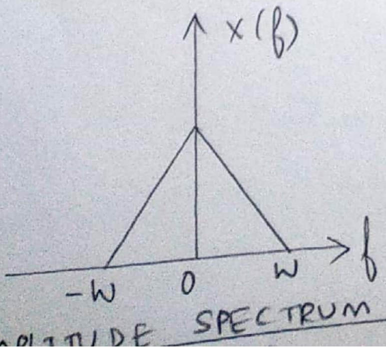
$\hat{x}(t)$ is hilbert transform of $x(t)$

Taking fourier transform of $x_+(t)$ is given by,

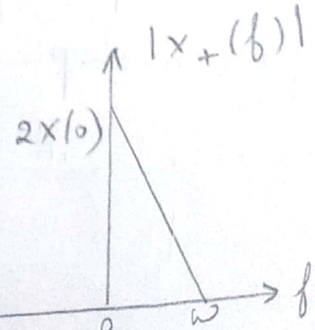
$$\boxed{X_+(f) = X(f)\left[1 + sgn(f)\right]} \quad \text{(2)}$$

Substituting sgn(f) values in eqn (2) we get

$$X_+(f) = \begin{cases} 2X(f), & \text{for } f>0 \\ X(0) & \text{for } f=0 \\ 0 & \text{for } f<0 \end{cases}$$



AMPLITUDE SPECTRUM OF LOW PASS



AMPLITUDE SPECTRUM OF THE PRE ENVELOPE $x_+(t)$

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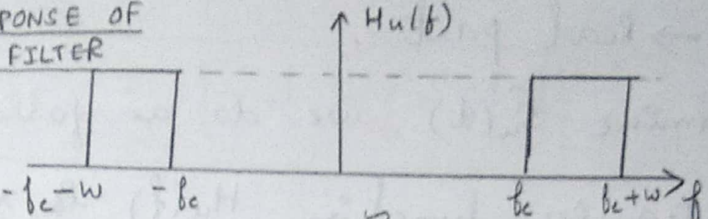
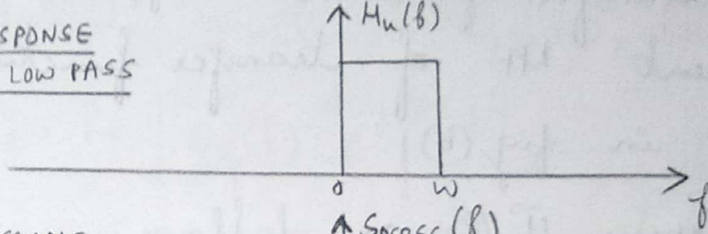
→ The SSB signal may be generated by passing a DSB-SC modulated wave through a Band pass filter of transfer function $H_u(f)$ (6)

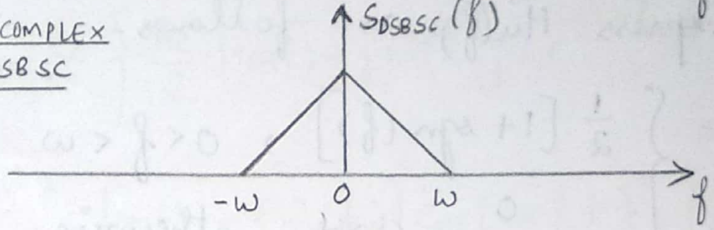
(a) FREQUENCY RESPONSE OF IDEAL BANDPASS FILTER TO SELECT UPPER SIDEBAND



(b) FREQUENCY RESPONSE OF EQUIVALENT LOW PASS FILTER



(c) SPECTRUM OF COMPLEX ENVELOPE OF DSBSC



→ The DSB-SC modulated wave is defined mathematically as,

$$\boxed{S_{DSBSC}(t) = A_c\, m(t) \cos(2\pi f_c t)}$$

where,

$m(t) \rightarrow$ message signal

$A_c \cos 2\pi f_c t \rightarrow$ Carrier signal

→ The low pass complex envelope of the DSB-SC modulated wave is expressed as,

$$\boxed{\tilde{S}_{DSBSC}(t) = A_c\, m(t)}$$

→ consider the SSB modulated wave $S_u(t)$ in which only the USB is retained.

→ It has quadrature and in-phase component

→ Then $\tilde{S}_u(t)$ is the complex envelope of $S_u(t)$ and we can write,

$$S_u(t) = Re\left[\tilde{S}_u(t)\exp(j2\pi f_c t)\right]$$

$$\boxed{S_u(t) = Re\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]} \quad \text{——— (1)}$$

where,
$Re \rightarrow$ Real part.

→ To determine $\tilde{S}_u(t)$, we do as follows.

(i) The BPF transfer function $H_u(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_u(f)$ as shown in fig (b)

We can express $\tilde{H}_u(f)$ as follows.

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2}\left[1+\mathrm{sgn}(f)\right], & 0<f<\omega \\ 0, & \text{otherwise} \end{cases} \quad \text{——— (2)}$$

where $\mathrm{sgn}(f)$ is the signum function

(ii) The DSB-SC modulated wave is replaced by its complex envelope. The spectrum of this envelope is as shown in fig (c)

ie, $$\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \quad \text{——— (3)}$$

(iii) The desired complex envelope $\tilde{S}_u(t)$ is determined by evaluating the IFT of the product $\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)$.

ie, $$\boxed{\tilde{S}_u(t) = IFT\left[\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right]} \quad \text{——— (4)}$$

Substituting eqn (2) and eqn (3) in eqn (4),

we get,

$$\tilde{s}_u(t) = \text{IFT}\left\{\frac{1}{2}\left[1+\text{sgn}(f)\right] \cdot A_c M(f)\right\}$$

$$\tilde{s}_u(t) = \text{IFT}\left\{\frac{A_c}{2}\left[M(f) + \text{sgn}(f)\,M(f)\right]\right\}$$

$$\therefore \boxed{\tilde{s}_u(t) = \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]} \quad \text{---- (5)}$$

$\hat{m}(t) \rightarrow$ Transformed function

$\longrightarrow$ Now, substitute eqn (5) in eqn (1), we get

WKT, $s_u(t) = \text{Re}\left[\tilde{s}_u(t) \cdot e^{j2\pi f_c t}\right]$ .... $\because$ (1)

$$s_u(t) = \text{Re}\left\{\frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right] \cdot e^{j2\pi f_c t}\right\}$$

$$s_u(t) = \text{Re}\left\{\frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right] \cos(2\pi f_c t) + j\sin(2\pi f_c t)\right\}$$

$$= \text{Re}\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + j\,m(t)\sin(2\pi f_c t) + j\hat{m}(t)\cos(2\pi f_c t) + j^2\hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$j^2 = -1$

$$= \text{Re}\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + j\,m(t)\sin 2\pi f_c t + j\hat{m}(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

Retain real part, remove j img part ??

$$\therefore \boxed{s_u(t) = \frac{A_c}{2}\left[\underbrace{m(t)\cos(2\pi f_c t)}_{\text{Inphase Component}} - \underbrace{\hat{m}(t)\sin(2\pi f_c t)}_{\text{Quadrature Component}}\right]} \quad \text{---- (6)}$$

Inphase Component ??　Quadrature Component ??

Eqn (6) shows that the SSB modulated wave contains only USB with an Inphase Component & Quadrature component.

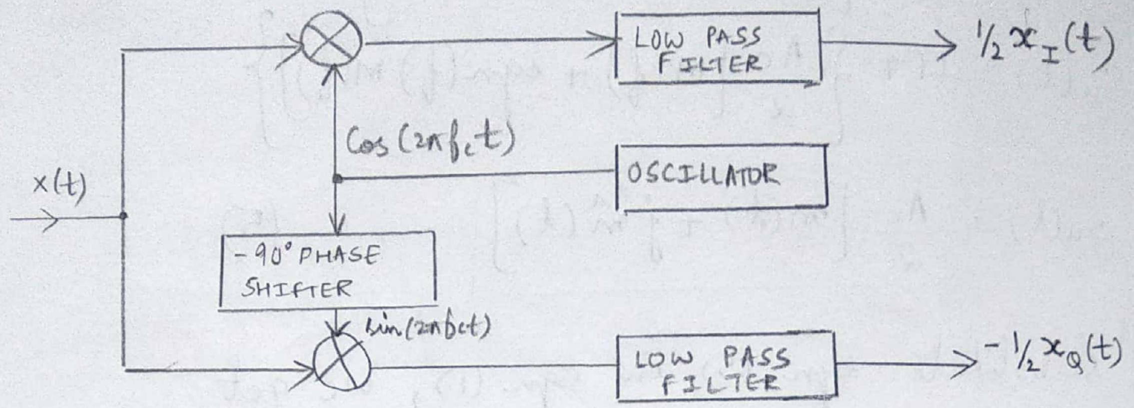
LSB only: $\Rightarrow s_l(t)$ denote SSB modulated wave with only LSB retained

1) Identify Transfer function $H_l(f)$ of Band pass filter the output of which equals $s_l(t)$ in response to DSBSC modulated wave

2) Determine transfer function $\tilde{H}_l(f)$ of equivalent LPF corresponding to $H_l(f)$

3) $s_l(t) = \frac{A_c}{2}\left[\underbrace{m(t)\cos(2\pi f_c t)}_{\text{In-phase}} + \underbrace{\hat{m}(t)\sin(2\pi f_c t)}_{\text{Quadrature}}\right]$

NOTE - IN-PHASE AND QUADRATURE PHASE COMPONENT -



SCHEME TO GENERATE IN PHASE AND QUADRATURE COMPONENT OF BAND PASS SIGNAL x(t)

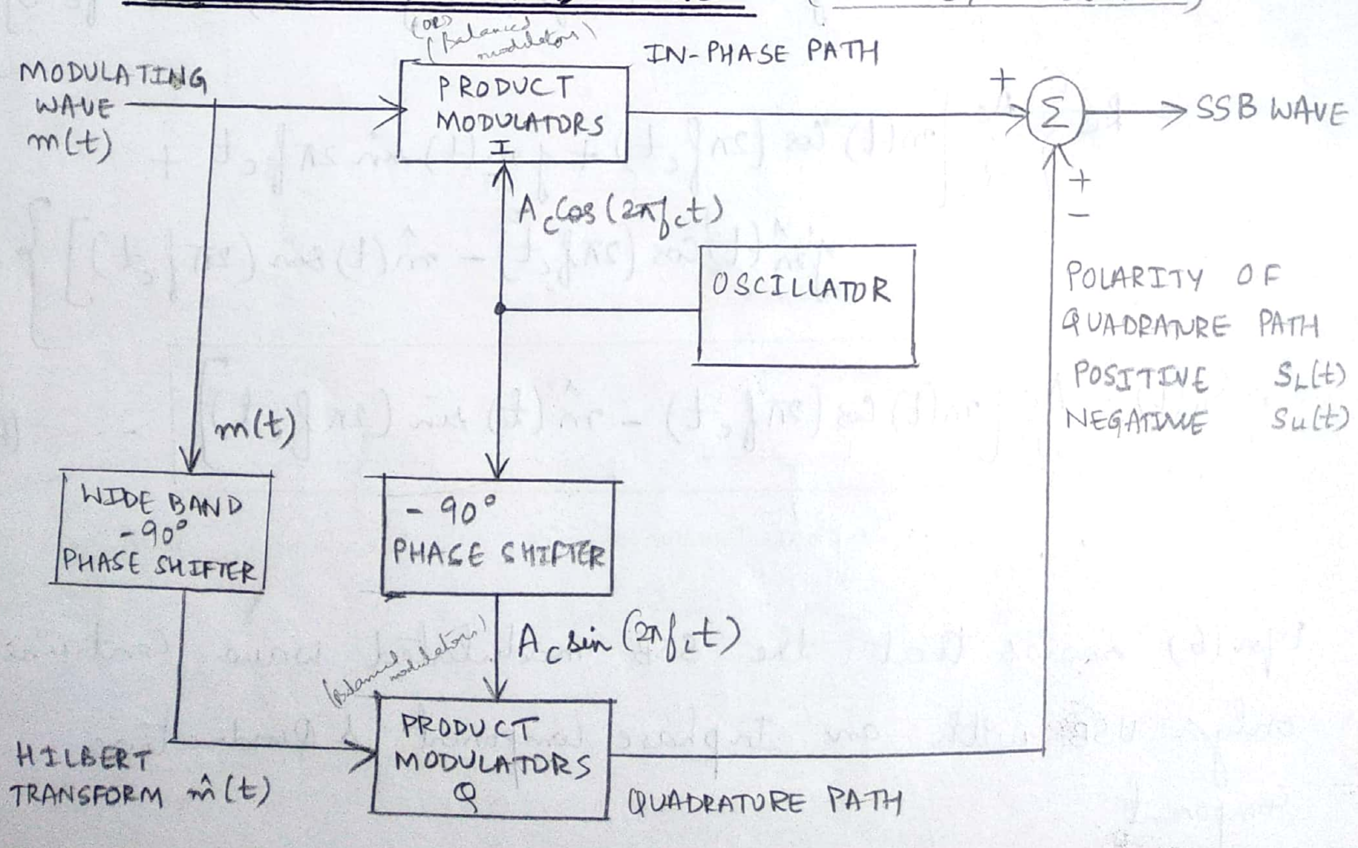
→ $x_I(t)$ and $x_Q(t)$ are low pass signals limited to the band $-W \le f \le W$

→ The Bandwidth of each filter is 'W'

→ In phase Component $x_I(t)$ → produced by multiplying $x(t)$ with $\cos(2\pi f_c t)$ and passing the product through a LPF

→ Quadrature Component $x_Q(t)$ → produced by multiplying $x(t)$ with $\sin(2\pi f_c t)$ and passing the product through identical LPF

★ PHASE DISCRIMINATION METHOD FOR GENERATING AN SSB MODULATED WAVE- (HARTLEY MODULATOR)



BLOCK DIAGRAM OF PHASE DISCRIMINATION METHOD

→ The SSB modulator uses two product modulators (8)
I and Q, supplied with carrier wave in phase quadrature to each other.

→ The message signal $m(t)$ and a carrier signal $A_c \cos(2\pi f_c t)$ is directly applied to the product modulator I, producing a DSBSC wave.

→ The hilbert transform $\hat{m}(t)$ (−90° phase shift) of $m(t)$ and carrier signal shifted by 90° are applied to the product modulator Q, producing DSB-SC wave.

→ The output of the product modulator 'I' is,

$$\boxed{S_I(t) = m(t) A_c \cos(2\pi f_c t)}$$

→ The output of the product modulator 'Q' is,

$$\boxed{S_Q(t) = \hat{m}(t) A_c \sin(2\pi f_c t)}$$

These signals $S_I(t)$ and $S_Q(t)$ are fed to a Summer

∴ The output of the summer is,

$$S(t) = S_I(t) \pm S_Q(t)$$

$$\boxed{S(t) = A_c m(t) \cos 2\pi f_c t \pm A_c \hat{m}(t) \sin 2\pi f_c t}$$

→ The plus (+) sign at the summing junction gives SSB with only LSB ie,

$$\boxed{S_L(t) = A_c m(t) \cos 2\pi f_c t + A_c \hat{m}(t) \sin 2\pi f_c t}$$

→ The minus (−) sign at the summing junction gives SSB with only USB ie,

$$\boxed{S_u(t) = A_c\, m(t) \cos 2\pi f_c t - A_c\, \hat{m}(t) \sin 2\pi f_c t}$$

NOTE – This SSB modulator is also known as the Hartley modulator.

★ DEMODULATION OF SSB WAVE -

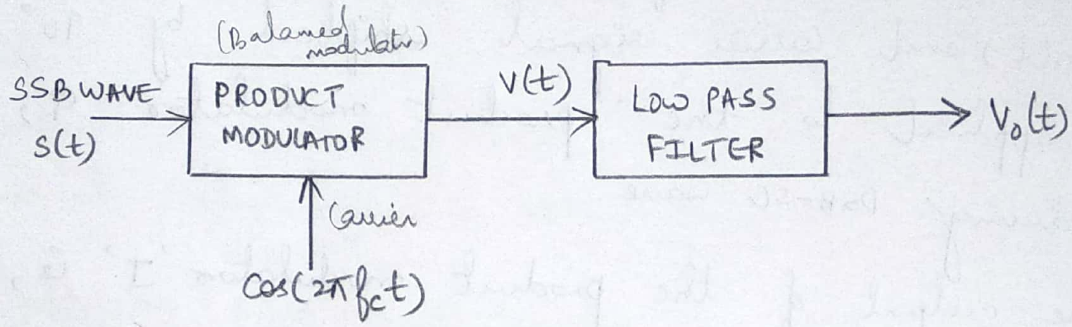


fig (i) COHERENT DETECTION OF SSB MODULATED WAVE

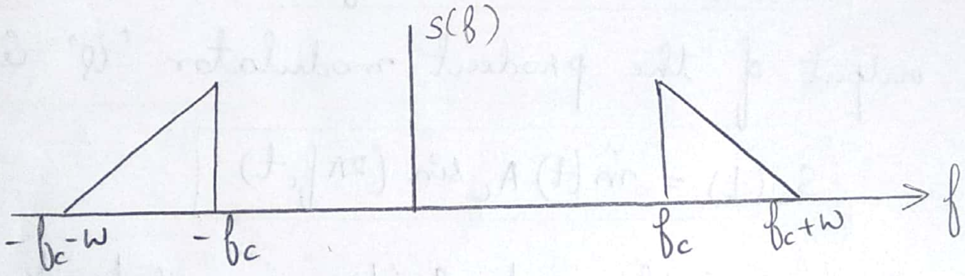


fig (ii) SPECTRUM OF SSB WITH UPPER SIDEBAND s(t)

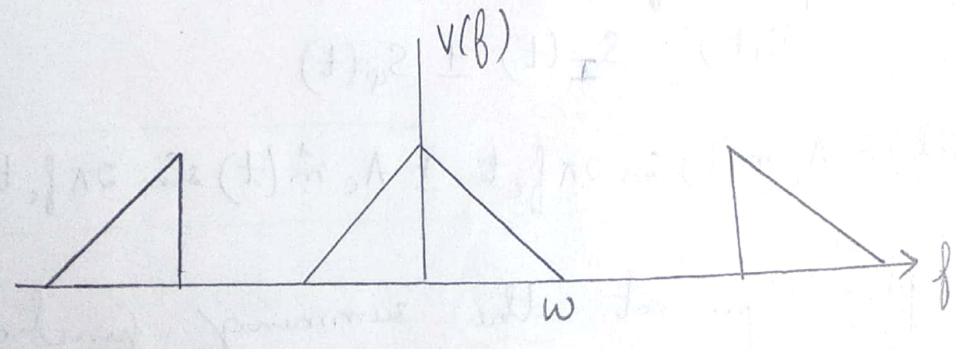


fig (iii) SPECTRUM OF OUTPUT OF PRODUCT MODULATOR

→ The baseband signal m(t) can be recovered from the SSB wave s(t) by using coherent detection.

→ The product modulator is having two inputs one input is the SSB modulated wave s(t) and another input is the locally generated carrier $\cos(2\pi f_c t)$ then low pass filtering the modulator output as in figure.

→ The product modulator output is given by,

$$\boxed{V(t) = s(t)\cdot\cos(2\pi f_c t)} \quad \text{---- (1)}$$

WKT,

$$\boxed{s(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right]} \quad \text{---- (2)}$$

substituting eqn (2) in eqn (1), we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t$$

$$V(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \cdot \cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t)\cos 2\pi f_c t \cdot \sin 2\pi f_c t$$

WKT,

$$\left\{\begin{aligned} \cos A\cdot\cos B &= \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \\ \cos A\cdot\sin B &= \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right] \end{aligned}\right\}$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(2\pi f_c + 2\pi f_c)t + \cos(\underbrace{2\pi f_c - 2\pi f_c}_{=0})t\right] \pm$$

$$\frac{A_c}{4}\hat{m}(t)\left[\sin(2\pi f_c + 2\pi f_c)t \mp \sin(\underbrace{2\pi f_c - 2\pi f_c}_{=0})t\right]$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t) + \underbrace{\cancel{\cos}(0)}_{=1}\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + \underbrace{\cancel{\sin}(0)}_{=0}\right]$$

W.K.T $\cos(0) = 1$, $\sin(0) = 0$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t) + 1\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + 0\right]$$

$$V(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)$$

$$V(t) = \underbrace{\frac{A_c}{4} m(t)}_{\text{scaled message signal}} + \underbrace{\frac{A_c}{4}\left[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)\right]}_{\text{unwanted Terms.}}$$

→ when $v(t)$ is passed through the filter, it will allow only the 1st term to pass through and will reject all other unwanted terms.

→ Thus, at the output of the filter we get the scaled message signal and the coherent SSB demodulation is achieved

$$\therefore \boxed{V_o(t) = \frac{A_c}{4} m(t)}$$

→ The detection of SSB modulated waves is based on the assumption that there is perfect synchronization between local carrier and that in the transmitter both in frequency and phase

→ Practically a phase error ϕ may arise in the locally generated carrier wave.

$\therefore$ The detector output is modified due to phase error as follows :

$$\boxed{V_o(t) = \frac{A_c}{4} m(t)\cos\phi \pm \frac{A_c}{4}\hat{m}(t)\sin\phi}$$

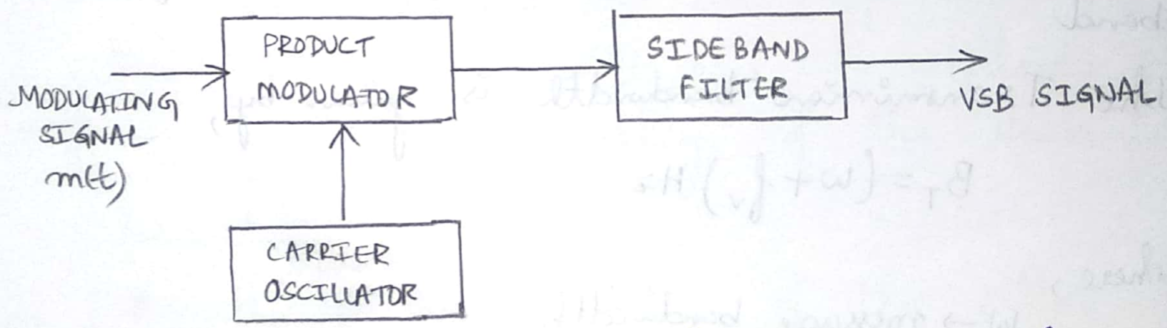
unwanted Terms

★ VESTIGIAL SIDE BAND MODULATION (VSB)

→ The very severe condition of frequency response requirements on the sideband filter in SSB-SC modulation can be relaxed by allowing a part of the unwanted sideband [called as vestige] to appear in the output of the modulator.

→ From this the design of the sideband filter is simplified to a great extent, but the bandwidth of the system is increased slightly (↑↑)

→ In VSB, one sideband and a part of the other sideband called as VESTIGE is also transmitted.



→ To generate a VSB signal, first we have to generate a DSB-SC signal and then pass it through a sideband filter.

This filter will pass the wanted sideband as it is a part of unwanted signal. sideband.

★ FREQUENCY DOMAIN DESCRIPTION OF VSB WAVE -

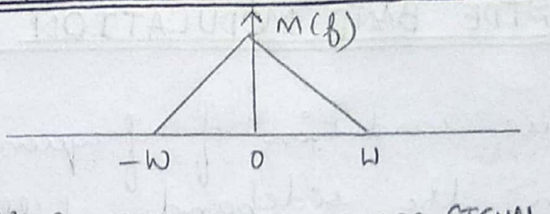


fig (a) SPECTRUM OF MESSAGE SIGNAL

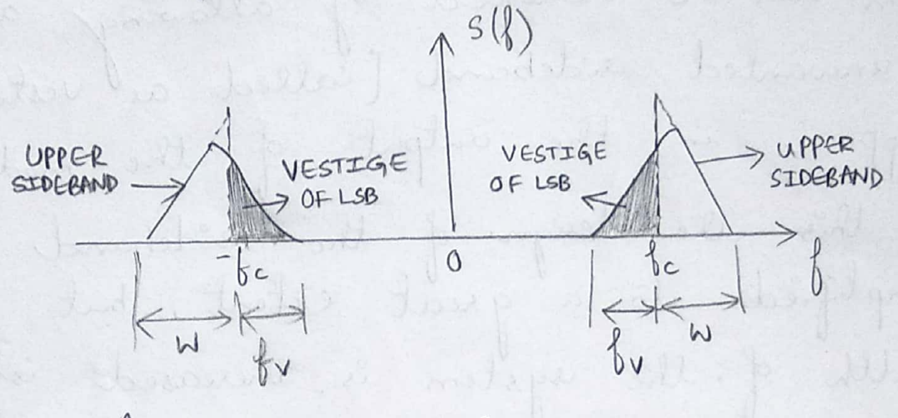


fig (b) SPECTRUM OF VSB MODULATED WAVE

→ fig (a) and (b) shows the spectrum of a VSB modulated wave $s(t)$ along with the message signal $m(t)$. Here lower sideband is modified into vestigial sideband

→ The Transmission bandwidth is given by,

$$B_T = (W + f_v)\ Hz$$

where,

$W \rightarrow$ message bandwidth

$f_v \rightarrow$ width of the vestigial sideband

ADVANTAGES OF VSB-

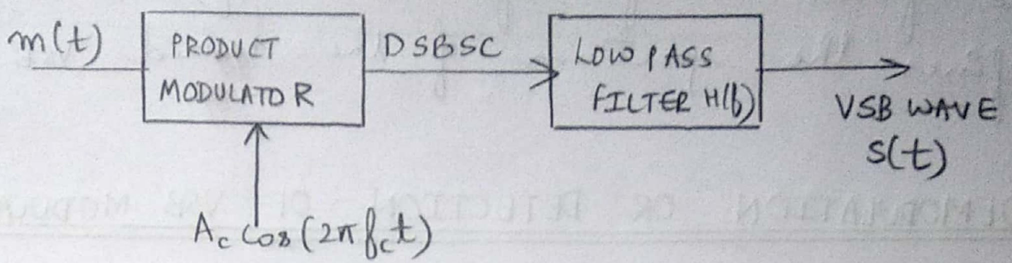
(1) The reduction in bandwidth which is as efficient as the SSB.

(2) Easy to design the filter.

APPLICATION OF VSB-

(1) VSB modulation main use is for the transmission of TV signals, as the video signals need larger transmission bandwidth.

★ GENERATION OF VSB MODULATED WAVE -



→ VSB modulation can be generated by passing a DSBSC wave through an appropriate filter of transfer function H(f)

→ The output of the product modulator is the DSB-SC wave is given by,

$$s(t) = m(t) \cdot c(t)$$

$$s(t) = m(t) A_c \cos(2\pi f_c t) \quad \text{--- (i)}$$

→ This DSB-SC signal is then applied to low pass filter. This filter will pass the wanted sideband as it is and the vestige of the unwanted sideband.

→ Let the transfer function of the filter be H(f). Hence the spectrum S(f) of the VSB modulated wave is given by,

Taking FT for eqn (i)

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)} \quad \text{--- (1)}$$

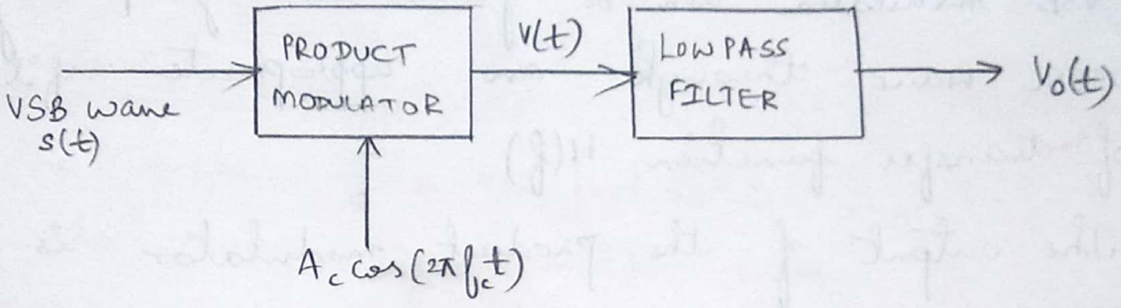
where,

M(f) is the fourier transform of the baseband signal m(t).

→ we should determine the specification of the filter transfer function $H(f)$ so that $S(f)$ defines the spectrum of the desired VSB wave $s(t)$

→

★ **DEMODULATION OR DETECTION OF VSB MODULATED WAVE.**



→ The demodulation of VSB modulated wave can be achieved by passing VSB wave $s(t)$ through a coherent detector

→ Thus, multiplying $s(t)$ by a locally generated carrier wave $A_c \cos(2\pi f_c t)$ which is synchronous with the carrier wave $A_c \cos(2\pi f_c t)$ in both frequency and phase, we get

$$\boxed{v(t) = A_c \cos 2\pi f_c t \cdot s(t)} \quad \text{------} (2)$$

Taking Fourier transform on both sides of eqn (2) we get,

$$V(f) = \frac{A_c}{2}\Big[\underbrace{S(f-f_c)}_{USB} + \underbrace{S(f+f_c)}_{LSB}\Big] \quad \text{——} (3)$$

WKT,

$$S(f) = \frac{A_c}{2}\big[M(f-f_c) + M(f+f_c)\big]\, H(f)$$

$\because f = f+f_c$ LSB

$$\underset{(LSB)}{S(f+f_c)} = \frac{A_c}{2}\big[M(f+\cancel{f_c}-\cancel{f_c}) + M(f+f_c+f_c)\big]\, H(f+f_c)$$

$$\boxed{S(f+f_c) = \frac{A_c}{2}\left[M(f) + M(f+2f_c)\right] H(f+f_c)} \quad \text{--- (4)}$$

$|||^{ly}$, $\because f = f - f_c$ USB

$$S(f-f_c) = \frac{A_c}{2}\left[M(f-f_c-f_c) + M(f+\cancel{f_c}-\cancel{f_c})\right] H(f-f_c)$$

$$\boxed{\underset{(USB)}{S(f-f_c)} = \frac{A_c}{2}\left[M(f-2f_c) + M(f)\right] H(f-f_c)} \quad \text{--- (5)}$$

→ Substituting eqn (4) and (5) in eqn (3), we get

$$V(f) = \frac{A_c}{2}\left\{\frac{A_c}{2}\left[M(f-2f_c) + M(f)\right] H(f-f_c) + \frac{A_c}{2}\left[M(f) + M(f+2f_c)\right] H(f+f_c)\right\}$$

$$= \frac{A_c}{4} M(f-2f_c) \cdot H(f-f_c) + \frac{A_c}{4} M(f) H(f-f_c) + \frac{A_c}{4} M(f) \cdot H(f+f_c) + \frac{A_c}{4} M(f+2f_c) H(f+f_c)$$

desired

$$V(f) = \underbrace{\frac{A_c}{4} M(f)\left[H(f-f_c) + H(f+f_c)\right]}_{\text{VSB wave}} + \underbrace{\frac{A_c}{4}\left[M(f-2f_c) H(f-f_c) + M(f+2f_c) H(f+f_c)\right]}_{\text{unwanted Term}} \quad \text{--- (6)}$$

→ Eqn (6) is passed through a LPF which eliminates unwanted term and passes only wanted term ie, VSB wave is given by,

$$\boxed{V_0(f) = \frac{A_c}{4} M(f)\left[H(f-f_c) + H(f+f_c)\right]} \quad \text{--- (7)}$$

→ The sp[...]

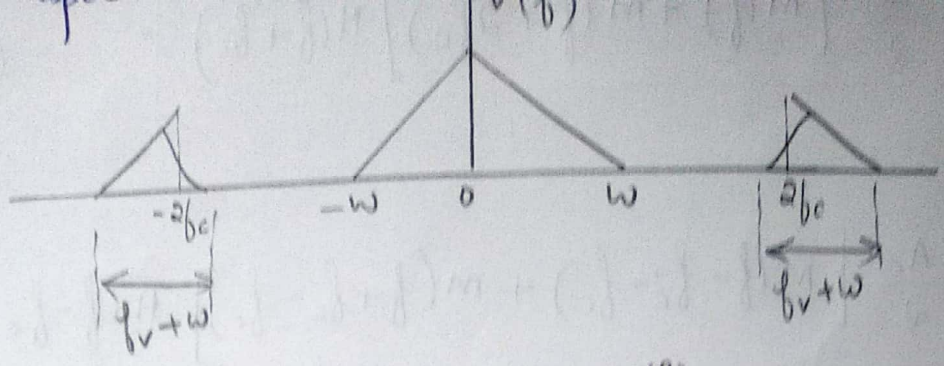


fig (a) SPECTRUM OF v(f)

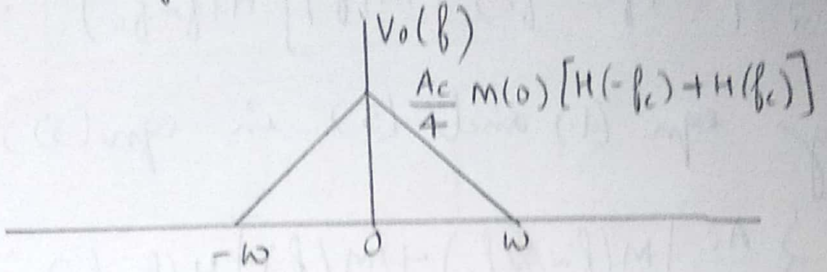


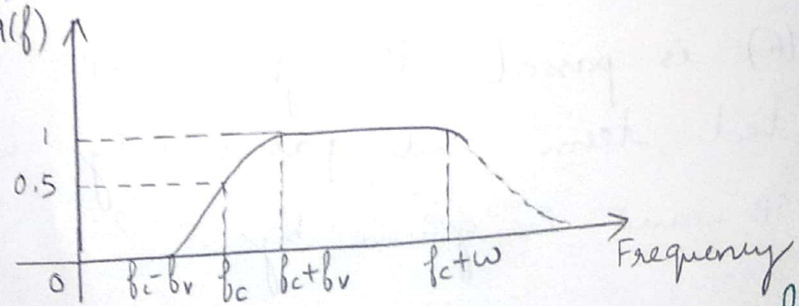
fig (b) SPECTRUM OF $V_o(f)$

→ To obtain the undistorted message signal $m(t)$ at the output of the demodulator, the transfer function $H(f)$ should satisfy the condition as follows -

$$\boxed{H(f-f_c) + H(f+f_c) = 2H(f_c)} \quad \text{—— (8)}$$

where, $H(f_c)$ is constant.

→ The condition in eqn (8) will be satisfied if the filter frequency response is as shown below,



NOTE - The design of VSB filter is less complicated to an SSB filter.

★ TIME DOMAIN DESCRIPTION OF VSB MODULATED WAVE -

→ The procedure to be followed for obtaining the time domain description of a VSB wave is similar to the one used for SSB.

(i) Let $s(t)$ = VSB modulated wave which contains full upper side band (USB) and vestige of lower sideband (LSB)

(ii) This VSB wave can be assumed to be generated by a low pass filter along with a DSB-SC signal.

(iii) Let the sideband filter has a transfer function $H(f)$ as shown in fig (a)

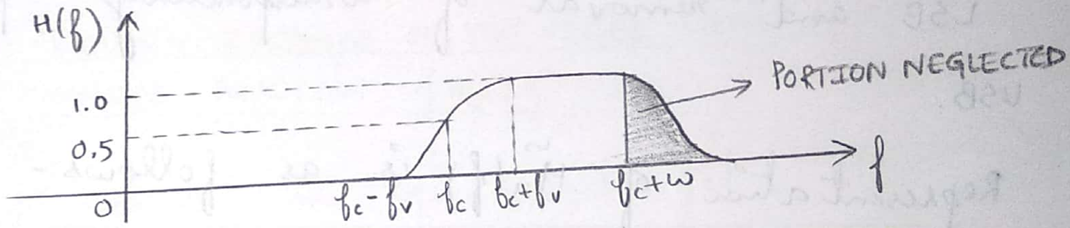


fig (a) FREQUENCY RESPONSE OF LOW PASS FILTER

(iv) The sideband filter can be replaced by an equivalent LPF with transfer function $\tilde{H}(f)$ as in fig (b)

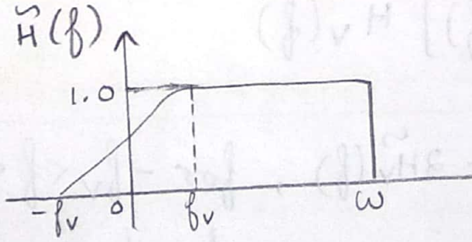


fig (b) FREQUENCY RESPONSE OF A LPF EQUIVALENT TO LOW PASS FILTER OF fig (a)

(V) We can represent $\tilde{H}(f)$ of fig (b) as the difference between two components $\tilde{H}_u(f)$ and $\tilde{H}_v(f)$ as follows -

$$\boxed{\tilde{H}(f) = \tilde{H}_u(f) - \tilde{H}_v(f)} \quad \text{------ (1)}$$

These two Components are plotted in fig (c) & fig (d)

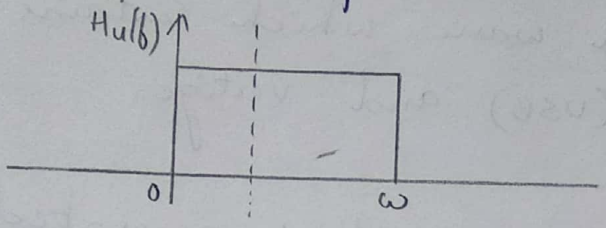


fig (c) FIRST COMPONENT OF $\tilde{H}(f)$ $\tilde{H}_u(f)$

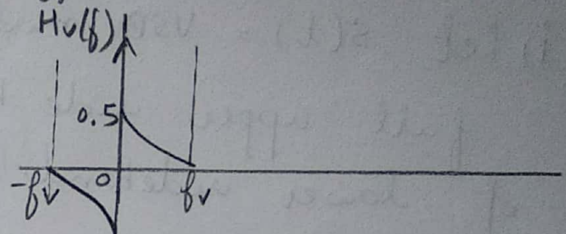


fig (d) SECOND COMPONENT OF $\tilde{H}(f)$ $\tilde{H}_v(f)$

(vi) → $\tilde{H}_u(f)$ represents the LPF equivalent to a BPF which rejects the LSB completly

$\tilde{H}_v(f)$ represents to the generation of vestige of LSB and removal of corresponding portion from USB.

(vii) Representation of $\tilde{H}_u(f)$ is as follows -

$$\boxed{\tilde{H}_u(f) = \begin{cases} \frac{1}{2}[1 + sgn(f)] & , \text{for } 0 < f < \omega \\ 0 & , \text{elsewhere} \end{cases}} \quad \text{---- (2)}$$

Substitute eqn (2) in eqn (1), we get

$$\tilde{H}(f) = \frac{1}{2}[1 + sgn(f)] \tilde{H}_v(f)$$

Redefining $\tilde{H}(f)$ ~~comp~~ transfer function

$$\boxed{\tilde{H}(f) = \begin{cases} \frac{1}{2}[1 + sgn(f) - 2\tilde{H}_v(f) & , \text{for } -f_v < f < \omega \\ 0 & , \text{elsewhere} \end{cases}} \quad \text{------ (3)}$$

(viii) In eqn (3), the signum function and the transfer function $\tilde{H}_v(f)$ are both odd functions of frequency.

∴ Inverse fourier transform of both are imaginary.

Thus, new transfer function is,

$$\boxed{H_Q(f) = \frac{1}{j}\left[sgn(f) - 2\tilde{H}_v(f)\right]} \quad ---- \quad (4)$$

(ix) Let $H_Q(t)$ be the inverse fourier transform of $H_Q(f)$ where $jH_Q(f)$ is plotted as in fig (e)

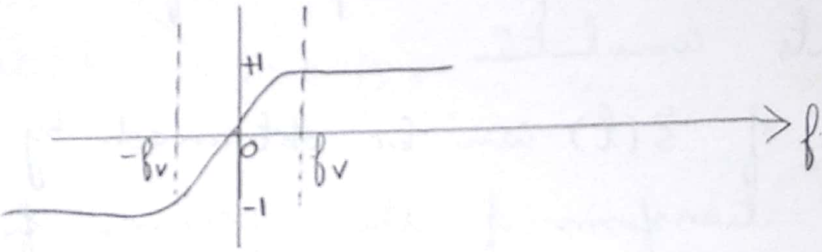


fig (e) FREQUENCY RESPONSE OF THE FILTER WITH TRANSFER FUNCTION $jH_Q(f)$

(x) Substituting eqn (4) in eqn (3), we get

$$\tilde{H}(f) = \begin{cases} \frac{1}{2}\left[1 + jH_Q(f)\right], & -f_v < f < W \\ 0 & , \text{otherwise} \end{cases}$$

NOTE - $jH_Q(f) = \left[sgn(f) - 2\tilde{H}_v(f)\right]$

$$H_Q(f) = \frac{1}{j}\left[sgn(f) - 2\tilde{H}_v(f)\right]$$

↓ From Here actual Time domain description of USB modulated signal

(Refer Next page)

→ Let us obtain the expression for VSB modulated wave where wave is expressed as,

$$\boxed{s(t) = \mathrm{Re}\left[\tilde{s}(t) \cdot e^{j2\pi f_c t}\right]} \quad \text{——— (1)}$$

where,

$\tilde{s}(t)$ is the complex envelope of $s(t)$

→ we get $\tilde{s}(t)$ at the output of the complex LPF showing a transfer function $\tilde{H}(f)$ when a DSB-SC wave is applied as its input

$$\therefore \boxed{\tilde{s}(t) = \tilde{h}(t) * \tilde{s}_{DSBSC}(t)}$$

where, $\tilde{h}(t)$ is the impulse response of the filter and represents convolution

→ The spectrum of $\tilde{s}(t)$ can be obtained by taking the fourier transform of the expression for $\tilde{s}(t)$

$$\boxed{\tilde{S}(f) = \tilde{H}(f) \cdot \tilde{S}_{DSB\text{-}SC}(f)} \quad \text{----- (2)}$$

WKT, $\tilde{S}_{DSBSC}(f) = A_c M(f)$ ——— (3)

Substituting eqn (3) in eqn (2), we get

$$\tilde{S}(f) = A_c M(f) \cdot \tilde{H}(f)$$

$$\tilde{S}(f) = A_c M(f) \left[\frac{1}{2}\left(1 + j H_Q(f)\right)\right]$$

$$\tilde{S}(f) = A_c M(f) \left[\frac{1}{2} + \frac{1}{2} j H_Q(f)\right]$$

$$\boxed{\tilde{S}(f) = \frac{A_c}{2} M(f) + j \frac{A_c}{2} H_Q(f) \cdot M(f)} \quad \text{——— (4)}$$

Taking IFT of eqn (4), we get,

$$\tilde{s}(t) = F^{-1}\left[\frac{A_c}{2} M(f)\right] + F^{-1}\left[j\frac{A_c}{2} H_Q(f) \cdot M(f)\right]$$

$$= \frac{A_c}{2} m(t) + j\frac{A_c}{2}\left[h_Q(t) * m(t)\right]$$

$$\tilde{s}(t) = \frac{A_c}{2}\left[m(t) + j\, h_Q(t) * m(t)\right]$$

$$\boxed{\tilde{s}(t) = \frac{A_c}{2}\left[m(t) + j\, m_Q(t)\right]} \quad \text{----} (5)$$

where $\boxed{m_Q(t) = h_Q(t) * m(t)}$

→ $m_Q(t)$ is the response produced by passing the message signal $m(t)$ through a LPF of impulse response $h_Q(t)$

Substituting eqn (5) in eqn (1) $\quad s(t) = Re\left[\tilde{s}(t) e^{j2\pi f_c t}\right]$

$$s(t) = Re\left[\frac{A_c}{2}\left(m(t) + j\, m_Q(t)\right) e^{j2\pi f_c t}\right]$$

WKT, $e^{j2\pi f_c t} = \cos(2\pi f_c t) + j \sin(2\pi f_c t)$

$$\therefore\ s(t) = Re\left[\frac{A_c}{2}\left[m(t) + j\, m_Q(t)\right] \cos(2\pi f_c t) + j \sin(2\pi f_c t)\right]$$

$$s(t) = Re\left[\frac{A_c}{2} m(t) \cos 2\pi f_c t + j\frac{A_c}{2} m(t) \sin 2\pi f_c t + j\frac{A_c}{2} m_Q(t) \cos 2\pi f_c t + \underset{(-1)}{j^2} \frac{A_c}{2} m_Q(t) \sin 2\pi f_c t\right.$$

$$= Re\left\{\underbrace{\frac{A_c}{2} m(t) \cos(2\pi f_c t)} + j \underbrace{\frac{A_c}{2} m(t) \sin(2\pi f_c t)} + j \underbrace{\frac{A_c}{2} m_Q(t) \cos(2\pi f_c t)} - \underbrace{\frac{A_c}{2} m_Q(t) \sin(2\pi f_c t)}\right.$$

→ Selecting only the real part, we get

$$\boxed{s(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right]} \quad \text{------ (6)}$$

eqn (6) is the expression for the VSB modulated wave in time domain

This represents the VSB wave with full USB and a vestige of LSB.

→ Further, the time domain description for the VSB modulated wave with full LSB and vestige of USB is as below,

$$\boxed{s(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + m_Q(t)\sin(2\pi f_c t)\right]}$$

★ ENVELOPE DETECTION OF VSB WAVE PLUS CARRIER-

→ VSB modulation is used in the commercial TV broadcasting in which along with VSB transmission a carrier signal of substantial size is transmitted.

∴ The modulated wave can be demodulated by using ENVELOPE DETECTOR.

WKT, The VSB modulated wave with full USB and a vestige of LSB is given by,

$$\boxed{s(t)_{VSB} = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right]} \quad \text{---- (1)}$$

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→ Now, adding carrier component $A_c \cos(2\pi f_c t)$ to eqn (1) scaled by a factor k_a, modifies the modulated wave applied to the envelope detector input as,

$$s(t) = A_c \cos 2\pi f_c t + k_a s_{VSB}(t)$$

$$s(t) = \frac{A_c}{2} k_a \left[m(t) \cos(2\pi f_c t) - m_Q(t) \sin(2\pi f_c t) \right] + A_c \cos(2\pi f_c t)$$

$$s(t) = \frac{A_c}{2} k_a m(t) \cos(2\pi f_c t) - \frac{A_c}{2} k_a m_Q(t) \sin(2\pi f_c t) + \underbrace{A_c \cos(2\pi f_c t)}_{\text{Grouped}}$$

$$\therefore\ s(t) = \underbrace{A_c}_{\text{Component}} \cos(2\pi f_c t) \underbrace{\left[1 + \frac{k_a}{2} m(t) \right]}_{\text{In-phase}} - \underbrace{\frac{k_a A_c}{2} m_Q(t) \sin(2\pi f_c t)}_{\text{Quadrature}}$$

where constant 'k_a' determines the percentage modulation

→ The envelope detector output is denoted by $s_o(t)$

$$\# \quad s_o(t) = \sqrt{[\text{Inphase component}]^2 + [\text{Quadrature Component}]^2}$$

$$s_o(t) = \sqrt{A_c^2 \left[\frac{1 + k_a m(t)}{2} \right]^2 + A_c^2 \left[\frac{k_a}{2} m_Q(t) \right]^2}$$

$$s_o(t) = \sqrt{A_c^2 \underbrace{\left[1 + \frac{k_a m(t)}{2} \right]^2}_{\text{Take common}} \left\{ 1 + \left[\frac{k_a/2\, m_Q(t)}{1 + k_a/2\, m(t)} \right]^2 \right\}}$$

$$s_o(t) = A_c \left[1 + \frac{k_a}{2} m(t) \right] \sqrt{1 + \left[\frac{k_a/2\, m_Q(t)}{1 + k_a/2\, m(t)} \right]^2}$$

$$\therefore\ \boxed{s_o(t) = A_c \left[1 + \frac{k_a}{2} m(t) \right] \left\{ 1 + \left[\frac{k_a/2\, m_Q(t)}{1 + \frac{k_a}{2} m(t)} \right]^2 \right\}^{1/2}} \quad \text{—— (3)}$$

eqn (3) indicates that the distortion is contributed by the quadrature component $m_Q(t)$

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→ This distortion can be reduced using two methods -

(i) Reducing the percentage modulation to reduce ka

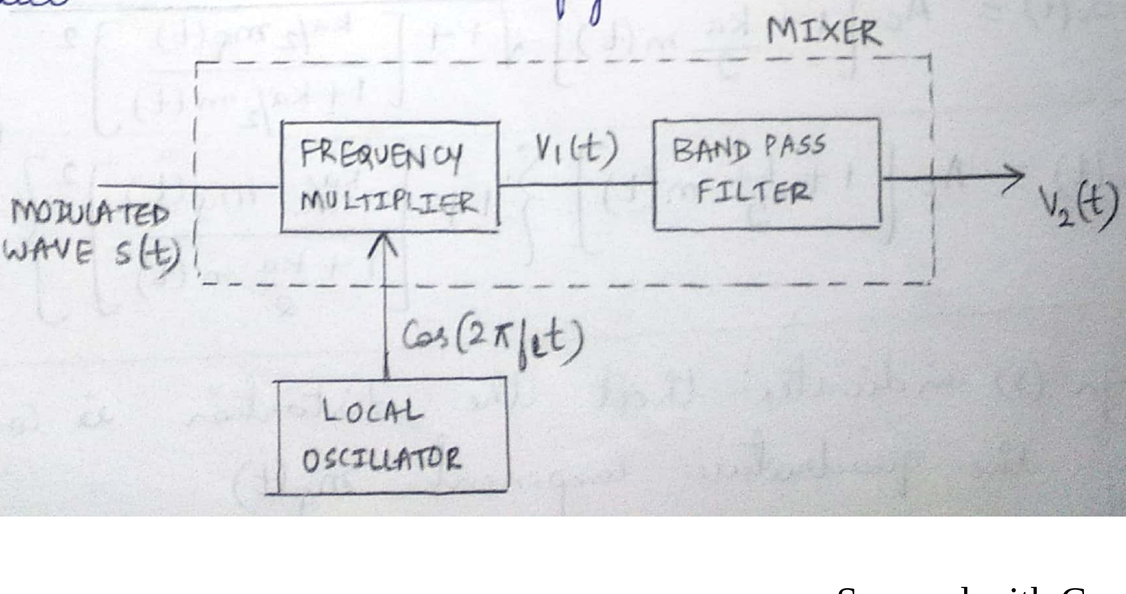
(ii) Increasing the width of the vestigial sideband to reduce $m_Q(t)$

★ FREQUENCY TRANSLATION

→ In the communication systems it is necessary to translate the modulated wave upward or downward in frequency, so that it occupies a new frequency band.

→ Shifting of complete frequency band maintaining the total bandwidth same, carrying the complete information is called 'FREQUENCY TRANSLATION'

→ This frequency translation is accomplished by multiplication of the signal by a locally generated carrier wave & then filtering the product terms as in fig (i)



→ Suppose we have to translate this modulated wave downward in frequency. To perform such translation it is necessary to change 'f_c' to a new value 'f_0'

where, $\boxed{f_0 < f_c}$

This is achieved as below,

(I) ★ FREQUENCY TRANSLATION TO LOWER FREQUENCY (fig Behind page)

STEP-1 :

Multiply the DSB-SC wave $s(t)$ by a locally generated carrier $\cos 2\pi f_L(t)$ as in fig (i)

∴ The output of the product modulator is given by,

$$V_1(t) = s(t) \cdot \cos 2\pi f_L t \qquad (\because s(t) = m(t) \cos 2\pi f_c t)$$

$$V_1(t) = m(t) \cos \underset{A}{2\pi f_c t} \cdot \cos \underset{B}{2\pi f_L t}$$

WKT,

$$\cos A \cdot \cos B = \frac{1}{2} \cos(A-B) + \frac{1}{2} \cos(A+B)$$

$$\therefore \boxed{V_1(t) = \frac{m(t)}{2} \cos 2\pi (f_c - f_L)t + \frac{m(t)}{2} \cos 2\pi (f_c + f_L)t} \quad \text{----- (2)}$$

{ Taking Fourier Transform for eqn (2),

$$V_1(f) = \frac{1}{4}\left\{M\left[f-(f_c-f_L)\right] + M\left[f+(f_c-f_L)\right]\right\} + \frac{1}{4}\left\{M\left[f-(f_c+f_L)\right] + M\left[f+(f_c+f_L)\right]\right\}$$

}

The spectrum of $V_1(f)$ is as shown in fig (c)

→ Consider a DSBSC signal $s(t)$ generated by using the information signal $m(t)$ [∵ 0 to W Hz] and the carrier $c(t) = A_c \cos 2\pi f_c t$ expressed as,

$$\boxed{s(t) = m(t)\cos(2\pi f_c t)} \quad \text{—— (1)} \qquad m(t)\, A_c \cos 2\pi f_c t)$$

where, $m(t)$ is limited to frequency band $-W \leq f \leq W$

{Taking Fourier Transform on Both sides of eqn (1)

$$S(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] \Big\}$$

$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$

→ The spectrum of $s(t)$ occupies the bands $(f_c - W)$ to $(f_c + W)$ and $(-f_c - W)$ to $(-f_c + W)$ as in fig (b)

(I)

★ FREQUENCY TRANSLATION TO LOWER FREQUENCY

:- DOWNWARD TRANSLATION

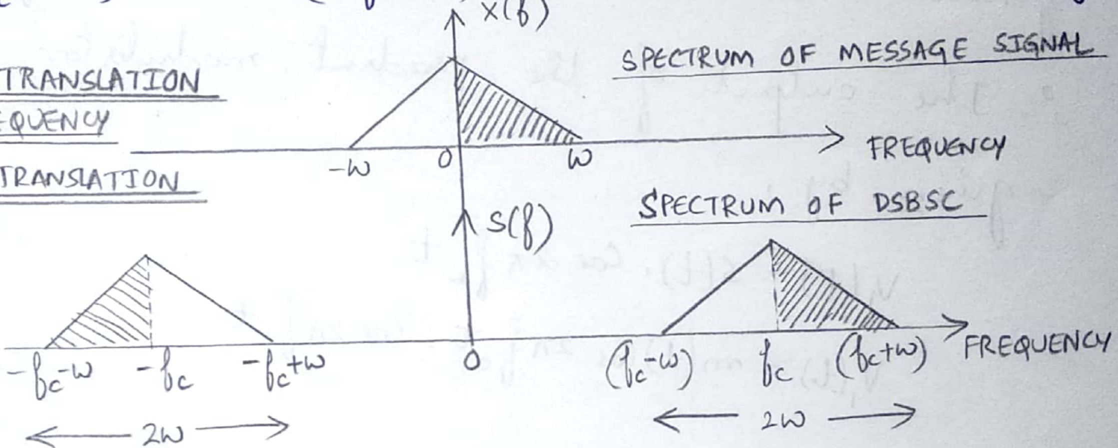


fig (a) FREQUENCY TRANSLATION PROCESS ILLUSTRATED USING SPECTRUM OF MESSAGE SIGNAL AND DSBSC WAVE

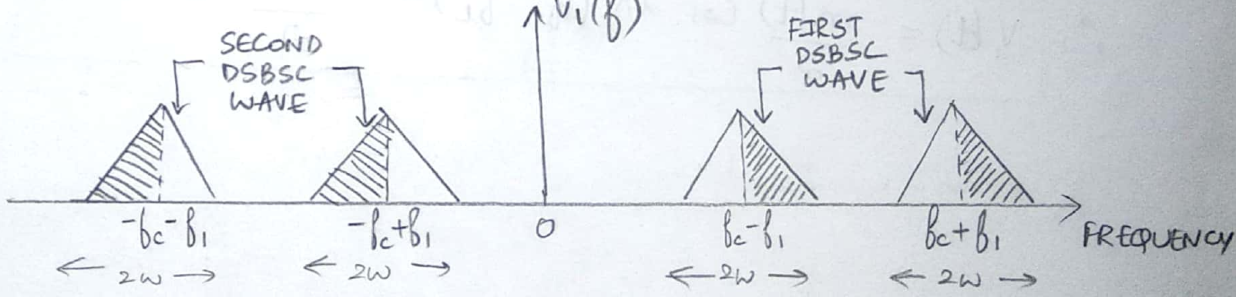


fig (b) SPECTRUM OF THE SIGNAL OBTAINED BY MULTIPLYING THE DSBSC WAVE WITH LOCAL CARRIER

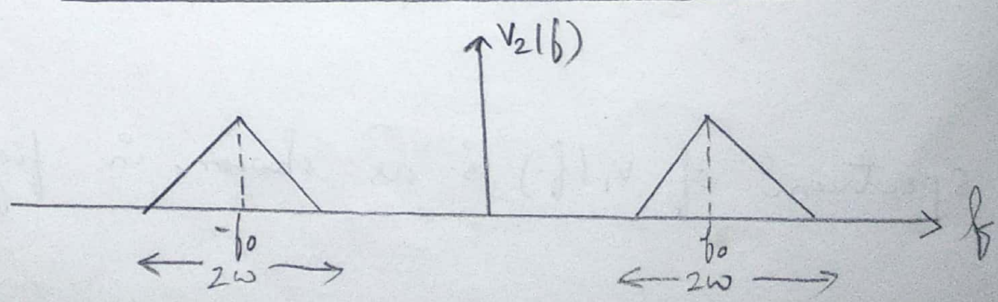


fig (c) SPECTRUM OF DSBSC WAVE TRANSLATED DOWNWARDS IN FREQUENCY

STEP 2:

Pass the multiplier output through a BPF

The carrier frequency is $\boxed{f_0 = f_c - f_L}$ → DOWNWARD TRANSLATION

The BPF are designed to pass the signal having Bandwidth $BW = 2W$ and center frequency 'f_0'

Thus, the output of the BPF $V_2(t)$ is given by,

$$V_2(t) = \frac{m(t)}{2} \cos 2\pi \underbrace{(f_c - f_L)}_{f_0} t$$

$$\boxed{V_2(t) = \frac{1}{2} m(t) \cos 2\pi f_0 t} \quad \text{----- (3)} \quad \because f_c - f_L = f_0$$

{ Taking fourier Transform on Both sides of eqn (3), we get

$$V_2(f) = \frac{1}{4} \left\{ M(f - f_0) + M(f + f_0) \right\}$$

}

The spectrum of $V_2(t)$ ie $V_2(f)$ is as in fig (d)

II ★ FREQUENCY TRANSLATION TO HIGHER FREQUENCY :-

UPWARD TRANSLATION -

(Step 1 & 2 Same as downward)

→ WKT, the output of the multiplier is given by,

$$\boxed{V_1(t) = \frac{1}{2} m(t) \cos 2\pi (f_c - f_L) t + \frac{1}{2} m(t) \cos 2\pi (f_c + f_L) t} \quad \text{----- (2)}$$

→ Designing a BPF with center frequency

$\boxed{f_0 = (f_c + f_L)}$ → upward translation

Bandwidth $BW = 2W$

→ The output of the BPF is given by,

$$V_2(t) = \frac{1}{2} m(t) \cos 2\pi (f_c \pm f_L) t$$

$$\boxed{V_2(t) = \frac{1}{2} m(t) \cos 2\pi (f_0) t} \quad \text{------} \quad (3) \quad \because f_0 = f_c + f_L$$

{ Taking Fourier transform of eqn (3) both sides, we get

$$V_2(f) = \frac{1}{4}\left[M(f-f_c) + M(f+f_0)\right]$$

}

The spectrum of $V_2(t)$ ie $V_2(f)$ is as in below fig.

FREQUENCY TRANSLATION TO HIGHER FREQUENCY :-

UPWARD TRANSLATION

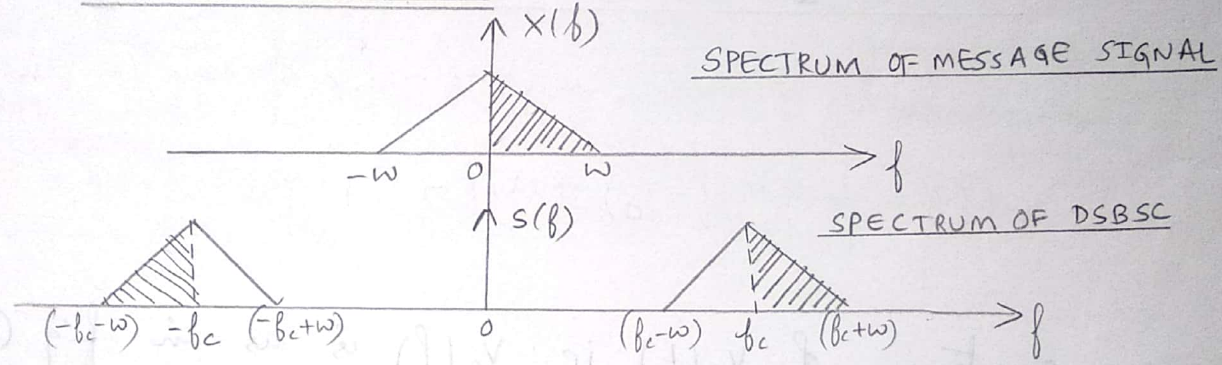


fig (a) FREQUENCY TRANSLATION PROCESS ILLUSTRATED USING SPECTRUM OF MESSAGE SIGNAL AND DSBSC WAVE

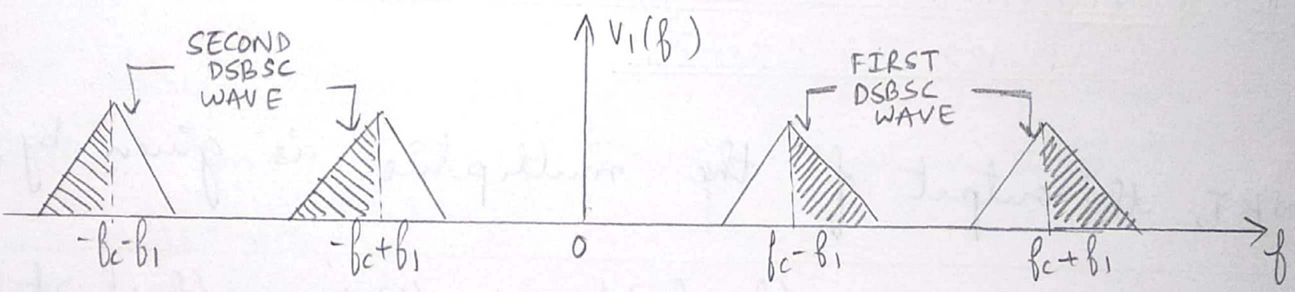


fig (b) SPECTRUM OF THE SIGNAL OBTAINED BY MULTIPLYING THE DSBSC WAVE WITH LOCAL CARRIER

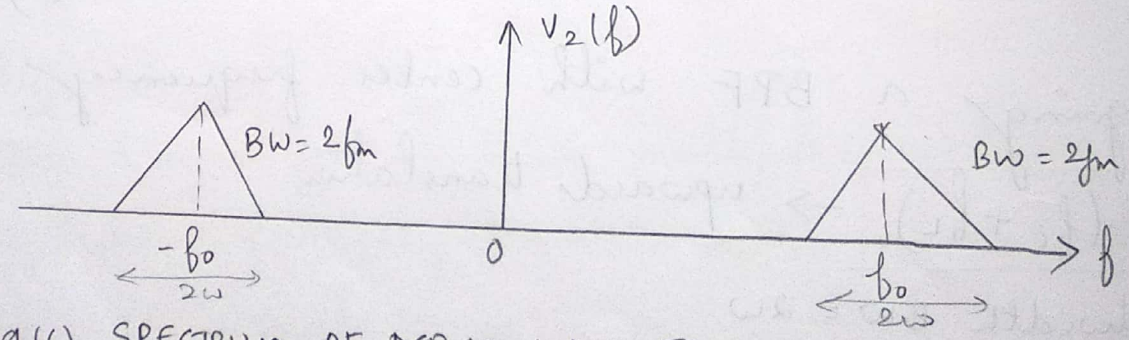
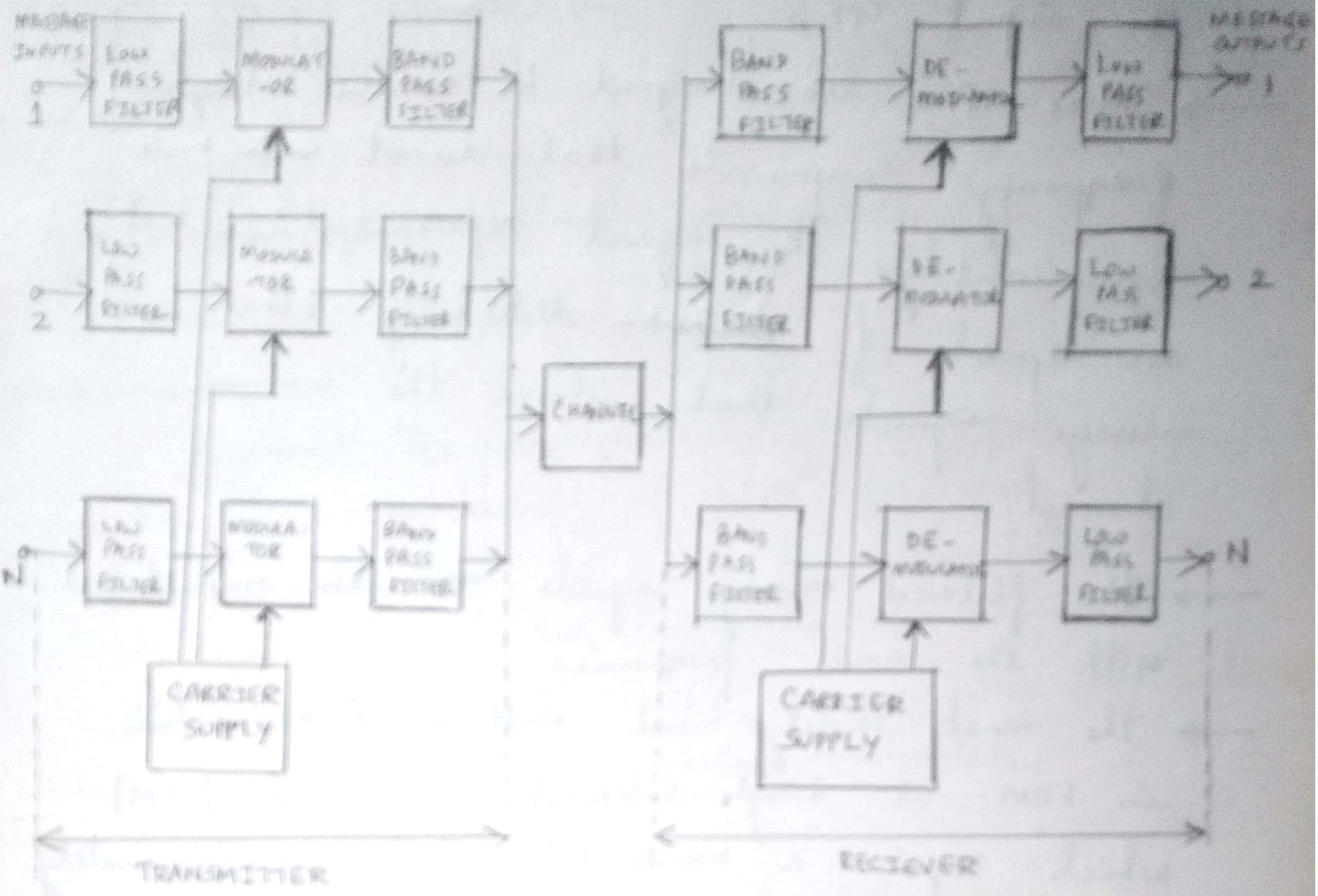


fig (c) SPECTRUM OF DSBSC WAVE TRANSLATED UPWARDS IN FREQUENCY

★ FREQUENCY DIVISION MULTIPLEXING [FDM]



BLOCK DIAGRAM OF FDM

→ MULTIPLEXING - This is a system in which more than one signal can be transmitted simultaneously on a common line

→ In Time division multiplexing [TDM] system, different time periods are alotted for different signals

→ In Frequency division multiplexing [FDM] system different frequency bands are alotted for different signals

→ The input message signals assumed to be of the low pass type are passed though the input LPF's

→ These LPF's are designed to remove high frequency components that do not contribute significantly to signal representation but are capable of disturbing other message signals that share the common channel.

→ The filtered message signals are then modulated with the carrier frequencies.

→ The most widely used method of modulation in FDM is single Sideband modulation [SSM] which requires a bandwidth that is approximately equal to that of original message signal.

→ The Band pass filters [BPF] following the modulators are used to restrict the band of each modulated wave to its prescribed range.

→ The resulting BPF outputs are next combined in parallel to form the input to the COMMON CHANNEL

→ At the receiving end, BPF's are connected to the common channel in parallel to separate the message signals on the frequency occupancy basis

→ Finally, the original message signals are recovered by individual demodulator.

TRANSMISSION BANDWIDTH

→ consider an FDM system using SSB modulation to transmit 24 independent voice inputs.

→ Assume a bandwidth of 4KHz for each voice input. Thus, in order to accomodate an FDM system using SSB modulation to transmit the 24 voice inputs, the communication channel must provide the transmission bandwidth :-

$$BW = n \times f_m$$

where, 'n' is the number of voice signals

Ex:-

For 24 voice inputs having a bandwidth of 4KHZ for each voice inputs, the transmission bandwidth is given by,

$$BW = 24 \times 4KHZ$$

$$\therefore \underline{BW = 96KHZ}$$

AGES -

large number of signals [channel] can be transmitted simultaneously

(2) FDM does not need synchronization between the transmitter and receiver for proper operation

(3) Demodulation of FDM is easy.

DISADVANTAGES -

(1) The communication channel must have a very large bandwidth

(2) Large number of modulators and filters are required.

(3) Cross Talks are a main disadvantage in FDM

(4) All the FDM channels get affected due to wideband fading.

★ COMPARISONS OF AMPLITUDE MODULATION TECHNIQUES -

SL NO	PARAMETER	DSB-FC STANDARD AM	DSB-SC	SSB	VSB
1.	POWER	High	Medium	Less	Less than DSB-SC but greater than SSB
2.	BANDWIDTH	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
3.	CARRIER SUPPRESSION	No	Yes	Yes	NO
4.	SIDEBAND TRANSMISSION	NO	NO	one sideband completly	one sideband suppressed partly.
5.	TRANSMISSION EFFICIENCY	Minimum	Moderate	Maximum	Moderate
6.	RECEIVER COMPLEXITY	Simple	Complex	Complex	Simple
7.	MODULATION TYPE	Non-linear	Linear	Linear	Linear
8.	APPLICATIONS	Radio Communication	Linear Radio Communication	Linear point to point mobile communication	Television

ANGLE MODULATION

NANDITHA KRISHNA

★ BASIC DEFINITIONS-

ANGLE MODULATION - INTRODUCTION

→ "This is the process in which angle of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping constant the amplitude of the carrier wave"

→ This is another method of modulating a sinusoidal carrier wave.

NOTE- In this method of modulation, the amplitude of the carrier wave is maintained constant.

→ There are Two Types of angle modulation

(i) Frequency modulation
(ii) phase modulation

→ Let the modulated wave be expressed in the general form as follows -

$$\boxed{s(t) = A_c \cos[\theta(t)]} \text{ ---- (1)}$$

where,

$A_c \rightarrow$ Carrier amplitude ($\therefore$ maintained constant)

$\theta(t) \rightarrow$ angular argument $\theta(t)$ varied by a message signal $m(t)$

→ This variation of $\theta(t)$ due to $m(t)$ can be expressed mathematically if we know the type of angle modulation.

If $\theta(t)$ changes by 2π radians then we say that a complete oscillation has occured.

→ If $\theta(t)$ increases monotonically with time, then the average frequency is in HERTZ [Hz] over an interval from t to $(t+\Delta t)$ is given by,

$$\boxed{f_{\Delta t}(t) = \frac{1}{2\pi} \frac{\theta(t+\Delta t) - \theta(t)}{\Delta t}} \quad \text{----} (2)$$

→ The instantaneous frequency of the angle modulated wave $s(t)$, is given by,

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$f_i(t) = \lim_{\Delta t \to 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \, \Delta t}\right]$$

$$\therefore \boxed{f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}} \quad \text{----} (3)$$

Eqn (3) is the basic definition of derivative of a function.

→ For an unmodulated Carrier, angle $\theta(t)$ is given by,

$$\boxed{\theta(t) = 2\pi f_c t + \phi_c(t)}$$

where,

The angular frequency of the carrier is ω_c, where $\omega_c = 2\pi f_c$ and ϕ_c is the value of $\theta(t)$ at $t=0$

(1) PHASE MODULATION [PM]

→ This is defined as that form of angle modulation in which the angular argument $\theta(t)$ is varied LINEARLY with the message signal $m(t)$.

It is given as shown,

$$\boxed{\theta(t) = 2\pi f_c t + k_p m(t)}$$

where,

$\omega_c t = 2\pi f_c t$ → represents the angular argument of the modulated carrier

k_p → constant, represents the phase sensitivity of the modulator, radians/volt.

→ The phase-modulated wave $s(t)$ is given by,

$$s(t) = A_c \cos[\theta(t)]$$

$$\therefore \boxed{s(t) = A_c \cos[2\pi f_c t + k_p m(t)]}$$

FEATURES OF PHASE MODULATION -

(1) The envelope of PM wave is a constant and equal to the amplitude of the unmodulated carrier

(2) The zero crossings of PM wave no longer have a perfect regularity in their spacing like AM wave. This is because instantaneous frequency of PM wave is proportional to time derivative of $m(t)$

(2) FREQUENCY MODULATION [FM]

→ This is defined as that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied LINEARLY with message signal $m(t)$.

It is given as shown,

$$\boxed{f_i(t) = f_c + k_f\, m(t)} \quad \text{----- (1)}$$

Where,

f_c → frequency of the unmodulated carrier

k_f → frequency sensitivity of the modulator expressed in hertz/volt.

WKT,
$$\frac{d\theta}{dt} = 2\pi f_i(t) \quad \text{---- (2)} \qquad \therefore f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$= \frac{d\theta(t)}{dt} = 2\pi f_i(t)$$

Integrating on Both sides of eqn (2) w.r.t 't'

$$\int \frac{d\theta(t)}{dt} = \int 2\pi f_i(t)$$

$$\theta(t) = \int_0^t 2\pi f_i(t)\, dt \quad \text{---- (3)}$$

$$\theta(t) = \int_0^t 2\pi \left[f_c + k_f m(t)\right] dt$$

$$= \int_0^t 2\pi f_c\, dt + \int_0^t 2\pi k_f\, m(t)\,.dt$$

$$= 2\pi f_c \int_0^t (1)\, dt + 2\pi k_f \int_0^t m(t)\,.dt$$

$$\therefore \boxed{\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt} \quad \text{---- (4)}$$

→ The FM wave in Time domain is written as,

$$s(t) = A_c \cos\left[\theta(t)\right] \quad \text{--- (5)}$$

Substituting eqn (4) in eqn (5), we get

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]}$$

→ The consequence of allowing the angular argument $\theta(t)$ to become dependent on the message signal $m(t)$ or on its integral is that the 'ZERO CROSSINGS' of a PM wave or FM wave no longer have a perfect regularity in their spacing.

NOTE1- ZERO CROSSINGS refer to the instants of time at which a waveform changes from a negative to positive value or positive to negative value.

NOTE2 - Envelope of a PM or FM wave is constant. Envelope of an AM wave is dependent on the message signal.

RELATIONSHIP BETWEEN FM AND PM -

→ In Both FM and PM, the instantaneous angle $\theta(t)$ changes, but in a different manner.

→ The expressions for the FM and PM waves in the time domain are as follows -

(i) PM wave :-

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]} \quad \text{--- (1)}$$

(ii) FM wave :-

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]} \quad \text{--- (2)}$$

→ comparing expressions (1) and (2) we can say that FM wave is actually a PM wave having a modulating signal $\int_0^t m(t)$ instead of $m(t)$

→ FM wave can be generated by first integrating $m(t)$ and then using the result as the input to a phase modulator as in fig (i)

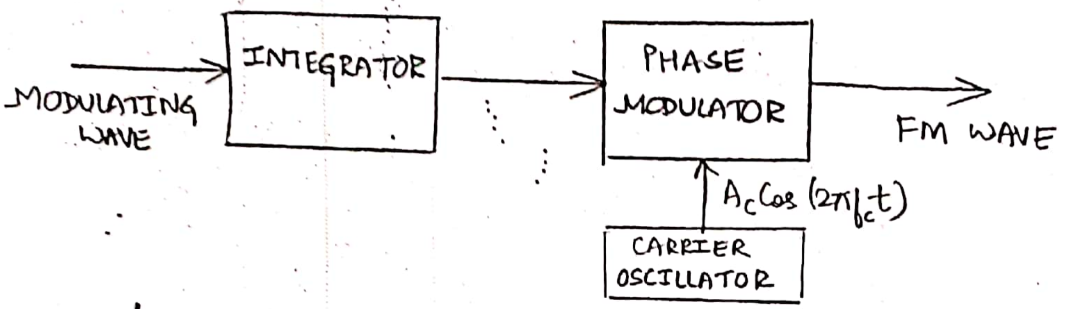


fig (i) GENERATING FM WAVE USING PHASE MODULATOR

→ PM wave can be generated by first differentiating $m(t)$ and then using the result as the input to a frequency modulator as in fig (ii)

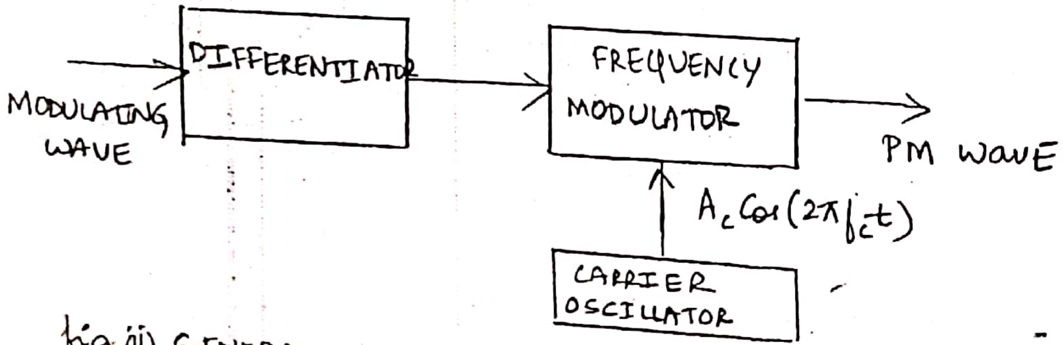


fig (ii) GENERATING PM WAVE USING FREQUENCY MODULATOR

★ **EXAMPLE - SQUARE MODULATION** (To show FM & PM waves) ④

→ consider two full cycles of square modulating wave m(t). as in fig (a)

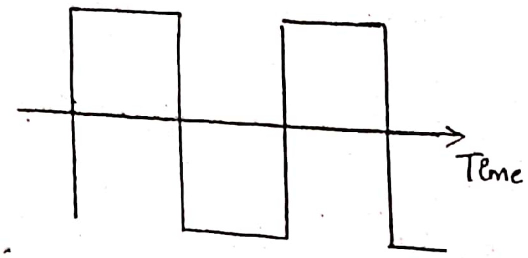


fig (a) SQUARE MODULATING WAVE m(t)

→ delta function

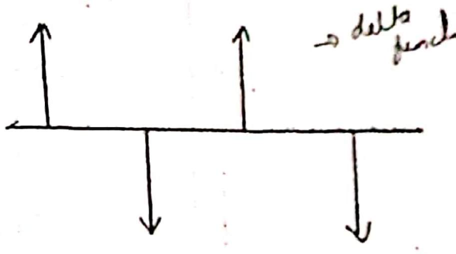


fig (c) DERIVATIVE OF m(t) WITH RESPECT TO TIME

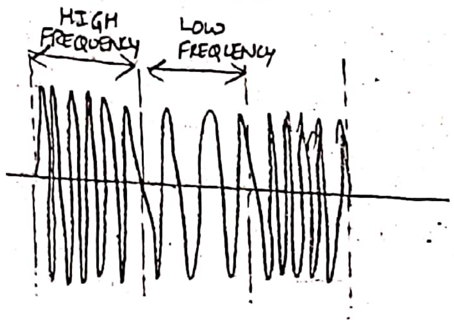


fig (b) FREQUENCY - MODULATED WAVE (FM WAVE)

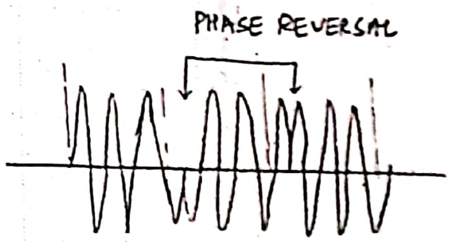


fig (d) PHASE - MODULATED WAVE (PM WAVE)

→ The FM wave produced by this modulating wave is as in fig (b)

→ To plot the PM wave produced by the square modulating wave m(t), the derivative $\frac{dm(t)}{dt}$ is plotted, which is differentiated version of m(t) as in fig (c).

This $\frac{dm(t)}{dt}$ is a train of alternate (+ve or -ve) delta functions (periodic sequence)

→ From this the desired PM wave is plotted as in fig (d)

★ FREQUENCY MODULATION-

→ The FM wave s(t) is defined by the equation $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$ is a non-linear function of the modulating wave m(t)

∴ Frequency modulation is a Non-linear modulation process.

→ The spectrum of an FM wave is not related in a simple manner to that of modulating wave m(t).

Thus, to study the spectral properties of an FM wave the approach is to start with single Tone modulation.

Instantaneous Value of FM Voltage:

SINGLE-TONE FREQUENCY MODULATION- ★

→ The Frequency modulated wave in the time domain is given by,

$$\boxed{s(t) = A_c \cos[\theta(t)]} \quad \text{--- (1)}$$

→ The sinusoidal modulating signal is defined by,

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \quad \text{--- (2)}$$

→ The Instantaneous frequency of FM signal is given by,

$$f_i(t) = f_c + k_f\, m(t)$$

$$f_i(t) = f_c + \underbrace{k_f A_m}_{} \cos(2\pi f_m t).$$

$$\boxed{f_i(t) = f_c + \Delta f \cos(2\pi f_m t)} \quad \text{---} (3)$$

where,

$\Delta f = k_f A_m$ → Called as Frequency deviation

→ Frequency deviation Δf represents the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency f_c

$\Delta f = f_c - f_i(t)$

→ Frequency deviation Δf is proportional to the amplitude of the modulating signal and is independent of the modulating frequency.

$\Delta f \propto A_m$ $\quad$ Δf is independent of f_m

→ W.K.T,

The angular velocity $w_i(t)$ is the rate of change of $\theta(t)$

$w = 2\pi f$

$$w_i(t) = \frac{d}{dt}\theta(t)$$

$$2\pi f_i(t) = \frac{d}{dt}\theta(t) \quad \text{-----} (4)$$

Integrating eqn (4) w.r.t dt

$$\int_0^t \frac{d\theta(t)}{dt} = \int_0^t 2\pi f_i(t)\, dt$$

$$\theta(t) = \int_0^t 2\pi f_i(t)\, dt \quad \text{---} (5)$$

Substituting eqn (3) in eqn (5)

$$\theta(t) = \int_0^t 2\pi \left[\underbrace{f_c + \Delta f \cos(2\pi f_m t)}_{f_i(t)} \right] dt$$

$$= \int_0^t 2\pi f_c \, dt + \int_0^t 2\pi \Delta f \underbrace{\cos(2\pi f_m t)}_{\cos at \, dt \;\; \because a = 2\pi f_m} dt$$

$$\theta(t) = 2\pi f_c t + \cancel{2\pi} \Delta f \cdot \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}$$ $\rightarrow \because$ WKT, $\int_0^t \cos at \, dt = \frac{\sin at}{a}$

$$\theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

$$\boxed{\theta(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)} \quad \text{--- (6)}$$

where,

$$\underset{\text{phase deviation}}{\beta} = \frac{\Delta f}{f_m}$$

Substituting eqn (6) in eqn (1), we get

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]}$$

Here from the above equation, parameter β represents the phase deviation of the FM wave]

[ie, maximum departure of the angular argument $\theta(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier]

MODULATION INDEX (β)

→ Modulation index is defined as the ratio of frequency deviation 'Δf' to the modulating frequency 'f_m'

ie $\beta = \frac{\text{Frequency Deviation}}{\text{Modulating Frequency}}$

$$\boxed{\beta = \frac{\Delta f}{f_m}}$$

→ In FM the modulation index can be greater than 1

→ The modulation index is very important in FM because it decides the bandwidth of the FM wave and also the number of sidebands having significant amplitude.

★ CLASSIFICATION OF FREQUENCY MODULATION (FM) Types/Properties of FM

(1) NARROW BAND FM - Property 1

→ A NBFM is the FM wave with a Small Bandwidth

→ The modulation index 'β' of NBFM is small as compared to one radian $\beta < 1$ rad

→ Thus NBFM has a narrow bandwidth which is equal to twice the message bandwidth

(2) WIDE BAND FM - Property 2

→ The WBFM has Infinite bandwidth and hence called as wideband FM

→ The WBFM has much larger value of 'β' which is theoretically infinite

For larger values of β, the FM wave ideally contains the carrier and an infinite number of sidebands located symmetrically around the carrier.

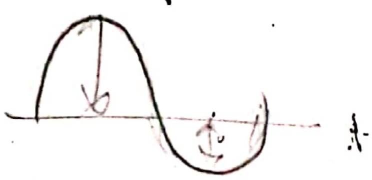
(3) CONSTANT AVERAGE POWER - Property 3

→ The envelope of an FM wave is constant, so that the average power of such a wave dissipated in 1 ohm resistor is also constant

→ The FM wave $s(t)$ has a constant envelope equal to A_c

$s(t) = A_c$

∴ power dissipation $= \dfrac{A_c^2}{2R}$



→ The average power dissipated by $s(t)$ in a 1 ohm resistor is given by,

$$P = \frac{A_c^2}{2(1)}$$

$$\therefore \boxed{P = \frac{A_c^2}{2}}$$

→ The average power of a single Tone FM wave $s(t)$ may be expressed as series,

$$\boxed{P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)}$$

But, $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

Thus, $P = \frac{A_c^2}{2}(1)$

$\therefore \boxed{P = \frac{A_c^2}{2}}$

★ (1) NARROW-BAND FREQUENCY MODULATION -

→ For small values of the modulation index β compared to 1 radian, the FM wave assumes a narrow band form consisting of a carrier, a upper side frequency component and a lower side frequency component

→ For small values of β (modulation index), we have

$$\left\{\begin{aligned} J_0(\beta) &\simeq 1 \\ J_1(\beta) &\simeq \frac{\beta}{2} \\ &\vdots \\ J_n(\beta) &\simeq 0, \quad n>1 \end{aligned}\right\}$$

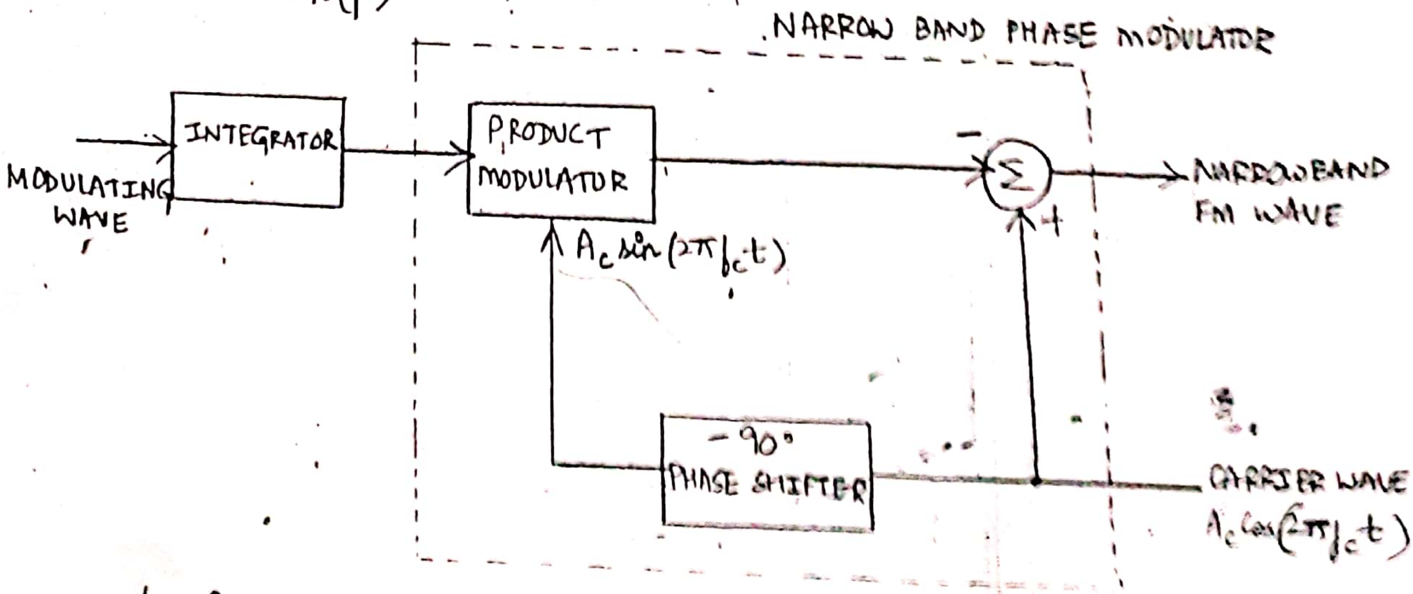


fig (i) BLOCK DIAGRAM FOR GENERATING NARROW BAND FM SIGNAL

→ consider a frequency modulated wave given as;
(OR) Time domain expression for an FM wave is,

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]} \quad ----(1)$$

using the trignometric identity

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

where, $A = 2\pi f_c t$

$B = \beta \sin(2\pi f_m t)$

$$\boxed{s(t) = A_c\left[\cos(2\pi f_c t) \cdot \cos(\beta \sin 2\pi f_m t) - \sin(2\pi f_c t) \cdot \sin(\beta \sin 2\pi f_m t)\right]} \quad ----(2)$$

In NBFM, β is small

∴ It is possible to approximate as below

$$\boxed{\begin{aligned} &\cos(\beta \cdot \sin 2\pi f_m t) \simeq 1 \\ &\sin(\beta \sin 2\pi f_m t) \simeq \beta \sin 2\pi f_m t \end{aligned}} \quad ----(3)$$

Substituting eqn (3) in eqn (2), we get

$$s(t) = A_c \cos 2\pi f_c t \,(1) - A_c \sin 2\pi f_c t \cdot (\beta \sin 2\pi f_m t)$$

$$\boxed{s(t) = A_c \cos 2\pi f_c t - \beta A_c \cdot \underbrace{\sin 2\pi f_c t}_{A} \cdot \underbrace{\sin 2\pi f_m t}_{B}} \quad ----(4)$$

W.K.T,

$$\sin A \cdot \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

$$s(t) = A_c \cos 2\pi f_c t - \left[\frac{\beta A_c}{2} \cos 2\pi (f_c - f_m)t - \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m)t\right.$$

$$\boxed{s(t) = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m)t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m)t} \quad ----(5)$$

WKT, the amplitude modulated wave is given by, ($\because$ previous topic of AM wave)

$$\boxed{s(t)_{AM} = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t} \quad \text{--- (6)}$$

→ comparing eqn (5) and (6), we see that the only difference between NBFM wave and AM wave is the sign reversal of the lower sideband (LSB)

$\therefore$ NBFM also requires the same bandwidth as that of AM

→ Taking fourier transform on Both sides of eqn (5) we get,

$$\boxed{\begin{aligned} S(f) = {} & \frac{A_c}{2} \left[\delta(f - f_c) + \delta(f + f_c) \right] \\ & - \frac{\beta A_c}{4} \left\{ \delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)) \right\} \\ & + \frac{\beta A_c}{4} \left\{ \delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) \right\} \end{aligned}}$$

→ The transmission bandwidth of a NBFM wave is $2f_m$ $\therefore B_T = 2f_m$

<u>NOTE</u> - The NBFM wave and conventional AM are identical but there will be no amplitude variations in FM.

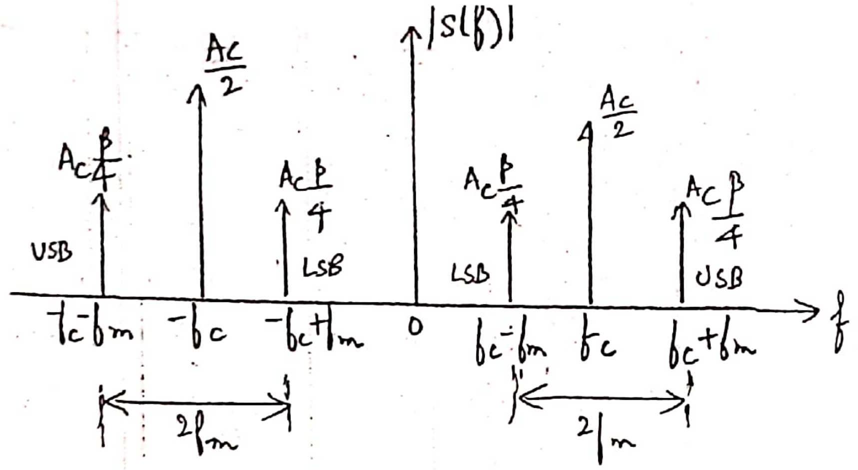


fig (ii) SPECTRAL CONTENT OF NBFM WAVE FOR SINGLE TONE MODULATION

★ (2) **WIDE BAND FREQUENCY MODULATION-** Spectrum analysis

→ For large values of the modulation index β compared to one radian, the FM wave contains a carrier and an infinite number of side frequency components located symmetrically around the carrier.

→ Amplitude of the carrier component contained in a wideband FM wave varies with the modulation index β in accordance with $J_0(\beta)$

→ It is possible to obtain the spectrum of a wideband FM signal by expanding the FM wave as a Fourier Series.

→ The FM wave for sinusoidal modulation is given by,

$$S(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right] \quad \text{---(1)}$$

Taking real part of eqn (1)

ie, eqn (1) has no imaginary part but has only real part

$\because$ $\boxed{\theta = 2\pi f_c t + \beta \sin 2\pi f_m t}$

$\Rightarrow$ $s(t) = Re\left[A_c e^{j\theta}\right]$ $s(t) = A_c \cos(\theta)$

$s(t) = Re\left[A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\right]$

$s(t) = Re\left[A_c e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right]$

$s(t) = Re\left[e^{j2\pi f_c t} \cdot A_c e^{j\beta \sin 2\pi f_m t}\right]$

$$\boxed{s(t) = Re\left[e^{j2\pi f_c t} \cdot \hat{s}(t)\right]} \quad \text{------ (2)}$$

where,

$$\boxed{\hat{s}(t) = A_c e^{j\beta \sin 2\pi f_m t}} \quad \text{------ (3)}$$

$\rightarrow$ $\hat{s}(t)$ is a periodic time function with a fundamental frequency 'f_m'.

This can be expressed using complex fourier series as,

$$\boxed{\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}} \quad \text{------ (4)}$$

where,

C_n is a complex fourier coefficient given by,

$$\boxed{C_n = f_m \int_{-1/2 f_m}^{1/2 f_m} \hat{s}(t) \cdot e^{-j2\pi n f_m t} \cdot dt} \quad \text{------ (5)}$$

Substituting eqn (3) in eqn (5),

we get,

$$\boxed{C_n = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} \cdot dt}$$ --- (5) NEW

$$C_n = A_c f_m \int_{-1/2f_m}^{1/2f_m} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} \cdot dt$$

Let $\boxed{x = 2\pi f_m t}$ ------ (i)

Differentiate eqn (i) w.r.t 't'

$$\frac{dx}{dt} = 2\pi f_m (1)$$

$$dt = \frac{dx}{2\pi f_m} \quad \text{--- (i)}$$

Giving the limits

W.K.T $x = 2\pi f_m t$

When $t = -\frac{1}{2f_m}$

$$x = \cancel{2}\pi \cancel{f_m} \cdot \left(\frac{-1}{\cancel{2}\cancel{f_m}}\right)$$

$$\boxed{x = -\pi}$$

when $t = \frac{1}{2f_m}$

$$x = \cancel{2}\pi \cancel{f_m} \cdot \left(\frac{1}{\cancel{2}\cancel{f_m}}\right)$$

$$\boxed{x = \pi}$$

$$C_n = A_c f_m \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \frac{dx}{2\pi f_m} \quad \because dt = \frac{dx}{2\pi f_m} \text{ -- (i)}$$

$$C_n = A_c \cdot \frac{\cancel{f_m}}{2\pi \cancel{f_m}} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot dx$$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot dx$$

$$\boxed{C_n = A_c J_n(\beta)} \quad \text{---- (6)}$$

where,

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin x - nx)}\, dx$$

$J_n(\beta)$ is a Bessel function of the 1st kind, n^{th} order with an argument β

Substituting eqn (6) in eqn (4) $\left\{\because \hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t} \ldots (4)\right\}$

we get, $$\boxed{\hat{S}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{j2\pi n f_m t}} \quad \text{----} \ (7)$$

Substituting eqn (7) in eqn (2) $\left\{\because s(t) = Re\left[\hat{s}(t).\, e^{j2\pi f_c t}\right] - (2)\right\}$

$$s(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t} . \, e^{j2\pi f_c t}\right]$$

$$s(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi[f_c + n f_m] t}\right]$$

W.K.T,

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$Re\left[e^{j\theta}\right] = \cos\theta \quad ; \quad \theta = 2\pi(f_c + n f_m)t$$

|||ly

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) . \cos\left[2\pi(f_c + n f_m)t\right]} \quad \text{------} \ (8)$$

Giving the values for n between $-\infty$ to $+\infty$

ie, $n = 0, 1, -1, 2, -2$ ------ $+\infty, -\infty$

$$s(t) = A_c \left[J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\cos 2\pi(f_c+f_m)t + J_{-1}(\beta)\cos 2\pi(f_c-f_m)t \right.$$
$$+ J_2(\beta)\cos 2\pi[f_c+2f_m]t + J_{-2}(\beta)\cos 2\pi[f_c-2f_m]t$$
$$\left. + J_3(\beta)\cos 2\pi[f_c+3f_m]t + J_{-3}(\beta)\cos 2\pi[f_c-3f_m]t + \cdots \right] \quad \text{---- (9)}$$

$$s(t) = A_c \left\{ J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\left[\cos 2\pi(f_c+f_m)t - \cos 2\pi(f_c-f_m)t\right] \right.$$
$$+ J_2(\beta)\left[\cos 2\pi(f_c+2f_m)t - \cos 2\pi(f_c-2f_m)t\right]$$
$$\left. + J_3(\beta)\left[\cos 2\pi(f_c+3f_m)t - \cos 2\pi(f_c-3f_m)t\right] + \cdots \right\}$$

→ Thus, the modulator signal has a carrier component and an infinite number of side frequencies $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$, ---- $f_c \pm n f_m$

→ Taking fourier transform on Both sides of eqn (9) (a) we get,

$$S(f) = \frac{A_c}{2} J_0(\beta)\left[\delta(f-f_c) + \delta(f+f_c)\right] +$$
$$\frac{A_c}{2} J_1(\beta)\left\{\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right\} +$$
$$\frac{A_c}{2} J_{-1}(\beta)\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\} + \cdots\cdots +$$
$$\cdots + \frac{A_c}{2} J_n(\beta)\left\{\delta[f-(f_c+nf_m)] + \delta[f+(f_c+nf_m)]\right\} +$$
$$\frac{A_c}{2} J_{-n}(\beta)\left\{\delta[f-(f_c-nf_m)] + \delta[f+(f_c-nf_m)]\right\} \quad \text{---(10)}$$

→ Now, plotting spectrum for above eqn (10)

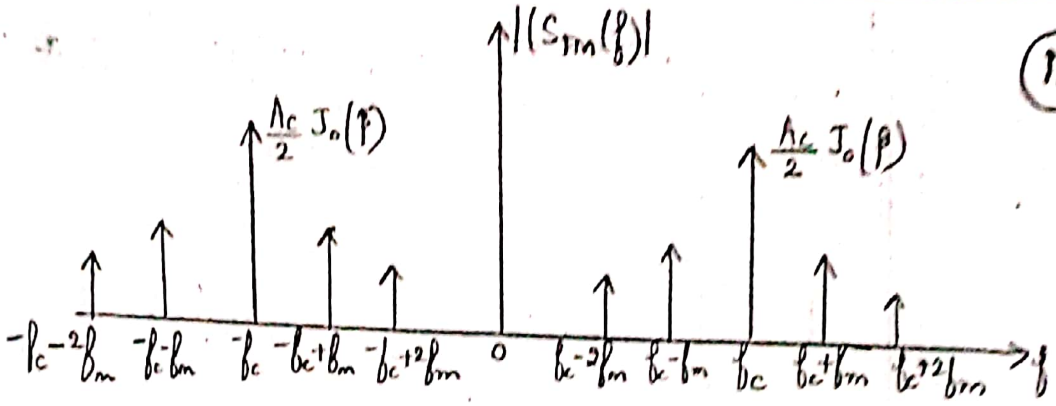


fig (1) AMPLITUDE SPECTRUM OF FM SIGNAL

NOTE - The amplitude of side frequency component depends upon the Bessel function
(Bessel variations will be as a function of 'β' fixing the values of 'n'.)

★ TRANSMISSION BANDWIDTH OF FM WAVES -

→ FM wave contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal is similarly infinite in extent.

→ Practically, FM wave is limited to a finite number of significant side frequencies compatible with a specified amount of distortion

→ we may specify an effective bandwidth required for the transmission of an FM wave.

→ consider the case of an FM wave generated by a single tone modulating wave of frequency f_m,
Here, the side frequencies are separated from the carrier frequency f_c, by an amount greater

than the frequency deviation Δf decrease rapidly towards zero, so that the bandwidth always exceeds the total frequency excursion.

→ For large values of modulation index β, the bandwidth approaches, is only slightly greater than the total frequency excursion $2\Delta f$.

→ For small values of modulation index β, the spectrum of FM wave is limited to the carrier frequency f_c and one pair of side frequencies at $f_c \pm f_m$, so that bandwidth approaches $2f_m$.

→ The Transmission bandwidth of an FM wave generated by a single tone modulating wave of frequency f_m as,

$$B = 2\Delta f + 2f_m$$

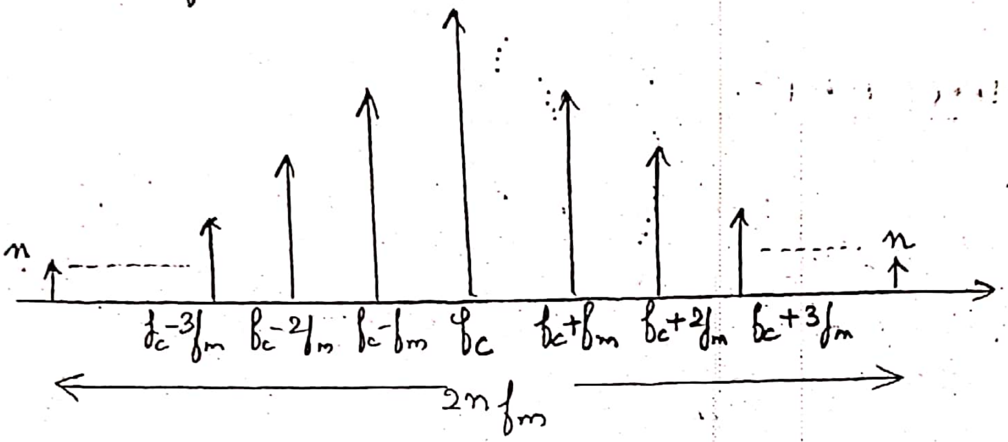
$$B = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This relation is known as CARSON'S RULE

NOTE - The 99% Bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side frequencies is greater than 1% of the carrier amplitude obtained when the modulation is removed.

→ We define the transmission bandwidth as $2n_{max} f_m$, where f_m is the modulation frequency n_{max} is the maximum value of the integer n that satisfies the requirement $|J_n(\beta)| > 0.01$.

→ The value of n_{max} varies with the modulation index β.



→ For a non-sinusoidal modulating wave $m(t)$, with its highest frequency component W, the bandwidth required is estimated using worst case analysis.

→ The quantity DEVIATION ratio is defined as the ratio of peak frequency deviation Δf to the highest modulating frequency W,

ie, $D = \dfrac{\Delta f}{W}$ — peak freq deviation / highest modulating freq.

$\therefore \Delta f = DW$

→ The Bandwidth B is using deviation ratio by replacing the value of $\Delta f = DW$ & $f_m = W$

WKT, $\therefore B = 2\Delta f + 2f_m \Rightarrow 2(\Delta f + f_m)$

$B = 2(DW + W)$

$B = 2(D+1)W.$

★ GENERATION OF FM WAVES:-

→ There are two basic methods of generating FM waves :-

(1) Indirect method or Armstrong method or Stereo FM

(2) Direct method or Direct FM

(1) INDIRECT METHOD -

→ In this method of producing frequency modulation the modulating wave is first used to generate a narrow band FM wave (NBFM) and then Frequency multiplier are used to increase the frequency deviation which results to generation of wideband FM wave (WBFM).

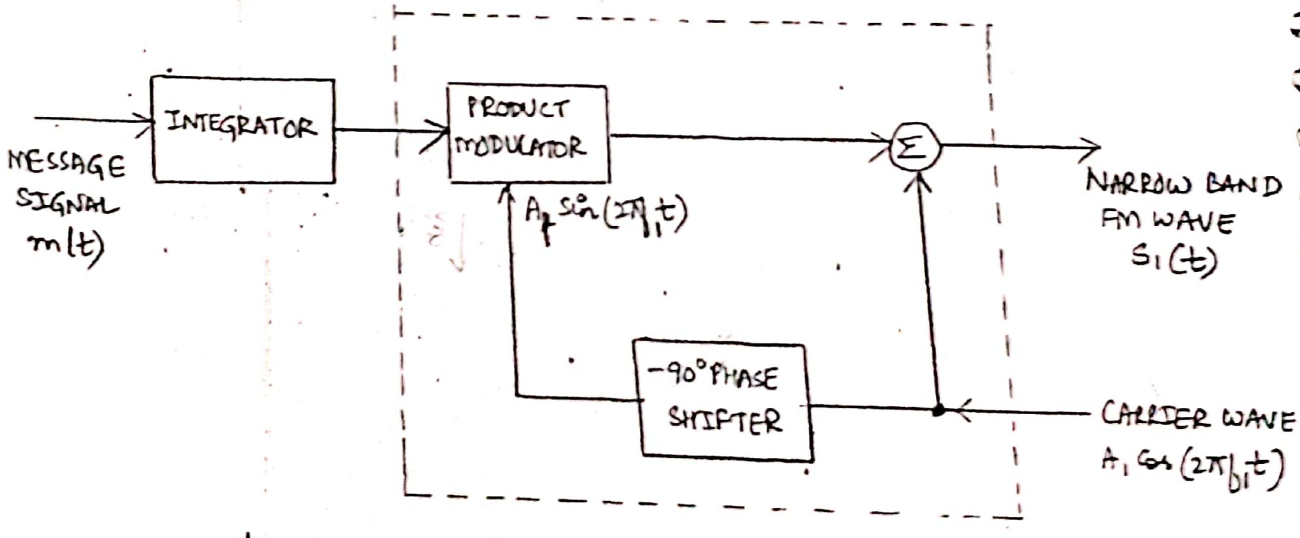
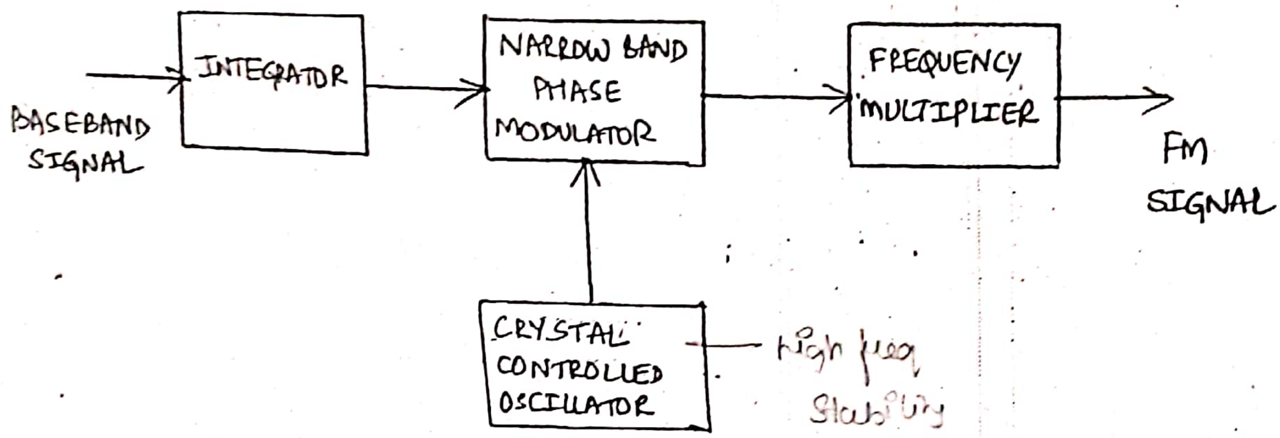


fig (i) NARROW BAND PHASE MODULATOR

→ Here in this method, the message signal m(t) is first passed through an integrator before applying it to the phase modulator as in fig(i)

→ The carrier signal is generated by using crystal oscillator because it provides very high frequency stability



fig(ii) INDIRECT METHOD OF GENERATING WIDE BAND FM SIGNAL

→ The operation of indirect method is divided into two parts as follows -

(i) Generate a NBFM using a phase modulator

(ii) using the frequency multipliers and mixer to obtain the required values of frequency deviation and modulation index (ie WBFM)

NBFM

→ In order to minimize the distortion in the phase modulator, the maximum phase deviation or the modulation index β is kept small which results in a NBFM signal

→ Let $s_1(t)$ be the NBFM wave, then we have

$$\boxed{s_1(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\cdot dt\right]} \quad \text{--- (1)}$$

where, freq modulator

$f_c \rightarrow$ frequency of the crystal oscillator

$K_f \rightarrow$ frequency sensitivity constant in Hz/volts.

→ For a single tone modulation signal defined by,

$$m(t) = A_m \cos 2\pi f_m t \quad ,$$

Then eqn (1) becomes,

$$\boxed{s_1(t) = A_c \cos\left[2\pi f_c t + \beta_1 \sin 2\pi f_m t\right]} \quad \text{--- (2)}$$

where,

$\beta_1 \rightarrow$ modulation index for single tone modulation. This is kept below 0.3 radians to minimize the distortions

→ The instantaneous frequency of eqn (2) is,

$$\boxed{f_i(t) = f_c + K_f\, m(t)}$$

(ii) GENERATION OF WBFM :-

→ The output of the narrow band phase modulator is then multiplied by a frequency multiplier, producing the desired WBFM wave as in fig (iii).

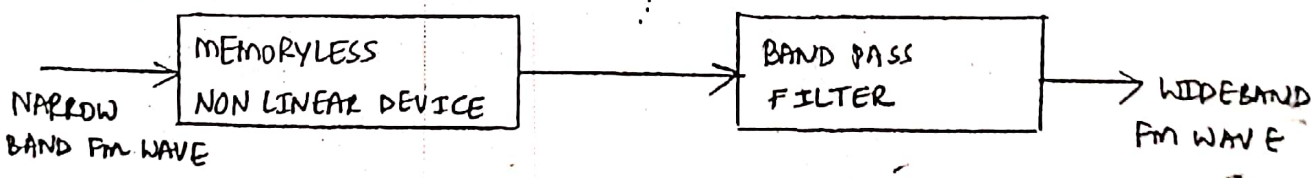


fig (iii) FREQUENCY MULTIPLIER.

→ A frequency multiplier consists of a memoryless non linear device followed by a BPF as in fig (iii)

→ The input-output relation of such a non-linear device may be expressed in the general form,

$$v(t) = a_1 s_1(t) + a_2 s_1^2(t) + \cdots + a_n s_1^n(t) \quad \text{---- (3)}$$

Where,

$a_1, a_2, \cdots a_n$ are coefficients

$n \rightarrow$ highest order of non linearity

Substituting eqn (1) in eqn (3) and simplifying, we find the frequency modulated wave having carrier frequencies f_c, $2f_L$ ---- nf_L with frequency deviation Δf_L, $2\Delta f_L$ --- $n\Delta f_L$

→ The BPF has two functions to perform -

(i) To pass the FM wave centered at carrier frequency nf_c and having the frequency deviation $n\Delta f_c$

(ii) To supress all other FM spectra

→ The output of the frequency multiplier produces the desired WBFm wave having the following the time - domain description

$$\boxed{s(t) = A_c \cos\left[2\pi n f_c t + 2\pi n k_f \int_0^t m(t) \cdot dt\right]} \quad \text{---- (4)}$$

Whose instantaneous frequency is,

$$\boxed{f_i'(t) = nf_c + nk_f m(t)}$$

(2). DIRECT METHOD -

→ In direct FM, the carrier frequency 'f_c' is directly varied in accordance with the amplitude of the modulating signal. $f_c \propto A(t)$

NOTE - Direct FM is not feasible, practically as it involves maintaing high frequency stability of the carrier with adequate frequency deviation

→ In direct FM systems, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device called as "VOLTAGE CONTROLLED OSCILLATOR" [VCO]

f_i of c $\propto m(t) \longrightarrow$ V.C.O

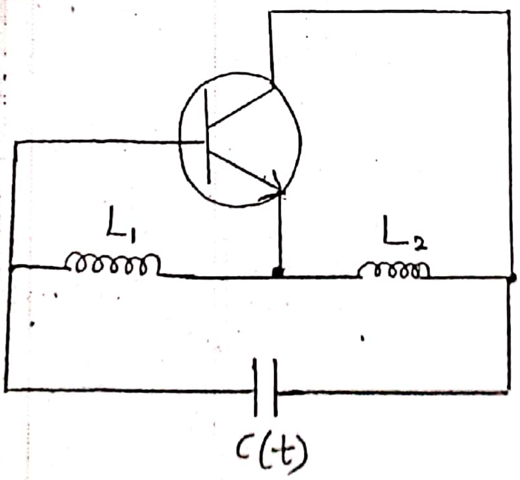
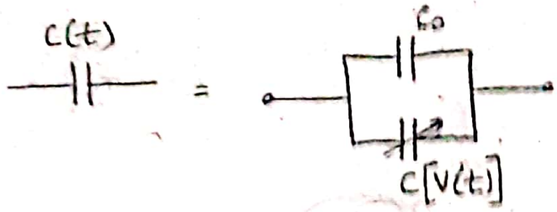


fig (i) HARTLEY OSCILLATOR

→ fig (i) shows a hartley oscillator in which the frequency determining capacitive component of the network in the oscillator consists of a fixed capacitor shunted by a voltage variable capacitor.

(i) $\because$



$\therefore C(t) = C_0 + C[V(t)]$

→ The frequency of oscillation of the hartley oscillator is given by,

$$\boxed{f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}} \quad \text{------ (1)}$$

Where,

$C(t) = C_0 + C[V(t)]$

L_1 and L_2 → are the two inductances in the frequency determining the oscillator

→ Assume that for a ~~sinusoidal modulating wave of frequency 'f_m'~~, the capacitance $C(t)$ is expressed as,

$C[V(t)] = \Delta C \cos 2\pi f_m t$

$$\boxed{C(t) = C_0 + \Delta C \cdot \cos(2\pi f_m t)} \quad \text{---- (2)}$$

Where,

C_0 → is the total capacitance in the absence of modulation ie, $f_m = 0$

ΔC → is the maximum change in capacitance

→ Substituting eqn (2) in eqn (1), we get

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0 + \Delta C \cos(2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left[1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]}}$$

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$$f_i(t) = f_0 \frac{1}{\sqrt{1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)}}$$

where,

$$f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}} \rightarrow \text{unmodulated frequency of oscillation}$$

$$f_i(t) = f_0 \frac{1}{\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{1/2}}$$

$$f_i(t) = f_0 \left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{-1/2}$$

$$f_i(t) = f_0 \left[1-\frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right]$$

Let $\boxed{-\frac{\Delta C}{2C_0} = \frac{\Delta f}{f_0}}$

$$f_i(t) = f_0 \left[1+\frac{\Delta f}{f_0}\cos(2\pi f_m t)\right]$$

$$f_i(t) = f_0 + \frac{\cancel{f_0}\Delta f}{\cancel{f_0}} \cdot \cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_0 + \Delta f \cdot \cos(2\pi f_m t)} \quad \text{---} (3)$$

→ Equation (3) is the instantaneous frequency of an FM wave assuming sinusoidal modulation.

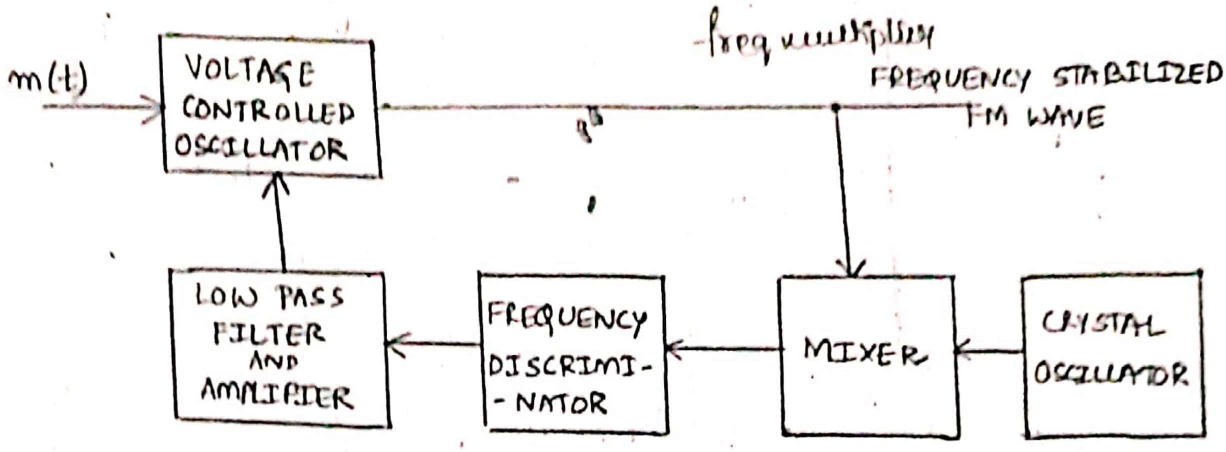

fig (ii) FEEDBACK SCHEME FOR THE FREQUENCY STABILIZATION OF A FREQUENCY MODULATOR

→ In order to generate a WBFM with the required frequency deviation we use fig (ii)
It consists of VCO, frequency multiplier and mixers.

→ This configuration provides good oscillator stability, constant proportionality between output frequency change to input voltage change, and the necessary frequency deviation to achieve WBFM.

→ The output of the FM generator (Freq multiplier) is applied to a mixer together with the output of a crystal controlled oscillator. and the difference frequency term is extracted.

→ The mixer output is next applied to the frequency discriminator and then low pass filtered.

→ A frequency discriminator is a device whose output voltage has an instantaneous amplitude that is proportional to the instantaneous frequency of the FM wave applied to its input.

→ When the FM transmitter has exactly the correct carrier frequency, the low pass filter output is zero.

→ The deviations of the transmitter carrier frequency from its assigned value will cause the frequency discriminator - filter combination, to develop a dc output voltage) with a polarity determined by the sense of the transmitter frequency drift.

→ This dc voltage after suitable amplification is applied to voltage controlled oscillator of the FM transmitter so as to modify the frequency of oscillator in a direction to restore the carrier frequency to its required value.

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