FOUR FOURCES OF FLIGHT

The four forces of flight - lift, drag, weight and thrust are denoted by L, D, W and T resply for an airplane in level flight. The free stream velocity V_∞ is always in the direction of the local flight of the airplane. The a/p lift and drag are $\perp^r$ and llel to V_∞ resply. 'L' and 'D' are aerodynamic forces of the complete a/p. 'W' always acts towards the centre of the earth for the level-flight case and W is always $\perp^r$ to V_∞. The thrust is produced by whatever flight propulsion device is powering the airplane.

Condition : Level flight.

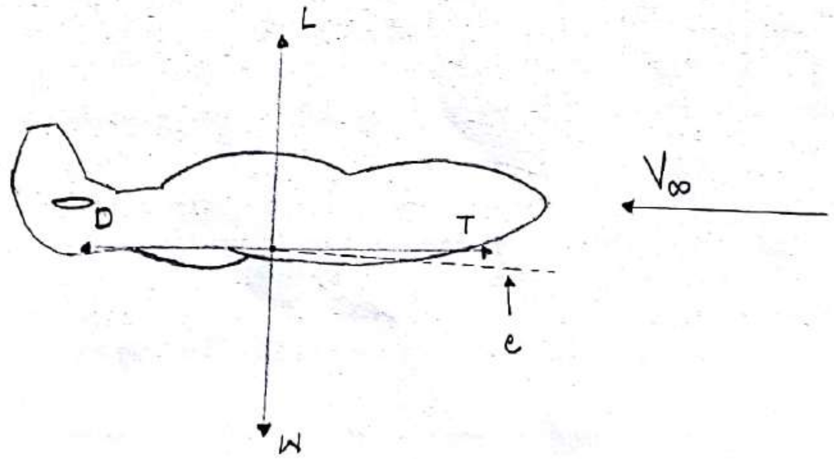


fig : level flight

Condition : Climbing flight.

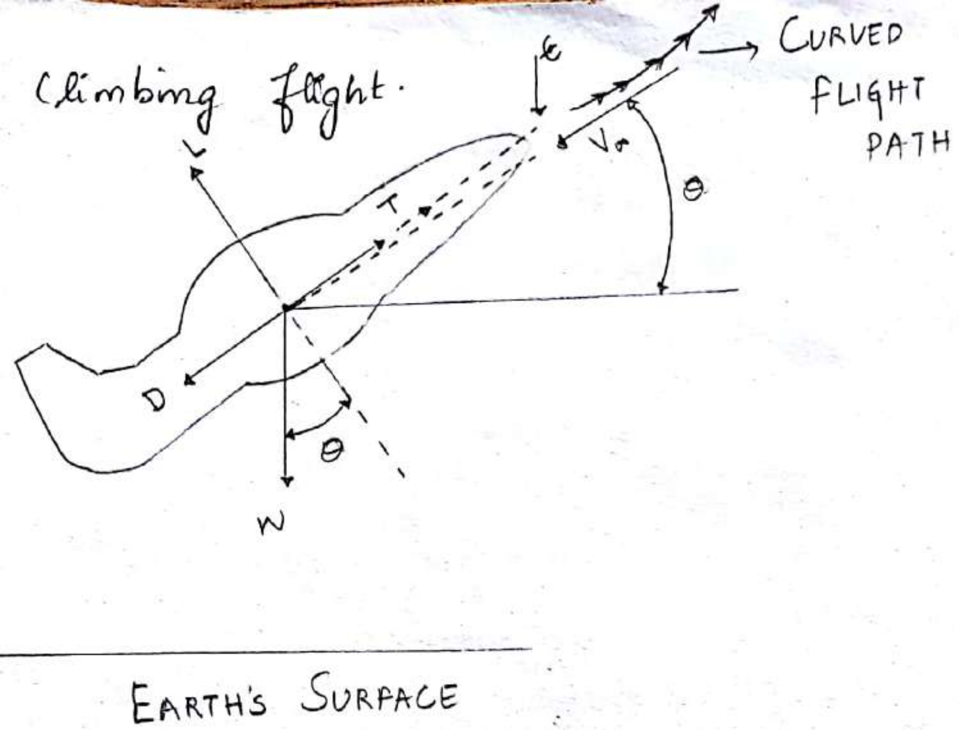


fig: Climbing flight

At any given instant as the a/p moves in the curved flight path, the local instantaneous angle of the flight path, relative to the horizontal is θ. Hence V_∞ is inclined at angle θ, which is called the local climb angle of the a/p. The direction of W is inclined at the angle θ relative to the lift.

Condition : Climbing flight and rolled through angle φ.

The climbing flight is rotated about the longitudinal axis. The a/p roll (bank) through the roll angle φ.

In the side view; the lift is rotated away from the local vertical through the angle φ.

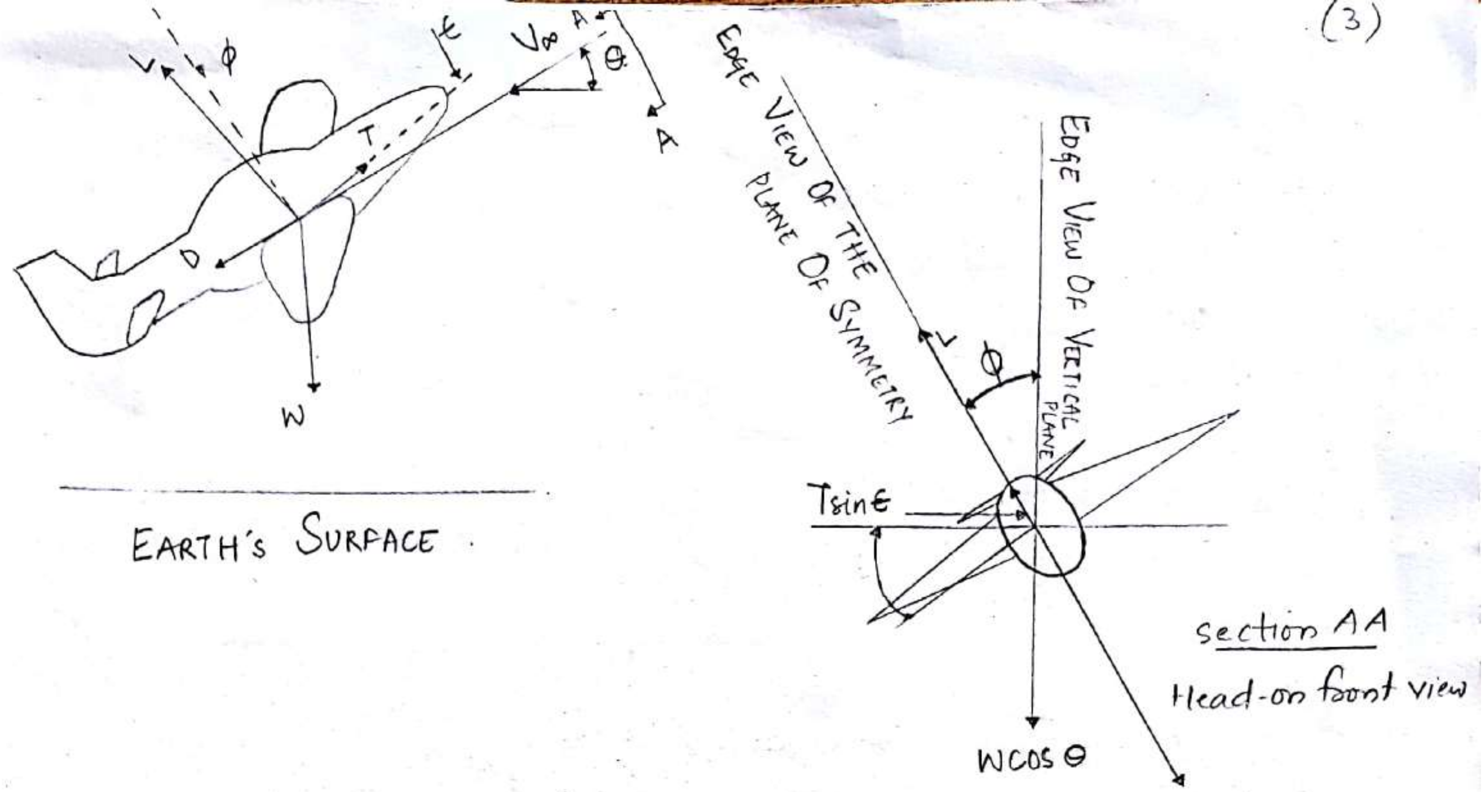


Fig: A/p in climbing flight and rolled through angle ϕ.

In the head-on front view, the lift 'L' is clearly inclined to the vertical at the angle ϕ. The thrust 'T' is inclined to the flight path direction through the angle 'ϵ'. In the head-on front view, T projects as the component 'Tsin ϵ'; this component is also rotated away from the vertical through angle ϕ.. The weight 'W' is always directed ~~downward~~ in the local vertical direction. In the head on front view, the weight projects as the component $W\cos\theta$. In the sideview, drag D is $\parallel^{el}$ to the local relative wind. In the head on view, D is parallel to V_∞.

EQUATIONS OF MOTION

The eqns of motion for an a/p is explained with Newton's Second law;

$$F = ma \longrightarrow ①$$

where $F \longrightarrow$ force

$a \longrightarrow$ acceleration.

Eqn ① is a vector eqn. To represent the above eqn in scalar form, choose an arbitrary direction in space, denoted by 's'.

Let F_s and a_s be the components of F and a resply in s direction.

$$F_s = ma_s \longrightarrow ②$$

Visualize the motion of the a/p along its curved flight path in three-dimensional space.

(*) Fig for airplane in climbing flight and rolled through angle ϕ.

Replace the fig; with a point mass at its center of gravity, with the four forces of flight acting through this point.

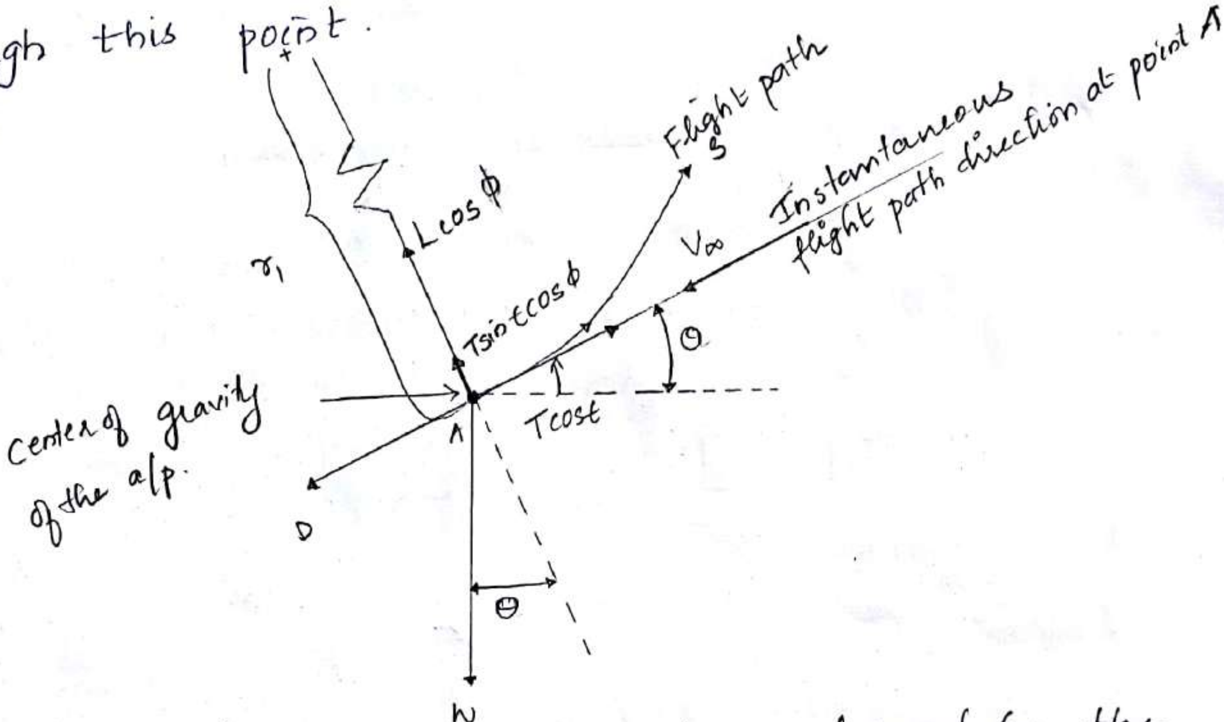


fig: Forces projected into the plane formed by the local free stream velocity V_∞ and the vertical

→ The component of lift in this plane is $L\cos\phi$.

→ The thrust is represented by its components $T\cos\epsilon$ and $T\sin\epsilon\cos\phi$. $\parallel^{el}$ & $\perp^{r}$ to V_∞.

The curvilinear motion of the ~~plane~~ airplane along the curved flight path, can be expressed by Newton's second law, by first taking components $\parallel^{el}$ to the flight path and then taking components $\perp^{r}$ to the flight path.

(6)

The components of force parallel to the flight path is :

$$F_{\parallel} = T\cos\epsilon - D - W\sin\theta \longrightarrow (3)$$

Acceleration parallel to the flight path is :

$$a_{\parallel} = \frac{dV_\infty}{dt} \longrightarrow (4)$$

Newton's second law, taken $\parallel^{el}$ to the flight p

$$ma_{\parallel} = F_{\parallel}$$

$$\boxed{m\frac{dV_\infty}{dt} = T\cos\epsilon - D - W\sin\theta} \longrightarrow (5)$$

In the direction perpendicular to the flight path, the component of force is

$$F_{\perp} = L\cos\phi + T\sin\epsilon\cos\phi - W\cos\theta$$

The radial acceleration of the curvilinear motion, $\perp^r$ to the flight path is ;

centrifugal force $= mr\omega^2$
$= mr\left(\frac{V}{r}\right)^2$
$F = \frac{mV^2}{r}$

$$a_{\perp} = \frac{V_{\infty}^2}{r_1}$$

where $r_1 \rightarrow$ local radius of curvature of the flight path.

Hence Newton's second law, taken perpendicular to the flight path is

$$ma_{\perp} = F_{\perp}$$

$$\boxed{\frac{mV_{\infty}^2}{r_1} = L\cos\phi + T\sin\epsilon\cos\phi - W\cos\theta} \rightarrow (6)$$

For the figure of airplane in climbing flight and rolled through angle ϕ, visualize a horizontal plane — a plane parallel to the flat earth.

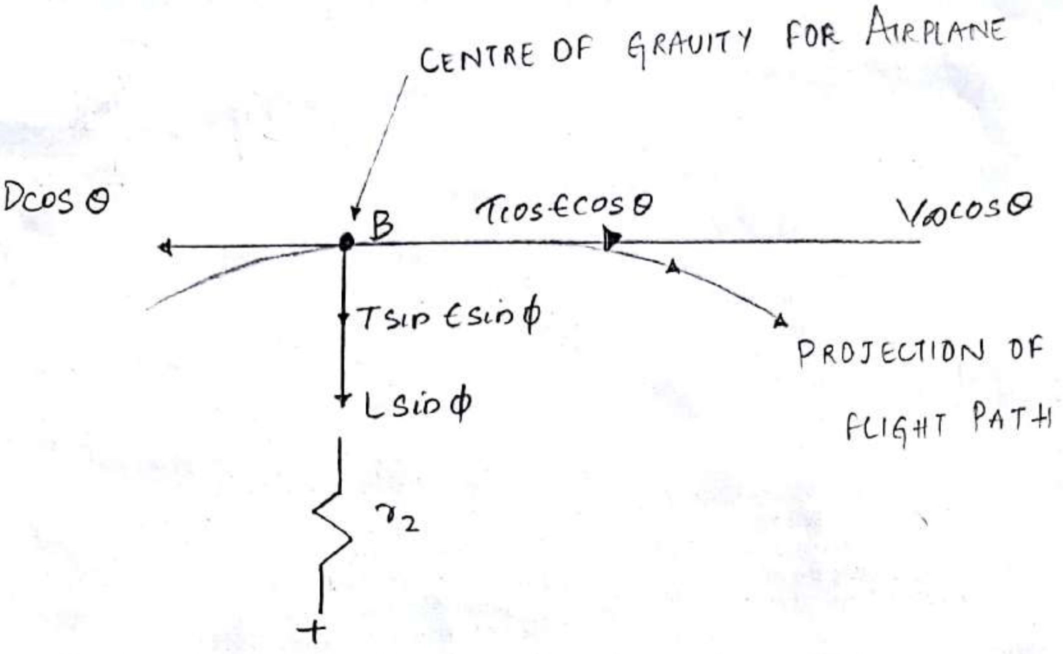


fig: Forces projected into the horizontal plane parallel to the flat earth.

The instantaneous location of the airplane's center of gravity (c.g) is the large dot represented by 'B'. The velocity vector of the airplane projects into this horizontal plane as the component $V_\infty \cos\theta$, tangent to the projected flight path at the c.g location. The local radius of curvature of the flight path in the horizontal plane is shown as r_2.

The projection of the lift vector in the horizontal plane is ~~Lco~~ $L\sin\phi$ and is perpendicular to the flight path. The components of the thrust vector in the horizontal plane are $T\sin\epsilon\sin\phi$ and $T\cos\epsilon\cos\theta$. perpendicular and parallel, resply, to the projected flight path. The component of drag in this plane is $D\cos\theta$. Since the weight acts perpendicular to the horizontal, its component is zero.

Consider the force components that are perpendicular to the flight path at the instantaneous location of the center of gravity. The sum of these forces are denoted by F_2.

$$F_2 = L\sin\phi + T\sin\epsilon\sin\phi$$

The instantaneous radial acceleration along the ~~direction ⊥r to the flight path in the horizontal~~ ~~plan.~~ curvilinear path;

$$a_2 = \frac{(V_\infty\cos\theta)^2}{r_2}$$

From Newton's second law taken along the direction perpendicular to the flight path in the horizontal plane;

$$\boxed{m\frac{(V_\infty\cos\theta)^2}{r_2} = L\sin\phi + T\sin\epsilon\sin\phi} \rightarrow (7)$$

The equations (5), (6), & (7) describes the translational motion of an airplane through three dimensional space over a flat earth. They are called the equations of motion for the a/p.

THRUST AVAILABLE AND THRUST REQUIRED CURVES.

The thrust available (T_A) is the thrust provided by the powerplant of the a/p. There are various flight propulsive devices such as reciprocating engine / propeller, turbojet, turbofan, turboprop etc.. These devices reliably and efficiently provide thrust inorder to propel the a/c. Hence T_A is associated with powerplant of an a/p.

Consider an a/p in steady, level flight at any given velocity and altitude. To maintain the speed and altitude, thrust must be generated to exactly overcome the drag and to keep the airplane going. This is the thrust required to maintain these flight conditions. The thrust - required (T_R) depends on the velocity, the altitude, and the aerodynamic shape, size and weight of the a/p. It is an airframe associated feature. The thrust required (T_R) is indeed equal to the drag of the a/p — it is the thrust required to overcome the aerodynamic drag.

Thrust required curves :-

A plot showing the variation of T_R with free stream velocity V_∞ is called thrust required curve. It is one of the essential elements in the analysis of a/p performance. A thrust required curve pertains to a given a/p at a given standard altitude. Since the thrust required is equal to the drag of the a/p, the thrust required curve is the plot of drag versus velocity for a a given airplane at a given altitude.

Graphical approach.

Consider a given a/p flying at a given altitude in steady, level flight. Let the physical characteristics of the a/p be

W → weight
AR → aspect ratio
S → wing planform area.

The drag polar for the a/p;

$$C_D = C_{D,0} + KC_L^2 \longrightarrow (1)$$

where C_D → total drag coefficient
$C_{D,0}$ → zero lift drag coefficient
KC_L^2 → drag due to lift.

In eqn (1) $C_{D,0}$ and K are known for the given a/p.

Steps to calculate thrust required:-

1. Choose a value of 'V_∞'.

2. For the chosen V_∞ calculate C_L

$$L = W = \frac{1}{2} \rho_\infty V_\infty^2 S C_L$$

$$\therefore C_L = \frac{2W}{\rho_\infty V_\infty^2 S} \longrightarrow (2)$$

(A) $\left\{ \begin{array}{l} \text{Total Drag} = \text{Zero lift drag} + \text{drag due to lift} \\ D = \frac{1}{2}\rho_\infty V_\infty^2 S C_{D} + \frac{1}{2}\rho_\infty V_\infty^2 S K C_L^2 \end{array} \right\}$

(3) Calculate C_D using eqn (1).

(4) Calculate drag.

$$D = \frac{1}{2}\rho_\infty V_\infty^2 S C_D \longrightarrow (3)$$

Drag required using eqn (3) is equal to the thrust required (T_R). This 'T_R' corresponds to the velocity chosen in step (1). This combination (T_R, V_∞) is one point on the thrust required curves.

(5) Repeat steps 1 to 4 for a large number of different values of V_∞, thus generating enough points to plot the thrust required curve.

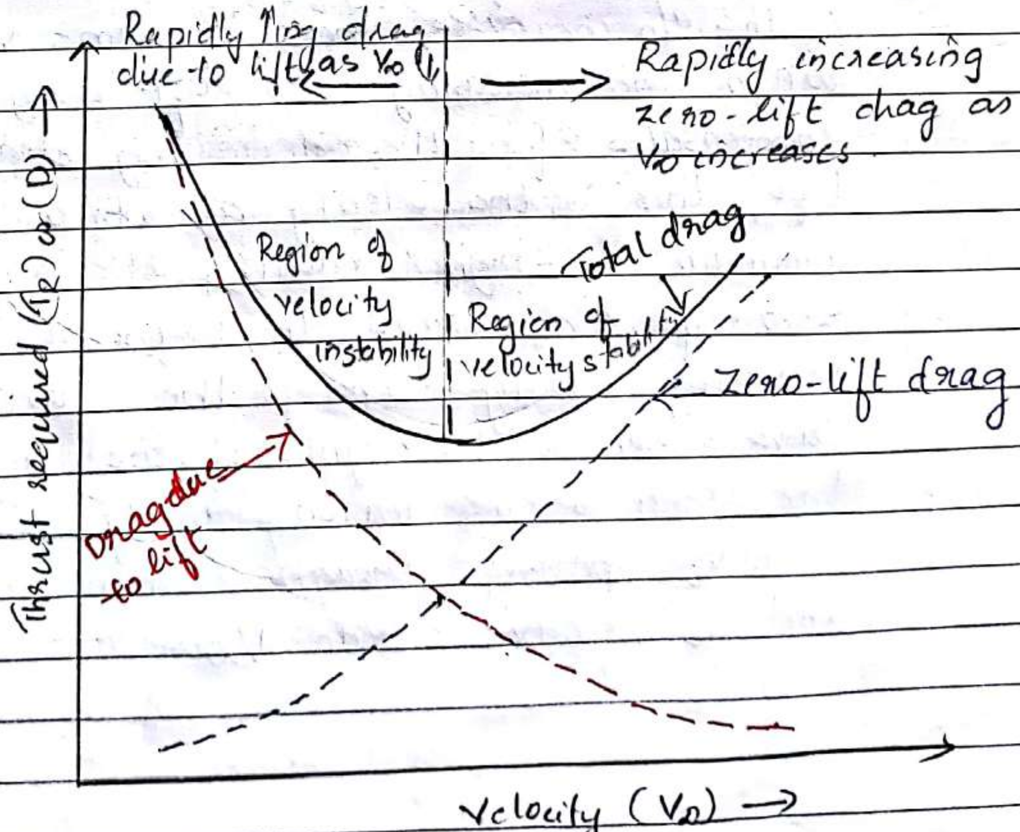


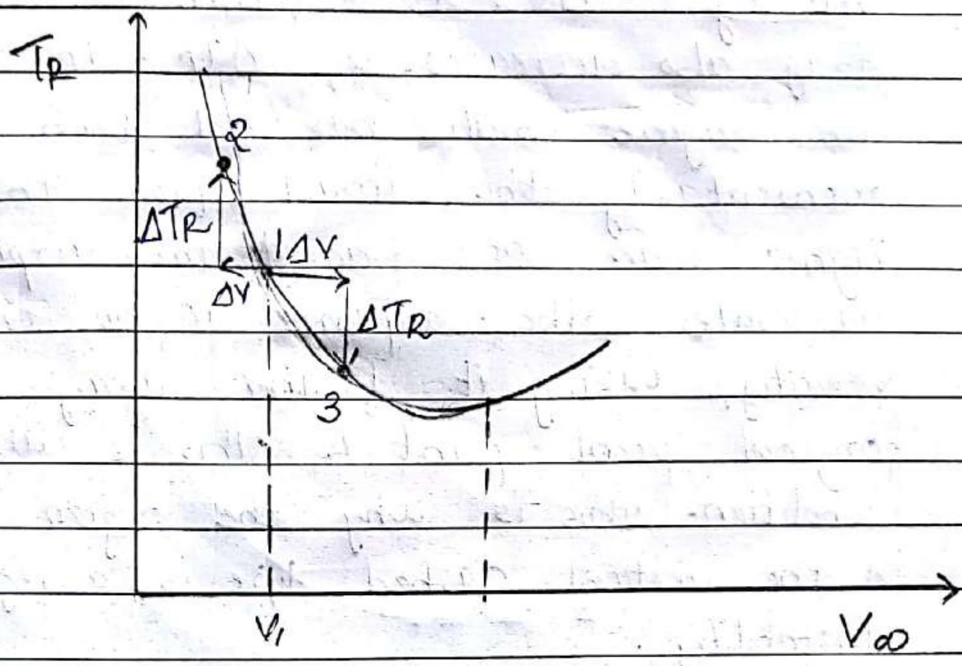
fig: Thrust required curves.

At low velocity, C_L is high and the total drag is dominated by the drag due to lift. Since the drag due to lift is proportional to the square of C_L, and C_L decreases rapidly as V_∞ increases, the drag due to lift rapidly decreases, inspite of the fact that the dynamic pressure $\frac{1}{2}\rho_\infty V_\infty^2$ is increasing. This is why the T_R curve first decreases as V_∞ increases. This part of the curve is shown to the left of the vertical dashed line - the region where the drag due to lift increases rapidly as V_∞ decreases.

In contrast, as in eqn (A) of total drag, the zero-lift drag increases as the square of V_∞. At high velocity, the total drag is dominated by the zero-lift drag. Hence as the velocity of the a/p increases, there is some velocity at which the increasing zero-lift drag exactly compensates for the decreasing drag due to lift; this is the velocity at which T_R is a minimum. At higher velocities, the rapidly increasing zero-lift drag causes T_R to increase with increasing velocity - this is the part of the curve shown ~~to~~ the right of the vertical dashed line. That is the reason, T_R first decreases with V_∞, passing through a minimum value and the increases with V_∞.

PREPARED BY : AKHILA RUPESH

→ Region left to the dashed line → Region of velocity instability



~~→ Region right to the dashed line → Region of~~

The airplane velocity is denoted by V_1. For steady flight, the engine throttle is adjusted such that the thrust from the engine is exactly equal to T_R.

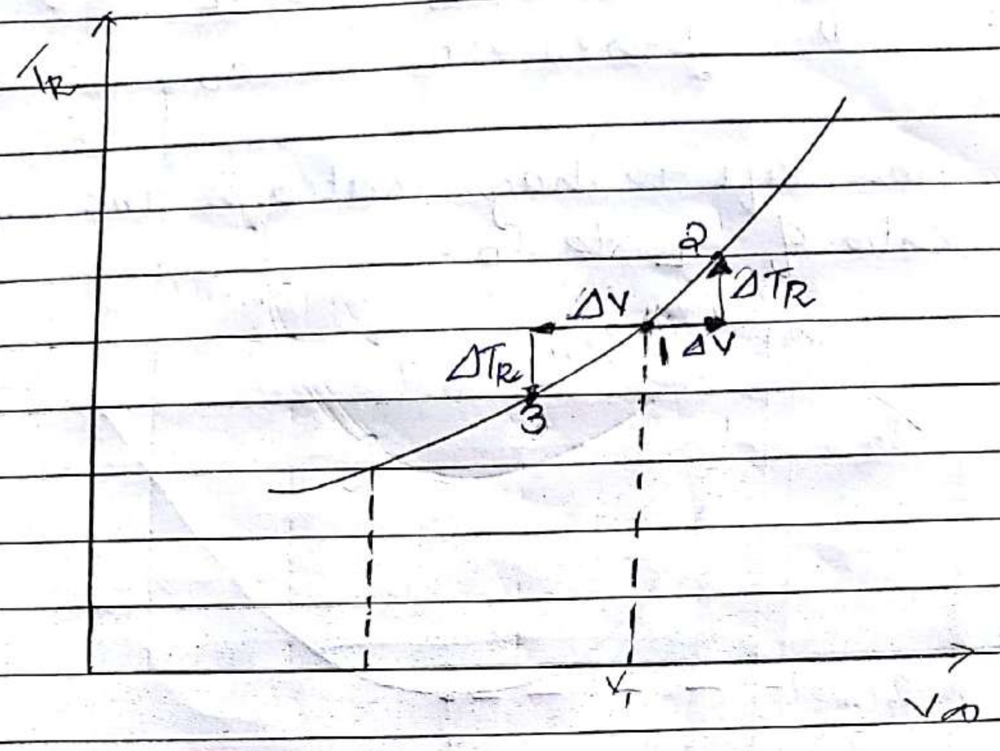
Assume the a/p is perturbed in some fashion by a horizontal gust, which momentarily decreases V_∞ for the a/p to velocity V_2. This decrease in velocity $\Delta V = V_2 - V_1$ causes an increase in T_R. But the engine throttle has not been touched and momentarily the drag of the airplane is higher than the thrust from the engine. This further slows down the airplane and takes it even farther away from its original point, point 1. This is an unstable condition.

Similarly, if the perturbation momentarily increases V_∞ to V_3, where the increase in velocity is $\Delta V = V_3 - V_1$, then T_R decreases, hence drag also decreases, ie, $\Delta T_R = T_{R3} - T_{R1}$. Again the engine throttle has not been touched, and momentarily the thrust from the engine is higher than the drag of the airplane. This accelerates the airplane to an even higher velocity, taking it farther away from its original point, point 1. This is also an unstable condition. This is why the region to the left of the vertical dashed line is a region of velocity instability.

→ Region right to the dashed line → Region of velocity stability.

A momentary increase in velocity $\Delta V = V_2 - V_1$ causes a momentary increase in T_R, hence drag increases. Since the throttle is not touched, momentarily the drag will be higher than the engine thrust, and the airplane will slow down that is, it will tend to return back to its original point 1. This is a stable condition.

Similarly, a momentary decrease in velocity $\Delta V = V_3 - V_1$ causes a momentary decrease in T_R, hence drag decreases. Since the throttle is not touched, momentarily the drag will be less than the engine thrust and the airplane will speed up, that is, it will tend to return to its original point 1.



This is also a stable condition.

Analytical Approach

For a steady level flight;

$T_R = D$

(×) & (÷) by W

$(\because W = L)$

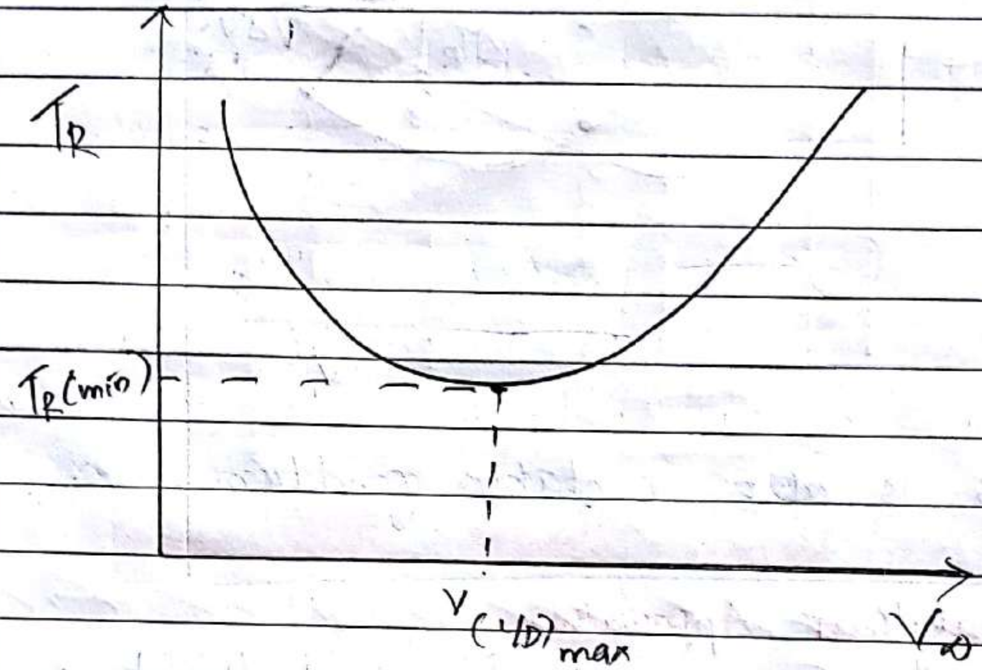
$$T_R = \frac{D}{W} \cdot W = W \cdot \frac{D}{L}$$

$$\boxed{T_R = \frac{W}{(L/D)}}$$

For an airplane with fixed weight, T_R decreases as L/D increases. Minimum T_R occurs when L/D is maximum. The lift-to-drag ratio is one of the most important parameters affecting airplane performance.

$$\frac{L}{D} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 S C_L}{\frac{1}{2}\rho_\infty V_\infty^2 S C_D} = \frac{C_L}{C_D}$$

The lift to drag ratio is the same as the ratio of C_L to C_D.



THRUST AVAILABLE AND MAXIMUM VELOCITY

Thrust Available :- Thrust available (T_A) is completely associated with the flight propulsive device.

* Propeller - Driven Aircraft

The component of aerodynamic force generated on a propeller is the thrust of the propeller. For a propeller / reciprocating engine combination, this propeller thrust is the thrust available, T_A. For a turboprop engine, the propeller thrust is augmented by the jet exhaust. The combined propeller thrust and jet thrust is the thrust available 'T_A' for the turboprop.

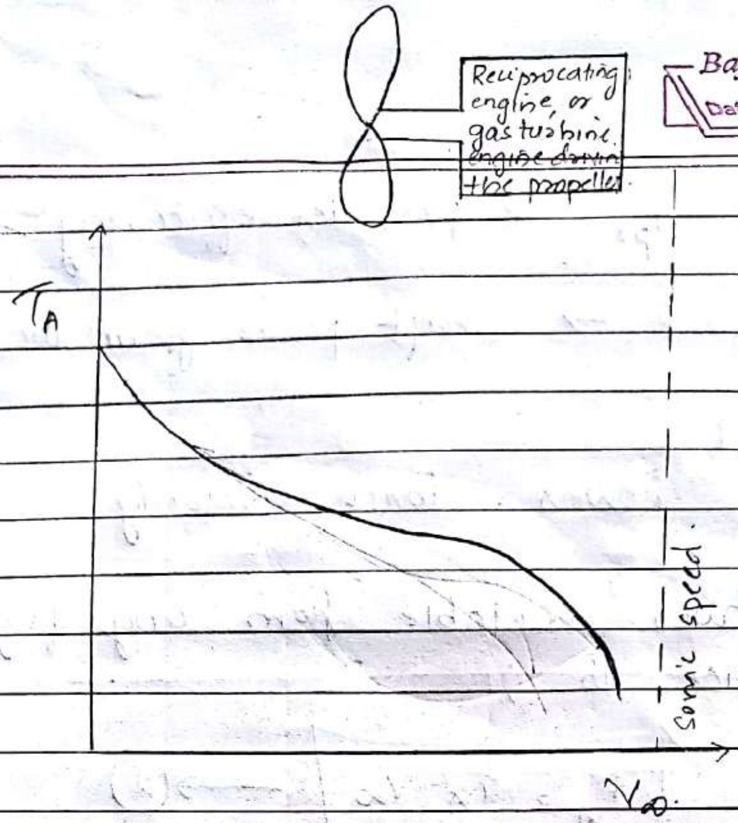


fig: Variation of thrust available versus velocity for a propeller - driven a/c.

The thrust is highest at zero velocity [called the Static thrust] and decreases with an increase in V_∞. The thrust rapidly decreases as V_∞ approaches sonic speed, this is because the propeller tips encounter compressibility problems at high speeds, including the formation of shock waves. This is why, propeller-driven a/c have been limited to moderate sonic speeds.

The propeller is attached to a rotating shaft which delivers power from a reciprocating piston engine or a gas turbine. The values of T_A for a propeller - driven a/p can be readily obtained from the power ratings. The power available from a propeller / reciprocating engine combination is given by

$$\boxed{P_A = \eta_{pr} P} \longrightarrow (1)$$

where $\eta_{pr} \rightarrow$ propeller efficiency

$P \rightarrow$ shaft power from the piston engine

W.K.T

Power = Force x velocity

$\therefore$ Power available from any flight propulsive device is ;

$$\boxed{P_A = T_A V_\infty} \longrightarrow (2)$$

~~Sodas~~ Comparing eqn (1) & (2)

$$\eta_{pr} P = T_A V_\infty$$

$$\boxed{T_A = \frac{\eta_{pr} P}{V_\infty}} \longrightarrow (3).$$

This is the thrust available for a propeller/ reciprocating engine combination.

Similarly ; for a turboprop engine, the thrust available is given by

$$\boxed{T_A = \frac{\eta_{pr} P_{es}}{V_\infty}} \longrightarrow (4)$$

where $P_R \rightarrow$ shaft power for a piston engine.

$P_{es} \rightarrow$ equivalent shaft power for a turboprop.

Both eqn ③ & ④ shows that T_A decreases as V_∞ increases.

* Jet-propelled aircraft.

Turbojet and turbofan engines are rated in terms of thrust.

a) Turbojet engine:

For turbojet engine; thrust equation is given by

$$T = (\dot{m}_{air} + \dot{m}_{fuel}) V_j - \dot{m}_{air} V_\infty + (p_e - p_\infty) A_e$$

where $\dot{m}_{air} \rightarrow$ mass flow of air

$\dot{m}_{fuel} \rightarrow$ mass flow of fuel

$p_e \rightarrow$ gas pressure at exit of the nozzle

$p_\infty \rightarrow$ ambient pressure

$A_e \rightarrow$ exit area of the nozzle.

$$T_A = \dot{m}_{air} (V_j - V_\infty) + \dot{m}_{fuel} \cdot V_j + (P_e - P_\infty) A_e$$

The value of V_j is a function of the internal compression and combustion processes taking place inside the engine.

Hence the value $(V_j - V_\infty)$ tends to decrea as V_∞ increases. With V_∞ increasing but V_j remaining same, the value of T is decreased.

→ At subsonic speeds: V_j and V_∞ tends to cancel and T_A will be a weak function of V_∞, ie,

$$\boxed{T_A \approx \text{constant with } V_\infty}$$

→ At super sonic speeds

$$\boxed{\frac{T_A}{(T_A)_{Mach\,1}} = 1 + 1.18\,(M_\infty - 1)}$$

b) ~~Turbofan engine~~.

the effect of altitude on T_A is given by

$$\boxed{\frac{T_A}{(T_A)_0} = \frac{\rho}{\rho_0}}$$

where

$(T_A)_0$ → thrust available at sea level

ρ_0 → standard sea-level density.

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b) Turbofan engine.

Turbofan engine combine the high thrust of a turbojet with the high efficiency of a propeller.

→ For high-by pass-ratio turbofans, the thrust ~~available~~ decreases with increasing velocity.

$$\boxed{\frac{T_A}{(T_A)_{V=0}} = A M_\infty^{-n}}$$

where $(T_A)_{V=0}$ → static thrust available at standard sea level.

A fn → function's of altitude obtained by correlating the data for a given engine.

→ For low-bypass-ratio of turbofan the thrust variation with velocity is much closer to that of a turbojet; almost constant at subsonic speeds and increasing with velocity at super sonic speeds.

The altitude variation of thrust for a high-bypass-ratio civil turbofan is given by

$$\boxed{\frac{T_A}{(T_A)_0} = \left[\frac{\rho}{\rho_0}\right]^m}$$

where $(T_A)_0$ → thrust available at sea level

m → depends on engine design, ~~is~~ usually near 1.

Maximum Velocity :-

Consider an a/p flying at any given altitude. For steady, level flight at a given velocity V_1, the value of T_A is adjusted such that $T_A = T_R$ at that velocity. This is denoted by point 1. The pilot of the a/p can adjust (T_A) by adjusting the engine throttle in the cockpit. For point 1, the engine is operating at partial throttle, and the resulting value of T_A is denoted by $(T_A)_{partial}$.

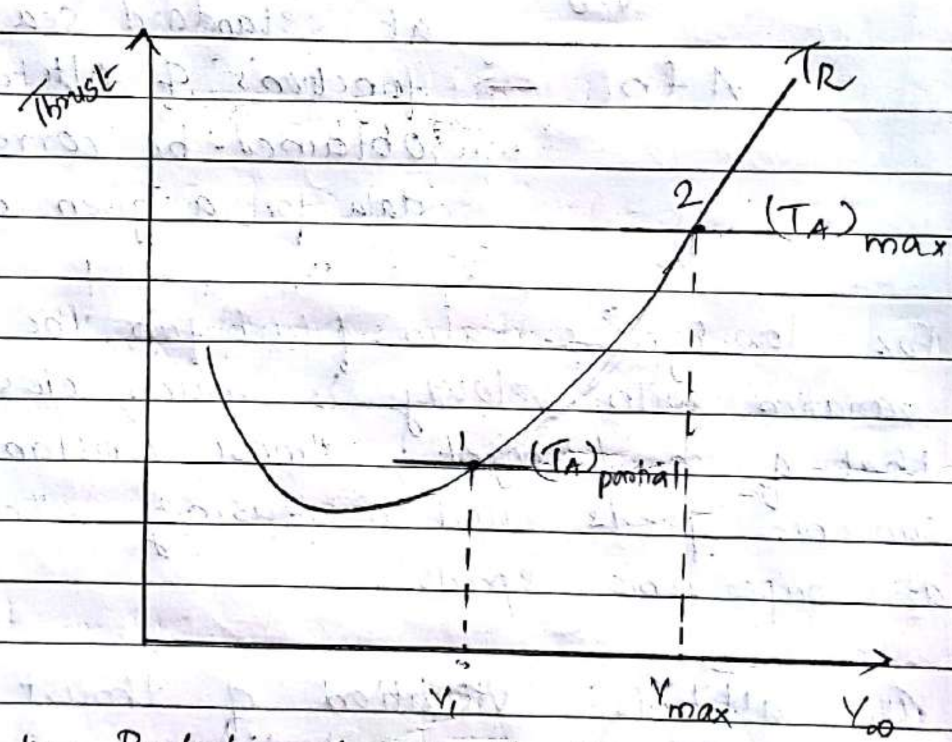


fig: Partial and full throttle conditions, intersection of the thrust available and thrust required curves.

When the throttle is pushed all the way forward, maximum thrust available is produced $(T_A)_{max}$. The a/p will accelerate to higher velocit and T_R will increase. until $T_R = (T_A)_{max}$ denoted by point 2. When the a/p is at point 2, any further increase in velocity requires more thrust than is available from the power-plant. Hence for steady level flight, point 2 defines the maximum velocity V_{max} at which the given a/p can fly at the given altitude.

By definition, the thrust available curve is the variation of T_A with velocity at a given throttle setting and altitude. For the throttle full forward, $(T_A)_{max}$ is obtained. The maximum thrust available curve is the variation of $(T_A)_{max}$ with velocity at a given altitude.

For turbojet and low-bypass-ratio turbofans, at subsonic speeds, the thrust is essentially constant with velocity. Hence for such power plants, the thrust available curve is a horizontal line. In steady level flight the maximum velocity of the a/p is determined by high speed intersection of the T_R & T_A curves.

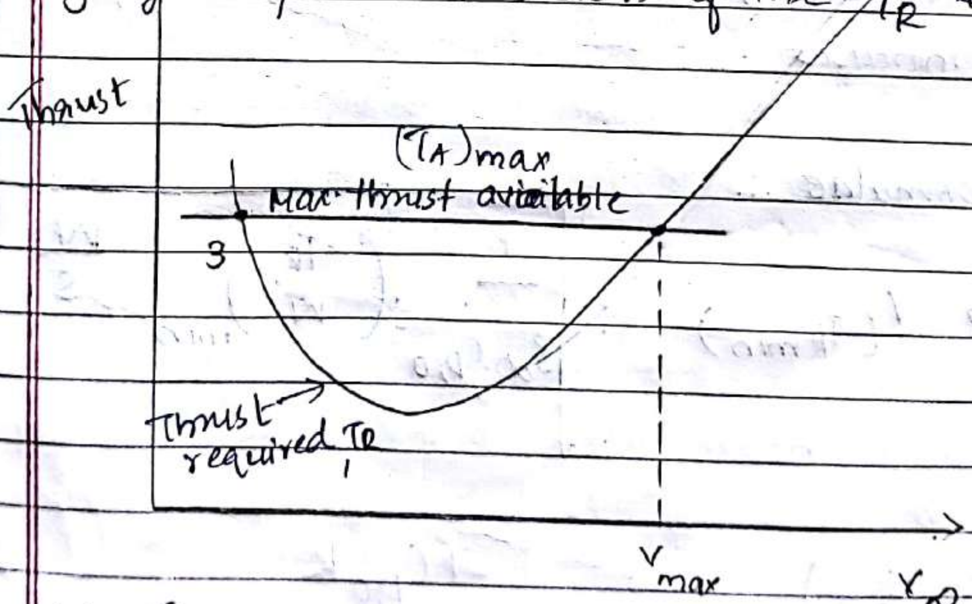


fig: Thrust available curve for a turbojet and low bypass ratio turbofan is essentially constant with velocity at subsonic speeds. The high speed intersection of the $(T_A)_{max}$ curve and T_R curve determines the V_{max}.

For steady level flight,

$$T_R = T_A$$

For flight at V_{max};

$$T_R = (T_A)_{max}$$

$$\boxed{V_{max} = \left\{ \frac{\left[(T_A)_{max}/W\right](W/S) + (W/S)\sqrt{\left[(T_A)_{max}/W\right]^2 - 4C_{D,0}K}}{\rho_\infty C_{D,0}} \right\}^{1/2}}$$

where $\left[(T_A)_{max}/W\right]$ → Max Thrust to weight ratio

$[W/S]$ → wing loading

∴ V_{max} increase with increase in $\left[(T_A)_{max}/W\right]$ and $[W/S]$ and decreases as $C_{D,0}$ and/or K increases.

Formulae :

→ $V_{(T_{R\,min})} = \sqrt{\dfrac{1}{\rho_\infty C_{D,0}} \left(\dfrac{T_R}{W}\right)_{min} \dfrac{W}{S}}$

→ $\left(\dfrac{T_R}{W}\right)_{min} = \sqrt{4C_{D,0}K}$

→ $V_{(T_{R\,min})} = \sqrt{\dfrac{2}{\rho_\infty} \sqrt{\dfrac{K}{C_{D,0}}} \dfrac{W}{S}}$

$\rightarrow AR = \frac{b^2}{S} = \frac{b}{a}$

$\rightarrow k = \frac{1}{\pi e AR}$

POWER AVAILABLE AND POWER REQUIRED CURVES :

Power Available (P_A)

P_A is the power provided by the power provided by the power plant of the a/p.

$$\boxed{P_A = T_A V_\infty}$$

Power required (P_R)

P_R is the dot product of T_R and V_∞

$$\boxed{P_R = T_R \cdot V_\infty = D V_\infty}$$

Power required curves.

A graphical plot of P_R versus V_∞ for a given a/p at a given altitude is called power required curve.

$$P_R = T_R V_\infty \cancel{\approx} D V_\infty \longrightarrow (1)$$

W.K.T, $T_R = \frac{W}{L/D} = \frac{W}{C_L/C_D}$

$$P_R = \frac{W}{(C_L/C_D)} \cdot V_\infty \longrightarrow (2)$$

For steady level flight,

$$L = W.$$

$$\therefore L = W = \frac{1}{2} \rho_\infty V_\infty^2 S C_L$$

$$V_\infty^2 = \frac{2W}{\rho_\infty S C_L}$$

$$\boxed{V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}} \longrightarrow (3)$$

Sub. (3) in (2)

$$P_R = \frac{W}{(C_L/C_D)} \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$\boxed{P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}}$$

$$\Rightarrow \boxed{P_R \propto \frac{C_L^{3/2}}{C_D}}$$

Hence, minimum power required occurs when the a/p is flying such that $C_L^{3/2}/C_D$ is a maximum value.

→ At minimum P_R;

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{1}{4}\left(\frac{3}{K C_{D,0}^{3/2}}\right)^{3/4}$$

→ Zero-lift drag equals one-third of the drag due to lift.

→ The velocity at which P_R is a minimum occurs at

$$V_{(C_L^{3/2}/C_D)_{max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}}\,\frac{W}{S}\right)^{1/2}$$

This velocity is less than that for minimum T_R, where C_L/C_D is a maximum, Indeed

$$V_{(C_L^{3/2}/C_D)_{max}} = 0.76\, V_{(C_L/C_D)_{max}}.$$

POWER AVAILABLE AND MAXIMUM VELOCITY

$$\boxed{P_A = T_A V_\infty}$$

* Propeller - Driven aircraft

Case I: Power available for a propeller / reciprocating engine combination is given by

$$\boxed{P_A = \eta_{pr} P}$$

The velocity and altitude effects on 'P' for a piston engine are :

1. Power 'P' is reasonably constant with V_∞

2. For an unsupercharged engine,

$$\boxed{\frac{P}{P_0} = \frac{\rho}{\rho_0}}$$

where P & ρ → shaft power output & density respectively at altitude

P_0 & ρ_0 → shaft power output & density respectively at sea level.

Taking into account the temperature effect,

$$\boxed{\frac{P}{P_0} = 1.132 \frac{\rho}{\rho_0} - 0.132}$$

3. For a supercharged engine, P is essentially constant with altitude up to the critical design altitude of the supercharger. Above this critical altitude, P decreases with ρ_0 replaced by the density at the critical altitude, ρ_{crit}.

Case II : The power available for a turboprop is given by

$$\boxed{P_A = \eta_{pr} P_{es}}$$

The velocity and altitude variations of P_A for turboprop are :

1. Power available 'P_A' is reasonsably constant with V_∞ or M_∞.

2. The altitude effect is given by

$$\boxed{\frac{P_A}{P_{A,0}} = \left(\frac{\rho}{\rho_0}\right)^n} \quad n = 0.7$$

* Jet propelled aircraft

Turbojet and turbofan engines are rated in terms of thrust

$$\boxed{P_A = T_A V_\infty}$$

Case I: The variation of P_A with velocity and altitude is reflected through the variation of T_A. Hence for a turbojet engine:

1. At subsonic speeds, T_A is essentially constant. Hence P_A is directly proportional to T_A.

For supersonic speeds,

$$\frac{T_A}{(T_A)_{Mach 1}} = 1 + 1.18 (M_\infty - 1)$$

In this case P_A for supersonic speeds is a non linear function of V_∞.

2. The effect of altitude on T_A is same as that of effect of P_A

$$\frac{P_A}{(P_A)_0} = \frac{\rho}{\rho_0}$$

Case II: ~~Turbofan~~ The variation of P_A for a turbofan is reflected through the variation of T_A. Hence for a turbofan:

1. The Mach number variation of the thrust is given by

$$\frac{T_A}{(T_A)_{V=0}} = A M_\infty^{-n}$$

2. The altitude variation for turbofan thrust

$$\frac{T_A}{(T_A)_0} = \left(\frac{\rho}{\rho_0}\right)^m$$

Hence the variation of P_A with altitude is

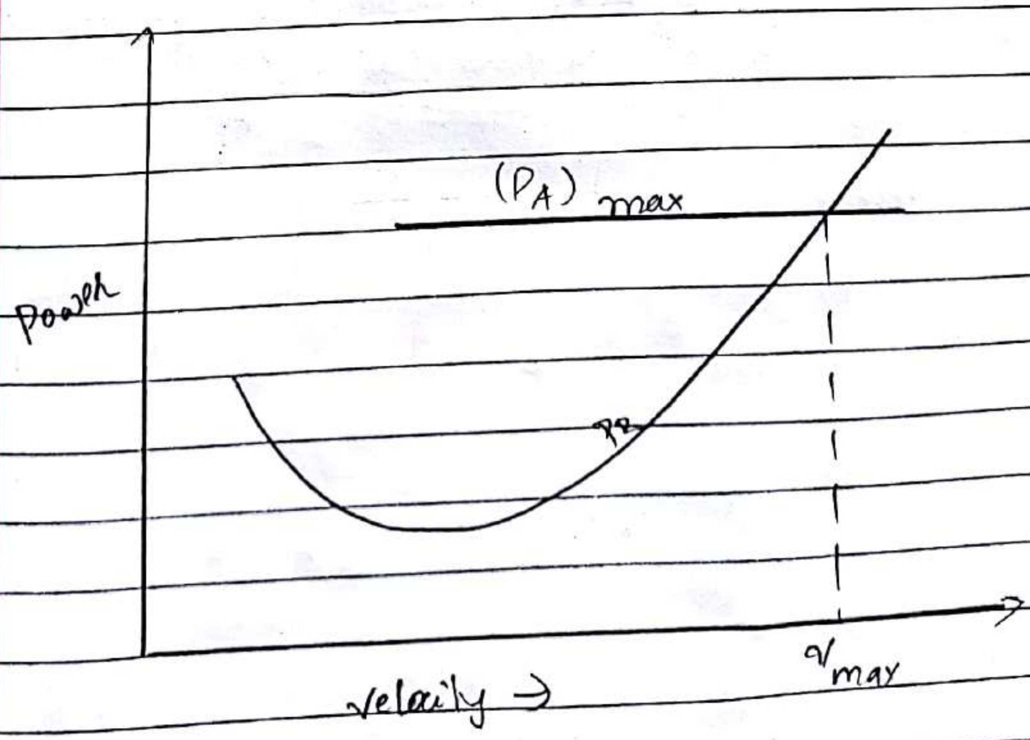
$$\boxed{\frac{P_A}{(P_A)_0} = \left(\frac{\rho}{\rho_0}\right)^m}$$

~~Power~~

Maximum Velocity

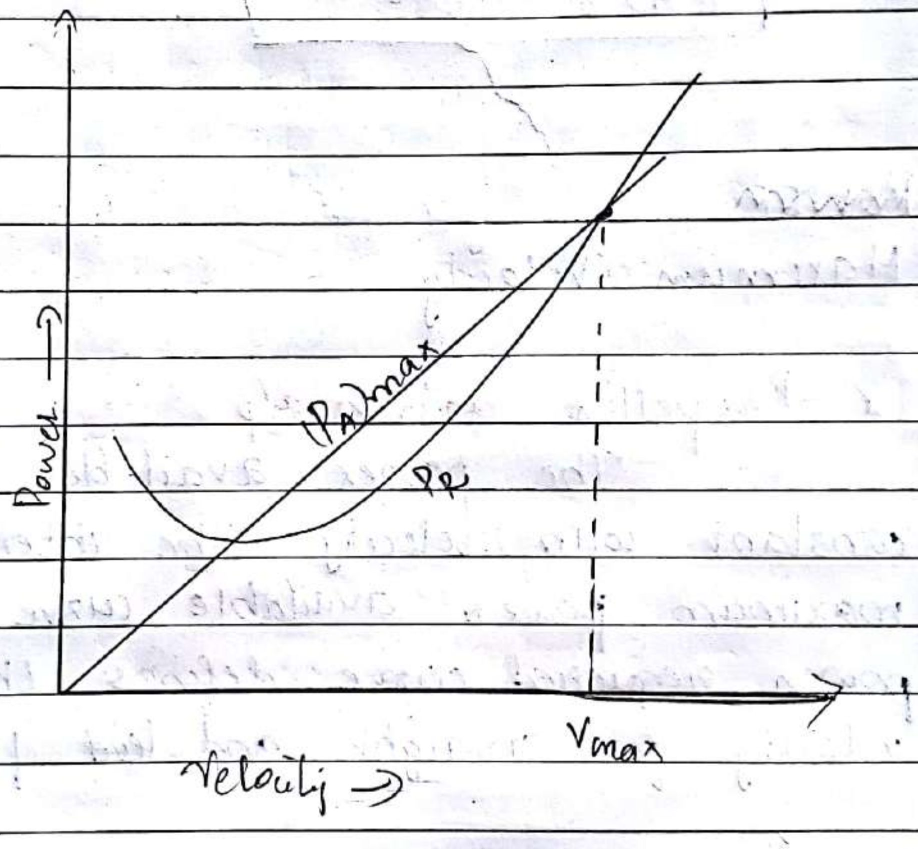
* Propeller driven a/p

The power available P_A is essentially constant with velocity. The intersection of the maximum power available curve and the maximum power required curve defines the maximum velocity for straight and level flight.



* Jet -propelled a/p

Assuming T_A is constant with velocity the power available at subsonic varies linearly with V_∞. The power P_R is also plotted. The high speed intersection of the maximum power available curve and power required curves defines the maximum velocity for straight and level flight.



Formula used for Numericals :

① $L = W = \frac{1}{2} \rho_\infty V_\infty S C_L$

② $C_D = C_{D,0} + K C_L^2$

③ $K = \frac{1}{\pi e AR}$

④ $AR = \frac{b^2}{S}$

⑤ $\frac{L}{D} = \frac{C_L}{C_D}$

⑥ $T_R = \frac{W}{(L/D)}$

⑦ $\left(\frac{T_R}{W}\right)_{min} = \sqrt{4 C_{D,0} K}$

⑧ $V_{(T_R min)} = \left(\frac{2}{\rho_\infty} \sqrt{\frac{K}{C_{D,0}}} \frac{W}{S}\right)^{1/2}$

⑨ $\left(\frac{L}{D}\right)_{max} = \frac{1}{\sqrt{4 C_{D,0} K}}$

Numericals.

Q:1) A private aeroplane with propeller driven engine has the following characteristics.

wing span = 10.912 m → (b)

wing area = 16.165 m^2 (S)

Normal gross weight = 13127.5 N (W)

Fuel capacity = 65 gal of aviation gasoline

Powerplant = one piston engine of 230 hp at sea level.

Parasite drag coefficient, $C_{D,0} = 0.025$

Oswald efficiency factor, $e = 0.8$

Propeller efficiency = 0.8

Assume $V_\infty = 60.96$ m/s.

Find C_L, C_D, L/D, T_R. Plot thrust required cu

Soln: To find C_L

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{13127.5 \times 2}{1.225 \times (60.96)^2 \times (16.165)}$$

$$= 0.3568$$

To find C_D

$$C_D = C_{D,0} + KC_L^2$$

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.8 \times AR}$$

$$AR = \frac{b^2}{S} = \frac{(10.912)^2}{16.165} = 7.366$$

$K = 0.054$

$C_D = 0.025 + 0.054(0.3568)^2$

$= 0.03187$

To find L/D

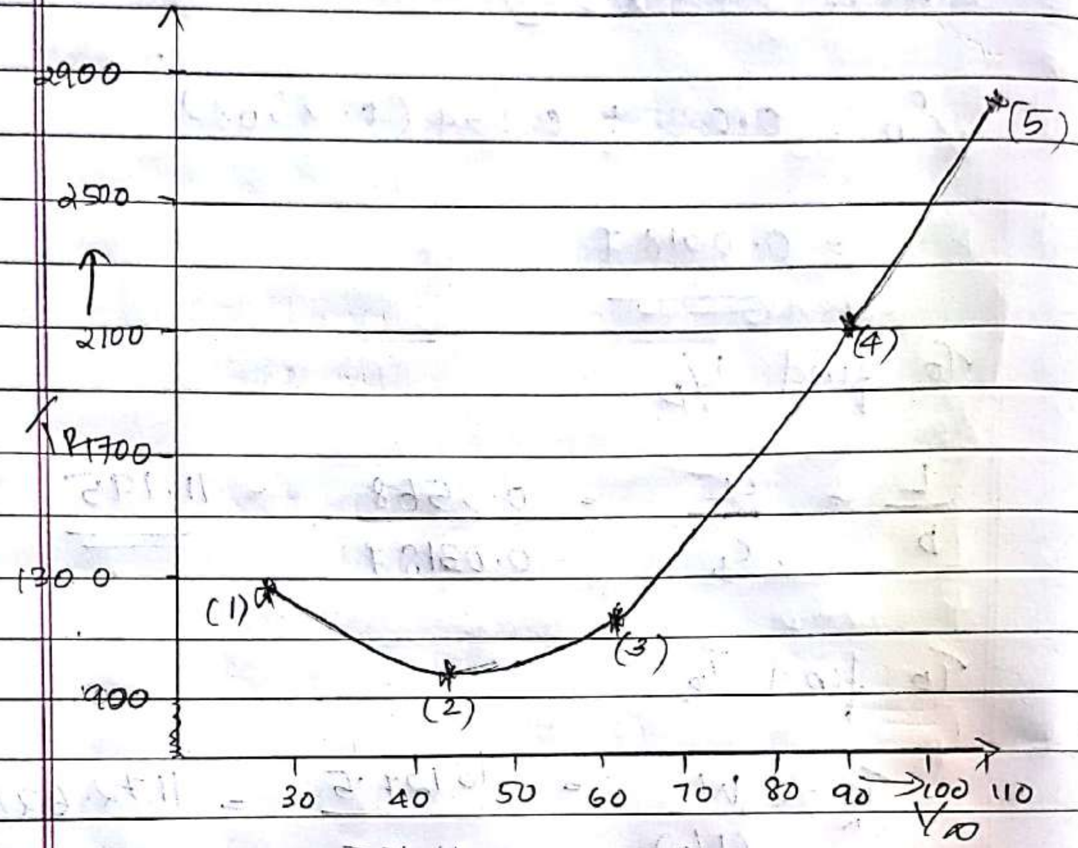
$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.3568}{0.03187} = 11.195$$

To find T_R

$$T_R = \frac{W}{(L/D)} = \frac{13127.5}{11.195} = 1172.62\,N$$

	V_0 (m/s)	C_L	C_D	L/D	T_R (N)
(1)	30.48	1.43	0.135	10.6	1238.44
(2)	45.72	0.634	0.047	13.6	965.257
(3)	60.96	0.3568	0.03187	11.195	1172.62
(4)	91.44	0.159	0.026	6.01	2184.28
(5)	106.68	0.116	0.026	4.53	2891.90

$$\left\{ \begin{array}{ll} lb \rightarrow \text{libra} = \text{pound}. & 1\,ft = 0.000305\,km \\ 1\,slug = 14.594\,kg & 1\,lb = 0.454\,kg \end{array} \right\}$$



Q:2) For Gulfstream IV at the conditions stated, calculate minimum thrust required and the velocity at which it occurs. The data given are

$W = 73000\,lb$, $S = 950\,ft^2$, $\rho_\infty = 8.9068 \times 10^{-4}\,slug/ft^3$

$C_{D,0} = 0.015$, $k = 0.08$.

Soln: To find minimum thrust required $(T_R)_{min}$

$$\left(\frac{T_R}{W}\right)_{min} = \sqrt{4 C_{D,0} k}$$

$$= \sqrt{4 \times 0.015 \times 0.08}$$

$$= 0.0693$$

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$$T_{R_{min}} = 0.0693 \times W$$

$$= 0.0693 \times 73{,}000$$

$$= 50589 \text{ lb}$$

To find velocity for minimum T_R

$$V_{(T_R)_{min}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}}\,\frac{W}{S}\right)^{1/2}$$

Wind loading; $\frac{W}{S} = \frac{73000}{950} = 7684 \text{ lb/ft}^2$

$$V_{(T_R)_{min}} = \left(\frac{2}{8.9068\times10^{-4}}\sqrt{\frac{0.08}{0.015}} \times 76.84\right)^{1/2}$$

$$= 631.24 \text{ ft/s}$$

Q:8) A jet powered executive a/c has the following characteristics

wingspan = 16.25 m

wing area = 29.54 m^2

Normal gross weight = 8.8176.75 N

Parasite drag coefficient = 0.02

Oswald's efficiency factor = 0.81

Calculate C_L, C_D, L/D, T_R

Assume $V_\infty = 152.4$ m/s.

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Soln:

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{88176.75}{\frac{1}{2}(1.225)(152.4)^2(29.54)}$$

$$= 0.210$$

$$AR = \frac{b^2}{S}$$

$$= \frac{(16.25)^2}{29.54} = 8.93$$

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$

$$= 0.02 + \frac{(0.21)^2}{\pi(0.81)(8.93)} = 0.022$$

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.21}{0.022} = 9.55$$

$$T_R = \frac{W}{(L/D)} = \frac{88176.75}{9.55} = 9233.167 \text{ N}$$

Q:4) For the Gulf stream W at the conditions given below, calculate the minimum power required and the velocity at which it occurs.

Altitude = 30,000 ft

$\rho_\infty = 8.9068 \times 10^{-4}$ slug/ft^3

$W = 73,000$ lb

$S = 950$ ft^2

$C_{D,0} = 0.015$

$k = 0.08$

$AR = 5.92$

$$\boxed{1hp = 550\,ft\,lb/s = 746\,W}$$

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$$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{1}{4}\left(\frac{3}{kC_{D,0}^{1/3}}\right)^{3/4} = \frac{1}{4}\left(\frac{3}{(0.08)(0.015)^{1/3}}\right)^{3/4}$$

$$= 10.83$$

$$\left(\frac{C_D^2}{C_L^3}\right)_{min} = \left(\frac{1}{10.83}\right)^2 = 8.526\times10^{-3}$$

$$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$$

$$(P_R)_{min} = \sqrt{\frac{2W^3}{\rho_\infty S}\left(\frac{C_D^2}{C_L^3}\right)_{min}}$$

$$= \sqrt{\frac{2(13000)^3(8.526\times10^{-3})}{(8.9068\times10^{-4})\,950}} = 2.8\times10^6\,ft\text{-}lb/s$$

$$(P_R)_{min} = 5091\,hp$$

$$V_{(C_L^{3/2}/C_D)_{max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{k}{3C_{D,0}}}\frac{W}{S}\right)^{1/2}$$

$$= \left[\frac{2}{8.9068\times10^{-4}}\sqrt{\frac{0.08}{3(0.015)}}(76.84)\right]^{1/2}$$

$$= 470.6\,ft/s$$

ALTITUDE EFFECTS ON POWER AVAILABLE AND POWER REQUIRED

With regard to P_R, curves at altitude could be generated by repeating the calculation of power, with ρ_∞ appropriate to the given altitude.

Let the subscript 'o' designate sea-level condition

$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}}$$

$$P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

At altitude, where density is ρ, these relations are

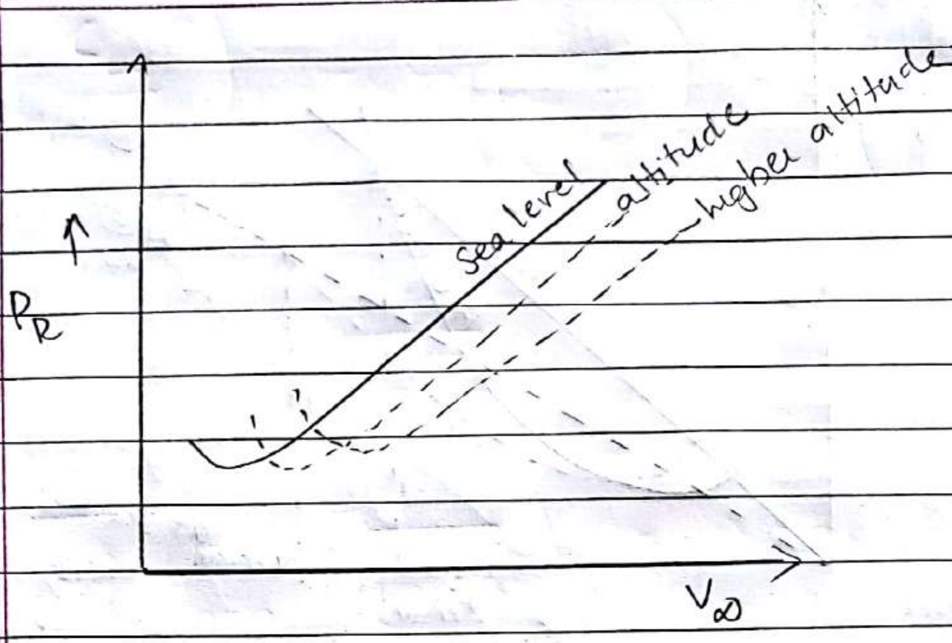
$$V_{alt} = \sqrt{\frac{2W}{\rho S C_L}}$$

$$P_{R,alt} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

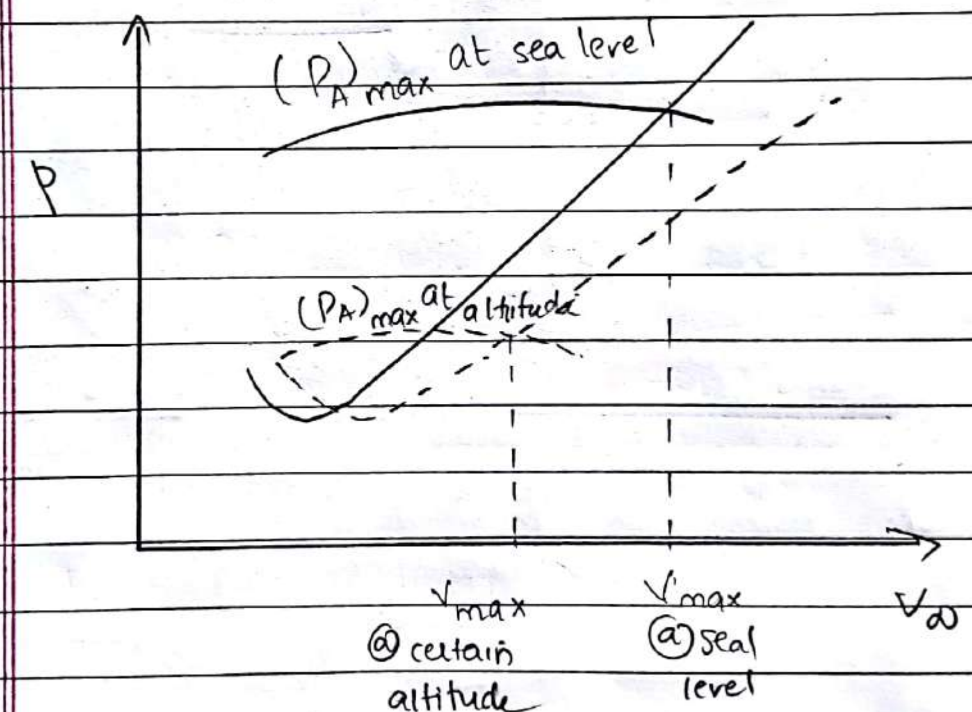
Let C_L remain fixed, hence C_D is fixed

$$V_{alt} = V_0 \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

$$P_{R,alt} = P_{R,0} \left(\frac{\rho_0}{\rho}\right)^{1/2}$$



→ Propeller driven a/p



The minimum velocity is determined either by stalling or by the low speed intersection of the power curves. These velocity considerations are important part of the a/p performance indeed

→ Jet propelled a/p